A DIFFERENT LOOK ON CYCLIC PREFIX FOR SC/FDE

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Abstract - In this work we investigate the use of a known symbol sequence – a so-called unique word (UW) – instead of the well-known cyclic prefix (as it is used in OFDM (Orthogonal Frequency Division Multiplexing) systems) for a single carrier system with frequency domain equalization (SC/FDE). The considered SC/FDE system is similar to the one proposed in the IEEE 802.16.3c target group. It is shown that the UW fulfils the theorem of cyclic convolution required for the efficient implementation of the SC/FDE structure. We will elaborate on the advantages that result for equalization, channel estimation, and synchronisation. Compared to the advantages, the disadvantages in terms of BER behaviour and bandwidth efficiency are rather modest.

Keywords – SC/FDE, OFDM, Cyclic Prefix, WLAN

I. INTRODUCTION

The wireless communications market undergoes a tremendous growth. One of the most challenging problems in high data rate wireless transmission is to overcome the time dispersion caused by multipath propagation. The common problem with conventional time domain equalizer structures is the fact that the computational complexity grows at least quadratically with the bit rate [1].

One of the major advantages of OFDM (Orthogonal Frequency Division Multiplexing) is its capability of using low complexity frequency domain equalization instead of costly time-discrete convolution operations. This advantage is gained at the expense of a cyclic prefix (CP), which is necessary to cope with time dispersive channels. The equalization complexity of the FFT based OFDM concept grows slightly faster than linearly with the bit rate.

A promising but compared to OFDM rarely investigated solution for broadband communication systems is the concept of single carrier transmission with frequency domain equalization (SC/FDE) [2]-[5]. This approach combines the properties of OFDM and single carrier transmission advantageously. Due to the use of FFT operations the receiver complexity is kept significantly below the complexity of conventional single carrier systems with time domain equalizer structures and due to the use of single carrier modulation schemes the constraints on the transceiver's analog components are more relaxed compared to OFDM systems. Different proposals have subsequently been made for SC/FDE, especially in the IEEE 802.16 Wireless MAN (Metropolitan Area Network) standardization process [6],[7].

This paper is organized as follows: In the next section we will introduce the baseband processing steps of the investigated SC/FDE system. In section III the concept of cyclic prefix (CP), that is used for OFDM and SC/FDE, and the concept of using a known sequence (unique word, UW) is described in detail. It will be shown that the concept of UW fulfils the theorem of cyclic convolution. Next we compare both concepts in terms of BER behaviour and bandwidth efficiency. Finally the advantages in terms of synchronization, channel estimation, and equalization are pointed out when using the propagated structure.

II. SC/FDE

Fig. 1 shows the block diagram of the investigated system as proposed in [5]. The SC/FDE signal processing starts with encoding the binary input data. In our investigations we are using the standard rate 1/2, constraint length 7 convolutional encoder with generator polynomials (133, 171). After coding the binary data is converted to QAM values and a guard period is added between successive blocks. The insertion of a guard period anticipates the blockwise processing needed in the receiver when using FFT operations. After pulse shaping (Root Raised Cosine Pulses) and digital-to-analog conversion the resulting I/Q signals are up-converted to RF.

In the receive path, after passing the RF part and the analog-to-digital conversion, time and frequency synchronization is performed first. For each symbol the cyclic prefix has to be removed before the equalization can be performed. If the guard interval T_G is longer than the duration T_h of the channel impulse response, there is no interference between the information symbols of successive blocks. The task of the frequency domain equalizer is to eliminate ISI within the individual blocks.

The equalized QAM values are then demapped onto soft bipolar values and decoded by a Viterbi decoder.

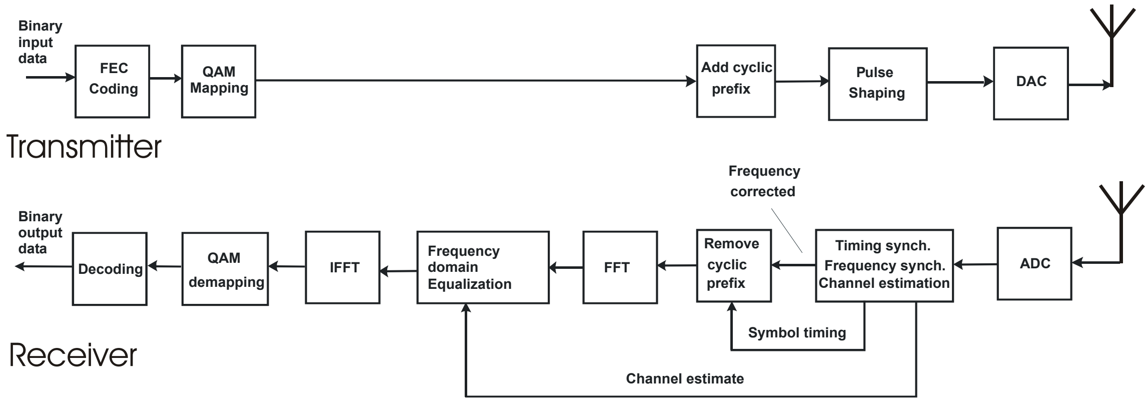

Fig. 1. Baseband block diagram of the simulated SC/FDE system.

III. CYCLIC PREFIX VERSUS UNIQUE WORD

A. Cyclic Prefix

Frequency domain equalization for single carrier systems is based on the equivalence between the convolution of two sequences in the time domain and the product of their Fourier transforms. Because of the use of FFT operations, the received signals have to be processed blockwise. In [2] it has been proposed for the first time not only to apply blockwise processing at the received signal but to perform a blockwise transmission similar to OFDM and insert a CP (cyclic prefix) between successive blocks. Due to the cyclic extension of the transmitted blocks, the convolution of one cyclically extended transmitted block and the channel impulse response $h(t)$ can be calculated by a circular convolution corresponding to the frequency domain relation

$$R(nf_0) = H(nf_0)S(nf_0) + N(nf_0) \qquad (1)$$

for $n \in Z$ and $f_0=1/T_{FFT}$. Here the functions $R(f)$, $S(f)$ and $H(f)$ are related to the time domain signals $r(t)$ (one period of the received data block), $s(t)$ (the original, non cyclically extended block) and $h(t)$ by the continuous Fourier transform. $N(f)$ is the Fourier transform of the additive noise. Fig. 2 shows the transmitted data structure.

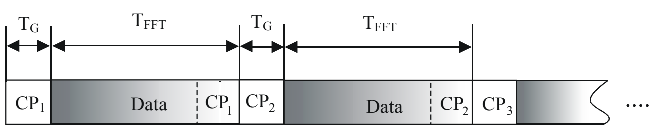

Fig. 2. Transmitted data structure using a Cyclic Prefix.

The CP is less useful for other purposes like channel estimation, equalization, or synchronization as long as the content of the CP is not known and varies with every single block. The overhead induced by the CP could be used in a more efficient way if its content would be known before and could be chosen in a proper way.

B. Unique Word

We will now give a mathematical description of the investigated SC/FDE system when using a UW instead of the traditional CP. Fig. 3 depicts the structure of one transmitted block, which consists of the original data sequence of N_s symbols and the sequence of the UW with N_G symbols. The overall duration of $N=N_S+N_G$ symbols is $T_{FFT}=NT$.

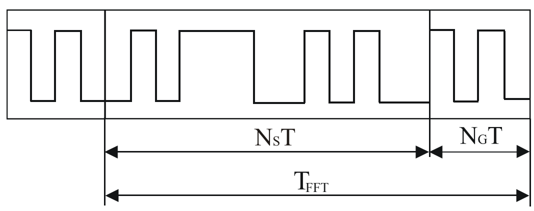

Fig. 3. Transmitted block using the concept of UW.

Instead of the cyclic prefix, a known sequence is part of every processed block. Let us denote $s_{Data,i}(t)$ to be the continuous-time representation of the symbol sequence of the i-th transmitted block with $s_{Data,i}(t)=0$ for $t \notin [0,T_{FFT}-T_G]$, whereas $s_i(t)$ defines the extended block

$$s_i(t) = \begin{cases} s_{Data_i}(t) & \text{for } t \in [0, T_{FFT} - T_G] \\ uw(t) & \text{for } t \in [T_{FFT} - T_G, T_{FFT}] \end{cases}. \qquad (2)$$

For the further development we define $\hat{s}_i(t)$ to be

$$\hat{s}_i(t) = \begin{cases} s_i(t) & \text{for } t \in [0, T_{FFT}] \\ uw(t+T_{FFT}) & \text{for } t \in [-T_G, 0] \\ 0 & elsewhere \end{cases} \qquad (3)$$

Note that this representation includes not only the UW within T_{FFT} but also the UW from the previous block. With this extended block the linear convolution of the i-th block with the channel impulse response becomes a circular convolution ($\otimes$) and the received block $\hat{r}_i(t)$ fulfils the condition

$$\hat{r}_i(t) = \hat{s}_i(t) * h(t) = s_i(t) \otimes h(t) \qquad (4)$$

within the interval $[-T_G+T_h,\ T_{FFT}]$. When restricting the received block to the time interval $[0,\ T_{FFT}]$ and applying the theorem of circular convolution to (4) we obtain the essential relation

$$\hat{R}_i(nf_0) = S_i(nf_0) \cdot H(nf_0) = R_i(nf_0) \qquad (5)$$

for $f_0=1/T_{FFT}$ and $n \in Z$. $R_i(f)$ denotes the Fourier transform of the received block $r_i(t)$, which would result from transmission of the original block $s_i(t)$ over the channel $h(t)$. The frequency domain relation (5) shows that the concept of UW is comparable to the concept of CP. Fig. 4 shows the data structure, as it is necessary when using a UW instead of a CP.

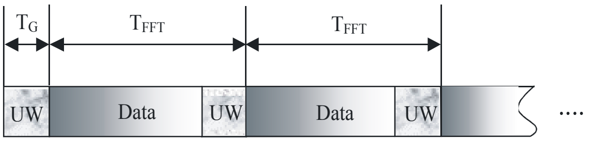

Fig. 4. Transmitted data structure using a Unique Word.

Two main differences are obvious when comparing the two concepts:

1) The UW is not random as the CP.

2) The UW is not removed at the receiver but is available after the equalization in the time domain.

C. Comparison of CP and UW in Terms of Bandwidth Efficiency and BER Behaviour

The bandwidth efficiency is reduced for a SC/FDE by the roll-off factor r on the one hand and by the guard period on the other hand. A simple estimation of the bandwidth efficiency of the described SC/FDE-CP system without taking coding into account can be given as:

$$\eta_{CP} = \frac{M}{1+r}\left(\frac{T_{FFT}}{T_{FFT}+T_G}\right)\left[\frac{bit/s}{Hz}\right]. \qquad (6)$$

M describes the number of bits per symbol. In comparison to this, the bandwidth efficiency of the new structure has to be calculated as follows:

$$\eta_{UW} = \frac{M}{1+r}\left(\frac{T_{FFT}-T_G}{T_{FFT}}\right)\left[\frac{bit/s}{Hz}\right]. \qquad (7)$$

This leads to an additional degradation of 4% in terms of bandwidth efficiency, assuming T_G to be 20% of $T_{FFT.}$ Furthermore a loss in terms of the BER behaviour is expected, based on the changed relation between T_{FFT} and T_G. In comparison to a single carrier system with time domain equalization a loss of about 0.75 dB for the CP and about 0.9 dB for the UW are expected, as a result of the additional overhead.

IV. APPLICATION OF THE NEW STRUCTURE – AN OVERVIEW

Transmission over multipath channels makes initial channel estimation necessary for synchronization and equalization purposes. Due to the fact that the UW is known (and can be chosen in a preferred way) and is processed as any information data, it can be used for equalization, channel estimation, or synchronization purposes. In this paper some results will be given for synchronization and equalization.

A. Synchronization and Channel Estimation

Reliable synchronization and channel estimation are indispensable criterions for high data rate wireless transmission. Initial channel estimation and block synchronization can be done by a preamble at the beginning of every burst (e.g. Hiperlan 2, IEEE 802.11a), while time varying channels, clock-frequency-offsets or carrier-frequency-offsets make tracking necessary. Tracking is mainly based on the insertion of pilot symbols or sequences. Implementing the structure of UW, pilot sequences are available automatically.
The consequence of a clock-frequency-offset is, that the sampling time between transmitter and receiver varies and leads to a rising displacement of the FFT-window. An auto-correlation (as stated in (8)) of two consecutive, received UWs may result in distinctive correlation peaks if the symbols of the UW are chosen as to have good correlation properties (e.g. Pseudo noise sequences, Barker sequences).

$$\varphi_{rr}(k) = \left| \sum^{N_G} \{ r_k \cdot r^*_{k+N} \} \right| \qquad (8)$$

r_k* indicates the complex conjugate of r_k. Nevertheless it is to mention, that if the time duration of the UW is too short on the one hand and due to the fact that the UW is corrupted by the cannel and noise on the other hand, a simple correlation of two successive UWs do not show reliable enough correlation peaks – the autocorrelation properties of the investigated sequences are partly lost.
Methods that are not based on special sequences were developed for OFDM and can be implemented for SC/FDE too. In [8],[9] it is shown that the log-likelihood function for the time instant θ, given the observation r_k, is:

$$\Lambda(\theta) = \Re e\left\{ 2 \sum_{k=\theta}^{\theta+N_G-1} r_k \cdot r^*_{k+N} \right\} - \frac{SNR}{SNR+1} \cdot \sum_{k=\theta}^{\theta+N_G-1} |r_k|^2 + |r_{k+N}|^2 \qquad (9)$$

The maximum likelihood estimation of θ, given r_k, is the argument maximizing (9). This approach is independent of the used sequence itself in general, but can be combined with sequences that show good correlation properties, as it is possible when using a UW. This method shows well defined correlation peaks, especially if a continued, weighted averaging over UWs is carried out. Fig. 5 shows the result that indicates the beginning of every FFT-window very precisely. The simulations haven been performed for $E_b/N_0 =$ 12 dB and multi path conditions. The indoor radio channels have been created, using the model of the 802.11a standardization process.

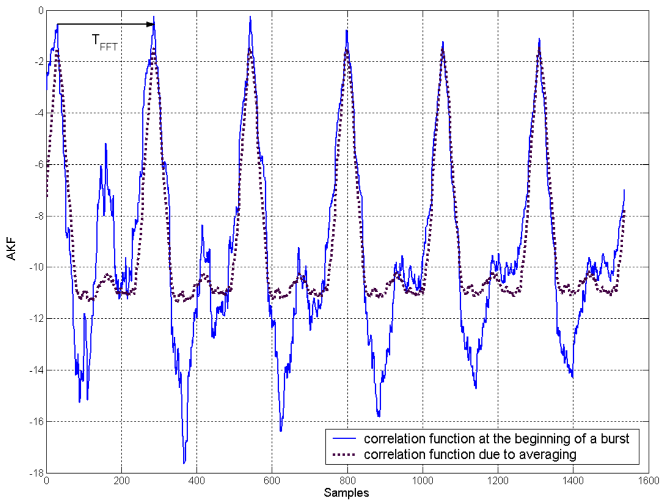

Fig. 5. Synchronization and tracking of the FFT-window.

The correlation of two consecutive UWs can be used additionally to compensate a carrier-frequency offset. Due to a carrier-frequency offset, the phase of two symbols of successive UWs varies by an amount of $2\pi\Delta fNT$.

$$r_{k+N} = r_k e^{-j2\pi\Delta fNT} \qquad (10)$$

To reduce the influence of noise, averaging of the measured phase difference is necessary, as defined in (11). The estimated carrier-frequency offset is given by

$$\Delta f_{averaged} = \frac{\arg(\sum^{NG} r_k \cdot r^*_{k+N})}{2\pi NT}. \qquad (11)$$

The rotation of the phase during one block is only resolvable as long it is less than ±π. This leads to a correctable frequency offset of 117 kHz for the investigated system (T_{FFT}=4.27 µs). Figure 6a) shows an I/Q diagram due to a frequency offset of 95 kHz, while Figure 6b) shows the I/Q diagram where the carrier-frequency offset has been estimated and corrected. For both simulations E_b/N_0=12 dB was used.

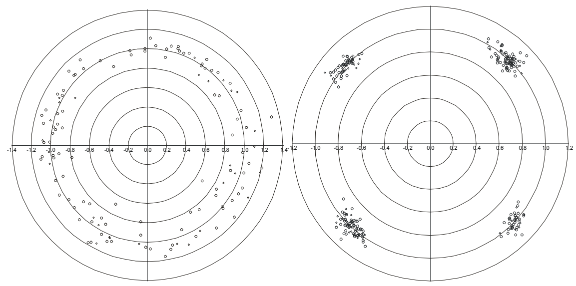

Fig. 6. a) I/Q Diagram due to carrier-frequency-offset, b) corrected carrier-frequency offset.

While for synchronization purposes the influence of the channel impulse response is undesirable because the correlation properties are lost partly, this influence of the channel is required if a UW is used for channel estimation. As long as the duration T_G of the CP or UW is longer than the duration T_h of the channel impulse response *h(t)*, the entire channel information is included in a UW. At least two adjacent UWs are necessary to implement a channel estimation [10]. Frank-Zadoff or Chu sequences fit best for this application. An advantageous implementation could be an initial channel estimation and a continued tracking with the used UW.

B. Equalization

A significant improvement in terms of BER can be reached when using decision feedback equalization (DFE). The combination of a frequency domain feed forward filter (FFF) and a time domain decision feedback filter (DFF) combines the advantages of reduced computational complexity and improved performance.

The problem with the DFE and the concept of CP is, that due to the cyclic convolution symbols at the beginning of an equalized block are dependent on symbols from the end of the same block that are not known yet and consequently can not be fed back. This problem is solved by using the concept of UW. A symbol at the beginning of a block is still dependent on symbols at the end of the block, but these symbols are part of a UW and are known in advance. Fig. 7

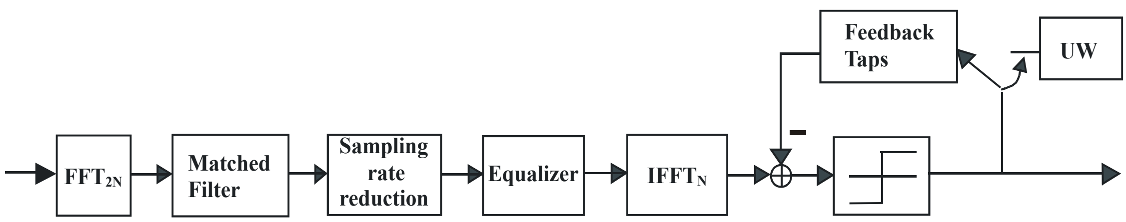

Fig. 7. Decision Feedback Equalizer.

shows the possible concept of a frequency domain feed forward filter and a time domain decision feedback filter. Decision feedback equalizers have to face the problem of error propagation – a wrong decided symbol leads to higher error probabilities for the following decisions and may reduce the benefits in terms of BER significantly. The concept of UW makes error propagation beyond one FFT-block impossible since the last symbols of every block (UW) are always decided correctly.

Exemplary bit error rate simulations have been performed for indoor multipath conditions. Fig. 8 shows the simulation results for different types of equalizers. An additional improvement of up to 6 dB at a BER of 10^{-5} is reached in comparison to linear equalizers when using the structure shown in Fig. 7. In comparison to this, the DFE that is based on CP suffers from a higher BER at the beginning of every block and stronger error propagation. Nevertheless it is essential to mention that the possible reached gain depends strongly on the radio indoor channel.

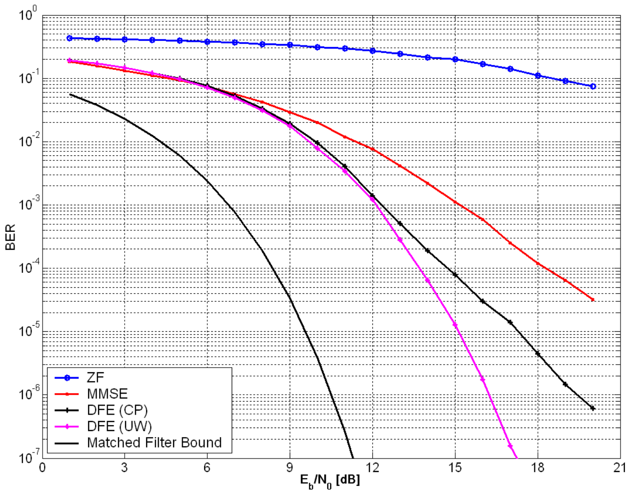

Fig. 8. BER simulation results for a linear Zero Forcing (ZF) Equalizer, a linear Minimum Mean Square Error (MMSE) and an MMSE Decision Feedback Equalizer (DFE) using CP and UW.

V. CONCLUSION

It has been shown that for the concept of UW the theorem of cyclic convolution is fulfilled and that it leads only to minor degradations in terms of bandwidth efficiency and BER behaviour. The advantages that can be derived from this structure compensate the disadvantages by far. The advantages are based on the knowledge of the UW, that it does not change during transmission and the fact that it is processed instead of removed at the receiver. The investigated structure supports easy channel estimation, synchronization and tracking of time and frequency offset. It further more enables a correct realization of a DFE. It can be concluded that the adoption of techniques initially associated with OFDM, like the cyclic prefix or frequency domain equalization, by conventional single carrier systems can boost the latter to make it equivalent to OFDM in principle and outperform it in some points.

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