Analysis of the Packet Loss Process for Multimedia Traffic

György Dán, Viktória Fodor and Gunnar Karlsson
KTH, Royal Institute of Technology,
Department of Microelectronics and Information Technology
Electrum 229, 16440 Kista, Sweden
{gyuri,viktoria,gk}@imit.kth.se

Abstract

The average loss probability is not sufficient to determine the effects of loss on the perceived quality for multimedia traffic. Nor is it sufficient for studying the possible ways of improving it, for example by forward error correction (FEC) and error concealment. However it is difficult to model the queuing behavior analytically for such traffic. It has been shown that for real-time communications, when buffers are small for delay reasons, short range dependence dominates the loss process and so the Markov-modulated Poisson process (MMPP) might be a reasonable source model. In this paper we present mathematical models for the loss process of the MMPP+M/D/1/K and the MMPP+M/M/1/K queues; we validate the models via simulations and use them to evaluate the effects of the packet size distribution on the packet loss process and the related FEC performance. We conclude that the packet size distribution affects the packet loss process and thus the efficiency of forward error correction. This conclusion is mainly valid in access networks where a single multimedia stream might affect the multiplexing behavior.

1 Introduction

For stream-type multimedia communications, as opposed to elastic traffic, the average packet loss is not the only measure of interest. The number of losses in a block of packets has a great impact both on the user-perceived visual quality and on the possible ways of improving it, for example by forward error correction and error concealment.

Forward error correction (FEC) has been proposed to recover from information losses in real-time applications, where the latency introduced by retransmission schemes is not acceptable [4, 9]. FEC increases the redundancy of the transmitted stream and recovers losses based on the redundant information. There are two main directions of FEC design to recover from packet losses. One solution, proposed by the IETF and implemented in Internet audio tools is to add a redundant copy of the original packet to one of the subsequent packets [10]. The other set of solutions, considered in this paper, use block coding schemes based on algebraic coding, e.g. Reed-Solomon coding [16]. The error correcting capability of RS codes with k data packets and c redundant packets is c if data is lost. The performance of an FEC scheme is largely affected by the characteristics of the loss process, e.g. the probability of loosing more than c packets in a block of $k+c$ packets.

In this paper we present two models to analyze the packet loss process of a bursty source, for example VBR video, multiplexed with background traffic in a single multiplexer with a finite queue. We consider exponential distributed and constant packet sizes. We model the bursty source by an L-state Markov-modulated Poisson process (MMPP) while the background traffic is modeled by a Poisson process. We validate the models via simulations and compare the packet loss process and the related FEC performance in case of the two packet size distributions.

With regard to the queueing performance, the exponential packet size distribution (PSD) is considered to be a worst case, while the deterministic PSD is the best case. The packet loss process has been investigated before in the case of exponential PSD [7], but other PSDs have not been considered. Thus it is not clear how the PSD affects the loss process in a multiplexer and hence the related FEC performance.

It is well known that compressed media, primarily VBR video, exhibits a self-similar nature [3]. Yoshihara et al. use the superposition of 2-state IPPs to model self-similar traffic in [20] and compare the loss probability of the resulting MMPP/D/1/K queue with simulations. They found that the approximation works well under heavy load conditions and it gives an upper bound on the packet loss probabilities. Ryu and Elwalid [18] showed that short term correlations have dominant influence on the network performance under realistic scenarios of buffer sizes for real-time traffic. Thus the MMPP may be a practical model to derive approximate results for the queueing behavior of LRD traffic such as real-time VBR video, especially in the case of small buffer sizes. Furthermore it is a popular source model for packetized voice

[14], and hence the results shown in this paper can be applied to voice over IP performance evaluation as well. Recently Cao et al. [6] showed that the traffic generated by a large number of sources tends to Poisson as the load increases due to statistical multiplexing and hence justifies the Poisson model for the background traffic.

The paper is organized as follows. Section 2 gives an overview of the previous work on the modeling of the loss process of a single server queue. In Section 3 we describe our models to calculate the loss probability in a block of packets. In Section 4 we validate our models by simulations and compare the results obtained with the two PSDs. In Section 5 we conclude our work.

2 Related Work

In [7], Cidon et al. present an exact analysis of the packet loss process in an M/M/1/K queue, that is the probability of losing j packets in a block of n packets, and show that the distribution of losses may be bursty compared to the assumption of independence. They also consider a discrete time system fed with a Bernoulli arrival process describing the behavior of an ATM multiplexer. In [13], Gurewitz et al. present explicit expressions for the above quantities of interest for the M/M/1/K queue. In [2] the multidimensional generating function of the probability of j losses in a block of n packets is obtained and an easy-to-calculate asymptotic result is given under the condition that $n \leq K + j + 1$.

The above models consider exponentially distributed service times. Models with general and deterministic service times have been proposed for calculating various measures of queueing performance. In [1], Ait-Hellal et al. present an asymptotic result for a system where the service times and the interarrival times are stationary ergodic, in particular they show that if the block lengths k and redundancy j is large enough, then the frame loss probabilities can be made arbitrarily small. The conditional loss probability (CLP) is derived for the N*IPP/D/1/K queue in [19] and it is shown that the CLP can be orders of magnitude higher than the loss probability. Kawahara et al. consider a discrete time system fed with an interrupted Bernoulli process and bursty background traffic modeled by a Markov modulated Bernoulli process in [16]. They use the model to investigate the effects of the burstiness of the tagged source and the buffer size on the effectiveness of FEC in ATM networks.

In [14] the performance of the MMPP/G/1/K queue was evaluated considering the superposition of multimedia and data traffic at a single server queue, and the corresponding delay distribution was given. The waiting time and queue length distribution of the N/G/1/K queue (N stands for the Neuts process) was derived in [5] including the MMPP/G/1/K queue as a special case. Even though the waiting time and queue length distribution of the MMPP/G/1/K queue has been derived, a more thorough analysis of the packet loss process has not yet been done.

3 Model Description

We consider a single multiplexer fed by two sources, a Markov-modulated Poisson process (MMPP) and a Poisson process, representing the tagged source and the background traffic respectively. We assume that the sources feeding the system are independent. The MMPP is described by the infinitesimal generator Q with elements r_{ij} and the arrival rate matrix $\Lambda = diag\{\lambda_1, \ldots, \lambda_L\}$, where λ_i is the average arrival rate while the underlying Markov chain is in state i. The Poisson process modeling the background traffic has average arrival rate λ. The superposition of the two sources can be described by a single MMPP with arrival rate matrix $\hat{\Lambda} = \Lambda \oplus \lambda = \Lambda + \lambda I = diag\{\hat{\lambda}_1, \ldots, \hat{\lambda}_L\}$, and infinitesimal generator $\hat{Q} = Q$, where $\oplus$ is the Kronecker sum.

Packets arriving from both sources have the same size distribution. The packets are stored in a buffer that can host up to K packets, and are served according to a FIFO policy.

Every n consecutive packets from the tagged source form a block, and we are interested in the probability distribution of the number of lost packets in a block arriving from the MMPP in the steady state of the system. Thus our purpose is to calculate the probability $P(j,n), n \geq 1, 0 \leq j \leq n$ of j losses in a block of n packets. Throughout the calculations we use notations similar to those in [7].

3.1 Exponential Packet Size Distribution

First we consider a system with exponential service time distribution and average service time $1/\mu$. We define the probability $P^a_{i,l}(j,n)$, $0 \leq i \leq K$, $l = 1 \ldots L$, $n \geq 1$, $0 \leq j \leq n$ as the probability of j losses in a block of n packets, given that the number of packets in the system is i just before the arrival epoch of the first packet in the block and the first packet of the block is generated in state l of the MMPP. As the first packet in the block is arbitrary,

$$P(j,n) = \sum_{l=1}^{L}\sum_{i=0}^{K}\Pi(i,l)P^a_{i,l}(j,n) \tag{1}$$

$\Pi(i,l)$, the steady state distribution of number of packets in the queue as seen by an arriving packet can be derived from the steady state distribution of the $MMPP/M/1/K$ queue as

$$\Pi(i,l) = \frac{\pi(i,l)\lambda_l}{\sum_{l=1}^{L}\lambda_l\sum_{i=0}^{K}\pi(i,l)}, \tag{2}$$

where $\pi(i,l)$, $0 \le i \le K$, $l = 1\ldots L$ denotes the probability that there are i packets in the queue and the $MMPP$ is in state l in the steady state of the $MMPP/M/1/K$ queue [12].

The probabilities $P^a_{i,l}(j,n)$ can be derived according to the following recursion. The recursion is initiated for n = 1 with the following relations

$$\begin{aligned} P^a_{i,l}(j,1) &= \begin{cases} 1 & j=0 \\ 0 & j\ge 1 \end{cases} & i \le K-1 \\ P^a_{i,l}(j,1) &= \begin{cases} 0 & j=0, j\ge 2 \\ 1 & j=1 \end{cases} & K-1 < i. \end{aligned} \tag{3}$$

Using the notation $p_m = \frac{\lambda_m}{\lambda_m+\lambda}$ and $\overline{p}_m = \frac{\lambda}{\lambda_m+\lambda}$, for $n \ge 2$ the following equations hold

$$P^a_{i,l}(j,n) = \sum_{m=1}^{L}\sum_{k=0}^{i+1} Q^{l,m}_{i+1}(k)\{p_m P^a_{i+1-k,m}(j,n-1) + \overline{p}_m P^s_{i+1-k,m}(j,n-1)\} \tag{4}$$

for $0 \le i \le K-1$ and

$$P^a_{i,l}(j,n) = \sum_{m=1}^{L}\sum_{k=0}^{K} Q^{l,m}_{K}(k)\{p_m P^a_{K-k,m}(j-1,n-1) + \overline{p}_m P^s_{K-k,m}(j-1,n-1)\} \tag{5}$$

for $i = K$. $P^s_{i,l}(j,n)$ is given by

$$P^s_{i,l}(j,n) = \sum_{m=1}^{L}\sum_{k=0}^{i+1} Q^{l,m}_{i+1}(k)\{p_m P^a_{i+1-k,m}(j,n) + \overline{p}_m P^s_{i+1-k,m}(j,n)\} \tag{6}$$

for $0 \le i \le K-1$ and

$$P^s_{i,l}(j,n) = \sum_{m=1}^{L}\sum_{k=0}^{K} Q^{l,m}_{K}(k)\{p_m P^a_{K-k,m}(j,n) + \overline{p}_m P^s_{M-k,m}(j,n)\} \tag{7}$$

for $i = K$. The probability $P^s_{i,l}(j,n), 0 \le i \le K, l = 1\ldots L, 0 \le j \le n$ is the probability of j losses in a block of n packets, given that the number of packets in the system is i just before the arrival of a packet from the background traffic and the MMPP is in state l. In equations (4) to (7) $Q^{l,m}_i(k)$ denotes the joint conditional probability of the events that out of i packets k leave during an interarrival time and the next arrival occurs in state m of the underlying Markov chain, given that the last arrival occurred in state l. A way to calculate $Q^{l,m}_i(k)$ is given in Appendix B.

The procedure of computing $P^a_{i,l}(j,n)$ is as follows. First we calculate $P^a_{i,l}(j,1), i = 0\ldots K$ from the initial conditions (3). Then in iteration k we first calculate $P^s_{i,l}(j,k), k = 1\ldots n-1$ using equations (6) and (7) and the probabilities $P^a_{i,l}(j,k)$, which have been calculated during iteration $k-1$. Then we calculate $P^a_{i,l}(j,k+1)$ using equations (4) and (5).

3.2 Deterministic Packet Size Distribution

Next, we consider a system with constant packet sizes of transmission time D. We define the probability $P^a_{x,l}(j,n), 0 \le x \le KD, l = 1\ldots L, n \ge 1, 0 \le j \le n$ as the probability of j losses in a block of n packets, given that the remaining workload in the system is x just before the arrival of the first packet in the block and the first packet of the block is generated in state l of the MMPP. As the first packet in the block is arbitrary,

$$P(j,n) = \sum_{l=1}^{L}\int_{x=0}^{KD} V(x,l)P^a_{x,l}(j,n)dx. \tag{8}$$

An approximation for $V(x,l)$, the workload distribution of the steady state queue as seen by an arriving packet, can be given based on the steady state distribution of the $MMPP/E_r/1/K$ queue as outlined in Appendix A.

The probabilities $P^a_{x,l}(j,n)$ can be derived according to the following recursion. The recursion is initiated for n = 1 with the following relations

$$P^a_{x,l}(j,1) = \begin{cases} 1 & j=0 \\ 0 & j\ge 1 \end{cases} \qquad x \le (K-1)D,$$

$$P^a_{x,l}(j,1) = \begin{cases} 0 & j=0, j\geq 2 \\ 1 & j=1 \end{cases} \quad (K-1)D < x. \tag{9}$$

Using the notation $p_m = \frac{\lambda_m}{\lambda_m+\lambda}$ and $\overline{p}_m = \frac{\lambda}{\lambda_m+\lambda}$, for $n \geq 2$ the following equations hold.

$$\begin{aligned} P^a_{x,l}(j,n) &= \sum_{m=1}^{L} \int_0^{x+D} f_{l,m}(t)\{p_m P^a_{x+D-t,m}(j,n-1) \\ &\quad + \overline{p}_m P^s_{x+D-t,m}(j,n-1)\}dt \\ &\quad + \int_{x+D}^{\infty} f_{l,m}(t)\{p_m P^a_{0,m}(j,n-1) + \overline{p}_m P^s_{0,m}(j,n-1)\}dt \end{aligned} \tag{10}$$

for $0 \leq x \leq (K-1)D$ and for $(K-1)D < x$

$$\begin{aligned} P^a_{x,l}(j,n) &= \sum_{m=1}^{L} \int_0^{x} f_{l,m}(t)\{p_m P^a_{x-t,m}(j-1,n-1) \\ &\quad + \overline{p}_m P^s_{x-t,m}(j-1,n-1)\}dt \\ &\quad + \int_{x}^{\infty} f_{l,m}(t)\{p_m P^a_{0,m}(j-1,n-1) + \overline{p}_m P^s_{0,m}(j-1,n-1)\}dt. \end{aligned} \tag{11}$$

$P^s_{x,l}(j,n)$ is given by

$$\begin{aligned} P^s_{x,l}(j,n) &= \sum_{m=1}^{L} \int_0^{x+D} f_{l,m}(t)\{p_m P^a_{x+D-t,m}(j,n) \\ &\quad + \overline{p}_m P^s_{x+D-t,m}(j,n)\}dt \\ &\quad + \int_{x+D}^{\infty} f_{l,m}(t)\{p_m P^a_{0,m}(j,n) + \overline{p}_m P^s_{0,m}(j,n)\}dt \end{aligned} \tag{12}$$

for $0 \leq x \leq (K-1)D$ and for $(K-1)D < x$

$$\begin{aligned} P^s_{x,l}(j,n) &= \sum_{m=1}^{L} \int_0^{x} f_{l,m}(t)\{p_m P^a_{x-t,m}(j,n) \\ &\quad + \overline{p}_m P^s_{x-t,m}(j,n)\}dt \\ &\quad + \int_{x}^{\infty} f_{l,m}(t)\{p_m P^a_{0,m}(j,n) + \overline{p}_m P^s_{0,m}(j,n)\}dt. \end{aligned} \tag{13}$$

The probability $P^s_{x,l}(j,n), 0 \leq x \leq KD, l = 1\ldots L, n \geq 1, 0 \leq j \leq n$ is the probability of j losses in a block of n packets, given that the remaining workload in the system is x just before the arrival of a packet from the background traffic and the MMPP is in state l. In (10) to (13) $f_{l,m}(t)$ denotes the interarrival-time distribution of the joint arrival process and is given in Appendix B.

3.3 Numerical Evaluation

The above set of integral equations can be solved using numerical integration. The finite integrals in equations (10) to (13) are calculated numerically while the infinite integrals - as the integrand only depends on t in $f_{l,m}(t)$ - can be evaluated analytically as shown in Appendix B (27). We introduce Δ the step size for the numerical integration such that $D = N\Delta$, and so instead of equations (9-13) we can write

$$\begin{aligned} P^a_{i,l}(j,1) &= \begin{cases} 1 & j=0 \\ 0 & j\geq 1 \end{cases} \quad i \leq (K-1)N, \\ P^a_{i,l}(j,1) &= \begin{cases} 0 & j=0, j\geq 2 \\ 1 & j=1 \end{cases} \quad (K-1)N < i. \end{aligned} \tag{14}$$

For $n \geq 2$ the following recursive equations hold.

$$\begin{aligned} P^a_{i,l}(j,n) &= \sum_{m=1}^{L} \sum_{\tau=0}^{i+N} f_{l,m}(\tau\Delta) c_\tau^{i+N} \{p_m P^a_{i+N-\tau,m}(j,n-1) \\ &\quad + \overline{p}_m P^s_{i+N-\tau,m}(j,n-1)\} \\ &\quad + \int_{i\Delta+D}^{\infty} f_{l,m}(t)\{p_m P^a_{0,m}(j,n-1) + \overline{p}_m P^s_{0,m}(j,n-1)\}dt \end{aligned} \tag{15}$$

for $0 \leq i \leq (K-1)N$ and for $(K-1)N < i$

$$\begin{aligned} P^a_{i,l}(j,n) &= \sum_{m=1}^{L} \sum_{\tau=0}^{i} f_{l,m}(\tau\Delta) c_\tau^{i} \{p_m P^a_{i-\tau,m}(j-1,n-1) \\ &\quad + \overline{p}_m P^s_{i-\tau,m}(j-1,n-1)\} \\ &\quad + \int_{i\Delta}^{\infty} f_{l,m}(t)\{p_m P^a_{0,m}(j-1,n-1) + \overline{p}_m P^s_{0,m}(j-1,n-1)\}dt. \end{aligned} \tag{16}$$

$P^s_{i,l}(j,n)$ is given by

$$P^s_{i,l}(j,n) = \sum_{m=1}^{L} \sum_{\tau=0}^{i+N} f_{l,m}(\tau\Delta) c_\tau^{i+N} \{p_m P^a_{i+N-\tau,m}(j,n) \tag{17}$$

$$+\overline{p}_m P^s_{i+N-\tau,m}(j,n)\}$$
$$+\int_{i\Delta+D}^{\infty} f_{l,m}(t)\{p_m P^a_{0,m}(j,n)+\overline{p}_m P^s_{0,m}(j,n)\}dt$$

for $0 \leq i \leq (K-1)N$ and for $(K-1)N < i$

$$\begin{aligned} P^s_{i,l}(j,n) &= \sum_{m=1}^{L}\sum_{\tau=0}^{i} f_{l,m}(\tau\Delta)c^i_\tau\{p_m P^a_{i-\tau,m}(j,n) \\ &\quad +\overline{p}_m P^s_{i-\tau,m}(j,n)\} \\ &\quad +\int_{i\Delta}^{\infty} f_{l,m}(t)\{p_m P^a_{0,m}(j,n)+\overline{p}_m P^s_{0,m}(j,n)\}dt, \end{aligned} \quad (18)$$

where the coefficient c^i_τ is the τ^{th} weighting coefficient in the i degree numerical integration. Through carefully choosing the numerical method by increasing N, the error induced by the numerical integration decreases at least proportional to $(\frac{1}{N})^5$.

The procedure of computing $P^a_{i,l}(j,n)$ is as follows. First we calculate $P^a_{i,l}(j,1), i=0\ldots KN$ from the initial conditions (14). Then in iteration k we first calculate $P^s_{i,l}(j,k), k=1\ldots n-1$ using equations (17) and (18) and the probabilities $P^a_{i,l}(j,k)$, which have been calculated during iteration $k-1$. Then we calculate $P^a_{i,l}(j,k+1)$ using equations (15) and (16).

4 Performance Analysis

In this section we validate the two models described in Section 3 via simulations and use them to evaluate the effects of the PSD on the packet loss process. The average packet length of both the tagged and the background traffic is set to 188 bytes, as given for the transport stream in the MPEG-2 standard [15]. Note that increasing the average packet length is equivalent to decreasing the link speed, and thus the particular fixed value of the average packet length does not limit the generality of the results presented here. The considered link speeds are 10 Mbps, 22.5 Mbps and 45 Mbps. The queueing delay is set to around 0.5 ms in all cases, resulting in queue lengths from 5 to 20 packets depending on the link speed. Both in the analytical models and in the simulations we consider a 3 state MMPP, with an average bitrate of 540 kbps, arrival intensities $\lambda_1=116/s, \lambda_2=274/s, \lambda_3=931/s$ and transition rates $r_{12}=0.12594, r_{21}=0.25, r_{23}=1.97, r_{32}=2$. These values were derived from an MPEG-4 encoded video trace. The simulations were performed in ns-2, the simulation time was between 20 thousand and 120 thousand seconds.

We use three measures to compare the packet loss process. The first one is a commonly used measure of closeness, the Kullback-Leibler distance [17] defined for two distributions as

$$d(p_1,p_2)=\sum_{j=0}^{n} P_1(j,n)log_2\frac{P_1(j,n)}{P_2(j,n)}, \quad (19)$$

The Kullback-Leibler distance is the same as the relative entropy of p_1 with respect to p_2. It is not a true metric, as it is not symmetric and does not satisfy the triangle inequality, but it is always non-negative and equals zero only if $p_1=p_2$.

The second measure is based on the gain that can be achieved by using FEC. Given the probabilities $P(j,n)$ the uncorrected loss probability for an RS(k,c+k) scheme can be calculated as

$$P^{k,c+k}_{loss}=\frac{1}{c+k}\sum_{j=c+1}^{c+k} jP(j,c+k). \quad (20)$$

Based on the uncorrected packet loss probability we define the FEC gain as the ratio of the average loss probability without the use of FEC and the uncorrected loss probability when using FEC: $f(k,c+k)=P_{loss}/P^{k,c+k}_{loss}$.

The third measure is the average loss run length. Given the probabilities $P(j,n)$ the average loss run length can be calculated as

$$E[B]=\sum_{n=1}^{\infty} P(B\geq n)=\sum_{n=1}^{\infty} P(n,n). \quad (21)$$

A higher value of the average loss run length results in fewer loss bursts given a particular average loss probability. Though the independence of the loss process is favorable for applications that employ some form of FEC, other applications not using FEC might prefer losses to occur in bursts, for example those that use reversible variable length coding to localize error propagation.

4.1 Constant average load

In this section we consider the case when the average load in the network is constant and compare the packet loss process, the efficiency of FEC and the average loss run length for the two PSDs. Figures 1-3 show the uncorrected loss probability without using FEC (FEC(1,1)) and using two different FEC schemes for the three considered link speeds as a function of the average load. Figures 4-6 show the average loss run length for the three considered link speeds as a function of the average load. Comparing the results given by the analytical models and the simulations shows that the models presented give accurate results. Figures 7 and 8 show the probability of loosing j packets in a block of 11 and 22 packets respectively at an average load of $\rho = 0.8$. Figures 9 and 10 show the Kullback-Leibler distance of $P(j,11)$ and $P(j,22)$ respectively. Comparing the figures we conclude that results obtained for the different link speeds show the same properties and thus where possible we will only show results for the 10 Mbps link. Figures 11 and 12 show the FEC gain for FEC(10,11) and FEC(20,22) on a 10 Mbps link. Comparing the figures we conclude that FEC(10,11) and FEC(20,22) behave similarly, and thus in the following we will only show figures for FEC(20,22) for brevity.

The figures show that there is a significant difference between the results with the two packet size distributions: the uncorrected loss probability and the average burst size are lower while the gain achievable by using FEC is higher in the case of the deterministic PSD. The difference however is partly due to the different average loss probabilities. We eliminate the effects of the average loss probability in the following subsection.

4.2 Constant average packet loss

In this section we consider the case when the average loss probability in the network is constant despite of the different PSDs. In order to compare the packet loss process at a certain average loss probability, we take the results from simulations with the deterministic PSD and decrease the background traffic of the mathematical model with exponential PSD to match the average packet loss probability. Figure 13 shows the probability of loosing j packets in a block of 22 packets. The average load of the scenarios with deterministic PSD is $\rho = 0.8$, while the average load of the scenarios with exponential PSD is set to match the average loss probability of the corresponding scenarios with deterministic PSD and the same link speed by changing the background traffic intensity. Figure 14 shows the Kullback-Leibler distance between the results obtained with the two distributions as a function of the average loss probability. The distance between the results with the two PSDs decreased significantly (three orders of magnitude) compared to Figure 10. Figure 15 shows the average loss run length on a 10 Mbps link. Figure 16 shows the FEC gain on a 10 Mbps link. Comparing these figures to Figures 4 and 12 respectively also shows that the difference between the results obtained with the different distributions is lower at a particular average loss probability than at a particular average load level. Thus the observed difference in Subsection 4.1 was partly due to the different average loss probabilities at a particular average load level.

The remaining difference between the packet loss processes and the related FEC performance (a factor of three in the considered scenario) can be due to the difference in the level of statistical multiplexing (the background traffic intensity was decreased and as a result the packet loss process became less independent) and to the difference between the packet size distributions. Figures 17 and 18 show the FEC gain on a 22.5 Mbps and a 45 Mbps link respectively. Comparing the figures we can see that the difference between results with the two PSDs in terms of FEC gain decreases as the link speed increases (from 10 Mbps to 45 Mbps). The reason for this is that the higher the link speed the less the background traffic has to be changed to keep the average loss probability constant, and thus the change in the level of statistical multiplexing decreases.

4.3 Isolating the effects of the packet size distribution

In the following subsection we separate the effects of the level of statistical multiplexing and the PSD. We do it by changing the arrival intensity of both the background traffic and the tagged stream in the mathematical model with exponential PSD in order to match the average loss probability given by the simulations with deterministic PSD. Thus we keep both the average loss probability and the level of statistical multiplexing constant (doing so is equivalent to matching the average loss probability through decreasing the link speed). Figure 19 shows the probability of loosing j packets in a block of 22 packets. The average load of the scenarios with deterministic PSD is $\rho = 0.8$, while the average load of the scenarios with exponential PSD is set to match the average loss probability of the corresponding scenarios with deterministic PSD and the same link speed by changing the intensity of both the tagged and the background traffic. Figure 20 shows the Kullback-Leibler distance as a function of the average loss probability on a 10 Mbps link for $P(j,22)$. Comparing this to Figure 14 we can see a further significant decrease in the distance of the distributions. Figure 21 shows the average loss run length as a function of the average loss probability. Comparing this to Figure 15 shows a decrease between the results with the two distributions. The same effect can be seen comparing Figure 22 to Figure 16, which shows the FEC gain on a 10 Mbps link as a function of the average loss probability for FEC(20,22). Thus the difference in the FEC gain considering a particular average loss probability is mainly due to the different levels of statistical multiplexing and in a lower extent to the different PSDs.

5 Conclusion

In this paper we presented analytical models to evaluate the loss process of a single multiplexer fed by a bursty source and Poisson background traffic for two different packet size distributions and used extensive simulations to validate them. We used the models to compare the packet loss process and the related FEC performance in the case of exponential and deterministic packet size distributions. Based on the results we conclude that the deterministic packet size distribution not only yields a lower average loss probability compared to the exponential packet size distribution but also a more independent packet loss process (lower average loss run length and more efficient FEC). We showed that the difference between the results at a particular average load level is mainly due to the different average loss probabilities while at a particular average loss probability it is mainly due to the different levels of statistical multiplexing. We showed that considering a particular average loss probability the difference between results obtained with the two distributions decreases as the link speed increases. Thus the packet size distribution has a bigger influence on the packet loss process in access networks. In those networks even single multimedia streams can have a significant impact on the characteristics of the aggregate traffic and thus improve the multiplexing performance by minimizing the variance of their packet size distribution. Furthermore, we conclude that applications experiencing a certain average loss probability in the network can have rough estimates on the performance of FEC independent of the packet size distribution in the network. The results presented here help to dissolve the mistrust in the reliability of the potential of FEC to improve the transmission quality of individual streams. Better understanding of the potential of FEC can promote its widespread use in the future.

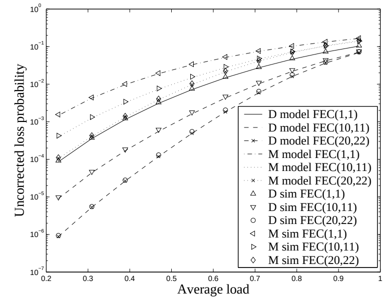

Figure 1: Uncorrected loss probability vs average load on a 10 Mbps link using various FEC schemes.

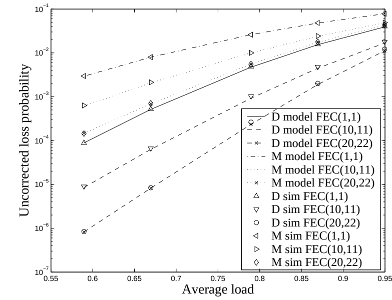

Figure 2: Uncorrected loss probability vs average load on a 22.5 Mbps link using various FEC schemes.

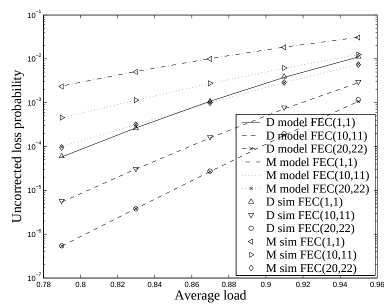

Figure 3: Uncorrected loss probability vs average load on a 45 Mbps link using various FEC schemes.

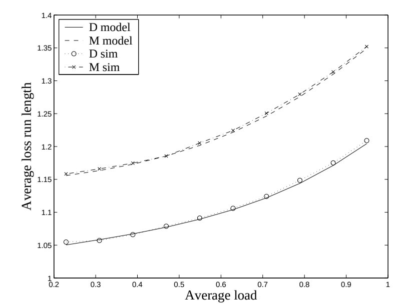

Figure 4: Average loss run length vs average load on a 10 Mbps link.

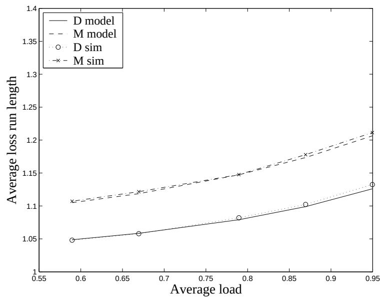

Figure 5: Average loss run length vs average load on a 22.5 Mbps link.

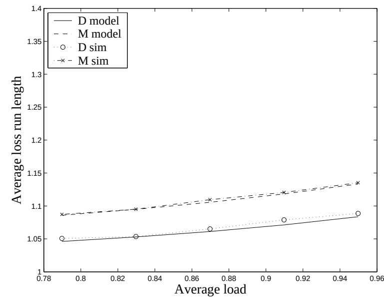

Figure 6: Average loss run length vs average load on a 45 Mbps link.

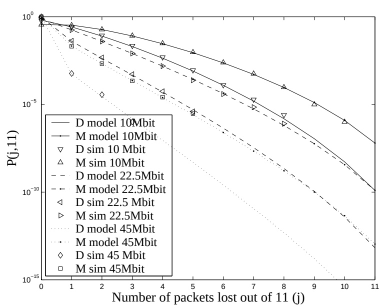

Figure 7: Probability of loosing j packets in a block of 11 packets at average load level $\rho = 0.8$.

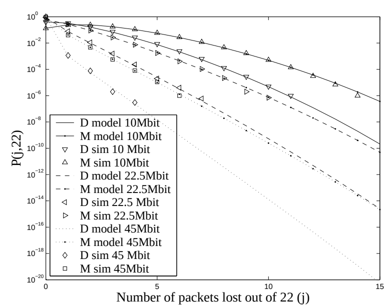

Figure 8: Probability of loosing j packets in a block of 22 packets at average load level $\rho = 0.8$.

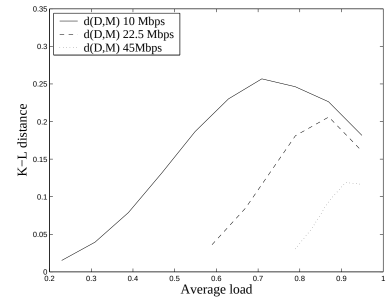

Figure 9: Kullback-Leibler distance vs average load for P(j,11).

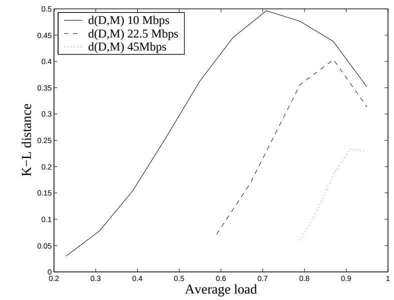

Figure 10: Kullback-Leibler distance vs average load for P(j,22).

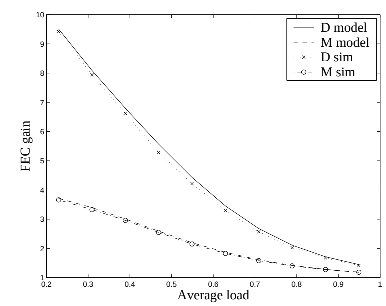

Figure 11: FEC gain vs average load for FEC(10,11) on a 10 Mbps link.

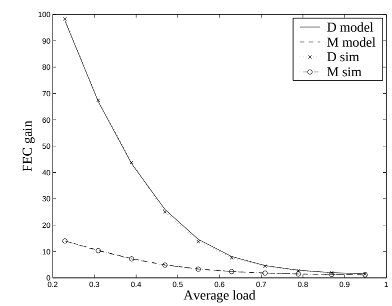

Figure 12: FEC gain vs average load for FEC(20,22) on a 10 Mbps link.

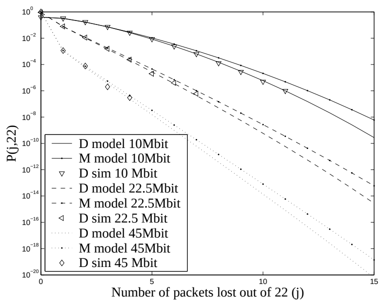

Figure 13: Probability of loosing *j* packets in a block of 22 packets. The average loss probability of the scenarios with the same link speed is equal.

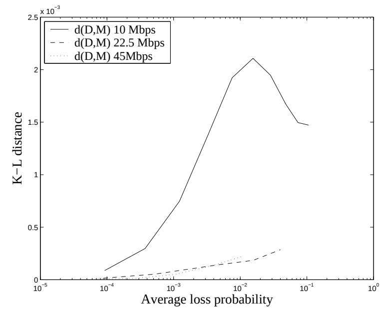

Figure 14: Kullback-Leibler distance vs average loss probability for P(j,22).

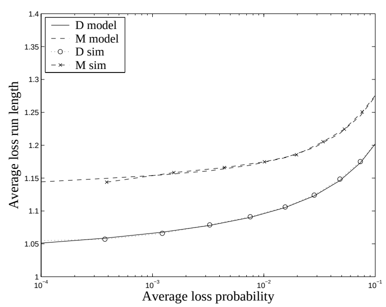

Figure 15: Average loss run length vs average loss probability on a 10 Mbps link.

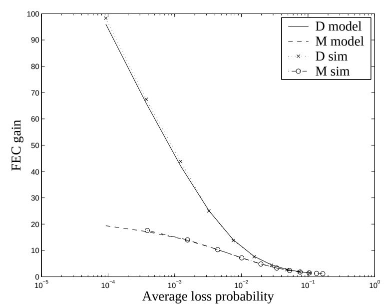

Figure 16: FEC gain vs average loss probability on a 10 Mbps link for FEC(20,22).

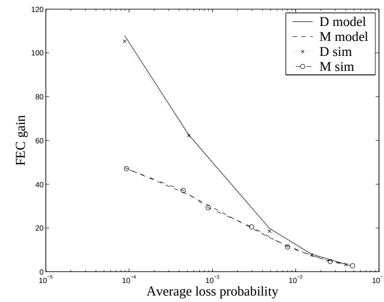

Figure 17: FEC gain vs average loss probability on a 22.5 Mbps link for FEC(20,22).

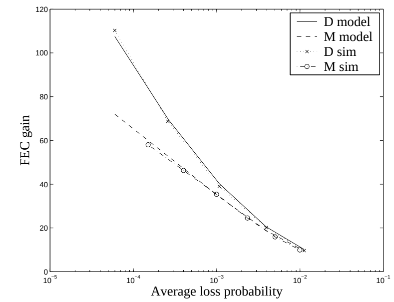

Figure 18: FEC gain vs average loss probability on a 45 Mbps link for FEC(20,22).

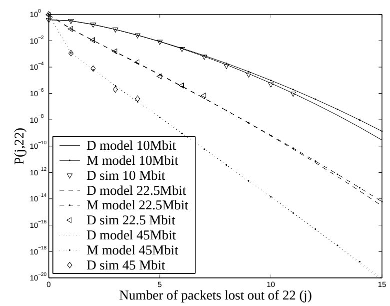

Figure 19: Probability of loosing *j* packets in a block of 22 packets. The average loss probability of the scenarios with the same link speed is equal. (same level of statistical multiplexing).

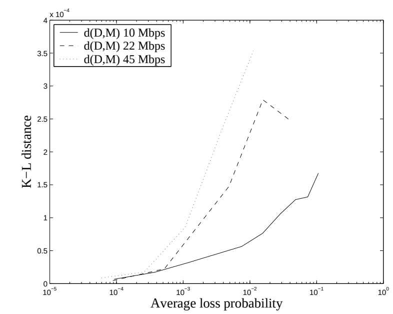

Figure 20: Kullback-Leibler distance vs average loss probability for P(j,22) (same level of statistical multiplexing).

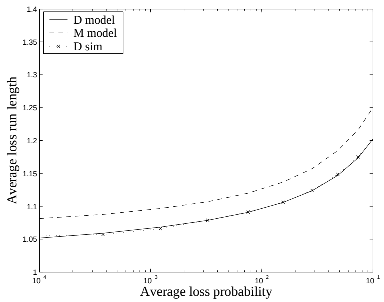

Figure 21: Average loss run length vs average loss probability on a 10 Mbps link (same level of statistical multiplexing).

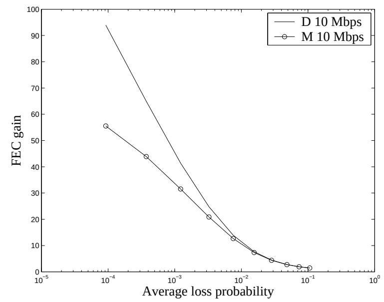

Figure 22: FEC gain vs average loss probability on a 10 Mbps link for FEC(20,22) (same level of statistical multiplexing).

6 Appendices

A Workload distribution

The Laplace transform of the virtual waiting time distribution for the MMPP/G/1/K queue is given in [5]. Following the arguments presented there one can derive the Laplace transform of the workload distribution

$$V(s) = \frac{1}{[\mu - \overline{\pi_0}(\hat{Q}-\hat{\Lambda})^{-1}\overline{e}]}\{\overline{\pi_0}[-s(sI-\hat{\Lambda}+\hat{Q})^{-1}(\hat{Q}-\hat{\Lambda})^{-1}$$

$$+\sum_{k=1}^{N-1} T[\hat{\Lambda}S]^{k-1}(sI+\hat{Q})S[G^*(s)]^k - [G^*(s)]^{N-1}\sum_{k=0}^{N-1}\overline{\pi_k}T[\hat{\Lambda}S]^{N-k-1}]]$$

$$+[G^*(s)]^{N-1}\sum_{k=1}^{N-1}\overline{\pi_k}\sum_{j=N-k}^{\infty}\left[\sum_{k=0}^{j}A_k T[\hat{\Lambda}S]^{n-k} - G^*(s)T[\hat{\Lambda}S]^n\right]\},$$

where $S=(\hat{\Lambda}-sI-\hat{Q})^{-1}$, $T=(sI-\hat{\Lambda}+\hat{Q})^{-1}$, $G^*(s)$ is the Laplace transform of the service time distribution. A_k is an $L\times L$ matrix whose (l,m)th element denotes the conditional probability of the MMPP reaching phase m and having k arrivals during a service time, starting from phase l. Instead of calculating the inverse Laplace transform of the above expression we use an approximation based on the steady state distribution of the remaining exponential stages in an $MMPP/E_r/1/K$ queue, where E_r denotes an r stage Erlang distribution. r is chosen to be cN, where $c \geq 1$ is an arbitrary whole number and N is defined in Section 3.3. Given $\pi(k,l)$, $0\leq k\leq cNK$, $1\leq l\leq L$, the steady state distribution of the remaining exponential stages in the $MMPP/E_r/1/K$ queue we calculate the queue length distribution as seen by an arriving packet as

$$\Pi(k,l) = \frac{\pi(k,l)\lambda_l}{\sum_{l=1}^{L}\lambda_l\sum_{i=0}^{cNK}\pi(i,l)}. \tag{22}$$

Given the queue length distribution as seen by an arriving packet, $\Pi(k,l)$, $0\leq k\leq cNK$, $1\leq l\leq L$, the workload distribution $V(i,l)$, $0\leq i\leq NK$,$1\leq l\leq L$ is approximated by

$$V(i,l) = \begin{cases} \Pi(0,l) & i=0 \\ \sum_{k=(i-1)c+1}^{ic}\Pi(k,l) & 0<i<=NK. \end{cases} \tag{23}$$

B Interarrival-time distribution

The probability $Q_i^{l,m}(k)$ denotes the joint conditional probability that between two arrivals from the joint arrival stream there are k exponential service completions out of i and the state of the MMPP at the moment of the arrival is m given that at the time of the last arrival the MMPP was in state l. $Q_i^{l,m}(k)$ can be expressed as

$$\begin{array}{ll} Q_i^{l,m}(k) = P^{l,m}(k) & if k<i \\ Q_i^{l,m}(k) = \sum_{j=i}^{\infty}P^{l,m}(j) & if k=i, \end{array} \tag{24}$$

where $P^{l,m}(k)$ denotes the joint probability of having k service completions with exponentially distributed service times between two arrivals and the next arrival coming in state m of the MMPP given that the last arrival came in state l.

The z-transform $P^{l,m}(z)$ of $P^{l,m}(k)$ is given by

$$P^{l,m}(z) = \sum_{k=0}^{\infty}\left(\int_0^{\infty}\frac{(\mu t)^k}{k!}e^{-\mu t}f^{l,m}(t)dt\right)z^k = f^{l,m*}(\mu-\mu z), \tag{25}$$

where $f^{l,m}(t)$ is the joint distribution of the interarrival-time and the probability that the next arrival is in state m given that the last arrival was generated in state l of the MMPP. The Laplace transform of $f^{l,m}(t)$ is denoted with $f^{l,m*}(s)$ and is given by [11]

$$f^{l,m*}(s) = \mathcal{L}\left\{e^{(\hat{Q}-\hat{\Lambda})x}\hat{\Lambda}\right\} = (sI - \hat{Q} + \hat{\Lambda})^{-1}\hat{\Lambda}. \tag{26}$$

The inverse Laplace-transform of (26) can be expressed analytically by partial fraction decomposition as long as $L \leq 4$.

$$f^{l,m}(t) = \sum_{j=1}^{L} B_j^{l,m} e^{\beta_j t}, \tag{27}$$

where β_j are the roots of $t(s) = det[sI - \hat{Q} + \hat{\Lambda}]$. Based on (27) the infinite integrals in equations (15),(16), (17) and (18) can be calculated as

$$\int_x^\infty f_{l,m}(t)dt = -\sum_{j=1}^{L} \frac{B_j^{l,m}}{\beta_j} e^{\beta_j x}. \tag{28}$$

Using the substitution $\alpha_j = 1 + \beta_j/\mu$ and $A_j^{l,m} = B_j^{l,m}/(\mu\alpha_j)$ one can calculate $P^{l,m}(k)$ based on (25)

$$P^{l,m}(k) = \sum_{j=1}^{L} A_j^{l,m} \frac{1}{\alpha_j^k}. \tag{29}$$

Given the probability $P^{l,m}(k)$ one can express $Q_i(k)$ as

$$Q_i^{l,m}(k) = \begin{cases} \sum_{j=1}^{L} A_j^{l,m} \left(\frac{1}{\alpha_j}\right)^k & 0 \leq k < i \\ \sum_{j=1}^{L} \frac{A_j^{l,m}}{1-1/\alpha_j} \left(\frac{1}{\alpha_j}\right)^i & k = i. \end{cases} \tag{30}$$

A detailed description of the calculation of $Q_i^{l,m}(k)$ can be found in [8].

References

[1] O. Ait-Hellal, E. Altman, A. Jean-Marie, and I. A. Kurkova, "On loss probabilities in presence of redundant packets and several traffic sources". *Performance Evaluation*, vol. 36-37,485–518, (1999).

[2] E. Altman and A. Jean-Marie, "Loss probabilities for messages with redundant packets feeding a finite buffer". *IEEE Journal on Selected Areas in Comm.*, vol. 16, no.5,779–787, (1998).

[3] J. Beran, R. Sherman, M. Taqqu, and W. Willinger, "Long-range dependence in variable-bit-rate video traffic". *IEEE Trans. on Communications*, vol. 43, no.2/3/4,1566–1579, (1995).

[4] E. Biersack, "Performance evaluation of forward error correction in ATM networks". In *Proc. of ACM SIGCOMM*, 248–257, (1992).

[5] C. Blondia, "The N/G/1 finite capacity queue". *Commun. Statist. - Stochastic Models*, vol. 5, no.2,273–294, (1989).

[6] J. Cao, W. S. Cleveland, D. Lin, and D. X. Sun, "Internet traffic tends toward poisson and independent as the load increases". In *Nonlinear Estimation and Classification*. Springer, (2002).

[7] I. Cidon, A. Khamisy, and M. Sidi, "Analysis of packet loss processes in high speed networks". *IEEE Trans. on Inform. Theory*, vol. IT-39, no.1,98–108, (1993).

[8] G. Dán. "Analysis of the loss process in an MMPP+M/M/1/K queue". TRITA-IMIT-LCN R 04:02, KTH/IMIT/LCN, (2004).

[9] G. Dán and V. Fodor, "Quality differentiation with source shaping and forward error correction". In *Proc. of MIPS'03*, 222–233, (2003).

[10] P. Dube and E. Altman, "Utility analysis of simple FEC schemes for VoIP". In *Proc. of Networking 2002*, 226–239, (2002).

[11] W. Fischer and K. Meier-Hellstern, "The markov-modulated poisson process MMPP cookbook". *Performance Evaluation*, vol. 18, no.2,149–171, (1992).

[12] D. Gross and C. M. Harris, *Fundamentals of Queueing Theory*. Wiley, New York, (1998).

[13] O. Gurewitz, M. Sidi, and I. Cidon, "The ballot theorem strikes again: Packet loss process distribution". *IEEE Trans. on Inform. Theory*, vol. IT-46, no.7,2599–2595, (2000).

[14] H. Heffes and D. M. Lucantoni, "A markov modulated characterization of packetized voice and data traffic and related statistical multiplexer performance". *IEEE Journal on Selected Areas in Comm.*, vol. 4, no.6,856–868, (1986).

[15] D. Hoffman and G. Fernando. "RTP payload format for MPEG1/MPEG2 video". RFC 2250, (1998).

[16] K. Kawahara, K. Kumazoe, T. Takine, and Y. Oie, "Forward error correction in ATM networks: An analysis of cell loss distribution in a block". In *Proc. of IEEE INFOCOM*, 1150–1159, (1994).

[17] S. Kullback, *Information Theory and Statistics*. Wiley, New York, (1959).

[18] B. Ryu and A. Elwalid, "The importance of long-range dependence of VBR video traffic in ATM traffic engineering: Myths and realities". In *Proc. of ACM SIGCOMM*, 3–14, (1996).

[19] H. Schulzrinne, J. Kurose, and D. Towsley, "Loss correlation for queues with bursty input streams". In *Proc. of IEEE ICC*, 219–224, (1992).

[20] T. Yoshihara, S. Kasahara, and Y. Takahashi, "Practical time-scale fitting of self-similar traffic with markov-modulated poisson process". *Telecommunication Systems*, vol. 17, no.1-2,185–211, (2001).

György Dán received the M.Sc. degree in Informatics from the Technical University of Budapest, Hungary in 1999 and the M.Sc. degree in Economics from the Budapest University of Economic Sciences, Hungary in 2003. He worked as consultant in the field of access networks, streaming media and videoconferencing 1999-2001. He was a PhD student at the Technical University of Budapest 1999 to 2002 and guest researcher at KTH, Royal Institute of Technology, Stockholm, Sweden 2001 to 2002. Currently he is a PhD student at the Department of Microelectronics and Information Technology of KTH, the Royal Institute of Technology working on traffic control for real-time multimedia traffic in packet switched networks.

Viktória Fodor received the M.Sc. and Ph.D. degrees in computer engineering from the Technical University of Budapest in 1992 and 1999, respectively. Since 1998 she has been with the Department of Microelectronics and Information Technology, KTH, Royal Institute of Technology where she is an Assistant Professor. Dr. Fodor currently serves in the program committee of the 3rd International Workshop on QoS in Multiservice IP Networks and the 1st International Conference on Multimedia Service Access Networks. Her research interests are in the areas of multimedia networking and traffic control.

Gunnar Karlsson is since 1998 professor at the Department of Microelectronics and Information Technology of KTH, the Royal Institute of Technology in Sweden, where he leads the Laboratory for Communication Networks. He has previously worked for Swedish Institute of Computer Science (SICS) from 1992 to 1998 and IBM Zurich Research Laboratory from 1989 to 1992. His Ph.D. is from Columbia University (1989), New York, and the M.Sc. from Chalmers University of Technology in Gothenburg (1983). He has been visiting professor at EPFL, Switzerland, and the Helsinki University of Technology in Finland. His research interests lie within the general field of multimedia networking.