Link Scheduling Algorithms for Wireless Mesh Networks

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***Abstract*—We provide an overview of link scheduling algorithms in Spatial Time Division Access (STDMA) wireless mesh networks. These algorithms can be classified into three categories: those based only on a communication graph model of the network, those based on communication graph and verifying Signal to Interference and Noise Ratio (SINR) threshold conditions at receivers and those based on an SINR graph model of the network. We outline a framework for modeling STDMA networks. We review representative research works from each of these classes. Finally, we describe the relative merits and demerits of each class of algorithms.**

I. INTRODUCTION

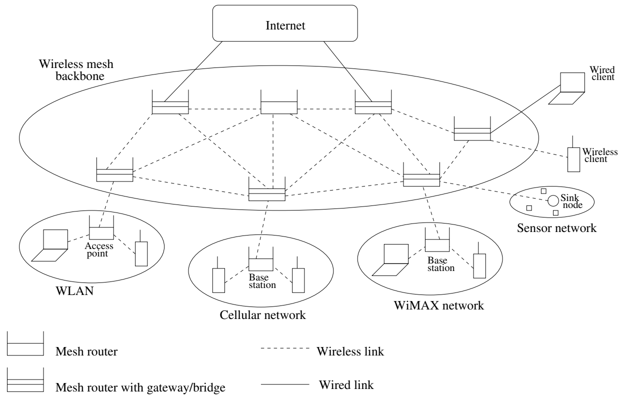


Fig. 1. Wireless mesh network, adapted from [1].

Wireless and mobile communications have revolutionized the way we communicate over the past decade. This impact has been felt both in voice communications and wireless Internet access. The ever-increasing need for applications like video and images have driven the need for technologies like 3^{rd} Generation Partnership Project Long Term Evolution (3GPP LTE), 3^{rd} Generation Partnership Project 2 (3GPP2), IEEE 802.16 Worldwide Interoperability for Microwave Access (WiMAX) networks and IEEE 802.11 Wireless Local Area Networks (WLANs) which promise broadband data rates to wireless users.

Wireless networks can be broadly classified into cellular networks and ad hoc networks. A wireless ad hoc network is a collection of wireless nodes that can dynamically self-organize into an arbitrary topology to form a network without necessarily using any pre-existing infrastructure. Based on their application, ad hoc networks can be further classified into Mobile Ad Hoc Networks (MANETs), wireless mesh networks and wireless sensor networks.

A Wireless Mesh Network (WMN) can be considered to be an infrastructure-based ad hoc network with a mesh backbone carrying most of the traffic. WMNs have been recently advocated to provide connectivity and coverage, especially in sparsely populated and rural areas. For example, several Wireless Community Networks (WCNs) are operational in Europe, Australia and USA [2]. Peer to peer wireless technology is also being developed by companies such as [3]. WMNs are dynamically self-organized and self-configured, with nodes in the network automatically establishing an ad hoc network and maintaining mesh connectivity [1]. An example of a WMN is shown in Figure 1. Typically, a node in a WMN can be a Mesh Router (MR) or a Mesh Client (MC). An MR consists of gateway/bridge functions and the capability to support mesh networking. MRs have little or no mobility and form a wireless backbone for MCs. The gateway/bridge functionalities in MRs aid in the integration of WMNs with heterogeneous networks such as Ethernet [4], cellular networks, WLANs [5], WiMAX networks [6] and sensor networks. WMNs are witnessing commercialization in various applications like broadband home networks, enterprise networks, community networks and metropolitan area networks. Moreover, WMNs diversify the functionalities of ad hoc networks, instead of just being another type of ad hoc network. These additional functionalities necessitate novel design principles and efficient algorithms for the realization of WMNs.

Significant research efforts are required to realize the full potential of WMNs. Among the many challenging issues in the design of WMNs, the design of the physical as well as the Medium Access Control (MAC) layers is important, especially from a perspective of achieving high network throughput. At the physical layer, techniques like adaptive modulation and coding, Orthogonal Frequency Division Multiplexing (OFDM) [7], [8] and Multiple Input Multiple Output (MIMO) techniques [9] can be used to increase the capacity of a wireless channel and achieve high data transmission rates. At the MAC layer, various solutions like directional antenna based MAC [10], MAC with power control [11] and multi-channel MAC [12] have been proposed in the literature.

In a WMN, a packet from a source MR to destination MR is usually relayed via intermediate MRs due to transmission power constraints. This requires sequential link transmissions and thus a WMN is multihop in nature. Moreover, multiple (concurrent) link transmissons corresponding to different source-destination pairs are required to achieve high network throughput. This leads us to the problem of routing and link scheduling which can be solved jointly [13] or sequentially. For simplicity, we assume that the routes in a WMN are pre-determined by a routing algorithm and focus on link scheduling aspects only. Specifically, we focus on MAC layer design for Spatial Time Division Multiple Access (STDMA) wireless networks.

An STDMA wireless network can be thought of as a mesh network wherein we allow concurrent communication between nodes that are "reasonably far" from each other, i.e., we exploit spatial reuse. An STDMA network abstracts the wireless mesh backbone (consisting of MRs) in a WMN. Since most traffic is carried by the WMN backbone, techniques that deliver high network throughput in an STDMA network can be easily translated to WMNs. Such techniques have the potential of achieving high network throughput in networks such as WiMAX mesh networks.

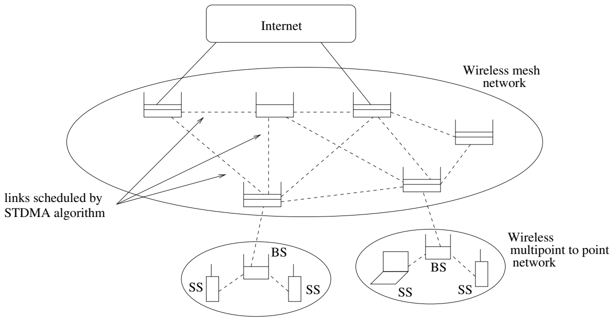


Fig. 2. Potential applications of link scheduling in wireless networks.

An STDMA link schedule describes the transmission rights for each time slot in such a way that communicating entities assigned to the same slot do not "collide". Link scheduling algorithms can be implemented at the MAC layer of wireless mesh networks, as shown in Figure 2.

An STDMA link schedule should be so designed that, in every time slot, all packets transmitted by the scheduled transmitters are received successfully at the corresponding (intended) receivers. Two models have been proposed in literature for specifying the criteria for successful packet reception. According to the protocol interference model [14], a packet is received successfully at a receiver only if its intended transmitter is within the *communication range* and other unintended transmitters are outside the *interference range* of the receiver. In essence, the protocol interference model mandates a "silence zone" around every scheduled receiver in a time slot. On the other hand, according to the physical interference model [14], a packet is received successfully at a receiver only if the SINR at the receiver is no less than a certain threshold, called *communication threshold*.

Link scheduling algorithms that employ the protocol interference model seek to minimize the schedule length so as to maximize network throughput. They model the network by a communication graph and employ novel techniques to color all the edges of the graph using minimum number of colors [15]. Consequently, such algorithms have the advantage of low computational complexity (in general). However, these algorithms do not consider SINR threshold conditions at a receiver and can lead to low network throughput.

On the other hand, link scheduling algorithms that employ the physical interference model, provide a reasonably accurate representation of the wireless network and aim to maximize the number of successful packet transmissions per time slot. These algorithms take into account wireless channel effects like propagation path loss, fading and shadowing, as well as SINR conditions at a receiver. However, such algorithms tend to have higher computational complexity.

In this paper, we outline a framework for modeling STDMA link scheduling algorithms. We consider a general representation of an STDMA wireless network, i.e., this model is not specific to any technology or protocol. This abstraction lends simplicity to the network model and helps us understand the design of scheduling algorithms for the network. Since the problem of determining an optimal link schedule is NP-hard [15], researchers have proposed various heuristics to obtain close-to-optimal solutions. In our view, such heuristics can be broadly classified into three categories: algorithms based on modeling the network by a two-tier or communication graph, "hybrid" algorithms based on modeling the network by a communication graph and verifying SINR conditions and algorithms based on modeling the network by an SINR graph.

The rest of this paper is structured as follows. In Section II, we describe the system model of an STDMA wireless network and explain the protocol and physical interference models. In Section III, we elucidate the equivalence between a point to point link schedule for an STDMA network and the colors of edges of the communication graph model of the network. This is followed by a review of research work on point to point link scheduling algorithms based on the protocol interference model. In Section IV, we describe the limitations of algorithms based on the protocol interference model from a perspective of maximizing network throughput in wireless networks. We review research work on link scheduling algorithms based on the physical interference model in Sections V and VI. Specifically, Section V reviews algorithms based on communication graph model of the network and SINR conditions, while Section VI reviews algorithms based on an SINR graph model of the network.

II. SYSTEM MODEL

We consider a general model of an STDMA wireless network with N immobile store-and-forward nodes in a two-dimensional plane, where N is a positive integer. Nodes are indexed as $1, 2, \ldots, N$. In a wireless network, a link is an ordered pair of nodes (t, r), where t is a transmitter and r is a receiver. We assume equal length packets. Time is divided into slots of equal duration. During a time slot, a node can either transmit, receive or remain idle. The slot duration equals the amount of time it takes to transmit one packet over the wireless channel. We make the following additional assumptions:

- Synchronized nodes: All nodes are synchronized to slot boundaries.
- Homogeneous nodes: Every node has identical receiver sensitivity, transmission power and thermal noise characteristics.
- Backlogged nodes: We assume a node to be continuously backlogged, i.e., a node always has a packet to transmit and cannot transmit more than one packet in a time slot.

Let:

$$\begin{aligned}(x_j, y_j) &= \text{Cartesian coordinates of node } j =: \mathbf{r}_j,\\ P &= \text{power with which a node transmits its packet},\\ N_0 &= \text{thermal noise power spectral density},\\ D(j,k) &= \text{Euclidean distance between nodes } j \text{ and } k.\end{aligned}$$

The received signal power at a distance D from the transmitter is given by $\frac{P}{D^\beta}$, where β is the path loss exponent. An STDMA link schedule is a mapping from the set of links to time slots. We only consider static link schedules, i.e., link schedules that repeat periodically throughout the operation of the network. Let C denote the number of time slots in a link schedule, i.e., the *schedule length.* For a given time slot i, j^{th} communicating transmitter-receiver pair is denoted by $t_{i,j} \to r_{i,j}$, where $t_{i,j}$ denotes the index of the node which transmits a packet and $r_{i,j}$ denotes the index of the node which receives the packet. Let M_i denote the number of concurrent transmitter-receiver pairs in time slot i. A link schedule for the STDMA network is denoted by $\Psi(\mathcal{S}_1, \cdots, \mathcal{S}_C)$, where

$$\begin{aligned}\mathcal{S}_i &:= \{t_{i,1} \to r_{i,1}, \cdots, t_{i,M_i} \to r_{i,M_i}\}\\ &= \text{set of transmitter-receiver pairs which can}\\ &\ \text{communicate concurrently in time slot } i.\end{aligned}$$

Note that a link schedule repeats periodically throughout the operation of the network. More specifically, transmitter-receiver pairs that communicate concurrently in time slot i also communicate concurrently in time slots $i + C$, $i + 2C$ and so on. Thus, $\mathcal{S}_i = \mathcal{S}_{i \pmod{C}}$. Finally, note that all transmitters and receivers are stationary.

Every link schedule must satisfy the following:

- Operational constraint: During a time slot, a node can transmit to exactly one node, receive from exactly one node or remain idle, i.e.,

$$\begin{aligned}&\{t_{i,j}, r_{i,j}\} \cap \{t_{i,k}, r_{i,k}\} = \phi\\ &\forall\ i = 1, \ldots, C\ \forall\ 1 \leqslant j < k \leqslant M_i.\end{aligned} \tag{1}$$

As an illustration, consider the STDMA wireless network shown in Figure 3(a). It consists of six nodes whose coordinates (in meters) are $1 \equiv (-40, 5)$, $2 \equiv (0, 0)$, $3 \equiv (95, 0)$, $4 \equiv (135, 0)$, $5 \equiv (-75, 0)$ and $6 \equiv (0, -75)$. An example link schedule for this STDMA network is shown in Figure 3(b). Note that this schedule is only one of the several possible schedules and is given here only for illustrative purposes. The schedule length is $C = 8$ time slots and the schedule is defined by $\Psi(\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4, \mathcal{S}_5, \mathcal{S}_6, \mathcal{S}_7, \mathcal{S}_8)$, where

$$\begin{aligned}\mathcal{S}_1 &= \{t_{1,1} \to r_{1,1}\}\\ &= \{1 \to 2\},\\ \mathcal{S}_2 &= \{t_{2,1} \to r_{2,1},\ t_{2,2} \to r_{2,2},\ t_{2,3} \to r_{2,3}\}\\ &= \{3 \to 4,\ 5 \to 1,\ 2 \to 6\},\\ \mathcal{S}_3 &= \{t_{3,1} \to r_{3,1}\}\\ &= \{3 \to 2\},\\ \mathcal{S}_4 &= \{t_{4,1} \to r_{4,1},\ t_{4,2} \to r_{4,2},\ t_{4,3} \to r_{4,3}\}\\ &= \{4 \to 3,\ 6 \to 2,\ 1 \to 5\},\\ \mathcal{S}_5 &= \{t_{5,1} \to r_{5,1},\ t_{5,2} \to r_{5,2}\}\\ &= \{2 \to 5,\ 6 \to 1\},\\ \mathcal{S}_6 &= \{t_{6,1} \to r_{6,1}\}\\ &= \{2 \to 1\},\\ \mathcal{S}_7 &= \{t_{7,1} \to r_{7,1},\ t_{7,2} \to r_{7,2}\}\\ &= \{1 \to 6,\ 5 \to 2\},\\ \mathcal{S}_8 &= \{t_{8,1} \to r_{8,1}\}\\ &= \{2 \to 3\}.\end{aligned}$$

After 8 time slots, the schedule repeats periodically, as shown in Figure 3(b).

A scheduling algorithm is a set of rules that is used to determine a link schedule $\Psi(\cdot)$. Usually, a scheduling algorithm needs to satisfy certain objectives.

Consider j^{th} receiver in time slot i, i.e., receiver $r_{i,j}$. The power received at $r_{i,j}$ from its intended transmitter $t_{i,j}$ (signal power) is $\frac{P}{D^\beta(t_{i,j}, r_{i,j})}$. Similarly, the power received at $r_{i,j}$ from its unintended transmitters (interference power) is $\sum_{\substack{k=1\\k\neq j}}^{M_i} \frac{P}{D^\beta(t_{i,k}, r_{i,j})}$. Thus, the Signal to Interference and Noise Ratio (SINR) at receiver $r_{i,j}$ is given by

$$\mathrm{SINR}_{r_{i,j}} = \frac{\frac{P}{D^\beta(t_{i,j}, r_{i,j})}}{N_0 + \sum_{\substack{k=1\\k\neq j}}^{M_i} \frac{P}{D^\beta(t_{i,k}, r_{i,j})}}. \tag{2}$$

Without considering the interference power, the Signal to Noise Ratio (SNR) at receiver $r_{i,j}$ is given by

$$\mathrm{SNR}_{r_{i,j}} = \frac{P}{N_0 D^\beta(t_{i,j}, r_{i,j})}. \tag{3}$$

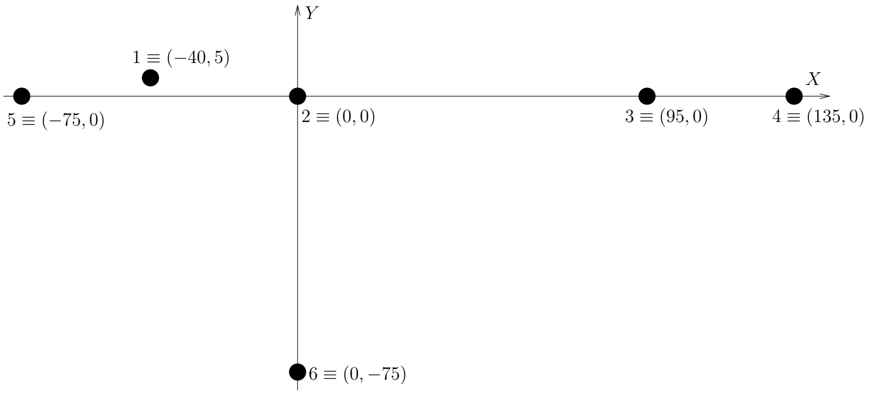
According to the *protocol interference model* [14], transmission $t_{i,j} \to r_{i,j}$ is successful if:

1) the SNR at receiver $r_{i,j}$ is no less than a certain threshold γ_c, termed as the *communication threshold.* From (3), this translates to

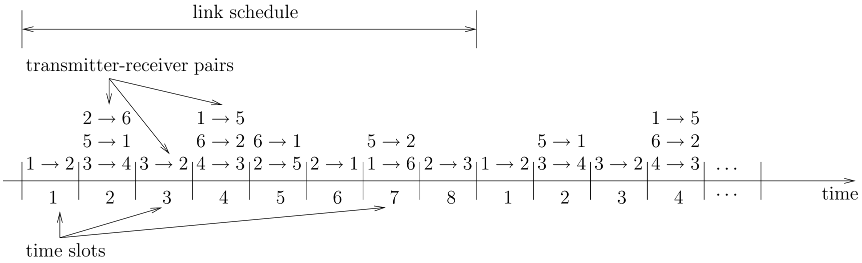
$$D(t_{i,j}, r_{i,j}) \leqslant \left(\frac{P}{N_0 \gamma_c}\right)^{\frac{1}{\beta}} =: R_c, \tag{4}$$

where R_c is termed as communication range, and

2) the signal from any unintended transmitter $t_{i,k}$ is received at $r_{i,j}$ with an SNR less than a certain threshold



(a) An STDMA wireless network with six nodes.



(b) A link schedule for network shown in Figure 3(a).

Fig. 3. Example of STDMA network and link schedule.

γ_i, termed as the *interference threshold*. From (3), this translates to

$$D(t_{i,k}, r_{i,j}) > \left(\frac{P}{N_0\gamma_i}\right)^{\frac{1}{\beta}} =: R_i$$
$$\forall\ k = 1, \dots, M_i,\ k \neq j, \tag{5}$$

where R_i is termed as interference range.

In essence, the transmission on a link is successful if the distance between the nodes is less than or equal to the *communication range* and no other node is transmitting within the *interference range* from the receiver.

The STDMA network is denoted by $\Phi(N, (\mathbf{r}_1, \dots, \mathbf{r}_N), P, \gamma_c, \gamma_i, \beta, N_0)$. Note that $0 < \gamma_i < \gamma_c$, thus $R_i > R_c$. The relation $R_i = 2R_c$ is widely assumed in literature [16], [17], [18], [19].

According to the *physical interference model* [14], the transmission on a link is successful if the SINR at the receiver is greater than or equal to the communication threshold γ_c. More specifically, the physical interference model states that transmission $t_{i,j} \rightarrow r_{i,j}$ is successful if:

$$\frac{\frac{P}{D^{\beta}(t_{i,j}, r_{i,j})}}{N_0 + \sum_{\substack{k=1\\k\neq j}}^{M_i} \frac{P}{D^{\beta}(t_{i,k}, r_{i,j})}} \geqslant \gamma_c. \tag{6}$$

Note that the physical interference model is less restrictive but more complex. Usually, this representation has been employed to model mesh networks with TDMA like access mechanisms [20].

A link schedule $\Psi(\cdot)$ is *conflict-free* if the SINR at every intended receiver does not drop below the communication threshold, i.e.,

$$\text{SINR}_{r_{i,j}} \geqslant \gamma_c\ \ \forall\ i = 1, \dots, C,\ \ \forall\ j = 1, \dots, M_i. \tag{7}$$

III. Link Scheduling based on Protocol Interference Model

A. Equivalence of Link Scheduling and Graph Edge Coloring

In this section, we describe the communication and two-tier graph representations of an STDMA wireless network. We explain the equivalence between a link schedule for the STDMA network and the colors of edges of the communication graph representation of the network, and illustrate this equivalence with an example.

The STDMA network $\Phi(\cdot)$ can be modeled by a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of vertices and $\mathcal{E}$ is the set of edges. Let $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$, where vertex v_j represents node j in $\Phi(\cdot)$. In the graph representation, if node k is within node j's communication range, then there is an edge from v_j to v_k, denoted by $v_j \xrightarrow{c} v_k$ and termed as communication edge. Similarly, if node k is outside node j's communication range but within its interference range, then there is an edge from v_j to v_k, denoted by $v_j \xrightarrow{i} v_k$ and termed as interference edge. Thus, $\mathcal{E} = \mathcal{E}_c \cup \mathcal{E}_i$, where $\mathcal{E}_c$ and $\mathcal{E}_i$ denote the set of communication and interference edges respectively. The *two-tier graph* representation of the STDMA network $\Phi(\cdot)$ is defined as the graph $\mathcal{G}(\mathcal{V}, \mathcal{E}_c \cup \mathcal{E}_i)$ comprising of all vertices and both communication and interference edges. The *communication graph* representation of the STDMA network $\Phi(\cdot)$ is defined as the graph $\mathcal{G}_c(\mathcal{V}, \mathcal{E}_c)$ comprising of all vertices and communication edges only. We will illustrate these representations with an example.

Parameter	Symbol	Value
transmission power	P	10 mW
path loss exponent	β	4
noise power spectral density	N_0	-90 dBm
communication threshold	γ_c	20 dB
interference threshold	γ_i	10 dB

TABLE I
SYSTEM PARAMETERS FOR STDMA NETWORKS SHOWN IN FIGURES 3(A), 7 AND 12.

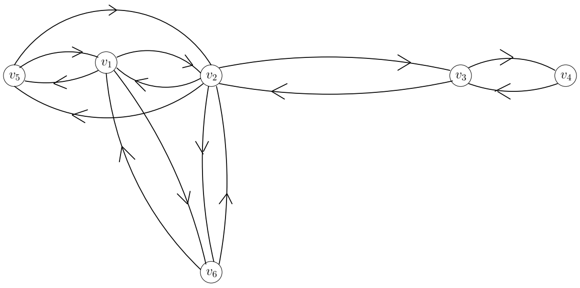


Fig. 4. Communication graph model of STDMA network described by Figure 3(a) and Table I.

Consider the STDMA wireless network $\Phi(\cdot)$ whose deployment is shown in Figure 3(a). The system parameters for this network are given in Table I. From (4) and (5), it can be easily shown that $R_c = 100$ m and $R_i = 177.8$ m. The corresponding communication graph representation $\mathcal{G}_c(\mathcal{V}, \mathcal{E}_c)$ is shown in Figure 4. The communication graph comprises of 6 vertices and 14 directed communication edges. The vertex and communication edge sets are given by

$$\begin{aligned} \mathcal{V} &= \{v_1, v_2, v_3, v_4, v_5, v_6\}, && (8) \\ \mathcal{E}_c &= \{v_1 \xrightarrow{c} v_2, v_2 \xrightarrow{c} v_1, v_1 \xrightarrow{c} v_5, v_5 \xrightarrow{c} v_1, v_1 \xrightarrow{c} v_6, \\ &\quad v_6 \xrightarrow{c} v_1, v_2 \xrightarrow{c} v_5, v_5 \xrightarrow{c} v_2, v_2 \xrightarrow{c} v_6, v_6 \xrightarrow{c} v_2, \\ &\quad v_2 \xrightarrow{c} v_3, v_3 \xrightarrow{c} v_2, v_3 \xrightarrow{c} v_4, v_4 \xrightarrow{c} v_3\}. && (9) \end{aligned}$$

The two-tier graph model $\mathcal{G}(\mathcal{V}, \mathcal{E}_c \cup \mathcal{E}_i)$ of the STDMA network $\Phi(\cdot)$ is shown in Figure 5. The two-tier graph comprises of 6 vertices, 14 directed communication edges and 10 directed interference edges. The vertex and communication edge sets are given by (8) and (9) respectively, while the interference edge set is given by

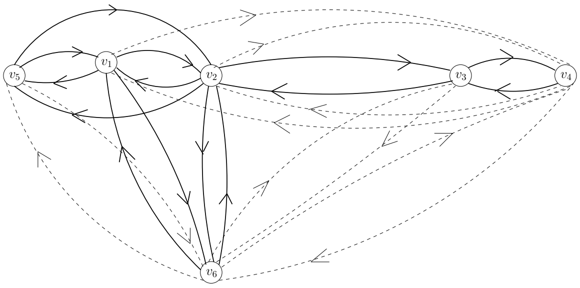


Fig. 5. Two-tier graph model of STDMA network described by Figure 3(a) and Table I.

$$\begin{aligned} \mathcal{E}_i &= \{v_1 \xrightarrow{i} v_4, v_4 \xrightarrow{i} v_1, v_2 \xrightarrow{i} v_4, v_4 \xrightarrow{i} v_2, \\ &\quad v_3 \xrightarrow{i} v_6, v_6 \xrightarrow{i} v_3, v_4 \xrightarrow{i} v_6, v_6 \xrightarrow{i} v_4, \\ &\quad v_5 \xrightarrow{i} v_6, v_6 \xrightarrow{i} v_5, \}. && (10) \end{aligned}$$

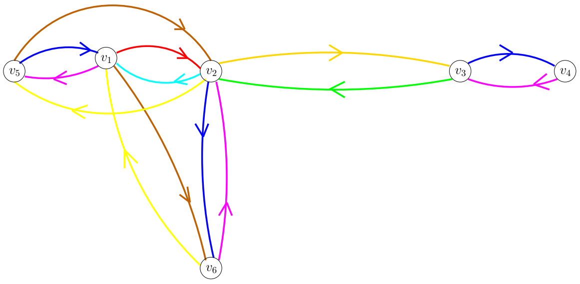


Fig. 6. Edge coloring of communication graph shown in Figure 4 corresponding to the link schedule shown in Figure 3(b).

Given the above representations, a link schedule $\Psi(\cdot)$ for an STDMA wireless network $\Phi(\cdot)$ can be considered as equivalent to assigning a unique color to every edge in the communication graph, such that transmitter-receiver pairs with the same color transmit simultaneously in a particular time slot. For the example network considered, the link schedule shown in Figure 3(b) corresponds to coloring of the edges of the communication graph shown in Figure 6. Time slots 1, 2, 3, 4, 5, 6, 7 and 8 in $\Psi(\cdot)$ correspond to colors red, blue, green, magenta, yellow, cyan, brown and gold in $\mathcal{E}_c$ respectively. Note that a coloring algorithm that uses the least number of colors also minimizes the schedule length. This aspect is further addressed in subsequent sections.

B. *Review of Algorithms*

In this section, we provide an overview of past research in the field of STDMA link scheduling algorithms based on the protocol interference model. The protocol interference model is widely studied in literature because of its simplicity. It has been usually employed to model networks such as Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) based WLANs[1] [20], [18]. Centralized algorithms [15], [21], [22], [13], [18] as well as distributed algorithms [23] have been proposed for generating link schedules based on the protocol interference model.

A link scheduling algorithm based on the protocol interference model utilizes a communication or two-tier graph model of the STDMA network to determine a link schedule [24], [25]. Algorithms based on the protocol interference model for assigning links to time slots (equivalently, colors) require that two communication edges $v_i \xrightarrow{c} v_j$ and $v_k \xrightarrow{c} v_l$ can be colored the same if and only if:

i) vertices v_i, v_j, v_k, v_l are all mutually distinct, i.e., there is no *primary edge conflict*, and

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[1] Consider an IEEE 802.11 based WLAN wherein CSMA with RTS/CTS/ACK is used to protect unicast transmissions. Due to carrier sensing, a transmission between nodes j and k may block all transmissions that are within a distance of R_i from either j (due to sensing RTS and DATA) or k (due to sensing CTS and ACK).

ii) $v_i \rightarrow v_l \notin \mathcal{G}(\cdot)$ and $v_k \rightarrow v_j \notin \mathcal{G}(\cdot)$, i.e, there is no *secondary edge conflict*.

The first criterion is based on the operational constraint (1). The second criterion states that a node cannot receive a packet if it lies within the interference range of any other transmitting node. A scheduling algorithm utilizes various graph coloring methodologies to obtain a non-conflicting link schedule, i.e., a link schedule devoid of primary and secondary edge conflicts.

To maximize the throughput of an STDMA network, algorithms based on the protocol interference model[2] seek to minimize the total number of colors used to color all the communication edges of $\mathcal{G}(\cdot)$. This will in turn minimize the schedule length. It is well known that for an arbitrary communication graph, the problem of determining a minimum length schedule (optimal schedule) is NP-hard [15], [22]. Hence, the approach followed in the literature is to devise algorithms that produce close to optimal (sub-optimal) solutions. The efficiency of a sub-optimal algorithm is typically measured in terms of its computational (run time) complexity and performance guarantee (approximation factor).

The concept of STDMA for wireless networks was formalized in [21]. The authors assume a multihop packet radio network with fixed node locations and consider the problem of assigning an integral number of slots to every link in an STDMA cycle (frame). To solve this problem, they model the network by a communication graph, determine a set of maximal cliques and then assign a certain number of slots to all the links in each maximal clique. Finally, the authors develop a fluid approximation for the mean system delay and validate it using simulations.

In [22], the authors consider pre-specified link demands in a spread spectrum packet radio network. They formulate the problem as a linear optimization problem and use the ellipsoid algorithm [26] to solve the problem. They assume that the desired link data rates are rational numbers and develop a strongly polynomial algorithm[3] that computes a minimum length schedule. Finally, they consider the problem of link scheduling to satisfy pre-specified end-to-end demands in the network. They formulate this problem as a multicommodity flow problem and describe a polynomial time algorithm that computes a minimum length schedule. As pointed out by the authors, their algorithm is not practical due to its high computational complexity.

A significant work in link scheduling under protocol interference model is reported in [15], in which the authors show that tree networks can be scheduled optimally, oriented graphs[4] can be scheduled near-optimally and arbitrary networks can be scheduled such that the schedule is bounded by a length proportional to the graph thickness[5] times the optimum number of colors.

In [15], the authors propose ArboricalLinkSchedule, an algorithm that uses a *fresh* set of colors to color each successive oriented graph. Consequently, their algorithm leads to a higher numbers of colors, especially if the number of oriented graphs is large. The authors employ such a heuristic primarily to upper bound the number of colors used by the algorithm ([15], Lemma 3.4) and consequently obtain bounds on the running time complexity and performance guarantee of the algorithm ([15], Theorem 3.3). Though their algorithm has nice theoretical properties such as low computational complexity, it can be shown that it may yield a higher number of colors *in practice* ([27], Chapter 3), which can lead to lower network throughput.

In [19], the authors investigate throughput bounds for a given wireless network and traffic workload under the protocol interference model. They use a conflict graph[6] to represent interference constraints. The problem of determining maximum throughput for a given source-destination pair under the flexibility of multipath routing is formulated as a linear program with flow constraints and conflict graph constraints. They show that this problem is NP-hard and describe techniques to compute lower and upper bounds on throughput. Finally, the authors numerically evaluate throughput bounds and computation time of their heuristics for simple network scenarios and IEEE 802.11 MAC (bidirectional MAC).

In [18], the authors investigate joint link scheduling and routing under the protocol interference model for a wireless mesh network consisting of static mesh routers and mobile client devices. Assuming that $l(u)$ denotes the aggregate traffic demand on node u, they consider the problem of maximizing λ, such that at least $\lambda l(u)$ amount of traffic can be routed from each node u to a fixed gateway node. Since this problem is NP-hard, the authors propose heuristics based on linear programming and re-routing flows on the communication graph. The algorithm in [18] consists of five steps: solve linear program, channel assignment, post processing, flow scaling and interference free link scheduling. They derive the worst case bound of their algorithm and evaluate its performance via simulations.

Another work which jointly investigates link scheduling and routing under protocol interference model is reported in [13]. The authors consider wireless mesh networks with half duplex and full duplex orthogonal channels, wherein each node can transmit to at most one node and/or receive from at most k nodes ($k \geqslant 1$) during any time slot. They investigate the joint problem of routing and scheduling to analyze the achievability of a given rate vector between multiple source-destination pairs. The scheduling algorithm is equivalent to

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[2] Link scheduling algorithms based on the protocol interference model are sometimes referred to as "graph based algorithms" in literature [24], [25]. This term is slightly confusing since scheduling algorithms based on the physical interference model also construct graphs prior to determining a link schedule.

[3] An algorithm is strongly polynomial if (a) the number of arithmetic operations (addition, multiplication, division or comparison) is polynomially bounded by the dimension of the input, and (b) the precision of numbers appearing in the algorithm is bounded by a polynomial in the dimension and precision of the input.

[4] An in-oriented graph is a directed graph in which every vertex has at most one outgoing edge. An out-oriented graph is a directed graph in which every vertex has at most one incoming edge.

[5] The thickness of a graph $\mathcal{G}(\cdot)$ is the minimum number of planar graphs into which $\mathcal{G}(\cdot)$ can be partitioned.

[6] Under the protocol interference model, the conflict graph $F(V_F, E_F)$ is constructed from the communication graph $\mathcal{G}_c(\mathcal{V}, \mathcal{E}_c)$ as follows. Let l_{ij} denote the communication edge $v_i \xrightarrow{c} v_j$. Vertices of $F(\cdot)$ correspond to directed edges l_{ij} in $\mathcal{E}_c$. In $F(\cdot)$, there exists an edge from vertex l_{ij} to vertex l_{pq} if any of the following is true: (a) $D(i,q) \leqslant R_i$ or (b) $D(p,j) \leqslant R_i$.

an edge-coloring on a multi-graph representation[7] and the corresponding necessary conditions lead the routing problem to be formulated as a linear optimization problem. The authors describe a polynomial time approximation algorithm to obtain an ϵ-optimal solution of the routing problem using the primal dual approach. Finally, they evaluate the performance of their algorithms via simulations.

Algorithms based on the protocol interference model represent the network by a communication or two-tier graph and employ a plethora of techniques from graph theory [28] and approximation algorithms [29], [30] to devise heuristics which yield a minimum length schedule. Consequently, such algorithms have the advantage of low computational complexity (in general).

IV. LIMITATIONS OF ALGORITHMS BASED ON PROTOCOL INTERFERENCE MODEL

Due to its inherent simplicity, the protocol interference model has been traditionally employed to represent a wide variety of wireless networks. However, it leads to low network throughput in wireless mesh networks. To emphasize this point, we provide examples to demonstrate that algorithms based on the protocol interference model can result in schedules that yield low network throughput.

Intuitively, the protocol interference model divides the deployment region of the STDMA wireless network into "communication zones" and "interference zones". This transforms the scheduling problem to an edge coloring problem for the communication graph representation of the network. However, this simplification can result in schedules that do not satisfy the SINR threshold condition (7).

Specifically, algorithms based on the protocol interference model do not necessarily maximize the throughput of an STDMA wireless network because:

1) They can lead to high cumulative interference at a receiver, due to hard-thresholding based on communication and interference radii [24], [25]. This is because the SINR at receiver $r_{i,j}$ decreases with an increase in the number of concurrent transmissions M_i, while the communication radius R_c and the interference radius R_i have been defined for a single transmission only.

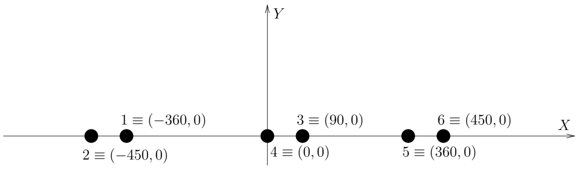

Fig. 7. An STDMA wireless network with six nodes.

For example, consider the STDMA wireless network whose deployment is shown in Figure 7. The network consists of six labeled nodes whose coordinates (in meters) are $1 \equiv (-360, 0)$, $2 \equiv (-450, 0)$, $3 \equiv (90, 0)$, $4 \equiv (0, 0)$, $5 \equiv (360, 0)$ and $6 \equiv (450, 0)$. The system parameters are shown in Table I, which yield $R_c = 100$ m and $R_i = 177.8$ m. The two-tier graph model of the STDMA network is shown in Figure 8; note that interference edges are absent. Consider the transmission requests $1 \to 2$, $3 \to 4$ and $5 \to 6$, which correspond to communication edges of the subgraph shown in Figure 9. The communication edges $v_1 \xrightarrow{c} v_2$, $v_3 \xrightarrow{c} v_4$ and $v_5 \xrightarrow{c} v_6$ shown in Figure 9 do not have primary or secondary edge conflicts. To minimize the number of colors, such an algorithm will color these edges with the same color, as shown in Figure 10. Equivalently, transmissions $1 \to 2$, $3 \to 4$ and $5 \to 6$ will be scheduled in the same time slot, say time slot i. However, our computations show that the SINRs at receivers $r_{i,1}$, $r_{i,2}$ and $r_{i,3}$ are 21.26 dB, 18.42 dB and 19.74 dB respectively. Figure 11 shows the nodes of the network along with the labeled transmitter-receiver pairs, receiver-centric communication and interference zones and the SINRs at the receivers. From the SINR threshold condition (6), transmission $t_{i,1} \to r_{i,1}$ is successful, while transmissions $t_{i,2} \to r_{i,2}$ and $t_{i,3} \to r_{i,3}$ are unsuccessful. This leads to low network throughput.

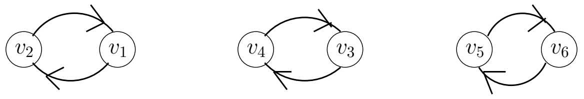

Fig. 8. Two-tier graph model of the STDMA wireless network described by Figure 7 and Table I.

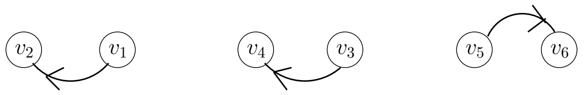

Fig. 9. Subgraph of two-tier graph shown in Figure 8.

2) Moreover, these algorithms can be extremely conservative and result in higher number of colors.
For example, consider the STDMA wireless network whose deployment is shown in Figure 12. The network consists of four labeled nodes whose coordinates (in meters) are $1 \equiv (0, 0)$, $2 \equiv (50, 0)$, $3 \equiv (220, 0)$ and

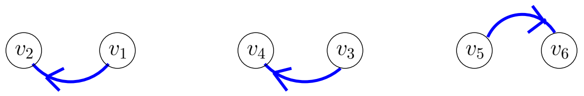

Fig. 10. Coloring of subgraph shown in Figure 9.

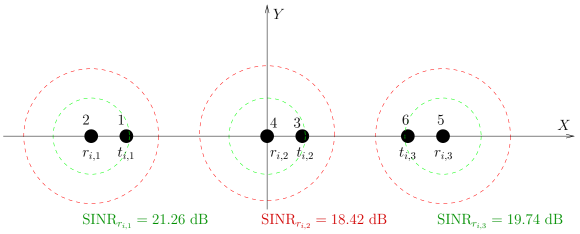

Fig. 11. Link scheduling algorithms based on protocol interference model can lead to high interference.

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[7] A multi-graph is a directed graph in which multiple edges can emanate from a vertex v_i and terminate at another vertex v_j $(v_j \neq v_i)$.

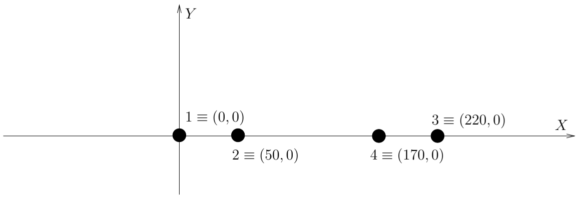

Fig. 12. An STDMA wireless network with four nodes.

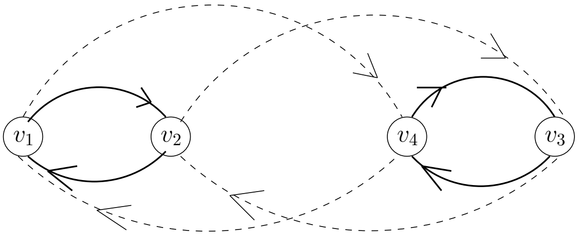

Fig. 13. Two-tier graph model of STDMA wireless network described by Figure 12 and Table I.

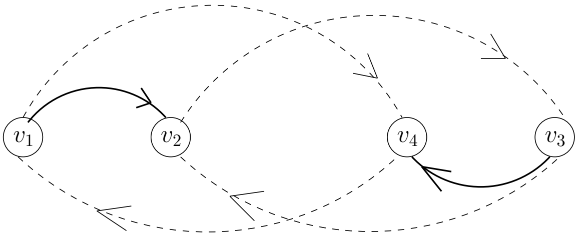

Fig. 14. Subgraph of two-tier graph shown in Figure 13.

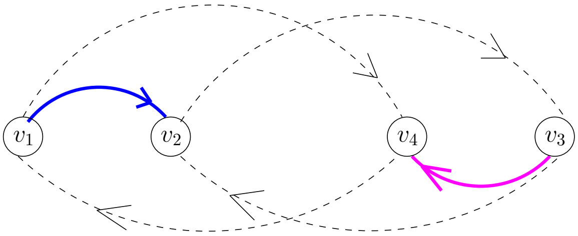

Fig. 15. Coloring of subgraph shown in Figure 14.

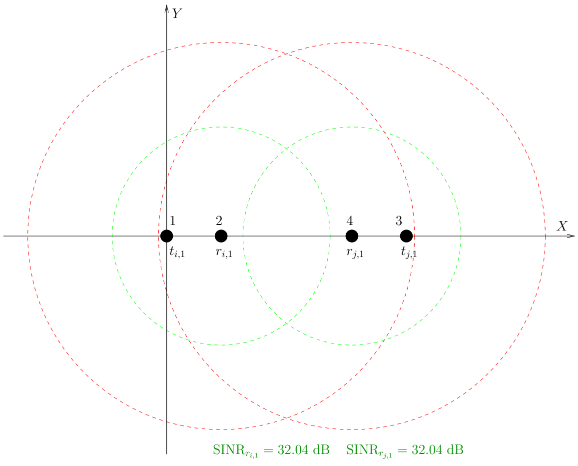

Fig. 16. Link scheduling algorithms based on protocol interference model can lead to higher number of colors.

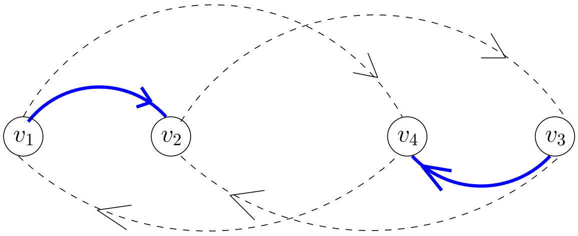

Fig. 17. Alternative coloring of subgraph shown in Figure 14.

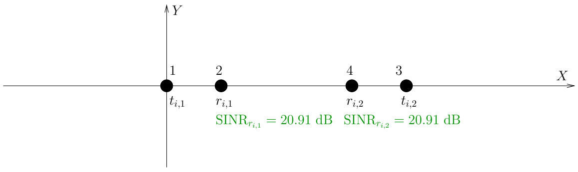

Fig. 18. A link schedule corresponding to Figure 17 that yields lower number of colors.

$4 \equiv (170, 0)$. The system parameters are shown in Table I, which lead to $R_c = 100$ m and $R_i = 177.8$ m. The two-tier graph model of the STDMA network is shown in Figure 13. Consider the transmission requests $1 \longrightarrow 2$ and $3 \longrightarrow 4$, which correspond to communication edges of the subgraph shown in Figure 14. The communication edges $v_1 \overset{c}{\longrightarrow} v_2$ and $v_3 \overset{c}{\longrightarrow} v_4$ shown in Figure 14 have secondary edge conflicts. Hence, such an algorithm will typically color these edges with different colors, as shown in Figure 15. Equivalently, a link scheduling algorithm based on the protocol interference model will schedule transmissions $1 \longrightarrow 2$ and $3 \longrightarrow 4$ in different time slots, say time slots i and j respectively, where $i \neq j$. Our computations show that the resulting SINRs at receivers $r_{i,1}$ and $r_{j,1}$ are both equal to 32.04 dB. Figure 16 shows the nodes of the network along with the labeled transmitter-receiver pairs, receiver-centric communication and interference zones and SINRs at the receivers. Observe that, with an algorithm based on the protocol interference model, the SINRs at both receivers are well above the communication threshold of 20 dB. Alternatively, consider an algorithm (perhaps based on the physical interference model) that schedules transmissions $1 \longrightarrow 2$ and $3 \longrightarrow 4$ in the same time slot, say time slot i. The corresponding edge coloring is shown in Figure 17. Our computations show that the resulting SINRs at receivers $r_{i,1}$ and $r_{j,1}$ are both equal to 20.91 dB, which are also above the communication threshold. Figure 18 shows the nodes of the network along with the labeled transmitter-receiver pairs and SINRs at the receivers. In essence, with the alternate algorithm, both transmissions $t_{i,1} \longrightarrow r_{i,1}$ and $t_{i,2} \longrightarrow r_{i,2}$ are successful, since signals powers are so high at the receivers that strong interferences can be tolerated. In summary, a link scheduling algorithm based on the protocol interference model will typically schedule the above transmissions in different slots and yield lower network throughput compared to the alternate algorithm.

3) Lastly, these algorithms are not aware of the topology of the network, i.e., they determine a link schedule without being cognizant of the exact positions of the transmitters and receivers.

Since link scheduling algorithms based on the protocol interference model yield low throughput, researchers have propounded algorithms based on the physical interference model to improve the throughput of STDMA wireless networks. To achieve higher throughput, one possible technique is to model the STDMA network by a communication graph and check SINR threshold conditions during assignment of links to time slots; this is the approach most commonly employed, for example in [20], [24], [31]. The other technique is to incorporate SINR threshold conditions into a special graph model of the network; this approach is more challenging and is considered in research work such as [27], [32], [33]. Research papers which employ the former approach are reviewed in Section V, while research papers which employ the latter approach are reviewed in Section VI.

V. LINK SCHEDULING BASED ON COMMUNICATION GRAPH MODEL AND SINR CONDITIONS

In this section, we examine recent research in link scheduling based on modeling the STDMA network by a communication graph and verifying SINR conditions at the receivers. Though algorithms based on this model [17], [34], yield higher throughput, they usually result in higher computational complexity than algorithms based on the protocol interference model.

In [20], the authors investigate throughput improvement in an IEEE 802.11 like wireless mesh network with CSMA/CA channel access scheme replaced by STDMA. For a successful packet transmission, they mandate that two-way communication be successful, i.e., a packet transmission is defined to be successful if and only if both data and acknowledgement packets are received successfully. Under this "extended physical interference model", they present a greedy algorithm which computes a link transmission schedule in a centralized manner. Assuming uniform random node distribution and using results from occupancy theory [35], they derive an approximation factor for the length of this schedule relative to the shortest schedule.

The throughput performance of link scheduling algorithms based on two-tier graph model $\mathcal{G}(\mathcal{V}, \mathcal{E}_c \cup \mathcal{E}_i)$ has been analyzed under physical interference conditions in [24]. The authors determine the optimal number of simultaneous transmissions by maximizing a lower bound on the throughput and subsequently propose Truncated Graph-Based Scheduling Algorithm (TGSA), an algorithm that provides probabilistic guarantees for network throughput. Though the analysis presented in [24] is mathematically elegant and based on the Edmundson-Madansky bound [36], [37], their algorithm may not yield high network throughput. This is because the partitioning of a maximal independent set of communication edges into multiple subsets (time slots) is arbitrary and not based on network topology, which can lead to significant interference in certain regions of the network [27].

The performance of algorithms based on the protocol interference model versus those based on communication graph model and SINR conditions is evaluated and compared in [25]. To generate a non-conflicting link schedule based on the protocol interference model, the authors use a two-tier graph model with certain SINR threshold values chosen based on heuristics and examples. To generate a conflict-free link schedule based on the physical interference model, the authors employ a method suggested in [38] which describes heuristics based on two path loss models, namely terrain-data based ground wave propagation model and Vogler's five knife-edge model. Their simulations results indicate that, under a Poisson arrival process, algorithms based on the protocol interference model result in higher average packet delay than algorithms based on communication graph model and SINR conditions.

In [34], the authors investigate the tradeoff between the average number of concurrent transmissions and sustained data rate per node for an IEEE 802.11 wireless network. They show that spatial reuse depends only on the ratio of transmit power to carrier sense threshold [5]. Keeping the carrier sense threshold fixed, they propose a distributed power and rate control algorithm based on interference measurement and evaluate its performance via simulations.

In [17], the authors investigate mitigation of inter-flow interference in an IEEE 802.11e wireless mesh network from a temporal-spatial diversity perspective. Measurements of received signal strengths are used to construct a virtual coordinate system to identify concurrent transmissions with minimum inter-flow interference. Based on this new coordinate system, one of the nodes, designated as gateway node, determines the scheduling order for downlink frames of different connections. Through extensive simulations with real-life measurement traces, the authors demonstrate throughput improvement with their algorithm.

From a perspective of maximizing network throughput observed by the physical layer, it is useful to consider a performance metric that takes into account SINR threshold condition (6) as the criterion for successful packet reception. In [31], we propose such a performance metric, spatial reuse, which is defined as:

$$\sigma \quad = \quad \frac{\sum_{i=1}^{C} \sum_{j=1}^{M_i} I(\mathrm{SINR}_{r_{ij}} \geqslant \gamma_c)}{C}. \tag{11}$$

Hence, spatial reuse equals the number of successfully scheduled links per time slot according to the physical interference model. A high value of spatial reuse directly translates to higher network throughput.

The fact that the interference at a receiver is an increasing function of the number of concurrent transmissions in a time slot limits the value of spatial reuse. More specifically, if too many transmissions are scheduled in a single time slot, the interference at some receivers will be high enough to drive the SINRs below the communication threshold, leading to lower spatial reuse. Therefore, for a given STDMA network, there are certain fundamental limits (upper bounds) on the spatial reuse.

In [31], we consider link scheduling in STDMA wireless mesh networks. The algorithm is based on modeling the network by a communication graph, partitioning the graph into minimum number of subgraphs using Matroids [39] and then coloring the edges in each subgraph while checking for SINR threshold conditions.

Algorithms based on representing the network by a communication graph and verifying SINR threshold conditions yield higher network throughput than algorithms based on the protocol interference model. However, this is achieved at the cost of higher computational complexity. Furthermore, the gains in throughput may not be significant enough to justify the increase in computational complexity. This has prompted researchers to solve the link scheduling problem in a more fundamental manner. They have proposed an altogether different model of the network, termed as SINR graph model, and developed heuristics. Such algorithms are reviewed in the following section.

VI. LINK SCHEDULING BASED ON SINR GRAPH MODEL

In literature, many authors refer to algorithms based on communication graph model and checking SINR conditions as "algorithms based on physical interference model". In this paper, only algorithms that embed SINR threshold conditions into an appropriate graph model of the network are referred to as "algorithms based on the physical interference model". Though the physical interference model is more realistic, algorithms based on this model [27], [32], [33] have, in general, higher computational complexity than algorithms based on the protocol interference model.

Link scheduling for power-controlled STDMA networks under the physical interference model is analyzed in [32]. The authors define scheduling complexity as the minimum number of time slots required for strong connectivity of the graph[8] constructed from the link schedule. They develop an algorithm employing non-linear power assignment[9] and show that its scheduling complexity is polylogarithmic in the number of nodes. In a related work [33], the authors investigate the time complexity of scheduling a set of communication requests in an arbitrary network. They consider a "generalized physical model" wherein the actual received power of a signal can deviate from the theoretical received power by a multiplicative factor. Their algorithm successfully schedules all links in time proportional to the squared logarithm of the number of nodes times the static interference measure [40]. However, the algorithms in [32], [33] can result in arbitrarily high transmission power at some nodes.

In [19], the authors provide a general framework for computation of throughput bounds for a given wireless network and traffic workload. Specifically, to represent interference constraints, they describe the following technique to construct a weighted conflict graph $F(V_F, E_F)$. Let $S_{ij} := \frac{P}{D^{\beta}(i,j)}$ denote the received signal power at node j due to the transmission from node i. In $F(\cdot)$, a vertex corresponds to a directed link l_{ij} (equivalently, node pair (i,j)) provided $\frac{S_{ij}}{N_0} \geqslant \gamma_c$. $F(\cdot)$ is a perfect graph wherein the weight w_{ij}^{pq} of the directed edge from vertex l_{pq} to vertex l_{ij} is given by $w_{ij}^{pq} = \frac{S_{pj}}{\frac{S_{ij}}{\gamma_c} - N_0}$. The authors describe methods to compute lower and upper bounds on throughput and the issues involved therein.

Analogous to a conflict graph, an SINR graph representation[10] of an STDMA wireless network has been proposed by us in [27]. The authors of [19] have not proposed any specific link scheduling algorithm and used the weighted conflict graph only to compute bounds on network throughput. We use an SINR graph representation of the network under the physical interference model and develop a link scheduling algorithm with lower time complexity.

To summarize, we compare representative link scheduling algorithms from each of these classes. For performance comparison, we assume system parameters from Table I and a uniform distribution of nodes in a circular area of radius 500 m. Figure 19 shows a representative performance comparison of the three classes of algorithms in terms of spatial reuse. Observe that algorithms based on SINR graph (SINRGraphLinkSchedule [27]) achieve better performance than algorithms based on communication graph and SINR conditions (GreedyPhysical [20]), which in turn perform better than algorithms based only on communication graph (ArboricalLinkSchedule [15]). However, this is achieved at the cost of successively higher computational complexity, as elucidated in Table II, where,

v = number of vertices in communication graph,
e = number of edges,
θ = graph thickness,
ρ = maximum vertex degree.

Overall, we observe the tradeoff between accuracy of the network representation, spatial reuse and algorithm running time complexity in the three classes of algorithms. For a more accurate network representation, higher throughput achieved, but at a cost of higher running time complexity.

VII. CONCLUSIONS

In this paper, we have provided a brief glimpse into three classes of link scheduling algorithms, each with its relative merits and demerits. For example, algorithms based on the protocol interference model have low computational complexity and are simple to implement, but yield low network throughput. On the other hand, algorithms based on SINR graph representation have higher computational complexity and are more cumbersome to implement, but achieve higher network throughput. Also, there exist algorithms based on communication graph and SINR conditions whose performance characteristics lie between these two classes. Hence, in general, these three classes of algorithms exhibit a tradeoff between complexity and performance. Finally, algorithms based on the protocol interference model are better suited to

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[8] A directed graph $\mathcal{G}(\cdot)$ is strongly connected if there exists a directed path from every vertex to every other vertex.

[9] In uniform power assignment, all nodes transmit with the same transmission power. In linear power assignment [32], a node transmits with minimum power required to satisfy the SINR threshold condition at the receiver, i.e., transmission power equals $N_0 \gamma_c D^{\beta}$. Non-linear power assignment refers to a power assignment scheme that is neither uniform nor linear.

[10] The SINR graph is analogous to a line graph [28] constructed from the communication graph representation of the network.

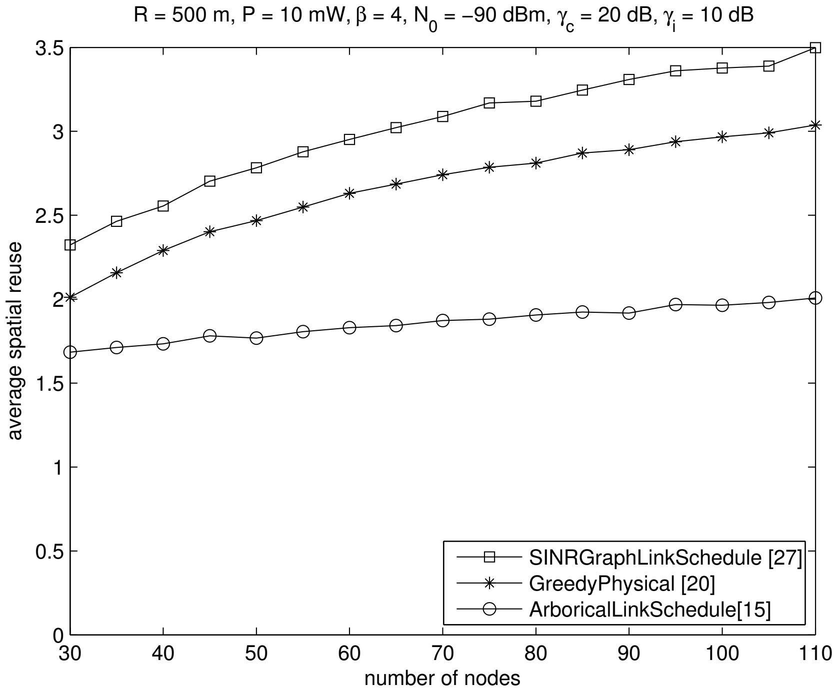

Fig. 19. Spatial reuse vs. number of nodes.

Transmission Power	Wireless Network Model	Link Scheduling Algorithms	Representative Computational Complexity
uniform	communication graph	[15]	$O(ev \log v + v\theta\rho^2)$
	communication graph and SINR conditions	[24], [20], [31]	$O(ev \log v + ev\theta)$
	SINR graph	[27]	$O(e^3)$
non-uniform	SINR graph	[32], [33]	$O((\log v)^4)$

TABLE II
COMPUTATIONAL COMPLEXITY OF REPRESENTATIVE LINK SCHEDULING ALGORITHMS.

model WLANs, while the latter two classes of algorithms are better suited to model wireless mesh networks.

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