Dynamics and Control of Non-holonomic Two Wheeled Inverted Pendulum Robot

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Abstract

In most wheeled mobile robots, at least one wheel is the auxiliary wheel aka caster. It must move smoothly without causing the robot to be interrupted if they are to truly do their job. Looking at it practically, however, they don't work sometimes. That is, the wheels frequently slid or slip when they were dragged, even though they were designed to roll without sliding or slipping. In order to free from this problem, one of the solutions is getting rid of it. Then, the total number of wheels attached on the robot changes, moreover, the mechanical characteristic of the robot having only two driving wheels without caster will be altered to that the robot is supposed to move and balance its body with only two driving wheels. Therefore, for this inverted pendulum type robot, it is necessary to investigate whether it is valid proposal or not and what mechanical characteristics it has. For doing this, dynamics of this kind of robot was governed to provide lots of information that will be helpful for design and control. And experiments with various motions were carried out to show its practical validity.

Keywords: Wheeled mobile robot, Inverted pendulum, Non-holonomic system

1 Introduction

Several kinds of wheels are attached to wheeled mobile robot, but they can fall into one of two categories: driving and auxiliary wheels. The former ones are rotated to permit the robot to move with torque being applied to the axles of those driving wheels. On the other side, the latter ones are equipped merely to ease the movement of the robot and suspend its body, and no driving torque is applied to their axles.

In most wheeled mobile robots, at least one wheel is the auxiliary wheel. In order to be free from problems of auxiliary wheels, it would be desirable to make the operation of those better. Getting rid of them, however, would also be another feasible idea. Going a step forward, those auxiliary wheels might be replaced with something different from them.

For this two wheeled mobile robot, it is to be questioned that what will happen if their auxiliary wheel was removed instead of replacing those wheels with something different or improving its performance without taking away. One of the changes due to getting rid of those wheels would be the total number of wheels attached on the robot. Moreover, the mechanical characteristic of the robot having only two driving wheels without any other auxiliary ones will be completely altered because there are no elements that can suspend and balance the body except for the driving wheels. That is, the robot is supposed to move and balance its body with only two driving wheels. Therefore, for this inverted pendulum type robot, it is necessary to investigate whether it is a valid proposal or not and what mechanical characteristics it has.

In 1992, Konayagi it al. [1] built an autonomous self-contained inverted pendulum robot and proposed two dimensional trajectory control algorithm for that kind of robot. In 1996, Ha and Yuta [2] proposed another inverted pendulum type self-contained mobile robot. They succeeded in balancing and trajectory control using a simplified model of two dimensional inverted pendulum. In 2000, Segway [3] developed a human transporter whose speed and direction were controlled solely by the rider's shifting weight and a manual turning mechanism on one of its handlebars. In February 2002, Grasser et al. [4] presented same mechanism with three dimensional model.

Hardware design and fabrication of the robot will be presented in chapter 2. Then, the equation of motion of the designed robot is derived in chapter 3, considering its constraints. Simulation results to test the model and to design the controller are presented in chapter 4. Finally, implementing results to the real system will be showed.

2 System Design and Fabrication

As shown in Figure 1, the two wheeled inverted pendulum robot is organized with main body, gear sets, drive wheels, motors, motor controllers, feedback sensors, and PC. The main body is made of aluminum plates and bars, and there is room to put the necessary electronic boards and sensors on the body. The body is equipped with two DC motors, and the gear of the motors is in contact with that of the wheels in the right and left sides. These motors are powered by the battery patch attached on the bottom of the robot, and they are controlled by the motion control processor. Two incremental encoders were adopted to monitor the behavior of the motors. In addition to those two encoders, gyroscope and tilt sensor were mounted on the body of the robot to measure the inclination angle and angular velocity of the robot. All these parts constitute the robot and they are controlled throughout by the host PC.

Like figure 2, the DC motors connected with the drive wheels are to provide torque to the axles for the upright balancing and navigation of the vehicle.

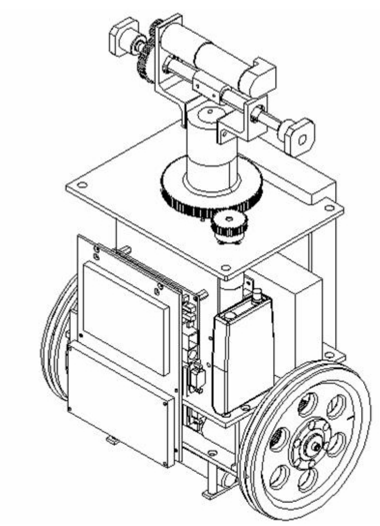

Figure 1. The designed robot

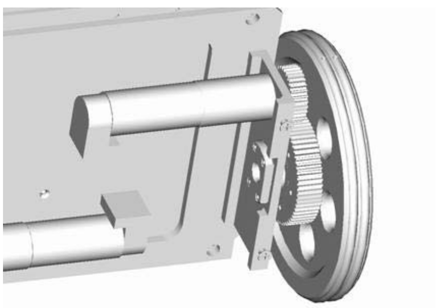

Figure 2. The interconnection between the wheel and the DC motor

Two kinds of sensors such as tilt sensor and gyroscope were used to measure the tilt angle of the robot and its time derivative. The important considerations in selecting those sensors were its resolution, bandwidth, linear range, sensitivity, and size, as listed in Table 1. In practice, normally a physical measurement of a control system is detected using a feedback sensor, and then its derivative or integration is calculated from that detection. However, it is possible that the numerical integration of the gyroscope output implies drift error. Thus, in this control system, the tilt angle and angular velocity of the robot's body are measured independently by using tilt sensor and gyroscope, respectively.

Table 1. The feedback sensors

	Tilt Sensor	Gyroscope
Resolution	0.05 (° rms)	0.1 (°/sec)
Bandwidth	125 (Hz)	7 (Hz)
Linear Range	± 20 (°)	± 80 (°/sec)
Sensitivity	35 ± 2 (mV/°)	20.4 (mV/°/sec)
Size (mm)	19.1×47.63×25.4	37×46×18.5

3 Dynamic Model

The equations of motion need to be derived to carry out the control and further researches including dynamic analysis. However, not only the derivation itself is important, but also it is essential to establish the exact dynamic model. Two different approaches have been adopted to formulate more exact dynamic model, i.e., Kane's dynamical equations and Lagrange's equations of motion.

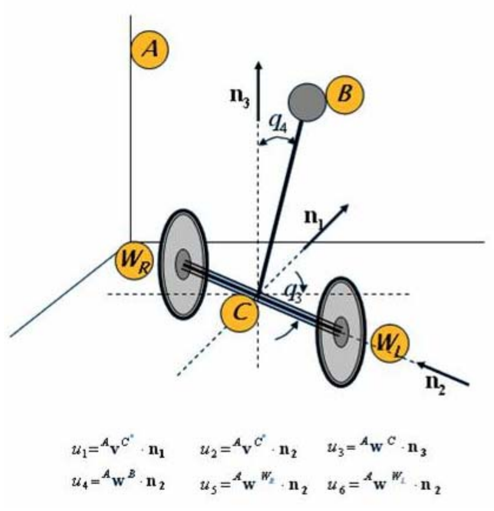

Figure 3. Coordinate system

As illustrated in Figure 3, the system is composed of three rigid bodies such as the right drive wheel W_R, the left drive wheel W_L, and the body B that is assumed to be a particle having total mass of the body in the point of the body's center of gravity. Each of them can be described with three Cartesian coordinates in a reference frame A, so the total number of the Cartesian coordinates of the system is 9.

Finally, for a non-holonomic inverted pendulum vehicle system possessing 3 degrees of freedom in A, 3 equations of motion is derived like followings. To test the reliability of the equations, the results by Lgrange's equations can be compared to that by Kane's dynamical equations. The detailed comparison can be leaved out for convenience. However, the results from both approaches were completely identical. Therefore, the derived equations of motion can be said to be reliable.

$$\begin{aligned} & M_b\dot{u}_1 + \frac{2}{R^2} I_{W_1}\dot{u}_1 + 2M_W\dot{u}_1 + M_bL\dot{u}_4 \cos q_4 \\ & \quad - M_bLu_3^2 \sin q_4 - M_bLu_4^2 \sin q_4 = \frac{1}{R}(\tau_R + \tau_L) \\ & M_bL^2\dot{u}_3 \sin^2 q_4 - M_bLu_1u_3 \sin q_4 + 2M_Wb^2\dot{u}_3 + I_b\dot{u}_3 + 2I_{W_2}\dot{u}_3 \\ & \quad + 2M_bL^2u_3u_4 \cos q_4 \sin q_4 + \frac{2b^2}{R^2} I_{W_1}\dot{u}_3 = \frac{b}{R}(\tau_R - \tau_L) \\ & M_bL\dot{u}_1 \cos q_4 - M_bL^2u_3^2 \sin q_4 \cos q_4 + M_bL^2\dot{u}_4 \\ & \quad + J_b\dot{u}_4 - M_bgL \sin q_4 = -(\tau_R + \tau_L) \end{aligned} \tag{1}$$

4 Controller Design

For the proposed system, the equations of motion were established. Thus, it can be linearized and then the state-space equations can be written as in Eq. (2), where the state vector and inputs are defined as in Eq. (3) and the matrices A, B are identified as in Eqs. (4), (5) and C is 6×6 I matrix and D is 6×2 null matrix.

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu} \\ \mathbf{y} &= \mathbf{Cx} + \mathbf{Du} \end{aligned} \tag{2}$$

$$\mathbf{x} = \begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_3 \\ \dot{q}_3 \\ q_4 \\ \dot{q}_4 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} \tau_R \\ \tau_L \end{bmatrix} \tag{3}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.885 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 25.77 & 0 \end{bmatrix} \tag{4}$$

$$B = \begin{bmatrix} 0 & 0 \\ 3.213 & 3.213 \\ 0 & 0 \\ 31.22 & -31.22 \\ 0 & 0 \\ -12.19 & -12.19 \end{bmatrix} \tag{5}$$

The adopted controller is LQR, the optimal controller which asymptotically stabilizes the feedback system of the augmented system and minimizes the performance index J. The cost function J is stated in Eq. (6) and the solution of this problem is derived by the matrix Riccati equation as in Eq. (7). The state feedback gain matrix is computed as Eq. (8) and also the stability is confirmed by the fact that the closed loop poles are located in LHP as in Eq. (9). And the simulation of upright balancing was carried out with the inclined initial condition.

$$J = \int_0^T \left(\vec{x}' Q \vec{x} + u' R u \right) dt \tag{6}$$

$$\begin{aligned} K(t) &= R^{-1} B' P(t) \\ -\dot{P} &= A'P + PA + Q - PBR^{-1}B'P \end{aligned} \tag{7}$$

$$K = \begin{pmatrix} -3.1623 & -4.1756 & 1.5811 & 0.7481 & -11.7320 & -2.0988 \\ -3.1623 & -4.1756 & -1.5811 & -0.7481 & -11.7320 & -2.0908 \end{pmatrix} \tag{8}$$

$$C.L.poles = \begin{pmatrix} -11.4653 + 7.3506i \\ -11.4653 - 7.3506i \\ -1.1780 + 0.8080i \\ -1.1780 - 0.8080i \\ -44.1030 \\ -2.2389 \end{pmatrix} \tag{9}$$

5 Control

The designed controller is implemented on the actual control hardware. Figure 4 shows inclination angle and its velocity during the up-right balancing. With the constant initial inclined angle, the robot started over to the stable balancing state.

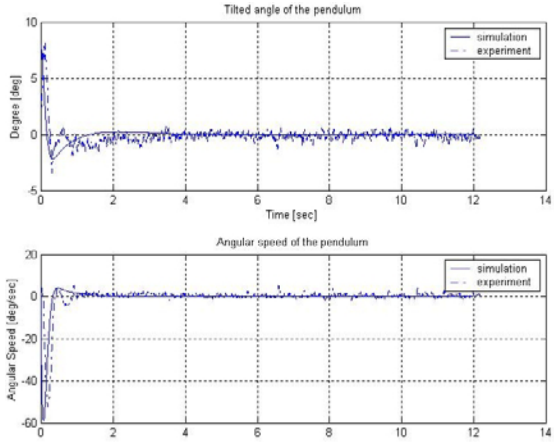

Figure 4. Up-right balancing

Figure 5 shows the results of rectilinear velocities while doing the motion. The robot follows the reference quite well, even though there exists the reaction time delay to the reference.

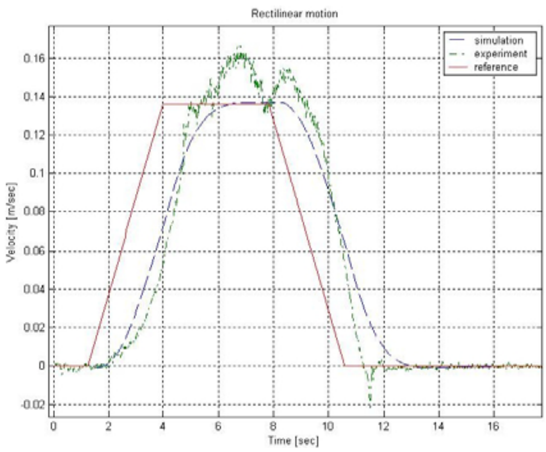

Figure 5. Rectilinear motion

Figure 6 show the results of the spinning angular velocity while doing the motion on the fixed world position. This confirms the fact that wheels without casters has a merit of mobility.

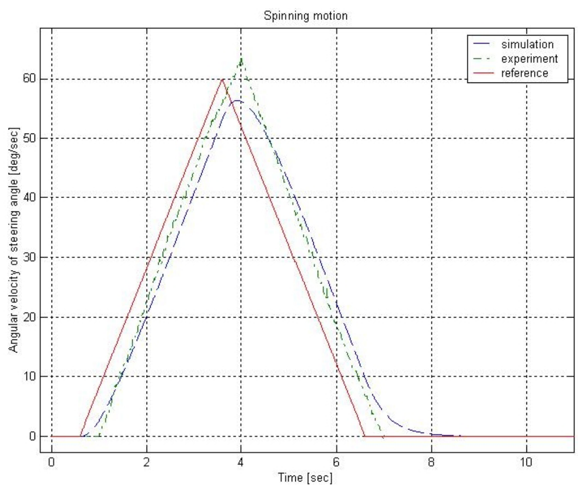

Figure 6. Spinning motion

6 Conclusion

The auxiliary wheels were removed from the general wheeled mobile robot and the system became the non-holonomic two wheeled inverted pendulum robot. The equations of motion were found by Lagrange's equations and Kane's dynamical equations, and both approaches provided the same results. Thus, the derived equations of motion can be said to be reliable. Based on the derived equations of motion, the controller was selected as LQR. Using this, the state-feedback gain matrix was found, and it was applied to the simulation. To implement the designed controller on the actual control hardware, the torque was modeled as a function of the angular velocity and acceleration of the drive wheel.

One of the two ultimate reasons why the dynamic model was derived is to construct the control system based on not trial and error but the theoretical model governing the system. To carry out more exact dynamic analysis, more exact dynamic model should be found. Therefore, the dynamic model of the two wheeled inverted pendulum robot on uneven terrain will be constructed considering the friction condition between the drive wheels and ground.

Acknowledgements

This work was supported in part by the Brain Korea 21 Project.

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