Testing the Separation Hypothesis with Farmer Heterogeneity: A New Use of the Structural Break Test

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Abstract

Agricultural households respond differently to market imperfections: production decisions are not separated from preferences or utility. Therefore, in order to correctly model household decisions it is important to test for this separation. Many innovative tests have been developed in the literature but most of them assume no heterogeneity across the farmers (i.e. all farmers are assumed to behave in the same way) and so these tests identify only one model for every farmer in the sample. By using a structural break test from time-series econometrics and by developing a search dimension in cross-section data that can act like a time dimension in time-series data, the test in this paper can allow for heterogeneity across the farmers. Thus, it is able to identify the correct model for each farmer in the sample.

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Introduction

It is well recognized that rural households especially in the developing countries are systematically exposed to market imperfections. These imperfections lead to what has been called non-separability. A household behaves as in the non-separation model if household's production decisions (e.g. choices of labor, production inputs and outputs) are affected by its preferences (e.g. consumer preferences, demographic composition). By contrast, in a separation model, the household behaves as a pure profit maximizing producer. The household's production decisions are not affected by its preferences[1].

Because of these differences the analysis of household behavior and policy effects could yield different results between the separation and non-separation model. Eswaran and Kotwal (1986) show that under the non-separation model, households with different asset positions use different strategies of labor deployment, and use it in turn to establish social classes. Taylor and Adelman (2002) analyze the impact of Mexican trade and transfer policies. They find that these policy shocks under imperfections in the labor and food market have, contrary to expectations of policy makers, remarkably small impact on production and rural incomes. Lofgren and Robinson (2002) show that large transaction costs in market participation by households in response to price and productivity shocks create discontinuities which differ markedly from smooth responses normally expected in the separation model. Le (2006) studies the trade liberalization effect on household welfare in Vietnam and finds that the welfare change under the separation model is almost twice as large as the welfare change under the non-separation model.

The significant differences in household behaviors and policy effects between the separation and non-separation model call for separation hypothesis tests to identify the correct model. The first set of tests uses the relationship between production decisions and preferences. These tests yield mixed results depending on the country in the study. For example, Lopez (1984) rejects the separation model with Canadian data, Benjamin (1992) cannot reject it with a sample of Javanese rural households, Bowlus and Sicular (2003) arrive at the same conclusion as Benjamin with a sample from China, but Grimard (2000) rejects it for Côte d'Ivoire.

The second set of tests uses the relationship between shadow wages and market wages (Jacoby 1993, Skoufias 1994, Lambert and Magnac 1994, Bhattacharyya and Kumbhakar 1997, Abdulai and Regmin 2000). Under the separation model, the shadow wages are the same as the market wages while under the non-separation model the shadow wages are different from the market wages. All of these tests reject the equality between shadow wages and market wages, and hence reject the separation model.

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[1] A comprehensive survey of agricultural household models can be found in Singh et. al. (1986). Recent progresses in these models are well documented in Janvry and Kanbur (2006).

Although popular in the literature, these tests have one common shortcoming: they do not allow heterogeneity across the farmers. In other words, all farmers are assumed to behave in the same way. This is described in Janvry and Sadoulet (2006) as follows:

"*The more fundamental weakness of these studies is that they do not recognize heterogeneity in the household population, even though it is so clearly emphasized in the theory … households' responses, especially in terms of their decision to participate or not to the market, are specific to each. In this context, the result one can expect from a global estimation (test) is unclear at best.*"

Recognizing heterogeneity across the farmers, this paper proposes a test which can identify the correct model for each farmer, and hence allow for heterogeneity across farmers. The method comes from structural break literature in time-series econometrics which dates back to Chow (1960) and is developed further by Andrews (1993) and Bai (1997, 1998). The purpose of the method is to identify structural change (or regime switching) in time-series variables such as GDP output, interest rate and inflation. Although widely used in time-series data, the method has never been used for cross-section data since there is no time dimension in cross-section data.

Based on the standard agricultural household model, this paper develops a search dimension in cross-section data which can act like a time dimension in time-series data and so the method can be applied to the separation hypothesis test with cross-section data. Given a sample of rural farmers in India, the test identifies 23% of the farmers behaving as in the separation model (80% are men, 20% are women) and 77% of the farmers behaving as in the non-separation model (84% are men, 16% are women). Future research on the effects of the government policies on farmers' decisions and welfare can use this method to accurately model different behaviors of farmers in the sample.

The paper is organized into three parts: *Part 1* presents the theoretical model and a brief review of previous studies on this topic. *Part 2* propose a new test to address farmer heterogeneity. *Part 3* describes data and results.

I. Theoretical model and literature review

The theoretical model is based on the standard time allocation model. A farmer maximizes utility function defined over leisure (l), consumption (c) and a vector of preference shifters A (e.g. number of children and number of adults): $U(c,l;A)$, subject to a budget constraint: $c = pQ + wm$, where p is the price of farm output and w is the market wage. In this budget constraint the farmer receives income from two activities: working in his farm to receive the farm output (Q) and working in the labor market to receive the market wage (w). Thus his labor supply (h) includes farm labor (L) and market labor

(m): $h = L + m$. The sum of labor supply and leisure is the total stock of time: $T = h + l$, T could be 24 hours/day.

The production function for the farm output is: $Q(L, F)$, where F is a vector of the fixed inputs (e.g. land and farm equipment)[1]. The common assumption is that $Q_L(L = 0, F) = \infty$ which means the farmer always has some farm labor ($L > 0$). Both functions U and Q are assumed to have the standard properties of the utility and production function.

The imperfections are introduced into the model as the upper and lower constraints on the market labor $0 \leq m \leq M$ where M is the maximum number of hours a farmer can work in the labor market. Note that for some farmers the upper constraint M can be zero, in which case m is zero. The farmer faces imperfections if either the lower constraint or the upper constraint is binding ($m = 0$ or $m = M$) and then behaves as in the non-separation model (NM). By contrast, the farmer faces no imperfection if neither lower nor upper constraints are binding ($0 < m < M$) and then behaves as in the separation model (SM). Further details about the NM and SM will be provided later.

The above household maximization problem can be summarized as follows:

$$\underset{L,m}{Max}\ U(pQ(L,F) + wm, T - L - m; A)$$
$$\text{subject to:}\quad 0 \leq m \leq M$$

In order to solve this problem we first set up the Lagrangian function:

$$\underset{L,m}{Max}\ U\left(pQ(L,F) + wm, T - L - m; A\right) + \delta_1 m + \delta_2 (M - m)$$

where δ_1 and δ_2 are the Lagrange multipliers.

The first order condition (FOC) for L is:

$$U_c pQ_L(L,F) - U_l = 0$$

and the Kuhn-Tucker conditions for the lower and upper constraints on m are:

$$\text{if } 0 < m < M \text{ then } U_c w - U_l = 0$$
$$\text{if } m = 0 \text{ then } U_c w - U_l + \delta_1 = 0 \text{ and } \delta_1 > 0$$
$$\text{if } m = M \text{ then } U_c w - U_l - \delta_2 = 0 \text{ and } \delta_2 > 0$$

[1] For brevity, the variable inputs are not included in the production function. The results would be the same if they were included.

The FOC can be rewritten as follows:

$$w^* = pQ_L(L,F) \tag{1}$$

$$\text{where } w^* = \frac{U_l\left(pQ(L,F)+wm,\, T-L-m;\; A\right)}{U_c\left(pQ(L,F)+wm,\, T-L-m;\; A\right)} \tag{2}$$

w^* is called the shadow wage or the opportunity cost of time and it is the key variable in this time allocation model[1]. Using w^*, the Kuhn-Tucker conditions can also be rewritten as:

$$\text{if } 0 < m < M \text{ then } w^* = w \tag{3}$$

$$\text{if } m = 0 \text{ then } w^* > w \tag{4}$$

$$\text{if } m = M \text{ then } w^* < w \tag{5}$$

A farmer behaves as in the SM if (3) is correct and in the NM if either (4) or (5) is correct. Our job is to identify the correct model for each farmer. We can see from the Kuhn-Tucker conditions that if the pair of m and M or the pair of w^* and w are observed for each farmer then the correct model can be identified easily. The problem is that only m and w are observed while M and w^* are not, so it is not possible to know the correct model by observing the data. However, based on the observed m the sample can divided into two groups:

Group 1: Farmers with $m = 0$

The behavior of farmers in this group is captured in (4) or (5) since M can be zero. We do not know if (4) or (5) is correct, but both of them belong to the NM, so all farmers in this group can be easily assigned to the NM. For farmers in this group, $w^* \neq w$ and (w^*, L) can be found by substituting (1) into (2):

$$pQ_L(L,F) = \frac{U_l\left(pQ(L,F)+wm,\; T-L-m, A\right)}{U_c\left(pQ(L,F)+wm,\; T-L-m, A\right)} \qquad \text{where } m = 0$$

This equation means that the choice of L depends on the preference shifter A, and so does w^*. In other words, the choices of (w^*, L) are not separated from A, which is why this is called the NM.

[1] The FOC in (1) states that the farmer always sets his shadow wage to the marginal product of labor in the farm. This important point has been used widely in empirical studies to calculate the shadow wages.

Group 2: Farmers with $m > 0$

The behavior of farmers in this group is captured in (3) or (5). Because these equations belong to different models, we cannot know which model is correct. However, we know the following:

- If (5) is correct (NM is the correct model) then $w^* < w$ and (w^*, L) can be found by substituting (1) into (2) as we did for group 1:

  $$pQ_L\left(L,F\right) = \frac{U_l\left(pQ\left(L,F\right)+wm,\ T-L-m,A\right)}{U_c\left(pQ\left(L,F\right)+wm,\ T-L-m,A\right)} \qquad \text{where } m = M \ .$$

  Similar to farmers in group 1, this equation means that the choices of (w^*, L) are not separated from A.

- If (3) is correct (SM is the correct model) then $w^* = w$, plug this into equation (1) to have: $pQ_L\left(L,F\right) = w$. This equation means that the choice of L does not depend on the preference shifter A, so does w^*. In other words the choices of (w^*, L) are separated from A, which is why this is called the SM.

Theoretical model summary

Given the discussion so far, the following relationships can be used to identify the correct model for each farmer:

- If $m = 0$ then this farmer always behaves as in the NM and we have:

  $w^* \neq w$ and (w^*, L) depend on A.

- If $m > 0$ then there are two possibilities:
  - if $w^* < w$ or (w^*, L) depend on A then this farmer behaves as in the NM
  - if $w^* = w$ or (w^*, L) do not depend on A then this farmer behaves as in the SM.

Since m is observed from the data, farmers with $m = 0$ can be easily assigned to the NM. The problem only arises for farmers with $m > 0$, as we do not know the correct model for them. In the literature, the right model for these farmers are identified from the relationship between (w^*, L) and A (in the first set of tests) or the relationship between w^* and w (in the second set of tests). In the following sections, these relationships are referred to as the "theoretical model summary".

Note that the imperfections are introduced into the model as the lower and upper constraints on the market labor ($0 \le m \le M$). These imperfections are chosen since they are cited in several studies about agricultural households (Jacoby 1993, Scoufias 1994, Sonoda and Maruyama 1999, Abdulai and Regmi 2000 among others). For other imperfections such as credit constraint, transactions or search costs, work location preferences, it is not difficult to show that the above theoretical model summary is basically unchanged. Interested readers can look at Singh et. al. (1986) for a survey of this agricultural household model and Janvry and Sadoulet (2006) for recent progress.

Global versus local tests

Most of the tests in the literature are global in the sense that every farmer in the sample is assign to only one model, either SM or NM. These global tests are not appropriate since farmers are different from one another, some suffer from imperfection and hence behave as in the SM while others do not suffer and hence behave as in the NM. The result from a global test, say the SM is the correct model, only implies that farmers in the SM are likely to outnumber farmers in the NM. This information is useful but what we really want to know is who is in the SM and who is in the NM so that the behavior of each farmer in the sample can be modeled accurately. Therefore, the appropriate test should be a local test which is able to identify the correct model for each farmer.

Review of the tests in the literature

We have seen from the theoretical model summary that the shadow wage is the key variable to identify the correct model. In the SM, $w^* = w$ while in the NM $w^* \neq w$ and w^* depends on A, so Benjamin (1992) started by assuming the following function for the shadow wage: $w^* = w(1+\alpha_1 A)$ where $\alpha_1 = 0$ for the SM and $\alpha_1 \neq 0$ for the NM[1]. Using this function and a Cobb-Douglas production function, Benjamin derives this regression for the farm labor demand[2]:

$$\log(L) = \log(\beta_0) + \beta_1 \log(p) + \beta_2 \log(p_z) + \beta_3 \left(\log(w) + \alpha_1 A\right) + \beta_4 \log(F)$$

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[1] In the specification of the shadow wage: $w^* = w(1+\alpha A)$, all exogenous variables in the model, including the fixed input F, should be included as arguments. However, as the aim of the test is to study the relationship between L and A, it is sufficient to include only w and A (Benjamin 1992, page 303).

[2] The farm labor demand is derived from maximizing the farm profit: $pQ - w^* L - p_z z$ where z is a vector of variable inputs, Q is a Cobb-Douglas function: $Q = \alpha L^{\alpha_L} F^{\alpha_F} z^{\alpha_z}$. Also, in the transformation $\log(w^*)$ or $\log\left[w(1+\alpha A)\right]$ is first-order approximated by $\log(w) + \alpha A$.

where p_z is a vector of the variable input prices. Due to the possible measurement error in w, instruments are necessary to consistently estimate this regression.

The test for the SM is whether $\alpha_1 = 0$, so the correct model is identified from the relationship between (w^*, L) and A. Using a sample of rural farmers with $m > 0$ in Java, Benjamin concluded that the SM is the correct model for every farmer.

Jacoby (1993) proposed a different test. He notices from (1) that w^* can be calculated from the marginal product of labor (MPL), so a production function, either Cobb-Douglas or Translog, is estimated first to find the MPL and then the following regression is performed to compare w^* and w:

$$w^* = b_0 + b_1 w$$

Like the Benjamin test, instruments are necessary in this regression due to the possible measurement error in w. The test for the SM is whether $b_0 = 0$ and $b_1 = 1$, so the correct model is identified from the relationship between w^* and w. Using a sample of rural farmers with $m > 0$ in Peru, Jacoby concludes that the NM is the correct model for every farmer.

Based on the Jacoby test, Lambert and Magnac (1994) were the first to develop a local test which is able to identify the correct model for each farmer. The authors also start by estimating the production function and then calculate MPL or w^* as in Jacoby paper. However, they also collect the standard error of w^* for each farmer. The innovation of the paper is that instead of regressing w^* on w, it directly compares w^* to w for each farmer by the t-test:

$$H_0 : w^* = w$$
$$H_1 : w^* \neq w$$

This is a local test in which a correct model can be found for each farmer. Using a sample of rural farmers with $m > 0$ in Cote d'Ivoire, Lambert and Magnac rejected the SM for 90% of men and 50% of women. However, the test results are not robust to measurement error in w[1]. This can be seen from the null hypothesis: $H_0 : w^* = w$. The t-test on this hypothesis can be performed for each farmer if the information about the standard errors of both w^* and w is available. The authors can calculate the standard error in w^* but not the standard error in w which arises from the measurement error. Unfortunately, the measurement error in w is usually present in most household survey data.

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[1] The results from the Benjamin and Jacoby tests are robust to measurement error in w since they use instruments in the regression.

II. New tests

We start with this production function:

$$Q = L^{\lambda} z_1^{\lambda_1} f(z,F) e^{\varepsilon} \tag{6}$$

where z_1 is one variable input (e.g. fertilizer), z is a vector of other variable inputs (e.g. hired labor, seed), $f()$ is a function, e is the natural base logarithm and ε is the random weather shock which is normalized to have $E\left(e^{\varepsilon}\right) = 1$ [1].

This production function is similar to the commonly used Cobb-Douglas function. However, it is more flexible since there are no assumptions on the functional form of $f()$ or on the factors to be included in z and F. That means the functional form for $f()$ can be anything (e.g. Cobb-Douglas, Translog or something else) and the factors in z and F can be anything as well. Additionally, both $f()$ and F can vary across households. For example, $f()$ can be a Cobb-Douglas function for farmers in this region but a Translog function for farmers in the other region due to the differences in weather conditions[2]. Finally, this production function is not estimated, so we can avoid the well-known endogeneity problem that is usually associated with the estimation of the production function.

Due to random weather shocks the farmer does not know the real output Q, so his MPL is based on his expectation about Q:

$$\begin{aligned} w^* &= MPL \\ w^* &= p\frac{\partial E(Q)}{\partial L} = p\lambda\frac{Qe^{-\varepsilon}}{L}e^{\kappa_1} \end{aligned} \tag{7}$$

where κ_1 is added to represent any possible measurement error.

And the input z_1 will be used until its marginal product equals its price (p_1):

$$p_1 = p\frac{\partial E(Q)}{\partial z_1} = p\lambda_1\frac{Qe^{-\varepsilon}}{z_1}e^{\kappa_2} \tag{8}$$

where κ_2 is any possible measurement error.

[1] Like previous papers, household labor and hired labor enter the production function separately.
[2] Different weather conditions usually require different production technologies.

For farmers with $m > 0$, we know the following from the theoretical model summary:

$$\begin{aligned} &\text{SM: } w^* = we^{\mu} \\ &\text{NM: } w^* < we^{\mu} \end{aligned} \tag{9}$$

where μ is any possible measurement error.

And following Benjamin, this function is assumed for the shadow wage[1]:

$$w^* = \left(1 + \alpha_1 A\right) we^{\mu} \tag{10}$$

where $\alpha_1 = 0$ for the SM and $\alpha_1 \neq 0$ for the NM.

Using information from (7) to (10), several equations can be derived to develop new tests:

First, taking logs on both sides of (7) and using (9) to have:

$$\begin{aligned} &\text{SM: } \log\left(pQ/wL\right) = -\log\left(\lambda\right) + \xi_1 \\ &\text{NM: } \log\left(pQ/wL\right) < -\log\left(\lambda\right) + \xi_1 \end{aligned} \tag{11}$$

where $\xi_1 = \mu + \varepsilon - \kappa_1$

Second, dividing (7) by (8) and then using w^* in (10) gives us[2]:

$$\log\left(p_1 z_1/wL\right) = \alpha_0 + \alpha_1 A + \xi_2 \tag{12}$$

where $\xi_2 = \mu + \kappa_2 - \kappa_1$, $\alpha_0 = -\log\left(\lambda_1/\lambda\right)$, $\alpha_1 = 0$ for the SM and $\alpha_1 \neq 0$ for the NM.

The following section presents two global tests and the subsequent section presents a local test based on the information in (11) and (12).

Global tests

The first global test comes directly from the regression of (12):

$$\log\left(p_1 z_1/wL\right) = \alpha_0 + \alpha_1 A + \xi_2 \tag{13}$$

If α_1 is significantly different from zero then the SM is rejected for everyone in the sample. This test is similar to the Benjamin test because the correct model is identified from the relationship between the preference shifters (A) and the production decisions in the farm. In the following paragraphs, this global test will be referred to as the modified Benjamin test.

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[1] A more general function of the shadow wage will be considered later.

[2] In the transformation, $\log\left(1 + \alpha_1 A\right)$ is first-order approximated by $\alpha_1 A$, similarly done in Benjamin.

The second global test uses the information in (11) which can be rewritten as:

$$\log\left(pQ/L\right) = -\log\left(\lambda\right) + \lambda_1 \log\left(w\right) + \xi_1 \tag{14}$$

where $\lambda_1 = 1$ for the SM and $\lambda_1 \neq 1$ for the NM.

Due to the measurement error in market wages, instruments are necessary to consistently estimate this regression. If λ_1 is significantly different from 1 then the SM is rejected for everyone in the sample. This test is similar to the Jacoby test because the correct model is identified from the relationship between the shadow wages and the market wages (since (14) is derived from (11) which originally comes from (9)). In the following paragraphs, this global test will be referred to as the modified Jacoby test.

Compared to the Benjamin and Jacoby tests, these two modified tests can avoid the regression of the production function (in Jacoby test) and the farm labor demand function (in Benjamin test). Thus, we can avert problems usually associated with these regressions such as unobserved farm characteristics and endogeneity problem. Also, the production function in (6) is more flexible than the commonly used Cobb-Douglas function. However, like any global test, these tests are unable to allow for heterogeneity across the farmers. If these global tests reject the SM then this result only implies that farmers in the NM are likely to outnumber farmers in the SM. In order to know who belongs to what model, we need to develop a local test, which is the main contribution of the paper.

Local test

The method for this test comes from the structural break literature in time-series econometrics, which dates back to Chow (1960). The purpose of this method is to identify whether there is any change in a vector of parameters (β) in this regression:

$$y_t = \beta X_t + e_t$$

where *t* is the index for time, *t* = $\{1, T\}$ and T is the sample size, X is a vector of independent variables and y is the dependent variable.

Given time-series data, Chow splits the sample into two sub-samples at a breakdate (T_1): one sub-sample from 1 to T_1 and the other from $T_1 + 1$ to T. Then the parameters are estimated separately for each sub-sample: $\beta = \beta_1$ for the first sub-sample and $\beta = \beta_2$ for the second sub-sample. Finally, the equality between β_1 and β_2 is tested using the classic *F* statistic. The limitation of the Chow test is that the breakdate must be known priori.

In the 1990s, Andrew (1993) and Bai (1997, 1998) provided an elegant test for an unknown breakdate. Although technically complicated, the testing procedure is quite simple. First, the sample is split into two sub-samples at a breakdate. Second, the parameters in the two sub-samples are estimated separately by ordinary least squares (OLS)[1]. Also, the Chow statistic and the full-sample sum of squared errors (*sse* $=\sum e_t^2$) are collected at this step. Third, the first two steps are repeated for every possible breakdate in the sample by moving the breakdate along the time dimension. Finally, the largest Chow statistic will be compared to a critical value presented in Andrew (1993). If it is larger than the critical value, then the stability in the parameters is rejected. The breakdate that gives the smallest *sse* is the structural break[2]. All observations before the structural break belong to one sub-sample with $\beta = \beta_1$ and all observation after the structural break belong to another sub-sample with $\beta = \beta_2$.

Note that parameters are assumed to change at the structural break, which is one particular point in the time dimension. In fact, there is usually a transitional period where the parameters gradually change from β_1 to β_2, and so the observations in this period do not fit well to any sub-samples, either before or after the structural break. However, as long as this transitional period is reasonably short, all observations before the structural break can be assigned to one sub-sample and all observations after the break can be assigned to another sub-sample.

Now we go back to our test of the separation hypothesis with cross-section data. There is a similar structural break issue in (12). We want to divide the sample into two sub-samples (separation and non-separation) with a structural change in a vector of parameters (α_1). These parameters change from zeros in the SM to some values different from zeros in the NM. The problem is that while the search for the structural break can be easily implemented in time-series data by moving the breakdate along the time dimension, for cross-section data this time dimension does not exist.

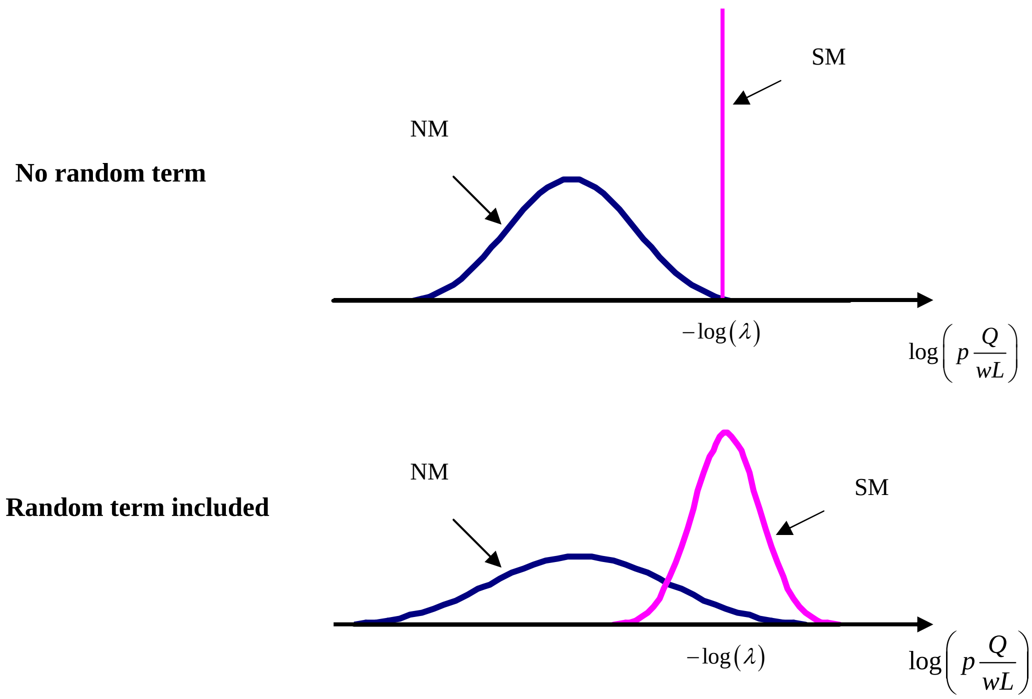
The question is how to set up a search dimension in cross-section data which can act like a time dimension in time-series data where the breakpoint (the breakpoint instead of breakdate is used for cross-section data) can move along this dimension and then the whole sample can be split into two sub-samples: one for the SM and the other for the NM. This question can be answered by using information in (11).

If there is no random term ($\xi_1 = 0$) in (11) then $\log\left(pQ/wL\right)$ for the farmers in the SM will focus at a single point, which is $-\log\left(\lambda\right)$, and $\log\left(pQ/wL\right)$ for the farmers in the NM will spread over every

[1] Some other regression methods are possible.
[2] The breakdate that gives the smallest *sse* can differ from the one that gives the largest Chow statistic (Hansen 2001, page 121).

point below $-\log(\lambda)$. The upper panel of Figure 2 displays a hypothetical histogram of $\log(pQ/wL)$ in this case. The spike at $-\log(\lambda)$ is the location of the farmers in the SM.

***Figure 2**: Hypothetical histogram of* $\log(pQ/wL)$



If the random term is included ($\xi_1 \neq 0$), $\log(pQ/wL)$ for farmers in the SM, instead of focusing at a single point, will be distributed around $-\log(\lambda)$. On the other hand, $\log(pQ/wL)$ for farmers in the NM will spread not only below $-\log(\lambda)$ but some may go over $-\log(\lambda)$. The lower panel of Figure 2 displays a hypothetical histogram of $\log(pQ/wL)$ in this case. As a result of the random term, there are some overlaps between the SM and NM.

We can see that farmers with a low value of $\log(pQ/wL)$ are likely to belong to the NM and farmers with a high value of $\log(pQ/wL)$ are likely to belong to the SM. Thus the sorted $\log(pQ/wL)$ can be used as a search dimension for our separation hypothesis test. The breakpoint can move along this dimension to search for the structural break. Farmers with $\log(pQ/wL)$ below the structural break belong to the NM while farmers with $\log(pQ/wL)$ above the break belong to the SM.

Because of the overlap, it is quite possible for some farmers below the structural break to actually belong to the SM. Vice versa, it is possible for some farmers above the structural break to actually belong to the NM. The accuracy of the test depends on this overlap, and the results cannot be considered reliable if the overlap is not small. This overlap is similar to the transitional period in time-series data. The observations in the overlap or the transitional period do not fit well with any sub-samples. In time-series data, it is reasonable to assume that the transitional period lasts for a relatively short period of time, but for cross-section data, the overlap cannot be assumed small since the random term, the cause of the overlap, can have a big variance. The empirical section below will examine the overlap and show that the overlap is relatively small based on our sample of Indian farmers.

Testing procedure

Having sorted $\log\left(pQ/wL\right)$ as the search dimension, the above Andrew and Bai testing procedure can be used to identify parameters changes in (12). Here is the summary of the procedure:

Step 1: Calculate $\log\left(pQ/wL\right)$ for every farmer and then sort the data by $\log\left(pQ/wL\right)$. Let N_1 be the breakpoint which moves from 5% to 95% of the sample size (N), a trimming level of 5%[1]. Due to this breakpoint, the sample is split into two sub-samples: one from 1 to N_1 and the other from N_1+1 to N .

Step 2: For each breakpoint N_1 we estimate (12) separately for the two sub-samples by OLS, and then have two sets of estimated parameters: $\alpha^N=\left(\alpha_0^N,\alpha_1^N\right)$ for the sub-sample from 1 to N_1 , and $\alpha^S=\left(\alpha_0^S,\alpha_1^S\right)$ for the sub-sample from N_1+1 to N . The superscript N and S stand for non-separation and separation respectively. The Chow statistic and the full-sample sum of squared errors (sse) are also collected at this step:

$$\text{Chow statistic} = \left(\alpha^N-\alpha^S\right)'\left(\text{cov}\left(\alpha^N\right)+\text{cov}\left(\alpha^S\right)\right)^{-1}\left(\alpha^N-\alpha^S\right)$$

$$sse=\sum_{i=1}^{N_1}\left(\log\left(\frac{p_{1i}z_{1i}}{w_iL_i}\right)-\alpha_0^N-\alpha_1^N A_i\right)^2+\sum_{i=N_1+1}^{N}\left(\log\left(\frac{p_{1i}z_{1i}}{w_iL_i}\right)-\alpha_0^S-\alpha_1^S A_i\right)^2$$

Step 3: Pick the largest Chow statistic and compare it to the critical value presented in Andrew (1993). If the statistic is larger than the critical value, then there are structural changes in the parameters and the structural break happens at the breakpoint with the lowest *sse*. At this structural break the sample is split into two sub-samples: the one below the structural break belongs to the NM and the other above the structural break belongs to the SM.

[1] Trimming is necessary to ensure that there are enough observations in each sub-sample to estimate the parameters. In papers using time series data with a limited number of observations, the trimming levels can be as high as 10%, 15% or 20%.

III. Data and Results

The data comes from the 1997-98 survey of living conditions of two states in India, Uttar Pradesh (UP) and Bihar. The survey questionnaire is modeled after the World Bank's Living Standard Measurement Study surveys. Data were collected through household and village level questionnaires in 120 villages, which are drawn from a sample of 25 districts in UP and Bihar[1].

A total of 2250 households were interviewed during the survey. After eliminating missing values and households with no one working on the farm, there is a sample of 1513 households with 2378 farmers. A farmer is defined as a person who is 18 years or older and who has some hours working on his own farm.

Based on the observed m the farmers in the sample can be divided into two groups. The first group includes 1417 farmers with $m = 0$, and the second group includes 961 farmers with $m > 0$. As mentioned in the theoretical model, the farmers in the first group are always assigned to the NM. The problem only arises for the farmers in the second group. We do not know who should be assigned to what model. The tests will focus on this group. Before presenting the test results, the following will talk about the calculation of variables in (11) and (12).

In (11) the value of the farm output (pQ) for each household is calculated from the sum of every farm product produced by the household. The farm labor (L) is the sum of the farm labor of every individual in the household[2]. While farm output and farm labor are calculated at the household level, the market wage (w) is measured at the individual level. For farmers with more than one job in the labor market, their market wages are calculated from the average of all the jobs. In (12) fertilizer is chosen as the variable input (z_1) since most farm households use this input in production and the value of this input ($p_1 z_1$) is readily available in the data. We can see from the calculation of these variables that measurement and reporting errors are usually inevitable. Following the Benjamin test, the variables in the preference shifter (A) include log(household size), the number of adult males, the number of adult females and the number of children. The sensitivity of the results to this choice of variables will be analyzed later. These shifters are usually recorded without measurement or reporting errors. Table 1 provides descriptive information about these variables.

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[1] The data is weighted so it is important to take the weight into estimation to avoid bias.

[2] The test results are almost unchanged when this sum of farm labor is adjusted for the difference between men and women: $L = L_{men} + rL_{women}$ where r is an adjustment factor which can be calculated from market wage ratio between women and men.

***Table 1**: Descriptive information about the variables*

Variable	*Description*	*Unit*	*Mean*	*Std.Dev.*
pQ	Household farm income	Rupee	19446.3	25672.8
L	Household farm labor	Hours	1775.1	1880.9
$p_1 z_1$	Value of fertilizer	Rupee	1759.4	2015.2
w	Market wage	Rupee/hour	8.1604	14.6673
A includes:				
Household size		Person	7.1450	3.9068
Number of adult males		Person	1.9116	1.0747
Number of adult females		Person	1.7380	1.0031
Number of children		Person	3.3797	2.1371

The global test results

For the modified Benjamin test, the regression in (13) is estimated and the separation hypothesis is rejected if the estimated coefficients are significantly different from zero. For the modified Jacoby tests, the regression in (14) is estimated and the separation hypothesis is rejected if the estimated coefficient is significantly different from 1. The sample for both regressions is limited to farmers with $m > 0$ (961 observations). Table 2 shows the results of these regressions.

***Table 2**: The global tests*

Explained variables: $\log\left(p_1 z_1 / wL\right)$ in the modified Benjamin test

$\log\left(pQ/L\right)$ in the modified Jacoby test

Modified Benjamin test

Explanatory variables	*Coefficients*
Log(household size)	0.0405 (0.1354)
Number of adult males	-0.1333*** (0.0353)
Number of adult females	0.0758* (0.0406)
Number of children	0.0074 (0.0231)
Constant	-1.9543*** (0.1405)
P-value: F-test for all coefficients (excluding constant) to be zero	0.0013

Modified Jacoby test

Explanatory variables	*Coefficients*
Log(w)	0.6684*** (0.0573)
Constant	1.0041*** (0.1063)
P-value: t-test for the coefficient on Log(w) to be 1	0.0000

Note: The standard errors are in the parentheses.
*** and * imply a significant difference from zero at 1% and 10% respectively.

In the modified Benjamin test, the coefficients of the number of adult males and the number of adult females are significantly different from zero. The F-test for all coefficients (excluding constant) to be zero strongly rejects the null hypothesis. In the modified Jacoby test, the coefficient on the Log(market wage) is significantly different from 1. Therefore, both global tests strongly reject the separation hypothesis for this sample of Indian farmers.

Because of heterogeneity across farmers, these results only imply that farmers in the NM probably outnumber farmers in the SM. In order to know who belongs to what model, we need to perform the local test.

The local test results

Although the test can be limited to farmers with $m > 0$ as in the global tests, there are two reasons to use all farmers in this local test. First, in this sample of Indian farmers there are as many as 1417 farmers with $m = 0$, so we can lose a significant number of observations by dropping them from the sample. Second, we know from the theoretical model that these farmers behave as in the NM. This is useful information and should be used to improve the power of the local test.

However, these farmers do not have wage information which is required to conduct the test (both (11) and (12) require w). In order to impute the wages for this group, a Mincerian wage regression is estimated using farmers with wage information: $\log(w) = \beta X$ where X is a vector of explanatory variables including education level, age, age squared, gender, district dummy variables and a constant. Table 3 shows the results of the Mincerian wage regression. All coefficients are significant with the expected sign. Men earn more than women. Education increases wages. Age does as well, but the effect is diminishing.

Table 3*: Mincerian wage regression*

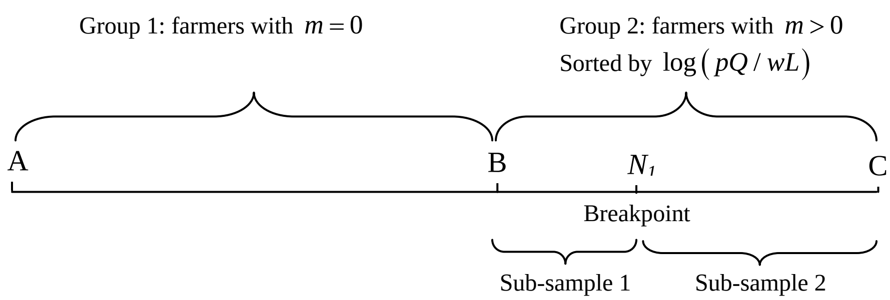
Explained variable: *Log(w)*

Explanatory variables	*Coefficients*
Gender	0.3474*** (0.0732)
Education	0.1086*** (0.0117)
Age	0.0457*** (0.0117)
Age squared	-0.0005*** (0.0001)
Constant	0.8474*** (0.2788)

Note: The standard errors are in the parentheses. Dummy variables for districts are included in the regression but not shown here. The number of observations is 961.

Since all farmers in the sample are used in the test, the above testing procedure for the local test needs to be adjusted slightly. First of all, the whole sample is separated into two groups: group 1 includes farmers with $m = 0$ (1417 farmers) and group 2 includes farmers with $m > 0$ (961 farmers). In step 1 of the testing procedure, farmers in group 2 are sorted by $\log(pQ/wL)$. Given a breakpoint N_1, this group is split into two sub-samples: sub-sample 1 below the breakpoint and sub-sample 2 above the breakpoint. The division of farmers into groups and sub-samples is shown in Figure 2.

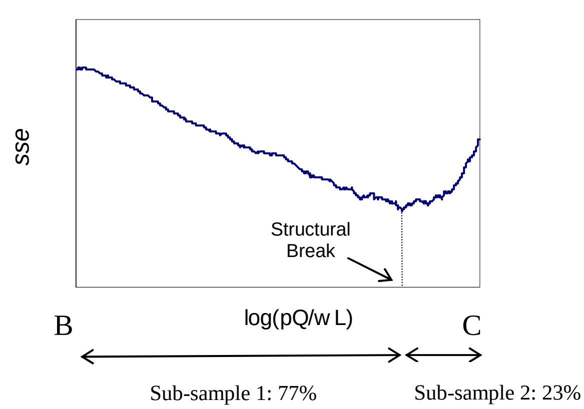
Figure 2: *Division of farmers in the data*



The segment AB includes farmers in group 1 with $m = 0$, who we already know always behave as in the NM. The segment BC includes farmers in group 2 with $m > 0$ for whom we want to know the correct model for each of them. The breakpoint (N_1) moves along BC and splits group 2 into two sub-samples. Sub-sample 1 with lower values of $\log(pQ/wL)$ is likely to belong to the NM while sub-sample 2 with higher values of $\log(pQ/wL)$ is likely to belong to the SM.

In step 2, two OLS regressions are estimated: one for farmers in sub-sample 1 and group 1 (from A to N_1) and the other for farmers in sub-sample 2 (from N_1 to C). Given these two regressions, the Chow statistic and the *sse* for each breakpoint can be calculated. The results are that the highest Chow statistic is 677 which is much higher than the critical value of 24.2 in Andrews (1993), so the stability in the parameters is strongly rejected. The values for the *sse* is presented in Figure 3.

Figure 3: *sse shape*



The structural break occurs at the breakpoint that gives the smallest *sse*. The result is that for farmers with $m > 0$, 23% are assigned to sub-sample 1 which belongs to the SM (80% are men, 20% are women) and 77% are assigned to sub-sample 2 which belongs to the NM (84% are men, 16% are women). The farmers in the NM clearly outnumber those in the SM, which is consistent with the results from the global tests.

Some may worry about the sensitivity of the structural break to the choice of the explanatory variables in the regression (12). Recall that these explanatory variables come from the specification of the shadow wage function in (10). Following the Benjamin test, the shadow wage is specified as a function of the preference shifters (A) only. In principle, the shadow wage should be a function of all exogenous variables in the model, so it should include not only preference shifters but also the fixed inputs (F) as follows: $w^* = \left(1 + \alpha_1 A + \alpha_2 F\right) w e^{\mu}$. Then, the regression in (12) becomes:

$$\log\left(p_1 z_1 / wL\right) = \alpha_0 + \alpha_1 A + \alpha_2 F + \xi_2$$

Given the data availability, land areas and land quality are included in the regression to represent F. The test is redone with this new regression equation, but the results are almost unchanged. The stability in the parameters is rejected and a similar portion of the farmers is assigned to the SM and NM. Also, other choices of the variables in A produce similar results. Thus, the test results are not sensitive to the choice of the explanatory variables in the regression.

Is $\log\left(pQ/wL\right)$ a good search dimension?

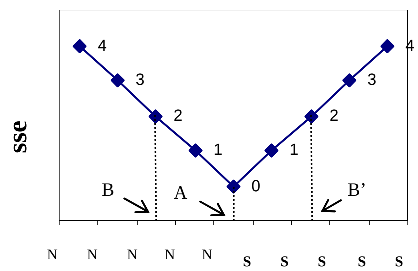
The search dimension is crucial to separate farmers into the SM and the NM. The test results strongly depend on it. In a good search dimension, there is a small overlap between the SM and NM. Most farmers, who are assigned to a model (e.g. SM) by the local test, truly belong to the model, while only a small number of them belong to the other model. By contrast, in a poor search dimension, there is a big overlap between the SM and NM. The results of the local test become unreliable since many farmers, who are assigned to one model, actually belong to the other.

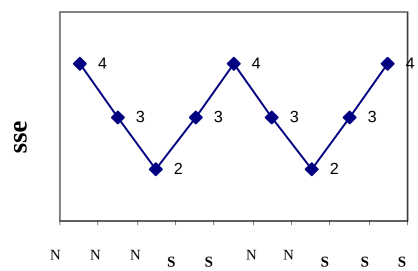
In order to examine the search dimension, we first inspect the shape of the *sse* and second look at the results of the global tests for farmers in the SM and NM separately.

Inspection of the sse shape

The *sse* shape is different between a good and a poor search dimension. Figure 4 clearly demonstrates this point. In this figure, there is a hypothetical sample of 10 farmers: 5 in the SM and 5 in the NM. These farmers are sorted by a search dimension and their positions are shown on the horizontal axis. Farmers in the SM are labeled as "s" and farmers in the NM are labeled as "N". The breakpoint moves along the search dimension to calculate the *sse*. The numbers in the graph (0,1,2,3,4) show the number of wrongly-assigned farmers for the corresponding breakpoint.

Figure 4: *Hypothetical sse shapes*





In a perfect search dimension, there is no overlap as shown on the horizontal axis of the left side of Figure 4: the 5 farmers in the NM are located next to one another, so are the 5 farmers in the SM. The *sse* calculated at the structural break (point A on figure 4) achieves its minimum since there are no wrongly-assigned farmers. If the breakpoint moves away from the structural break by say 2 observations (point B or B' on Figure 4) then 2 farmers are assigned to the wrong model, so the regression errors in (12) for these 2 farmers must be bigger. That means the *sse* calculated at this breakpoint must be greater than the minimum *sse*.

Using the same logic, we can see that when the breakpoint moves away from the structural break, there are more wrongly-assigned farmers and *sse* gets bigger. Therefore, in a perfect search dimension, *sse* will have a V-shape with only one minimum located at the structural break.

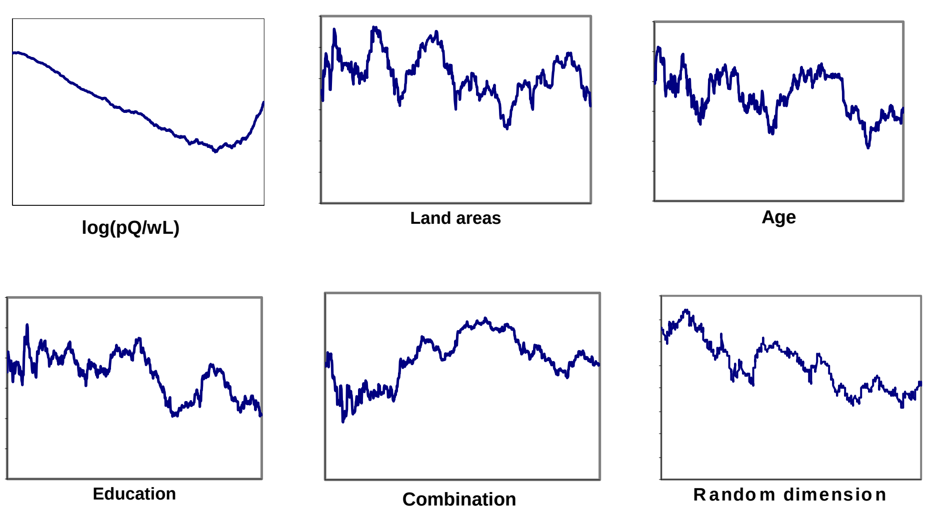
If the search dimension is poor, there is an overlap as shown on the horizontal axis of the right side of Figure 4: the 2 farmers in the SM are not located next to other farmers in the SM, as is the case with the 2 farmers in the NM. As the breakpoint moves along the search dimension from left to right, the number of wrongly-assigned farmers goes down and up and then down and up again, so it causes *sse* to go down and up quite often.

In practice, it is impossible to have a perfect search dimension. However, in a relatively good search dimension with a reasonably small overlap, *sse* still achieves its minimum at the structural break with a minimum number of wrongly-assigned farmers. As the breakpoint moves away from the structural break, there are more wrongly-assigned farmers and *sse* gets bigger, so a V-shape for *sse* is still expected. By contrast, in a poor search dimension, the *sse* shape looks random with many ups and downs.

An equivalent inspection of the sse shape is also conducted in time-series data but the aim is to examine not the search dimension but the number of structural breaks (Bai 1997, Hansen 2001). This equivalence between time-series and cross-section data comes from the fact that the *sse* shape is determined by two factors: the number of the structural breaks and the search dimension. In the separation hypothesis with cross-section data, there is only one structural break so the *sse* shape depends only on the search dimension. That means *sse* shape can be used to separate a good and a poor search dimension. In time-series data, the search dimension, which is the time dimension, is already good (the transitional period is usually short), so the *sse* shape depends only on the number of the structural breaks. That means *sse* shape can be used to identify the number of structural breaks in the sample.

Given this discussion about the *sse* shape, we can conclude from Figure 3 that $\log\left(pQ/wL\right)$ is a good search dimension with a small overlap since it does have a V-shape. Besides $\log\left(pQ/wL\right)$ there are several other choices for the search dimension such as education, age, land area or some combination of these. Taking education as an example, farmers with high education are less likely to be constrained in the labor market (i.e. the upper constraint $m \le M$ is less likely to be binding to them) so they should behave as in the SM. By contrast, farmers with low education are more likely to be constrained in the labor market, so they should behave as in the NM. The breakpoint can move along the education dimension to split the sample into two sub-samples: one for the SM and the other for the NM. The same logic can apply to age and land areas variable. The more the farmers have in these variables the less likely they are constrained in the labor market. In order to know if these variables can be used as a search dimension, we can inspect their *sse* shapes in Figure 5.

Figure 5*: sse shapes for different search dimensions*



Note: In these charts the vertical axis is the *sse*. The variable on the horizontal axis is used as the search dimension

The last chart (random dimension) is the *sse* shape when no variable is used as the search dimension. In this chart, each farmer in group 2 is randomly assigned a unique number from 1 to the size of the group which is 961. The farmers in this group are then sorted by this random dimension and *sse* is calculated by moving the breakpoint along this random dimension.

The second chart on the bottom row of Figure 5 uses a combination of land areas, age and education as the search dimension. The combination is implemented in the following way. First, all variables are standardized to the same mean and variance (e.g. mean=10 and variance=1) and then they are combined using the Frobenius norm: $\sqrt{(land\ area)^2 + (edu)^2 + (age)^2}$. The higher the norm, the more likely the farmer belongs to the SM, and vice versa, the lower the norm the more likely the farmer belongs to the NM.

The results in Figure 5 show that the *sse* shapes of land areas, age, education and combination of these variables look quite random with many ups and downs. These shapes look as random as using the random dimension, so they are obviously not a good search dimension. Only $\log(pQ/wL)$ with a clear V-shape for *sse* is a good one.

Separate global tests for farmers in the SM and NM

Besides inspecting the shape of the *sse*, we can also verify if $\log(pQ/wL)$ is a reliable search dimension by applying global tests to the farmers in the SM and NM separately. In a good search dimension, most farmers assigned to a model (e.g. SM) by the local test truly belong to the model while in a poor search dimension many farmers assigned to this model actually belong to the other model. Using this idea, we can examine the search dimension by applying the global tests separately to farmers in the SM and NM.

For farmers assigned to the SM by the local test, if most of the farmers truly belong the SM then the coefficients in the modified Benjamin test must be zero and the coefficient in the modified Jacoby test must be 1. Table 4 shows the results of these two tests. In the modified Benjamin test, no coefficient is significantly different from zero. Also, the F-test for all coefficients (excluding constant) to be zero cannot reject the null hypothesis. In the modified Jacoby test, the coefficient on $\log(w)$ is not significantly different from 1. Thus both global tests confirm that most farmers, who are assigned to the SM by the local test, truly belong to the SM.

Table 4: *Global tests for farmers in the SM*

Explained variables: $\log\left(p_1 z_1 / wL\right)$ in the modified Benjamin test

$\log\left(pQ/L\right)$ in the modified Jacoby test

Modified Benjamin test

Explanatory variables	*Coefficients*
Log(household size)	0.2565 (0.3978)
Number of male adults	0.0415 (0.0967)
Number of female adults	0.0585 (0.1092)
Number of children	-0.0743 (0.0673)
Constant	-1.1072*** (0.4198)
P-value: F-test for all coefficients (excluding constant) to be zero.	0.3595

Modified Jacoby test

Explanatory variables	*Coefficients*
Log(w)	1.0629*** (0.1680)
Constant	1.9172*** (0.2439)
P-value: t-test for the coefficient on Log(w) to be 1.	0.6401

Note: The standard errors are in the parentheses, *** implies a significant difference from zero at 1% level.

Now, for farmers assigned to the NM by the local test, if they truly belong to the NM then the coefficients in the modified Benjamin test must be different from zero and the coefficient in the modified Jacoby test must be different from 1. Table 5 shows the results of these two tests.

Table 5: *Global tests for farmers in the NM*

Explained variables: $\log\left(p_1 z_1 / wL\right)$ in the modified Benjamin test

$\log\left(pQ/L\right)$ in the modified Jacoby test

Modified Benjamin test

Explanatory variables	*Coefficients*
Log(household size)	-0.0165 (0.1342)
Number of male adults	-0.1144*** (0.0349)
Number of female adults	0.1028*** (0.0401)
Number of children	0.0115 (0.0230)
Constant	-2.0700*** (0.1391)
P-value: F-test for all coefficients (excluding constant) to be zero.	0.0031

Modified Jacoby test

Explanatory variables	*Coefficients*
Log(w)	0.7018*** (0.0537)
Constant	0.7896*** (0.1018)
P-value: t-test for coefficient on Log(w) to be 1.	0.0000

Note: The standard errors are in the parentheses, *** implies a significant difference from zero at 1% level.

In the modified Benjamin test, the coefficients of the number of adult males and the number of adult females are significantly different from zero. Also, the F-test for all coefficients to be zero strongly rejects the null hypothesis. In the modified Jacoby test, the coefficient on $\log(w)$ is significantly different from 1. Therefore, both global tests confirm that most farmers, who are assigned to the NM by the local test, truly belong to the NM.

In summary, based on the inspection of the *sse* shape and the results of the global tests for each sub-sample, we can conclude that $\log(pQ/wL)$ is a reliable search dimension with a small overlap.

Conclusion

This paper offers a new method to test the separation hypothesis. The new test is able to capture heterogeneity across the farmers. The results are not sensitive to the choices of explanatory variables in the regression and are derived from a relatively flexible production function. Also, the search dimension, a crucial factor to identify the right model for each farmer, is shown to be reliable with a small overlap. Future research on the effects of the government policies on farmers' decisions and welfare can use this method to accurately model different behaviors of farmers in the sample.

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