

Capacitated Cell Planning of 4G Cellular Networks

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Abstract

Optimal planning of cellular networks is an important ingredient in the effort to provide advanced cellular services at a reasonable cost. Cell planning includes planning a network of base stations to provide the required coverage of the service area with respect to current and future traffic requirements, available capacities, interference, and the desired QoS. The forthcoming fourth generation (4G) cellular networks are projected to provide a wide variety of new services, including high-quality voice, high-definition video, and high-data-rate wireless channels. Cell planning for these networks requires new approaches, combining appropriate modeling of the technologies dependent characterizations and novel algorithm techniques.

In this paper we provide such an approach; we rigorously model important aspects of the 4G networks, define the budget limited cell planning problem and the minimum-cost cell planning problem, and provide the first approximation algorithms for these problems. We then show how the theoretical results can be used to derive practical planning of cellular networks. Our results indicate that a theoretical study together with optimized practical implementations outperform previous commonly used techniques, and achieve a close-to-optimal planning of real cellular networks.

1 Introduction

Cell planning is an important step in the design, deployment, and management of cellular networks. Cell planning includes planning a network of base stations that provides a (full or partial) coverage of the service area with respect to current and future traffic requirements, available capacities, interference, and the desired QoS. Cell planning is employed not only when a new network is built or when a modification to a current network is made, but also (and mainly) when there are changes in the traffic demand, even within a small local area (e.g., building a new mall in the neighborhood or opening new highways). Cell planning which is capable of responding to local traffic changes and/or to make use of advance technological features at the planning stage is essential for a cost effective design of 4G systems.

Although the detailed structure of 4G systems is as of yet not well defined, there is a clear consensus regarding some of the important aspects of the technologies to be implemented in these systems¹. 4G systems are planned to provide even higher transmission rates and larger capacity than current 3G (IMT-2000 based) systems, both in terms of the number of users as well as in terms of traffic volume. Most likely, 4G systems will be designed to offer bit rates of 100 Mbit/s (peak rate in mobile environment) to 1 Gbit/s (fixed indoors) with a 5 MHz frequency bandwidth.

¹See International Telecommunication Union (ITU) website <http://www.itu.int/home/index.html>.

The system capacity is expected to be at least 10 times larger than current 3G systems. In addition, these objectives should be met together with a drastic reduction in the cost (1/10 to 1/100 per bit) [see [34], for example]. Such high frequencies yield a very strong signal degradation and suffer from significant diffraction resulting from small obstacles, hence forcing the reduction of cell size (in order to decrease the amount of degradation and to increase coverage), resulting in a significantly larger number of cells compared to previous generations. 4G systems will have cells of different sizes: picocells (e.g., an in-building small base station with an antenna on the ceiling), microcells (e.g., urban street, up to 1km long with base stations above rooftops at 25m height), and macrocells (e.g., non-line-of-sight urban macro-cellular environment). Each such cell is expected to service users with different mobility patterns, possibly via different radio technologies. Picocells can serve slow mobility users with relatively high traffic demands. They can provide high capacity coverage with hot-spot areas coverage producing local solutions for these areas. Even though these cells do not have a big RF impact on other parts of the network, they should be taken into consideration during the cell planning stage since covering hot-spot areas may change the traffic distribution. At the same time, microcells and macrocells can be used to serve users with high mobility patterns (highway users) and to cover larger areas. Hence, it is important to be able to choose appropriate locations for potential base stations and to consider different radio technologies, in order to achieve maximum coverage (with low interference) at a minimum cost.

The increased number of base stations, and the variable bandwidth demand of mobile stations, will force operators to optimize the way the *capacity* of a base station is utilized. In contrast to previous generations, the ability of a base station to successfully satisfy the service demand of all its mobile stations is going to be highly limited and will mostly depend on its infrastructure restrictions, as well as on the service distribution of its mobile stations. To the best of our knowledge, we are not aware of any cell planning approach which takes the capacity of base stations into account.

Base stations and mobile terminals are expected to make extensive use of adaptive antennas and smart antennas. In case the system will have the ability to distinguish between different users (by their geographic positions), adaptive antennas will point a narrow lobe to each user, reducing interference while at the same time maintaining high capacity. Smart antenna systems combine an antenna array with a digital signal-processing capability, enabling base stations to transmit and receive in an adaptive, spatially-sensitive manner. In other words, such a system can automatically change the directionality of its radiation patterns in response to its signal environment. This can dramatically increase the performance characteristics (such as capacity) of a wireless system. Hence, future methods for cell planning should be able to include a deployment of smart antennas and adaptive antennas in their optimization process. Note that current advanced tools for cell planning already have capabilities for electrical modifications of tilt and azimuth.

This paper rigorously studies algorithmic aspects of cell planning problems, incorporating the anticipated 4G technologies into cell planning, and presenting new methods for solving these problems. In particular, our approach emphasizes, for the first time, the algorithmic aspects of better utilization of the capacities of base stations - a fundamental optimization parameter in 4G systems. Our new techniques are based on novel modeling of technology dependent characterizations as well as approximation algorithms that provide provable good solutions. In addition, methods presented in this paper are also applicable to current networks and various radio technologies.

1.1 Formulation and background

Consider a set $I = \{1, 2, \dots, m\}$ of possible configurations of base stations and a set $J = \{1, 2, \dots, n\}$ of clients.² Each *base station configuration* (abbreviated *base stations*) containing

²Notice that when planning cellular networks, the notion of “clients” sometimes means mobile-stations and sometimes it represents the total traffic demand created by a cluster of mobile-stations at a given location. In this

the geographical location, typical antenna pattern (as well as its adopted model for propagation), azimuth, tilt, height and any other relevant parameters of a base station antenna that together with the technology determine the coverage area of the antenna and the interference pattern (for example, two such configurations with the same parameters except the tilt will be considered as *different*). Each base station $i \in I$ has capacity w_i , installation cost c_i , and every client $j \in J$ has a demand d_j . The demand is allowed to be simultaneously satisfied by more than one base station. Each base station i has a *coverage area* represented by a set $S_i \subseteq J$ of clients admissible to be covered (or satisfied) by it; this base station can satisfy at most w_i demand units of the clients in S_i . Computing S_i , as a preprocessing stage to the cell planning itself, is based on the physical properties of the antenna, the power setting, terrain information, and the corresponding area-dependant propagation models. An optional way of generating the collection of S_i 's is to list for each base station i the set of clients who “see” i as their “best” server (or either “best” or “secondary”). Such computations are usually done using simulators and are outside the scope of this paper.

When a client belongs to the coverage area of more than one base station, interference between the servicing stations may occur. These interferences are modeled by a penalty-based mechanism and may reduce the contribution of a base station to a client. We denote by $Q(i, j)$ the net contribution of base station i to client j , for every $j \in J$, $i \in I$, after incorporating all relevant interference. We formulate cell planning problems using the abstract notation of $Q(i, j)$ to be *independent* of the adopted model of interference. Interference models are discussed in Section 1.2.

Using this formulation, we define two cell planning problems. The *budgeted cell planning problem* (BCPP) asks for a subset of base stations $I' \subseteq I$ whose cost does not exceed a given budget B and the total number of *fully* satisfied clients is maximized. That is, an optimal solution to BCPP needs to maximize the number of clients for which $\sum_{i \in I'} Q(i, j) \geq d_j$. The *minimum-cost cell planning problem* (CPP) is to find a subset $I' \subseteq I$ of minimum cost that satisfies at least γ of the demands of all the clients, for a given constant $0 < \gamma \leq 1$.

Many special cases of both cell planning problems have been extensively studied in the literature. In most cases the problem is NP-hard and finding an optimal solution for it (when applied to real networks) is infeasible in a reasonable running time. Thus, much of the work deals with only a limited aspect of the cell planning problem, and different heuristics solutions. In this paper we address this hardness using approximation algorithms. A *δ -approximation algorithm* is a polynomial-time algorithm that always finds a feasible solution for which the objective function value is within a proven factor of δ of the optimal solution.

1.2 How to model the interference?

Interference handling is an important issue in planning and management of cellular networks. Basically, interference is caused by simultaneous signal transmissions in different cells (inter-cell). In this section we overview interference in the forthcoming 4G systems and present a new approach of incorporating interference in cell planning.

In narrowband systems (e.g., IS-136, GSM), transmissions within a cell are restricted to separate narrowband channels. Furthermore, neighboring cells use different narrowband channels for user transmissions. This requires splitting of the total bandwidth and reduces the *frequency reuse* in the network. However, the network can now be simplified and approximated by a collection of point-to-point non-interfering links, and the physical-layer issues are essentially point-to-points ones. Notice that since the level of interference is kept minimal, the point-to-point links typically have high signal-to-interference-plus-noise ratios (SINRs).

In contrast, wideband systems design uses the following strategy: all transmissions are spread over the entire bandwidth. The key feature of these systems is *universal frequency reuse*, that is, the

paper we support both forms of representations.

same spectrum is used in every cell. However, simultaneous transmissions can now interfere with each other and links typically operate at low SINRs. Two of the systems that differ, by a design choice, in how the users' signal are spread are CDMA and OFDM. The Code Division Multiple Access (CDMA) system is based on direct-sequence spread-spectrum. Here, users' information bits are coded at a very low rate and modulated by pseudonoise sequences. In such systems (e.g., IS-95), simultaneous transmissions, both intra-cell and inter-cell, cause interference. In the Orthogonal Frequency Division Multiplexing (OFDM) system, on the other hand, users' information is spread by hopping in the time-frequency grid. Here, the transmissions within a cell is kept orthogonal but adjacent cells share the same bandwidth and inter-cell interference still exists. This system has the advantage of the full frequency reuse of CDMA while retaining the benefits of the narrowband system where there is no intra-cell interference (see Chapter 4 in [32]).

Interference is typically modeled, for cell planning proposes, by an *interference matrix* which represents the impact of any base station on other base stations, as a result of simultaneous coverage of the same area (see Appendix 6B in [9]). Next, we generalize this model to also include the geographic position of this (simultaneous) coverage.

Let P be an $m \times m \times n$ matrix of *interference*, where $p(i_1, i_2, j) \in [0, 1]$ represents the fraction of i_1 's service which client j loses as a result of interference with i_2 (defining $p(i, i, j) = 0$ for every $i \in I$, $j \in J$, and $p(i, i', j) = 0$ for every $j \notin S_{i'}$)³. This means that the interference caused as a result of a coverage of a client by more than one base station depends on the geographical position of the related "client" (e.g., in-building coverage produces a different interference than a coverage on highways using the same set of base stations). As defined above, $Q(i, j)$ is the contribution of base station i to client j , taking into account the interference from all relevant base stations. We describe here two general models for computing $Q(i, j)$.

Let x_{ij} be the fraction of the capacity w_i of base station i that is supplied to client j . Recall that $I' \subseteq I$ is the set of base stations selected to be opened, the contribution of base station i to client j is defined to be

$$Q(i, j) = w_i x_{ij} \cdot \prod_{i' \neq i: i' \in I'} (1 - p(i, i', j)). \quad (1)$$

Notice that, as defined by the above model, it is possible for two distinct base stations, say α and β , to interfere with each other within a geographical area, namely "client" j (i.e., $p(\alpha, \beta, j) > 0$), although $j \notin S_\beta$.

Since (1) is a high-order expression we use the following first-order approximation, while assuming that the p 's are relatively small,

$$\prod_{i' \in I'} (1 - p(i, i', j)) = (1 - p(i, i'_1, j))(1 - p(i, i'_2, j)) \dots \approx 1 - \sum_{i' \in I'} p(i, i', j). \quad (2)$$

Combining (1) and (2) we get

$$Q(i, j) \approx \begin{cases} w_i x_{ij} (1 - \sum_{i' \neq i \in I'} p(i, i', j)), & \sum_{i' \in I'} p(i, i', j) < 1 \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

Consider, for example, a client j belonging to the coverage areas of two base stations i_1 and i_2 , and assume that just one of these base stations, say i_1 , is actually participating in j 's satisfaction (i.e., $x_{i_1 j} > 0$ but $x_{i_2 j} = 0$). According to the above model, the mutual interference of i_2 on i_1 's contribution ($w_1 x_{i_1 j}$) should be considered, although i_2 is not involved in the coverage of client j .

In most cellular wireless technologies, this is the usual behavior of interference. However, in some cases a base station can affect the coverage of a client *if and only if* it is participating in its demand satisfaction. The contribution of base station i to client j in this case is defined by

³For simplicity, we do not consider here interference of higher order. These can be further derived and extended from our model.

$$Q(i, j) \approx \begin{cases} w_i x_{ij} \left(1 - \sum_{i' \neq i \in I_j} P(i, i')\right), & \sum_{i' \neq i \in I_j} P(i, i') < 1 \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

where I_j is the set of base stations that participate in the coverage of client j , i.e., $I_j = \{i \in I : x_{ij} > 0\}$. Notice that in this model the interference function does not depend on the geographic position of the clients.

1.3 Our contributions

In this paper we study two of the most important cell planning problems in the perspective of future 4G cellular networks: BCPP and CPP. We combine a theoretical study of approximation algorithms for these problems with a simulation study of the practical effectiveness of these algorithms when compared to the current state of the art.

The budgeted cell planning problem. We present the first study of BCPP. We show, in Section 3.1, that approximating BCPP is NP-hard. Then we define a restrictive version of BCPP, the $k4k$ -budgeted cell planning, by making an additional assumption which is valid in most practical scenarios. The additional property is that every set of k -opened base stations can fully satisfy at least k clients, for every integral value of k . In Section 3.2 we show that this problem remains NP-hard and, through a rigorous study of the optimal solution, we present, in Section 3.4.3, an $\frac{e-1}{3e-1}$ (≈ 0.240) factor approximation algorithm for this problem.

From a theoretical point of view, our study contains two independent results on the *client assignment problem* (CAP) and the *budgeted maximum assignment problem* (BMAP). Given a set of base stations with its capacities, and a set of clients with its demands, the client assignment problem asks for the maximum number of clients that can be satisfied each by exactly one base station. Notice that in this problem we consider the set of base stations as (already) opened and no installation cost is specified. We show, in Section 3.4.1, that this problem is NP-hard and can be approximated within a factor of $1/2$ of the optimum. The second problem, BMAP, is a generalization of CAP, and assume that every base station has an installation cost, and a general budget is also given. A $\frac{1}{2}(1 - \frac{1}{e})$ -approximation algorithm is presented in Section 3.4.2 for this problem. This algorithm generalizes the result of the well-known study on the budgeted maximum coverage problem [19] in the sense that the problem considered in [19] does not include, among others, neither capacities nor non-uniform demands.

The minimum-cost cell planning problem. We present an $O(\log W)$ -approximation algorithm for the non-interference (i.e., P is the zero matrix) case of the CPP, where W is the largest capacity over all base stations selected for opening. To the best of our knowledge this is the first approximation algorithms for this special case of the problem. This generalizes the results for the hard capacitated set cover problem [10] in the case where both hard capacities and non-uniform demands are present .

Simulation results and practical aspects. In order to verify that our algorithms for CPP perform well in practice, two different simulation sets were conducted with scenarios relevant to 4G technologies (Section 5). Each of these simulations has the goal of minimizing the total cost and minimizing the total number of antennas (sectors). In both simulation sets our results indicate that the practical algorithms derived from our theoretical scheme can generate solutions that are very close to the optimal solutions and much better than the proved worst-case theoretical bounds. Moreover, our algorithms achieve a significantly better lower bound on the solution cost than that achieved by the commonly used greedy approaches [29, 33].

2 Previous work

Cell planning is one of the most studied problems in the context of optimization of cellular networks. Previous works dealt with a wide variety of special cases (e.g., cell planning without interference, frequency planning, uncapacitated models, antenna-type limitations, and topological assumptions regarding coverage) and the objectives are mostly of minimum-cost type. The techniques used in these works range from meta-heuristics (e.g., genetic algorithms, simulated annealing, etc.) [6,13,18,21,23–27,31,37] and greedy approaches [2,3,28,33,38], through exponential-time algorithms that compute an optimal solution [22], to approximation algorithms for special cases of the problem [1,16,17,22]. A comprehensive survey of various works on cell planning problems appears in [7] and a comparison between optimization methods for cell planning of 3G systems appears in Chapter 14 of [29].

A solution for an uncapacitated version of CPP was presented by Hurley in [18]. In his paper, Hurley [18] uses local improvements simulated annealing procedures to optimize multi-objective function (full coverage, minimum total cost of sites, lower bound on the total satisfied traffic demands, minimum interference level, and upper bound on the number of handovers permitted). However, base stations are assumed to have unlimited capacity and no theoretical analysis or guarantee on the quality of the solutions is given.

A restricted approach for cell planning is described in a series of papers (e.g., [25,27], and [26]). These papers are characterized by an integer programming formulation for various optimization problems: planning location for a given number of base stations, penalizing multiple coverage, minimizing interference between base stations, counting reused frequencies, minimizing the number of blocked channels, minimizing the number of blocked downlink connections, and maximizing the uniquely served traffic. Optimization of these objectives is done via simulated annealing and branch-and-bound heuristics. Unfortunately, most of the solutions for these problems cannot be applied for 4G technologies.

The problem of planning location for base stations (BSLP) is one of the most studied cases CPP. In this problem clients are usually assumed to have unit demand and base stations are assumed to have an installation cost and ability to cover any number of clients (from their coverage area). The objective is, in general, to minimize the total cost of opening base station in order to provide connectivity to all the clients. A local improvement procedure using a naive simulated annealing is used in [31]. In their work, no specific model is used and interferences are not supported (although the work is addressing wideband cellular technologies). Genetic algorithms are used in [21,23] and [37] to solve this problem. In these papers multi-objective optimization is performed, no interferences are assumed [21], and wideband cellular technologies are not supported ([23], and [37]). Tabu search approach is used in [24] and [6] for an integer programming formulation of the problem.

Greedy algorithms for combinatorial formulations of BSLP are presented in [28,33,38]. In these papers interference is not supported, no traffic demands are assumed (i.e., clients are assumed to have unit demand), and no performance guarantee is analyzed (or claimed).

Approximation algorithms for BSLP are used in [1,16], and [17]. A PTAS for two combinatorial problems are presented in [16] and [17]: Maximizing the number of totally satisfied mobile clients, and minimizing the number of installed base stations, while a bicriteria $O(\log n, \log n)$ -approximation algorithm is described in [1] for BSLP. In these papers many restrictive assumptions are made: the model is limited to only a metric space (which is not necessarily true in practice), no traffic demands are supported, and base stations are assumed to have an unlimited capacity of coverage. In addition, in [16] and [17] it is assumed that only omnidirectional pattern antennas are used, and no location of base stations is allowed below a minimal distance between these base stations (which is also not practical as in the case of installing several antennas on one site).

An exact $2^{O(\sqrt{m} \log m)}$ -time algorithm is described in [22] for the base station location problem.

No interferences are assumed by this somewhat efficient exponential algorithm, while topological restrictions are also adopted (geographical distances are in a Euclidean space and omnidirectional patterns of the same radius are assumed for all antennas).

Base station location problems together with frequency planning are addressed in [13]. This problem is solved via Lagrangian relaxation based heuristic, an integer programming formulation, and then performing local improvements heuristics. No traffic demands are assumed in this work, interferences are via reuse only, and only frequency assignment-based cellular technologies are supported.

CPP is very closely related to the family of facility location problems, one of the most well-studied problems in combinatorial optimization (see [36] for an excellent survey). In the traditional *facility location problem* we wish to find optimal locations for facilities (or base stations) in order to serve a given set of client locations; we are also given a set of locations in which facilities may be built, where building a facility in location $i \in I$ incurs a cost of c_i ; each client $j \in J$ must be assigned to one facility, thereby incurring a cost of c_{ij} , proportional to the distance between locations i and j ; the objective is to find a solution of minimum total (assignment + opening) cost. In the *k-median problem*, facility costs are replaced by a constraint that limits the number of facilities to be k and the objective is to minimize the total assignment costs. These two classical problems are min-sum problems, in that the sum of the assignment costs goes into the objective function. The *k-center problem* is the min-max analogue of the *k-median problem*: one builds facilities at k locations out of a given number of locations, so as to minimize the maximum distance from a given location to the nearest selected location. Theoretically speaking, CPP is a “new” type of discrete location problem. Observe that this new problem is not a special case of any of the known min-sum discrete location problems (e.g., there is no connection cost between base stations and clients) nor a “special NP-hard case” of a minimum-cost flow problem (e.g., how to model the interference?).

3 The budgeted cell planning problem

In this section⁴ we study the problem of cell planning under budget constraint. To the best of our knowledge, despite the extensive research on non-budgeted cell planning problems, there is no explicit study in the literature of BCPP (in both theoretical and, surprisingly, also in practical settings).

BCPP is closely related to the well-known budgeted maximum coverage problem. Given is a budget B and a collection of subsets \mathcal{S} of a universe U of elements, where each element in U has a specified weight and each subset has a specified cost. The *budgeted maximum coverage problem* asks for a subcollection $\mathcal{S}' \subseteq \mathcal{S}$ of sets, whose total cost is at most B , such that the total weight of elements covered by \mathcal{S}' is maximized. This problem is the “budgeted” version of the set cover problem in which one wishes to cover all the elements of U using a minimum number of subsets of \mathcal{S} . The budgeted maximum coverage problem is a special case of BCPP in which elements are clients with unit demand, every set $i \in I$ corresponds to a base station i containing all clients in its coverage area $S_i \subseteq J$, and $w_i \geq |S_i|$ for all base stations in I . In this setting, budgeted maximum coverage is the case (in the sense that a solution to BCPP is optimal if and only if it is optimal for the budgeted maximum coverage) when there are no interference (i.e., P is the zero matrix). For the budgeted maximum coverage problem, there is a $(1 - \frac{1}{e})$ -approximation algorithm [4, 19], and this is the best approximation ratio possible unless $NP=P$ [14, 19]. Interestingly enough, we show in the next section that our generalization makes this problem hard to approximate.

BCPP is also closely related to the budgeted unique coverage version of set cover. In the *budgeted unique coverage problem* elements in the universe are uniquely covered, i.e., appear in

⁴Parts of the results described in this section appeared in [8].

exactly one set of S' . As with the budgeted maximum coverage problem, this problem is a special cases of BCPP. In this setting, budgeted unique coverage is when the interference is taking to be the highest (i.e., the impact matrix is a matrix of 1's, except the diagonal where elements are equals to zero). For the budgeted unique coverage problem, there is an $\Omega(1/\log n)$ -approximation algorithm [12] and, up to a constant exponent depending on ϵ , $O(1/\log n)$ is the best possible ratio assuming $\text{NP} \not\subseteq \text{BPTIME}(2^{n^\epsilon})$ for some $\epsilon > 0$.

Another closely related problem to BCPP is the problem of maximizing a nondecreasing submodular set function with a budget constraint. Let $U = \{1, \dots, n\}$, let c_u be a nonnegative weight, $u \in U$, and let B be a nonnegative budget. The problem of maximizing a nondecreasing submodular set function with a budget constraint can be described as

$$\max_{S \subseteq U} \left\{ f(S) : \sum_{u \in S} c_u \leq B \right\},$$

where $f(S)$ is a nonnegative nondecreasing submodular polynomially computable set function (a set function is submodular if $f(S) + f(T) \geq f(S \cup T) + f(S \cap T)$ for all $S, T \subseteq U$ and nondecreasing if $f(S) \leq f(T)$ for all $S \subseteq T$). For this problem, there is a $(1 - \frac{1}{e})$ -approximation algorithm [30], and as this problem is a generalization of the budgeted maximum coverage ($c_u = 1$, for all $u \in U$, and $f(S)$ denotes the maximum weight that can be covered by the set S), this ratio is the best achievable. Although this problem seems, at least from a natural perspective, to be closely related to BCPP, observe that set (covering) requirements are not submodular, in general, when interferences are involved. Consider, for example, an instance of BCPP in which $I = \{1, 2, 3\}$ with $w_1 = w_2 = 1$ and $w_3 = 1/4$, a single client with $d = 2$ that can be satisfied by all base stations, and symmetric interference $p(1, 3) = p(2, 3) = 1/2$, while $p(1, 2) = 0$. Taking $S = \{1\} \cup \{3\}$ and $T = \{2\} \cup \{3\}$ we have $f(S) + f(T) \not\geq f(S \cup T) + f(S \cap T)$, where $f(S)$ is defined to be the maximum number of fully satisfied clients that can be covered by the set S of base stations.

3.1 Inapproximability

As mentioned earlier, the budgeted maximum coverage problem is a special case of BCPP. Khuller, Moss, and Naor [19] showed that the greedy method of picking at each step the most effective set until either no element is left to be covered or the budget limitation is exceeded, results, together with a combination of the enumeration technique, in a $(1 - \frac{1}{e})$ -approximation algorithm for this problem. Unfortunately, a natural attempt to adopt the ideas from [19] to the more general setting of BCPP fails, as stated by the next theorem.

Theorem 1. *It is NP-hard to find a feasible solution to the budgeted cell planning problem.*

Proof. The proof is via a reduction from the subset sum problem. Given an instance of the subset sum problem, i.e., a set of natural numbers $A = \{a_1, a_2, \dots, a_n\}$ and an additional natural number $T = \frac{1}{2} \sum_{i=1}^n a_i$. We build an instance of BCPP with $I = \{1, 2, \dots, n\}$, $|J| = 1$ and $w_i = c_i = a_i$ for every $i \in I$; the budget and the single client's demand are $B = d = T$ and no interference are assumed.

It is easy to see that the client is satisfied if and only if there exists $S \subseteq A$ with $\sum_{i \in S} a_i = T$. Since there is only a single client, any polynomial-time approximation algorithm must produce a full coverage, solving the subset sum problem in polynomial time. \square

3.2 The $k4k$ -budgeted cell planning problem

In light of the above inapproximability result, we turn to define a restrictive version of BCPP which is nevertheless general enough to capture interesting practical cases. To this end we use the fact that typically the number of base stations in cellular networks is much smaller than the number

of clients. Notice that when planning cellular networks, the notion of “clients” sometimes means mobile-clients and sometimes it represents the total traffic demand created by many mobile-clients at a given location. Our models support both forms of representations. Moreover, when there is a relatively large cluster of antennas in a given location, this cluster is usually designed to meet the traffic requirements of a high-density area of clients. Thus, for both interpretations of “clients” the number of satisfied clients is always much bigger than the number of base stations. Followed by the above discussion, we define the *k4k-budgeted cell planning problem* (*k4k-BCPP*) to be BCPP with the additional property that every set of k base stations can fully satisfy at least k clients, for every integer k (and we refer to this property as “*k4k* property”). First, we show that this problem remains hard.

Theorem 2. *The k4k-budgeted cell planning problem is NP-hard.*

Proof. Via a reduction from the budgeted maximum coverage problem. Consider an instance of the budgeted maximum coverage problem, that is, a collection of subsets $\mathcal{S} = \{S_1, \dots, S_m\}$ with associated costs $\{c_i\}_{i=1}^m$ over a domain of elements $X = \{x_1, \dots, x_n\}$, and a budget L .

We can construct an instance of *k4k-BCPP* such that an optimal solution to this problem gives an optimal solution to the budgeted maximum coverage problem. First, we construct a bipartite graph of elements vs. sets, derived from the budgeted maximum coverage instance: there is an edge (x_i, S_j) if and only if element x_i belongs to the set S_j . The instance of *k4k-BCPP* is as follows: the set of clients is $\{x_1, \dots, x_n\} \cup \{y_1, \dots, y_m\}$, where each of the x_j ’s has a unit demand and each of the y_r ’s has zero demand. The set of potential base stations is $\{S_1, \dots, S_m\}$, each with opening cost c_i and capacity $w_i = |S_i|$. The admissible clients of base station S_i are the elements of S_i and all the y ’s, the budget is $B = L$, and there is no interference.

Clearly, a solution to *k4k-BCPP* is optimal if and only if the corresponding solution of the budgeted maximum coverage instance is optimal. \square

In the remainder of this section we assume that the interference model is the one defined in Equation (4).

3.3 The structure of BCPP solutions

Our algorithm is based on a combinatorial characterization of the solution set to BCPP (and in particular to *k4k-BCPP*). The following lemma is a key component in the analysis of our approximation algorithm.⁵

Lemma 3. *Every solution to the k4k-budgeted cell planning problem can be transformed to a solution in which the number of clients that are covered by more than one base station is at most the number of opened base stations. Moreover, this transformation leaves the number of fully satisfied clients as well as the solution cost unchanged.*

Proof. Consider a solution $\Delta = \{I', J', \mathbf{x}\}$ to the *k4k-BCPP*, where $I' \subseteq I$ is the set of base stations selected for opening, $J' \subseteq J$ is the set of fully satisfied clients, x_{ij} ’s are the base station-client coverage rates, and $J'' \subseteq J'$ is the set of clients that are satisfied by more than one base station. Without loss of generality we may assume that every client has a demand greater than zero, since there is no need for “covering” clients with zero demand. We associate the weighted bipartite graph $G_\Delta = (I' \cup J', E)$ with every such solution. In this graph, $(i, j) \in E$ has weight $w(i, j) = w_i x_{ij}$ if and only if $x_{ij} > 0$, and $w(i, j) = 0$, otherwise. Two cases need to be considered:

1. If G_Δ is acyclic then we are done (i.e., no transformation is needed); in this case $|J''| < |I'|$.
To see this, let T be a forest obtained from G_Δ by fixing an arbitrary base station vertex

⁵This Lemma is also true for non-*k4k* versions of the BCPP.

as the root (in each of the connected components of G_Δ) and trimming all client leaves. These leaves correspond to clients who are covered, in the solution, by a single base station. Since the distance, from the root, to every leaf of each tree is even, the number of internal client-vertices is at most the number of base station-vertices, hence $|J''| < |I'|$.

2. Otherwise, we transform $G_\Delta = (I' \cup J', E)$ into an acyclic bipartite graph $G_{\Delta'} = (I' \cup J', E')$ using a cycle canceling algorithm. For simplicity, we first describe the following algorithm for the no interference case.

Algorithm A [CYCLE CANCELING WITHOUT INTERFERENCE]. As long as there are cycles in G_Δ , pick a cycle C and let γ be the weight of a minimum-weight edge on this cycle. Take a minimum-weight edge on C and, starting from this edge, alternately, in clockwise order along the cycle, decrease and increase the weight of every edge by γ .

It is easy to verify that upon termination every client receives, and every base station supplies, the same amount of demand units as before. Moreover, the only changes here are the values of the x_{ij} 's. Hence, Algorithm A preserves the number as well as the identity of the satisfied clients. Since in each iteration at least one edge is removed, $G_{\Delta'}$ is acyclic, thus yielding $|J''| < |I'|$ as in the former case.

When there are interferences, using Algorithm A to keep the number (as well as the identity) of satisfied clients unchanged is possible only when modifying the x_{ij} 's does not affect the $Q(i, j)$ of any client. Otherwise, this algorithm can no longer guarantee that the number of the satisfied clients will remain the same.

To overcome this problem we generalize the method of cycle canceling. Consider a cycle $C = (v_1, \dots, v_k = v_1)$ in G_Δ , such that odd vertices correspond to base stations. Let v_i be any client-vertex in C . Now suppose the base station which corresponds to v_{i-1} increases its supply to v_i by α units of demand. The basic idea of the generalization is to compute the exact number of demand units the base station which corresponds to v_{i+1} *must* subtract from its coverage, in order to preserve the satisfaction of that client, taking into account all the demand (with its interference) supplied by base station vertices which are outside the cycle.

Notice that increasing a certain $w(v_i, v_{i+1})$ *does not necessarily* increase the supply to client v_i . When interference are considered, it could actually happen that increasing $w(v_i, v_{i+1})$ *decreases* the supply to v_i (if the new interference penalties outweigh the increased supply). Similarly, decreasing some $w(v_i, v_{i+1})$ could actually increase the supply to v_i . However, one can assume for optimal solutions that these cases do not occur (as the solution could be transformed into an equivalent solution where such edges have $w(v_i, v_{i+1}) = 0$).

To demonstrate the idea of canceling cycles when interference exist let us assume, for simplicity, that there is only a single base station which is not on the cycle, denoted by v_o , which participates in the coverage of client v_i . Then, the total contribution of base stations v_{i-1} , v_{i+1} , and v_o to the coverage of client v_i is, by (1),

$$\delta(v_i) = Q(v_o, v_i) + Q(v_{i+1}, v_i) + Q(v_{i-1}, v_i).$$

Given that the supply of base station v_{i-1} to client v_i is increased by α units of demand (i.e., $w'(v_{i-1}, v_i) = w(v_{i-1}, v_i) + \alpha$, where w' is the updated weight function of the edges), base station v_{i+1} must decrease its supply to this client by β units of demand (i.e., $w'(v_{i+1}, v_i) = w(v_{i+1}, v_i) - \beta$) in order to preserve the satisfaction of client v_i (assuming v_o 's supply remains

the same). Then, the value of β can be computed via a solution to the following equation (in variable β),

$$\delta'(v_i) = Q'(v_0, v_i) + Q'(v_{i+1}, v_i) + Q'(v_{i-1}, v_i) = \delta(v_i).$$

Notice that our cycle canceling algorithms are used for the proof of existence and such computations are not necessary for the execution of our approximation algorithm.

Algorithm B [CYCLE CANCELING WITH INTERFERENCE]. As long as there are cycles in G_Δ , pick a cycle $C = (v_1, \dots, v_k = v_1)$ where odd vertices represent base stations. As before, every edge $e_i = (v_i, v_{i+1})$ on the cycle has a weight $w(v_i, v_{i+1})$ associated with it, representing the amount of demand supplied by the base-station-vertex in e_i to the client-vertex in e_i . For simplicity, let d'_i denote this value.

We recursively define a sequence of weights $\{y_i\}_{i=1}^{k-1}$, with alternating signs which represent a shift in the demand supply of base stations to clients along the cycle. Start by setting $y_1 = \epsilon$, representing an increase of ϵ to the demand supplied by the base-station-vertex v_1 to the client-vertex v_2 . This increase of supply may not all be credited to vertex v_2 due to interference, some of which is possibly due to base stations which are outside the cycle, and which contribute to v_2 's satisfaction. Set y_2 to be the maximal decrease in the demand supplied by base-station-vertex v_3 to client-vertex v_2 , such that v_2 remains satisfied. This amount of demand is now "available" to base-station-vertex v_3 , hence we allow v_3 to supply this surplus to client-vertex v_4 . We continue in this manner along the cycle.

If, by repeating this procedure, we end up with $|y_{k-1}| \geq \epsilon$, then we say the cycle is ϵ -adjustable. Otherwise, redefine the values of $\{y_i\}_{i=1}^{k-1}$ in a similar manner, but in reverse order, i.e., starting from y_{k-1} and ending with y_1 . However, it is easy to verify that the cycle is ϵ -adjustable in at least one direction, for some value of ϵ .

Let ϵ_{\max} be the largest value for which the cycle is adjustable, and consider its corresponding values of y_i , $i = 1, \dots, k-1$. Note that the y_i 's have alternating signs, and for any client-vertex v_i , $y_i = -y_{i-1}$. Define the quotients $z_i = d'_i/y_i$ for every $i = 1, \dots, k-1$, and let $z_{\min} = \min_{z_i < 0} |z_i|$. Now increase the amount of demand supplied on every edge on the cycle to be $w'(v_i, v_{i+1}) = y_i \cdot z_{\min}$, where w' is the updated weight function of the edges, as before.

Two important invariants are maintained throughout our cycle-canceling procedure. The first is that $w'(i, j) \geq 0$ for every edge (i, j) of the cycle. The second is that there exists at least one edge $e = (i, j)$ on the cycle for which $w'(i, j) = 0$. Therefore Algorithm B preserves the number and the identity of the satisfied clients and $G_{\Delta'}$ is also a solution. Since at each iteration at least one edge is removed, $G_{\Delta'}$ is acyclic and $|J''| < |I'|$ as before.

□

3.4 An $\frac{\epsilon-1}{3\epsilon-1}$ -approximation algorithm

The main difference between $k4k$ -BCPP and other well-studied optimization problems is the existence of interferences. In order to overcome the difficulties caused by interferences, we use Lemma 3. We distinguish between two cases according to whether or not the optimal solution has the following property: there are many clients that are covered by more than one base station. If the optimal solution has this property, then by opening the maximum number of base stations and applying the $k4k$ property, we get a good approximation. Otherwise, we reduce the problem

Algorithm 1 GREEDY APPROXIMATION FOR CAP

```

1: for all clients in a non-decreasing order of their demand do
2:   Let  $j$  be the current client.
3:   Find the first base station in the given order that can cover  $j$ .
4:   if it exists then
5:     Assign  $j$  to this base station.
6:   else {all base stations cannot cover  $j$  due to capacity constraints}
7:     Leave client  $j$  uncovered.
8:   end if
9: end for

```

to the problem of finding a feasible set of base stations such that the number of clients that can be covered, each by exactly one base station, is maximized. Although this problem is still NP-hard, we show how to approximate it using the greedy approach and ideas similar to the ideas of [19].

3.4.1 The client assignment problem

Prior to using the greedy approach to solve the $k4k$ -BCPP one must answer the next question: how many clients can be covered by a set S of opened base stations, and how many more can be covered if an additional base station i is to be opened next? Formally, for a given set of base stations, I' , let $N(I')$ be the number of clients that can be satisfied, each by exactly one base station (we assume no interference, or interference of the second kind). We refer to the problem of computing $N(\cdot)$ as the *Client Assignment Problem* (CAP).

Lemma 4. *The function $N(\cdot)$ is not submodular.*

Proof. Consider the following example: $I = \{1, 2, 3\}$, $J = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $S_1 = J$, $S_2 = \{1, 2, 3\}$, $S_3 = \{4, 5, 6\}$. The demands are: $d_1 = d_2 = d_3 = d_4 = d_5 = d_6 = 4$, $d_7 = 3$, $d_8 = d_9 = d_{10} = 9$, and the capacities are: $w_1 = 30$, $w_2 = w_3 = 12$.

Let $S = \{1, 2\}$ and $T = \{1, 3\}$. One can verify that $N(S) = N(T) = 8$, $N(S \cap T) = 7$ and $N(S \cup T) = 10$. \square

Theorem 5. *CAP is NP-hard.*

Proof. We reduce the PARTITION problem to CAP. Given is a set of positive integers $\{a_1, \dots, a_n\}$ whose sum is T , where T is even, we construct the following instance of CAP: $I = \{i_1, i_2\}$, $J = \{1, \dots, n, n+1, n+2\}$, $w_1 = w_2 = \frac{T+1}{2}$, $S_1 = J \setminus \{n+2\}$, $S_2 = J \setminus \{n+1\}$, and for $j \leq n$, $d_j = a_j$, and $d_{n+1} = d_{n+2} = 0.5$. It is easy to verify that there exists a partition of $\{a_1, \dots, a_n\}$ into two subsets whose sum is $T/2$ if and only if it is possible to cover all the clients in the above CAP instance. \square

Lemma 4 and Theorem 5 indicate the difficulty in applying a greedy approach for solving $k4k$ -BCPP. Informally speaking, submodularity guarantees that a greedy choice made at some point stays a greedy choice even when taking into account subsequent steps. Without submodularity it is not clear whether greediness is the right approach. Moreover, Theorem 5 implies that we cannot efficiently compute the best (in the greedy sense) base station to open in a single step. These two difficulties prevent us from using the generalization of [19] proposed by Sviridenko [30] to approximate $k4k$ -BCPP, as the algorithm of Sviridenko can be used to approximate only submodular (polynomially-computable) functions.

In order to overcome these problems, we use an approximation for CAP. We note that CAP is a special case of the well studied *General Assignment Problem* (GAP), which can be approximated up to a constant factor (see [15] for a $\frac{e-1}{e}$ -approximation). Nevertheless, for our application we

will need an approximation algorithm with submodularity-like properties, and hence we use the following simple greedy algorithm. The algorithm gets as an input an *ordered* set of base stations $I' = \{i_1, \dots, i_k\}$.

Denote by $N_A(I')$ the number of clients covered by Algorithm 1 when it is given I' as an input. In what follows we use (I', i) (for $i \notin I'$) to denote an input to Algorithm 1 in which base station i is ordered *last*, that is, after all the base stations in I' . We also denote by $BS_{I'}(j)$ the base station that covers client j when Algorithm 1 gets I' as its input (let $BS_{I'}(j) = \emptyset$ when client j is not covered).

Theorem 6. *Algorithm 1 is a $\frac{1}{2}$ -approximation to CAP, that is, for every I' , $N_A(I') \geq \frac{N(I')}{2}$.*

Proof. Let OPT denote the optimal solution. Fix a base station i , and let J_{OPT} denote the set of clients covered by i in OPT , but are not covered at all by Algorithm 1. Order the clients in J_{OPT} in a non-decreasing order of their demand, and let j_1, j_2, \dots, j_k be this order. Denote the clients that Algorithm 1 chooses to cover using base station i by j'_1, j'_2, \dots, j'_h (where these are ordered according to the order they are covered by the algorithm). Notice that $d_{j'_h} \leq d_{j_1}$ and $d_{j_1} + \sum_{\ell=1}^h d_{j'_\ell} > w_i$, since otherwise the algorithm would cover j_1 using i .

For each client $j_t \in J_{OPT}$, let

$$z(j_t) = \min \left\{ \ell \mid \sum_{r=1}^{\ell} d_{j'_r} + \sum_{r=1}^t d_{j_r} > w_i \right\}.$$

One can think of $z(j_r)$ as the reason that j_r is not covered by i . Feasibility of OPT implies that $\sum_{\ell=1}^k d_{j_\ell} \leq w_i$, and hence $z(j_t)$ is well-defined. Moreover, since $d_{j'_h} \leq d_{j_1}$, for every j_t and j_s , $r \neq s$, we have that $z(j_t) \neq z(j_s)$. This implies that $k \leq h$, and the theorem follows. \square

We need the following two properties of Algorithm 1.

Lemma 7 (Monotonicity of Algorithm 1). *For every set of base stations I' and every base station $i \notin I'$, $N_A(I' \cup \{i\}) \geq N_A(I')$.*

Proof. Assume to the contrary that $N_A(I' \cup \{i\}) < N_A(I')$ and consider the first client j during the execution with the input $I' \cup \{i\}$ that is not covered by $BS_{I' \cup \{i\}}(j)$ or by a base station that precedes $BS_{I'}(j)$ in the given order of base stations. During the execution with the input I' , base station $BS_{I'}(j)$ had at least d_j unused capacity when Algorithm 1 considered client j . During the execution with the input $I' \cup \{i\}$, base station $BS_{I'}(j)$ had at most $d_j - 1$ unused capacity when Algorithm 1 considered client j . Hence, there exists at least one client j' with demand at most d_j that is covered by $BS_{I' \cup \{i\}}(j)$ during the execution with the input $I' \cup \{i\}$ and that is not covered by $BS_{I'}(j)$ during the execution with the input I' . Since j' is considered by the algorithm before client j , it could be covered by $BS_{I'}(j)$ during the execution with the input I' (we have at least d_j units of capacity available that are later used to cover j). Hence we conclude that that $BS_{I'}(j')$ precedes $BS_{I'}(j)$, which is a contradiction to the way we chose j . \square

Lemma 8. *For every set of base stations I' and every base stations $i_1, i_2 \notin I'$,*

$$N_A(I', i_1, i_2) - N_A(I', i_1) \leq N_A(I', i_2) - N_A(I').$$

Proof. Note that since i_2 is the last base station in the order, it cannot change the clients that are covered by the rest of the base stations. Hence, $N_A(I', i_1, i_2) - N_A(I', i_1)$ is simply the number of clients covered by i_2 when the algorithm is given (I', i_1, i_2) as an input, and $N_A(I', i_2) - N_A(I')$ is the number of clients covered by i_2 when the algorithm is given (I', i_2) as an input. By the definition of the algorithm, the set of clients that the algorithm will try to cover by i_2 when i_1 is available is a subset of the clients that the algorithm will try to cover by i_2 when i_1 is not available. Hence, the lemma follows from the greediness of the algorithm with respect to i_2 . \square

Algorithm 2 BUDGETED MAXIMUM ASSIGNMENT

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1: For every ordered  $I' \subseteq I$ ,  $c(I') \leq B$  and  $|I'| < 3$ , compute  $N_A(I')$ . Let  $I_1$  be the subset with
   the highest value of  $N_A$  computed.
2: for every ordered  $I' \subset I$ ,  $c(I') \leq B$  and  $|I'| = 3$  do
3:    $U \leftarrow I \setminus I'$ 
4:   repeat
5:     Select  $i \in U$  such that maximizes  $\frac{N_A(I',i) - N_A(I')}{c_i}$ .
6:     if  $c(I') + c_i \leq B$  then
7:        $I' \leftarrow (I', i)$ 
8:     end if
9:      $U \leftarrow U \setminus \{i\}$ .
10:  until  $U = \emptyset$ 
11:  if  $N_A(I') > N_A(I_1)$  then
12:     $I_1 \leftarrow I'$ 
13:  end if
14: end for
15: Output  $I_1$ .

```

3.4.2 The budgeted maximum assignment problem

In this section we present an approximation algorithm for the following problem: find a subset I' of base stations whose cost is at most the given budget and that maximizes $N(I')$. We refer to this problem as the *budgeted maximum assignment problem* (BMAP). Algorithm 2 and its analysis generalize the ideas of [19] in the sense that the problem considered in [19] includes neither capacities nor non-uniform demands. Notice that we use here Algorithm 1 to compute $N_A(\cdot)$, and hence all subsets of base stations are ordered.

If the optimal solution has less than three base stations, we will consider it in the first step of the algorithm, and get at least $\frac{1}{2}$ of its value. It is left to take care of the case that the optimal solution has at least three base stations. In this case, we order the base stations in OPT by selecting at each step the set in OPT that maximizes the difference in the value of $N_A(\cdot)$. Let Y be the first three base stations according to this order, and let Y' be the set of base stations that are added to Y when the algorithm considers Y as its initial set of opened base stations. For the rest of the discussion, reorder $OPT \setminus Y$ according to the order the base stations were considered by Algorithm 2.

Let ℓ be the number of base stations opened by the algorithm before the first base station from $OPT \setminus Y$ is considered but not added to Y' because its addition would violate the budget constraint. Denote these base stations by i_1, \dots, i_ℓ , and let $i_{\ell+1}$ be the first base station from $OPT \setminus Y$ is considered but not added to Y' . In what follows, let $G_h = \bigcup_{k=1}^h i_k$ (in this order). Notice that by fixing Y to be the three first base stations in the input to Algorithm 1, we also fix the clients that are going to be assigned to them by the algorithm, regardless of what other base stations are opened. Hence, in what follows we abuse notation and for I' such that $Y \cap I' = \emptyset$ we denote by $N_A(I')$ the number of clients that are covered by I' when Algorithm 1 is given the input Y, I' .

Lemma 9. For each i_k , $k = 1, \dots, \ell + 1$ we have: $\frac{c_{i_k}}{B} \cdot (N_A(OPT \setminus Y) - N_A(G_{k-1})) \leq N_A(G_k) - N_A(G_{k-1})$.

Proof. For each base station $i \in OPT \setminus (Y \cup G_{k-1})$, we have $\frac{N_A(G_{k-1}, i) - N_A(G_{k-1})}{c_i} \leq \frac{N_A(G_{k-1}, i_k) - N_A(G_{k-1})}{c_{i_k}}$. Hence,

$$(N_A(G_{k-1}, i) - N_A(G_{k-1})) \leq c_i \cdot \frac{N_A(G_k) - N_A(G_{k-1})}{c_{i_k}}.$$

Summing up over all $i \in OPT \setminus (Y \cup G_{k-1})$ we get:

$$\sum_{i \in OPT \setminus (Y \cup G_{k-1})} (N_A(G_{k-1}, i) - N_A(G_{k-1})) \leq \sum_{i \in OPT \setminus (Y \cup G_{k-1})} c_i \cdot \frac{N_A(G_k) - N_A(G_{k-1})}{c_{i_k}}.$$

Since $\sum_{i \in OPT \setminus (Y \cup G_{k-1})} c_i \leq B$,

$$\sum_{i \in OPT \setminus (Y \cup G_{k-1})} (N_A(G_{k-1}, i) - N_A(G_{k-1})) \leq B \cdot \frac{N_A(G_k) - N_A(G_{k-1})}{c_{i_k}}.$$

By Lemma 8 we get:

$$N_A(G_{k-1}, OPT \setminus (Y \cup G_{k-1})) - N_A(G_{k-1}) \leq B \cdot \frac{N_A(G_k) - N_A(G_{k-1})}{c_{i_k}},$$

and by Lemma 7 we get

$$N_A(OPT \setminus Y) - N_A(G_{k-1}) \leq B \cdot \frac{N_A(G_k) - N_A(G_{k-1})}{c_{i_k}},$$

which completes the proof. \square

Lemma 10. For each $i_k, k = 1, \dots, \ell + 1$ we have:

$$N_A(G_k) \geq \left[1 - \prod_{h=1}^k \left(1 - \frac{c_{i_h}}{B} \right) \right] \cdot N_A(OPT \setminus Y)$$

Proof. By induction on k . For $k = 1$, $\frac{N_A(i_1)}{c_{i_1}} \geq \frac{N_A(OPT \setminus Y)}{B}$ by Lemma 9. Suppose the lemma is true for $k - 1$, we show it holds for k .

$$\begin{aligned} N_A(G_k) &= N_A(G_{k-1}) + [N_A(G_k) - N_A(G_{k-1})] \\ &\geq N_A(G_{k-1}) + \frac{c_{i_k}}{B} \cdot (N_A(OPT \setminus Y) - N_A(G_{k-1})) \\ &= \left(1 - \frac{c_{i_k}}{B} \right) \cdot N_A(G_{k-1}) + \frac{c_{i_k}}{B} \cdot N_A(OPT \setminus Y) \\ &\geq \left(1 - \frac{c_{i_k}}{B} \right) \cdot \left[1 - \prod_{h=1}^{k-1} \left(1 - \frac{c_{i_h}}{B} \right) \right] \cdot N_A(OPT \setminus Y) + \frac{c_{i_k}}{B} \cdot N_A(OPT \setminus Y) \\ &= \left[1 - \prod_{h=1}^k \left(1 - \frac{c_{i_h}}{B} \right) \right] \cdot N_A(OPT \setminus Y). \end{aligned}$$

Here, the first inequality follows from Lemma 9, and the second inequality follows from the induction hypothesis. \square

Theorem 11. Algorithm 2 is a $\frac{e-1}{2e}$ -approximation for BMAP.

Proof. Notice that for positive c_1, \dots, c_r such that $\sum_{i=1}^r c_i = A$, the function $(1 - \prod_{i=1}^r (1 - \frac{c_i}{A}))$ is minimized when $c_1 = \dots = c_r = \frac{A}{r}$. From Lemma 10 and the fact that the total cost of $G_{\ell+1}$,

Algorithm 3 $k4k$ -BUDGETED CELL PLANNING

- 1: Let I_1 be the output of Algorithm 2.
 - 2: Let I_2 be a set of base stations of maximum size having a total opening cost less than or equal to B .
 - 3: **if** $N_A(I_1) < |I_2|$ **then**
 - 4: Output I_2 and a set of $|I_2|$ clients that can be covered using the oracle.
 - 5: **else**
 - 6: Output I_1 and the clients covered by Algorithm 1 for these base stations.
 - 7: **end if**
-

$c(G_{\ell+1})$, is more than B , we get:

$$\begin{aligned}
 N_A(G_{\ell+1}) &\geq \left[1 - \prod_{h=1}^{\ell+1} \left(1 - \frac{c_{i_h}}{B} \right) \right] \cdot N_A(OPT \setminus Y) \\
 &\geq \left[1 - \prod_{h=1}^{\ell+1} \left(1 - \frac{c_{i_h}}{c(G_{\ell+1})} \right) \right] \cdot N_A(OPT \setminus Y) \\
 &\geq \left[1 - \left(1 - \frac{1}{\ell+1} \right)^{\ell+1} \right] \cdot N_A(OPT \setminus Y) \\
 &\geq \left(1 - \frac{1}{e} \right) \cdot N_A(OPT \setminus Y) .
 \end{aligned}$$

From Lemma 8 we know that $N_A(G_\ell) + N_A(i_{\ell+1}) \geq N_A(G_{\ell+1})$, hence we get:

$$N_A(G_\ell) + N_A(i_{\ell+1}) \geq \left(1 - \frac{1}{e} \right) \cdot N_A(OPT \setminus Y) .$$

By the way we ordered OPT and by Lemma 8 we have that

$$N_A(i_{\ell+1}) \leq \frac{1}{3} N_A(Y) .$$

We combine the two last inequalities and get:

$$\begin{aligned}
 N_A(Y \cup G_\ell) &= N_A(Y) + N_A(G_\ell) \\
 &\geq N_A(Y) + \left(1 - \frac{1}{e} \right) \cdot N_A(OPT \setminus Y) - N_A(i_{\ell+1}) \\
 &\geq N_A(Y) + \left(1 - \frac{1}{e} \right) \cdot N_A(OPT \setminus Y) - \frac{1}{3} N_A(Y) \\
 &\geq \left(1 - \frac{1}{3} \right) N_A(Y) + \left(1 - \frac{1}{e} \right) \cdot N_A(OPT \setminus Y) \\
 &\geq \left(1 - \frac{1}{e} \right) \cdot N_A(OPT) \\
 &\geq \frac{e-1}{2e} \cdot N(OPT) ,
 \end{aligned}$$

where the last inequality follows from Theorem 6. This completes the proof. □

3.4.3 Approximating $k4k$ -BCPP

We are now ready to present a $\frac{e-1}{3e-1}$ -approximation algorithm for $k4k$ -BCPP.

Theorem 12. *Algorithm 3 is a $\frac{e-1}{3e-1}$ -approximation algorithm for the $k4k$ -budgeted cell planning problem.*

Proof. Let \tilde{n} be the number of covered clients in the solution obtained by Algorithm 3, and let n^* be the maximum number of satisfied clients as obtained by the optimal solution. In the latter, let n_1^* denote the number of clients that are satisfied by a single base station, and n_2^* denote the number of clients satisfied by more than one base station. Let I^* denote the set of base stations opened (by the optimal solution) for satisfying these $n^* = n_1^* + n_2^*$ clients.

Let $N(OPT)$ denote the value of the optimal solution for the BMAP instance. It holds that $N(OPT) \geq n_1^*$. For the solution I_1 we know that

$$\tilde{n} \geq N_A(I_1) \geq \frac{e-1}{2e} N(OPT) \geq \frac{e-1}{2e} n_1^*. \quad (5)$$

We get:

$$\frac{3e-1}{2e} \tilde{n} = \tilde{n} + \frac{e-1}{2e} \tilde{n} \quad (6)$$

$$\geq \tilde{n} + \frac{e-1}{2e} |I^*| \quad (7)$$

$$\geq \frac{e-1}{2e} n_1^* + \frac{e-1}{2e} n_2^* \quad (8)$$

$$= \frac{e-1}{2e} n^* \quad (9)$$

where Inequality (7) follows from the fact that $\tilde{n} \geq |I_2| \geq |I^*|$ and the $k4k$ property, and Inequality (8) is based on (5) and Lemma 3. \square

4 The minimum-cost cell planning problem

Recall that the minimum-cost cell planning problem asks for a subset of base stations $I' \subseteq I$ of minimum cost that satisfies at least γ of the demands of all the clients, for a given constant $0 < \gamma \leq 1$, while the maintaining the capacity of every base station.

Let z_i denote the indicator variable of an opened base station, i.e., $z_i = 1$ if base station $i \in I$ is selected for opening, and $z_i = 0$ otherwise. Consider the following integer program for this problem (\mathbf{IP}_1).

In the first set of constraints (10) we ensure that at least γ of the demand d_j of every client j is satisfied, while the second set (11) ensure that the ability of every open base station to satisfy the demands of the clients is limited by its capacity (and that clients can be satisfied only by opened base stations). The contribution $Q(i, j)$ of base station i to client j , taking into account interference

$$\min \sum_{i \in I} c_i z_i \quad (\mathbf{IP}_1)$$

$$\text{s.t.} \quad \sum_{i: j \in S_i} Q(i, j) \geq \gamma \cdot d_j, \quad \forall j \in J \quad (10)$$

$$\sum_{j \in J} x_{ij} \leq z_i, \quad \forall i \in I \quad (11)$$

$$0 \leq x_{ij} \leq 1, \quad \forall i \in I, j \in S_i \quad (12)$$

$$x_{ij} = 0, \quad \forall i \in I, j \notin S_i \quad (13)$$

$$z_i \in \{0, 1\}, \quad \forall i \in I$$

$$\min \sum_{i \in I} c_i z_i \tag{LP_2}$$

$$\text{s.t.} \sum_{i \in I} w_i x_{ij} \geq \gamma \cdot d_j, \quad \forall j \in J \tag{14}$$

$$\sum_{j \in J} x_{ij} \leq z_i, \quad \forall i \in I \tag{15}$$

$$0 \leq x_{ij} \leq 1, \quad \forall i \in I, j \in S_i \tag{16}$$

$$x_{ij} = 0, \quad \forall i \in I, j \notin S_i$$

$$0 \leq z_i \leq 1, \quad \forall i \in I \tag{17}$$

from other base stations, can be modeled as in (3) or (4), or any other predefined behavior of interference. However, because of the way $Q(i, j)$'s are computed, the integer program (\mathbf{IP}_1) is not linear when interference exists. Without loss of generality we may assume that every client in the input has demand at least 1, as the used units can be scaled accordingly and there is no need for “covering” the clients with zero demand. Lastly, we use the following assumption as well.

Assumption 1. *The values $\{w_i\}_{i \in I}$ and $\{\gamma d_j\}_{j \in J}$ are integers.*

When there are no interference, \mathbf{IP}_1 becomes much simpler. (\mathbf{LP}_2) is its linear programming relaxation, in which the last set of integrality constraints (13) is relaxed to allow the variables z_i to take rational values between 0 and 1.

In fact, \mathbf{LP}_2 is a minimum-cost flow problem. To see that, consider the network (G, u, c') , which is defined as follows.

- The graph $G = (V, E)$, where $V = I \cup J \cup \{s\}$ and $E = \{(i, j) \mid i \in I, j \in S_i\} \cup \{(s, i) \mid i \in I\} \cup \{(j, s) \mid j \in J\}$.
- The vertex capacity function u , where $u(s) = \infty$, $u(i) = w_i$ for $i \in I$ and $u(j) = \gamma d_j$ for $j \in J$.
- The vertex cost function c' , where $c'(i) = \frac{c_i}{w_i}$ for $i \in I$, $c'(j) = 0$ for $j \in J$ and $c'(s) = -1 - \max_{i \in I} c'(i)$.

Accordingly, Assumption 1 yields that there is an optimal solution to the above flow problem, in which the flow in every edge is integral (specifically, any open base station i that serves a client j contribute at least one unit of the client's demand). Moreover, this solution can be computed efficiently using the known algorithms for minimum-cost flow [5]. We denote the solution to \mathbf{LP}_2 which correspond to that flow by $\{\bar{z}, \bar{x}\}$. Let $\bar{I}_j = \{i \in I : \bar{x}_{ij} > 0\}$, for every client $j \in J$. Note that by this definition it follows that for every $i \in \bar{I}_j$ we have that

$$w_i \bar{x}_{ij} \geq 1. \tag{18}$$

Next we introduce our approximation algorithms for CPP with no interference. The algorithm is based on the greedy approach and achieves an approximation of $O(\log W)$, where $W = \max_{i \in I} \{w_i\}$. Unlike the criterion used by other known greedy heuristics, our greedy algorithm chooses to open a base station which maximizes the increase in the maximum demand that can be satisfied by the entire set of the opened base stations.

A greedy $O(\log W)$ -approximation algorithm

In this section we present a greedy algorithm for CPP with no interference. It generalizes the algorithm of Chuzhoy and Naor [10] for set cover with hard capacities. For the sake of completeness, we briefly introduce their notations and the main analysis ideas as well as the necessary changes.

Algorithm 4 MINIMUM-COST CELL PLANNING (GREEDY)

```

1:  $I' \leftarrow \emptyset$ .
2: while  $f(I') < \gamma \sum_{j \in J} d_j$  do
3:   Let  $i = \arg \min_{i \in I: f_{I'}(i) > 0} \frac{c_i}{f_{I'}(i)}$ .
4:    $I' \leftarrow I' \cup \{i\}$ .
5: end while
6: return  $I'$ .
  
```

For a subset of base stations, $H \subseteq I$, let $f(H)$ denote the maximum total demand (in demand units, where the clients need not be fully covered) that can be satisfied by the base stations in H . For $i \in I$, define $f_H(i) = f(H \cup \{i\}) - f(H)$. Note that when there are no interferences, we can calculate $f(H)$ using the following linear program:

$$\max \sum_{j \in J} \sum_{i \in H} w_i x_{ij} \quad (\text{LP}_3)$$

$$\text{s.t.} \quad \sum_{i \in H} w_i x_{ij} \leq \gamma \cdot d_j, \quad \forall j \in J \quad (19)$$

$$\sum_{j \in J} x_{ij} \leq 1, \quad \forall i \in H \quad (20)$$

$$0 \leq x_{ij} \leq 1, \quad \forall i \in H, j \in S_i \quad (21)$$

$$x_{ij} = 0, \quad \forall i \in H, j \notin S_i$$

We need the following lemma.

Lemma 13. *Let $H \subseteq I$ be a subset of the base stations, and let H_1 and H_2 be a partition of H into two disjoint sets. Then, there exists a solution to IP_1 in which the base stations in H_1 satisfy a total of $f(H_1)$ demand units.*

Proof. Assume that we are given a solution to IP_1 , $\{\tilde{\mathbf{z}}\}, \{\tilde{\mathbf{x}}\}$, such that the base stations in H_1 satisfy a total of less than $f(H_1)$ demand units. Let \mathbf{x} be an optimal solution to LP_3 for H_1 . Iteratively, update $\{\tilde{\mathbf{z}}\}, \{\tilde{\mathbf{x}}\}$ as follows: while $\sum_{i \in H_1} \sum_{j \in J} w_i \tilde{x}_{ij} < f(H_1)$,

1. Let $i \in H_1$ be a base station such that $\sum_{j \in J} \tilde{x}_{ij} < \sum_{j \in J} x_{ij}$ (notice there must exist such i).
2. Let $j \in J$ be a client such that $\tilde{x}_{ij} < x_{ij}$.
3. Let $\Delta = w_i \cdot \min \{x_{ij} - \tilde{x}_{ij}, \sum_{j \in J} x_{ij} - \sum_{j \in J} \tilde{x}_{ij}\}$.
4. If there exists a base station $i' \in H_2$ such that $\tilde{x}_{i'j} > 0$, let $\delta = \min \{w_{i'} \tilde{x}_{i'j}, \Delta\}$ and set $\tilde{x}_{ij} \leftarrow \tilde{x}_{ij} + \frac{\delta}{w_i}$ and $\tilde{x}_{i'j} \leftarrow \tilde{x}_{i'j} - \frac{\delta}{w_{i'}}$.
5. Else, there exists a base station $i' \in H_1$ such that $\tilde{x}_{i'j} > x_{ij}$. Let $\delta = \min \{w_{i'} \cdot (\tilde{x}_{i'j} - x_{ij}), \Delta\}$ and set $\tilde{x}_{ij} \leftarrow \tilde{x}_{ij} + \frac{\delta}{w_i}$ and $\tilde{x}_{i'j} \leftarrow \tilde{x}_{i'j} - \frac{\delta}{w_{i'}}$.

One can easily verify that the above process halts with a feasible solution with the desired property. \square

Let i_1, i_2, \dots, i_k be the base stations that were chosen by Algorithm 4 to the solution, in the order they were chosen. Let I'_ℓ be the solution at the end of iteration ℓ of the algorithm. Let OPT be a set of base stations that comprises an optimal solution, $\{\bar{\mathbf{z}}, \bar{\mathbf{x}}\}$.

Next, we inductively define for each iteration ℓ and $i \in \text{OPT} \setminus I'_\ell$ a value $a_\ell(i)$, so that the following invariant holds: it is possible to cover all the clients using the base stations in $\text{OPT} \cup I'_\ell$ with the capacities $a_\ell(i)$ for $i \in \text{OPT} \setminus I'_\ell$ and w_i for $i \in I'_\ell$.

Let $a_0(i) = \sum_{j \in J} w_j x_{ij}$. The invariant holds trivially. Consider the ℓ th iteration. By the induction hypothesis and Lemma 13, there exists a solution $\{\mathbf{z}, \mathbf{x}\}$ of \mathbf{IP}_1 such that the base stations in I'_ℓ satisfy a total of exactly $f(I'_\ell)$ demand units and each base station $i \in \text{OPT} \setminus I'_\ell$ satisfies at most $a_{\ell-1}(i)$ demand units. For each $i \in \text{OPT} \setminus I'_\ell$ let $a_\ell(i) = \sum_{j \in J} w_j x_{ij}$.

In what follows, we charge the cost of the base stations that are chosen by Algorithm 4 to the base stations in OPT . If $i_\ell \in \text{OPT}$, we do not charge any base station for its cost, since OPT also pays for it. Otherwise, we charge each $i \in \text{OPT} \setminus I'_\ell$ with $\frac{c_{i_\ell}}{f_{I'_{\ell-1}}(i_\ell)} \cdot (a_{\ell-1}(i) - a_\ell(i))$. Notice that the total cost of i_ℓ is indeed charged.

Consider a base station $i \in \text{OPT}$. If $i \in I'$, let h denote the iteration in which it was added to the solution. Else, let $h = k + 1$. For $\ell < h$, it follows from the definition of $a_{\ell-1}(i)$ that $f_{I'_{\ell-1}}(i) \geq a_{\ell-1}(i)$. By the greediness of Algorithm 4 it holds that:

$$\frac{c_{i_\ell}}{f_{I'_{\ell-1}}(i_\ell)} \leq \frac{c_i}{f_{I'_{\ell-1}}(i)} \leq \frac{c_i}{a_{\ell-1}(i)},$$

and the total cost charged upon i is:

$$\begin{aligned} \sum_{\ell=1}^{h-1} \frac{c_{i_\ell}}{f_{I'_{\ell-1}}(i_\ell)} \cdot (a_{\ell-1}(i) - a_\ell(i)) &\leq c_i \sum_{\ell=1}^{h-1} \frac{(a_{\ell-1}(i) - a_\ell(i))}{a_{\ell-1}(i)} \\ &= c_i \cdot H(a_0(i)) \\ &= c_i \cdot O(\log a_0(i)) \\ &= c_i \cdot O(\log w_i), \end{aligned}$$

where $H(r)$ is the r th harmonic number. This completes the analysis.

5 Simulation results and practical aspects

In the previous sections, we proved theoretical bounds on the performance of our algorithm. In this section we discuss the practicality of the models and evaluate our algorithm through simulations. The simulation results are comprised of two separate sets of simulations: planning “greenfield” networks and planning UMTS networks in a real urban environment.

5.1 On greedy algorithms for minimum-cost cell planning

As mentioned earlier in this paper, various algorithmic approaches have been studied for the minimum-cost cell planning problem. Methods include, among others, genetic algorithms, tabu search, branch-and-bound, and simulated annealing. These methods do not have a guaranteed polynomial running time and the quality of their solutions depend on the duration of the execution. For our comparisons, we would like to consider algorithms with polynomial running time. The greedy technique (e.g., [28, 33, 38]) is a good practical (and very popular in real-life planning [29, Chapter 14]) candidate for such a comparison (the works of [28] and [38] use methods that are very similar to [33]).

The SCBPA (Set Cover Base stations Positioning Algorithm) model due to Tutschku [33] is perhaps the most natural approach. Its aim is to provide sufficient coverage for a planning area using as few base stations as possible. Usually, a set \mathcal{S} of potential base stations is given, each of them serves a certain sub-area $S_i \subseteq J$, $i \in \mathcal{S}$, of the planning area J , and each has an installation

cost c_i . The goal is to cover the entire area J with a minimum-cost subset of base stations. This is essentially the minimum-weight set cover problem.

The model for the minimum-weight set cover problem neither supports capacities for base stations nor does it support client demands. Hence, this model cannot be used for solving cell planning problems for B3G/4G networks. Therefore, we define a *natural generalization* of the Tutschku's SCBPA algorithm specifically designated for capacitated versions of cell planning problems; we call this generalization *Extended-SCBPA algorithm* (ESCBPA). The ESCBPA greedy algorithm picks at each step a base station that maximizes the ratio between the amount of non-covered demands this station can satisfy and its cost. The algorithm stops when all the clients are fully satisfied (or alternatively, if not enough capacity is left). Notice that when there is no restriction on the capacity of the base stations, this algorithm becomes identical to the one described in [33].

From a computational point of view the SCBPA is an $O(\log n)$ -approximation algorithm for an n -clients CPP [11, 35], while its generalization, ESCBPA, can be arbitrarily bad. To see this, consider an instance with three base stations $I = \{1, 2, 3\}$ of unit capacity and opening costs of $c_1 = 1, c_2 = C$, and $c_3 = \epsilon$ ($C > 1$ and $\epsilon > 0$), and a set of two clients $J = \{1, 2\}$, each has a unit demand. In addition, assume that $S_1 = \{1\}$, $S_2 = \{2\}$, and $S_3 = \{1, 2\}$. First, the ESCBPA algorithm will select to open the third base station and client 1, for example, is the one who is satisfied. Then, the second base station will be picked and client 2 will be satisfied. The total cost of this solution is $C + \epsilon$. However, by opening the first base station (and satisfying client 1), and the third (now satisfying client 2), the optimal solution achieves a full coverage with a total cost of $1 + \epsilon$. Hence the performance guarantee of the ESCBPA algorithm is unbounded. Moreover, one can show that ESCBPA might not produce a feasible solution to CPP (that is, to satisfy all the demand of the clients), although such a solution exists.

Our greedy algorithm (described in Section 4) has a different greedy criterion: it chooses to open a base station which maximizes the increase in the maximum demand that can be satisfied by the entire set of the so-far-opened base stations. To emphasize the different behavior of these two greedy approaches, consider the following example. We have two base stations $I = \{1, 2\}$ with capacities $w_1 = 1 + 3\epsilon$ and $w_2 = 1 + 2\epsilon$, for a given $\epsilon > 0$. The opening costs of these two base stations are taken to be the same. There are five clients $J = \{1, \dots, 5\}$ with $d_1 = d_2 = 1 + \epsilon, d_3 = 1$, and $d_4 = d_5 = \epsilon$. In addition, $S_1 = \{1, 2, 3\}$ while $S_2 = \{3, 4, 5\}$. The ESCBPA will pick the base station with the largest amount of net capacity (base station 1), yielding a satisfaction of at most one single client; our greedy algorithm chooses the base station that can satisfy the largest amount of non-covered demand (base station 2), resulting in a full coverage of all his 3 clients. However, when there are no upper limitations on the capacity of the base stations, both the ESCBPA and our greedy algorithm are the same.

5.2 An LP-rounding based $O(W\sqrt{\log n})$ -approximation algorithm

Many NP-hard combinatorial optimization problems can be formulated as an integer programming problem, which can be subsequently relaxed into a linear programming problem (whose solution is a lower bound of the integer program). However, the optimal solution to the linear programming problem in general does not coincide with the solution to the initial integer programming problem. A basic approach is to solve the linear program and then convert the fractional solution obtained into an integer solution, trying to ensure that in this process the cost does not increase too much.

In this section we present an LP-rounding based approximation algorithm for the non-interference variant of CPP. Our algorithm is based on solving the LP-relaxation (\mathbf{LP}_3), and randomly rounding the fractional solution to an integer solution. The integer solution it produces is within a factor of $O(W\sqrt{\log n})$ of the optimum, where $W = \max_{i \in I} \{w_i\}$, as before.

Notice that from a worst-case viewpoint, our $O(\log W)$ -approximation algorithm (Section 4) is much better than the algorithm presented in this section. The purpose of discussing Algorithm 5

Algorithm 5 MINIMUM-COST CELL PLANNING (LP ROUNDING)

-
- 1: Calculate $\{\bar{z}, \bar{x}\}$ as explained above.
 - 2: $W \leftarrow \max_{i: z_i > 0} \{w_i\}$; $\lambda \leftarrow \Theta(W\sqrt{\log n})$.
 - 3: **for all** $i \in I$ **do**
 - 4: $z_i \leftarrow 1$ with probability $\min\{1, \lambda \cdot \bar{z}_i\}$.
 - 5: **for all** $j \in J$ **do**
 - 6: $x_{ij} \leftarrow \frac{\bar{x}_{ij}}{\bar{z}_i} z_i$.
 - 7: **end for**
 - 8: **end for**
 - 9: **return** $\{z\}$ and $\{x\}$.
-

is to examine how this algorithm is performed in practice. For space considerations, we cite our results without proofs.

Theorem 14. *Algorithm 5 finds a subset $I' \in I$ of base stations that satisfies (10)-(13) with high probability, and whose expected cost is no more than $O(W\sqrt{\log n})$ times the optimal cost, where $W = \max_{i \in I} \{w_i\}$.*

In the following two sets of simulations, we compare ESCBPA algorithm and our techniques (Sections 4 and 5.2) to the value of the optimal solution of the linear program \mathbf{LP}_3 (which is a lower bound on the optimal solution for the minimum-cost cell planning problem).

5.3 Planning “greenfield” networks

In this set of simulations we built a network consisting of an $n \times n$ -grid of clients’ locations (demand points that we considered as *bins*). Each bin is assumed to have traffic demand generated according to the Erlang distribution with parameter 1/30 (which is realistic in urban areas in busy hours). We consider a random set of 10% of all grid points to be possible locations for positioning base stations. For each such possible location, we consider the set of 8 optional sectorized antennas (with horizontal azimuth values of $0^\circ, 45^\circ, \dots, 315^\circ$ and with an open angle of $30^\circ, 60^\circ$ and 120°). The radius of each antenna was chosen up to 5 bins length, but under considerations of urban areas (i.e., relatively small cells). The height and the tilt of each antenna are taken to be fixed. So, for the $n \times n$ -grid of clients we have a set of $24 \times 0.1n^2$ possible configurations of antennas. Each antenna has a coverage area reflected from its geometric parameters. Both the capacity and the installation cost of each antenna are assumed to be a linear function of the sum of demands of all of the clients within its coverage area. Notice that we assume that no interference exist in this scenarios. The cell planning task here is to choose a minimum-cost subset of antenna-configurations in order to satisfy the traffic demands of all the clients.

Figure 1(a) depicts the ratio between the solution cost and the LP cost as a function of network size of each of the three methods. We ran our simulation on network size of 25 – 144 bins (clients). Our results show that our approximation algorithms achieves a significant lower value of the solution cost than the greedy heuristic. Among our two algorithms, the $O(\log W)$ -approximation algorithm was better (up to a factor of 50%) than the $O(W\sqrt{\log n})$ -approximation algorithm, where W is equal here to the maximum number of bins associated with a base station (e.g., $\max_{i \in I} |S_i|$). Moreover, we are far away from the corresponding theoretical worst-case behavior of $O(\log W)$, and in fact, all simulation results are within a factor of 2.5 of the optimum lower bound. Since the $O(\log W)$ -approximation algorithm is expected to produce a close-to-optimal solutions when the capacities are relatively low, our results indicate that such a behavior is also apparent when the capacities of the base stations are proportional to the sum of the traffic demands of their clients (under the anticipated RF conditions derived from the way the S_i ’s are created in the pre-processing stage).

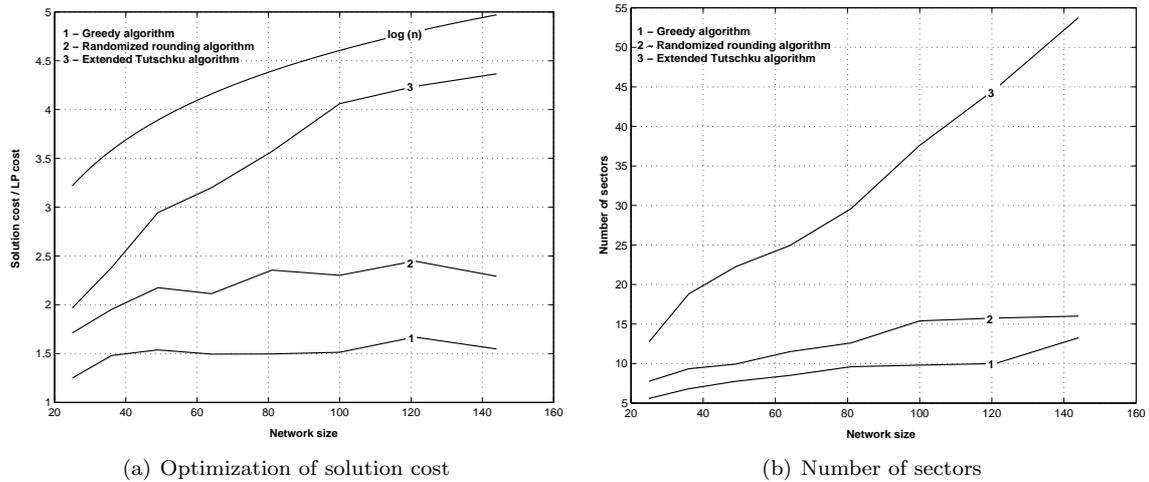


Figure 1: Planning B3G networks

Figure 1(b) depicts the number of sectors selected for deployment as a function of the network size for the three algorithmic methods. We see that our $O(\log W)$ -approximation algorithm selects approximately 15%-20% of the number of sectors selected by the ESCBPA algorithm. Notice that when the objective was the minimize the total number of sectors, both approximation algorithms gave relatively similar results.

5.4 Planning UMTS network in Helsinki

In the second simulation set we study the theoretical bounds of our approximation algorithms on real networks. We compare our algorithm to the optimal solution of the corresponding LP on microcellular and picocellular UMTS networks in a real urban environment.

We analyze an area of 800m² in the city center of Helsinki, Finland (a square area with (386200,6674800) as the lower left corner and (387000,6675600) as the upper right corner (Figure 2), given in UTM coordinates), using data taken from [20]⁶.

Table 1: Cell planning in Helsinki

	Extended Tutschku	Greedy algorithm	Randomized rounding
Solution cost / LP ₃ cost	2.987	1.683	1.886
Number of sectors	32	15	21

Table 1 shows that our $O(\log W)$ -approximation algorithm was better than the ESCBPA algorithm and our $O(W\sqrt{\log n})$ -approximation algorithm (in both objectives). In this real-data scenario our $O(\log W)$ -approximation algorithm reached the same level of coverage with approximately half the number of sectors ESCBPA algorithm did. Notice that all three algorithms output relatively close-to-optimal solutions (1.68 to 2.98 times the lower bound on the optimal solution).

⁶Although the data in [20] does not contain capacities for base stations, base stations in this scenario were limited in their capacities in a similar way as in Simulation A.

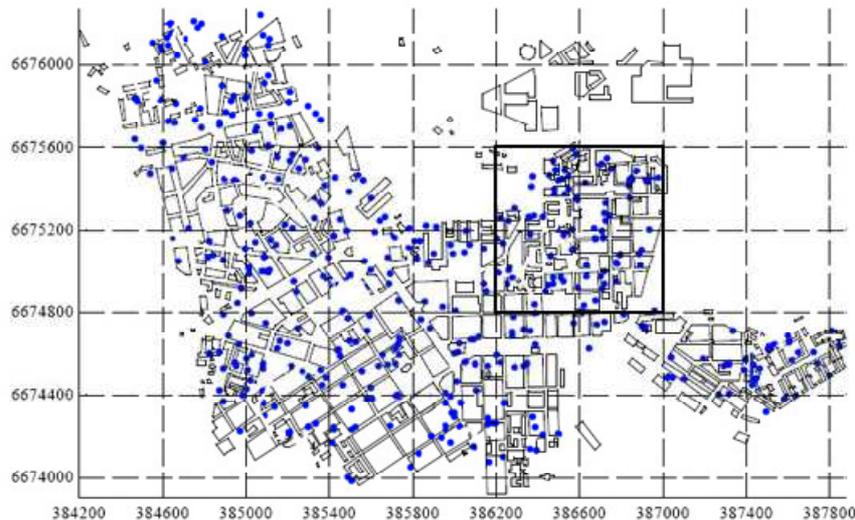


Figure 2: Building contours map of Helsinki

6 Conclusions and open problems

In this work, we describe new techniques for solving two important cell planning problems: the budgeted cell planning (BCPP) and the minimum-cost cell planning problems (CPP). As far as we know, previous work do not provide performance guarantee for these problems. Our formulation is very flexible and it allows the incorporation of several 4G cellular networks characterizations (such as smart antennas, capacities for base stations, interference, and different-size-cells). Due to the NP-hardness of the problems, we resorted to polynomial-time approximation algorithms.

We show that although BCPP is NP-hard to approximate, we can still cover all useful scenarios by adopting a very practical assumption, called the $k4k$ -property, satisfied by every real cellular network, and we give a fully combinatorial $\frac{e-1}{3e-1}$ -approximation algorithm for this problem.

We obtained a combinatorial $O(\log W)$ -approximation algorithm addressed a non-interference version of CPP, where W is the largest capacity over all base stations selected for opening. An LP-based $O(W\sqrt{\log n})$ -approximation algorithm is also presented for this problem.

We simulated our algorithms to the CPP and compared its performance to the results produced by previous works. Two different simulation sets were conducted with scenarios relevant to 4G technologies. In both sets of simulation our results indicate that practical algorithms derived from the theoretical scheme can provide solutions that are close to the optimal solution and much better than our proved theoretical bound.

The main open problem is a development of an approximation algorithm for CPP while taking into account interferences.

Another open problem is the *minimum-cost site planning problem*. In this problem, in addition to what was given in CPP, we also have a set of potential geographic locations (sites) for installing a cluster of antennas (or base stations). Usually, such a cluster comprises of antennas of different pattern, power, and direction, and these are addressed to service a large area with relatively several different behavior subareas (e.g., a close high-density mall in one side and a farther low-density neighborhood, at the other side of the site). The problem is motivated by the operator's need to reduce the number of geographical sites in which its equipment is installed since their rental is relatively high.

The minimum-cost site planning problem can be seen as two-level CPP for which the "sites" can be seen as the first level and the base stations, selected to be installed on these sites, as the

second level. Since this problem is a natural generalization of CPP, we believe that solving CPP will make the solution for the former much more accessible.

The results presented in this paper show that a theoretical approach based on approximation algorithms can model capacities, non-uniform demands, and interference, and provide both theoretically bounded and practical good algorithms for various cell planning problems. This is a significant step towards focused planning of 4G networks, and turning approximation algorithms to be a major player in efficient practical solutions to many planning and covering problems in cellular networks.

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