COMSOL MULTIPHYSICS®

USER'S GUIDE

VERSION 4.0a

COMSOL

Benelux
COMSOL BV
Röntgenlaan 19
2719 DX Zoetermeer
The Netherlands
Phone: +31 (0) 79 363 4230
Fax: +31 (0) 79 361 4212
info@comsol.nl
www.comsol.nl

Denmark
COMSOL A/S
Diplomvej 376
2800 Kgs. Lyngby
Phone: +45 88 70 82 00
Fax: +45 88 70 80 90
info@comsol.dk
www.comsol.dk

Finland
COMSOL OY
Arabiankatu 12
FIN-00560 Helsinki
Phone: +358 9 2510 400
Fax: +358 9 2510 4010
info@comsol.fi
www.comsol.fi

France
COMSOL France
WTC, 5 pl. Robert Schuman
F-38000 Grenoble
Phone: +33 (0)4 76 46 49 01
Fax: +33 (0)4 76 46 07 42
info@comsol.fr
www.comsol.fr

Germany
COMSOL Multiphysics GmbH
Berliner Str. 4
D-37073 Göttingen
Phone: +49-551-99721-0
Fax: +49-551-99721-29
info@comsol.de
www.comsol.de

India
COMSOL Multiphysics Pvt. Ltd.
Esquire Centre
C-Block, 3rd Floor
No. 9, M. G. Road
Bangalore 560001
Karnataka
Phone: +91-80-4092-3859
Fax: +91-80-4092-3856
info@comsol.co.in

Italy
COMSOL S.r.l.
Via Vittorio Emanuele II, 22
25122 Brescia
Phone: +39-030-3793800
Fax: +39-030-3793899
info.it@comsol.com
www.it.comsol.com

Norway
COMSOL AS
Søndre gate 7
NO-7485 Trondheim
Phone: +47 73 84 24 00
Fax: +47 73 84 24 01
info@comsol.no
www.comsol.no

Sweden
COMSOL AB
Tegnérgatan 23
SE-111 40 Stockholm
Phone: +46 8 412 95 00
Fax: +46 8 412 95 10
info@comsol.se
www.comsol.se

Switzerland
COMSOL Multiphysics GmbH
Technoparkstrasse 1
CH-8005 Zürich
Phone: +41 (0)44 445 2140
Fax: +41 (0)44 445 2141
info@ch.comsol.com
www.ch.comsol.com

United Kingdom
COMSOL Ltd.
UH Innovation Centre
College Lane
Hatfield
Hertfordshire AL10 9AB
Phone: +44-(0)-1707 636020
Fax: +44-(0)-1707 284746
info.uk@comsol.com
www.uk.comsol.com

United States
COMSOL, Inc.
1 New England Executive Park
Suite 350
Burlington, MA 01803
Phone: +1-781-273-3322
Fax: +1-781-273-6603

COMSOL, Inc.
10850 Wilshire Boulevard
Suite 800
Los Angeles, CA 90024
Phone: +1-310-441-4800
Fax: +1-310-441-0868

COMSOL, Inc.
744 Cowper Street
Palo Alto, CA 94301
Phone: +1-650-324-9935
Fax: +1-650-324-9936

info@comsol.com
www.comsol.com

For a complete list of international representatives, visit
www.comsol.com/contact

Company home page
www.comsol.com

COMSOL user forums
www.comsol.com/support/forums

COMSOL Multiphysics User's Guide

Version: June 2010 COMSOL 4.0a

Part number: CM020002

CONTENTS

Chapter 1: Introduction

Chapter 2: The COMSOL Modeling Environment

Chapter 3: Global and Local Definitions

Chapter 4: Visualization and Selection Tools

Chapter 5: Geometry Modeling and CAD Tools

Chapter 6: Materials

Chapter 7: Building a COMSOL Model

Chapter 8: Overview of the Physics Interfaces

Chapter 9: Electromagnetics

Chapter 10: Acoustics

Chapter 11: The Chemical Species Transport Interfaces

Chapter 12: The Fluid Flow Interface

Chapter 13: Heat Transfer Interfaces

Chapter 15: Equation-Based Modeling

Chapter 16: Sensitivity Analysis

Chapter 17: Optimization

Chapter 18: Meshing

Chapter 19: Multiphysics Modeling

Chapter 20: Deformed Meshes

Chapter 21: Solving

Chapter 22: Results

Chapter 23: Running the COMSOL Software

Chapter 24: Glossary

1

Introduction

Welcome to COMSOL Multiphysics®! This *User's Guide* details features and techniques that help you throughout all of your COMSOL Multiphysics modeling in version 4.0a using the all-new COMSOL Desktop environment. In this book, for example, we detail procedures to build model geometries in COMSOL Multiphysics, create a mesh for the finite elements, create parameters and variables you can use within a model, add the physics and material properties, and solve and display the results. The explanations, tutorials, and examples show you, step by step, how to tap into many functions and capabilities available in the COMSOL environment.

This introductory section provides an overview of COMSOL Multiphysics and its product family.

The Documentation Set

The full documentation set that ships with COMSOL Multiphysics consists of the following titles:

- *COMSOL Quick Installation Guide*—basic information for installing the COMSOL software and getting started. Included in the DVD package.
- *Introduction to COMSOL Multiphysics*—information about version 4.0a and how to build models using its all-new desktop environment. Included in the DVD package.
- *COMSOL License Agreement*—the license agreement. Included in the DVD package.
- *COMSOL Installation and Operations Guide*—besides covering various installation options, it describes system requirements and how to configure and run the COMSOL software on different platforms, including client/server architectures as well as shared-memory and distributed (cluster) parallel versions.
- *COMSOL Multiphysics User's Guide*—the book you are reading, it covers the functionality of COMSOL Multiphysics across its entire range from geometry modeling to postprocessing, including the interfaces for physics and equations. It serves as a tutorial and a reference guide to using COMSOL Multiphysics.
- *COMSOL Multiphysics Reference Guide*—this book reviews geometry, mesh, solver, and postprocessing features and provides detailed information about the model object and the commands that you can use to access COMSOL Multiphysics functions from within MATLAB. Additionally, it describes some advanced features and settings in COMSOL Multiphysics and provides background material and references. This book is only available electronically in HTML and PDF formats.

In addition, each of the optional modules

- *AC/DC Module*
- *Acoustics Module*
- *Batteries and Fuel Cells Module*
- *CFD Module*
- *Chemical Reaction Engineering Module*
- *Earth Science Module*
- *Heat Transfer Module*

- *MEMS Module*
- *Plasma Module*
- *RF Module*
- *Structural Mechanics Module*

has a *User's Guide*.

The documentation for the optional CAD Import Module and LiveLinks to CAD packages is available in separate *User's Guides*, and the documentation for the optional Material Library in the *Material Library User's Guide*.

There *COMSOL LiveLink™ for MATLAB® Interface Guide* shows how to access all of COMSOL Multiphysics' capabilities from the MATLAB programming environment.

Note: The full documentation set is available in electronic formats—PDF and HTML—through the COMSOL help system.

Where Do I Access the Documentation and Model Library?

Note: If you are working directly from a PDF on your computer, the blue underlined links do not work to open a model or documentation referenced in a different user guide. However, if you are using the online help desk in COMSOL Multiphysics, these links work to other modules, model examples, and documentation sets.

The Documentation

The *COMSOL Multiphysics User's Guide* and *COMSOL Multiphysics Reference Guide* describe all the interfaces included with the basic COMSOL license. These guides also have instructions about how to use COMSOL Multiphysics, and how to access the documentation electronically through the COMSOL Multiphysics help desk.

To locate and search all the documentation, in COMSOL, select **Help>Documentation** from the main menu and either enter a search term or look under a specific module in the documentation tree.

The Model Library

Each model comes with a theoretical background and step-by-step instructions to create the model. The models are available in COMSOL as MPH-files that you can open for further investigation. Use both the step-by-step instructions and the actual models as a template for your own modeling and applications. SI units are used to describe the relevant properties, parameters, and dimensions in most examples, but other unit systems are available.

To open any model in COMSOL, select **File>Open Model Library** from the main menu, and then search either by name or browse by module name. If you also want to review the documentation explaining how to build a model, select **Help>Documentation** in COMSOL and again, search by name or browse by module.

If you have feedback or suggestions for additional models for the library (including those developed by you), feel free to contact us at suggest@comsol.com.

Typographical Conventions

All COMSOL manuals use a set of consistent typographical conventions that should make it easy for you to follow the discussion, realize what you can expect to see on the screen, and know which data you must enter into various data-entry fields. In particular, you should be aware of these conventions:

- A **boldface** font of the shown size and style indicates that the given word(s) appear exactly that way on the COMSOL Desktop (or, for toolbar buttons, in the corresponding tooltip). For instance, we often refer to the **Model Builder** window, which is the window that contains the model tree. As another example, the instructions might say to click the **Zoom Extents** button, and the boldface font indicates that you can expect to see a button with that exact label on the COMSOL Desktop.
- Click text highlighted in blue and underlined to go to other information in the PDF. When you are using the online help desk in COMSOL Multiphysics, these links also work to other modules, model examples, and documentation sets.
- The names of other items on the COMSOL Desktop that do not have direct labels contain a leading uppercase letter. For instance, we often refer to the Main toolbar; this horizontal bar containing many icons appears on top of the user interface. However, nowhere on the screen will you see the term “Main” referring to this toolbar.

- The symbol **>** indicates a menu item. For example, **Options>Results** is equivalent to: From the **Options** menu, choose **Results**.
- A `Code` (monospace) font indicates keyboard entries in the user interface. You might see an instruction such as "Type `1.25` in the **Current density** edit field." The monospace font also indicates code. This font also indicates variable names. An italic *`Code`* (monospace) font indicates user inputs and parts of names that can vary or be defined by the user.
- An *italic* font indicates the introduction of important terminology. Expect to find an explanation in the same paragraph or in the Glossary. The names of books in the COMSOL documentation set also appear using an italic font.

About COMSOL Multiphysics

COMSOL Multiphysics is a powerful interactive environment for modeling and solving all kinds of scientific and engineering problems. The all-new version 4 provides a powerful integrated desktop environment with a *Model Builder* where you get full overview of the model and access to all functionality. With this software you can easily extend conventional models for one type of physics into multiphysics models that solve coupled physics phenomena—and do so simultaneously. Accessing this power does not require an in-depth knowledge of mathematics or numerical analysis.

Using the built-in *physics interfaces* and the advanced support for material properties, it is possible to build models by defining the relevant physical quantities—such as material properties, loads, constraints, sources, and fluxes—rather than by defining the underlying equations. You can always apply these variables, expressions, or numbers directly to solid and fluid domains, boundaries, edges, and points independently of the computational mesh. COMSOL Multiphysics then internally compiles a set of equations representing the entire model.

You access the power of COMSOL Multiphysics as a standalone product through a flexible graphical user interface (GUI) or by script programming in Java or the MATLAB® language (requires the COMSOL LiveLink for MATLAB).

Using these physics interfaces, you can perform various types of studies including:

- Stationary and time-dependent (transient) studies
- Linear and nonlinear studies
- Eigenfrequency, modal, and frequency response studies

When solving the models, COMSOL Multiphysics uses the proven *finite element method (FEM)*. The software runs the finite element analysis together with adaptive meshing (if selected) and error control using a variety of numerical solvers. The studies can make use of multiprocessor systems and cluster computing, and you can run batch jobs and parametric sweeps. A more detailed description of this mathematical and numerical foundation is in the *COMSOL Multiphysics Reference Guide*.

New in version 4 is the concept of sequences, which means that COMSOL Multiphysics records all steps to create the geometry, mesh, studies and solver settings, and visualization and results presentation. It is therefore easy to parameterize any part of the model; simply change a node in the model tree and re-run the sequences. The program remembers and reapplies all other information and data in the model.

Partial differential equations (PDEs) form the basis for the laws of science and provide the foundation for modeling a wide range of scientific and engineering phenomena. Therefore you can use COMSOL Multiphysics in many application areas, for example:

- Acoustics
- Bioscience
- Chemical reactions
- Diffusion
- Electromagnetics
- Fluid dynamics
- Fuel cells and electrochemistry
- Geophysics
- Heat transfer
- Microelectromechanical systems (MEMS)
- Microwave engineering
- Optics
- Photonics
- Plasma physics
- Porous media flow
- Quantum mechanics
- Radio-frequency components
- Semiconductor devices
- Structural mechanics
- Transport phenomena
- Wave propagation

Many real-world applications involve simultaneous couplings in a system of PDEs—*multiphysics*. For instance, the electric resistance of a conductor often varies with temperature, and a model of a conductor carrying current should include resistive-heating effects. Chapter 19, "Multiphysics Modeling," discusses multiphysics modeling techniques. Many predefined multiphysics interfaces provide easy-to-use entry points for common multiphysics applications.

In its base configuration, COMSOL Multiphysics offers modeling and analysis power for many application areas. For several of the key application areas there are also

optional modules. These application-specific modules use terminology and solution methods specific to the particular discipline, which simplifies creating and analyzing models. The COMSOL 4.0a product family includes these modules, including the all-new Batteries and Fuel Cell Module, CFD Module, and Plasma Module:

- AC/DC Module
- Acoustics Module
- Batteries and Fuel Cells Module
- CFD Module
- Chemical Reaction Engineering Module
- Earth Science Module
- Heat Transfer Module
- MEMS Module
- Plasma Module
- RF Module
- Structural Mechanics Module

The CAD Import Module provides the possibility to import CAD data using the following formats: IGES, SAT (Acis), Parasolid, STEP, SolidWorks, Pro/ENGINEER, and Inventor. An additional add-on provides support for CATIA V5. There are also LiveLinks (bidirectional interfaces) for SolidWorks, Pro/ENGINEER, and Autodesk Inventor.

You can build models of all types in the COMSOL Multiphysics user interface. For additional flexibility, COMSOL also provides LiveLink for MATLAB, a seamless interface to MATLAB. This gives you the freedom to combine multiphysics modeling, simulation, and analysis with other modeling techniques. For instance, it is possible to create a model in COMSOL and then export it to Simulink as part of a control-system design.

We are delighted you have chosen COMSOL Multiphysics for your modeling needs and hope that it exceeds all expectations. Thanks for choosing COMSOL!

The COMSOL Modules

The optional modules

- AC/DC Module
- Acoustics Module
- Batteries and Fuel Cells Module
- CFD (Computational Fluid Dynamics) Module
- Chemical Reaction Engineering Module
- Earth Science Module
- Heat Transfer Module
- MEMS Module
- Plasma Module
- RF Module
- Structural Mechanics Module

are optimized for specific application areas. They offer discipline-standard terminology and physics interfaces, material libraries, specialized solvers, element types, and visualization tools.

The AC/DC Module

The AC/DC Module provides a unique environment for simulation of AC/DC electromagnetics in 2D and 3D. The AC/DC Module is a powerful tool for detailed analysis of coils, capacitors, and electrical machinery. With this module you can run static, quasi-static, transient, and time-harmonic simulations in an easy-to-use graphical user interface.

The available physics interfaces cover the following types of electromagnetics field simulations:

- Electrostatics
- Electric currents in conductive media
- Magnetostatics
- Low-frequency electromagnetics

Material properties include inhomogeneous and fully anisotropic materials, media with gains or losses, and complex-valued material properties. Infinite elements makes it possible to model unbounded domains. In addition to the standard results and visualization functionality, the AC/DC Module supports direct computation of lumped parameters such as capacitances and inductances as well as electromagnetic forces. With the multiphysics capabilities of COMSOL Multiphysics, you can couple simulations with heat transfer, structural mechanics, fluid flow formulations, and any other physical phenomena.

This module also provides interfaces for modeling electrical circuits and importing ECAD drawings.

The Acoustics Module

The Acoustics Module provides tailored physics interfaces for modeling of acoustics in fluids and solids. The module supports time-harmonic, modal, and transient studies for fluid pressure as well as static, transient, eigenfrequency, and frequency-response analyses for structures. The available physics interfaces include the following functionality:

- Frequency-domain and transient pressure acoustics
- Acoustic-structure interactions
- Aeroacoustics
- Boundary mode Acoustics
- Aeroacoustics with flow
- Compressible potential flow
- Solid mechanics
- Piezoelectricity

For the pressure acoustics applications, you can choose to analyze the scattered wave in addition to the total wave. PMLs (perfectly matched layers) provide accurate simulations of open pipes and other models with unbounded domains. The modeling domain can include dipole sources as well as monopole sources, and it is easy to specify point sources in terms of flow, intensity, or power. The module also includes modeling support for several types of damping. For results evaluation of pressure acoustics models, you can compute the far field.

Typical application areas for the Acoustics Module include:

- Automotive applications such as mufflers and car interiors

- Modeling of loudspeakers and microphones
- Aeroacoustics
- Underwater acoustics

Using the full multiphysics couplings within the COMSOL Multiphysics environment, you can couple the acoustic waves to, for example, an electromagnetic analysis or a structural analysis for acoustic-structure interaction.

Batteries and Fuel Cells Module

The Batteries and Fuel Cells Module provides customized physics interfaces for modeling of batteries and fuel cells. These physics interfaces provide tools for building detailed models of the configuration of the electrodes and electrolyte in electrochemical cells. They include descriptions of the electrochemical reactions and the transport properties that influence the performance of batteries, fuel cells, and other electrochemical cells.

The physics interfaces are organized in primary, secondary and tertiary current density distributions physics interfaces. These are available for solid nonporous electrodes and for porous electrodes. In addition to these generic physics interfaces, the Batteries and Fuel Cells Module contains a dedicated physics interface for the modeling of Li-ion batteries.

The tailored physics interfaces mentioned above are also complemented with extended functionality in other physics interfaces for chemical species transport, heat transfer, and fluid flow.

The physics interfaces for chemical species transport of neutral species are extended by adding nodes that directly couple to electrochemical reactions defined in the physics interfaces for electrochemical cells. A typical example is the transport and reactions of gaseous species in gas diffusion electrodes and gas channels in fuel cells.

The heat transfer physics interfaces include heat sources that describe ohmic losses in the electrodes and electrolyte and heat sources due to electrochemical reactions in electrochemical cells.

The fluid flow capabilities are extended for laminar flow, where the chemical species transport and the energy balances influence the properties of the flow.

CFD Module

The CFD Module is an optional package that extends the COMSOL Multiphysics modeling environment with customized user interfaces and functionality optimized for the analysis of all types of fluid flow. Ready-to-use interfaces let you model laminar and turbulent flows in single or multiple phases. Functionality for treating coupled free and porous media flow, stirred vessels, and fluid-structure interaction are also included.

- Laminar and turbulent flow
- Single-phase and multiphase flow
- Isothermal and non-isothermal flow
- Compressible and incompressible flow
- Newtonian and non-Newtonian flow

The ready coupling of heat and mass transport to fluid flow enables modeling of a wide range of industrial applications such as heat exchangers, turbines, separations units, and ventilation systems.

Together with COMSOL Multiphysics, the CFD Module takes flow simulations to a new level, allowing for arbitrary coupling to physics interfaces describing other physical phenomena, such as structural mechanics, electromagnetics, or even user defined transport equations. This allows for effortless modeling of any multiphysics application involving fluid flow.

Chemical Reaction Engineering Module

The Chemical Engineering and Reaction Engineering Modules have merged as of COMSOL 4.0a. The new module is called the Chemical Reaction Engineering Module.

The reaction engineering tools use reaction formulas to create models of reacting systems. In this context, a model means the material (mass), energy (heat), and momentum balances for a system. The Chemical Reaction Engineering Module not only defines these balances, it can also solve the material and energy balances for space-independent models (that is, for models where the composition and temperature in the reacting system vary only in time) and space-dependent models. This makes it possible to create models involving material, energy, and momentum balances in COMSOL Multiphysics directly from a set of reaction formulas.

Included in these models are the kinetic expressions for the reacting system, which are automatically or manually defined in the Chemical Reaction Engineering Module. You also have access to a variety of ready-made expressions in order to calculate a system's thermodynamic and transport properties.

The Chemical Reaction Engineering Module presents a powerful way of modeling equipment and processes in chemical engineering. It provides customized physics interfaces and formulations for momentum, mass, and heat transport coupled with chemical reactions for applications such as:

- Reaction engineering and design
- Heterogeneous catalysis
- Separation processes
- Fuel cells and industrial electrolysis
- Process control

COMSOL Multiphysics excels in solving systems of coupled nonlinear PDEs that can include:

- Heat transfer
- Mass transfer through diffusion, convection, and migration
- Fluid dynamics
- Chemical reaction kinetics
- Varying material properties

The multiphysics capabilities of COMSOL Multiphysics can fully couple and simultaneously model fluid flow, mass and heat transport, and chemical reactions.

In fluid dynamics you can model fluid flow through porous media, characterize flow with the incompressible Navier-Stokes equations. It is easy to represent chemical reactions by source or sink terms in mass and heat balances. These terms can be of arbitrary order.

The physics interfaces in this module cover the following areas:

- Chemical Species Transport
  - Reaction engineering
  - Transport of diluted species through diffusion, convection, and migration in electric fields
  - Transport of concentrated species using one of the following diffusion models: mixture-averaged, Maxwell-Stefan, or Fick's law
  - Nernst-Planck transport equations
- Heat Transfer
  - Heat transfer in fluids
  - Heat transfer in solids
  - Heat transfer in porous media
- Fluid Flow
  - Single-phase flow (incompressible Navier-Stokes equations)
  - Darcy's law
  - Brinkman equations
  - Free and porous media flow

The Earth Science Module

The earth and planets are giant laboratories that involve all manner of physics. The Earth Science Module combines physics interfaces for fundamental processes and links to COMSOL Multiphysics and the other modules for structural mechanics and electromagnetics analyses. New physics represented include heating from radiogenic decay that produces the geotherm, which is the increase in background temperature with depth. You can use the variably saturated flow interfaces to analyze unsaturated zone processes (important to environmentalists) and two-phase flow (of particular interest in the petroleum industry as well as steam-liquid systems). Important in earth sciences, the heat transfer and chemical transport interfaces explicitly account for physics in the liquid, solid, and gas phases.

The physics interfaces in this module cover the following areas:

- Darcy's law for hydraulic head, pressure head, and pressure. Also part of a multiphysics interface for poroelasticity (requires the Structural Mechanics Module or the MEMS Module).
- Fracture flow

- Solute transport in saturated and variably saturated porous media
- Richards' equation including nonlinear material properties using van Genuchten, Brooks and Carey, or user defined parameters.
- Heat transfer by conduction and convection in porous media with one mobile fluid, one immobile fluid, and up to five solids
- Brinkman equations
- Single-phase flow (incompressible Navier-Stokes equations)

The Earth Science Module Model Library contains a number of interesting examples, both single physics and multiphysics. This module combines new and existing physics in a form that earth scientists can readily use.

The Heat Transfer Module

The Heat Transfer Module supports all fundamental mechanisms of heat transfer, including *conductive*, *convective*, and *radiative* heat transfer (both *surface-to-surface* and *surface-to-ambient* radiation). Using the physics interfaces in this module along with inherent multiphysics capabilities of COMSOL Multiphysics you can model a temperature field in parallel with other physics—a powerful combination that makes your models even more accurate and representative of the real world.

Available physics interfaces include functionality for:

- General heat transfer, including conduction, convection, and surface-to-surface radiation
- Bioheat equation for heat transfer in biomedical systems
- Heat transfer in porous media
- Heat radiation in participating media
- Highly conductive layer for modeling of heat transfer in thin structures
- Nonisothermal incompressible fluid flow
- Turbulent flow using the k-ε and k-ω turbulence models

The Heat Transfer Module Model Library contains models, many with multiphysics couplings, that cover applications in electronics and power systems, process industries, and manufacturing industries. This Model Library also provides tutorial and benchmark models.

The MEMS Module

One of the most exciting areas of technology to emerge in recent years is MEMS (microelectromechanical systems), where engineers design and build systems with physical dimensions of micrometers. These miniature devices require multiphysics design and simulation tools because virtually all MEMS devices involve combinations of electrical, mechanical, and fluid-flow phenomena.

Available physics interfaces include:

- Solid mechanics for 2D plane stress and plane strain, axisymmetry, and 3D solids.
- Piezoelectric modeling for 2D plane stress and plane strain, axisymmetry, and 3D solids.
- Film damping and lubrication shells
- Electrokinetic flow
- General laminar flow, including Stokes flow and multiphase flow

The MEMS module also includes predefined multiphysics interfaces for thermal-structural, Joule heating with thermal stress or thermoelectromechanical (TEM), acoustic-structural, and fluid-structure interaction.

The MEMS Module Model Library contains a suite of models of MEMS devices such as sensors, actuators, and microfluidics systems. The models demonstrate a variety of multiphysics couplings and techniques for moving boundaries.

This module also provides interfaces for circuit modeling, a SPICE interface, and support for importing ECAD drawings.

Plasma Module

The Plasma Module is based on a series of scientific publications on numerical modeling of non-equilibrium discharges. These papers are referenced throughout the documentation and we encourage you to seek out these references for additional background information.

The Plasma Module can model low temperature, non-equilibrium discharges, such as:

- Inductively coupled plasmas (ICP).
- Capacitively coupled plasmas (CCP).
- Microwave plasmas.
- Light sources.

- Electrical breakdown.
- Space thrusters.
- DC discharges.
- Dielectric barrier discharges (DBD).
- Reactive gas generators.

The complexity of plasma modeling lies in the fact it combines elements of reaction engineering, statistical physics, fluid mechanics, physical kinetics, heat transfer, mass transfer and electromagnetics. The net result is a true multiphysics problem involving complicated coupling between the different physics. The COMSOL Multiphysics Plasma Module is designed to simplify the process of setting up a self consistent model of a low temperature plasma.

The Plasma Module includes all the necessary tools to model plasma discharges, beginning with a Boltzmann Equation, Two-Term Approximation solver that computes the electron transport properties and source coefficients from a set of electron impact collision cross sections. The Boltzmann Equation, Two-Term Approximation interface allows you to determine many of the interesting characteristics of a discharge by providing input properties such as the electric field and the electron impact reactions which make up the plasma chemistry, without solving a space dependent problem.

For space dependent models, the reactions and species which make up the plasma chemistry are conveniently managed in the model builder. Electron impact reactions can be defined directly in the model builder or by reading in a set of collision cross sections from file. When the fluid velocity and gas temperature are of interest, there are physics interfaces available for Laminar Flow and Heat Transfer. There are several options available when coupling the charged species transport to the electromagnetic fields. Poisson's equation for the electrostatic potential is always solved. For inductively coupled plasmas where induction currents are responsible for sustaining the plasma, the AC/DC Module is required. For wave heated discharges (microwave plasmas), the RF Module is required.

The Plasma Module also includes a set of predefined multiphysics couplings for the most common types of plasma reactors. This eliminates the need for you to manually implement all the complicated couplings between the different physics.

The RF Module

The RF Module provides a unique environment for the simulation of electromagnetic waves in 2D and 3D. With this module you can run harmonic, transient, and eigenfrequency simulations in an easy-to-use graphical user interface. For example, use the RF Module to simulate electromagnetic wave propagation in microwave components and photonic devices.

The RF Module is useful for component design in virtually all areas where you find electromagnetic waves, such as:

- Antennas
- Waveguides and cavity resonators in microwave engineering
- Optical fibers
- Photonic waveguides
- Photonic crystals
- Active devices in photonics

The available physics interfaces cover the following types of electromagnetics field simulations:

- In-plane, axisymmetric, and full 3D electromagnetic wave propagation
- Full vector mode analysis in 2D and 3D

Material properties include inhomogeneous and fully anisotropic materials, media with gains or losses, and complex-valued material properties. In addition to the standard postprocessing features, the RF Module supports direct computation of S-parameters and far-field patterns. You can add ports with a wave excitation with specified power level and mode type, and add PMLs (perfectly matched layers) to simulate electromagnetic waves that propagate into an unbounded domain. For time-harmonic simulations, you can use the scattered wave or the total wave. Using the multiphysics capabilities of COMSOL Multiphysics you can couple simulations with heat transfer, structural mechanics, fluid flow formulations, and other physical phenomena.

This module also provides interfaces for circuit modeling, a SPICE interface, and support for importing ECAD drawings.

The Structural Mechanics Module

The Structural Mechanics Module solves problems in structural mechanics and solid mechanics, adding special element types—beam, truss, shell, and plate elements—for engineering simplifications.

Available physics interfaces include:

- Solid mechanics for 2D plane stress and plane strain, axisymmetry, and 3D solids
- Piezoelectric modeling
- Beams in 2D and 3D, Euler theory
- Truss and cable elements
- Shells and plates

Supporting both linear and nonlinear materials, the module's study capabilities include static, eigenfrequency, time dependent (transient), frequency response, and parametric studies, as well as contact and friction.

The Structural Mechanics Module version 4.0a provides predefined interfaces for elastoplastic, hyperelastic, and viscoelastic materials. Materials can be isotropic, orthotropic, or fully anisotropic, and you can use local coordinate systems to specify material properties.

CAD Import and LiveLink Connections

COMSOL 4.0a includes packages for importing CAD drawings into COMSOL Multiphysics and LiveLinks, which provide bidirectional interfaces to SolidWorks®, Pro/ENGINEER®, and Autodesk Inventor®. The following products are available:

- CAD Import Module, for file import of CAD drawings in Parasolid, SAT (ACIS®), STEP, IGES, SolidWorks, Pro/ENGINEER, and Inventor formats.
- CATIA V5 Import Module, which requires the CAD Import Module, imports CAD drawings in CATIA V5 format.
- COMSOL LiveLink for SolidWorks, which provides a fully-associative bidirectional link to SolidWorks.
- COMSOL LiveLink for Pro/ENGINEER, which provides a fully-associative bidirectional link to Pro/ENGINEER.
- COMSOL LiveLink for Inventor, which provides a fully-associative bidirectional link to Autodesk Inventor.

LiveLink for MATLAB

The LiveLink for MATLAB gives you access to all modeling functionality in COMSOL and add-ons through MATLAB. The interface in the LiveLink is based on the COMSOL API. The product includes utility functions that lets you produce graphics in MATLAB figure windows, access COMSOL data, and to perform postprocessing.

Internet Resources

A number of Internet resources provide more information about COMSOL Multiphysics, including licensing and technical information. This section provides information about some of the most useful web links and email addresses.

COMSOL Web Sites

Main corporate web site: http://www.comsol.com/

Worldwide contact information: http://www.comsol.com/contact/

Online technical support main page: http://www.comsol.com/support/

COMSOL Support Knowledge Base, your first stop for troubleshooting assistance, where you can search for answers to any COMSOL questions: http://www.comsol.com/support/knowledgebase/

Product updates: http://www.comsol.com/support/updates/

CONTACTING COMSOL BY EMAIL

For general product information, contact COMSOL at info@comsol.com.

Send your COMSOL technical support questions to support@comsol.com. You will receive an automatic notification and a case number by email.

COMSOL Community

On the COMSOL web site, you find a user community at http://www.comsol.com/community/. The user community includes a discussion forum, a model exchange, news postings, and a searchable database of papers and presentations.

Note: To receive technical support from COMSOL for the COMSOL products, please contact your local COMSOL representative or send your questions to support@comsol.com.

2

The COMSOL Modeling Environment

The COMSOL Desktop provides a complete and integrated modeling environment for creating, analyzing, and visualizing multiphysics models. This section provides an overview of the COMSOL modeling environment as controlled by COMSOL Desktop and the tools and windows it provides. An introductory model provides step-by-step instructions that show how to use the COMSOL Desktop for setting up and solving a multiphysics model.

In this section:

The COMSOL Desktop Environment

This section describes the major components in the COMSOL Multiphysics environment. This is completely controlled by the *COMSOL Desktop*, which you can personalize to your own modeling needs and preferences. Primarily consisting of the *Model Builder* and *Graphics* windows, other dockable windows can be opened, closed, and placed according to the modeling settings you need to access and the GUI configuration you want to work in. You can save these configurations and the last opened configuration is always displayed when you open COMSOL Multiphysics again.

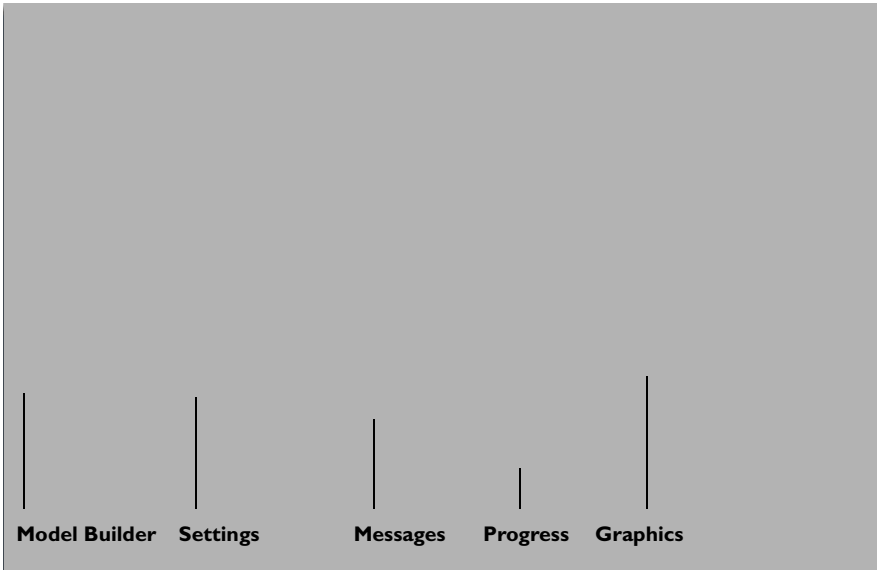


Figure 2-1: The COMSOL Desktop with its major windows in a widescreen layout.

The following sections describe how to configure the COMSOL Desktop layout and language and the main functionality of the major windows, most of which appear in Figure 2-1 above.

Changing the COMSOL Desktop Layout

To customize the COMSOL Desktop environment, you rearrange the windows by moving, resizing, detaching, or docking each window. Predefined layouts are also available from the **Options** menu.

For any window, perform the following tasks as required.

MOVING OR RESIZING THE WINDOW

- Click-and-drag the window tab (the tab is where the window name, **Model Builder** for example, displays) to where you want it.
- Right-click the window tab and select **Move>View** (to move a separate window) Move the mouse to where you want to the view or tab to display and left-click to confirm the move.
- Right-click the window tab and select **Move>Tab Group** (to move several tabbed windows) from the drop-down list. Move the mouse to where you want to the view or tab to display and left-click to confirm the move.

MAXIMIZING AND RESTORING A WINDOW'S ORIGINAL POSITION

- Double-click a window tab to maximize the window and then double-click it again to restore it.

DETACHING A WINDOW FOR MOVING AND RESIZING

- Right-click the window tab and select **Detach** so that you can move it and resize it as a separate window. Select **Detach** again to dock it on the COMSOL Desktop.

MINIMIZING OR MAXIMIZING THE WINDOW

- Click the **Minimize** or **Maximize** button in the top-right corner.
- Right-click the window tab and select **Minimize** or **Maximize** from the list.

RESIZING THE WINDOWS

- Hover your mouse over the window borders until a double arrow displays. Click-and-drag the borders between windows until the layout is how you want it.

SETTING THE LAYOUT TO WIDESCREEN OR REGULAR SCREEN

- Select **Options>Desktop Layout>**
  - **Widescreen Layout**: suitable for widescreen monitors.

    The **Model Builder** window and the **Settings** window display side by side.
  - **Regular Screen Layout**: suitable for monitors with a regular screen (4:3).

    The **Model Builder** window displays on top of the **Settings** window.

RESETTING THE DESKTOP TO DEFAULT SETTINGS

- Select **Options>Desktop Layout>Reset Desktop**.

  The default settings are restored either for a widescreen layout or a regular screen layout depending on your monitor.

Moving Between Windows and Sections on the COMSOL Desktop

You can use the mouse and the options in the previous section to select, move, and resize windows. In addition, the following keyboard shortcuts provide quick ways to navigate between the windows on the COMSOL Desktop and to switch focus from one window to another or between sections in the **Settings** window:

- Press Ctrl+Tab to switch focus to the next window on the desktop.
- Press Ctrl+Shift+Tab to switch focus to the previous window in the desktop.
- Press Ctrl+Alt+left arrow to switch focus to the **Model Builder** window.
- Press Ctrl+Alt+right arrow to switch focus to the **Settings** window.
- Press Ctrl+Alt+up arrow to switch focus to the previous section in the **Settings** window.
- Press Ctrl+Alt+down arrow to switch focus to the next section in the **Settings** window.

See also Summary of Keyboard Shortcuts.

Changing the COMSOL Desktop Language

1. Start COMSOL Multiphysics.
2. From the **Options** menu, select **Preferences**. For other preference settings, see Editing Model Preferences Settings.

3 Select an available language for the graphical user interface (GUI) from the **Language** list. Options are: Traditional Chinese, Simplified Chinese, English, French, German, Italian, Japanese, Korean, or Spanish.

4 Click **OK**.

A message displays indicating you must restart COMSOL Multiphysics for the changes to take effect.

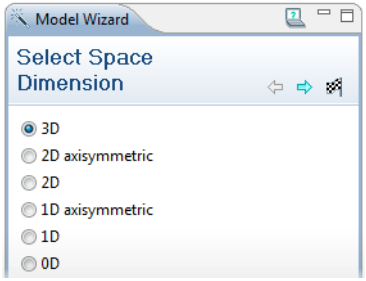
5 Click **OK,** exit and re-open COMSOL to display the GUI in the selected language.

The Model Wizard

Note: Also see The Model Wizard and Model Builder.

When you first open COMSOL Multiphysics or start creating a new model, the GUI displays the **Model Builder**, **Graphics**, and **Model Wizard** windows. The **Model Wizard** window ( ) contains a series of pages to help you build a model—**Select Space Dimension**, **Select Physics**, and **Select Study Type**.

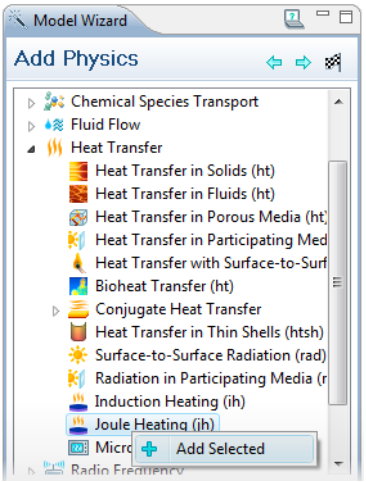
THE SELECT SPACE DIMENSION PAGE



Specify the final geometry dimension on the **Select Space Dimension** page—**3D**, **2D axisymmetric**, **2D**, **1D axisymmetric**, **1D**, or **0D**. 0D is used for physics interfaces modeling spatially homogeneous systems such as chemical reacting systems and electrical circuits. If you want to import a geometry, this is done in the **Model Builder**, but make sure you choose spatial dimensions that this geometry exists in.

THE ADD PHYSICS PAGE

On the **Add Physics** page, select one or more multiple physics interfaces to describe the model. Use the **Add Selected** ( ) or **Remove Selected** ( ) buttons to easily add or remove physics interfaces for *multiphysics models*.



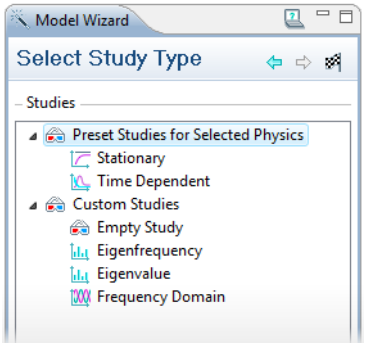
The tree organizes the available interfaces on the basis of *application areas* such as fluid flow, heat transfer, and structural mechanics. The physics found in the modules your license supports display in the different application areas. In some cases, licensing of a module adds physics interfaces to these application areas as well as attributes to existing physics interfaces, which are enhanced with additional functionality. The **Recently Used** branch lists the last five physics interfaces used in recent modeling sessions.

When selecting and adding a physics interface to the **Selected physics** section, you can review and optionally modify any dependent variable names in the **Dependent variables** section and, for some physics interfaces and PDE interfaces, specify the number of dependent variables. For other physics interfaces you can edit both the name of the field and the field components. Examples of fields with components are the displacement field in structural mechanics and the velocity field in fluid flow.

When you are finished adding the physics, click the **Next** button ( ) to go to the **Select Study Type** page.

THE SELECT STUDY TYPE PAGE

In the **Studies** tree, you select the type of study you want to perform. The **Select Study Type** page is dependent on the set of physics interfaces (and mathematics interfaces) selected to describe the model. Some study types are applicable to the physics described while others are not; but all are in some way available to be chosen. You can select the *study type* from one or more of the following **Studies**:



- **Preset Studies**—The study types preset by a physics interface (or mathematics interface) if only one has been chosen.
- **Preset Studies for Selected Physics**—The study types preset by all of the selected interfaces.
- **Custom Studies**—This branch contains study types not preset for all physics interfaces, including study types preset for some studies and other study types:
  - **Preset Studies for Some Physics**—The study types preset by some, but not all, of the selected interfaces.
  - Other studies—The study types that are not preset by any of the selected interfaces but exist for other interfaces in COMSOL Multiphysics or any of the modules that your license includes. There is also an empty study type.

Basic Steps to Build a Model

Note: Also see Geometry Modeling and CAD Tools.

1 In the **Model Wizard**, **Select a Space Dimension** for the model: **3D**, **2D axisymmetric**, **2D**, **1D axisymmetric**, **1D**, or **0D**. Not all physics interfaces are available for all space dimensions.

2 Click the **Next** button (⇨).

The **Add Physics** page opens.

3 There are several ways to select one or several *physics interfaces* to add to the model. There are also mathematics interfaces (for PDEs, ODEs, and DAEs) in the

Mathematics branch. Double-click the physics interface, click the **Add Selected** ( ) or **Remove Selected** ( ) buttons, or right-click and select **Add Selected**. The **Selected physics** section lists your choices.

Note: When selecting and adding physics or mathematics interfaces in the **Selected physics** section, you can review and optionally modify its dependent variable names in the **Dependent variables** section. For some interfaces, you also specify the number of dependent variables.

---

4 Click the **Next** button ( ).

The **Select Study Type** page opens.

5 Select a study type from the available options.

6 Click the **Next** button ( ) (when available), otherwise click the **Finish** button ( ) to confirm your selections and close the **Model Wizard**.

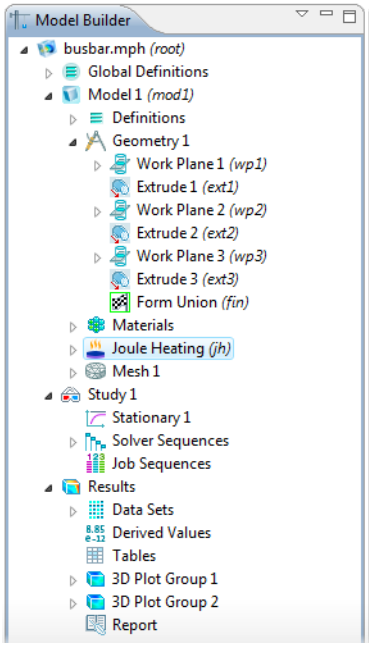
After clicking **Finish**, other nodes, such as **Materials** and **Meshes** automatically display in the **Model Builder** in the default sequence.

Note: You can click the **Finish** button ( ) anytime, and any operation you selected can be added to, changed, or deleted in the **Model Builder**.

The Model Builder Window

Note: Also see The Model Wizard and Model Builder.

You control the modeling procedure through the **Model Builder** window ( ). The Model Builder includes a *model tree* with all the functionality and operations for building and solving models and displaying the results. These are introduced to your modeling procedure by adding a *branch*, such as the *Geometry* branch. Branches can have further nodes that relate to their parent node. A *leaf* is considered to be the final node in a branching structure (tree) and usually has a number of *attributes* with their own *settings* that are characteristic to it. Branches and subbranches can also contain attributes and settings. See Figure 2-1.



BRANCHES IN THE MODEL TREE

You can proceed through your modeling procedure in the **Model Builder** by selecting the branches in the order suggested by their default positions, from the top down, or selecting and defining each branch as needed. The main branches include:

•The *Global Definitions branch*: for global definitions, for example, defining parameters and functions that you can use everywhere. See Global and Local Definitions.

•The *Model branches*: for defining models. A **Model** branch includes the associated subbranches of *Definitions*, for locally defining parts of your model, *Geometry*, *Materials*, *Physics*, and *Meshes*. A **Model** branch is furnished with functionality for local **Definitions** because several models can separately be defined in one multiphysics file, for example, when treating certain parts of your model in 2D and other parts in 3D, or when setting up a system model with several components.

- The *Study branch*: where you can set up study steps and solver configurations for solving a model using one or more study types for different analyses.
- The *Results branch*: for presenting and analyzing results. See Results.

One level below, each **Model** branch contains the following subbranches:

- Just as the **Global Definitions** branch collects parameters, variables, and functions accessible at all levels in the **Model Builder**, the *Definitions branch* (one per **Model**) collects the definitions of variables, functions, and other objects whose geometric scope is restricted to a single model. An example of the type of objects you can add under the **Definitions** branch is the *Selection* feature, which saves selections of geometric entities (boundaries, for example) that relate to a region or part of the overall geometry for reuse in multiple operations later in the modeling process. See Global and Local Definitions.
- The *Geometry branch* contains the definition of the model's geometry. See Creating a Geometry for Successful Analysis.

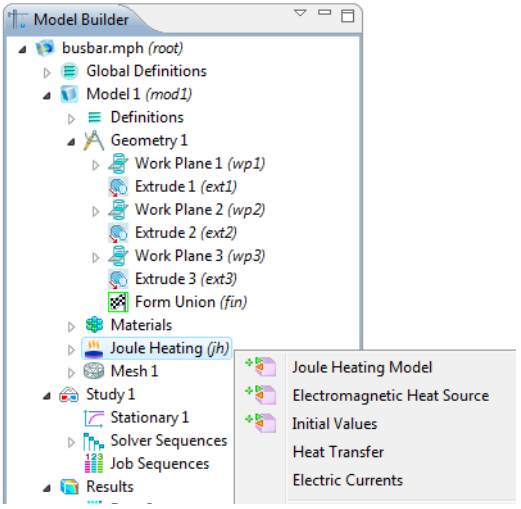
- Under the *Materials branch*, you can collect all material properties organized in **Materials** nodes with a defined geometric scope. Material properties required by any of the physics interfaces show up automatically in the defined material's **Settings** window. See Materials.
- Each physics interface you include in a model forms its own subbranch placed below the **Materials** branch.
- The *Mesh branch* collects all meshes defined for a model. If there is only a single mesh in a model, its **Mesh** node appears directly under the corresponding **Model** node. See Meshing.

SEQUENCING

COMSOL Multiphysics operates through *sequencing*. By adding nodes to a model branch in the **Model Builder** in a certain order, you set up sequences of operations, such as geometry, mesh, solver, and results sequences. The software records a sequence as a macro in the background. A solver sequence, for example, could define your model with one solver and then, using the returned solution, solve it with an alternative solver. If you make changes to a node, the software automatically updates other parts of the sequence that are dependent on this change. For most sequences, you run the sequence by right-clicking the top node for the branch and selecting **Build All** (geometry and mesh), **Compute** (studies), or **Plot** (plot groups), or by pressing F8.

OPENING A CONTEXT MENU TO ADD NODES

Right-click nodes in the **Model Builder** to open the *context menu* and add additional, and relevant, functionality, operations, or attributes to the model sequence. After selecting an option from the list, a **Settings** window opens to the right (by default) of the **Model Builder** window. In the context menu, a plus sign next to the icon means a node of that type is added to the **Model Builder**. On Windows, you can also open the context menu using



the Menu key or by pressing Shift+F10, on Linux by pressing Shift+F10, and on the Mac by pressing Ctrl+F10 (see Summary of Keyboard Shortcuts for a complete list of keyboard shortcuts).

MOVING NODES IN THE MODEL TREE

The order of the nodes in a branch affects the evaluation of the sequence. In the following branches it is possible to move nodes up and down to control the evaluation of the sequence:

- The **Material** nodes in the **Materials** branch.
- The nodes for physics interfaces (such as material models, boundary conditions, and sources) in the **Physics** branch, except the default nodes.
- The mesh nodes in the **Mesh** branch.
- The study step nodes in the **Study** branch.

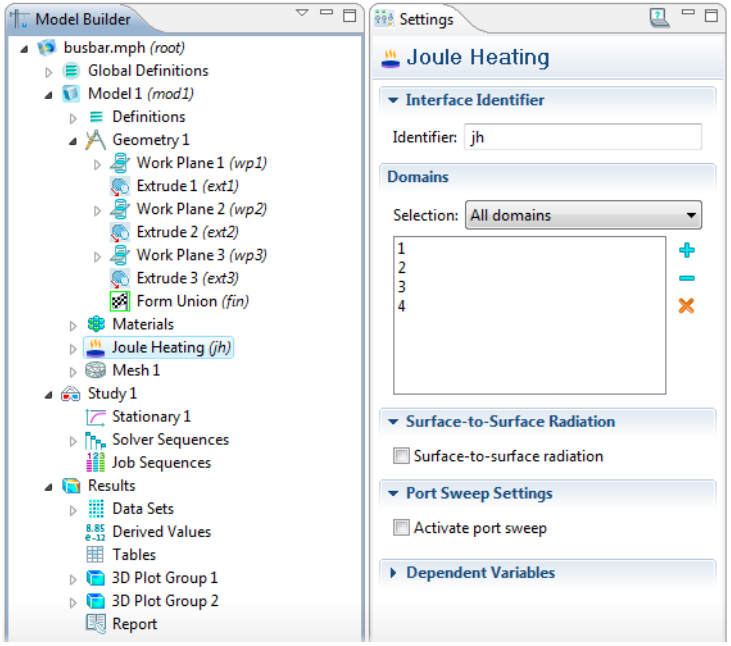
To move a node up or down, right-click the node and select **Move Up** () or **Move Down** (), or by using the corresponding keyboard shortcuts Ctrl+up arrow or Ctrl+down arrow.

GOING TO THE SOURCE NODE

In the **Settings** window for many nodes you reference other nodes in the model tree such as one of the following:

- A model
- A solution
- A study or study step
- A data set

Nodes where you refer to other nodes include plot groups, data sets, and solver; in such nodes' **Settings** windows, click the **Go to Source** button () to move to the node that the selection in the list next to the button refers to.



The **Settings** window ( ) is an interactive environment for defining an operation or model property while the model and changes are displayed in the **Graphics** window. When you click on any node in the **Model Builder**, the corresponding **Settings** window displays a page with the same name.

When an operation or property is updated in the **Settings** window, its effect on the model is displayed in the **Graphics** window either instantaneously or by clicking the **Build Selected** button ( ) or the **Plot** button ( ) available in some of the **Settings** window toolbars.

To select the parts of the model to define in a specific **Settings** window, select the relevant geometric entities either directly in the displayed model in the **Graphics** window or from the **Selection List** window.

The Graphics Window

The **Graphics** window () presents a graphical view of the geometry, mesh, and postprocess results of your model.

The **Graphics** window has useful tools for changing the view and selecting multiple entities. For more information, see Visualization and Selection Tools.

The Messages Window

Note: Also see Meshing and Solving.

The **Messages** window () contains information useful to you after an operation is performed. This includes:

- The number of mesh elements and degrees of freedom in your model.
- Solution times.
- Error messages. The messages are in chronological order and can be scrolled through.

The Progress Window

Note: Also see Meshing and Solving.

The **Progress** window () displays the progress of the meshing and model solving process. You can stop the solving process if it is determined that a model's solving time or likelihood for convergence is not progressing as expected.

The Results Window

The **Results** window () displays the results from integral and variable evaluations defined in Derived Values features and stored in Tables features. For details on how to use these features, see the section Plot Groups and Plotting in the *COMSOL Multiphysics Reference Guide* (or see Where Do I Access the Documentation and Model Library?).

The Help Window

The **Help** window provides access to *dynamic help* in the COMSOL Desktop and has these pages: **Related Topics**, **Contents**, **Search**, **Bookmarks**, and **Index**.

- *Related Topics*: A brief description of the respective branch, sub-branch, or leaf node that is active in the **Model Builder**. This is an interactive environment. Click on another node then the **Related Topics** window to update the contents instantly.

  To view more detailed information, including summaries of the different attributes and settings related to the node, click on one or more of the topics found under the **See also** section. This takes you into COMSOL Multiphysics' documentation.

Note: Once you enter the COMSOL Multiphysics documentation, the contents of the Help window are no longer updated when you change the active node or window. To return to the Related Topics page, either click the **Back** button or refresh the Help window as described in the next section.

  The **More results** section, if present, gives a shortcut to the Search page with a search string related to the active Model Tree node or COMSOL Desktop component.

- *Contents*: A tree-based menu structure of COMSOL Multiphysics' documentation.
- *Search*: A search engine for returning text strings, and combinations of same. Typing two text strings in the edit field returns all instances where both strings display in the same document (such as a section or example model in a manual). Quotation marks return a text string that displays exactly as it has been written, that is, with the words in the order given within the quotation marks. The "@" works as a wild-card.
- *Bookmarks*: A list of saved search criteria.
- *Index*: A tree-based menu structure of the documentation indexes.

OPENING OR REFRESHING THE HELP WINDOW

There are up to four different ways to open or refresh the **Help** window:

- Click the **Dynamic Help** button () in the active window's toolbar (not all windows have this toolbar).
- From the main menu, select **Help>Dynamic Help**.
- Press F1.

- Right-click a node in the **Model Builder** and select **Dynamic Help** ( (the last item in the menu).

  By default, the **Help** window is added to the right of the **Graphics** window.

If you select the **Strict Help Refresh** option on the **Help** menu, the **Help** window always updates its contents to shows the relevant context help information when clicking on a window tab or on a model tree node in the **Model Builder**, regardless of which part of the help system that you display in the **Help** window.

Note: The first time you access the help system during a modeling session COMSOL Multiphysics loads the help files. This might take a few seconds.

Note: The very first time you open the **Help** window on your computer, the operating system might issue a firewall security warning. To use Dynamic Help, you need to allow COMSOL Multiphysics access through the firewall.

The Model Library Window

Note: Also see Results for various examples of model plots.

Select **File>Open Model Library** to open the **Model Library** () window. Browse and load model files to the COMSOL Desktop. To locate any models using a specific feature, type a suitable keyword in the edit field and click **Search**.

The **Model Library** window's toolbar contain the following buttons:

- **Refresh** (): Click to update the Model Library tree, for example if you have saved a file in a folder under the Model Library root directory since you opened the **Model Library** window.

- **Set Model Library Root** ( ): Clicking this button launches the **Model Library Root** window where you can enter a new root directory for the Model Library or just inspect its current location.
- **Dynamic Help** ( ): Select a model in the Model Library tree and click this button to read a brief description of the model in the **Help** window. To read the full model documentation, including step-by-step modeling instructions, click the link under **See also** in the **Help** window.

To read the model documentation in PDF format, select the model in the tree and then click the **Model PDF** button. Alternatively, right-click the model node and select **Model PDF** from the context menu.

You can load a model to the COMSOL Desktop from the Model Library in three different ways:

- Double-click the model node in the tree.
- Select the model node, then click the **Open** button.
- Right-click the model node, then select **Open** from the context menu.

The Material Browser Window

In the **Material Browser** window ( ) you can browse the material libraries and load materials into your models. For more information about the **Material Browser** window and working with materials, see Materials.

Summary of Keyboard Shortcuts

The following table summarizes the available keyboard shortcuts on Windows and Linux and on the Mac:

SHORTCUT (WINDOWS, LINUX)	SHORTCUT (MAC)	ACTION
F1	F1	Display dynamic help for the selected node
F2	F2	Rename the selected node, file, or folder
F8	F8	Build the geometry, build the mesh, or update the plot
Del	Del	Delete a node
Ctrl+up arrow	Command+up arrow	Move a physics node (except default nodes), material node, mesh node, or study step node up one step in the sequence

SHORTCUT (WINDOWS, LINUX)	SHORTCUT (MAC)	ACTION
Ctrl+down arrow	Command+down arrow	Move a physics node (except default nodes), material node, mesh node, or study step node down one step in the sequence
Ctrl+Tab	Ctrl+Tab	Switch focus to the next window in the desktop
Ctrl+Shift+Tab	Ctrl+Shift+Tab	Switch focus to the previous window in the desktop
Ctrl+Alt+left arrow	Command+Alt+left arrow	Switch focus to the Model Builder window
Ctrl+Alt+right arrow	Command+Alt+right arrow	Switch focus to the Settings window
Ctrl+Alt+up arrow	Command+Alt+up arrow	Switch focus to the previous section in the Settings window
Ctrl+Alt+down arrow	Command+Alt+down arrow	Switch focus to the next section in the Settings window
Shift+F10 or (Windows only) Menu key	Ctrl+F10	Open context menu
Ctrl+Space	Ctrl+Space	Open list of predefined quantities for insertion in Expression edit fields for plotting and results evaluation.

3

Global and Local Definitions

This section describes the global and local (model) definitions features. Depending on the geometric scope, you add the feature nodes described in this section to either the **Global Definitions** node or under the **Definitions** node for a particular model. The definitions you can add include:

- About Global and Local Definitions
- Global Parameters
- Operators, Functions, and Variables Reference
- Variables
- Global and Local Functions
- Model Couplings
- Coordinate Systems
- Identity and Contact Pairs
- Probes

Also see Visualization and Selection Tools and Using Units.

About Global and Local Definitions

There are two types of definitions that can be defined in the **Model Builder**—**Global** and local **Definitions**.

Global Definitions

Right-click **Global Definitions** to add the following:

- **Parameters**: global, scalar values that you can use to parameterize any part of the model. Global Parameters
- **Variables**: expression variables that can be used anywhere to simplify the specifications of some properties. Variables
- **Functions** are predefined function templates for common function types such as step functions, ramps, and random functions. Global and Local Functions

Local Definitions

After adding any **Model** node to the **Model Builder**, right-click **Definitions** to add definitions with a local scope that are applied to that specific model. Add:

- **Variables**: expression variables used to simplify the specifications of some properties. Variables
- **Selection**: create a user defined set of geometric entities for reuse throughout the model. See User-Defined Selections.
- **View**: create a user defined view to visualize the model. See User-Defined Views.
- **Contact Pair**: specify two selections of boundaries that cannot penetrate each other under deformation. Identity and Contact Pairs
- **Identity Pair**: specify two selections of boundaries that overlap but belong to different parts of an assembly. Then assign a boundary condition to connect the physics in the two parts in a physics interface. Identity and Contact Pairs
- **Functions** are predefined function templates for common function types such as step functions, ramps, and random functions. Global and Local Functions
- **Model Couplings**. The coupling operators are useful for creating integrated quantities and identity mappings, adding nonlocal couplings, and many other applications. Model Couplings

- **Coordinate Systems**. You can create additional coordinate systems of different types for use in some of the physics interfaces. Coordinate Systems
- **Probes**. To monitor the development of a scalar-valued quantity (real or complex valued number) from a dynamic simulation (time-dependent, frequency, parametric). Probes

Building Expressions

When you define any expression, whether it is global or local, there are many available variables, mathematical functions, physical constants, and operators that you can use to define the expression.

You can also use many common mathematical and logical operators and functions along with constants to specify parameters and variables as well as equation coefficients and material properties.

- Variables
- Physical constants are predefined constants for universal constants such as the universal gas constant and the speed of light in vacuum.
- Operators
- Mathematical functions

Operators, Functions, and Variables Reference

Many common mathematical and logical operators, functions, and constants can be used to specify parameters, variables, equation coefficients, and material properties. These tables list the unary and binary operators (Table 3-1 and Table 3-2), special operators (Table 3-3), mathematical functions and constants (Table 3-4), and physical constants predefined as variables (Table 3-5) that can be used in COMSOL. Also see Model Couplings for information about coupling operators.

Unary and Binary Operators

TABLE 3-1: UNARY OPERATORS

OPERATOR	DESCRIPTION
+	unary plus
-	unary minus
!	logical not

TABLE 3-2: BINARY OPERATORS

OPERATOR	DESCRIPTION
+	plus
-	minus
*	multiply
/	divide
^	power
==	equal
!=	not equal
>	greater than
>=	greater than or equal to
<	less than
<=	less than or equal to

TABLE 3-2: BINARY OPERATORS

OPERATOR	DESCRIPTION
&&	logical and
||	logical or

Special Operators

There are special operators available to model and evaluate results. Many physics interfaces use these operators to implement equations and special functionality. See Table 3-3 and the detailed descriptions that follow.

TABLE 3-3: SPECIAL OPERATORS

OPERATOR	DESCRIPTION
`d(f,x)`	Differentiation operator. Differentiation of `f` with respect to `x`. See The Differentiation Operators for more information.
`pd(f,x)`	Differentiation operator. Differentiation of `f` with respect to `x`. No chain rule for dependent variables. See The Differentiation Operators for more information.
`test(`*`expr`*`)`	Test function operator. See The Test Operator for more information.
`nojac(`*`expr`*`)`	No contribution to the Jacobian. See The Nojac Operator for more information.
`up(`*`expr`*`)`	Evaluate expression as defined in adjacent upside. See The Up, Down, and Mean Operators for more information.
`down(`*`expr`*`)`	Evaluate expression as defined in adjacent downside. See The Up, Down, and Mean Operators for more information.
`mean(`*`expr`*`)`	Mean value of expression as evaluated on adjacent boundaries. See The Up, Down, and Mean Operators for more information.
`depends(`*`expr`*`)`	True if expression depends on the solution. See The Depends Operator for more information.
`islinear(`*`expr`*`)`	True if expression is a linear function of the solution. See The Islinear Operator for more information.
`if(`*`cond`*`,`*`expr1`*`,`*`expr2`*`)`	Conditional expression evaluating the second or third argument depending on the value of the condition. See The If Operator for more information.

TABLE 3-3: SPECIAL OPERATORS

OPERATOR	DESCRIPTION
`with`	Access any solution. See The With Operator for more information.
`at`	Access the solution at any time. See The At Operator for more information.
`ppr`	Accurate derivative recovery. See The Ppr and Pprint Operators for more information.
`pprint`	Accurate derivative recovery within each domain group. See The Ppr and Pprint Operators for more information.
`reacf`	Accurate integrals of reaction forces and fluxes. See The Reacf Operator for more information.
`adj(`*`expr`*`)`	Evaluate expression using the adjoint sensitivity. See The Adj Operator for more information.
`fsens(`*`expr`*`)`	Evaluate expression using the functional sensitivity. See The Fsens Operator for more information.
`sens(`*`expr`*`,`*`i`*`)`	Evaluate expression using the forward sensitivity for the parameter given by the second argument. See The Sens Operator for more information.
`realdot(a,b)`	Treat complex numbers a and b as real 2-vectors and return their dot product. See The Realdot Operator for more information.
`shapeorder(`*`variable`*`)`	The element order used for discretization of variable. See The Shapeorder Operator for more information.
`prev(`*`expr`*`,`*`i`*`)`	Evaluate expression at the *i*th previous time step. See The Prev Operator for more information.
`bdf(`*`expr`*`,`*`i`*`)`	Apply backward differentiation formula of order *i* on expression. See The Bdf Operator for more information.

THE DIFFERENTIATION OPERATORS

- Use the `d` operator to differentiate a variable with respect to another variable. For example, `d(T,x)` means differentiation of T with respect to x.
- Use the `d` operator in the model settings. If it contains any of the dependent variables, it typically makes the model nonlinear.

- You can use the `d` operator during postprocessing to visualize and compute derivatives. Some space derivatives are also available using predefined variables. For example, `uxx`, `d(ux,x)`, and `d(d(u,x),x)` are equivalent.
- The `pd` operator works in a similar way to the `d` operator but does not apply the chain rule for dependent variables. The expression `d(E,TIME)` computes the reference time derivative of the expression `E`.

Examples of Using the Differentiation Operators

The expressions `d(u^2,u)` and `pd(u^2,u)` both equal `2*u`—`d` also takes the space and time variables into account and treats their derivatives accordingly. In other words, if u is the dependent variable and x and t are the space coordinate and time, respectively, then `d(u+x,x)` equals `ux+1` (`ux` is the space derivative of u with respect to x), while `pd(u+x,x)` equals `1`, because u is considered to be independent of anything but itself in the case of `pd`. Equivalently, `d(u,t)` equals `ut`, while `pd(u,t)` is zero.

THE TEST OPERATOR

- The `test` operator is available for modeling using the weak formulation of the equations.
- This operator creates the test function for the variable that it operates on. As an alternative to the operator `test(u)`, you can use `u_test`, that is, append `_test` to your dependent variable name. For an expression, $\text{test}(F(u, \nabla u))$, the test operator is equivalent to:

$$\sum_i \text{test}(u_i)\frac{\partial}{\partial u_i}F(u_i, \nabla u_i) + \text{test}(\nabla u_i)\frac{\partial}{\partial \nabla u_i}F(u_i, \nabla u_i)\,,$$

  for all dependent variables u_i.

THE NOJAC OPERATOR

- The `nojac` operator makes sure that any expression that it operates on is excluded from the Jacobian computation. This is useful if a Jacobian contribution is not strictly necessary and the computational requirements for it are high.
- The k-ε turbulence model is an example where the use of the `nojac` operator improves performance.

THE UP, DOWN, AND MEAN OPERATORS

- COMSOL Multiphysics can evaluate expressions on both sides of a domain. These operators are available only on boundaries.

- For an expression or a variable that is discontinuous across a boundary, the value is different on either side, and COMSOL Multiphysics normally displays the mean values on the boundary.
- Use the `up`, `down`, and `mean` operators to evaluate an expression on the upside or downside of the boundary or the mean value on the boundary. If the upside or downside is outside of the geometry, or if the variables in the expression are not active on that side, the `up` or `down` operator returns 0.

For more information about the upside and downside of a boundary, see Tangent and Normal Variables.

THE DEPENDS OPERATOR

- The `depends` operator returns 1 if the expression that it operates on depends on the solution; otherwise it returns 0.
- Use this operator to check user defined expressions for dependency on the solution.

THE ISLINEAR OPERATOR

- The `islinear` operator returns 1 if the expression that it operates on is a linear function of the solution; otherwise it returns 0.
- Use this operator to check user defined expressions for linearity with respect to the solution. The stationary solver does this automatically to choose between a linear or a nonlinear solver.

THE IF OPERATOR

- The `if(`*`cond`*`,`*`expr1`*`,`*`expr2`*`)` operator implements a conditional expression.
- The first argument is a condition that COMSOL Multiphysics treats as a Boolean expression. If—at a particular evaluation point—`cond` is true, then the second argument is evaluated, otherwise the third argument is evaluated. That is, only one branch is evaluated at any evaluation point.
- Use the `if` operator to avoid illegal operations. For example, `if(x==0,1,sin(x)/x)` is a valid and continuous expression for all values of x, including $x = 0$.

THE WITH OPERATOR

- The `with` operator can access any solution during results evaluation.
- For time-dependent problems, parametric problems, and eigenvalue problems, this makes it possible to use the solution at (1) any of the time steps, (2) any parameter value, or (3) any eigensolution in an expression used for plotting or data evaluation.

- Use the solution number as the first input argument. The second input argument is the expression that you want to evaluate using this solution. For example, `with(3,u^2)` provides the square of the third eigensolution for an eigenvalue problem.
- For example, use the `with` operator to verify that two eigensolutions are orthogonal.

THE AT OPERATOR

- The `at` operator can access a solution to a time-dependent problem at any time. COMSOL Multiphysics provides the solution at that time using interpolation.
- The first input argument is the time. The second input argument is the expression that you want to evaluate using this solution. For example, `at(12.5,u)` is the solution at 12.5 s.

THE PPR AND PPRINT OPERATORS

- When you apply the `ppr` operator on an expression, COMSOL uses polynomial-preserving recovery to evaluate all variables in the expression that are discretized using Lagrange shape functions; see Accurate Derivative Recovery.

  For example, if `e = ux + vy`, then `ppr(e^2)=(ppr(ux)+ppr(vy))^2`.
- The `pprint` operator similarly applies polynomial-preserving recovery within each group of domains with equal settings. You can use these operators to get an estimate of the discretization error in the gradient.

  For example `ux-pprint(ux)` in a 1D model.

Note: If you apply these operators on expressions that are used when solving the model, COMSOL computes the Jacobian approximately by ignoring the operator. For example, the Jacobian of `ux-pprint(ux)` is 0.

THE REACF OPERATOR

- The reaction force operator (`reacf`) is available when computing integrals of reaction forces or fluxes.
- Apply the `reacf` operator on the names of dependent variables when doing a surface integration. For example, in structural mechanics, with dependent variable u and v corresponding to x- and y-displacements, use `reacf(u)` and `reacf(v)` to access integrals of the reaction forces in the x- and y-direction, respectively.

- You can disable access to the operator by clearing the **Reaction forces** check box in the **Output** section in the solver's **Settings** window.
- The reaction force operator is not available when using weak constraints.

THE ADJ OPERATOR

- When you apply the adjoint sensitivity operator (`adj`) to an expression, COMSOL uses the adjoint sensitivity solution instead of the primal solution for the evaluation.
- The adjoint sensitivity solution is available for results when the sensitivity solver has been used with the adjoint sensitivity method, and for the dependent variables that have been solved for.

THE FSENS OPERATOR

- When you apply the functional sensitivity operator (`fsens`) to an expression, COMSOL uses the functional sensitivity solution for the evaluation.
- The functional sensitivity solution is available for postprocessing when the sensitivity solver has been used with either the adjoint or the forward sensitivity method, and for the sensitivity variables that have been solved for.

THE SENS OPERATOR

- When you apply the forward sensitivity operator (`sens`) to an expression, COMSOL uses the forward sensitivity solution instead of the primal solution for the evaluation.
- The forward sensitivity solution is available for postprocessing when the sensitivity solver has been used with the forward sensitivity method, and for the dependent variables that have been solved for.
- For scalar sensitivity variables, you can access the corresponding forward sensitivity solution by giving the sensitivity variable name as the second argument to this operator. For example, with the dependent variable u and the scalar sensitivity parameter q, you access the forward sensitivity solution $\partial u/\partial q$ as `sens(u,q)`.
- For a sensitivity parameter that is not scalar, a more elaborate syntax specifying a unique degree of freedom must be used. This is done by giving an integer as the second argument, corresponding to the global degree of freedom number for the requested sensitivity parameter.

THE REALDOT OPERATOR

- The expression `realdot(a,b)` treats complex numbers a and b as if they were real-valued vectors of length 2 and returns their dot product. You can also think of the operator call as a shorthand form of `real(a*conj(b))`. This expression,

however, is not an analytical function of its complex arguments and therefore has no unique partial derivatives with respect to a and b.

- The difference between `realdot(a,b)` and `real(a*conj(b))` is that the partial derivatives of the former with respect to a and b are defined as $\mathrm{conj}(b)$ and $\mathrm{conj}(a)$, respectively, while for the latter expression, the partial derivatives are $\mathrm{real}(\mathrm{conj}(a))$ and $\mathrm{real}(a)$.
- The difference between the partial derivative definitions is important during sensitivity analysis of frequency-response problems (scalar or vector Helmholtz equations).
- Common objective function quantities like power and energy must be redefined in terms of `realdot(a,b)` rather than `real(a*conj(b))` for the sensitivity solver to compute correct derivatives. This applies also to the absolute value, `abs(a)`, via the definition $|a|^2 = \mathrm{realdot}(a,a)$.

THE SHAPEORDER OPERATOR

- The expression `shapeorder(`u`)` gives the element order used for discretization of the variable u.
- The argument u must be a dependent variable (or a partial derivative of a dependent variable).
- It is an error to apply the `shapeorder` operator to, for example, an expression, a constant, or a space coordinate.

THE PREV OPERATOR

- When the time discrete solver is used, it stores the solution at a number of previous time steps.
- The expression `prev(`*expr*,*i*`)` evaluates *expr* using the solution obtained *i* time steps before the current time step.
- The operator can be used in equations as well as for results evaluation.
- When used in equations, the `prev` operator makes it possible to discretize time derivatives. For example, to discretize `ut` (the time derivative of u) with the formula known as the *backward Euler method*, use the expression `(u-prev(u,1))/timestep`. Here, `timestep` is the size of the time step used to reach the current solution `u`. The `prev` operator is also applicable for `timestep`. For example,

`prev(timestep,1)` is the size of the time step used to reach the solution at the previous time step.

- When using the `prev` operator, sufficiently many previous time steps must be stored. You can specify the number of previous time steps to store in the time discrete solver (time discrete levels) in the **Number of time discrete levels** field in the **General** section of the **Time Discrete** solver's **Settings** window. Evaluating an expression at a previous time step that has not been stored results in an error.

THE BDF OPERATOR

- You can use the `bdf` operator to approximate time derivatives when the time discrete solver is used.
- The expression `bdf(`*expr*`,`*i*`)` results in a discretization of the time derivative of *expr* using a backward differentiation formula.
- The second argument, *i*, determines the order of accuracy of the discretization. Currently, first order and second order is available, so allowed values are $i = 1$ and $i = 2$. A second-order formula requires access to two previous time steps. Because this is not possible at the initial step, the evaluation at the initial step always uses the first-order formula.
- The `bdf` operator can be implemented using the `prev` operator. For example, you can obtain the first-order backward differentiation formula, also known as the backward Euler method, through `bdf(u,1) = (u-prev(u,1))/timestep`.

Mathematical Functions

Note: The following functions are only available when solving and not when evaluating parameters and variables in the user interface: `acosh`, `acoth`, `acsch`, `asech`, `asinh`, `atanh`, `besselj`, `bessely`, `besseli`, `besselk`, and `erf`.

TABLE 3-4: MATHEMATICAL FUNCTIONS

FUNCTION	DESCRIPTION	SYNTAX EXAMPLE
`abs`	absolute value	`abs(x)`
`acos`	inverse cosine	`acos(x)`
`acosh`	inverse hyperbolic cosine	`acosh(x)`
`acot`	inverse cotangent	`acot(x)`
`acoth`	inverse hyperbolic cotangent	`acoth(x)`

TABLE 3-4: MATHEMATICAL FUNCTIONS

FUNCTION	DESCRIPTION	SYNTAX EXAMPLE
`acsc`	inverse cosecant	`acsc(x)`
`acsch`	inverse hyperbolic cosecant	`acsch(x)`
`arg`	phase angle	`arg(x)`
`asec`	inverse secant	`asec(x)`
`asech`	inverse hyperbolic secant	`asech(x)`
`asin`	inverse sine	`asin(x)`
`asinh`	inverse hyperbolic sine	`asinh(x)`
`atan`	inverse tangent	`atan(x)`
`atan2`	four-quadrant inverse tangent	`atan2(y,x)`
`atanh`	inverse hyperbolic tangent	`atanh(x)`
`besselj`	Bessel function of the first kind	`besselj(a,x)`
`bessely`	Bessel function of the second kind	`bessely(a,x)`
`besseli`	modified Bessel function of the first kind	`besseli(a,x)`
`besselk`	modified Bessel function of the second kind	`besselk(a,x)`
`ceil`	round up to nearest integer	`ceil(x)`
`conj`	complex conjugate	`conj(x)`
`cos`	cosine	`cos(x)`
`cosh`	hyperbolic cosine	`cosh(x)`
`cot`	cotangent	`cot(x)`
`coth`	hyperbolic cotangent	`coth(x)`
`csc`	cosecant	`csc(x)`
`csch`	hyperbolic cosecant	`csch(x)`
`eps`	floating point relative accuracy	`eps`
`erf`	error function	`erf(x)`
`exp`	exponential	`exp(x)`
`floor`	round down to nearest integer	`floor(x)`
`gamma`	gamma function	`gamma(x)`
`i`, `j`	imaginary unit	`i`
`imag`	imaginary part	`imag(u)`
`inf`	infinity	`inf`
`log`	natural logarithm	`log(x)`

TABLE 3-4: MATHEMATICAL FUNCTIONS

FUNCTION	DESCRIPTION	SYNTAX EXAMPLE
log10	common logarithm (base 10)	log10(x)
log2	base 2 logarithm	log2(x)
max	maximum of two arguments	max(a,b)
min	minimum of two arguments	min(a,b)
mod	modulo operator	mod(a,b)
NaN, nan	not-a-number	nan
pi	pi	pi
psi	psi function and its derivatives	psi(x,k)
range	create a range of numbers	range(a,step,b)
real	real part	real(u)
round	round to closest integer	round(x)
sec	secant	sec(x)
sech	hyperbolic secant	sech(x)
sign	signum function	sign(u)
sin	sine	sin(x)
sinh	hyperbolic sine	sinh(x)
sqrt	square root	sqrt(x)
tan	tangent	tan(x)
tanh	hyperbolic tangent	tanh(x)

Physical Constants

Reference

The NIST Reference on Constants, Units, and Uncertainty, http://physics.nist.gov/constants

Physical constants are fundamental, universal constants that represent physical quantities. COMSOL Multiphysics includes the most widely used physical constants as predefined variables. Also see Variables.

Table 3-5 lists all supported physical constants with their names, symbol (variable name), value, and SI unit. When defined as variables, all variable names get the suffix `_const` that indicates that they are physical constants. For example, `mu0_const` is the

variable name for the permeability of vacuum. The values are taken from Ref. 1 and include the SI unit.

TABLE 3-5: PHYSICAL CONSTANTS

NAME	SYMBOL (VARIABLE)	VALUE
Acceleration of gravity	`g`	`9.80665[m/s^2]`
Avogadro constant	`N_A`	`6.02214179e23[1/mol]`
Boltzmann constant	`k_B`	`1.3806504e-23[J/K]`
Characteristic impedance of vacuum (impedance of free space)	`Z0`	`376.730313461...[ohm] (mu0*c)`
Electron mass	`me`	`9.10938215e-31[kg]`
Elementary charge	`e`	`1.602176487e-19[C]`
Faraday constant	`F`	`96485.3399[C/mol]`
Fine-structure constant	`alpha`	`7.2973525376e-3`
Gravitational constant	`G`	`6.67428e-11[m^3/(kg*s^2)]`
Molar volume of ideal gas (at 273.15 K and 1 atm)	`V_m`	`22.413996e-3[m^3/mol]`
Neutron mass	`mn`	`1.674927211e-27[kg]`
Permeability of vacuum (magnetic constant)	`mu0`	`4*pi*1e-7[H/m]`
Permittivity of vacuum (electric constant)	`epsilon0`	`8.854187817e-12[F/m]`
Planck's constant	`h`	`6.62606896e-34[J*s]`
Planck's constant over 2 pi	`hbar`	`1.054571628e-34[J*s]`
Proton mass	`mp`	`1.672621637e-27[kg]`
Speed of light in vacuum	`c`	`299792458[m/s]`
Stefan-Boltzmann constant	`sigma`	`5.670400e-8[W/(m^2*K^4)]`
Universal gas constant	`R`	`8.314472[J/(mol*K)]`
Wien displacement law constant	`b`	`2.8977685e-3[m*K]`

It is important to be careful using physical constants if the unit system is not in SI units. For example, the default values for the permittivity of vacuum and the permeability of vacuum require that you provide all other quantities in SI units and that you use meter (m) for the geometry length. If you draw the geometry using another length scale, you need to change the numerical values for the physical quantities accordingly. For example, if you draw the geometry using μm as the length scale, you must use $\varepsilon_0 = 8.854187817 \cdot 10^{-18}$ F/μm and $\mu_0 = 4\pi \cdot 10^{-13}$ H/μm.

Global Parameters

Under the **Global Definitions** node, enter values in the **Parameters** table to define parameters used throughout the model making it possible to parameterize, for example, a geometric dimension. Parameters are scalar numbers and they are the same for all geometries and domains, they can depend on other parameters, and can contain any mathematical functions.

You can use parameters in the following contexts:

- In dimensions for geometric primitives or other geometry features
- As parameters for the mesh generators
- As parameters to control some aspects of the solution process
- To quickly evaluate a mathematical expression, including unit conversion
- Dimensions in the geometry
- As mesh-size parameters
- In physics settings, expressions, and coupling operators
- In expressions when evaluating results

Defining, Saving, or Importing Global Parameters

1 In the **Model Builder**, right-click **Global Definitions** and select **Parameters**.

The **Parameters** page opens in the **Settings** window and a node is added under **Global Definitions**.

2 Enter parameters manually or import from a text file (see Loading Parameters from a Text File below).

a In the **Parameters** table or the field under the table, enter a parameter **Name**.

b In the **Expression** column or field, enter an expression that defines the parameter value—a scalar number.

The parameter value can be a function of other parameters as well as mathematical constants and functions, and it can use unit syntax.

The **Value** column displays the value of the parameter in the base SI units.

c In the **Description** column or field, enter an optional description.

3 Add more parameters as required to the rows in the table.

Organize the table with the **Move Up** ( ), **Move Down** ( ), or **Delete** ( ) buttons or right-click the cell and select **Move Up**, **Move Down**, or **Delete**.

4 *Optional:* You can save the parameters to a text file to reuse in other models. Click the **Save to File** button ( ) and enter a **File Name**, including the extension .txt. Click to **Save** the text file.

The information is saved in tab-separated columns in the same order as displayed on screen.

Loading Parameters from a Text File

1 In the **Model Builder**, right-click **Global Definitions** and select **Parameters**.

The **Parameters** page opens in the **Settings** window and a node is added under **Global Definitions**.

2 Load data from a spreadsheet program or other format by saving the file with a .txt extension. Data must be separated by tabs or colons.

3 In the **Settings** window, click the **Load from File** button ( ).

4 Navigate to the text (.txt) file to load and click **Open**.

The data loads to the table. If there is other data already in the table, it adds it after the last row. Move or edit as required.

Variables

Using the **Variables** (a=) node's **Settings** window you can create symbolic *variables* that are defined using an expression. Depending on the type of variable, you can use a variable in a local or global context to simplify model definitions and for convenient access to quantities related to the expression that defines the variable. This section describes the syntax and names for variables that you define or that are predefined in a COMSOL Multiphysics model. Variables can be applied both at the global and local levels.

About Global and Local Variables

You can define *global variables* or *local variables* in the same way as parameters and functions. Global variables are applied to all the models in a file, whereas local variables are valid in the entire model (geometry) or for specific domains, boundaries, edges, or points. This provides the flexibility to create variables with different meanings in different parts of the model.

THE BENEFITS OF DEFINING VARIABLES

Defining variables simplifies model specification and results analysis. For example, variables can make a model easier to understand by using short names for complicated expressions. You can also use expressions that contain other variables, which are then computed when solving the model.

Using variables can also improve visualization and results analysis. For example, if you need to view a field variable throughout a model, but it has different names in different domains, you can create unique names for the field variables contained in different domains and then plot the variable to show the entire field in all domains. If you have provided a description for a variable that you use as the expression in a plot, it becomes the default description for the plot.

To specify a coefficient or a material property, you can type a value or an expression directly in the corresponding field. Such expressions can contain:

- Numerical values.
- Units (see Using Units).
- Mathematical and logical functions (see Mathematical Functions).

- Space coordinates, time, and the dependent variables in any physics interface in the model as well as their space derivatives and time derivatives.
- Physical constants—predefined universal constants (see Physical Constants).
- Parameters, expression variables, coupling operators, and functions you have defined, including external functions and MATLAB functions (requires the COMSOL LiveLink to MATLAB). See Operators, Functions, and Variables Reference.
- COMSOL Multiphysics functions and operators such as `d` and `mean`.

You can use these types of variables, constants, and expressions in all features for the physics interfaces; many types of variables are also available anywhere in the model.

About Predefined Physics Interface Variables

Physics interface variables are predefined variables that the physics interfaces introduce. They are typically functions of the dependent variables. Many of these variables are available as **Predefined quantities** lists in the settings for plots and other results analysis features.

Physics interface variables are physics-interface specific and consist of physically meaningful combinations of material properties and other variables. These variables are predefined for each physics interface.

To access physics interface variables, use a variable scoping syntax that uses the interface identifier to indicate the physics interface that they belong to.

Variable Naming Convention and Scope

COMSOL Multiphysics uses variable scoping to control the access to variables within a model and variables in other models within the same MPH-file, for example. To access variables, use the following scoping mechanisms:

- To refer to the top level of the model tree, use `root`.
- To refer to variables in a model branch, use its model identifier, such as `mod1`.
- To refer to variables in a physics interface, use its interface identifier, such as `solid`.
- COMSOL evaluates the physics interface variables in the model scope so you need to prepend the interface identifier to access these variables. For example, `solid.disp` refers to the total displacement in a **Solid Mechanics** interface with the interface identifier `solid`.

- The dependent variable (field variable) are unique within a model, and you do not need the interface identifier to access them. For example, you can type just `T` to access the temperature in a **Heat Transfer** interface.
- When you refer to a variable you only need to provide the part of the full scope that makes the variable unique. For example, within a **Solid Mechanics** interface `solid` in Model 1 `mod1`, it is sufficient to type `solid.disp`, but `mod1.solid.disp` and the full scope `root.mod1.solid.disp` are also correct. To access the same variable from another model, use `mod1.solid.disp` or `root.mod1.solid.disp`. The same mechanism applies to variables defined within a model. To access a global parameter `param1`, you can use `param1` directly or `root.param1`.

For information about the interface identifiers, see the sections about the physics interfaces in this book and the *User's Guides* for the modules.

Variable Classification and Geometric Scope

COMSOL Multiphysics provides a set of variables you can use in expressions when specifying a model and for visualizing and postprocessing the solution. A number of variables are common to all physics interfaces in a model, for example, the space coordinate variables x, y, and z.

Every physics interface also has its own set of variables to represent quantities relevant to the physics or equations that it covers. Characteristics of variables include:

- Parameters and geometric variables are always available.
- The choice of physics interfaces and the dimension of the geometries in the model affect the set of available field variables and special variables.
- Equations can be active in different domains, which also affects the set of available variables. Variables corresponding to certain equation terms are available only in the particular part of the geometry (such as domains, boundaries, or points) where the equation is active.
- Variables defined on boundaries, edges, and points are active if the adjacent domain is active.

Variables are divided into the following general categories:

- *Parameters* (see Global Parameters) and *expression variables* (user defined parameters)
- Variables for time, frequency, and eigenvalues. See Variables for Time, Frequency, and Eigenvalues.

- *Physical constants* (predefined universal constants)
- *Geometric variables* (variables that characterize geometric properties). See Geometric Variables.
- *Field variables* (dependent variables and variables derived from them). For example, shape function variables (see Shape Function Variables) and physics interface variables (see About Predefined Physics Interface Variables).
- *Coupling operators*. See Model Couplings.

Specifying Varying Coefficients and Material Properties

When assigning varying material properties, for example, you can enter expressions that use:

- Space and time variables
- Results from other physics interfaces in the model
- Results from the current physics interface
- Interpolation from gridded data that is entered into a table or stored in a file
- Interpolation from measured data

There are three categories of properties described next—space-varying, time-varying, and nonlinear. These are the most common for coefficients and material properties that vary during an analysis.

SPACE-VARYING PROPERTIES

- Model properties can vary in space for all study types and solvers (except for applications that do not involve a geometry).
- When defining these properties, use the names of the space variables (independent variables)—normally x, y, and z. The space variables in axisymmetric models are normally called r and z.
- Use space variables in expression variables and coupling operators. For example, you might type the expression `K0+0.1*x` in a field if thermal conductivity varies with x.
- You need to use space variables when using interpolation commands with gridded and measured data sets.

TIME-VARYING PROPERTIES

Model properties can vary in time for time-dependent (transient) analyses. Use the reserved variable name for time, `t`. See The Time Variable.

NONLINEAR PROPERTIES

In a nonlinear model, the model properties depend on the variables for which you solve (the dependent variables). For example, you might enter an expression like `K0+0.1*T^2` in a field if thermal conductivity varies quadratically with temperature T.

Variables for Time, Frequency, and Eigenvalues

The following variables are predefined to represent time (`t`), frequency (`freq`), and eigenvalue (`lambda`).

THE TIME VARIABLE

- For time-dependent problems use the time variable (`t`) with the unit seconds (s).
- It can be part of any expression in the point, edge, boundary, and domain settings, as well as in postprocessing.
- It is always scalar, even when the solution contains more than one output time.
- For stationary models, the value of `t` is 0.
- The value of `t` for results evaluation corresponds to the selection made in the **Time** list in the **Data** sections for the visualization and data evaluation nodes in the **Results** branch in the **Model Builder**. See Results

THE FREQUENCY VARIABLE

- The frequency variable (`freq`) is the global frequency for models in the frequency domain (time-harmonic models and frequency response analysis, for example).

THE EIGENVALUE VARIABLE

- When specifying an eigenvalue problem, use the eigenvalue variable (`lambda`) like any other variable.
- The eigenvalue solver extracts the Jacobian, the damping matrix, and the mass matrix through Taylor expansion of the problem with respect to the eigenvalue variable around a specified eigenvalue linearization point (which is zero by default).
- Other solvers treat the eigenvalue variable as a constant with value zero, unless it is set by an eigenvalue solution used as initial solution.
- After solving an eigenvalue problem, the eigenvalue name is available as a scalar variable for use in postprocessing expressions.
- To choose between different eigenvalues, select one from the **Eigenvalue** list in the **Data** sections for the visualization and data evaluation features in the **Results** section

of the **Model Builder**. The value of the eigenvalue variable corresponds to the selection made in the **Eigenvalue** list. See Results

- For many physics interfaces, the default is to use an eigenfrequency study and compute and display the eigenfrequencies rather than the eigenvalues.

Geometric Variables

The variables that characterize geometric properties are listed in Table 3-6, with detailed descriptions for some of the variables following the table.

TABLE 3-6: GEOMETRIC VARIABLES

VARIABLE	DESCRIPTION
`dom`	The domain number, the boundary number, the edge number, or the vertex (point) number. This variable takes only integer values.
`dvol`	The volume scale factor variable, `dvol`, is the determinant of the Jacobian matrix for the mapping from local (element) coordinates to global coordinates. For 3D domains, this is the factor that the software multiplies volumes by when moving from local coordinates to global coordinates. In 2D and 1D domains, it is an area scaling factor and length scaling factor, respectively. If a moving mesh is used, `dvol` is the mesh element scale factor for the undeformed mesh. The corresponding factor for the deformed mesh is named `dvol_spatial`.
`h`	Available on all geometric entities, represents the mesh element size (in the material/reference frame), that is, the length of the longest edge of the element.
`meshtype`	The mesh type index for the mesh element. This is the number of edges in the element.
`meshelement`	The mesh element number.
`nx`, `ny`, `nz`	See Normal Variables
`qual`	A mesh quality measure.

TABLE 3-6: GEOMETRIC VARIABLES

VARIABLE	DESCRIPTION
`reldetjac` `reldetjacmin`	The determinant of the Jacobian matrix for the mapping from local (element) coordinates to global coordinates, normalized to that of a first-order element. It is the determinant of the Jacobian matrix for the mapping from the straight mesh element to the possibly curved element used when solving. Use this variable to measure the difference in shape between a curved element and the corresponding straight element. The variable `reldetjacmin` is a scalar for each element defined as the minimum value of the `reldetjac` variable for the corresponding element. A `reldetjacmin` value less than zero for an element means that the element is wrapped inside-out; that is, the element is an *inverted mesh element*.
s, s1, s2	See Parameterization Variables.
t*x* and t*y* (2D) t1*x*, t1*y*, t1*z* (3D edges) 2*x*, t2*y*, t2*z* (3D surfaces)	See Tangent Variables.
x, *y*, *z* *r*, *z*	See Space Coordinate Variables.

Note: When entering the **Space Coordinate**, **Parameterization**, **Tangent**, and **Normal** geometric variables, replace the letters highlighted below in *italic font* with the actual names for the dependent variables (solution components) and independent variables (spatial coordinates) in the model.

For example, replace u with the names of the dependent variables in the model, and replace x, y, and z with the first, second, and third spatial coordinate variable, respectively. x_i represents the ith spatial coordinate variable. If the model contains a deformed mesh or the displacements control the spatial frame (in solid mechanics, for example), you can replace the symbols x, y, z with either the spatial coordinates (`x`, `y`, `z` by default) or the material/reference coordinates (`X`, `Y`, `Z` by default).

The variables `dvol`, `h`, `qual`, `reldetjac`, and `reldetjacmin` are based on the mesh viewed in the material/reference frame. If you have a moving mesh, the corresponding

variables for the mesh viewed in the spatial frame have a suffix `_spatial`, that is, `dvol_spatial`, etc. If you use a deformed geometry, the corresponding variables for the original, undeformed mesh have a suffix `_mesh`.

SPACE COORDINATE VARIABLES

- The space coordinate variables (independent variables) x, y, z are available for all domain types.
- For a Cartesian geometry the default names are x, y, z.
- For axisymmetric geometries the space coordinates are normally called r and z.
- You can change the default names of the space coordinates when adding a geometry to a model (see Creating Composite Geometry Objects).
- If a deformed mesh is used, x, y, z can be both the *spatial coordinates* (`x`, `y`, `z` by default) and the *material/reference coordinates* (`X`, `Y`, `Z` by default); see Mathematical Description of the Mesh Movement.
- If the model includes a deformed mesh, the variables x`TIME`, y`TIME`, z`TIME` represent the mesh velocity. To access these variables, replace x, y, and z with the names of the spatial coordinates in the model (`x`, `y`, and `z` by default).

PARAMETERIZATION VARIABLES

The surface-boundary parameterization variables are useful for defining distributed loads and constraints such as a parabolic velocity profile. The available parameterization variables are:

- The curve parameter `s` (or `s1`) in 2D. Use a line plot to visualize the range of the parameter, to see if the relationship between x, y (the space coordinates) and `s` is nonlinear, and to see if the curve parameterization is aligned with the direction of the corresponding boundary. In most cases it runs from 0 to 1 in the direction indicated by the arrows shown on the boundaries in boundary selection mode. You can use `s` on boundaries when specifying boundary conditions.
- The arc length parameter `s1` available on edges in 3D. It is approximately equivalent to the arc length of the edge. Use a line plot to visualize to see the values of `s1`.
- The surface parameters `s1` and `s2` in 3D are available on boundaries. In many cases they can be difficult to use because the relationship between x, y, and z (the space coordinates) and `s1` and `s2` is nonlinear. Often it is more convenient to use expressions with x, y, and z for specifying distributed boundary conditions. To see the values of `s1` and `s2`, plot them using a surface plot.

TANGENT AND NORMAL VARIABLES

The tangent and normal variables are components of the tangential and normal unit vectors.

Tangent Variables

- In 2D, tx and ty define the curve tangent vector associated with the direction of the boundary.
- In 3D, the tangent variables t1x, t1y, and t1z are defined on edges. The tangent variables t1x, t1y, t1z, t2x, t2y, and t2z are defined on surfaces according to

$$(t_{ix}, t_{iy}, t_{iz}) = k_i\left(\frac{\partial x(s_1, s_2)}{\partial s_i}, \frac{\partial y(s_1, s_2)}{\partial s_i}, \frac{\partial z(s_1, s_2)}{\partial s_i}\right), \quad i = 1, 2$$

  These most often define two orthogonal vectors on a surface, but the orthogonality can be ruined by scaling geometry objects. The vectors are normalized; k_i is a normalizing parameter in the expression just given.

If you use a deformed mesh, the tangent variables are available only for the undeformed mesh. In that case, replace x, y, and z with the names of the reference coordinates (X, Y, Z by default).

Normal Variables

- In 1D, nx is the outward unit normal pointing out from the domain.
- In 2D, nx and ny define a normal vector pointing outward relative to the domains.
- In 3D, nx, ny, and nz define a normal vector pointing outward relative to the domains.

Direction of the Normal Component on Interior Boundaries

To get control of the direction of the normal component on interior boundaries, the following variables are available:

- In 1D:
  - un$x$, the outward unit normal seen from the upper domain
  - dn$x$, the outward unit normal seen from the lower domain
- In 2D:
  - un$x$ and un$y$ for the up direction
  - dn$x$ and dn$y$ for the down direction

  The upside is defined as the left side with respect to the direction of the boundary.

- In 3D:
  - un*x*, un*y*, and un*z* for the up direction
  - dn*x*, dn*y*, and dn*z* for the down direction

To visualize any of these vector variables use arrow plots on surfaces or lines.

If a deformed mesh is used, the normal variables are available both for the deformed configuration and for the undeformed configuration. In the first case, replace x, y, and z with the spatial coordinate names (x, y, and z by default). In the second case, replace x, y, and z with the material/reference coordinate names (X, Y, and Z by default).

Normal Vector Variables Representing Element Surface Normals

A similar set of variables—n*x*mesh, un*x*mesh, and dn*x*mesh, where x is the name of a space coordinate—use the element shape function and are normal to the actual element surfaces rather than to the geometry surfaces.

Shape Function Variables

Shape function variables and boundary-coupled shape variables are related to the *shape functions* of the finite elements. The shape function variables are related to the degrees of freedom defined by a finite element.

A finite element defines a number of variables, typically a dependent variables and its derivatives. Such variables are called *shape functions variables* because they are computed directly from shape functions and the degrees of freedom.

When you select a physics interface, you enter names for the dependent variables—these names are used to construct the finite elements. The dependent variable name is the basis for additional variable names that the finite elements generate.

Note: When entering the shape function variables, replace the letters highlighted below in *italic font* with the actual names for the dependent variables (solution components) and independent variables (spatial coordinates) in the model.

For example, replace u with the names of the dependent variables in the model, and replace x, y, and z with the first, second, and third spatial coordinate variable, respectively. x_i represents the ith spatial coordinate variable. If the model contains a deformed mesh or the displacements control the spatial frame (in solid mechanics, for

example), you can replace the symbols x, y, z with either the spatial coordinates (x, y, z by default) or the material/reference coordinates (X, Y, Z by default).

AN EXAMPLE OF LAGRANGE ELEMENT VARIABLES

For the Lagrange element, which is the element type used by most physics interfaces, Table 3-7 lists the available variable names, assuming you gave the name u as the argument to the shape function, and that the names x, y, and z are provided for the independent variables.

TABLE 3-7: LAGRANGE ELEMENT VARIABLE NAMES

ENTITY TYPE\ SPACE DIMENSION	1D	2D	3D
POINT		u	u
EDGE			u, *u*Tx, *u*Ty, *u*Tz
BOUNDARY	*u*, *u*Tx, *u*t, *u*Txt	*u*, *u*Tx, *u*Ty, *u*t, *u*Txt, *u*Tyt	*u*, *u*Tx, *u*Ty, *u*Tz, *u*t, *u*Txt, *u*Tyt, *u*Tzt
DOMAIN	*u*, *ux*, *uxx*, *u*t, *ux*t, *uxx*t, *u*tt, *ux*tt, *uxx*tt,	*u*, *ux*, *uy*, *uxx*, *uxy*, *uyx*, *uyy*, *u*t, *ux*t, *uy*t, *uxx*t, *uxy*t, *uyx*t, *uyy*t, *u*tt, *ux*tt, *uy*tt, *uxx*tt, *uxy*tt, *uyx*tt, *uyy*tt	*u*, *ux*, *uy*, *uz*, *uxx*, *uxy*, *uxz*, *uyx*, *uyy*, *uyz*, *uzx*, *uzy*, *uzz*, *u*t, *ux*t, *uy*t, *uz*t, *uxx*t, *uxy*t, *uxz*t, *uyx*t, *uyy*t, *uyz*t, *uzx*t, *uzy*t, *uzz*t, *u*tt, *ux*tt, *uy*tt, *uz*tt, *uxx*tt, *uxy*tt, *uxz*tt, *uyx*tt, *uyy*tt, *uyz*tt, *uzx*tt, *uzy*tt, *uzz*tt,

For example, with a fluid-flow interface, you get the set of variables indicated in Table 3-7 for u, v, w, and p, respectively.

- The variables *ux*, *uy*, and *uz* are the components of the gradient ∇u, that is, the 1st-order spatial derivatives.
- The variables *uxx*, *uxy*, *uxz*, *uyx*, *uyy*, *uyz*, *uzx*, *uzy*, and *uzz* are the 2nd-order space derivative components. They are meaningful only if the degree of the polynomial shape function used in an element is high enough.

- For elements with 2nd-degree polynomial shape functions (2nd-order elements), the polynomial degree of the 2nd-order derivatives is zero; that is, the second derivatives are constant in each element.
- For element orders lower than two, the second derivatives evaluate to zero regardless of the values of the 2nd-order derivatives of the true solution.

If the model uses a deformed mesh, each finite element is associated with a certain frame (the spatial frame or the material frame). The frame determines the names of the variables generated by the finite element. For instance, if the spatial frame is used, the Lagrange element computes derivatives with respect to the spatial coordinates, `ux`, `uy`, `uz`. If the material frame is used, the Lagrange element computes derivatives with respect to the material coordinates `uX`, `uY`, `uZ`.

THE TIME DERIVATIVES OF THE DEPENDENT VARIABLES

The variable u`t` is the time derivative of the dependent variable u. You can also form mixed space-time derivatives as $u x_i$`t`, for example, `uxt`,

$$\frac{\partial^2 u}{\partial x \partial t}.$$

Note: The `t` must be last in a mixed derivative. You can also use second time derivatives such as `utt` or `uxtt` (but not higher derivatives in time).

If the model contains a deformed mesh, there is, in addition to the usual time derivative `ut`, the *mesh time derivative* `uTIME`. This also holds for mixed space-time derivatives.

TANGENTIAL DERIVATIVE VARIABLES

On boundaries, edges, and points you also have access to *tangential derivative variables*. They have names such as u`Tx`, u`Ty`, and u`Tz`. Using these variables, it is possible to create models with phenomena on boundaries, edges, or points as described with PDEs.

The tangential derivative variables represent the Cartesian components of the tangential projection of the gradient of shape function variables:

$$(\nabla u)_T = (I - \mathbf{n}\mathbf{n}^T) \cdot \nabla u \, .$$

In this equation, $(\nabla u)_T$ is the tangential gradient, which consists of the tangential derivatives in each spatial direction, $\boldsymbol{I}$ is the unity tensor, $\mathbf{n}$ is the outward unit normal vector, and ∇u is the gradient of u.

VARIABLE INHERITANCE

On boundaries, edges, and points, gradients and second derivatives of the shape functions are available by *inheritance*; that is, the average of the values of the variables from the adjacent domains are computed. This process can progress for several levels.

For example, `ux` is the average on a boundary from the adjacent domains, then the average on an edge from the adjacent boundaries, and finally, the average at the points from the adjacent edges.

If possible, avoid using variable inheritance for gradients and second derivatives in a model. Instead, use the tangential derivative variables for equation-based modeling on boundaries.

For computations of integrals of reaction forces and fluxes, use the `reacf` operator (see The Reacf Operator).

For high accuracy reaction forces and fluxes in other circumstances, use weak constraints on boundaries instead of directly accessing the gradient through inheritance (see Computing Accurate Fluxes).

BOUNDARY COUPLED SHAPE VARIABLES

Use the `up` and `down` operators (see The Up, Down, and Mean Operators) to access values on either side of the boundary between domains.

Let u be the name of the dependent variable and x be the name of a space coordinate. Then the variables in Table 3-8 are available.

TABLE 3-8: BOUNDARY COUPLED SHAPE VARIABLES

VARIABLE	MEANING
uux	The value of ux taken from the adjacent domain in the up direction. If ux is not defined on that domain, the variable uux is undefined.
dux	The value of ux taken from the adjacent domain in the down direction. If ux is not defined on that domain, the variable dux is undefined.

Entering Ranges and Vector-Valued Expressions

You can enter ranges and vector-valued expressions such as extra grid-line coordinates using the following formats:

- A space-separated or comma-separated list of values: `10, 15, 23, 29.7, 30`.
- A delimited space-separated list using curly braces; for example, as an argument to a function: `cos({0 pi/4 pi/2})`.
- Equally-spaced values using the `range` function as in `range(start value,step_size,end value)`. For example, `range(0,0.2,3)` creates the values 0, 0.2, 0.4,..., 2.6, 2.8, 3.0. The step size is 1 if you provide only start and end values and skip the step value. Note that *`start_value`* can be either smaller or larger than *`end_value`*. In the latter case, the step size must be negative.

Combine these formats in a single expression to create an array of values that contain an arbitrary number of segments with differently spaced values mixed with other freely specified values.

EXAMPLES USING THE RANGE FUNCTION

- `range(a,(b-a)/(n-1),b)` gives a list of *n* equally-spaced increasing values in the range [`a`, `b`] if `b >a` or decreasing values in the range [`b`, `a`] if `a >b`.
- `10^range(-3,3)` gives the exponentially increasing sequence 10^{-3}, 10^{-2}, ..., 10^{3}.
- `1^range(1,10)` gives a sequence of length 10 where all elements equal 1. Multiplying the vector `1^range(1,n)` by a constant value `a` gives a vector of *n* elements all equal to `a`.
- `0^range(1,5)` gives the sequence `0 0 0 0 0`.

USING THE RANGE DIALOG BOX TO GENERATE ARRAYS

A convenient way to generate vectors of values is to use the **Range** dialog box, which you open by clicking the **Range** button ( ) next to most of the edit fields that accept vectors of values. In that dialog box, use the **Entry method** list to select **Step** to enter a step size or **Number of values** to specify the number of values in the array. You specify the starting and ending values for an array of values in the **Start** and **Stop** edit fields, respectively. In addition you enter the step size in the **Step** edit field or the number of values in the **Number of values** edit field, depending on the setting in the **Entry method** list. By default, the spacing of the values is linear, but you can select a function to apply to all values. To do so, choose one of the available arithmetic and trigonometric functions from the **Function to apply to all values** list. For example, select **exp10** to

create an array of exponentially increasing values. The list includes the following functions:

- The default value **None**, which means linear spacing using the `range` function directly with the values that you specify.
- The exponential functions **exp10** (base-10 exponential function) and **exp** (base-e exponential function), which create exponentially-spaced values using the specified range of values as powers of 10 and of the mathematical constant e, respectively.
- The trigonometric functions **cos** (cosine) and **sin** (sine), which create sinusoidally varying values.
- The square root function **sqrt**, which creates a vector with values that are the square roots of the values that you specify.

Click **Replace** to replace the contents in the edit field with the values that you have specified in the **Range** dialog box. Click **Add** to add the range of values to the end of the existing values in the associated edit field. That way you can create more complex ranges.

MODELING FEATURES THAT SUPPORT RANGES AND VECTOR-VALUED EXPRESSIONS

The following modeling features support ranges and vector-valued expressions:

- Extra grid lines in the **Axis** node's **Settings** window.
- Interval coordinates when using the **Interval** node's **Settings** window for 1D geometries.
- The times for output from the time-dependent solver and the list of parameter values in the **Settings** windows for study step nodes for time-dependent and stationary solvers and for parametric sweeps.
- The contour levels, the streamline start-point coordinates, and the coordinates in arrow plots. Whenever you specify a number of coordinates in **Settings** windows for plots, COMSOL Multiphysics uses scalar expansion—if one component is the same for all coordinates, you can enter a single number in the corresponding text field. For example, to get 101 linearly spaced coordinates from $y = 6$ to $y = 7$ along $x = 3$, enter it as the single scalar `3` for x and then `range(6,0.01,7)` for y. Thus, you need not enter 101 similar values for x.
- Element distribution in the meshing settings.

Adding Global Variables to the Model Builder

1 Open or create a model file.

2 In the **Model Builder**, right-click **Global Definitions** and select **Variables**.

The **Variables** page opens in the **Settings** window and a **Variables** node is added to the model under **Global Definitions**.

Adding Local Variables to Individual Models

1 Open or create a model file.

2 In the **Model Builder**, right-click **Definitions** and select **Variables**.

The **Variables** page opens in the **Settings** window and a **Variables** node is added to the model under **Model>Definitions**.

Assigning Geometric Scope to a Variable

Note: The **Global Definitions Variables** page does not have a **Geometric Scope** section.

1 Create a local variable for a model.

2 In the **Geometric Scope** section, select the scope in the **Geometric entity level** list. The options are **Entire Geometry**, **Domain**, **Boundary**, **Edge** (3D only), and **Point**.

If you select a level other than **Entire Geometry**, a **Selection** list for geometric entities of the selected type (boundaries, for example) displays. Also see Selecting and Deselecting Geometric Entities for detailed information.

Adding Variable Definitions

1 Create a global or local **Variable**.

2 On the **Variables** page, in the **Variables** table, click the first cell in the **Name** column. Enter a variable name.

3 In the **Expression** column, enter an expression that defines the variable. Expressions can contain space coordinates and solution components that are also available within the same scope.

The **Unit** column displays the unit of the variable.

4 In the **Description** column, enter an optional description.

5 Add more variables as required to the rows in the table.

Click any cell to move or delete the corresponding row. Click the **Move Up** ( ), **Move Down** ( ), or **Delete** ( ) buttons or right-click the cell and select **Move Up**, **Move Down**, or **Delete**.

Editing Variable Definitions

- Click on any cell and enter changes directly in the table; or click any cell in the parameter row to edit. The **Name**, **Expression**, and **Description** fields under the table autofill with the same values. Edit the data in these fields.

Saving Variable Definitions to a Text File

1 Enter variables into the table.

2 Click the **Save to File** button ( ).

3 Enter a **File Name**, including the extension .txt. Click **Save** ( ).

The information is saved in tab-separated columns in the same order as displayed on screen.

Loading Variable Definitions from a Text File

1 In the **Model Builder**, right-click **Global Definitions** or **Definitions** and select **Variables**.

2 Load data from a spreadsheet program or other format by saving the file with a .txt extension. Data must be separated by tabs or colons.

3 In the **Settings** window, click the **Load from File** button ( ).

4 Navigate to the text (.txt) file to load and click **Open**.

The data loads to the table. If there is other data already in the table, it adds it after the last row. Move or edit as required.

Summary of Common Predefined Variables

The following table summarizes the most important predefined variables that are generally available in all COSMOL Multiphysics models. Some are only available in certain geometries or in time-dependent models, for example.

TABLE 3-9: COMMON PREDEFINED VARIABLES

TYPE OF VARIABLES	VARIABLES
Time	`t`
Frequency	`freq`
Eigenvalues	`lambda`
Position	`x`, `y`, `z`, `r`, `X`, `Y`, `Z`, `R`
Edge/surface parameters	`s`, `s1`, `s2`
Edge/surface normals	`n`, `nx`, `ny`, `nz`, `nr`
Edge tangents	`tx`, `ty`, `tz`, `tr`
Surface tangents	`t1x`, `t1y`, `t1z`, `t2x`, `t2y`, `t2z`
Edge/surface upward normals	`un`, `unx`, `uny`, `unz`
Edge/surface downward normals	`dn`, `dnx`, `dny`, `dnz`
Numerical constants	`eps`, `i`, `j`, `pi`
Mesh information	`h`, `dom`, `meshtype`, `meshelement`, `dvol`, `qual`, `reldetjac`, `reldetjacmin`

Global and Local Functions

There are three broad categories of functions—**Analytic**, **Interpolation**, and **Piecewise**—and a number of other functions as listed in Table 3-10. You can also create external function interfaces to include functions written in C and MATLAB. Functions can be global or local in scope, although two functions can only be defined globally.

TABLE 3-10: GLOBAL AND LOCAL FUNCTION DESCRIPTIONS

FUNCTION	DESCRIPTION AND USE	GO TO
Analytic	An analytic function is defined by a symbolic expression. It extends variables with the ability to bind parameters during function calls. In other words, do not need to know the actual variable names in an expression when you write the function.	page 104
External (Global only)	An external function interfaces to other external functions written in COMSOL. Then use those functions as any other functions defined in COMSOL Multiphysics. For example, for a user-created shared library (depending on the platform, a DLL, .so, or .dylib file).	page 106
Gaussian Pulse	The Gaussian pulse function is the common bell-shaped curve. It has a shape that is similar to a Gaussian (normal) distribution.	page 107
Interpolation	An interpolation function is defined by a table or file containing the values of the function in discrete points. The interpolation data can be both structured, (defined on a grid) or unstructured (defined on a generic point cloud).	page 107
MATLAB (Global only)	Use a MATLAB function feature to interface to functions written in MATLAB. These functions can be used as any other function defined in COMSOL Multiphysics. MATLAB functions are evaluated in MATLAB.	page 111

TABLE 3-10: GLOBAL AND LOCAL FUNCTION DESCRIPTIONS

FUNCTION	DESCRIPTION AND USE	GO TO
Piecewise	A piecewise function is created by splicing together several functions, each defined on one interval. Define the argument, extrapolation and smoothing methods, and the functions and their intervals. This function has one variable with different definitions on different intervals, which must not overlap or have any holes between them.	page 111
Ramp	A ramp function is a linear increase with a user defined slope that begins at some specified time. This function has one variable (the time t, for example).	page 114
Random	A random function generates white noise with uniform or normal distribution and has one or more arguments to simulate white noise. Its distribution is either uniform or normal. To generate a random function of the space coordinates x, y, and z, for example, use this function with three input variables—it returns the same value each time you call it for a given set of input arguments. To do a Monte Carlo simulation, add one or more additional input arguments that vary using during a parametric sweep, for example.	page 114
Rectangle	A rectangle function is 1 in an interval and 0 everywhere else. This function (also called top hat or boxcar) is useful for band-pass filtering; use it to select values within an interval. It can also simulate a signal that is turned on during an interval or a load that is active on a part of a boundary, for example. This function has one variable (the time t, for example).	page 115
Step	A step function is a sharp transition from 0 to some other value (amplitude) at some location (a certain time, for example). Create a single step function with a certain amplitude from an initial level to a final level at a start location. This function has one variable (the time t, for example).	page 115

TABLE 3-10: GLOBAL AND LOCAL FUNCTION DESCRIPTIONS

FUNCTION	DESCRIPTION AND USE	GO TO
Triangle	A triangle function is a linear increase and linear decline within an interval and 0 everywhere else. Use the triangle function for band-pass filtering; that is, use it to select values within an interval. This function has one variable (the time *t*, for example).	page 116
Waveform	A waveform function is a periodic function with one of several characteristic shapes: sawtooth, sine, square, or triangle. This function has one variable (the time *t*, for example).	page 116

OTHER FUNCTION CHARACTERISTICS

Smoothing

Many functions type have smoothing as an optional choice. Smoothing makes a function more well-behaved and more suitable for use in finite element modeling. It replaces jumps with smooth transitions that eliminates discontinuities and can represents the inertia present in real applications.

Plotting

You can create plots of all functions of 1–3 variables using the **Plot** button () in the upper-right corner of the **Settings** window. For analytic functions you must first define a range for the arguments in the **Plot Parameters** section.

Adding Functions to Materials

You can define **Analytic**, **Interpolation**, and **Piecewise** functions for materials. See Materials for information about materials. Then follow the instructions as described in this section.

Defining an Analytic Function

1 Open or create a model file.

2 In the **Model Builder**, right-click **Global Definitions** or **Definitions** and select **Functions>Analytic** ().

The **Analytic** page opens in the **Settings** window and a node is added in the **Model Builder**.

3 Enter a **Function name**.

4 Under **Parameters**:

a In the **Expression** field, enter the mathematical expression that defines the functions, such as `a+b*cos(c)`.

b Enter **Arguments** to the analytic function as comma-separated entries. For example, `a,b,c`.

c From the **Derivatives** list, select **Automatic** or **Manual** to enable COMSOL to evaluate the derivatives of all functions. COMSOL Multiphysics must know the derivative of a function if you are using a variable that depends on the solution in a function argument.

- Select **Automatic** to compute the derivatives symbolically.
- Select **Manual** to enter the derivatives with respect to one or more of the arguments to the function. For derivatives that you do not define, COMSOL uses 0 as the value of the derivative. In this example, enter `1,cos(c),d(a+b*cos(c),c),0` in the associated edit field.

  Notice the use of the `d` operator (see Special Operators for more information).

5 Under **Periodic Extension**:

a Select the **Make periodic** check box to make the function periodic and extend its definition within an interval to the whole real axis.

b Define the interval in the **Lower limit** and **Upper limit** fields.

6 Under **Advanced**, select the **May produce complex output for real arguments** check box if the defined function works similarly to `sqrt`; that is, it sometimes returns complex values for a real-valued input.

7 Under **Plot parameters**, in the **Plot Parameters** table or the fields under it, enter the **Argument** name, a **Lower limit**, and an **Upper limit**.

Use the **Move Up** ( ), **Move Down** ( ), or **Delete** ( ) buttons to rearrange and delete rows in the table, or right click in any cell and select these options. Click the **Plot** button ( ) to generate a preview plot of the function and set ranges for all the arguments.

Note: This is only available globally on the **Global Definitions** node. Also see the *COMSOL Multiphysics Reference Guide* for details about the interface to the library (or see Where Do I Access the Documentation and Model Library?).

1 Open or create a model file.

2 In the **Model Builder**, right-click **Global Definitions** and select **Functions>External** ( ).

The **External** page opens in the **Settings** window and a node is added in the **Model Builder**.

3 Under **Functions**, in the **Library** field, enter a **Library** (the complete network path) or **Browse** to locate a library to import.

4 For each row in the **Functions** table, enter a **Function** name (`myfun`, for example).

5 For each **Function** in the table, enter a space-separated or comma-separated list of the names of its input **Arguments** (`x y`, for example).

Use the **Move Up** ( ), **Move Down** ( ), or **Delete** ( ) buttons and the fields under the table to edit the table contents.

6 Under **Derivatives**, enter data in this section based on the following.

a In the **Derivatives** table, each row contains a partial derivative of a function with respect to one of its arguments.

b The entries in the **Function** column must occur in the function table, and the entries in the **Argument** column must occur among the arguments listed for that function in the function table.

c The **Partial derivative** column contains expressions for the partial derivatives. Partial derivatives that are not specified in the table default to 0.

Use the **Move Up** ( ), **Move Down** ( ), or **Delete** ( ) buttons and the fields under the table to edit the table contents.

7 Under **Advanced**:

a Enter a value in the **Initialization data** field. The value is sent to the library when it is loaded.

b Select the **Thread safe** check box to declare that the function is a thread-safe pure function (that is, a function that always returns the same results using the same

input argument values and that do not have any side effects or output). Selecting this check box can then improve performance.

8 Click the **Plot** button ( ) to generate a preview plot of the function.

Defining a Gaussian Pulse Function

1 Open or create a model file.

2 In the **Model Builder**, right-click **Global Definitions** or **Definitions** and select **Functions>Gaussian Pulse** ( ).

The **Gaussian Pulse** page opens in the **Settings** window and a node is added in the **Model Builder**.

3 Enter a **Function name**.

4 Under **Parameters**:

a Enter a **Location** value for the Gaussian pulse mean s_0.

b Enter a **Standard deviation** σ of the normal distribution.

The Gaussian pulse has the same characteristics as the normal distribution: it is a pulse with a shape that is similar to a normal or Gaussian distribution as a function:

$$y(s) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(s-s_0)^2}{2\sigma^2}}$$

In the equation above, s is the input variable, s_0 is the location (mean), and σ is the standard deviation. This function is a function of one variable (the time t, for example).

5 Click the **Plot** button ( ) to generate a preview plot of the function.

Defining an Interpolation Function

1 Open or create a model file.

2 In the **Model Builder**, right-click **Global Definitions** or **Definitions** and select **Functions>Interpolation** ( ).

The **Interpolation** page opens in the **Settings** window and a node is added in the **Model Builder**.

3 Enter a **Function name**.

4 Under **Parameters,** select **Table** or **File** from the **Data source** list.

- If you select **Table**, enter a **Function name** and enter coordinates t and function values $f(t)$ into the table cells. Use the **Move Up** (⇧), **Move Down** (⇩), or **Delete** (✕) buttons to rearrange and delete rows in the table.
- If you select **File** to import function data:

a Enter a **Filename** (the complete network path) or **Browse** to locate a file to import.

b From the **Data format** list select **Spreadsheet**, **Grid**, or **Sectionwise**.

- A *spreadsheet* file has coordinates and function values; a CSV (comma-separated value) file exported from a spreadsheet program is in this format.

```
%Coordinates
One to three columns containing x, y (optional), and
z (optional)
%Elements
Triangulation where each row contains the row indices of the
points in the Coordinates section that make up one element
(triangular in 2D, tetrahedral in 3D)
%Data (funname)
Column of data values for each point
```

  It is possible to include more than one function in the file as long as a `%Data` header separates them one from the other.

- A *grid* file

```
% Grid
x grid points separated by spaces
y grid points separated by spaces (optional)
z grid points separated by spaces (optional)
% Data
Data values separated by spaces
```

  Each row contains values for different x grid points for fixed values of y and z. The rows first increase the y grid value and then the z grid value. The grid points can also represent another independent variable that the data values depend on. For example, the "grid points" can be temperature values and the data values the thermal conductivity at these temperatures. It is important to use a comment line starting with `%` to separate the grid points or other interpolation points and the data values that are associated with these coordinates or interpolation points. You

can use the `postwriteinterpfile` function to create a file of this format from MATLAB.

It is possible to include more than one function in the file as long as a `%Data` header separates them one from the other.

- A *sectionwise* file contains structured mesh and function data; COMSOL can export such files.

```
%Header (optional)
Columns containing x, y (optional), and z (optional) followed
by data columns.
```

You can include function names in the header. In that case, the input columns must be labeled with `x`, `y`, and `z`, respectively, depending on input dimension. For example, a file with the following content creates two 2D functions named `myfun1` and `myfun2`:

```
% x y myfun1 myfun2
0 0 0.12 0.34
0 1 0.52 1.50
1 0 0.67 0.91
```

If the file does not include any header to indicate the function dimension, the software assumes that it is identical to the largest geometry dimension present in the model. A file with four columns, for example, is interpreted as one 3D function in a 3D model, two 2D functions in a 2D model, and three 1D functions in a 1D model.

c If you selected **Spreadsheet**, enter a **Number of arguments**.

For all file types, enter data into the table. Add a **Function** name and its **Position in file**. The first function in the file has position 1, the following has position 2, and so on. Use the **Move Up** ( ), **Move Down** ( ), or **Delete** ( ) buttons to rearrange and delete rows in the table. Also see the example below.

5 Under **Interpolation and Extrapolation**:

The interpolation setting controls how the program evaluates the function between the discrete points where it is defined by the table or file, and the behavior of the function outside the domain where it is defined by the table or file.

a Select one of the following methods from the **Interpolation** list:

- For functions of one variable select **Nearest neighbor**, **Linear**, **Piecewise cubic**, or **Cubic spline**.

Piecewise-cubic interpolation uses a piecewise-cubic Hermite polynomial with continuous first derivatives. It preserves the shape of the data and respects

monotonicity. The **Cubic-spline** method also performs interpolation with a piecewise cubic polynomial. Here, even second derivatives are continuous; however, the method does not necessarily respect monotonicity.

- For functions of more than one variable, select **Nearest neighbor** or **Linear**. The other options are not supported.

b Select one of the following methods from the **Extrapolation** list to specify how to treat arguments outside the grid or mesh of points.

- **Constant**. Uses the value from the closest point inside the grid (for structured interpolation) or the value from the closest mesh element (for unstructured interpolation). The interpolation function evaluates the polynomial from the closest grid point at the actual point where a value is requested.
- **Linear**. The function is linear outside the grid with a continuous function value and continuous first derivative at the boundary of the grid. **Piecewise cubic** or **Cubic spline** must be selected from the **Interpolation** list in Step 1.
- **Nearest function**. Evaluates the polynomial from the closest grid point at the actual point where a value is requested.
- **Specific value**. Uses a single value, such as zero or NaN (Not a Number), everywhere outside the grid or mesh. Enter the value in the **Values outside range** edit field.

Note: Unstructured interpolation supports using a constant or a specific value only.

---

6 Click the **Plot** button ( ) to generate a preview plot of the function.

EXAMPLE OF IMPORTING A FILE DATA SOURCE INTO A PARAMETER TABLE

You have a file named `temp.txt`, which contains temperature measurements in nine points in the plane:

```
10  3  310
20  3  309
30  3  314
10  6  302
20  6  307
30  6  311
10  9  307
20  9  308
20  9  314
```

The data columns contain x-coordinates, y-coordinates, and temperature values, respectively. To use this file as an interpolation function called `tempfun`, perform the following steps.

1. Select **File** from the **Data source** list.
2. Enter a **Filename** (the complete network path) or **Browse** to locate a file to import.
3. From the **Data format** list select **Spreadsheet**.
4. Enter a **Number of arguments**. In this example, enter `2`.
5. Enter the **Function** name `tempfun`.
6. Enter its **Position in file** as `1`. The first function in the file has position 1, the following has position 2, and so on.

   The position in file for a function is the column after the space coordinates where it is defined. In this example with two arguments (space coordinates), the third column is Position 1 in the file.
7. If desired, adjust the interpolation and extrapolation settings in the **Interpolation and Extrapolation** section.

You can now use the function `tempfun` with `x` and `y` as input arguments in a 2D model to get the interpolated value for the temperature at any position.

About MATLAB Functions

You can use MATLAB functions from COMSOL Multiphysics. This requires the LiveLink for MATLAB product. See the section Defining a MATLAB Function in the *LiveLink for MATLAB User's Guide*.

Defining a Piecewise Function

1. Open or create a model file.
2. In the **Model Builder**, right-click **Global Definitions** or **Definitions** and select **Functions>Piecewise** ( ).

   The **Piecewise** page opens in the **Settings** window and a node is added in the **Model Builder**.
3. Enter a **Function name**.

4 Under **Parameters**:

a Enter a piecewise function name in the **Argument** field.

b Select one of the following methods from the **Extrapolation** list to control what happens when the function is evaluated in a point that does not belong to any interval.

- **Constant**. Uses the function value from the endpoint of the closest interval. Uses the value from the start point of the first and the end point of the last interval on the corresponding sides.
- **None**. Evaluation fails outside of the intervals where it is defined. Trying to evaluate the function generates an error and evaluates to NaN (Not-a-Number).
- **Nearest function**. Evaluate the function from the closest interval. The function at the first or last interval (depending of the side) is used to evaluate at the actual point where a value is requested.
- **Specific value**. Enter a value in the **Values outside range** field. If selecting the specific-number method you can assign a single value (usually zero or NaN) to all points outside the intervals.

c Select one of the following from the **Smoothing** list: **No smoothing**, **Continuous function**, **Continuous first derivative**, or **Continuous second derivative**.

The functions in contiguous intervals may not evaluate to the same value where the intervals meet. Apply smoothing to make the piecewise function more well-behaved numerically. Functions from neighboring intervals are then blended close to where the intervals meet.

- If you selected **Continuous function**, **Continuous first derivative**, or **Continuous second derivative**, enter a value in the **Relative size of transition zone** field. Relative size meaning in relation to the size of the intervals on both sides of the border.

5 Under **Parameters**, for each cell in the **Intervals** table:

a Enter **Start** and **End** interval limits. The intervals must not overlap, and there cannot be any holes between intervals.

Note: The intervals need to be contiguous and in ascending order.

b Enter an expression defining the **Function** based on the **Function type**—**Polynomial**, **Exponential**, or **General**.

- Use a **Polynomial** to define functions using different polynomials on the intervals, where i can be any integer, also negative and zero:

$$f(x) = \sum_i a_{ij} x^i \qquad x_{j\text{start}} < x \le x_{j\text{end}}$$

- Use an **Exponential** to define functions using different exponential functions with a polynomial exponent on the intervals where i can be any non-negative integer:

$$f(x) = e^{\sum_i a_{ij} x^i} \qquad x_{j\text{start}} < x \le x_{j\text{end}}$$

The Polynomial and Exponential polynomial function types are defined using consecutive pairs specifying the polynomial order and the corresponding coefficient. For example, to specify the function

$$f(x) = 1.23 + 1.2 \cdot 10^3 x^3 - 6.58 x^5$$

enter `0 1.23 3 1.2e3 5 -6.58` in the **Function** column.

- Use a **General** function to create functions that use an analytic expression on the different intervals. You also need to specify the name of your argument in the **Argument** field.

  Enter a General expression using the argument you have specified in the **Argument** field. Enter the above function as `1.23+1.2e3*x^3-6.58*x^5` if you specify x as the argument.

  Use the **Move Up** ( ), **Move Down** ( ), or **Delete** ( ) buttons to rearrange and delete rows in the table.

6 Click the **Plot** button ( ) to generate a preview plot of the function.

Defining a Ramp Function

1 In the **Model Builder**, right-click **Global Definitions** or **Definitions** and select **Functions>Ramp** ( ).

The **Ramp** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter a **Function name**.

3 Under **Parameters**:

a Enter a **Location** value s_0 for the start of the ramp.The function evaluates to 0 for values less than its start location and increases linearly for values greater than the location.

b Enter a **Slope** k of the ramp.

c To ensure that the value never exceeds a certain point, select the **Cutoff** check box and enter a value.

For an input variable s, a start location s_0, and a slope k, the ramp function's value is 0 for $s<s_0$ and $k(s-s_0)$ for $s\geq s_0$.

4 Click the **Plot** button ( ) to generate a preview plot of the function.

Defining a Random Function

1 In the **Model Builder**, right-click **Global Definitions** or **Definitions** and select **Functions>Random** ( ).

The **Random** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter a **Function name**.

3 Under **Parameters**:

a Enter a **Number of arguments** to the random function.

b Select **Uniform** or **Normal** from the **Distribution** list.

- If you selected **Uniform**, enter a **Mean** and **Range**. The range is the difference between the largest and smallest values that the function can return.
- If you selected **Normal**, enter a **Mean** and **Standard deviation**.

4 Click the **Plot** button ( ) to generate a preview plot of the function.

Defining a Rectangle Function

1 In the **Model Builder**, right-click **Global Definitions** or **Definitions** and select **Functions>Rectangle** ( ).

The **Rectangle** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter a **Function name**.

3 Under **Limits**, enter a **Lower limit** and **Upper limit** to specify the interval for the rectangle function. For example, if the input argument is time, enter a start and end time. This function evaluates to 1 for values within an interval. Outside the interval it evaluates to 0.

4 Under **Smoothing**:

a To turn on the smoothing function select the **Size of transition zone** check box.

b Enter a value in the **Size of transition zone** field to control the amount of smoothing. Use smoothing to improve the behavior of the model by avoiding discontinuities that are difficult to handle numerically.

5 Click the **Plot** button ( ) to generate a preview plot of the function.

Defining a Step Function

1 In the **Model Builder**, right-click **Global Definitions** or **Definitions** and select **Functions>Step** ( ).

The **Step** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter a **Function name**.

3 Under **Parameters**:

a Enter a **Location** (s_0) of the step. The value of the step function is the initial level for input values that are smaller than the location of the step.

b In the **From** field, enter a start level (L_{start}).

c In the **End** field, enter a final level (L_{end}).

For an input variable s, a start location s_0, and initial level L_{start} and a final level L_{end}, the step function's value is L_{start} for $s < s_0$ and L_{end} for $s \geq s_0$. The amplitude of the step is $L_{\text{end}} - L_{\text{start}}$.

4 Under **Smoothing**:

a To turn on the smoothing function select the **Size of transition zone** check box.

b Enter a value in the **Size of transition zone** field to control the amount of smoothing. Use smoothing to improve the behavior of the model by avoiding discontinuities that are difficult to handle numerically.

5 Click the **Plot** button ( ) to generate a preview plot of the function.

Defining a Triangle Function

1 In the **Model Builder**, right-click **Global Definitions** or **Definitions** and select **Functions>Triangle** ( ).

The **Triangle** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter a **Function name**.

3 Under **Limits**, enter a **Lower limit** and **Upper limit** to specify the interval for the triangle function. For example, if the input argument is time, enter a start and end time. In the midpoint of the interval, this function evaluates to 1, and moving toward the interval boundaries it falls off to 0. Outside the interval it evaluates to 0.

4 Under **Smoothing**:

a To turn on the smoothing function select the **Size of transition zone** check box.

b Enter a value in the **Size of transition zone** field to control the amount of smoothing. Use smoothing to improve the behavior of the model by avoiding discontinuities that are difficult to handle numerically.

5 Click the **Plot** button ( ) to generate a preview plot of the function.

Defining a Waveform Function

1 In the **Model Builder**, right-click **Global Definitions** or **Definitions** and select **Functions>Waveform** ( ).

The **Waveform** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter a **Function name**.

3 Under **Parameters**, select a waveform from the **Type** list: **Sawtooth**, **Sine**, **Square**, or **Triangle**.

   a If you select **Sine**, enter a **Frequency**, **Phase**, and **Amplitude** value. Smoothing is not available for this waveform.

   b For **Sawtooth**, **Square**, or **Triangle** waveforms, the **Smoothing** check box is selected by default. Enter a value in the **Transition zone** field.

   If required, turn off smoothing by clearing the **Smoothing** check box.

4 For **Sawtooth**, **Square**, or **Triangle**, enter a **Frequency**, **Phase**, and **Amplitude** value.

5 Click the **Plot** button () to generate a preview plot of the function.

Model Couplings

Model couplings establish couplings between different parts of a model or between different models. A model coupling is defined by a coupling operator, taking an expression as its argument. When the operator is evaluated at a point in the destination, the value is computed by evaluating the argument on the source. All coupling types have a source and a destination.

The destination is where the operator can be evaluated. Its value is computed by evaluating the operator's argument at one or several points in the source. Both the source and the destination are purely geometrical objects, but the source is limited to one geometry whereas the destination is usually global. However, with extrusion and projection couplings, the final evaluation fails if the mapping fails, therefore it is the destination that must be well defined. Some operators have a global destination, which means that they are available everywhere.

About Coupling Operators

Coupling operators are useful for modeling coupled problems and are a generalization of ordinary expressions. They are defined by first selecting the source, where the argument of the operator is evaluated, and a destination. You don't need an evaluation expression to define coupling operators.

Coupling operators can:

- Make the value and the exact Jacobian of an expression available nonlocally (see Nonlocal Couplings and The Sparsity of the Jacobian.
- Take information from a boundary, for example, and make it available on other parts of a model (a domain, for example).
- Be used just for results evaluation and visualization purposes.
- Define nonlocal couplings including mesh transformations, integrals over domains, and projections.

There are three categories of coupling operators:

- *Extrusion*. These operators connect a source and a destination and take an expression as an argument. When the operator is evaluated at a point in the destination, its value is computed by evaluating the argument at a corresponding point in the source. See About Extrusion Model Coupling and Operators, Defining

a Boundary Similarity Model Coupling and Defining an Identity Mapping Model Coupling.

- *Projection*. These coupling operators evaluate a series of line integrals on the source domain, where the line positions depend on the positions of the evaluation points in the destination domain. See About Projection Model Couplings and Operators.
- *Scalar*. These operators define a scalar value such as an integration, the average over a set of geometric entities, and the maximum or minimum value of an expression. See About Scalar Model Couplings and Operators, Defining an Average Model Coupling, and Defining Maximum and Minimum Model Couplings.

NONLOCAL COUPLINGS AND THE SPARSITY OF THE JACOBIAN

The Jacobian for problems formulated using the finite element method is usually large but sparse. This is because the solution at each mesh node can depend at most on the degrees of freedom at the neighboring mesh elements. However, by introducing nonlocal model couplings using coupling operators, you create nonlocal dependencies that fill up the rows and columns of the affected source and destination nodes. These additional elements might make the Jacobian matrix only slightly less sparse, with marginal effects on solution speed; it can also make it a great deal less sparse, in which case memory use and CPU time involved in solving the problem increases considerably. For this reason, take particular care when introducing nonlocal couplings.

TABLE 3-11: COUPLING OPERATOR DESCRIPTIONS

COUPLING OPERATOR	DESCRIPTION	GO TO
Average	Computes the average of an expression over selected geometric entities. It can be evaluated anywhere in any model, and the value does not depend on the evaluation point. It is similar to the Integration operator; the difference being that the integral is divided by the volume, area, arc length, or number of points in the source, depending on the dimension.	page 140
Boundary Similarity	Maps an expression defined on a part of a boundary to another part of a boundary with the same shape. Available in 2D and 3D, it is defined by a similarity mapping from the destination to the source.	page 127

TABLE 3-11: COUPLING OPERATOR DESCRIPTIONS

COUPLING OPERATOR	DESCRIPTION	GO TO
General and Linear Extrusion	Maps an expression defined on a source domain to an expression that can be evaluated on the destination domain. Use a Linear Extrusion when the correspondence between evaluation points in the source and destination is linear (and in some to non-linear cases). Use a General Extrusion coupling define a nonlinear mapping.	page 120
General and Linear Projection	Projection operators evaluate a series of line or curve integrals on the source domain, where the line or curve positions depend on the positions of the evaluation points in the destination domain. The line or curve depends on the point where the operator is evaluated.	page 131
Identity Mapping	Maps between geometric entities which overlap, possibly when viewed in different frames. When it is evaluated at a specific set of coordinates in the destination frame, its argument is evaluated with the same coordinates in the source frame.	page 130
Integration	Integrates an expression over selected geometric entities. This is evaluated by integrating the argument over the source domain or summing the argument over the node points in the source domain. The operator can be evaluated anywhere in any model, and the value does not depend on the evaluation point.	page 137
Maximum and Minimum	These operators compute the maximum or minimum of an expression over selected geometric entities and give the maximum and minimum values of the arguments over the source domain. The operator can be evaluated anywhere in any model, and the value does not depend on the evaluation point.	page 141

About Extrusion Model Coupling and Operators

An extrusion coupling operator maps values from the source domain to the destination domain. When the domains are of the same space dimension, you typically have a point-wise mapping. When the destination domain has higher dimension than the source domain, the mapping is done by extruding point-wise values to the higher dimensions. Define the map between the source and destination as one of these

operators: general extrusion, linear extrusion, boundary similarity, or identity mapping.

ABOUT THE GENERAL EXTRUSION COUPLING OPERATOR

A **General Extrusion** coupling operator maps an expression defined on a source domain to an expression that can be evaluated on the destination domain. Use this to define a nonlinear mapping of this kind. These operators define a more *general extrusion* between source and destination than the linear extrusion. Specifically, when the destination has more space dimensions than the source, the operator performs extrusion of values.

ABOUT THE LINEAR EXTRUSION COUPLING OPERATOR

A **Linear Extrusion** coupling operator maps an expression defined on a source domain to an expression that can be evaluated on the destination domain. Use this to define a linear mapping of this kind. Linear extrusion can be used when the correspondence between evaluation points in the source and destination is linear, and in some non-linear cases. Otherwise, use a general extrusion coupling.

These operators define a *linear extrusion* that maps between geometric parts of the same dimension. The parts can exist in geometries of different space dimensions. For example, you can couple from edges in 2D to those in 3D; you can also couple 2D domains to 3D faces. In these cases you obviously need geometries of different space dimensions for the source and destination. You define the linear extrusion by specifying points in both the source and destination.

ABOUT THE BOUNDARY SIMILARITY COUPLING OPERATOR

The **Boundary Similarity** coupling operator is slightly different for 2D and 3D.

In 3D, the destination map is a similarity that maps a destination boundary, onto a set of source boundaries. The mesh is always viewed in the mesh frame.

By default, the algorithm automatically chooses a map when symmetries make several maps possible. To control this choice in 3D, add an **Edge Map**, **One Point Map**, or **Two Point Map** subfeature.

- *Edge Map:* Specify that a certain destination edge should be mapped onto a certain source edge. Their relative direction is given by the property direction. The edges must be adjacent to the given boundary.

- *One-Point Map:* Specify that a certain destination vertex should be mapped onto a certain source vertex.
- *Two-Point Map:* Specify that two destination vertices should be mapped onto two source vertices.

In 2D, it works the same except the destination map is a similarity that maps a destination edge onto a set of source edges and there are no subfeatures to add.

ABOUT SOURCE AND DESTINATION MAPPINGS

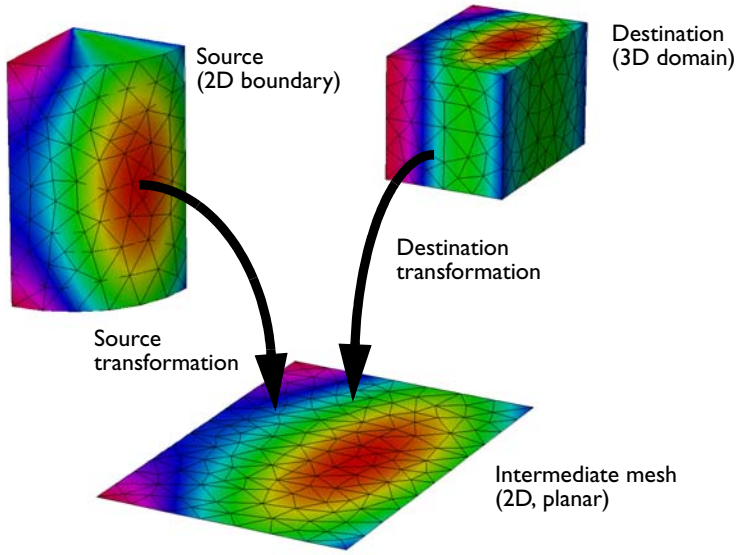


Figure 3-1: General extrusion mappings.

The definition of any extrusion model coupling involves two mesh maps, which are important to understand. The *source map* is a one-to-one mapping that maps the mesh of the physical source to an *intermediate mesh* embedded in a space of the same dimension as the source. The *destination map* is a mapping from the destination, where the operator can be evaluated, to the same space that contains the intermediate mesh.

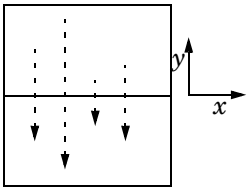
When you request the value of the coupling operator somewhere in the destination domain, the software transforms the destination points using the destination map. It compares the resulting coordinates to the elements in the intermediate mesh to find corresponding locations in the physical source. This means that the source map must

be inverted but not the destination map. The latter can in fact be noninvertible, which is, for example, sometimes the case for a general extrusion.

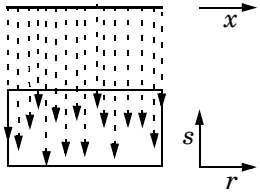
To avoid the need to solve a nonlinear system of equations for every destination point, the software assumes that the source map is linear on each element of the intermediate mesh. In practice, the map is often trivial and leaves the coordinates unchanged, but it can also rescale, stretch, bend, or reflect the mesh.

EXAMPLES OF EXTRUSION MODEL COUPLING

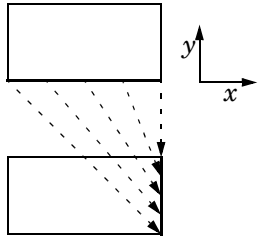
Note: All the graphics in these examples use **General Extrusion** model coupling.



One application of a **General Extrusion** coupling is to mirror the solution on the x-axis. This can be useful for postprocessing. The source map to enter is `x`, `y`, and the destination map is `x`, `-y`. This can also be done with **Linear Extrusion**.



Another **General Extrusion** example is to extrude the solution in the 1D geometry to a 2D domain along the s-axis. The source map is `x`, and the destination map is `r`.

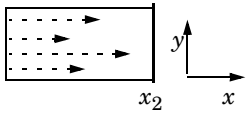


Another example maps values on the lower boundary of a rectangle that extends from $x = -1$ to $x = 1$ and from $y = 0$ to $y = 1$, to the right boundary on the same rectangle. The source map is `(x+1)/2` and the destination map is `y`. This can be done with **General** or **Linear Extrusion**, or with **Boundary Similarity**.

Extrusion Model Coupling—Example 2

Consider the case of a single rectangular domain where the source term in Poisson's equation comes from the inward flux over the right boundary for the corresponding y coordinate.

$$-\Delta u = \frac{\partial}{\partial n} u(x_2, y) \quad \text{on } \Omega$$
$$u = xy \quad \text{on } \partial\Omega$$



The figure to the left illustrates the extrusion process. The values of the influx on the boundary become available throughout the domain by extrusion along the y-axis. The source map is **y**, and the destination map is **y**.

Defining Extrusion Model Coupling

DEFINING A GENERAL EXTRUSION MODEL COUPLING

1 In the **Model Builder**, right-click **Definitions** and select **Model Couplings>General Extrusion**.

The **General Extrusion** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter an **Operator name** or use the default.

3 Under **Source Selection**, from the **Geometric entity level** list, select **Domain**, **Boundary**, **Edge** (3D only), or **Point**. Selection from this list turns on the button with the same name on the **Graphics** window.

4 Select **Manual** or **All** (**Domains**, **Boundaries**, **Edges**, or **Points**) from the **Selection** list.

- If you select **Manual**, select geometric entities in the **Graphics** window.
- Select **All** to add the applicable geometry to the **Selection** box.

Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required on the **Settings** window. Also see Selecting and Deselecting Geometric Entitiesfor detailed information.

5 Under **Destination Map**, specify the general extrusion destination map by entering expression in the **x-expression**, **y-expression**, and **z-expression** fields.

This maps each point in the destination to a point in the intermediate mesh, where the argument of the extrusion operator is evaluated. A general extrusion operator can be evaluated at any point where the destination map expressions are defined.

Note: The number of destination map expressions should be the same as the space dimension of the intermediate mesh. For example, if the intermediate mesh is in two dimensional space, leave the z-expression field empty.

6 Under **Source**:

a Select a **Source frame** to use on the source domain. In most cases the **Source** section default settings can be used.

b Optionally, select the **Use source map** check box and enter expressions in the **x-expression**, **y-expression**, and **z-expression** fields for the source map from the source to the intermediate mesh. The number of source map expressions should be the same as the number of destination map expressions. With the default source map expressions, the intermediate mesh can be considered identical to the source.

The dimensionality idim of the intermediate space is determined by the number of non-empty source and destination map expressions, which must be the same, and must also satisfy srcedim ≤ idim ≤ srcsdim, where srcedim is the dimension of the source selection and srcsdim is the dimension of the source geometry.

7 Under **Advanced**:

a Select an option from the **Mesh search method** list to specify what should happen if an evaluation point in the destination is mapped to a point outside the source.

- If **Closest point** is selected, the closest point in the source selection is used.
- If **Use tolerance** is selected the result depends on the other field definitions in this section.

b Enter a value in the **Extrapolation tolerance** field. If the mapped point is within a distance of the extrapolation tolerance times the mesh element size, the point is considered to be in the source. Otherwise, the mapping fails.

c Select the **Use NaN when mapping fails** check box if you want the operator to evaluate to NaN. Otherwise an error occurs.

DEFINING A LINEAR EXTRUSION MODEL COUPLING

1 In the **Model Builder**, right-click **Definitions** and select **Model Couplings>Linear Extrusion**.

The **Linear Extrusion** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter an **Operator name**.

3 Under **Source Selection**, from the **Geometric entity level** list, select **Domain**, **Boundary**, **Edge** (3D only), or **Point**. Selection from this list turns on the button with the same name on the **Graphics** window.

4 Select **Manual** or **All** (**Domains**, **Boundaries**, **Edges**, or **Points**) from the **Selection** list.

- If you select **Manual**, select geometric entities in the **Graphics** window.
- Select **All** to add the applicable geometry to the **Selection** box.

Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required. Also see Selecting and Deselecting Geometric Entities for detailed information.

5 Under **Source**:

a Select a **Source frame** to use on the source domain. In most cases the **Source** section default settings can be used.

b Optionally, select the **Use source map** check box and enter expressions in the **x-expression**, **y-expression**, and **z-expression** fields for the source map from the source to the intermediate mesh. With the default source map expressions, the intermediate mesh can be considered identical to the source.

6 Under **Source Vertices**, as required, enter values for **Source vertex 1**, **2**, **3**, and **4**.

Note: The selection of **Source Vertices** and **Destination Vertices** together define the linear mapping from the destination to the source.

- The number of source vertices entered should be the same as the number of destination vertices. This number must be at least one, and not more than $1 + \min(\mathrm{srcsdim}, \mathrm{dstsdim})$ where $\mathrm{srcsdim}$ and $\mathrm{dstsdim}$ are the dimensions of the source and destination geometries.
- If not all source vertices selections are used, the empty selections must be last.
- Click the **Activate Selection** button ( ) to define the source vertices. Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

7 Under **Destination**:

- The mapping from destination to source is defined as the following. First, the destination is orthogonally projected onto the linear space spanned by the

destination vertices. Then this linear space is mapped linearly to the source, so that each destination vertex is mapped to the corresponding source vertex.

a Select an option from the **Destination geometry** list if there is more than one geometry in the model.

- A linear extrusion operator can only be evaluated on the destination geometry and the destination vertices must be chosen in the destination geometry.

b Select an option from the **Destination frame** to evaluate the destination vertex coordinates.

8 Under **Destination Vertices**, as required, enter values for **Destination vertex 1**, **2**, **3**, and **4**. The number of destination vertices should be the same as the number of source vertices. If not all destination vertex selections are used, the empty selections must be last.

- Click the **Activate Selection** button () to define the source vertices. Use the **Add to Selection** (), **Remove from Selection** (), and **Clear Selection** () buttons as required.

9 Under **Advanced**:

a Select an option from the **Mesh search method** list to specify what should happen if an evaluation point in the destination is mapped to a point outside the source.

- If **Closest point** is selected, the closest point in the source selection is used.
- If **Use tolerance** is selected the result depends on the other field definitions in this section.

b Enter a value in the **Extrapolation tolerance**. If the mapped point is within a distance of the extrapolation tolerance times the mesh element size, the point is considered to be in the source. Otherwise, the mapping fails.

c Select the **Use NaN when mapping fails** check box if you want the operator to evaluate to NaN. Otherwise an error occurs.

DEFINING A BOUNDARY SIMILARITY MODEL COUPLING

1 In the **Model Builder**, right-click **Definitions** and select **Model Couplings>Boundary Similarity**.

The **Boundary Similarity** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter an **Operator name**.

3 Under **Source Boundaries**, select **Manual** or **All Boundaries** from the **Selection** list to define the source selection of a boundary similarity operator.

- If you select **Manual**, select geometric entities in the **Graphics** window.
- Select **All boundaries** to add all geometry to the **Selection** box.

Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required. Also see Selecting and Deselecting Geometric Entities for detailed information.

4 Under **Destination Boundary**, enter a single boundary as the destination.

Click the **Activate Selection** button ( ) to define the source vertices. Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

5 Under **Advanced**:

a *2D:* Select a relative **Direction** of the source and destination edges—**Automatic**, **Same**, or **Opposite orientation**. This specifies the relative direction of the source and boundary edges and allows you to specify which one of the two possible similarity mappings between the source and destination edge to use.

3D: Similar 3D direction functionality is provided by the subfeatures **One Point Map**, **Two-Point Map**, and **Edge Map** to exactly specify the similarity mapping between the source and destination when more than one possibility exists. See Defining One-Point, Two-Point, or Edge Maps (3D).

b Select the **Use source map** check box to have a nonlinear correspondence between the source and destination. The source map is specified by entering expressions in the **x-expression**, **y-expression**, and **z-expression** fields.

c Select an option from the **Mesh search method** list to specify what should happen if an evaluation point in the destination is mapped to a point outside the source.

- If **Closest point** is selected, the closest point in the source selection is used.
- If **Use tolerance** is selected the result depends on the other field definitions in this section.

d Enter a value in the **Extrapolation tolerance** field. If the mapped point is within a distance of the extrapolation tolerance times the mesh element size, the point is considered to be in the source. Otherwise, the mapping fails.

e Select the **Use NaN when mapping fails** check box if you want the operator to evaluate to NaN. Otherwise an error occurs.

Note: Only one map feature is allowed per boundary similarity coupling and only one source or destination point per field is allowed.

In the **Model Builder**, right-click the **Boundary similarity** node and select **One Point Map**, **Two-Point Map**, or **Edge Map**. Then follow the applicable instructions below.

- *One-Point Maps*

  a Under **Points**, click the **Activate Selection** button ( ) to select a **Point on source** adjacent to the source selection. Click in the **Graphics** window and select the point.

  b Click the **Activate Selection** button ( ) to select a **Point on destination** adjacent to the destination selection. Click in the **Graphics** window and select the point, which is mapped to the selected source point by the similarity mapping from destination to source.

  Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

- *Two-Point Maps*

  a Under **Source Points**, click the **Activate Selection** button ( ) to select a **First point on source**. Click in the **Graphics** window and select the point.

  b Click the **Activate Selection** button ( )to select a **Second point on source** adjacent to the source selection. Click in the **Graphics** window and select the point.

  c Under **Destination Points**, click the **Activate Selection** button ( ) to select **First point on destination**. Click in the **Graphics** window and select the point.

  d Click the **Activate Selection** button ( ) to select a **Second point on destination** adjacent to the destination selection. Click in the **Graphics** window and select the point. The destination points are mapped to the corresponding points on the source by the similarity mapping.

  Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

- *Edge Maps*

  a Under **Edges**, click the **Activate Selection** button ( ) to select a **Source edge**. Click in the **Graphics** window and select the edge.

  b Click the **Activate Selection** button ( ) to select a **Destination edge**. Click in the **Graphics** window and select the edge. The destination edge is mapped to the source edge by the similarity mapping from destination to source.

  Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

  c Under **Advanced**, select a relative **Direction** of the source and destination edges—**Automatic**, **Same**, or **Opposite orientation**. This specifies the relative direction of the source and boundary edges and allows you to specify which one of the two possible similarity mappings between the source and destination edge to use.

DEFINING AN IDENTITY MAPPING MODEL COUPLING

1 In the **Model Builder**, right-click **Definitions** and select **Model Couplings>Identity Mapping**.

  The **Identity Mapping** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter an **Operator name**.

3 Under **Source Selection**, from the **Geometric entity level** list, select **Domain**, **Boundary**, **Edge** (3D only), or **Point**. Selection from this list turns on the button with the same name on the **Graphics** window.

4 Select **Manual** or **All** (**Domains**, **Boundaries**, **Edges**, or **Points**) from the **Selection** list.

  - If you select **Manual**, select geometric entities in the **Graphics** window.
  - Select **All** to add all applicable geometric entities to the **Selection** box.

  Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required. Also see Selecting and Deselecting Geometric Entities for detailed information.

5 Under **Frames**:

  a Select a **Source frame** to use on the source geometric entity. In most cases the default **Spatial frame** can be used.

  b Select a **Destination frame** to use on the destination geometric entity. In most cases default **Spatial frame** can be used.

6 Under **Advanced**:

a Select an option from the **Mesh search method** list to specify what should happen if an evaluation point in the destination is mapped to a point outside the source.

- If **Closest point** is selected, the closest point in the source selection is used.
- If **Use tolerance** is selected the result depends on the other field definitions in this section.

b Enter a value in the **Extrapolation tolerance**. If the mapped point is within a distance of the extrapolation tolerance times the mesh element size, the point is considered to be in the source. Otherwise, the mapping fails.

c Select the **Use NaN when mapping fails** check box if you want the operator to evaluate to NaN. Otherwise an error occurs.

About Projection Model Couplings and Operators

The two types of *projection coupling operators*—**General Projection** and **Linear Projection**—evaluate a series of line integrals on the source domain, where the line positions depend on the positions of the evaluation points in the destination domain. In this way you can calculate the integral of an expression over one space variable for a range of different points along the other space axis, giving a result that varies over the latter space variable. For example, you can obtain the average along the y direction of a variable u defined on some 2-dimensional domain in the xy-plane by calculating the integral

$$\overline{u}(x) = \int u(x, y)\, dy$$

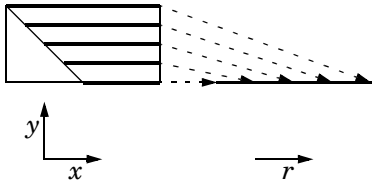
COMSOL Multiphysics uses a method whereby it first applies a one-to-one map to the mesh of the source domain. The last space dimension in the transformed mesh is the one integrated over, so the lines used to integrate are vertical in the transformed source mesh. The software takes the placement of the vertical lines in the transformed source mesh from the positions of the transformed destination evaluation points. It then carries out the integrals in the source domain over curves that correspond to the vertical lines in the transformed source mesh.

You can define the map between source and destination in two ways: as a *linear projection* or as a *general projection*.

EXAMPLES OF PROJECTION COUPLINGS

Note: All the graphics in these examples use **General Projection** model coupling. These can also be done using **Linear Projection**.

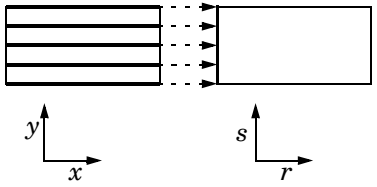
Projection Coupling Operators—Example 1



For each point r, the coupling operator returns the integral

$$v(r) = \int_{\substack{y = r/2 \\ (x,y) \in S_2}} u(x,y)dx$$

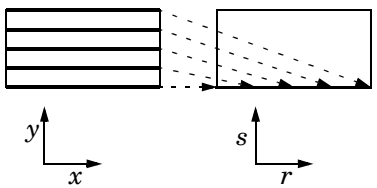
The source map is `y`, `x`, and the destination map is `r/2`.



For each point $(0, s)$, the coupling operator returns the integral

$$v(0,s) = \int_{\substack{y = s \\ (x,y) \in S_2}} u(x,y)dx$$

The source map is `y`, `x`, and the destination map is `s`.



For each point $(r, 0)$, the coupling operator returns the integral

$$v(r,0) = \int_{\substack{y = r/2 \\ (x,y) \in S_2}} u(x,y)dx$$

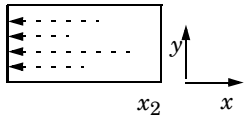
The source map is `y`, `x`, and the destination map is `r/2`.

Note: The integration can also sweep nonrectangular domains. The integrals include only the source domains; they exclude other domains and the external area.

Projection Coupling Operators—Example 2

Consider the case of a single rectangular domain with Poisson's equation. Integrate the solution squared along lines parallel to the x-axis and make the result available for postprocessing on the left boundary.

$$-\Delta u = 1 \quad \text{on } \Omega$$
$$u = 0 \quad \text{on } \partial\Omega$$



The figure illustrates the projection process. Project the integral of the solution squared on the boundary. The source map is `y`, `x` and the destination map is `y`. If the projection operator is called `genproj1`, the desired result is obtained by evaluating `genproj1(u^2)`.

Defining Projection Model Couplings

DEFINING A GENERAL PROJECTION MODEL COUPLING

A projection coupling operator evaluates an expression defined on a source domain by integration along lines or curves depending on the evaluation point in the destination domain. Use a **General Projection** operator to define integration along curves.

Note: It is only possible to use projection model coupling with simplex elements such as triangles and tetrahedra.

About the General Source and Destination Map Vertices

The **General Projection** operator is defined by mapping the source to an abstract intermediate space of dimension *srcedim*, and the destination to the subspace of dimension *srcedim* - 1 obtained by setting the last coordinate to 0. Here *srcedim* is the dimension of the source selection. To every point in the destination, there corresponds a vertical line in the intermediate space, obtained by allowing the last coordinate to vary while the remaining coordinates are given by the destination map. The set of points in the source selection which are mapped onto this line by the source

map, is a line or curve, and the projection operator is evaluated by integrating along this line or curve.

1 In the **Model Builder**, right-click **Definitions** and select **Model Couplings>General Projection**.

The **General Projection** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter an **Operator name**.

3 Under **Source Selection**, from the **Geometric entity level** list, select **Domain**, **Boundary**, **Edge** (3D only), or **Point**. Selection from this list turns on the button with the same name on the **Graphics** window.

4 Select **Manual** or **All** (**Domains**, **Boundaries**, **Edges**, or **Points**) from the **Selection** list.

- If you select **Manual**, select geometric entities in the **Graphics** window.
- Select **All** to add the applicable geometry to the **Selection** box.

Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required on the **Settings** window. Also see Selecting and Deselecting Geometric Entities for detailed information.

5 Under **Source Map**, specify the general projection source map by entering expressions in the **x-expression**, **y-expression**, and **z-expression** fields.

Note: The dimension of the intermediate space equals the dimension srcedim of the source selection. If the selection has lower dimension than the source geometry, specify only the first srcedim expressions.

You can use expressions containing space coordinates in the source geometry when defining the map. Remember that the map must be approximately linear within each mesh element.

6 Select a **Source frame** from the list.

7 Under **Destination Map**, enter an **x-expression** and **y-expression** for each coordinate except the last in the intermediate space.

The destination map has one field less than the source map. When defining the map it is permissible to use expressions containing space coordinates in the destination geometry. The destination mapping can be highly nonlinear or noninvertible.

- If the selection has lower dimension than the source geometry, specify only the first srcedim – 1 expressions. A general projection operator can be evaluated at any point where its destination map is defined.
- If the source selection has dimension 1, no destination map needs to be specified, and consequently this section is not shown if the source geometry is 1D. In this case, it is probably better to use an **Integration coupling**.

8 Under **Advanced**, enter an **Integration order** of the numerical integration method.

DEFINING A LINEAR PROJECTION MODEL COUPLING

Use a Linear Projection when the argument is to be integrated along a line, and the line depends linearly on the evaluation point.

The linear projection maps between source domains and destination domains of the nearest lower dimension. The domains can exist in geometries of different space dimensions. For example, you can couple from subdomains in 2D to edges in 3D; you can also couple 3D subdomains to 2D subdomains. Define the linear projection by specifying points in both the source and destination domains.

About the Linear Source and Destination Map Vertices

The **Source Vertices** and **Destination Vertices** together define a linear mapping from the destination to the source. An evaluation point in the destination geometry is first orthogonally projected onto the linear space spanned by the destination vertices (unless they span the entire space). The projected point is then mapped to the source geometry by a linear mapping taking each destination vertex to the corresponding source vertex. Let L be the line through this point which is parallel to a line through the first and the last source vertex.

If the source selection lies in the linear space spanned by the source vertices, the Linear Projection operator is evaluated by integrating along the line L. In general the operator is evaluated by integrating along the line or curve in the source selection which is mapped to L under orthogonal projection onto the linear space spanned by the source vertices.

1 In the **Model Builder**, right-click **Definitions** and select **Model Couplings>Linear Projection**.

The **Linear Projection** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter an **Operator name**.

3 Under **Source Selection**, from the **Geometric entity level** list, select **Domain**, **Boundary**, **Edge** (3D only), or **Point**. Selection from this list turns on the button with the same name on the **Graphics** window.

4 Select **Manual** or **All** (**Domains**, **Boundaries**, **Edges**, or **Points**) from the **Selection** list.

- If you select **Manual**, select geometric entities in the **Graphics** window.
- Select **All** to add the applicable geometry to the **Selection** box.

Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required on the **Settings** window. Also see Selecting and Deselecting Geometric Entities for detailed information.

5 Under **Source**, select a **Source frame** from the list to evaluate the coordinates of the source vertices in the selected frame.

Now specify the linear projection by giving a set of points in the source geometry and in the destination geometry. The order of the vertices is significant. COMSOL Multiphysics constructs a linear projection from the source domain to the destination domain using the subspaces spanned by the vertices. Denote the map rank by n, denote the source vertices by $x_0, x_1,..., x_n$, and denote the destination vertices by $x'_0, x'_1,\ldots, x'_n$. After padding the source and destination vertices' vectors with zeros as necessary, the software solves the following matrix equation for a *transformation matrix* T and a *translation vector* V:

$$\begin{aligned} x'_0 &= Tx_0 + V \\ x'_1 - x'_0 &= T(x_1 - x_0) \\ &\cdots \\ x'_n - x'_0 &= T(x_n - x_0) \end{aligned}.$$

For the projection model coupling you must have one more vertex in the source geometry than in the destination geometry.

6 Under **Source Vertices**, as required, enter values for **Source vertex 1**, **2**, **3**, and **4**.

Note: The selection of **Source Vertices** and **Destination Vertices** together define the linear mapping from the destination to the source.

- Select srcedim + 1 source vertices where srcedim is the dimension of the source selection. Depending on the dimension of the source selection, it may be that some of the last source vertex selections should be left empty.
- If not all source vertices selections are used, the empty selections must be last.
- Click the **Activate Selection** button ( ) to define the source vertices. Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

7 Under **Destination**, select an option from the **Destination geometry** list if there is more than one geometry in the model.

- A linear projection operator can only be evaluated on the destination geometry and the destination vertices must be chosen in the destination geometry.
- The destination vertex coordinates are evaluated in the selected **Destination frame**.

8 Under **Destination Vertices**, as required, enter values for **Destination vertex 1**, **2**, and **3**. The number of destination vertices should be one less than the number of source vertices. If not all destination vertex selections are used, the empty selections must be last.

- Select srcedim destination vertices where srcedim is the dimension of the source selection. Depending on the dimension of the source selection, it may be that some of the last destination vertex selections should be left empty.
- Click the **Activate Selection** button ( ) to define the source vertices. Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

9 Under **Advanced** you can enter an **Integration order** of the numerical integration method.

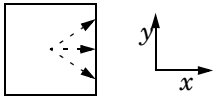
About Scalar Model Couplings and Operators

ABOUT INTEGRATION MODEL COUPLINGS AND OPERATORS

An integration coupling operator is the value of an integral of an expression over a set of geometric entities (domains, for example). Thus, the integration coupling operator is always a scalar quantity. The source of the operator can be an expression at a vertex

(point) or the integral of an expression over one or several domains, boundaries, or edges. Integration coupling operators have global destination, so they can be evaluated anywhere in the model. For example, the `dest` operator can be used to create convolution integrals.

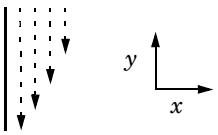
Examples of the Use of Integration Coupling Operators



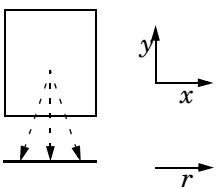
Consider Poisson's equation on a rectangular domain. The integral of the solution squared serves as the influx in a Neumann boundary condition on the right boundary. There is a Dirichlet boundary condition on the left boundary, and the top and bottom boundaries have zero influx.

$$-\Delta u = 1 \quad \text{on } \Omega$$
$$u = x \quad \text{on } \partial\Omega_1$$
$$\frac{\partial u}{\partial n} = 0 \quad \text{on } \partial\Omega_{2,3}$$
$$\frac{\partial u}{\partial n} = -\int_{\Omega} u^2 d\Omega \quad \text{on } \partial\Omega_4$$

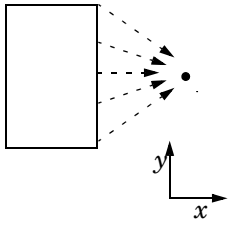
For example, define an integration coupling operator called `intop1`, with the rectangular domain as source. The influx for the Neumann boundary condition is obtained as `intop1(u^2)`.



A second example is when a scalar value from a vertex is used everywhere on a boundary to which the vertex belongs. In structural mechanics you can employ this type of coupling to formulate displacement constraints along a boundary in terms of the displacements of the end point. In electromagnetics the same technique can implement *floating contacts.*



Another example is to use the integral over a subdomain in a 2D geometry along a subdomain in another 1D geometry. This approach is helpful for process-industry models where two processes interact.



Finally, integration coupling operators can implement *integral constraints*. You first define a coupling operator at some vertex in such a way that it represents the value of the integral to be constrained. Then use a point constraint to set the coupling operator, and thereby the integral, to the desired value.

ABOUT AVERAGE MODEL COUPLINGS

Computes the average of an expression over selected geometric entities. It can be evaluated anywhere in any model, and the value does not depend on the evaluation point. It is similar to the Integration operator; the difference being that the integral is divided by the volume, area, arc length, or number of points in the source, depending on the dimension.

ABOUT MAXIMUM AND MINIMUM MODEL COUPLINGS

These operators compute the maximum or minimum of an expression over selected geometric entities and give the maximum and minimum values of the arguments over the source domain. The operator can be evaluated anywhere in any model, and the value does not depend on the evaluation point. You can give two arguments, and the returned value is the value of the second argument evaluated in the max/min of the first argument.

Defining Scalar Model Couplings

DEFINING AN INTEGRATION MODEL COUPLING

1 In the **Model Builder**, right-click **Definitions** and select **Model Couplings>Integration**.

The **Integration** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter an **Operator name**.

3 Under **Source Selection**, from the **Geometric entity level** list, select **Domain**, **Boundary**, **Edge** (3D only), or **Point**. Selection from this list turns on the button with the same name on the **Graphics** window.

4 Select **Manual** or **All** (**Domains**, **Boundaries**, **Edges**, or **Points**) from the **Selection** list.

- If you select **Manual**, select geometric entities on the **Graphics** window. The buttons are already activated.
- Select **All** to add the applicable geometry to the **Selection** box.

Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required on the **Settings** window. Also see Selecting and Deselecting Geometric Entities for detailed information.

5 Under **Advanced**, from the **Method** list select **Integration** or **Summation over nodes**. In most cases use integration. Summation over nodes is useful, for example, for computing reaction forces. If **Integration** is selected, enter a value in the **Integration order** field.

6 Under **Advanced**, when working with multiple frames, select a **Frame** from the list for the volume element to be used in the integration.

DEFINING AN AVERAGE MODEL COUPLING

1 In the **Model Builder**, right-click **Definitions** and select **Model Couplings>Average**.

The **Average** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter an **Operator name**.

3 Under **Source Selection**, from the **Geometric entity level** list, select **Domain**, **Boundary**, **Edge** (3D only), or **Point**. Selection from this list turns on the button with the same name on the **Graphics** window.

4 Select **Manual** or **All** (**Domains**, **Boundaries**, **Edges**, or **Points**) from the **Selection** list.

- If you select **Manual**, select geometric entities on the **Graphics** window. The buttons are already activated.
- Select **All** to add the applicable geometry to the **Selection** box.

Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required on the **Settings** window. Also see Selecting and Deselecting Geometric Entities for detailed information.

5 Under **Advanced**:

a Enter an **Integration order**.

b When working with multiple frames, select a **Frame** from the list for the volume element to be used in the average.

DEFINING MAXIMUM AND MINIMUM MODEL COUPLINGS

1 In the **Model Builder**, right-click **Definitions** and select **Model Couplings>Maximum** or **Minimum**.

The **Maximum** or **Minimum** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter an **Operator name**.

3 Under **Source Selection**, from the **Geometric entity level** list, select **Domain**, **Boundary**, **Edge** (3D only), or **Point**. Selection from this list turns on the button with the same name on the **Graphics** window.

4 Select **Manual** or **All** (**Domains**, **Boundaries**, **Edges**, or **Points**) from the **Selection** list.

- If you select **Manual**, select geometric entities on the **Graphics** window. The buttons are already activated.
- Select **All** to add the applicable geometry to the **Selection** box.

Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required on the **Settings** window. Also see Selecting and Deselecting Geometric Entities for detailed information.

5 Under **Advanced**, the **Point type** controls the choice of evaluation points—the result is more accurate with more points but more points also means a slower evaluation.

a From the **Point type** list, select **Node points**, **Lagrange points**, or **Integration points** to compute the maximum or minimum by evaluating the argument expression at a finite set of points in the source and taking the maximum or minimum of these values.

b If **Integration points** is selected, enter an **Integration order** of 1. The default is 4.

c If **Lagrange points** is selected, enter a **Lagrange order**. The default is 2.

Coordinate Systems

COMSOL Multiphysics uses a global Cartesian coordinate system by default to specify loads and constraints in all physics interfaces and on all geometric entity levels (points, edges, boundaries, and domains). Also see Space Coordinate Variables.

The Cartesian coordinate system space coordinates default to the following names in 2D and 3D (in 2D axisymmetric geometries, COMSOL Multiphysics uses cylindrical coordinates):

GEOMETRY	DEFAULT NAME OF SPACE COORDINATES
2D	$x\ y$
3D	$x\ y\ z$
Axial symmetry 2D	$r\ \varphi\ z$

In 3D, an image displays in the lower-left corner of the **Graphics** window 

to indicate this coordinate system.

User defined coordinate systems can be used on all geometric entity levels to simplify the modeling process. In some of the physics interfaces, you can use these coordinate systems to define orthotropic and anisotropic material properties that are not aligned with the global Cartesian coordinate system, for example. See Table 3-12.

TABLE 3-12: COORDINATE SYSTEM DESCRIPTIONS

COORDINATE SYSTEM	DESCRIPTION	GO TO
Boundary System,	2D and 3D. A local base vector system on 2D boundaries (t, n) and on 3D boundaries (t1, t2, n). Use it to apply loads that apply in a normal or tangential direction on a boundary that is not aligned with the global Cartesian coordinate system. This coordinate system is always available.	page 143
Base Vector System	1D, 2D, and 3D. Define this using a set of base vectors to form a coordinate system, which you can declare as orthonormal.	page 145
Cylindrical System	2D and 3D. Use this where rotational symmetry about the axis is required. Not applicable in geometries with 2D axisymmetry.	page 146

TABLE 3-12: COORDINATE SYSTEM DESCRIPTIONS

COORDINATE SYSTEM	DESCRIPTION	GO TO
Mapped System	1D, 2D and 3D. This can deal with spherical and cylindrical coordinates, and translated and rotated coordinate systems. Use this to create a system that defines a mapping from the frame coordinate system.	page 147
Rotated System	2D and 3D. Use this to define rotation about the out-of-plane direction in 2D and Euler angles in 3D.	page 148

About Boundary Coordinate Systems

Note: For 2D and 3D models, the **Boundary System** node is automatically added under **Definitions**.

The *boundary coordinate system* is available on boundaries in 2D and 3D. Boundary systems are not available in 1D. Use it to apply loads that apply in a normal or tangential direction on a boundary that is not aligned with the global Cartesian coordinate system. Common applications for this coordinate system include specifying pressure or normal displacement on a surface.

To specify the boundary coordinate system, you specify the direction of the normal and a direction that is projected onto the boundary, normalized, and used as the first tangent vector.

The normal direction is in most cases the outward-pointing normal vector. The general definition of the normal (for shells, for example), is the direction of the normal vector $\mathbf{n}$, which you can plot using the variables for its components (typically nx, ny, and nz). See Normal Variables for more information.

2D BOUNDARY COORDINATE SYSTEM

- In 2D the local coordinate system is defined by ($\mathbf{t}_1$, $\mathbf{n}$, $\mathbf{t}_o$), representing the tangential and normal direction of the boundary.

- This coordinate system is always right-oriented. For exterior boundaries the normal is by default directed out from the domain. The user can reverse the normal direction.
- The second tangent direction ($\mathbf{t}_0$) is the cross product between normal vector ($\mathbf{n}$) and the first tangent direction ($\mathbf{t}_1$). This method always gives a right-oriented orthonormal system, unless the tangent direction is parallel to the normal.

3D BOUNDARY COORDINATE SYSTEM

- In 3D the local coordinate system is defined by ($\mathbf{t}_1$, $\mathbf{t}_2$, n), representing two tangential directions and one normal direction. $\mathbf{t}_1$ and $\mathbf{t}_2$ depend on the parametrization of the geometry.
- This coordinate system is always right-oriented but not always orthogonal. For exterior boundaries the normal is by default directed out from the domain. The user can reverse the normal direction.
- Common applications for this coordinate system include specifying pressure or normal displacement on a surface.
- The second tangent direction ($\mathbf{t}_2$) is the cross product between the specified normal vector ($\mathbf{n}$) and the first tangent vector ($\mathbf{t}_1$). This method always gives a right-oriented orthonormal system, unless the tangent direction is parallel to the normal.

Defining a Boundary Coordinate System

Note: For 2D and 3D models, a **Boundary System** node is automatically added under **Definitions**.

1. In the **Model Builder**, right-click **Definitions** and select **Coordinate Systems>Boundary System**.

   The **Boundary System** page opens in the **Settings** window and a node is added in the **Model Builder**.
2. Enter a **Coordinate System Identifier**.
3. Under **Settings**, select a **Frame type**—**Reference configuration**, or **Deformed configuration** (the default).
4. In the **Coordinate names** table, the default names are entered—`t1`, `t2`, and `n` (for 3D models) or `t1`, `n`, and `to` (for 2D models). Click the table cells to edit the names.

Note: t_1 and t_2 depend on how the geometry was created and are usually perpendicular to each other.

5 To reverse the direction of the normal for the boundary system, select the **Reverse normal direction** check box.

6 Select an option from the **Create first tangential direction from** list: **Global Cartesian** or **Manual**.

7 If **Global Cartesian** is selected, select **1**, **2**, or **3** (that is, x, y, or z) from the **Axis** list.

8 If **Manual** is selected, default values are displayed for the local tangent variables `t1x`, `t1y`, and `t1z` (3D) or `t1x` and `t1y` (2D). Enter other values as required to define a tangent direction by specifying directions for a local tangent plane in the **x**, **y**, and **z** fields.

About Base Vector Systems

Define a **Base Vector System** using a set of base vectors to form a coordinate system, which you can declare as orthonormal.

A vector $\mathbf{F}$ in the base of the new base vector system defined by the base vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ becomes $\mathbf{F} = F_1\mathbf{u}_1 + F_2\mathbf{u}_2 + F_3\mathbf{u}_3$, and the transformation matrix between bases is

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix} \cdot \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{Bmatrix} |\mathbf{u}_i| = 1 \\ \mathbf{u}_i \cdot \mathbf{u}_j = \delta_{ij} \end{Bmatrix} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}^T \cdot \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

where the last equality holds when the base vector system is orthonormal.

Defining a Base Vector Coordinate System

1 In the **Model Builder**, right-click **Definitions** and select **Coordinate Systems>Base Vector System**.

The **Base Vector System** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter a **Coordinate System Identifier**.

3 Under **Settings**, in the **Coordinate names** table, the default names are entered—`x1`, `x2`, and `x3`. In planar 2D models, `x1` and `x2` are typically the in-plane coordinates, and `x3` is the out-of-plane coordinate.

4 Under **Base vectors**, define the base vectors in terms of the global Cartesian coordinates `x1`, `x2`, and `x3`; one base vector on each row (two for 2D and three for 3D).

5 *1D and 2D:* Select the **Plane Index**:

- *1D:* Select an option from the **In-plane index** list. The default is **1**.
- *2D*: Select an option from the **Out-of-plane index** list. The defaults are **3** for a plane 2D model and **2** for an axisymmetric 2D model.

For example, to map the first vector, `x1`, to the direction defined by $y = x$ in 2D, enter `1` in the fields under **x** and **y** on the **x1** row.

6 Under **Forced simplifications**, define properties for the base vector systems to simplify the coordinate transformations.

7 Select the **Orthonormal** check box to define the system as orthonormal.

About Cylindrical Coordinate Systems

A **Cylindrical Coordinate System** may be used in 2D and 3D where rotational symmetry about the axis is required. Note that the cylindrical coordinate system is not applicable in geometries with 2D axisymmetry.

The local coordinate system is defined by (r, φ, α) where r represents the radial distance from the longitudinal axis, φ is the azimuthal angle and α is the distance from origin along the longitudinal axis. In 2D one can only specify the origin, whereas in 3D one may specify the longitudinal axis direction, **a**, and the radial base vector, $\boldsymbol{e}_r(\varphi = 0)$, as well. These direction vectors are automatically normalized.

The definition of the cylindrical coordinates in terms of the global cartesian coordinates $\boldsymbol{r} = \boldsymbol{r}(x, y, z)$ are

$$\begin{bmatrix} r \\ \varphi \\ \alpha \end{bmatrix} = \begin{bmatrix} |\boldsymbol{r} - (\boldsymbol{r}_0 + \boldsymbol{a}(\boldsymbol{a} \cdot (\boldsymbol{r} - \boldsymbol{r}_0)))| \\ \operatorname{atan} \dfrac{(\boldsymbol{a} \times \boldsymbol{e}_r) \cdot (\boldsymbol{r} - \boldsymbol{r}_0)}{\boldsymbol{e}_r \cdot (\boldsymbol{r} - \boldsymbol{r}_0)} \\ \boldsymbol{a} \cdot (\boldsymbol{r} - \boldsymbol{r}_0) \end{bmatrix}$$

Defining a Cylindrical Coordinate System

1 In the **Model Builder**, right-click **Definitions** and select **Coordinate Systems>Cylindrical System**.

The **Cylindrical System** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter a **Coordinate System Identifier**.

3 Under **Settings,** select a **Frame type**—**Mesh**, **Material**, or **Spatial**. The default is **Spatial**.

4 In the **Coordinate names** table, the default **Coordinate names** are entered—`r`, `phi`, and `a`. In planar 2D models, `r` and `phi` are the in-plane coordinates, and `a` is the out-of-plane coordinate.

5 Enter the **Origin of system**. The default is an origin coinciding with the one from the global system.

6 For 3D enter the **Axis direction**. The default axis direction is the z direction in the global system.

7 In 3D enter the **Radial base vector direction** for azimuthal angle $\varphi = 0$. The default direction is the x direction in the global system.

About Mapped Coordinate Systems

Use a **Mapped Coordinate System** to create a system that defines a mapping from the frame coordinate system. This method can deal with spherical coordinates, cylindrical coordinates, and translated and rotated coordinate systems.

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} u_1(x, y, z) \\ u_2(x, y, z) \\ u_3(x, y, z) \end{bmatrix}$$

Defining a Mapped Coordinate System

1 In the **Model Builder**, right-click **Definitions** and select **Coordinate Systems>Mapped System**.

The **Mapped System** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter a **Coordinate System Identifier**.

3 Under **Settings,** select a **Frame type**—**Mesh**, **Material**, or **Spatial**. The default is **Spatial**.

4 In the **Coordinate names** table, the default names are entered—`x1`, `x2`, and `x3`. In planar 2D models, `x1` and `x2` are typically the in-plane coordinates, and `x3` is the out-of-plane coordinate.

5 Under **Coordinate mapping**, the **Coordinate** column displays the **Coordinate names** with the **Expression** column displaying the associated mapped coordinate.

6 If required, under **Forced simplifications**, select the **Orthonormal** check box.

About Rotated Coordinate Systems

Use the **Rotated Coordinate System** to define rotation about the out-of-plane direction in 2D and Euler angles in 3D.

On the **Rotated System** page, you define the rotation relative to the global Cartesian coordinate system. In 3D you specify the local coordinate system (x_1, y_1, z_1) using three consecutive Euler angles (rotation angles) α, β, and γ. See Figure 3-2.

In 2D you describe the rotated coordinate system by the rotation angle about the out-of-plane vector. In both cases you can define the origin of the coordinate system.

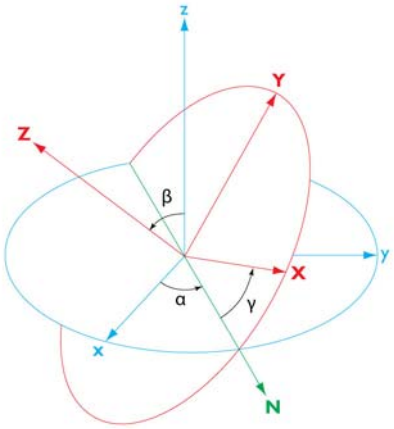


Figure 3-2: 3D Euler angles in a rotated coordinate system.

The transformation matrix for the 3D case is then

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\gamma - \sin\alpha\cos\beta\sin\alpha & -\cos\alpha\sin\gamma - \sin\alpha\cos\beta\cos\gamma & \sin\beta\sin\alpha \\ \sin\alpha\cos\gamma + \cos\alpha\cos\beta\sin\gamma & -\sin\alpha\sin\gamma + \cos\alpha\cos\beta\cos\gamma & -\sin\beta\cos\alpha \\ \sin\beta\sin\gamma & \sin\beta\cos\gamma & \cos\beta \end{bmatrix}^T \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Defining a Rotated Coordinate System

1 In the **Model Builder**, right-click **Definitions** and select **Coordinate Systems>Rotated System**.

The **Rotated System** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter a **Coordinate System Identifier**.

3 Under **Settings**, in the **Coordinate names** table, the default names are entered—`x1`, `x2`, and `x3`. In planar 2D models, `x1` and `x2` are typically the in-plane coordinates, and `x3` is the out-of-plane coordinate.

4 Enter a **Out of Plane Rotation** or **Euler Angles**:

2D Models:

a Enter the **Rotation about out-of-plane axis** (in radians).

b Select the coordinate from the **Out-of-plane index** field. The default index is **3** for planar 2D models and **2** for axisymmetric 2D models.

3D Models: Enter the **Euler angles (Z-X-Z)** (in radians) in the α, β, and γ fields.

5 In the **Origin of System** field, define the origin of the rotated coordinate system. Define it as a vector with two (for 2D) or three (for 3D) components.

The default is the origin for the global Cartesian coordinate system. Using another origin translates the coordinates in the rotated system by that distance from the global Cartesian origin.

Identity and Contact Pairs

Pairs are available for assemblies (that is, geometries created by not forming a union of all geometry objects as the final step), where you need to connect boundaries between parts. There are two types of pairs:

- *Identity pairs*: these pairs make the fields across two connected boundaries (one from each connecting part) continuous. This is equivalent to the continuity that you get by default on interior boundaries in a geometry created by forming a union. Some physics interfaces provide special boundary conditions for identity pairs.

Use an Identity Pair to specify two selections of boundaries that overlap but belong to different parts of an assembly. Then assign a boundary condition to connect the physics in the two parts in a physics interface. Identity pairs connect overlapping boundaries in different connecting parts of an assembly.

- *Contact pairs*: these pairs define boundaries where the parts may come into contact but cannot penetrate each other under deformation for modeling of structural contact and multiphysics contact.

Note: Contact modeling requires the Structural Mechanics Module or MEMS Module.

Use a Contact Pair to specify two selections of boundaries that cannot penetrate each other under deformation. The contact pairs defined boundaries for parts that may come into contact (boundaries that cannot penetrate each other under deformation).

Defining an Identity Pair

1 In the **Model Builder**, right-click **Definitions** and select **Identity Pair** ().

The **Identity Pair** page opens in the **Settings** window and a node is added under **Definitions**.

2 Enter a **Pair name**. It is used as a suffix in names of operators and variables defined by the pair.

The operator mapping an expression *E* on the source side to the destination side is denoted `src2dst_pn(E)`, where *pn* is the pair name. The *variable* `src2dst_pn` (defined on the destination) is `1` where there is a corresponding source point, and `0`

otherwise. The corresponding operator and variable for use on the source side are denoted `dst2src_pn`.

3 Under **Source Boundaries**:

a Click the **Activate Selection** button ( ) to define the source boundaries.

b Select **Manual** or **All Boundaries** for the boundaries on the source side.

If **Manual** is selected, click in the **Graphics** window to add boundaries to the **Selection** section.

Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

4 If required, click the **Swap Source and Destination** button ( ) to swap the source boundaries and the destination boundaries.

5 Under **Destination Boundaries**:

a Click the **Activate Selection** button ( ) to define the source boundaries.

b Select **Manual** or **All Boundaries** for the boundaries on the destination side.

These boundaries should overlap the source boundaries. The condition that connects the physics on the destination and source boundaries is specified in the physics interface. For example, it can be a constraint that constrains a dependent variables (temperature, for example) on the destination side to be equal to a dependent variable on the source side.

If **Manual** is selected, click in the **Graphics** window to add boundaries to the **Selection** section.

Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

6 If there are several frames in the model, the **Frame** section is available. Select the **Source frame** and the **Destination frame**. Source and destination points are connected if their coordinates in their respective frames are equal.

7 If required, click the **Swap Source and Destination** button ( ) to swap the source boundaries and the destination boundaries.

Defining a Contact Pair

1 In the **Model Builder**, right-click **Definitions**. Select **Contact Pair** ( ).

The **Contact Pair** page opens in the **Settings** window and a node is added under **Definitions**.

2 Enter a **Pair name**. It is used as a suffix in names of operators and variables defined by the pair.

The operator mapping an expression *E* on the source side to the destination side is denoted `mst2slv_`*pn*`(E)`, where *pn* is the pair name. Similarly, there is an operator `mst2slv_`*pn*`_mph` that is suited for use in multiphysics coupling.

The variable `geomgap_slv_`*pn* is the geometric gap between the source and the destination, seen from the destination side (following the normal of the destination boundary).

The corresponding operators and variables for use on the master side are denoted `slv2mst_`*pn*, `slv2mst_`*pn*`_mph`, `geomgap_mst_`*pn*.

3 Under **Source Boundaries**:

a Click the **Activate Selection** button ( ) to define the source boundaries.

b Select **Manual** or **All Boundaries** for the boundaries on the source side.

If **Manual** is selected, click in the **Graphics** window to add boundaries to the **Selection** section.

Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

4 If required, click the **Swap Source and Destination** button ( ) to swap the source boundaries and the destination boundaries.

5 Under **Destination Boundaries**:

a Click the **Activate Selection** button ( ) to define the source boundaries.

b Select **Manual** or **All Boundaries** for the boundaries on the destination side.

If you selected **Manual**, click in the **Graphics** window to add boundaries to the **Selection** section.

The contact algorithm constrains the destination boundaries so that they do not penetrate the source boundaries.

6 If required, click the **Swap Source and Destination** button ( ) to swap the source boundaries and the destination boundaries.

7 Under **Advanced**:

a The **Search method** defaults to **Fast**—the algorithm only keeps track of source and destination points that have a distance less than a certain *search distance*. Select **Direct** for a slower, but more robust, search.

b Select the **Manual control of search distance** check box to tune the search distance (SI unit: m). By default, the search distance is taken as 0.01 times the diagonal of the geometry's bounding box.

c If the **Manual control of search distance** check box is selected, enter a different value in the **Distance** field.

Probes

Probes monitor the development of a scalar-valued quantity (real or complex valued number) from a dynamic simulation (time-dependent, frequency, parametric), by two different techniques; tabulated data and 1D graph plots. It is meaningful to probe while solving, and to probe after the computation is finished.

Plot while solving is a technique used to halt the simulation and launch some predefined plot commands and then continue with the simulation.

Defining a Domain, Boundary, or Edge Probe

1 In the **Model Builder**, right-click **Definitions** and select **Probes>Domain Probe**, **Boundary Probe**, or **Edge Probe (3D)**.

The **Domain**, **Boundary**, or **Edge Probe** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Under **Probe Settings**, select an option from the **Type** list—**Average**, **Maximum**, **Minimum**, or **Integral**.

3 Enter a **Probe name**.

4 Under **Source Selection**, from the **Selection** list, select **Manual** or **All** (**Domains**, **Boundaries**, or **Edges**).

- If **Manual** is selected, select the domain, boundary or edge in the **Graphics** window.
- Select **All** to add the domain, boundary or edge to the **Selection** box.

Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required in the **Settings** window.

For general information about selections, see Using the Selection List Window.

5 Under **Expression**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c Select the **Description** check box to enter a description of the data set (or edit the default).

6 Under **Integration Settings**:

   a From the **Method** list, select **Integration** or **Summation**.

   b Enter a value in the **Integration order** field.

   c When working with multiple frames, select a **Frame** from the list for the volume element to be used in the integration.

Defining a Domain Point Probe

1 In the **Model Builder**, right-click **Definitions** and select **Probes>Domain Point Probe**.

   The **Domain Point Probe** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Under **Point Selection** select an option from the **Line entry method** list—**Point and surface normal**, **Point and direction**, **Two Points**, or **None**. If **None** is selected, go to step c.

   a If **Two Points** is selected, from the **Point being modified** list, select **First point** or **Second point**.

   b Enter a **Depth along line** or use the slider to select a value.

   c Enter **Coordinates**. For 2D enter **x** and **y** coordinates; for **3D** enter **x**, **y**, and **z** coordinates.

   d Select the **Snap to closest boundary** check box to snap the selected points to the grid.

3 If required, define a **Point Probe Expression**.

Note: A **Point Probe Expression** node is automatically added under **Domain Point Probe**. To add more nodes, right-click **Domain Point Probe** and select **Point Probe Expression**.

---

4 In the **Model Builder**, under **Domain Point Probe**, click **Point Probe Expression**.

   The **Point Probe Expression** page opens in the **Settings** window.

5 Enter a **Probe name**.

6 Under **Expression**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c Select the **Description** check box to enter a description of the data set (or edit the default).

Defining a Boundary Point Probe

1 In the **Model Builder**, right-click **Definitions** and select **Probes>Boundary Point Probe**.

The **Boundary Point Probe** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 In the **Graphics** window, click the boundary to add to the **Boundary Selection** section.

3 Under **Point Selection** enter the **Coordinates**. For 2D enter **x** and **y** coordinates; for **3D** enter **x**, **y**, and **z** coordinates.

4 Define a **Point Probe Expression**.

Note: A **Point Probe Expression** node is automatically added under **Boundary Point Probe**. To add more nodes, right-click **Boundary Point Probe** and select **Point Probe Expression**.

---

5 In the **Model Builder**, under **Boundary Point Probe**, click **Point Probe Expression**.

The **Point Probe Expression** page opens in the **Settings** window.

6 Enter a **Probe name**.

7 Under **Expression**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c Select the **Description** check box to enter a description of the data set (or edit the default).

Defining a Global Variable Probe

1 In the **Model Builder**, right-click **Definitions** and select **Probes>Global Variable Probe**.

The **Global Variable Probe** page opens in the **Settings** window and a node is added in the **Model Builder**.

2 Enter a **Probe name**.

3 Under **Expression**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c Select the **Description** check box to enter a description of the data set (or edit the default).

4

Visualization and Selection Tools

COMSOL Multiphysics provides a number of tools to visualize and control how you view models in the **Graphics** window.

In this section:

Working with 3D Geometry

COMSOL Multiphysics has many ways to zoom, rotate, and move around the **Graphics** window in order to adjust the view of a 3D model during its creation and analysis. There are also several ways to select each part of the geometry, the geometric entities, and create reusable **Selections** and **Views**.

While building 3D models, it is important to clearly understand what the object looks like and to easily select specific features within the geometry. For example:

- Highlight, select, or hide any part of the geometry using buttons, mouse clicks, keyboard shortcuts, or combinations of actions;
- Create user defined **Selections** to reuse throughout the model;
- View an object where the edges of a part are represented by solid lines *(wireframe rendering);*
- Capture a screen-shot of the **Graphics** window; and
- Control visualization quality with lighting and transparency.

COMSOL Multiphysics also provides many visual tools to help you create a model. For example:

- Different colors, thicker edges (with **OpenGL** and **Software** rendering only, not **DirectX**. See Editing Model Preferences Settings), or larger points to highlight the different geometric entities selected;
- A **Selection List** window to list all the specific geometric entities in the model and to locate and select complex geometric entities;
- Logical selection of overlapping objects in the **Graphics** window;
- A variety of **Show/Hide** and **Select** buttons in the **Graphics** toolbar;
- The option to create predefined **Views** of the geometry. and
- The ability to choose different view combinations of **Names**, **Identifiers**, **Tags**, and **Types** in the **Model Builder** (see Building a COMSOL Model).

About Geometric Entities

Conceptually, a geometry is a collection of bounded *geometric entities*. The entities are *connected manifolds*, that is, volumes, surfaces, curves, or points. Table 4-1 summarizes the technical terms used for these entities.

Geometric entities of the maximum dimension are called *domains*, while of the next highest dimension they are called *boundaries*. The boundaries are sometimes referred to as *faces* in 3D and *edges* in 2D. The *vertices* are also called *points*.

The following rules apply to domains:

- The (interiors of the) domains are disjointed. However, this is only strictly true if the finalization method is "form a union". When it is "form an assembly", domains may overlap (though that is normally considered a modeling error).
- Every geometric entity is bounded by entities of smaller dimension. In particular, a domain (in 3D, 2D, or 1D) is bounded by boundaries, edges (in 3D), and vertices (in 3D and 2D). A boundary (in 3D or 2D) is bounded by edges (in 3D) and vertices. An edge is bounded by vertices.

COMSOL Multiphysics geometric entities are comprised of *domains*, *boundaries*, *edges* (3D only), and *points*. For example, a 3D cube consists of one domain with six boundaries. The six boundaries have 12 edges and the edges connect at eight points (see Figure 4-1). This enables you to visualize a cube by displaying one or more of these four types. For instance, you can generate a wireframe plot by rendering only the cube edges.

TABLE 4-1: NAMES OF GEOMETRIC ENTITIES IN DIFFERENT SPACE DIMENSIONS

ENTITY DIMENSION	NAME IN 3D	NAME IN 2D	NAME IN 1D	NAME IN 0D
3D	domain			
2D	boundary	domain		
1D	edge	boundary	domain	
0D	vertex	vertex	boundary	domain

Geometry objects are *adjacent* if they connect directly to each other. Hence all boundaries, edges, and points on the cube are adjacent to the domain. An edge on the cube is adjacent to two boundaries and two points.

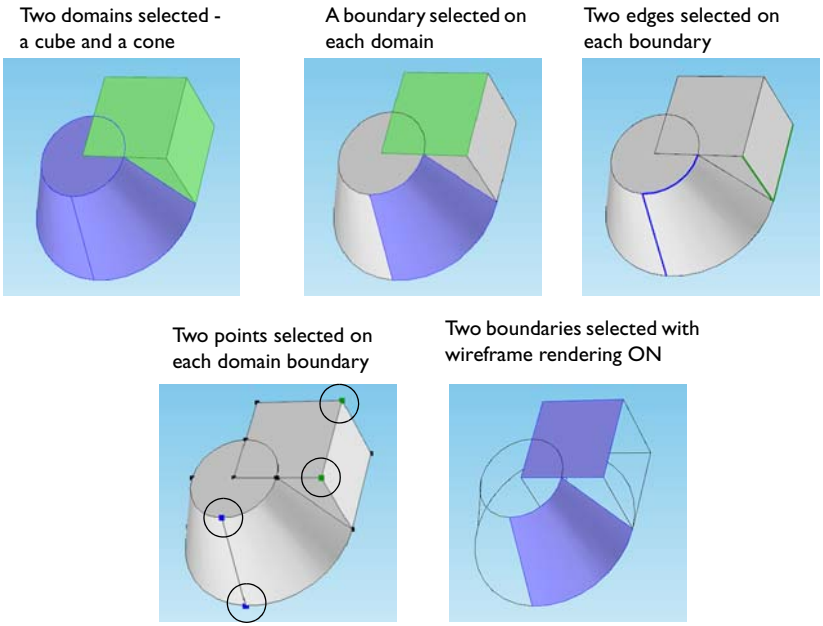


Figure 4-1: A 3D geometry is comprised of domains, boundaries, edges, and points.

About Selecting Geometric Entities

Throughout COMSOL there are many selection lists all based on the same principle—select a **Domain**, **Boundary**, **Edge**, or **Point** and use the same methods to add or remove these geometric entities. For example, a **Physics Interface** window for defining equations and material properties, boundary conditions, sources, and other parts of the model's physics, or the **Variables** window.

There are toolbar buttons, mouse click options, page settings, and keyboard shortcuts available to help you move, select, and highlight geometric entities. Often there is more than one way to do the same thing. The power of COMSOL is that every level of geometry can be treated individually. See Table 4-2 for a list of different ways to complete the same task.

Figure 4-2 shows the difference between a **Selection List** window and the selection lists that are found in the **Settings** window. Also see Figure 4-4 and Figure 4-5 for examples

how to use a combination of the **Selection List** window and these pages to select geometric entities.

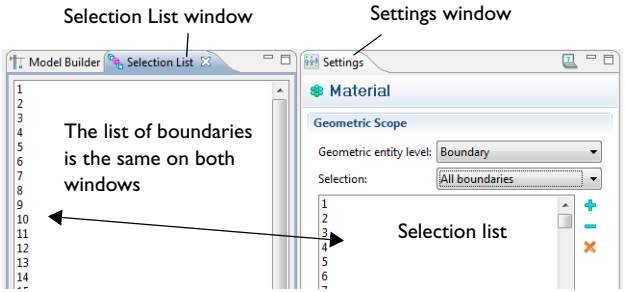


Figure 4-2: Selection List window compared to the Selection list in the Settings window.

VISUALIZATION CUES TO HELP WITH SELECTION

Geometric Entities Highlighted in Color in the Graphics Window

COMSOL Multiphysics highlights geometric entities at different stages of selection. Selected lines also have a thicker line width and selected points are larger. In Figure 4-1, each geometric entity is highlighted in red, blue, green, or with no highlight to indicate its status. It cycles between red (left-click to select the geometric entity), blue (right-click to add it to the selection), green (left-click to show that it is added), and no highlight (right-click to cancel the selection).

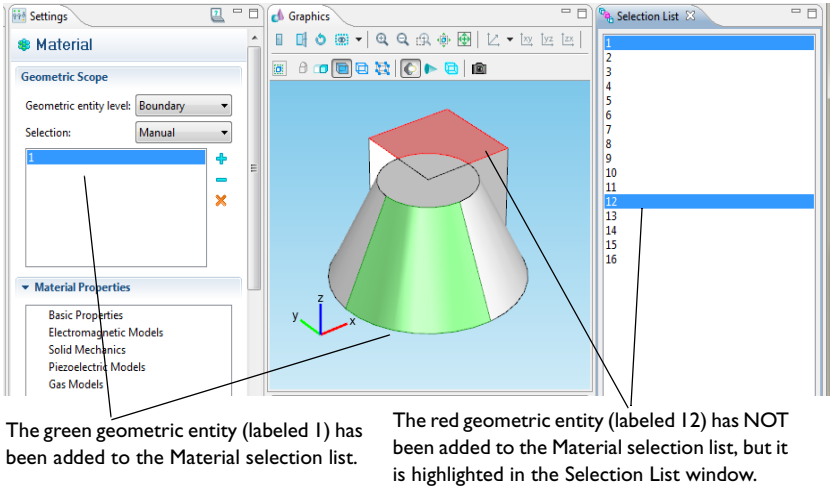


Figure 4-3: Selecting geometric levels - highlighted to indicate status.

Red The geometric entity is highlighted but not added to the model selection list. Left-click once with the mouse. If you right-click it locks all red geometric entities. See Figure 4-3 for an example. Notice how the COMSOL Desktop is rearranged to better view all windows while making a selection.

Blue The geometric entity has been added to the model selection list. The selection is highlighted in blue after you right-click once. See Figure 4-1 for an example.

Green After the geometric entity is added to the selection list, when you click anywhere (directly in the **Graphics** window, from the **Settings** page, or in the **Selection** window) it is highlighted green, which corresponds to the name of the geometric entity in the **Selection** section in the **Settings** window. See Figure 4-1 and Figure 4-3 for examples.

No highlight If the geometric entity is not highlighted it means it is not selected.

About Overlapping Objects

When you click on a 3D geometry comprised of several objects, geometric entities of the same type might overlap at the point where you click. Overlapping objects are selected starting with the closest geometric entity and ending with the overlapping entity the furthest away.

For example, if you click on a circle boundary that overlaps with a sphere boundary as well as a square boundary, the first click selects the circle boundary (the point of origin), then the second click selects the next closest boundary on the sphere, and the last click selects the boundary on the square. Then the cycle starts again with the point of origin, the circle boundary.

Pasting a Selection From File

If you have a list of geometric entities (boundaries, for example) in a file or document, you can copy the list and paste that selection into a selection list. This list of numbers could be a step in a COMSOL Multiphysics modeling instruction and can include commas and spaces as separators, ranges such as 10–34, and even words like “and.” When you have selected a list of numbers and copied it using Ctrl+C (from a text file or PDF file, for example), click the **Paste Selection** button ( ) next to the selection list where you want to add a selection from file. The selection that you paste into the selection list adds to any existing selections.

Using the Selection List Window

Use the **Selection List** window to make it easier to choose objects, for example, while working with complex geometries and when you need to easily locate a geometric entity that is not easily viewed. The **Selection List** is particularly useful when you know the geometric entity number to select; for example, when you are following **Model Library** instructions to build a model.

You open this window from the main menu, **Options>Selection List**, and COMSOL then determines the geometric entities listed based on where in the model you are working.

The **Selection List** window displays a variety of geometric entities. Click on any item to see it highlighted in the **Graphics** window and select items as described in Selecting and Deselecting Geometric Entities. For example, use the **Selection List** in these situations:

- When in the **Model Builder** under the **Geometry** node—the geometry objects are displayed in the **Selection List**, for example, **blk1** and **cone1.**
- When working in windows with **Selection** or **Geometric Scope** sections (a **Selection** window under a **Definitions** node for example), or anywhere you assign materials, physics, boundary conditions, and other model settings. The **Selection List** displays the specific **Geometric entity level** selected (domain, boundary, edge, or point). See Figure 4-4.
- When working in the **Geometry** node with **Chamfer** or **Fillet** features and you want to locate specific points. Specify the selection level by clicking the **Select Domains** ( ), **Select Boundaries** ( ), or **Select Points** ( ) buttons in the **Graphics** toolbar and add it to the **Vertices to fillet** or **chamfer**.

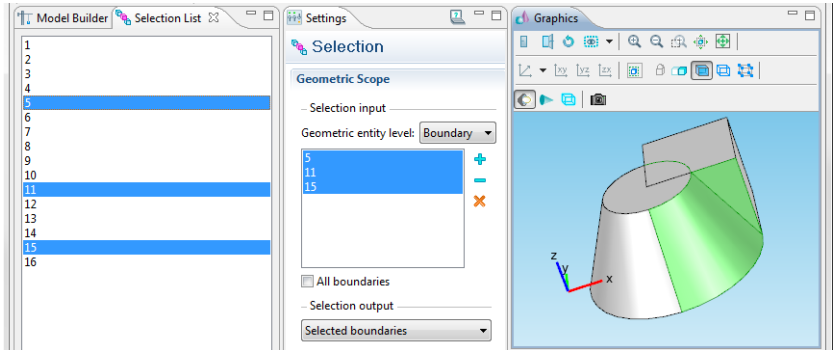


Figure 4-4: Using a combination of the Selection List and Selection window to choose boundaries on a 3D model.

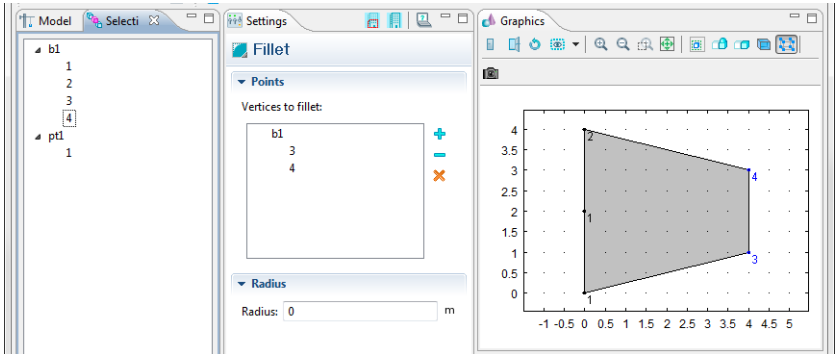


Figure 4-5: Using a combination of the Selection List and Selection window to choose fillet points to add to a 2D model.

AN EXAMPLE OF SELECTING BOUNDARIES WITH THE SELECTION LIST

This example demonstrates the selection of boundaries. When there is the possibility of overlapping geometric entities, it is recommended that you use the **Selection List** window to ensure the correct part of the geometry is selected.

1 From the main menu, select **File>Open Model Library**.

2 Navigate to the **COMSOL Multiphysics>Structural Mechanics>mast_diagonal_mounting** model file. Double-click to open it.

3 In the **Graphics** toolbar, click the **Select Boundaries** button ( ).

4 From the main menu, select **Options>Selection List**.

The **Selection List** window lists all the boundaries contained in all the domains.

There are many ways to select the boundaries:

- Click on any boundary contained in the **Selection List** window.
- Ctrl+click to select more than one boundary from the list at a time. The boundaries are highlighted at the same time in the **Graphics** window.
- Rotate the geometry as required and click on it to highlight boundary numbers in the **Selection List** window.
- In the **Selection List**, shift-click to select contiguous items.
- In the **Graphics** window, shift-click to add an object to the selection. If objects are overlapping, you may need to click more than twice to select all objects.

Selecting and Deselecting Geometric Entities

Note: The same methods are used when working with 2D geometric entities except there are no edges.

Add and remove 3D geometric entities (domains, boundaries, edges, or points) to selection lists in different ways, including buttons in the **Graphics** toolbar, using the **Selection List** window, clicking directly on the geometry, or clicking buttons in the **Settings** window. In the **Graphics** window the geometric entities are color highlighted as you make the selections and you can lock items as you select them.

At various stages of selecting geometric entities, it is also useful to lock a selection to prevent it from being removed by accident—for example, if you are making multiple selections among many overlapping objects. When an object is locked (that is to say, added to a selection list) it is highlighted in blue.

If you start by selecting the **Geometric entity level>Boundary**, and then select **All Boundaries**, the **Selection** list displays all these boundaries. If you make any changes to this list, for example, remove a boundary, the **Selection** list reverts to **Manual**. This option is useful, however, for locating and determining which items to add or remove from the selection and is the same as those geometric entities that display in the **Selection List** window.

See Table 4-2 for the many different ways to select geometric entities using toolbar buttons, mouse click options, page settings, and keyboard shortcuts.

TABLE 4-2: DIFFERENT WAYS TO SELECT GEOMETRIC ENTITIES

TASK	ACTION
Select any level of geometry:	In the **Graphics** window, click the geometric entity. In the **Settings** window, select from a selection list. Open the **Selection List** window and click on entity names.
Select all parts of the geometry:	From the main menu select **Edit>Select All** or press Ctrl+A. This highlights and selects all entities but does NOT confirm the selection or lock it if the model changes. To lock a selection, on some pages, the option to select the **All** (**domains**, **boundaries**, **edges**, or **points**) check box adds that geometric entity to the list. The selected items are highlighted in the **Graphics** window and all entities are kept selected even if the geometry changes. Open the **Selection List** window, shift+click to select all the entity names.
Deselect all parts of the geometry not added to a selection list:	From the main menu select **Edit>Deselect All**. In the **Graphics** window, click outside of the geometry or press Ctrl+D to deselect all selections whereas locked selections remain intact.
Select adjacent geometric entities (3D):	In the **Graphics** window, click a geometric entity, for example, an edge. Then click multiple times to cycle through all the edges that are next to an edge. Hold the Ctrl key down while clicking to confirm selection of all the adjacent geometric entities at the same time.
Select, move, and rotate at the same time:	Multiple mouse actions can be done at the same time in 3D. For example, use the mouse to rotate or move the object left and right to locate the geometric entity to add to a selection list, then right-click and left-click to add to the selection or click the **Select Box** button ().
Lock parts of the geometry during selection:	On some **Settings** windows, click the **Select all** (**domains**, **boundaries**, **edges**, or **points**) check box. See Creating a User-Defined Selection Node for an example.

TABLE 4-2: DIFFERENT WAYS TO SELECT GEOMETRIC ENTITIES

TASK	ACTION
Add items to a selection list:	For user defined selections this action must be completed on the selection page. See Creating a User-Defined Selection Node for information. In the **Graphics** window, left-click (red highlight) then right-click (blue highlight) a geometric entity. Or select one or more geometric entities and click the **Add to Selection** button ( ) in the **Settings** window. In the **Selection List** window, select the entity names to add, and click the **Add to Selection** button ( ) in the **Settings** window. You can also paste selections from a file. See Pasting a Selection From File for information.
Select multiple parts of the geometry:	In the **Graphics** window, click the **Select Box** button ( ) then click and hold the left mouse button to draw a square over the geometry. It is like a "rubberband" selecting all entities enclosed by this operation. Shift-click adds additional geometric entities to the **Selection list**. In the **Selection List** window, select the entity names to add using the Shift or Ctrl keys. Click the **Add to Selection** button ( ) in the **Settings** window to add the geometric entity to the **Selection list**.
2D only: Use an -OR operation with the current selection	In the **Graphics** window, press Ctrl+click on the geometric entity you want to select. Then click the **Select Box** button ( ) and click and hold the left mouse button to draw a square over the geometry. Presently selected objects are deselected when inside the **Select Box** and vice versa.
3D only: Use an -AND operation with the current selection	In the **Graphics** window, press Ctrl+click on the geometric entity you want to select. Then click the **Select Box** button ( ) and click and hold the left mouse button to draw a square over the geometry. Only the presently selected objects inside the rubber-band box stay selected. This makes it possible to select a detail from several selected objects.
Select only objects:	In the **Graphics** window, click the **Select Objects** button ( ). To select more than one at a time, press Ctrl+click or Shift+click.

TABLE 4-2: DIFFERENT WAYS TO SELECT GEOMETRIC ENTITIES

TASK	ACTION
Select only domains:	In the **Graphics** window, click the **Select Domains** button (). To select more than one at a time, press Ctrl+click or Shift+click. In the **Settings** window, select **Domain** from a **Selection list** and then click in the **Graphics** window. Only domains are highlighted.
Select only boundaries:	In the **Graphics** window, click the **Select Boundaries** button (). To select more than one at a time, press Ctrl+click or Shift+click. In the **Settings** window, select **Boundary** from a **Selection list** and then click in the **Graphics** window. Only boundaries are highlighted.
Select only edges:	In the **Graphics** window, click the **Select Edges** button (). To select more than one at a time, press Ctrl+click or Shift+click. In the **Settings** window, select **Edge** from a **Selection list** and then click in the **Graphics** window. Only edges are highlighted.
Select only points:	In the **Graphics** window, click the **Select Points** button (). To select more than one at a time, press Ctrl+click or Shift+click. In the **Settings** window, select **Point** from a **Selection list** and then click in the **Graphics** window. Only points are highlighted.

TABLE 4-2: DIFFERENT WAYS TO SELECT GEOMETRIC ENTITIES

TASK	ACTION
Remove a geometric entity from a selection list:	For user defined selections this action must be completed on the selection page. See Creating a User-Defined Selection Node for information. In the **Graphics** window, highlight or select the geometric entity and click the **Remove from Selection** button ( ) in the **Settings** window. In the **Graphics** window, left-click to highlight and select the geometric entity to remove. Any blue geometric entity turns green. To remove that geometric entity, right-click the mouse button. In the **Selection List** window, select the entity names to remove using the Shift or Ctrl keys. Click the **Remove from Selection** button ( ) in the **Settings** window to remove the geometric entity from the **Selection list**.
Clear all items from a selection list:	For user defined selections this action must be completed on the selection page. See Creating a User-Defined Selection Node for information. Click the **Clear Selection** button ( ) in the **Settings** window.

Zooming In and Out in the Graphics Window

TASK	ACTION
Zoom in and out:	Click the **Zoom In** button ( ) to zoom in. Click the **Zoom Out** button ( ) to zoom out. *3D only*: Click the middle mouse button and drag it forward and backward to zoom in and out of the object. The zoom is centered where the first click is made in the **Graphics** window.
Zoom into a general area of the geometry:	Click the **Zoom Box** button ( ) then click and drag to highlight a section of the geometry to zoom into.
Zoom into a geometric entity:	Click any domain, boundary, edge, or point. Click the **Zoom Selected** button ( ) to display to the extent of the window. This button is also available in connection with the selection lists for domains, boundaries, edges, and points in 3D models
Zoom out to the view the complete geometry:	Click the **Zoom Extents** button ( ) to zoom out and fit the complete geometry into the window.

Changing Views in the Graphics Window

TASK	ACTION
Change the view to the xy-, yz-, or zx-plane:	Click the **Go to View XY** ( ), **YZ** ( ), or **ZX** ( ) buttons. The first click selects the plane view with a positive normal direction. A second click on the same button switches to a negative normal direction.
Change the view to the default:	Click the **Go to Default 3D View** button ( ).
Display a user defined view:	After creating a **View** under the **Definitions** node, click the down arrow next to the **Go to View** button ( ) and select a user defined view from the list.

TASK	ACTION AND RESULT	OPERATION ORDER
Rotate the geometry about the axes:	Left-click and hold down the mouse button while dragging it in any direction. This rotates the scene around the axes parallel to the screen X- and Y-axes with origin in the scene rotation point.	left-click
Move the visible frame on the image plane in any direction, (like using a camera shift lens):	Right-click and hold down the mouse button while dragging it in any direction.	right-click
Rotate about the X- and Y axes in the image plane (tilt and pan the camera):	Press Ctrl and left-click. While holding down both buttons, drag the mouse in any direction. This places the rotation coordinate system in the camera and rotates around the axes parallel to the screen X- and Y-axes.	Ctrl+left-click
Move the camera in the plane parallel to the image plane:	Press Ctrl and right-click. While holding down both buttons, drag the mouse in any direction.	Ctrl+right-click
Rotate the camera around the axis:	Press Ctrl+Alt, then left-click. While holding down all buttons, drag the mouse in any direction.	Ctrl+Alt+ left-click
Move the scene in the plane orthogonal to the direction the camera looks at:	Right-click then press Ctrl. While holding down both buttons, drag the mouse in any direction.	right-click+Ctrl
Move the scene in the plane orthogonal to the axis between the camera and the scene rotation point:	Right-click the mouse and press Alt. While holding down both buttons, drag the mouse in any direction.	right-click+Alt
Move the camera into and away from the object (dolly in/out):	Click the middle mouse button and then press Ctrl. While holding down both buttons, drag the mouse in any direction.	middle-click+Ctrl

TASK	ACTION AND RESULT	OPERATION ORDER
Rotate the camera about its axis between the camera and the scene rotation point (roll direction):	Press Alt, then left-click. While holding down both buttons, drag the mouse in any direction.	Alt+left-click
Move the camera along the axis between the camera and the scene rotation point:	Press Alt, then middle-click. While holding down both buttons, drag the mouse in any direction.	Alt+middle-click

Lighting, Transparency, and Wireframe Rendering

Note: The **Scene Light** feature is available for the 3D View.

TASK	ACTION
Turn Scene Light ON or OFF:	In the **Graphics** window, click the **Scene Light** button (). See Figure 4-6. When creating a **View**, this action toggles the **Scene light** check box on the **View** page.
Turn Transparency ON or OFF:	In the **Graphics** window, click the **Transparency** button (). See Figure 4-6. When creating a **View**, this action toggles the **Transparency** check box on the **View** page.
Turn Wireframe Rendering ON or OFF:	In the **Graphics** window, click the **Wireframe Rendering** button (). See Figure 4-6. When creating a **View**, this action toggles the **Wireframe Rendering** check box on the **View** page. Also see Editing Model Preferences Settings to set the level of graphic detail to Wireframe and speed up the rendering of complex models or to improve visual appearance.

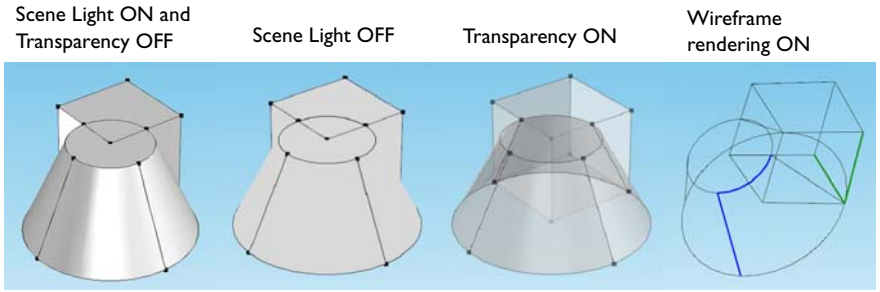


Figure 4-6: Scene light, transparency, and wireframe rendering examples applied to a geometry.

Hiding and Showing Geometric Entities

Note: Selection methods vary a little based on whether the model is 1D, 2D, or 3D.

Selecting an item in any **Selection list** highlights the corresponding geometric entities or objects in the **Graphics** window for selection or deselection. Only the geometric entities you can see in the **Graphics** window are available for selection; that is to say hidden objects cannot be selected.

TASK	ACTION
Hide selected geometric entities:	In the **Graphics** window, select any geometric entity (domain, boundary, edge, or point) and click the **Hide Selected** button ( ).
	When creating a **View**, right-click the **View** node and select **Hide Geometry Objects**. Select a **Geometric entity level** from the list to hide.
	When creating a **View**, right-click the **View** node and select **Hide Geometry Entities**. Select a **Geometric entity level** from the list to hide.
Show selected geometric entities:	In the **Graphics** window, select any geometric entity (domain, boundary, edge, or point) and click the **Show Selected** button ( ).

TASK	ACTION
Reset all hidden objects to the default:	In the **Graphics** window, click the **Reset Hiding** button ( ) to reset all hidden domains, boundaries, edges, or points to the default. If working on a user defined **View**, this removes any **Hide Geometry Objects** or **Entities** page added to a **View** node.
View hidden geometric entities:	In the **Graphics** window, click the **View Hidden Only** button ( ) to display only hidden domains, boundaries, edges, or points.
View unhidden geometric entities:	In the **Graphics** window, click the **View Unhidden** button ( ) to display any domains, boundaries, edges, or points not hidden.
View all geometric entities:	In the **Graphics** window, click the **View All** button ( ) to display all hidden and unhidden domains, boundaries, edges, or points.

User-Defined Selections

In the **Model Builder**, create **Selection** nodes under model **Definitions** to represent various parts of the geometry and simplify the process of assigning materials, model equations, boundary conditions, and other model features. Each user defined **Selection** can be reused during the model design and renamed with descriptive titles—for example, tube, wall, or fluid. Changes to the **Selection** (for example, by adding or removing a boundary) updates all features in the model that use that particular **Selection**.

Creating a User-Defined Selection Node

In addition to the following procedure, you can create a **Selection** node from an existing selection in any **Settings** window that contains a selection of geometric entities such as boundaries. To do so, click the **Create Selection** button () in the **Settings** window that contains the selection.

Use the following steps to create a user defined **Selection** node:

1. In the **Model Builder**, right-click any **Definitions** node under a **Model**, and choose **Selection**.

   A **Selection** page opens in the **Settings** window and a node with the same name is added to the **Model Builder**.
2. Right-click to **Rename** the **Selection** node as required.
3. Under **Geometric Scope** add geometric entities to the **Selection input.** See Figure 4-7 and Figure 4-8 for examples and Selecting and Deselecting Geometric Entities for the different ways to select geometric entities.
   a. Select the **All** (**domains**, **boundaries**, **edges**, or **points**) check box to add that geometric entity to the list. The selected items are highlighted in the **Graphics** window. Selecting this check box locks the entities selected even if the geometry changes.
   b. Select a **Geometric entity level**—**Domain**, **Boundary**, **Edge** (3D only), or **Point** to add or remove from the **Selection input** list.

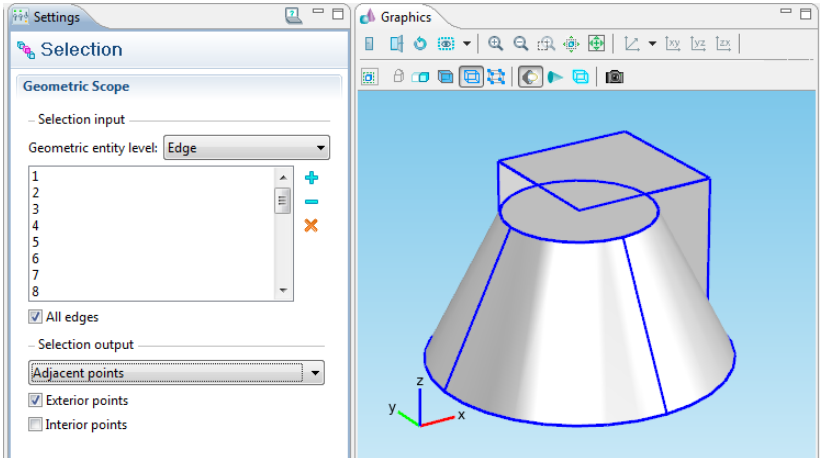


Figure 4-7: Adding all 3D edges to a selection.

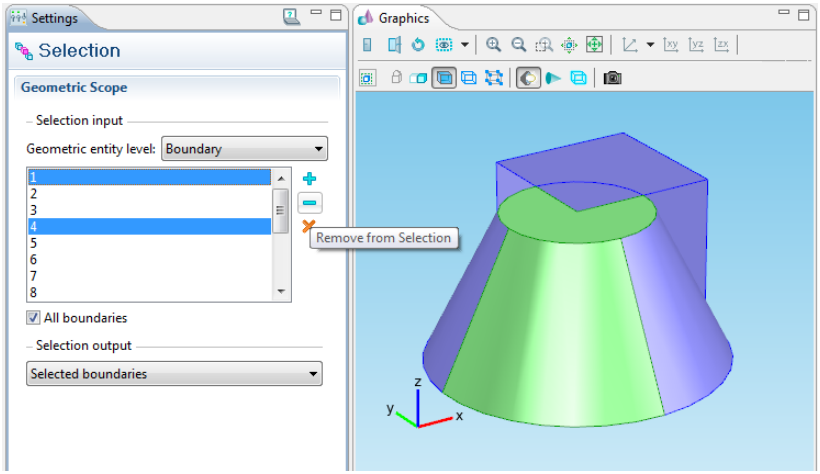


Figure 4-8: Selecting individual boundaries to remove from a selection list.

Note: For 2D geometries, there are no edges, and the related selection options are not available. For 1D models, there are no edges or points, and the related selection options are not available.

Note: If a particular selection is used elsewhere in the model, it is not possible to change the output because it can make it invalid by changing the output type, for example, from domains to boundaries.

4 In the **Selection output** section, define the geometric objects that the selection contains. The options available and defaults depend on your selection in the **Geometric entity level** list.

- If **Domain** is the input the default output is the **Selected domains**.

  Select other options as required. Select **Adjacent boundaries**, **Adjacent edges**, or **Adjacent points** to include boundaries, edges, or points next to the selected domains either as **Exterior** (the default) or **Interior boundaries**, **Interior edges**, or **Interior points**. Click to select or clear the check boxes as needed.

- If **Boundary** is the input, the default output is the **Selected boundaries**.

  Select other options as required. Select **Adjacent edges** or **Adjacent points** to include the edges next to the selected domains either as **Exterior** (the default) or **Interior edges** or **Interior points**. Click to select or clear the check boxes as needed.

- If **Edge** is the input, the default output is **Selected edges**.

  Select **Adjacent points** to include the points next to the selected domains either as **Exterior** (the default) or **Interior points**. Click to select or clear the check boxes as needed. The **Point** input is the same as the selection output.

EXAMPLE OF A USER DEFINED SELECTION

1 From the main menu, select **File>Open Model Library**.

2 Navigate to the **COMSOL Multiphysics>Structural Mechanics>mast_diagonal_mounting** model file. Double-click to open it.

3 Click the **Definitions** node under **Model 1**.

Several nodes display in the **Model Builder**.

4 Click the **Fixed face** and **Force faces** nodes to see how each are defined.

5 Click the **Face Load** node under **Solid Mechanics**.

The **Face Load** page opens in the **Settings** window. Under **Boundaries**, **Force faces** is the **Selection**. **Fixed face** is also available to be selected from the **Selection** list.

User-Defined Views

In the **Model Builder**, create a user defined **View** under model **Definitions**. Each user defined **View** is accessible in the **Graphics** window during model creation and analysis, making it easy to switch between views and find the best way to illustrate your model. You can rename the nodes with descriptive titles—for example, *transparency with scene light*.

Creating Views

Note: For some plots, the **View** node displays under **Results**. For example, when 2D axisymmetric revolved plots or 2D cut plane plots for a 3D models are created. For the majority of plots, use the **Definitions** node to create a **View**.

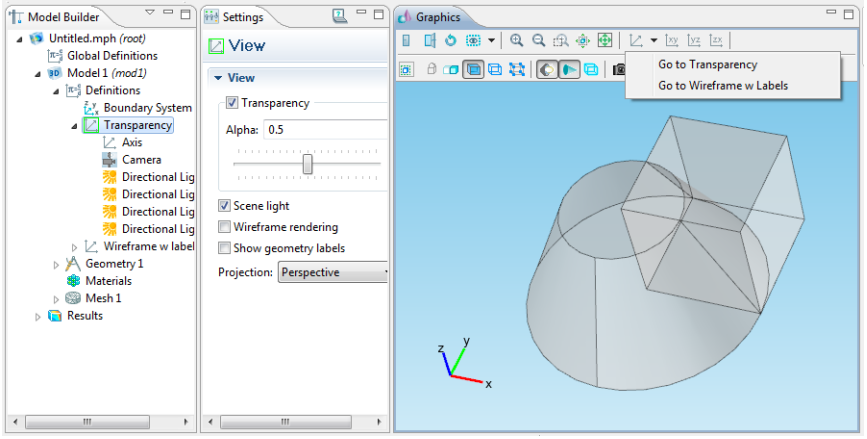


Figure 4-9: Selecting a predefined transparency view of the geometry.

Adding a 2D User-Defined View

ADDING THE NODE TO THE MODEL BUILDER

1 In the **Model Builder**, right-click **Definitions** and select **View**.

The **View** page opens in the **Settings** window and a node with the same name is added to the **Model Builder**.

2 On the **View** page, select the **Show geometry labels** check box to display the geometric short name in the **Graphics** window.

3 Select the **Lock axis** check box to keep the **Axis** settings from changing when using the mouse.

4 Right-click the **View** node to add **Hide Geometric Entities** and **Hide Geometry Object** nodes. See Hiding Geometry Objects in a User-Defined View and Hiding Geometric Entities in a User-Defined View

DEFINING THE VIEW AXIS FOR A 2D VIEW

1 In the **Model Builder**, under **Definitions>View** node, click the **Axis** node.

The **Axis** page opens in the **Settings** window.

2 Under **Axis**, enter **X** and **Y min** and **X** and **Y max** values.

3 The **Preserve aspect ratio** check box is selected by default. Click to clear if required.

4 Under **Grid**, enter **Extra X** and **Y** values.

Select the **Manual spacing** check box and enter **X, Y**, and **Z spacing** values.

Adding a 3D User-Defined View

ADDING THE NODE TO THE MODEL BUILDER

1 In the **Model Builder**, right-click **Definitions** and select **View**.

The **View** page opens in the **Settings** window and a node with the same name is added to the **Model Builder**.

2 On the **View** page, select the **Transparency** check box to turn the feature on. The **Transparency** button ( ) is activated in the **Graphics** window at the same time. Enter a value in the **Alpha** field or use the slider to select a transparency level.

Watch the changes in the **Graphics** window at the same time to help choose a transparency level.

3 The **Scene light** check box is selected by default. Click to clear the check box if required. The **Scene Light** button ( ) is turned on or off in the **Graphics** window at the same time.

Note: The type of **Scene Light** displayed depends on the other settings made on the **View** pages. For example, if the **Intensity** and **Color** are edited on the **Directional Light** page, this is what is shown when the **Scene Light** check box is selected.

4 Select the **Wireframe rendering** check box to view the edges of the object as solid lines. The **Wireframe Rendering** button () is turned on or off in the **Graphics** window at the same time.

5 Select the **Show geometry labels** check box to display the geometric short name in the **Graphics** window.

6 Select the **Show grid** check box to display a numbered grid in the **Graphics** window.

7 From the **Projection** list, select **Perspective** or **Orthographic** (parallel) as required.

8 Select the **Lock camera** check box to keep the **Camera** settings from changing when using the mouse.

9 Right-click the **View** node to add **Directional Light**, **Point Light**, **Spotlight**, **Headlight**, **Hide Geometric Entities** and **Hide Geometry Object** nodes.

Note: Four **Directional Light** nodes are automatically added to the **Model Builder**.

DEFINING THE CAMERA POSITION

1 In the **Model Builder**, under **Definitions** click to expand the **View** node where you want to define the camera position.

2 Click the **Camera** node.

The **Camera** page opens in the **Settings** window.

3 In the **Graphics** window, left-click and hold the mouse to orient the geometry on the axes. The corresponding coordinates are displayed in the **Settings** window under the

Position, **Target**, and **Up Vector** sections. Or enter **x, y**, and **z** coordinates in each section.

- The **Position** is the location of the camera and the **Target** default is 1 unit in front of the camera position. The **Up Vector** value assigns the orientation of the camera roll direction, usually in the z plane.

4 If **Perspective** is selected as the 3D view **Projection**, then under **Position**, enter a **Zoom angle** or use the **Zoom** buttons on the **Graphics** toolbar. This angle is half the viewing angle (in degrees) for the shortest side of the 3D canvas.

5 Right-click the mouse and shift the geometry left, right, up, or down as required. This shift operation moves the currently visible frame on the image plane. The corresponding coordinate values that display in the **Settings** window under **View Offset** are relative to the image width and height, respectively. Alternatively, enter **x** and **y** coordinates.

6 If **Orthographic** is selected as the 3D view **Projection**, then under **View Offset**, enter an **Orthographic scale**. This defines the size in scene length of the viewing block along the longest side of the canvas.

DEFINING A VIEW DIRECTIONAL LIGHT

1 In the **Model Builder**, under **Definitions** click to expand the **View** node where you want to define the directional light.

2 Click the **Directional Light** node.

The **Directional Light** page opens in the **Settings** window.

3 Under **Direction**, enter **x, y**, and **z** coordinates. This defines where the light comes from (negative to the light direction).

4 Under **Settings**, enter an **Intensity** or use the slider to select a value.

5 Select a **Color** or **Custom** to select a different option.

6 Select the **Show in camera coordinate system** check box to make the light rotate together with the camera.

DEFINING A VIEW POINT LIGHT

1 In the **Model Builder**, under **Definitions** click to expand the **View** node where you want to define the point light.

2 Right-click the **View** node and select **Point Light**.

The **Point Light** page opens in the **Settings** window and a node with the same name is added to the **Model Builder**.

3 Under **Position**, enter **x, y**, and **z** coordinates.

4 Under **Settings**, enter an **Intensity** or use the slider to select a value.

5 Select a **Color** or **Custom** to select a different option.

6 Select the **Show in camera coordinate system** check box to make the light rotate together with the camera.

DEFINING A VIEW SPOTLIGHT

1 In the **Model Builder**, under **Definitions** click to expand the **View** node where you want to define the spotlight.

2 Right-click the **View** node and select **Spotlight**.

The **Spotlight** page opens in the **Settings** window and a node with the same name is added to the **Model Builder**.

3 Under **Position**, enter **x, y**, and **z** coordinates.

4 Under **Direction**, enter **x, y**, and **z** coordinates.

5 Under **Settings**:

a Enter a **Spread angle** (in degrees).

b Enter an **Intensity** or use the slider to select a value.

c Select a **Color** or **Custom** to select a different option.

d Select the **Show in camera coordinate system** check box to make the light rotate together with the camera.

Hiding Geometry Objects in a User-Defined View

Use this feature to hide all geometric entities of a certain type in a **View** setting, for example, hide all objects, all domains, or all boundaries.

1 In the **Model Builder**, under **Definitions** click to expand the **View** node where you want to hide the geometry objects.

2 Right-click **View** and select **Hide Geometry Objects**.

The **Hide Geometry Objects** page opens in the **Settings** window and a node with the same name is added to the **Model Builder**.

3 Select a **Geometric entity level** to hide—**Object**, **Domain**, **Boundary**, **Edge**, or **Point**.

Use the **Add to Selection** (), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required in the **Settings** window.

Hiding Geometric Entities in a User-Defined View

Use this feature to hide specific geometric entities in a **View** setting, for example, hide an object, or four domains, or boundaries only on a specific domain.

1 In the **Model Builder**, under **Definitions** click to expand the **View** node where you want to hide the geometric entities.

2 Right-click **View** and select **Hide Geometric Entities**.

The **Hide Geometric Entities** page opens in the **Settings** window and a node with the same name is added to the **Model Builder**.

3 From the **Geometric entity level** list, select **Domain**, **Boundary**, **Edge**, or **Point**. Selection from this list activates the button with the same name in the **Graphics** window.

4 Select **Manual** or **All** (**Domains**, **Boundaries**, **Edges**, or **Points**) from the **Selection** list.

- If you select **Manual**, go the **Graphics** window and select geometric entities of your model.
- Select **All** to add the applicable geometry (**Domains**, **Boundaries**, **Edges**, or **Points**) to the **Selection** box. Use the **Create Selection** (), **Paste Selection** (), **Zoom Selected** (), **Add to Selection** (), **Remove from Selection** ( ), and **Clear Selection** () buttons as required.

Screenshots

Capturing a Screenshot

1. In the **Graphics** window, click the **Image Snapshot** button ().
2. Under **Image**, select a **Unit** to define the image size—**Pixels**, **Millimeters**, or **Inches**.
3. Select the **Lock aspect ratio** check box to maintain the calculation of the width and height (if one or the other is changed).
4. Enter a **Width** and **Height** and **Resolution**.
5. The **Antialiasing** check box is selected by default. Click to clear if required. Antialiasing determines if jagging in the image is smoothed or not.
6. Under **Layout**, the **Title**, **Legend**, and **Axes** check boxes are selected by default to display the information on the screenshot. Click **Include** to edit the selections.
7. Enter a **Font** size.
8. Select a **Background**—**Current** or **Color**.
9. Click **Color** to select a custom color.
10. Under **File**:
    a. Select a file **Format**—**PNG**, **BMP**, or **JPG**.
    b. Enter a file path or click **Browse**.

5

Geometry Modeling and CAD Tools

The CAD tools in COMSOL Multiphysics provide many possibilities to create geometries using solid and boundary modeling. This section covers geometry modeling in 1D, 2D, and 3D with examples of solid modeling, boundary modeling, Boolean operators, and other CAD tools in COMSOL Multiphysics. In addition, it shows how to use the tools for exploring geometric properties, such as volumes and surfaces. This section also provides information about using external CAD data.

In this section:

Creating a Geometry for Successful Analysis

Several techniques can ensure that a geometry results in a good mesh and gives reasonable solution times for the finite element analysis. They include the use of symmetry and eliminating small details, gaps, holes, and singularities.

USING SYMMETRIES

Using symmetry is one of the most effective ways to reduce the size of a finite element model.

MAKING THE GEOMETRY MATCH THE BOUNDARY CONDITIONS

In many solutions, the modeling domain is unbounded or too large for successful meshing and analysis. For these cases a suitable boundary condition can replace the exterior of the domain. It is important for the geometry to be large enough to validate these boundary conditions. For outflows in fluid-flow models, for example, the boundary should be perpendicular to the fully developed flow. Inspections and modifications of the solved model might be necessary to verify the validity of the boundary condition.

AVOIDING EXCESSIVELY SMALL DETAILS, HOLES, AND GAPS

Many geometries, especially those designed using a CAD system without finite element analysis, contain small holes, details, and gaps. These small features can make the domain unbounded and must be removed before analysis. Small details and holes can lead to large meshes and finite element models or even failure during mesh generation. Make sure the snapping feature is activated to avoid small gaps and mismatches between the geometry objects.

The CAD Import Module contains tools for automatic and interactive repair and defeaturing of 3D CAD data.

AVOIDING SINGULARITIES AND DEGENERACIES IN THE GEOMETRY

A singularity in a geometry is a sharp corner or angle that can create problems during meshing and analysis. In reality, a sharp reentrant corner leads to infinite stress values in a stress analysis of a perfectly elastic material. To avoid a singularity, round sharp corners using fillets. The stress value for a sharp corner is finite in the finite element analysis, but refinement of the mesh increases the stresses in the corner without limit.

A degeneracy in the geometry can occur during solid modeling. For example, fillet areas that taper to a point and the apex of a cone can become degenerate points. These degeneracies might cause problems for the mesh generator and problems during the analysis of the model. A common degeneracy in the geometry occurs when you create a 3D solid (for example, a cylinder) by rotation about an axis that touches the rotation area. In such cases, it is better to create the solid object by extruding a cross section or to use geometric 3D primitives.

ASSOCIATIVE GEOMETRY

Associative geometry is a concept for the automatic updating of applied physical properties, such as boundary conditions and equation coefficients, under geometric transformations. Thus, once you have defined the physical properties of a finite element model and return to the Geometry branch to modify the geometric model, COMSOL Multiphysics updates the physical properties according to the geometry modifications.

The associative geometry functionality utilizes geometry-mapping information between the groups of geometric entities (vertices, edges, boundaries, and domains) in the *finalized geometry* and the corresponding groups in the geometric model.

This geometry mapping is not always without ambiguities. Thus, COMSOL Multiphysics makes some heuristic decisions when mapping the physical properties between the finalized geometry (the object on which the physical properties are imposed) and the geometric model. In some cases the resulting updated physical properties might not be the ones that you would expect.

The COMSOL Multiphysics Geometry and CAD Environment

Overview of Geometry Modeling Concepts

In COMSOL Multiphysics you can use ***solid modeling*** or ***boundary modeling*** to create objects in 1D, 2D, and 3D. They can be combined in the same geometry (***hybrid modeling***).

During solid modeling, you form a geometry as a combination of solid objects using *Boolean operations* like union, intersection, and difference. Objects formed by combining a collection of existing solids using Boolean operations are known as *composite solid objects*. Boundary modeling is the process of defining a solid in terms of its boundaries, for example using lines to create a solid hexagonal domain in 2D. You can combine such a solid with *geometric primitives*—common solid modeling shapes like blocks, cones, spheres, rectangles, and circles, which are directly available in COMSOL Multiphysics.

In 3D, you can form 3D solid objects by defining 2D solids in *work planes* and then *extruding* and *revolving* these into 3D solids. It is also possible to *embed* 2D objects into the 3D geometry.

You can also overlay additional nonsolid objects on top of solid objects to control the distribution of the mesh and to improve postprocessing capabilities. For example, you can add a curve object to a geometry to control the element size in the vicinity of this curve, or add a point to guarantee a mesh vertex in a specific location or to create a time-dependent or parametric-value graph at that location in the geometry.

Once you have created a geometry, the *Model Builder* gives you an overview of the *geometry sequence* that creates the objects in a *Geometry branch*. The settings for the nodes making up the sequence can be changed at any time and the whole sequence is re-run instantaneously.

You can import 2D geometries from DXF files and 3D geometries from STL and VRML files. See Import in the *COMSOL Multiphysics Reference Guide* for details of how to import these CAD file formats (or see Where Do I Access the Documentation and Model Library?).

The CAD Import Module provides an interface for the import of CAD files in Parasolid, SAT (ACIS), Inventor, Pro/E, SolidWorks, STEP, and IGES formats. In addition, the CATIAV5 Import Module provides an interface for CATIAV5 files.

The products LiveLink for Inventor, LiveLink for Pro/ENGINEER, and LiveLink for SolidWorks, offer bidirectional links to these respective products. Using these, you can run parametric geometry sweeps driven from the COMSOL environment but operating directly on the geometries in the respective CAD package environments.

Geometry Toolbar

TABLE 5-1: GEOMETRY BUTTONS

BUTTON	NAME
	Draw Point
	Draw Line
	Draw Quadratic
	Draw Cubic
	Draw Square
	Draw Square (Center)
	Draw Rectangle
	Draw Rectangle (Center)
	Draw Circle
	Draw Circle (Center)
	Draw Ellipse
	Draw Ellipse (Center)

TABLE 5-1: GEOMETRY BUTTONS

BUTTON	NAME
	Split
	Delete
	Union
	Intersection
	Difference
	Convert to Solid
	Convert to Surface
	Convert to Curve
	Convert to Point
	Measure

About Selecting the Cartesian or Cylindrical Coordinate System

COMSOL Multiphysics uses a global Cartesian or cylindrical (axisymmetric) coordinate system. You select the geometry dimension and coordinate system when starting a new model in the *Model Wizard*. Variable names for the spatial coordinates are x, y, and z for Cartesian coordinates and r, φ, and z for cylindrical coordinates. These coordinate variables (together with the time parameter for time-dependent models) make up the *independent variables* in COMSOL Multiphysics models. Also see Coordinate Systems for more information.

THE COORDINATE SYSTEMS AND THE SPACE DIMENSION

The labels assigned to the coordinate system variables vary according to the space dimension:

- Models that you open using the space dimensions 1D, 2D, and 3D use the Cartesian coordinate independent variable labels x, y, and z.

- In 2D axisymmetric geometries, the x-axis represents the r label, which is the radial coordinate, while the y-axis represents the z label, the height coordinate.
- In 1D axisymmetric geometries, the default radial coordinate is labelled as r, and represented by the x-axis.

For axisymmetric cases the geometry model must fall in the positive half plane, that is to say $r \geq 0$.

Also see About Cylindrical Coordinate Systems and Defining a Cylindrical Coordinate System for more information.

Selecting the Coordinate System and Space Dimension

In the **Model Wizard**, select **3D**, **2D axisymmetric**, **2D**, **1D axisymmetric**, or **1D** from the **Select Space Dimension** list. You can do this before starting a new model or by right-clicking the root node in the **Model Builder** and selecting **Add Model** for creating models with multiple geometries. The **Model** node's icon indicates the space dimension:

-  for 3D geometries
-  for 2D axisymmetric geometries
-  for 2D geometries
-  for 1D axisymmetric geometries
- for 1D geometries
- for 0D geometries (space-independent models)

Note: Not all physics interfaces are available for all coordinate systems and space dimensions.

Creating Composite Geometry Objects

You can form a *composite geometry object* by combining objects with Boolean operations: **Union**, **Intersection**, **Difference**, or **Compose**. Also form complex geometries using Boolean formulas that include multiple objects and operations.

1 In the **Model Builder**, click the Geometry node, select objects in the **Graphics** window. Then click the **Union** ( ), **Intersection** ( ), or **Difference** ( ) buttons.

- OR -

In the **Model Builder**, right-click Geometry and select **Boolean Operations**.

2 Select **Union**, **Intersection**, **Difference**, or **Compose**.

A **Union**, **Intersection**, **Difference**, or **Compose** page opens in the **Settings** window.

3 For **Union**, **Intersection**, or **Compose**, in the **Input objects** section, add geometry from the **Graphics** window.

For **Difference**, click the **Activate Selection** button ( ) in the **Objects to add** or **Objects to subtract** sections. Then add or subtract geometry in the **Graphics** window.

Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required in the **Settings** window.

4 Select the **Keep input objects** check box to keep the initial objects.

5 Select the **Keep interior boundaries** check box as required.

6 Edit the **Relative repair tolerance** default if the geometry has larger defects, or if you need to resolve extremely small details.

During the creation of a composite object, COMSOL automatically repairs the generated geometry object by removing small edges and faces. Edit the default if required. The value is relative to the overall dimensions of the geometry. For example, if the dimensions are in meters, the default repair tolerance of 10^{-6} makes the geometry repair heal gaps that are smaller than a micrometer (10^{-6} m).

7 For **Compose** only, enter a **Set formula** that defines the Boolean operation. Set formulas are only available for solid objects. The symbols `+`, `-`, and `*` represent *set union*, *set difference*, and *set intersection*, respectively. Use these set formulas to form a composite object by combining the selected solid objects according to the set formula. For example, entering `(cyl1+sph1)*blk1` first forms the union of the cylinder, `cyl1`, and the sphere, `sph1`, and then forms the intersection of that union with the block, `blk1`.

8 Click the **Build Selected** button ( ).

Adding Affine Transformations to Geometry Objects

Mirroring, moving, rotating, and scaling are *affine transformations* applied to geometry objects.

1 Define an object.

2 In the **Model Builder**, right-click Geometry and select an option from the **Transforms** menu:

- **Mirror**—Specify the reflection axis or reflection plane.
- **Move**—Specify displacements in all space directions.
- **Rotate**—Specify the rotation angle with a rotation axis or center of rotation.
- **Scale**—Specify center of scaling and scaling factors in all space directions.

3 The **Mirror**, **Move**, **Rotate**, or **Scale** page opens in the **Settings** window.

4 Under **Input**, select the **Input objects** in the **Graphics** window. The objects display in the **Input objects** list. Use the **Add to Selection** (), **Remove from Selection** (), and **Clear Selection** () buttons as required.

5 Select the **Keep input objects** check box to use the selected geometry objects for further geometric operations.

Mirror

1 2D and 3D: Under **Point on Line of Reflection**, enter a point to be fixed during reflection by entering **x**, **y**, and **z** coordinates as required based on the model dimensions.

2 2D and 3D: Under **Normal vector to Line of Reflection**, enter a vector in the direction to reflect by entering **x**, **y**, and **z** coordinates as required based on the model dimensions.

3 ID: Under **Point of Reflection**, enter the coordinate of the point of reflection in the x field.

Move

- Under **Displacement**, enter the **x**, **y**, and **z** coordinates as required by the model dimension. To create several copies, enter a comma-separated list of displacements in the fields.

Rotate

1 Under **Rotation Angle**, enter the rotational angle in the **Rotation** field.

2 2D: Under **Center of Rotation**, enter **x**, and **y** coordinates.

3D: Under **Point on Axis of Rotation**, enter the **x**, **y**, and **z** coordinates.

3 3D: Under **Axis of Rotation**, select an **Axis type**—**Cartesian** or **Spherical**.

4 If **Cartesian** is selected, enter a direction vector in the **x**, **y**, and **z** fields.

If **Spherical** is selected, enter the angles **theta** (polar, zenith) and **phi** (azimuth).

Scale

1 Under **Scale Factor**, the **Scaling** default is **Isotropic**. Enter a **Factor**.

   If **Anisotropic** is selected as the **Scaling**, enter the **x**, **y**, and **z** coordinates as required by the model dimension.

2 Under **Center of Scaling**, enter the **x**, **y**, and **z** center point coordinates as required by the model dimension.

Creating an Array of Identical Geometry Objects

Create a rectangular or linear array of duplicates of geometry objects.

1 In the **Model Builder**, right-click Geometry and select **Transforms>Array**.

   The **Array** page opens in the **Settings** window.

2 Under **Input**, select the **Input objects** to duplicate in the **Graphics** window. The objects display in the **Input objects** list. Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

3 Under **Size**, the **Array type** defaults to **Rectangular** (2D) or **Three-dimensional** (3D). Enter the number of duplicates in each coordinate direction in the **x**, **y**, and **z** fields.

   If **Linear** is selected as the **Array type**, enter the number of duplicates in the **Size** field.

4 Under **Displacement**, enter the **x**, **y**, and **z** coordinates as required by the model dimension.

Copying and Pasting Geometry Objects

Make a displaced copy of a geometry objects.

1 In the **Model Builder**, right-click **Geometry** and select **Transforms>Copy**.

   The **Copy** page opens in the **Settings** window.

2 Under **Input**, select the **Input objects** to duplicate in the **Graphics** window. The objects display in the **Input objects** list. Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

3 **Keep input objects** is selected by default. Click to clear the check box if required.

4 Under **Displacement**, enter the **x**, **y**, and **z** coordinates as required by the model dimension.

Converting Geometry Objects

Transform one or more geometry objects into a different type of object by *converting* it, for example, converting from a solid to a curve. The object is then called a composite object, which then becomes a solid, face, curve, or point object depending on the target type chosen.

Converting an object is useful for closed domains in 2D and 3D—create solid domains by converting the surrounding curve object or face object to a solid object. This is necessary to apply physics and material properties to the domain.

Available conversions are:

2 D

- Convert a solid object into a curve or point object.
- Convert a curve object defining at least one closed domain into a solid object.
- Convert a curve object into a point object (2D and 3D).

3 D

- Convert a solid object into a surface, curve, or point object.
- Convert a surface object defining at least one closed domain into a solid object.
- Convert a surface object into a curve, or point object.
- Convert a curve object into a point object (2D and 3D).

1 In the **Model Builder**, click the Geometry node, select an object in the **Graphics** window. Then click the **Convert to Solid** ( ), **Convert to Surface** (3D) ( ), **Convert to Curve** ( ), or **Convert to Point** ( ) buttons.

- OR -

In the **Model Builder**, right-click **Geometry** and select an option from the **Conversions** menu—**Convert to Solid**, **Convert to Surface** (3D only), **Convert to Curve**, or **Convert to Point**.

The **Convert to Solid**, **Surface**, **Curve**, or **Point** page opens in the **Settings** window.

2 Under **Input**, select the **Input objects** to convert in the **Graphics** window. The objects display in the **Input objects** list. Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

3 Select the **Keep input objects** check box to use the selected geometry objects for further geometric operations.

4 Edit the **Relative repair tolerance** default if you experience problems with the convert operation. The absolute repair tolerance is the relative repair tolerance times the maximum coordinate of the input objects. Geometric entities that have a distance less than the absolute repair tolerance are merged.

5 Click the **Build Selected** button ( ).

Splitting Geometry Objects

The split operation splits an object into its entities:

- A solid splits into solids corresponding to its domains.
- A surface object splits into surface objects corresponding to its faces.
- A curve object splits into curve objects corresponding to its edges.
- A point object splits into point objects corresponding to its vertices.
- A general (mixed) object splits into solids (corresponding to the domains), surface objects (corresponding to faces not adjacent to a domain), curve objects (corresponding to edges not adjacent to a face or domain), and point objects (corresponding to vertices not adjacent to an edge, face, or domain).

1 In the **Model Builder**, click the **Geometry** node, select objects in the **Graphics** window. Then click the **Split** button ( ).

-OR-

In the **Model Builder**, right-click **Geometry** and select **Split**.

The **Split** page opens in the **Settings** window.

2 Under **Input**, select the **Input objects** to split in the **Graphics** window. The objects display in the **Input objects** list. Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

3 Select the **Keep input objects** check box to use the selected geometry objects for further geometric operations.

4 Click the **Build Selected** button ( ).

Deleting Objects and Entities

To delete objects or entities, click the Geometry node in the **Model Builder**, select objects or entities in the **Graphics** window, and click the **Delete** button ( ). Alternatively, use a **Delete Entities** feature according to the following.

1 In the **Model Builder**, right-click **Geometry** and select **Delete Entities**.

   The **Delete Entities** page opens in the **Settings** window.

2 Under **Input**, select a **Geometric entity level** to delete—**Object**, **Domain**, **Boundary**, **Edge**, or **Point**. Select the geometry in the **Graphics** window. Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

3 Click the **Build Selected** button ( ).

If you use the **Delete** button ( ) to delete objects, the software deletes the selected objects that correspond to primitive features by deleting their nodes from the sequence. If you delete objects that do not correspond to primitive features or if you delete geometric entities a **Delete Entities** node appears in the sequence.

Note: If you want to delete all objects created by a feature, it is better to right-click the feature, and select **Delete** or **Disable**.

Creating a 1D Geometry Model

1 Double-click the COMSOL icon to launch COMSOL.

The **Model Wizard** opens.

2 On the **Select Space Dimension** page, click the **1D** button.

3 Click the **Finish** button ().

4 In the **Model Builder**, right-click Geometry and select **Interval**.

The **Interval** page opens in the **Settings** window.

5 Select a **Number of intervals**—**One** or **Many**. Select **Many** to get an object consisting of a sequence of connected intervals.

- If **Many** is selected, in the **Points** field, enter a comma-separated list of coordinates.
- If **One** is selected, enter the interval endpoint coordinates in the **Left endpoint** (default= 0) and **Right endpoint** (default= 1) fields.

6 Click the **Build Selected** button ().

You can add points to the geometry to divide the domain into two domains.

7 In the **Model Builder**, right-click **Geometry** and select **Point**.

The **Point** page opens in the **Settings** window.

8 Under **Point**, enter the **x** coordinate.

9 Click the **Build All** button ().

Creating a 2D Geometry Model

This section describes how to build a 2D cross section of a heat sink and introduces 2D geometry operations in COMSOL. At this time, you do not model the physics that describe the operation of the heat sink.

Assume that you want to estimate the maximum amount of heat dissipated by a heat sink placed around a resistor for high-power applications. The heat sink consists of an extruded aluminum profile as in Figure 5-1. If you neglect the effects at the ends of the elongated heat sink, you can simplify the model and obtain a decent estimate of the heat dissipated by creating a 2D cross section.

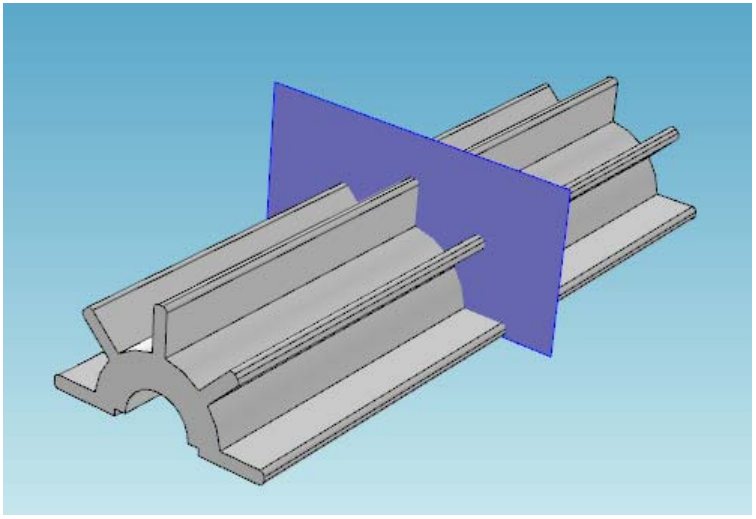


Figure 5-1: Example of a 3D heat sink model with cross section.

CREATING A BASIC 2D GEOMETRY MODEL

1 Double-click the COMSOL icon to launch COMSOL.

The **Model Wizard** opens.

2 On the **Select Space Dimension** page, click the **2D** button.

3 Click the **Finish** button ( ).

CREATING PARAMETERS FOR GEOMETRY PARAMETERIZATION

The following steps explain how to create two circles to form the core of the heat sink in Figure 5-1. To investigate different dimensions of the heat sink, parameterize the

geometry. Start by defining the radius of the outer arc of the heat sink, the radius of the inner arc, and the thickness and the length of the heat sink flanges.

1 In the **Model Builder**, right-click **Global Definitions** and select **Parameters**.

The **Parameters** page opens in the **Settings** window.

2 In the **Parameters** table, enter these settings:

NAME	EXPRESSION	VALUE	DESCRIPTION
R1	5e-3[m]	0.0050 m	Radius Circle 1
R2	2.5e-3[m]	0.0025 m	Radius Circle 2
d	1e-3[m]	0.0010 m	Height
L	5e-3[m]	0.0050 m	Width

ADDING TWO CIRCLES WITH PREDEFINED PARAMETERS

1 In the **Model Builder**, right-click **Geometry 1** and select **Circle**.

The **Circle** page opens in the **Settings** window.

2 Under **Size**, enter R1 in the **Radius** field.

3 Click the **Build Selected** button ().

A circle with radius R1 displays in the **Graphics** window.

4 Right-click **Geometry 1** and select **Circle**.

5 Under **Size**, enter R2 in the **Radius** field.

6 Click the **Build Selected** button ().

A circle with radius R2 displays in the **Graphics** window. Click the **Zoom Extents** button () to see both circles.

SUBTRACTING THE SMALLER CIRCLE

1 In the **Model Builder**, right-click **Geometry 1** and select **Boolean Operations>Difference**.

The **Difference** page opens in the **Settings** window.

2 Under **Difference**, click the **Activate Selection** button ( ) to activate the **Objects to add** list for choosing objects.

3 In the **Graphics** window, select the object **c1** (the larger circle) by left- and then right-clicking it.

C1 is highlighted in red, then blue and added to the **Objects to add** list.

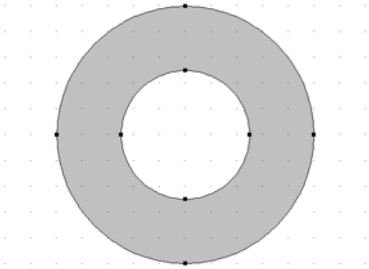
4 Click the **Activate Selection** button ()to activate the **Objects to subtract** list.

5 Select the object **c2** (the smaller circle) by left- and then right-clicking it.

C2 is highlighted in red, then blue and added to the **Objects to subtract** list.

6 Click the **Build Selected** button ( ).

The object **dif1** is created by subtracting the smaller circle from the larger circle.



INTERSECTING WITH RECTANGLE

1 In the **Model Builder**, right-click **Geometry 1** and select **Rectangle**.

2 Under **Size**:

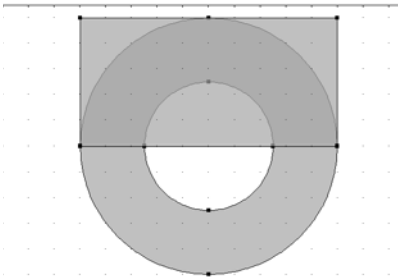
a In the **Width** field enter `2*R1`.

b In the **Height** field, enter `R1`.

3 Under **Position**, enter `-R1` in the **x** field.

4 Click the **Build Selected** button ( ).

The interaction operation creates the object r1 (not related to the circle radius), which coincides with the intersecting area of the two input objects.

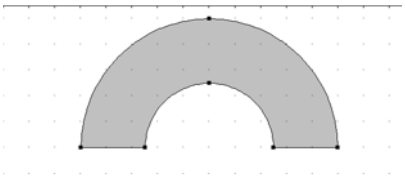


5 In the **Model Builder**, right-click **Geometry 1** and select **Boolean Operations> Intersection**.

6 Select both objects, **dif1** (the combined circle) and **r1** (the rectangle), by left-clicking and then right-clicking them.

Each object is highlighted in red, then blue and added to the **Input Objects** list.

7 Click the **Build Selected** button ( ) to create the object int1.



ADDING A RECTANGLE TO CREATE A FLANGE

1 In the **Model Builder**, right-click **Geometry 1** and select **Rectangle**.

2 Under **Size**:

a In the **Width** field, enter `L`.

b In the **Height** field, enter `d`.

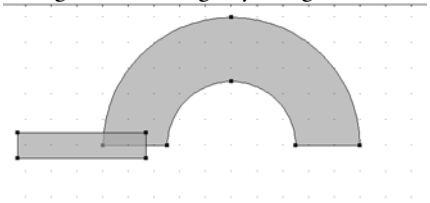
3 Under **Position**:

a In the **x** field enter `-(2/3*R1+L)`.

b In the **y** field enter `-d/2`.

4 Click the **Build Selected** button ( ).

The object r2 (not related to the circle radius) is created. Next, round the sharp edges of the flange by using fillets. Click the **Zoom Extents** button ( ).



ADDING A FILLET TO ROUND THE FLANGE EDGES

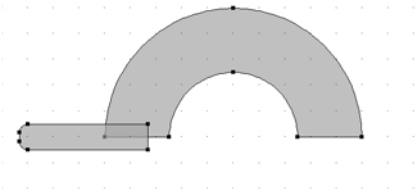
1 In the **Model Builder**, right-click **Geometry 1** and select **Fillet**.

2 Select **Vertices 1** and **4** (the left-hand corners) on object **r2** (the small rectangle).

3 Click the **Add to Selection** button ( ) to add these points to the **Vertices to fillet** section.

4 On the **Fillet** page, under **Radius**, enter `d/3` in the **Radius** field.

5 Click the **Build Selected** button ( ) to create object `fil1`.



ADDING ROTATE OPERATIONS TO CREATE FIVE FLANGES

Rotate the flange 45 degrees and keep the original input object to create five flanges on top of the heat sink.

Adding Rotate 1 to Create Object Rot1

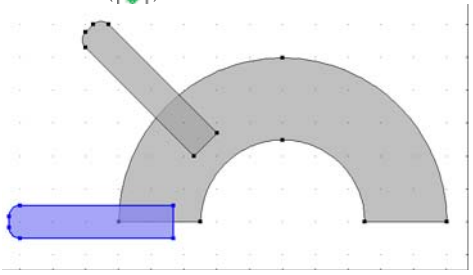
1 In the **Model Builder**, right-click **Geometry 1** and select **Transforms>Rotate**.

2 Select object **fil1** (the filleted rectangle) and add it to the **Input Objects** list.

3 On the **Rotate** page, under **Input**, select the **Keep input objects** check box.

4 Under **Rotation Angle**, enter `-45` in the **Rotation** field.

5 Click the **Build Selected** button ( ) to create object **rot1**. Click the **Zoom Extents** button ( ).



Adding Three More Rotations to the Model

1 In the **Model Builder**, right-click **Geometry 1** and select **Transforms>Rotate**.

2 Select the object **rot1** (the resulting rotated filleted rectangle) to add it to the **Input Objects** list.

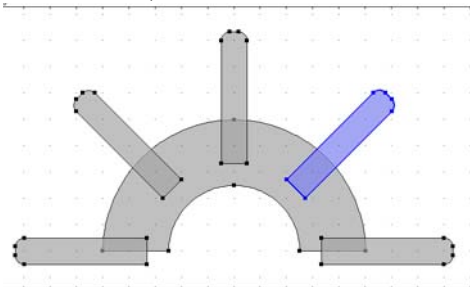
3 On the **Rotate** page, under **Input**, select the **Keep input objects** check box.

4 Under the **Rotation Angle** section, enter `-45` in the **Rotation** field.

5 Click the **Build Selected** button ( ). to create object **rot2**.

6 Repeat the above steps to create object **rot3** and **rot4**. Use object **rot2** to create **rot3** and object **rot3** to create **rot4**.

7 When done, click the **Zoom Extents** button ( ) to view the completed object.



REMOVING INTERIOR BOUNDARIES IN UNION OPERATIONS

1 In the **Model Builder**, right-click **Geometry 1** and select **Boolean Operations>Union**.

2 Select the objects **int1**, **fil1**, **rot1**, **rot2**, **rot3**, and **rot4**.

3 Under **Union**, click to clear the **Keep interior boundaries** check box to remove the internal boundaries in the union operation.

4 Click the **Build All** button ( ). Click the **Zoom Extents** button ( ). The final geometry is shown in Figure 5-2.

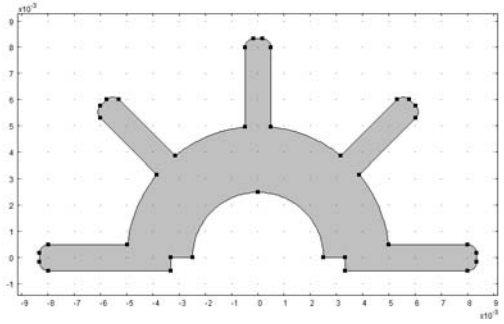


Figure 5-2: Final 2D object created in the Model Builder.

VIEWING THE GEOMETRY SEQUENCE

Figure 5-3 shows the geometry sequence used to create Figure 5-2. All primitive objects and the fillet operation are parameterized through the radius of the inner and outer heat sink arcs, the length and thickness of the flanges, and the radius of the fillets. You can change the parameter values in the **Parameters** table or for any object to create alternative heat sink geometries. The sequence still remains and when you click the **Build All** button ( ) a new geometry is created.

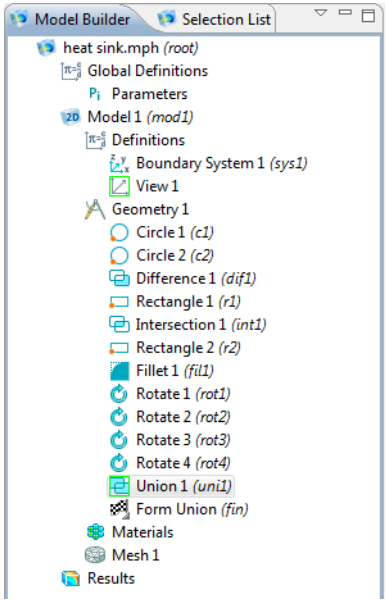


Figure 5-3: an example of a 2D geometry sequence.

RE-RUNNING THE GEOMETRY SEQUENCE WITH DIFFERENT PARAMETERS

1 In the **Model Builder**, under **Global Definitions**, click **Parameters**.

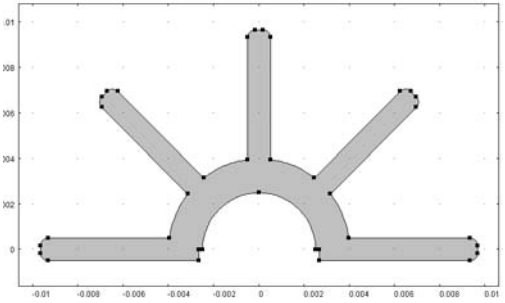
The **Parameters** page opens in the **Settings** window.

2 Under **Parameters**, enter the following settings in the table. Replace the previous data:.

NAME	EXPRESSION	VALUE	DESCRIPTION
R1	`4e-3[m]`	0.0040 m	Radius Circle 1
R2	`2.5e-3[m]`	0.0025 m	Radius Circle 2
d	`1e-3[m]`	0.0010 m	Height
L	`7e-3[m]`	0.0070 m	Width

3 In the **Model Builder**, click **Geometry 1**.

4 Click the **Build All** button ( ). Click the **Zoom Extents** button ( ) to view the geometry as defined by the new parameters.



Creating a 3D Geometry Model

Figure 5-4 shows the geometry of a heat sink used for cooling microprocessors. This section describes the steps to create this geometry and introduces 3D drawing tools and techniques.

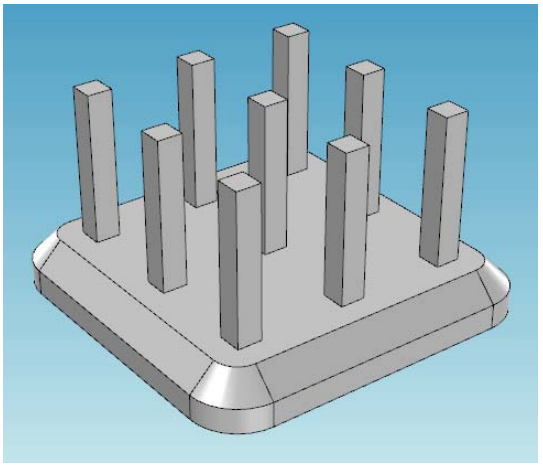


Figure 5-4: Example of a 3D heat sink model.

Creating 3D Geometries Using the Model Builder

CREATING A BASIC 3D GEOMETRY MODEL

1 Double-click the COMSOL icon to launch COMSOL.

The **Model Wizard** opens.

2 On the **Select Space Dimension** page, click the **3D** button.

3 Click the **Finish** button ( ).

CREATING PARAMETERS FOR GEOMETRY PARAMETERIZATION

1 In the **Model Builder**, right-click **Global Definitions** and select **Parameters**.

The **Parameters** page opens in the **Settings** window.

2 In the **Parameters** table, enter these settings:

NAME	EXPRESSION	VALUE	DESCRIPTION
L1	1.5e-2	0.015	Pillar thickness (in the heat sink)
L2	3e-3	0.0030	Pillar Length (in the heat sink)

USING WORK PLANES TO CREATE A BÉZIER POLYGON

Use work planes to create 2D geometries, which you then extrude or revolve to create 3D objects.

Creating a Bézier Polygon

1 In the **Model Builder**, right-click **Geometry 1** and select **Work Plane**.

The **Work Plane** page opens in the **Settings** window.

2 Under **Work Plane**, select **xz-plane** from the **Plane** list.

3 Under the **Work Plane 1** node, right-click **Geometry** and select **More Primatives>Bézier Polygon**.

4 On the **Bézier Polygon** page, under **Polygon Segments**, click **Add Linear**.

Segment 1 (linear) displays in the **Added segments** list.

5 Under **Control points**:

a In row **1**, enter `-2e-3` in the **x** field.

b In row **2**, enter `-4e-3` in the **x** field.

6 Click **Add Linear** to add **Segment 2 (linear)** to the **Added segments** list. Some of the **Control points** are automatically filled in with values; the control points from the previous line are already filled in as the starting points for the next line.

7 Under **Control points**, in row **2**, enter `2e-3` in the **y** field.

8 Click **Add Linear** to add **Segment 3 (linear)** to the **Added segments** list.

a In row **2**, enter `-2e-3` in the **x** field.

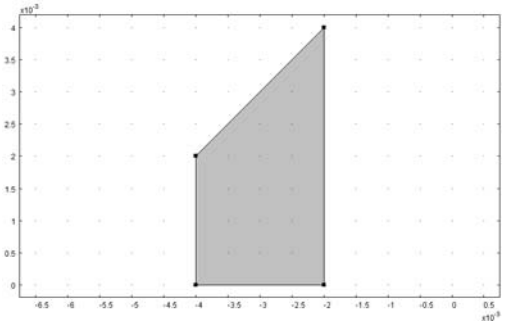
b In row **2**, enter `4e-3` in the **y** field.

9 Click **Add Linear** to add **Segment 4 (linear)** to the **Added segments** list.

10 Under **Control points**, in row **2**, enter `0` in the **y** field.

11 Click **Close Curve**.

12 Click the **Build Selected** button ( ) and the **Zoom Extents** button ( ).



REVOLVING A 2D OBJECT TO CREATE A 3D OBJECT

1 In the **Model Builder**, right-click **Work Plane 1** and select **Revolve**.

The **Revolve** page opens in the **Settings** window and the 2D Bézier Polygon displays in the **Graphics** window.

2 On the **Revolve** page, under **Revolution Angles**, enter 90 in the **End angle** field.

Note: The **Revolution Axis** corresponds to the position of the *y*-axis in the work plane's 2D coordinate system.

---

3 Under **General**, select the **Keep input objects** check box. **Work Plane 1** is required for the next steps.

4 Click the **Build Selected** button ( ) and the **Zoom Extents** button ( ) to view the object **rev1**.

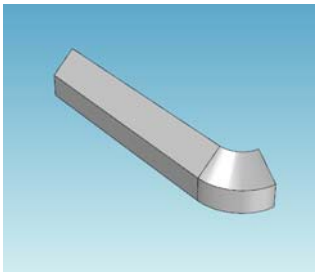
ADDING AN EXTRUSION AND UNION

1 In the **Model Builder**, right-click **Work Plane 1** and select **Extrude**.

The **Extrude** page opens in the **Settings** window.

2 Under **Distances from Work Plane**, enter `-2e-2` in the **Distances** row.

3 Click the **Build Selected** button ( ) and the **Zoom Extents** button ( )to view the object **ext1**.



4 In the **Model Builder**, right-click **Geometry 1** and select **Boolean Operations>Union**.

The **Union** page opens in the **Settings** window.

5 Select the objects **rev1** and **ext1** and add them to the **Input objects** section.

6 On the **Union** page, under **Union**, click to clear the **Keep interior boundaries** check box to remove the interior boundary between the corner section and the edge section of the heat sink.

7 Click the **Build Selected** button ( ). Objects **rev1** and **ext1** are combined to create object **uni1**.

ADDING A ROTATION TO THE 3D OBJECT

1 In the **Model Builder**, right-click **Geometry 1** and select **Transforms>Rotate**.

The **Rotate** page opens in the **Settings** window.

2 Select the object **uni1** and add it to the **Input objects** section under **Input.**

3 Select the **Keep input objects** check box to leave the input object intact as a rotation of the object is created.

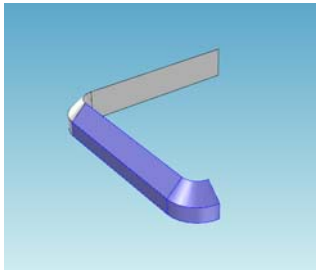
4 Under **Rotation Angle**, enter `-90` in the **Rotation** field.

5 Under **Point on Axis of Rotation**:

a In the **x** field, enter `1e-2`.

b In the **y** field, enter `1e-2`.

6 Click the **Build Selected** button ( ) and the **Zoom Extents** button ( ) to view the object **rot1**.



CREATING UNION 2

1 In the **Model Builder**, right-click **Geometry 1** and select **Boolean Operations>Union**.

The **Union** page opens in the **Settings** window.

2 Select the objects **uni1** and **rot1** and add them to the **Input objects** section on the **Union** page.

3 Under **Union**, click to clear the **Keep interior boundaries** check box.

4 Click the **Build Selected** button ( ) to create object **uni2**.

ADDING A SECOND ROTATION

1 In the **Model Builder**, right-click **Geometry 1** and select **Transforms>Rotate**.

The **Rotate** page opens in the **Settings** window.

2 Select the object **uni2** and add it to the **Input objects** section under **Input**.

3 Select the **Keep input objects** check box.

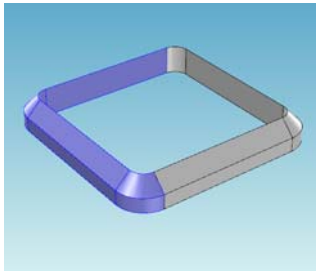
4 Under **Rotation Angle**, enter `-180` in the **Rotation** field.

5 Under **Point on Axis of Rotation**:

a In the **x** field, enter `1e-2`.

b In the **y** field, enter `1e-2`.

6 Click the **Build Selected** button ( ).



CREATING UNION 3

1 Right-click **Geometry 1** and select **Boolean Operations>Union**.

The **Union** page opens in the **Settings** window.

2 Select the objects **uni2** and **rot2** and add them to the **Input objects** section under **Union**.

3 Click to clear the **Keep interior boundaries** check box.

4 Click the **Build Selected** button ( ) to create object **uni3**.

CREATING WORK PLANE 2 AND ADDING A SQUARE

1 In the **Model Builder**, right-click **Geometry 1** and select **Work Plane**.

The **Work Plane** page opens in the **Settings** window and a **Work Plane 2** node is added in the **Model Builder**.

2 Click the **Build Selected** button ( ).

3 To the right of the **Settings** window, click the **Show Work Plane** button ( ). Note that you can use the projection of the 3D geometry on the xy-plane as a guide for creating the middle section of the heat sink base.

4 In the **Model Builder**, under **Work Plane 2**, right-click **Geometry** and select **Square**.

The **Square** page opens in the **Settings** window.

5 Under **Size**, enter `2.4e-2` in the **Side length** field.

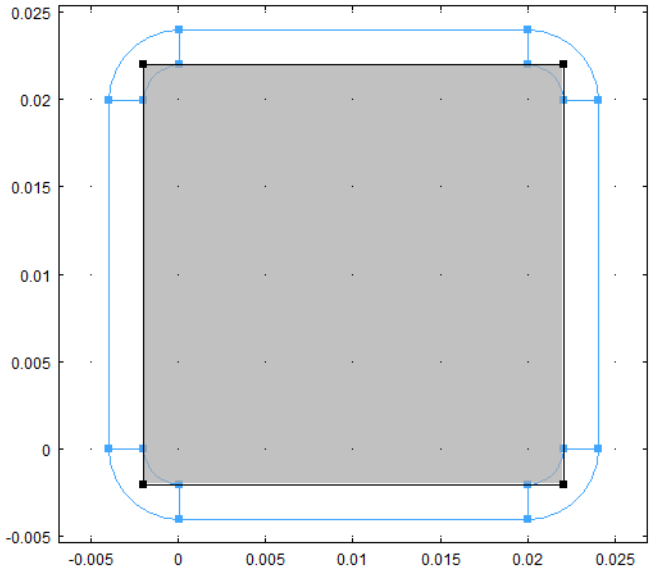
6 Under **Position**:

a Select **Center** from the **Base** list.

b In the **x** field, enter `1e-2`.

c In the **y** field, enter `1e-2`.

7 Click the **Build Selected** button ( ) and the **Zoom Extents** button ( ).



TRIMMING THE SQUARE TO FIT USING THE FILLET OPERATION

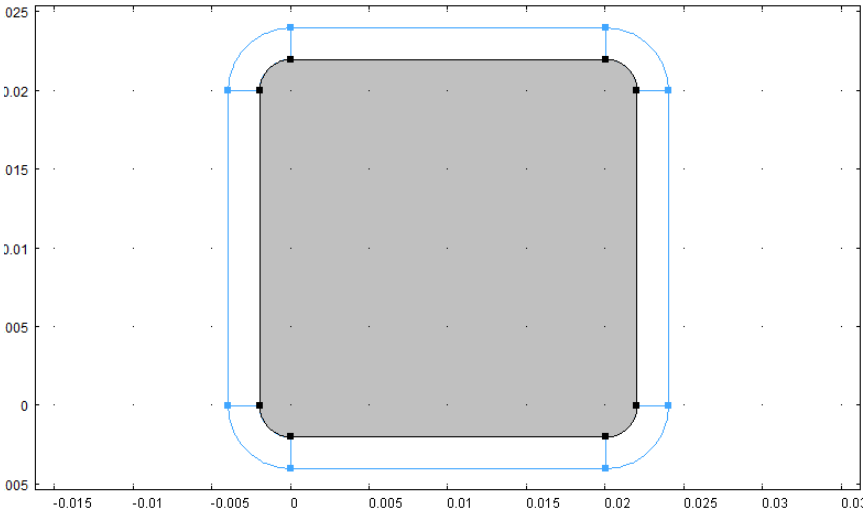
1 In the **Model Builder**, under **Work Plane 2**, right-click **Geometry** and select **Fillet**.

The **Fillet** page opens in the **Settings** window.

2 Add points 1, 2, 3, and 4 on the object **sq1** to the **Vertices to fillet** section under **Points**.

3 Under **Radius**, enter `2e-3` in the **Radius** field.

4 Click the **Build Selected** button.( )



ADDING EXTRUDE 2 AND COMBINING OBJECTS TO COMPLETE THE BASE

1 In the **Model Builder**, right-click **Geometry 1** and select **Extrude**.

The **Extrude** page opens in the **Settings** window.

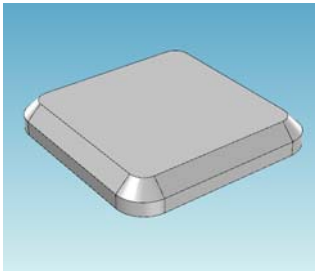
2 Under **Distances from Work Plane**, enter `4e-3` in the **Distances** row.

3 Click the **Build Selected** button ( ).

4 In the **Model Builder**, right-click **Geometry 1** and select **Boolean Operations>Union**.

5 Select the objects **uni3** and **ext2** to add to the **Input objects** section under **Union**.

6 Click the **Build Selected** button ( ) to create object **uni4**. This completes the base of the heat sink.



DRAWING THE UPPER PART OF THE HEAT SINK

Creating a Work Plane and a Square

1 In the **Model Builder**, right-click **Geometry 1** and select **Work Plane**.

The **Work Plane** page opens in the **Settings** window and a **Work Plane 3** node is added in the **Model Builder**.

2 Under **Work Plane**, enter `4e-3` in the **z-coordinate** field.

3 From the **3D projection** list, select **Entire 3D geometry** to visualize the projected edges of the heat sink's base in the work plane.

4 Click the **Build Selected** button ( ).

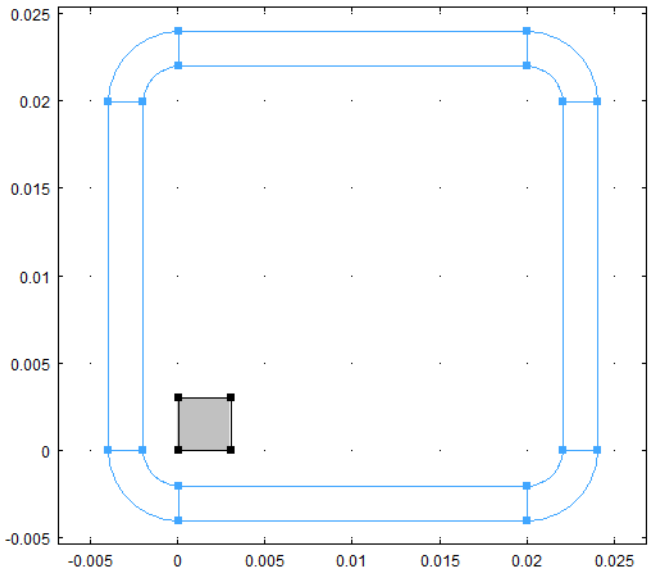
5 To the right of the **Settings** window, click the **Show Work Plane** button ( ).

6 In the **Model Builder**, under **Work Plane 3**, right-click **Geometry** and select **Square**.

The **Square** page opens in the **Settings** window

7 Under **Size**, enter `L2` in the **Side length** field.

8 Click the **Build Selected** button ( ) to create square **sq1** with side length L2.



ADDING ARRAYS

1 In the **Model Builder**, under **Work Plane 3**, right-click **Geometry** and select **Transforms> Array**.

2 Add the object **sq1** to the **Input objects** section under **Input**.

3 Under **Size**:

   a In the **x** field, enter `3`.

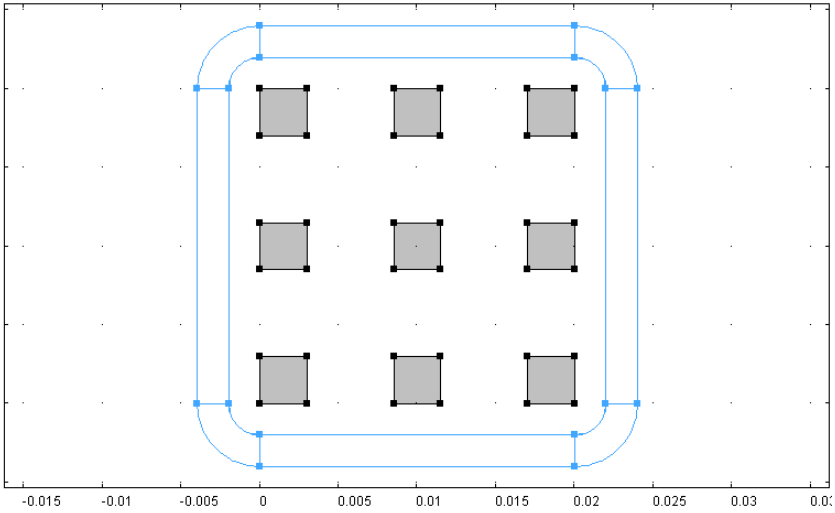
   b In the **y** field, enter `3`.

4 Under **Displacement**:

   a In the **x** field, enter `1e-2-L2/2`.

   b In the **y** field, enter `1e-2-L2/2`.

5 Click the **Build Selected** button ( ).



Adding Extrude 3 and Combining Objects (Union)

1 In the **Model Builder**, right-click **Geometry 1** and select **Extrude**.

   The **Extrude** page opens in the **Settings** window.

2 Under **Distances from Work Plane**, enter `L1` in the **Distances** row.

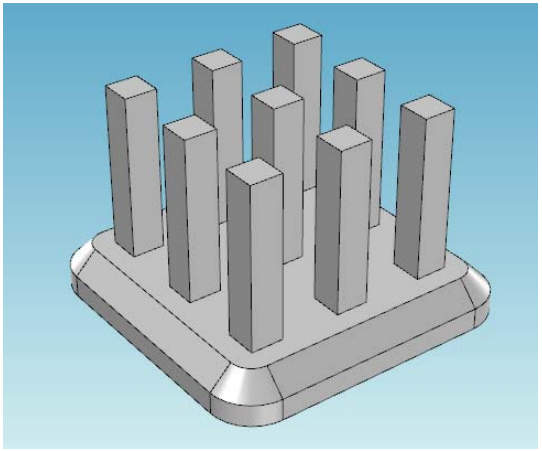
3 Click the **Build Selected** button ( ) and the **Zoom Extents** button ( ).

4 Right-click **Geometry 1** again and select **Boolean Operations>Union**.

The **Union** page opens in the **Settings** window.

5 Add all the objects to the **Input objects** list under **Union**.

6 Click the **Build All** button ( ) to complete the heat sink geometry.



THE GEOMETRY SEQUENCE

Figure 5-5 shows the list of the geometry in the **Model Builder** used to create Figure 5-4. You can edit any node for each of the drawing operations.

In this case, the upper part of the heat sink is parameterized, through the thickness and height of the heat sink pillars. You can edit the parameter values defined previously to change the heat sink geometry.

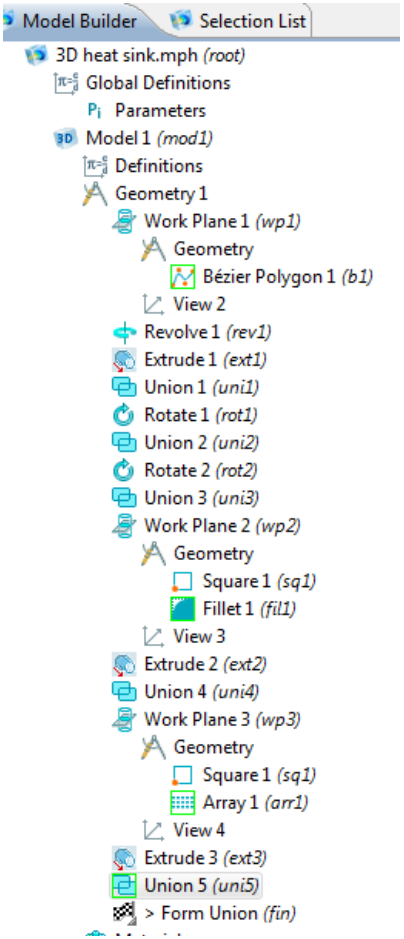


Figure 5-5: Sample 3D Model Builder Geometry Sequence

RE-RUNNING THE GEOMETRY SEQUENCE WITH DIFFERENT PARAMETERS

1 In the **Model Builder**, under **Global Definitions**, click **Parameters**.

The **Parameters** page opens in the **Settings** window.

2 Under **Parameters** enter the following settings in the table. Replace the previous data:

NAME	EXPRESSION	VALUE	DESCRIPTION
L1	1.2e-2	0.012	Pillar thickness (in the heat sink)
L2	2e-3	0.0020	Pillar Length (in the heat sink)

3 In the **Model Builder**, click **Geometry 1**.

4 Click the **Build All** button () and the **Zoom Extents** button () to view the geometry as defined by the new parameters.

6

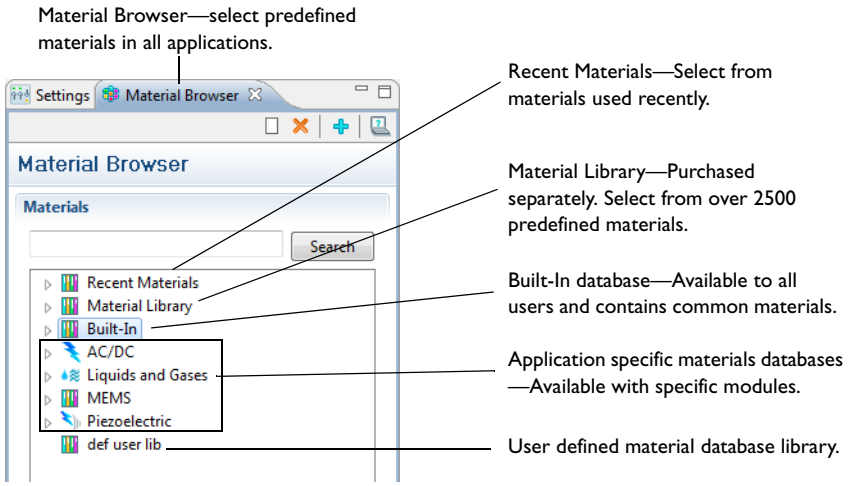
Materials

This section describes how to work with materials: adding them to the model, using material databases, and defining material properties.

In this section:

Materials Overview

Predefined Material Databases Included with COMSOL Modules



All COMSOL modules have predefined material data available to build models. The most extensive material data is contained in the separately purchased Material Library, but all modules contain commonly used or module-specific materials. For example, the **Built-In** database is available to all users but the **MEMS** database is included with the Acoustics Module, MEMS Module, and Structural Mechanics Module. You can also create your own materials and material libraries by researching and entering material properties yourself.

You access all the material databases (including the Material Library) from the **Material Browser**. These databases are briefly described below.

RECENT MATERIALS

From the **Recent Materials** folder ( ), you can select from a list of recently used materials, with the most recent at the top.

MATERIAL LIBRARY

An optional add-on database, the **Material Library** ( ), contains data for over 2500 materials and 20,000 property functions. You can search an online list of available

materials in the Material Library from the COMSOL website: http://www.comsol.com/products/material.

BUILT-IN

Included with COMSOL Multiphysics, the **Built-In** database ( ) contains common solid materials with electrical, structural, and thermal properties. See Predefined Built-In Materials for all COMSOL Modules for a list.

AC/DC

Included in the AC/DC Module, the **AC/DC** database ( ) has electric properties for some magnetic and conductive materials.

LIQUIDS AND GASES

Included in the Acoustics, Chemical Engineering, Earth Science, Heat Transfer, and MEMS Modules, the **Liquids and Gases** database ( ) includes transport properties and surface tension data for liquid/gas and liquid/liquid interfaces.

MEMS

Included in the Acoustics, MEMS, and Structural Mechanics Modules, the **MEMS** database ( ) has properties for MEMS materials—metals, semiconductors, insulators, and polymers.

PIEZOELECTRIC

Included in the Acoustics, MEMS, and Structural Mechanics Modules, the **Piezoelectric** database ( ) has properties for piezoelectric materials.

USER-DEFINED MATERIAL DATABASES

The **User-Defined Library** folder ( ) is where you create user defined material databases (libraries). When you create any new database, this also displays in the **Material Browser**. See Creating a User defined Materials Database.

Note: All material databases (including the **Material Library**) shipped with COMSOL Multiphysics and the optional modules are read-only.

USING THE MATERIALS IN THE PHYSICS SETTINGS

The physics set-up in a model is determined by a combination of settings in the **Materials** and physics interface pages. When you add the first material to a model, COMSOL automatically assigns that material to the entire geometry. The power of COMSOL is that you can select different geometric entities to have different materials. The following example uses the *heat_sink.mph* model file contained in the Heat Transfer Module Model Library.

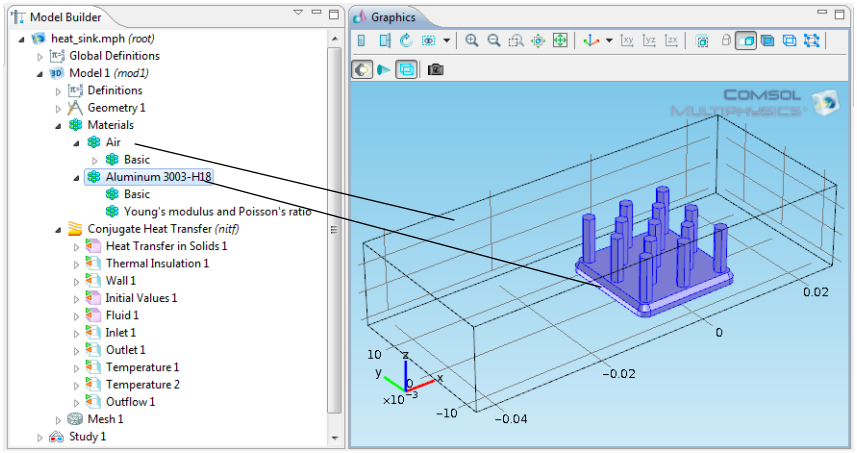


Figure 6-1: Assigning materials to a heat sink model. Air is assigned as the material to the box surrounding the heat sink, and aluminum to the heat sink itself.

If a geometry consists of a heat sink in a container, you can assign **Air** as the material in the container surrounding the heat sink and **Aluminum** as the heat sink material itself (see Figure 6-1). The **Conjugate Heat Transfer** physics interface, selected during model set-up, has a **Fluid** flow model, defined in the box surrounding the heat sink, and a **Heat Transfer** model, defined in both the aluminum heat sink and in the air box. The **Heat Transfer in Solids 1** settings use the material properties associated to the **Aluminum 3003-H18** materials node, and the **Fluid 1** settings define the flow using the **Air** material properties. The other nodes under **Conjugate Heat Transfer** define the initial and boundary conditions.

All physics interface properties automatically use the correct **Materials** properties when you use the default **From material** setting. This means that you can use one node to

define the physics across several domains with different materials; COMSOL then uses the material properties from the different materials to define the physics in the domains. If material properties are missing, the **Material Contents** section on the **Materials** page displays a stop icon to warn you about the missing properties or values. See About the Material Page.

EVALUATING AND PLOTTING MATERIAL PROPERTIES

{[MR: Add this when Erik D. has checked in support for mat1.rho etc. Notice special case of functions like mat1.def.rho(pA,T)]]

About the Material Browser Window and Page

The **Material Browser** window ( ) contains a number of databases with a broad collection of physical, elastic, electromagnetic, fluid, piezoelectric, and thermal properties of materials (Figure 6-2). Use the **Material Browser** to find predefined materials and add them to the **Model Builder**, or create your own custom material library.

Note: When you are using the **Material Browser**, the words *window* and *page* are interchangeable. For simplicity, the instructions refer only to the **Material Browser**.

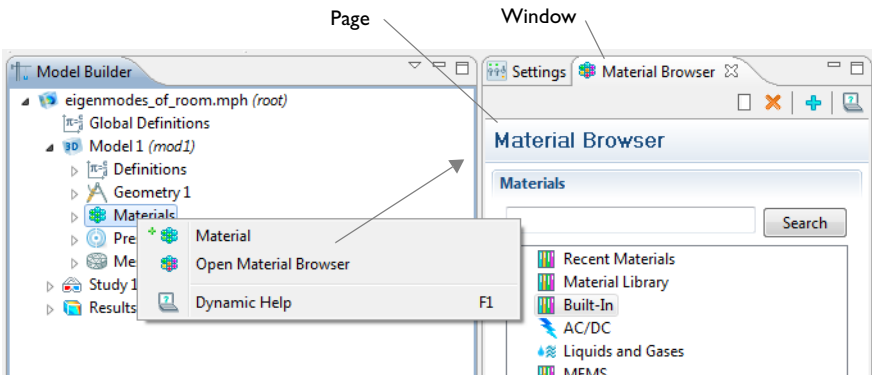


Figure 6-2: Opening the Material Browser page in the Material Browser window.

The **Material Browser** contains the following sections:

MATERIALS

Browse all the available material databases or search in the **Materials** section. **Search** a specific material by name, UNS number, or DIN number. You can also browse for a specific material by class (for example, in the Material Library) or physics module (for example, MEMS). When you locate the material you want, right-click to **Add Material to Model**.

INFORMATION

When you are browsing the material databases, in particular the **Model Library**, some materials include additional information—UNS number, DIN number, and/or Composition.

PROPERTIES

When you are browsing the material databases, this section lists predefined **Properties**. The table includes the default listings of **Property**, **Expression**, and the **Property groups** to which it belongs.

FUNCTIONS

When you are browsing the material databases, some materials have predefined **Functions** listed. The table includes the default listings of **Function**, **Type**, **Expression**, and the **Property groups** to which it belongs.

INPUT

When you are browsing the Material Library database, some materials have predefined **Input** listed. The table includes the default listings of **Input**, **Unit**, and **Property groups** to which it belongs.

About the Material Page

The **Material** page () summarizes the predefined or user defined material properties. It is where you add material properties specific to the model and assign the geometric entity level where the material is used in the geometry (domain, boundary, edge, or point) and the domains, for example, that consists of this material.

After adding a material, click the material node (for example, **Copper**) in the **Model Builder**. The **Material** page opens in the **Settings** window (Figure 6-3).

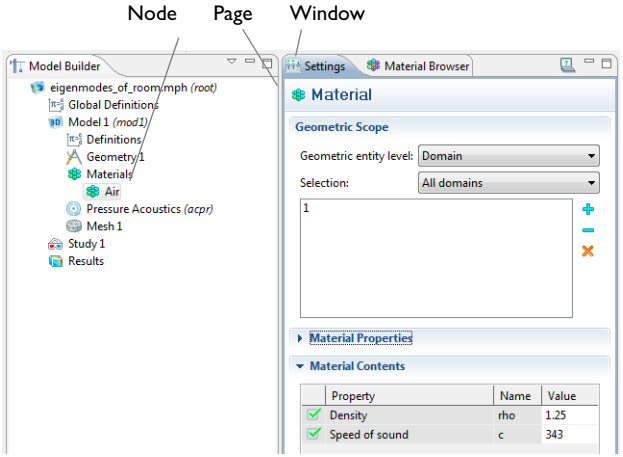


Figure 6-3: Opening the Material page in the Settings window.

The **Material** page contains the following sections:

GEOMETRIC SCOPE

Assign the material to a specific **Geometric entity level**—**Domain**, **Boundary**, **Edge** (3D only), or **Point**—on the geometry in the **Graphics** window (the geometry in the model).

MATERIAL PROPERTIES

Add additional material properties to the model if they are not already included in the default settings.

MATERIAL CONTENTS

View lists of all the default material data. The table lists the **Property**, **Name**, **Value**, **Unit**, and the **Property group**. The **Property group** corresponds to the nodes in the **Model Builder** with the same name (see Figure 6-4). If required, edit a default property **Value**.

The left column provides visual cues about the status of each property.

- A stop sign ( ) indicates that an entry in the **Value** column is required. It means that the material is used but undefined.
- A warning sign ( ) indicates that the material parameters are undefined and unused. An entry is only required if the material is used.

- A green check mark (✓) indicates that the property has a **Value** and is currently being used in the physics of the model.
- Properties with no indication in the left column are not currently used by any physics interface in the model.

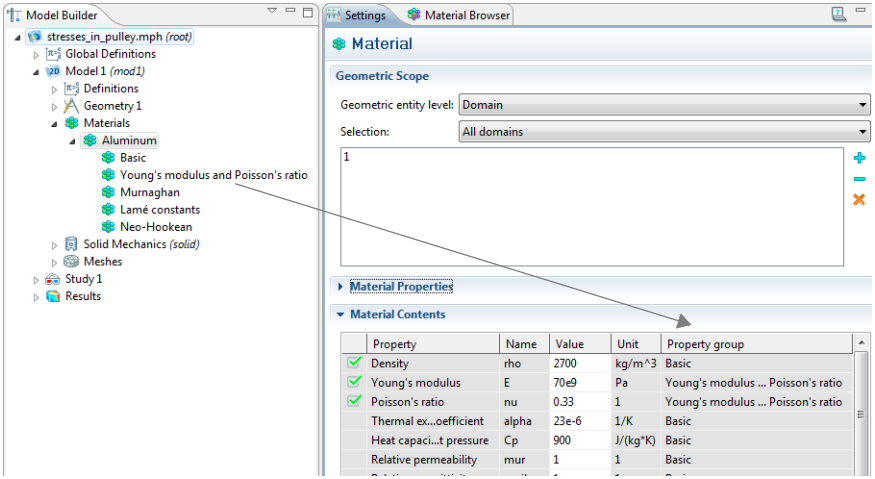


Figure 6-4: An example of Property Groups on the Material page.

About the Property Group Page

The **Property Group** page is where you add output and input properties, define local parameters, and enter expressions for a specific property group (Figure 6-5).

The **Property Group** page is in the **Settings** window and opens when you click the node (for example, **Basic**) under the material name in the **Model Builder**. The **Property Group** page is associated to other pages relating to the same material.

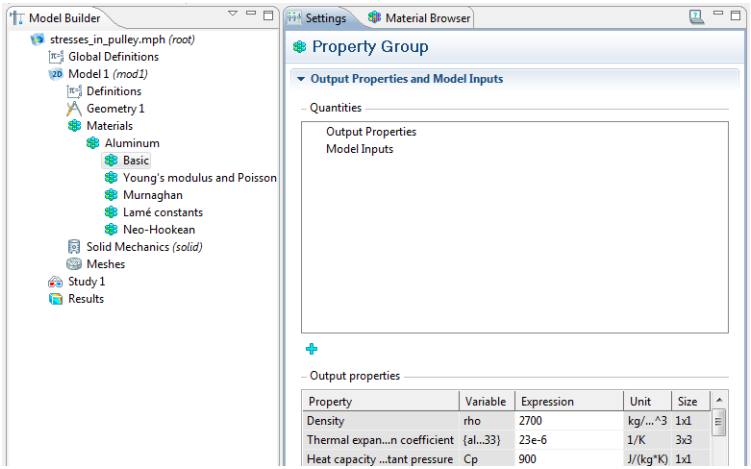


Figure 6-5: An example of a Basic Property Group page.

OUTPUT PROPERTIES AND MODEL INPUTS

View predefined material properties in the **Output properties** table. Add additional properties from the **Quantities** subsection to the **Output properties** or **Model Inputs** tables. If required, edit the expressions in the **Output properties** subsection's **Expression** column. The model inputs are physical quantities such as temperature that are used as inputs in the expressions that define the output properties.

LOCAL PARAMETERS

Enter a user defined **Parameter** and its corresponding **Expression** and organize the table as required.

About Automatic Adding of Property Groups to a Material

Material property groups are automatically added to the material node in the **Model Builder**. You can also add additional predefined property groups or create your own **User defined property group**. The available properties are grouped together differently according to the physical context. For example, the **Copper** node in Figure 6-4 contains the following default property groups: **Basic**, **Young's modulus and Poisson's ratio**, and **Linearized resistivity**.

Each of the property groups has a **Property Group** page. When you click a **Model Builder** node (for example, **Basic**), the **Property Group** page displays specific information about that property group. The property groups are summarized in a **Material Contents** table on the **Material** page.

Adding Predefined Materials

Note: The first material added to a model is assigned to entire geometry.

Working on the Material Browser

When you are using the **Material Browser**, the words *window* and *page* are interchangeable. For simplicity, the instructions refer only to the **Material Browser**.

OPENING THE MATERIAL BROWSER

1. Open or create a model file.
2. From the **Options** menu select **Material Browser** or right-click the **Materials** node and select **Open Material Browser**.

   The **Material Browser** opens by default to the right of the **Settings** window.

ADDING A PREDEFINED MATERIAL TO A MODEL

1. Open the **Material Browser**.
2. Under **Materials**, search or browse for materials.
   - Enter a **Search** term to find a specific material by name, UNS number (Material Library materials only), or DIN number (Material Library materials only). If the search is successful, a list of filtered databases containing that material displays under **Materials**.

Note: To clear the search field and browse, delete the search term and click **Search** to reload all the databases.

   - Click to open each database and browse for a specific material by class (for example, in the Material Library) or physics module (for example, MEMS).

Note: Always review the material properties to confirm they are applicable for the model. For example, **Air** provides temperature-dependent properties that are valid at pressures around 1 atm.

3 When you locate the material you want, right-click to **Add Material to Model**.

A node with the material name is added to the **Model Builder** and the **Material** page opens.

Working on the Material Page

VIEWING A SUMMARY OF THE MATERIAL PROPERTIES

1 Add a material and geometry to the model.

2 In the **Model Builder**, click the material node to open the **Material** page in the **Settings** window.

The **Material** page summarizes all the predefined data and has these sections:

- **Geometric Scope**: select the object **Domain**, **Boundary**, **Edge** (3D only), or **Point** to assign material to the geometry domain, boundary, edge or point.
- **Material Properties**: select and define specific material parameters to add to your model.
- **Material Contents**: lists all the default material property data. The table contains the **Property**, **Name**, **Value**, **Unit**, and the **Property group**. The **Property group** corresponds to the nodes in the **Model Builder** with the same name. If required, edit a default property **Value**.

ASSIGNING GEOMETRIC SCOPE TO A MATERIAL

1 Add a material and geometry to the model.

2 On the **Material** page, from the **Geometric entity level** list, select **Domain**, **Boundary**, **Edge** (3D only), or **Point**. Selection from this list activates the button with the same name in the **Graphics** window.

3 Select **Manual** or **All** (**Domains**, **Boundaries**, **Edges**, or **Points**) from the **Selection** list.

- If you select **Manual**, go the **Graphics** window and select geometric entities of your model.
- Select **All** to add the applicable geometry (**Domains**, **Boundaries**, **Edges**, or **Points**) to the **Selection** box. Click the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required.

ADDING PREDEFINED MATERIAL PROPERTIES

1 Add a material to the model.

2 In the **Model Builder**, click the material node to open the **Material** page.

Note: Review the default properties listed in the **Material Contents** table before adding new material properties.

---

3 Browse the **Material Properties** section to locate the property to add to the model. For example, right-click **Loss Tangent** under **Electromagnetic Models** and select **Add to Material** or click the **Add** button ( ).

See Figure 6-6. In this example, a **Loss Tangent** node is added to the **Model Builder**, and its associated properties are added to the **Material Contents** table. Any properties requiring **Values** are highlighted by red stop signs ( ) in the left-hand column.

4 Scroll through the **Material Contents** table to locate properties with a stop sign ( ) in the column to indicate it is missing data. Enter a **Value** into the table next to the applicable property. In the example, the **Loss tangent** and **Relative permittivity**

properties require data. The left column provides visual cues about the status of each property.

- A stop sign ( ) indicates that an entry in the **Value** column is required. It means that the material is used but undefined.
- A warning sign ( ) indicates that the material parameters are undefined and unused. An entry is only required if the material is used.
- A green check mark ( ) indicates that the property has a **Value** and is currently being used in the physics of the model.
- Any entry made on the **Material** page is also added to the **Property Group** page under **Output properties** - **Expression** column. Click the **Property Group** node (for example, **Loss tangent**) to open this page.

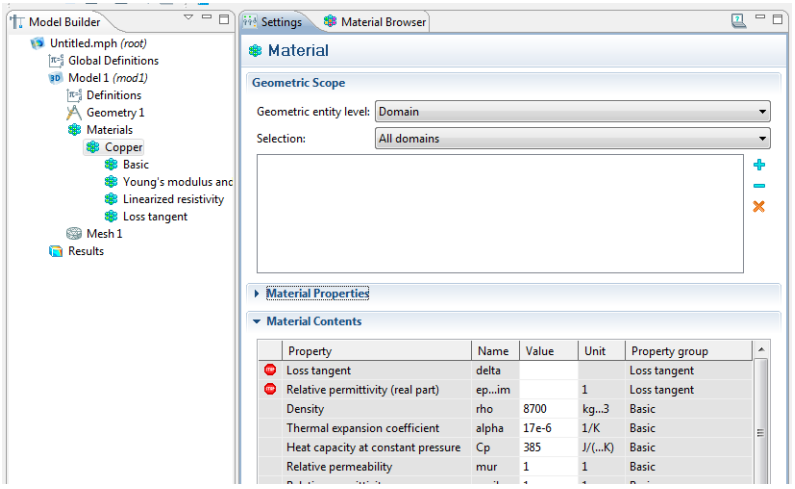


Figure 6-6: Adding a material property to a model. Two properties require data.

Note: To delete a property group you have added, right-click the property group node (in the **Model Builder**) and select **Delete**. The **Basic** property group cannot be deleted.

ADDING OUTPUT PROPERTIES TO PREDEFINED MATERIALS

Note: **Output Properties** is only accessible from the **Basic** material properties and with user defined property groups.

1 Add a material to the model.

2 In the **Model Builder**, click **Basic** to open the **Property Group** page. See Figure 6-4.

The **Property Group** tables are linked to other pages related to the selected material. Use the **Move Up** ( ), **Move Down** ( ), and **Delete** ( ) buttons to organize the tables as required.

3 Under **Output Properties and Model Inputs**, in the **Quantities** section:

a Browse the **Output Properties** list to locate the property to add. See About the Output Materials Properties for a list.

b Right-click the property to **Add** items to the **Output properties** table or click the **Add** button ( ).

c In the **Output properties** table, edit the **Expression** column or field.

Use the **Move Up** ( ), **Move Down** ( ), and **Delete** ( ) buttons to organize the table as required.

ADDING MODEL INPUTS TO PREDEFINED MATERIALS

1 Add a material to the model.

2 In the **Model Builder**, click the property group node (for example, **Basic**) to open the **Property Group** page.

The **Property Group** page tables display on other pages pertaining to the selected material.

3 Browse the **Model Inputs** list. Right-click the property to **Add** items to the **Model inputs** table. Or click the **Add** button ( ). See Model Input Properties for a list.

ADDING LOCAL PARAMETERS TO PREDEFINED MATERIALS

1 Add a material to the model.

2 In the **Model Builder**, click the property group node (for example, **Basic**) to open the **Property Group** page.

3 Under **Local Parameters**, enter **Parameters** and **Expressions** as required.

Use the **Move Up** ( ), **Move Down** ( ), and **Delete** () buttons to organize the table as required.

Note: Also see Adding a User-Defined Property Group.

Adding a User-Defined Property Group

1 Open or create a model file and add a material (predefined or user defined) to the model.

2 In the **Model Builder**, right-click any material node (for example, **Copper**) and select **User defined property group**.

A **User defined property group** node is added to the **Model Builder**, and a **Property Group** page opens in the **Settings** window.

3 Follow the instructions to define the property group—see Working on the Property Group Page.

User-Defined Materials and Libraries

User defined materials provide the flexibility needed to design your model and experiments using a combination of existing material properties or defining it yourself. You can also create your own materials database (library) to include materials you use often.

Adding User defined Materials

1 Open or create a model file.

2 In the **Model Builder**, right-click **Materials** and select **Material**.

A **Material** node is added to the **Model Builder**, and an undefined **Material** page opens in the **Settings** window.

3 Follow the instructions as required to define your material:

a Working on the Material Page

b Working on the Property Group Page

c Adding a User-Defined Property Group

d Adding a Function to the Material

RENAMING MODEL BUILDER NODES

If required, edit or add comments to the **Node Properties**.

1 In the **Model Builder**, right-click any node and select **Properties**.

2 On the **Node Properties** page, enter the **Name**, **Author**, **Version**, or **Comments** about the node. The **Tag** and **Date created** fields cannot be changed.

The node's **Name** can be edited by right-clicking directly on the node and selecting **Rename**.

Adding an External Material Library

1 In the **Material Browser**, under **Materials**, right-click any database and select **Add Material Library**.

The **Choose Material Library** window opens.

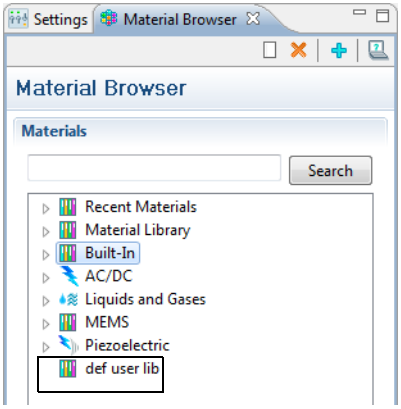
2 Navigate to a material library file and click **Save**. For example, MatWeb provides a service where you can export technical datasheets from MatWeb's collection in the

format for a COMSOL material library. For more information about this service, visit www.matweb.com.

Creating a User defined Materials Database

Note: Because there is a separately purchased **Materials Library**, the user defined "library" is referred to as a database instead.

When you first open the **Material Browser**, an empty material database (library) is available for you to start creating your own library of materials.



Adding a User defined Material Database to the Material Browser

If this empty database (library) is removed from this list, you can add it back to the **Material Browser**.

1. Open the **Material Browser**.
2. Right-click anywhere in the **Materials** section and select **Add Material Library**.
3. Navigate to the folder where the empty database is located. This varies based on your installation, but search for the file name *def_user_lib.mph*.
4. Click **Save**.
5. Right-click the database to rename it to something meaningful to you.
6. Repeat these steps to add additional user defined material databases. Rename each one as required.

RENAMING A USER DEFINED MATERIAL LIBRARY

Right click to rename the material database to something meaningful to you.

Creating a User defined Material Database

1. Open the **Material Browser**.
2. Right-click anywhere in the **Materials** section and select **Add Material Library**.
3. Navigate to the folder where the empty database is located. This varies based on your installation, but search for the file name *def_user_lib.mph*.
4. Click **Save**.
5. Right-click the database to rename it to something meaningful to you.

You can now add predefined materials or create your own material to add to the user defined database.

ADDING A USER DEFINED MATERIAL

1. In the **Model Builder**, right-click **Materials** and select **Material**.

   A **Material** node is added to the **Model Builder**, and an undefined **Material** page opens in the **Settings** window.
2. Follow the instructions as required to define your material:
   a. Working on the Material Page
   b. Working on the Property Group Page
   c. Adding a User-Defined Property Group
   d. Adding a Function to the Material
3. When you are done, right-click the new material and select **Add Material to Library**.

ADDING A PREDEFINED MATERIAL

1. Open the **Material Browser** and add any predefined material to the **Model Builder**.
2. Right click the material node and select **Add Material to Library**.
3. Right-click to **Rename Selected** material that has been added to your library.

REMOVING A MATERIAL FROM A USER DEFINED DATABASE

Open the **Material Browse**. Locate the material to remove. Right-click the material and select **Remove Selected**.

Material Properties Reference

You have access to the material properties for the predefined materials in most physics interfaces. Using this information you can either create your own material property group or define a completely new material.

On the **Basic>Property Group** page, add **Output properties** from the **Quantities** subsection. For all **Property Group** pages, you can add **Model inputs**.

About the Output Materials Properties

Note: Some of these properties are only applicable to physics interfaces in the add-on modules.

BASIC MATERIAL PROPERTIES

These material properties can be added to models from two pages— **Material** and **Property Group**. In both cases the properties belong to the **Basic** property group.

- When you access this information from the **Basic>Property Group** page, it is listed under **Quantities>Output Properties** and **Variable** is listed in the table.
- When you access this information from the **Material** page, it is listed under **Material Properties>Basic Properties** and **Name** is listed in the tables.

PROPERTY	NAME/VARIABLE	UNIT
Characteristic Acoustic Impedance	Z	Pa*s/m
Bulk Modulus	K	Pa
Compliance	C	1/Pa
Compressibility	chif	1/Pa
Piezoelectric Coupling d	d	C/N
Piezoelectric Coupling e	e	C/m^2
Density	rho	kg/m^3
Elevation	D	m^2/s
Elasticity	cE	Pa
Electric Conductivity	sigma	S/m

PROPERTY	NAME/VARIABLE	UNIT
Electron Mobility	mue	$m^2/(Vs)$
Surface Emissivity	epsilon rad	1
Heat Capacity at Constant Pressure	Cp	J/(kgK)
Permeability	kappa	m^2
Isotropic Structural Loss Factor	eta s	1
Mean Molar Mass	Mn	kg/mol
Poisson's Ratio	nu	1
Ratio of Specific Heats	gamma	1
Relative Permeability	mur	1
Relative Permittivity	epsilonr	1
Resistivity	res	Ω m
Shear Modulus	G	Pa
Speed of Sound	cp	m/s
Storage	S	1/Pa
Thermal Conductivity	k	W/(mK)
Coefficient of Thermal Expansion	alpha	1/K
Dynamic Viscosity	mu	(Pa)s
Young's Modulus	E	Pa

ELECTROMAGNETIC MODELS

These material property groups (including all associated properties) can be added to models from the **Material** page.

PROPERTY GROUP AND PROPERTY	NAME/VARIABLE	UNIT
LINEARIZED RESISTIVITY		
Reference resistivity	rho0	Ω m
Resistivity temperature coefficient	alpha	-
Reference temperature	Tref	K
LOSS TANGENT		
Loss tangent	delta	-
Relative permittivity (real part)	epsilonPrim	1
Electric conductivity	sigma	S/m
DIELECTRIC LOSSES		
Relative permittivity (imaginary part)	epsilonBis	1

PROPERTY GROUP AND PROPERTY	NAME/VARIABLE	UNIT
Relative permittivity (real part)	epsilonPrim	1
REFRACTIVE INDEX		
Refractive index	n	1
Refractive index, imaginary part	ki	-
MAGNETIC LOSSES		
Relative permeability (real part)	murPrim	-
Relative permeability (imaginary part)	murBis	-
HB CURVE		
Local Parameters	normB	-
BH CURVE		
Local Parameters	normH	-

SOLID MECHANICS

These material property groups (including all associated properties) can be added to models from the **Material** page.

PROPERTY GROUP AND PROPERTY	NAME/VARIABLE	UNIT
LINEAR ELASTIC MATERIAL MODEL		
YOUNG'S MODULUS AND POISSON'S RATIO		
Young's Modulus	E	Pa
Poisson's Ratio	nu	1
LAMÉ CONSTANTS		
Lamé Constant	lambLame	Pa
Lamé Constant	muLame	Pa
BULK MODULUS AND SHEAR MODULUS		
Bulk Modulus	K	Pa
Shear Modulus	G	Pa
PRESSURE-WAVE AND SHEAR-WAVE SPEEDS		
Pressure-wave Speed	cl	m/s
Shear-Wave Speed	ct	m/s
ORTHOTROPIC		
Young's Modulus	Evector	Pa
Poisson's Ratio	nuvector	1
Shear Modulus	Gvector	Pa
Loss factor for orthotropic Young's modulus	eta_Evector	Pa

PROPERTY GROUP AND PROPERTY	NAME/VARIABLE	UNIT
Loss factor for orthotropic shear modulus	eta_Gvector	Pa
ANISOTROPIC		
Elasticity matrix	D	Pa
Loss factor for elasticity matrix D	eta_D	Pa
ELASTOPLASTIC MATERIAL MODEL		
Yield stress level	sigmags	Pa
Kinematic tangent modulus	Ek	Pa
Isotropic tangent modulus	Et	Pa
VISCOELASTIC MATERIAL MODEL		
Long-term shear modulus	Gv	Pa
Bulk modulus	K	Pa
HYPERELASTIC MATERIAL MODEL		
NEO-HOOKEAN		
Lamé Constant	mu	Pa
Lamé Constant	lambda	Pa
MOONEY-RIVLIN		
Model parameters	C10	Pa
Model parameters	C01	Pa
MURNAGHAN		
Murnaghan third-order elastic moduli	l	Pa
Murnaghan third-order elastic moduli	m	Pa
Murnaghan third-order elastic moduli	n	Pa
Lamé Constant	lambLame	Pa
Lamé Constant	muLame	Pa

PIEZOELECTRIC MODELS

These material property groups (including all associated properties) can be added to models from the **Material** page.

PROPERTY GROUP AND PROPERTY	NAME/VARIABLE	UNIT
STRESS-CHARGE FORM		
Elasticity matrix cE	cE	Pa
Coupling matrix	eES	C/m^2
Relative permittivity	epsilonrS	1
STRAIN-CHARGE FORM		

PROPERTY GROUP AND PROPERTY	NAME/VARIABLE	UNIT
Compliance matrix sE	sE	1/Pa
Coupling matrix	dET	C/N
Relative permittivity	epsilonrT	1

GAS MODELS

This material property group (including all associated properties) can be added to models from the **Material** page.

PROPERTY GROUP AND PROPERTY	NAME/VARIABLE	
IDEAL GAS		
Heat capacity at constant pressure	Cp	J/(kg*K)
Ratio of specific heats	gamma	1
Mean molar mass	Mn	kg/mol
Specific gas constant	Rs	J/(kg*K)

Model Input Properties

The following input properties can be added to models from the **Property** page.

PROPERTY	NAME/VARIABLE
Temperature	T
Absolute Pressure	pA
Electric Field	{E1, E2, E3}
Magnetic Field	{H1, H2, H3}
Magnetic Flux Density	{B1, B2, B3}
Velocity Field	{u1, u2, u3}
Frequency	freq
Stress	{F1, F2, F3}

Predefined Built-In Materials for all COMSOL Modules

The **Built-In** database is available for all users and has data for these materials:

- Air
- Acrylic plastic
- Alumina

- Aluminum 3003-H18
- Aluminum 6063-T83
- Aluminum
- American red oak
- Beryllium copper UNS C17200
- Brick
- Cast iron
- Concrete
- Copper
- FR4 (Circuit Board)
- Glass (quartz)
- Granite
- High-strength alloy steel
- Iron
- Magnesium AZ31B
- Mica
- Molybdenum
- Nimonic alloy 90
- Nylon
- Lead Zirconate Titanate (PZT-5H)
- Silica glass
- Silicon
- Polysilicon
- Solder, 60Sn-40Pb
- Steel AISI 4340
- Structural steel
- Titanium beta-21S
- Tungsten
- Water, liquid

Material Property Groups Descriptions

The following material property groups are available for the entire COMSOL Multiphysics product line (some groups are only applicable to physics interfaces in some of the add-on modules):

Basic: Contains over 25 basic material properties for use with all materials. See About the Output Materials Properties for a complete list.

Linearized Resistivity: For electromagnetic models, to define the electric resistivity (and conductivity) as a linear function of temperature.

Loss Tangent: For electromagnetic models, allows you to specify a loss tangent for dielectric losses or take it from the material. Assumes zero conductivity.

Dielectric Losses: For electromagnetic models, allows you to specify the real and imaginary parts of the relative permittivity or take them from the material.

Refractive Index: For electromagnetic models, allows you to specify the real and imaginary parts of the refractive index or take them from the material. Note that this assumes a relative permeability of unity and zero conductivity.

Magnetic Losses: For electromagnetic models, allows you to specify the real and imaginary parts of the relative permeability or take them from the material.

HB Curve: For electromagnetic models, allows you to use a curve that relates magnetic flux density B and the magnetic field H.

BH Curve: For electromagnetic models, allows you to use a curve that relates magnetic flux density B and the magnetic field H.

Young's Modulus and Poisson's Ratio: In solid mechanics models, Young's Modulus defines the modulus of elasticity and is the spring stiffness in Hooke's law. Poisson's Ratio defines the normal strain in the perpendicular direction, generated from a normal strain in the other direction.

Lamé Constants: In solid mechanics models, these are linear elastic material property constants.

Bulk Modulus and Shear Modulus: In solid mechanics models, a linear elastic material property where the bulk modulus is a measure of the solid's resistance to volume

changes. The shear modulus is a measure of the solid's resistance to shear deformations.

Pressure-Wave and Shear-Wave Speeds: In solid mechanics models, a linear elastic material property where you specify the pressure-wave speed (longitudinal wave speed) and the shear-wave speed (transverse wave speed).

Orthotropic: In solid mechanics models, a linear elastic material property that has different properties in orthogonal directions; its stiffness depends on those properties.

Anisotropic: In solid mechanics models, a linear elastic material property that has different material properties in different directions; the stiffness comes from the symmetric elasticity matrix.

Elastoplastic Material Model: In solid mechanics models, an elastoplastic property group adds these equations: yield stress level, kinematic tangent modulus, and isotropic tangent modulus.

Viscoelastic Material Model: In solid mechanics models, viscoelastic materials are those that exhibit both elastic and viscous behavior when they deform. This property group adds the long-term shear modulus and bulk modulus equations.

Hyperelastic Material Model: In hyperelastic materials the stresses are computed from a strain energy density function. They are often used to model rubber, but also used in acoustoelasticity.

Neo-Hookean: In solid mechanics, a hyperelastic material property with two model parameters. The model is based on modified strain invariants.

Mooney-Rivlin: In solid mechanics, a hyperelastic material property with three model parameters. The model is based on modified strain invariants.

Murnaghan: In solid mechanics, a hyperelastic material property with five model parameters. The model is based on modified strain invariants and is typically used in acoustoelasticity.

Stress-Charge Form: In Piezoelectric models, used to express the constitutive relations using the elasticity matrix, coupling matrix, and relative permittivity equations.

Strain-Charge Form: In Piezoelectric models, used to express the constitutive relations using the compliance matrix, coupling matrix, and relative permittivity equations.

Ideal Gas: In gas models, this property group adds the heat capacity at constant pressure, ratio of specific heats, mean molar mass, and specific gas constant equations.

Other Predefined Materials in COMSOL Modules

- See the AC/DC Module documentation for list of common AC/DC materials.
- See the Acoustics Module or MEMS Module documentation for a list of Liquids and Gases, MEMS, and Piezoelectric materials.
- See the Chemical Reaction Engineering Module, Earth Science Module, or Heat Transfer Module documentation for a list of Liquids and Gases materials.
- See the Structural Mechanics Module documentation for a list of MEMS and Piezoelectric materials.

Using Functions

Adding a Function to the Material

Material functions are either automatically added to the **Model Builder** sequence (usually with materials from the **Model Library**) or you can add functions based on your own requirements:

1. Add a material to the model.
2. In the **Model Builder**, right-click a property group node, for example, **Basic**.
3. Select one of the following from the **Functions** list:
   - Select **Analytic** to extend properties with the ability to bind parameters during function calls. See Example of Defining an Analytic Function. Also see Analytic.
   - Select **Interpolation** to interpolate from data that can be both structured (defined on a grid) or unstructured (defined on a generic point cloud). See Interpolation.
   - Select **Piecewise** if there is one material property that has different definitions on different intervals.The intervals must not overlap, and there cannot be any holes between intervals. See Piecewise.

Note: Once a function is created, it can be used for any property in the same property group.

Example of Defining an Analytic Function

Assume that you want to define Young's modulus for a material as a function of pressure and temperature. You can name the function `young(p,T)` and use the expression `2e10+1e6*p-1e8*T` to define the function.

1. Open the **Material Browser**.
2. **Search** or browse for a material to add to the model.
   a. In the **Material Library**, open the **Cast Irons & Mold materials>Cast Irons** folder.
   b. From the list, right-click **A297 HI (UNS J94003)** to **Add Material to Model**.

   The material is added to the **Model Builder** with a **Basic** property group containing Piecewise functions. Observe that Young's Modulus is not automatically added.

3 Add a **Young's Modulus** property to the material.

a In the **Model Builder**, click the material node (in this example, **A297 HI**).

b On the **Material** page, under **Material Properties>Basic Properties**, right-click **Young's Modulus** and **Add to Material**.

A **Young's modulus** property is added to the undefined properties table in the **Basic** property group.

4 In the **Model Builder**, right-click **Basic** and select **Functions>Analytic**.

5 On the **Analytic** page, enter `young` in **Function Name**.

6 Under **Parameters**:

a In the **Expression** field enter `2e10+1e6*p-1e8*T`.

b In the **Arguments** field enter `p,T`.

c Select **Manual** from the **Derivatives** list.

d In the **Argument** column, enter `p,T`.

e In the **Partial derivative** column, enter `1e6,-1e8`.

The function `young` can now be used to define the Young's modulus in your material.

COMSOL Multiphysics must know the derivative of a function if you are using a variable that depends on the solution in a function argument. This is why in this example you use a **Manual** derivative. You can use the `d` operator to compute derivatives of a function.

If you use **Auto** while defining the **Derivatives** parameter, COMSOL Multiphysics uses the `d` operator on the expression to get the derivatives. In this case, under **Advanced**, select the **May produce complex output for real arguments** check box if the defined function works similarly to `sqrt`, that is, if it sometimes returns complex values for real-valued input.

7 Click the material node (in this example, **A297 HI**). On the **Material** page, under **Material Contents**, enter `young(p[1/Pa],T[1/K])[Pa]` in the **Value** column (in the Young's modulus row).

Click the **Basic** node to observe that the **Young's modulus** analytic function is defined on the **Property Group** page. See Figure 6-7.

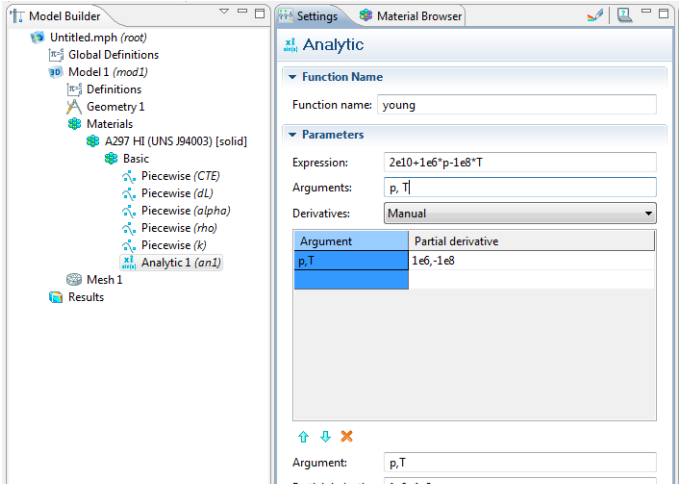


Figure 6-7: Adding a Young's Modulus property as an Analytic function.

7

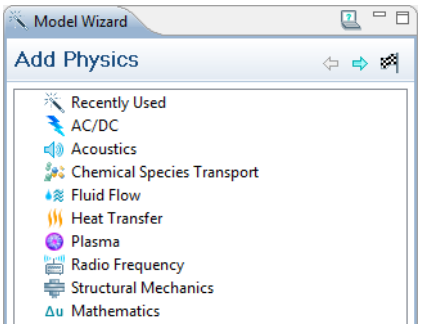
Building a COMSOL Model

This section explains a range of methods and topics used when building models in COMSOL Multiphysics: From working with the Model Wizard and fundamental concepts for the Model Builder to the use of units.

In this section:

The Model Wizard and Model Builder

When you open COMSOL to create a model, the **Model Wizard** opens by default to guide you through selecting the space dimension, physics interfaces, and study type.



Every physics interface is easy to use and contains predefined physics descriptions and equations for a variety of engineering and scientific disciplines, from **Acoustics** to **Structural Mechanics**. You can also define your own physics interface from the beginning using the options under **Mathematics**. The last step of the **Model Wizard** is to select a **Study Type**—**Stationary**, **Time Dependent**, **Eigenfrequency**, or some other study type.

You can create a new model or add physics at any time. Right-click the **Root** (top) node and select **Add Model** to open the **Model Wizard**, or right-click a **Model** node and select **Add Physics**.

After clicking the **Finish** button ( ) in the **Model Wizard**, the **Model Builder** window displays a model tree with a set of default nodes—**Definitions**, **Geometry**, **Materials**, **Mesh**, and nodes based on the physics interfaces selected (see Figure 7-1). The **Model** nodes and branches form the sequence of operations that define your model.

The Model Nodes in the Model Builder

The **Model** nodes define the scope for all parts of a model, and all these nodes together form the input for a model. The position of the nodes in the vertical direction represent the order of execution of operations. Model files can have many **Model** nodes. For example, if you are setting up a system model using a 2D simplification for certain components and full 3D description for other components, these can both be added to the **Model Builder** to represent the different model requirements.

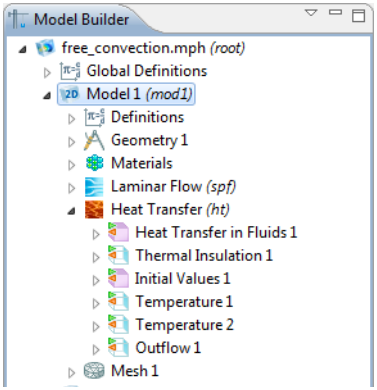


Figure 7-1: An example of the Model Builder default nodes.

These default nodes are normally added under a **Model** node:

- **Definitions**: Contains variables, selections, views, pairs, functions, probes, model couplings, and coordinate systems, that are defined locally for the model. See Global and Local Definitions for information about using these local **Definitions** and **Global Definitions**. Use **Global Definitions** to define **Parameters** and **Variables** with a global scope—that is, not specific to one **Model** node.
- **Geometry**: Contains the sequence of geometric objects and operations (or imported CAD data) that defines the model geometry. You can also draw your own geometry in the **Graphics** window.
- **Materials**: Contains the materials and material properties used as sources for material data in the model. See Materials for detailed information.
- **Physics**: Any added physics or mathematical interface displays as a node under **Model** (**Heat Transfer** in Figure 7-1 for example).
- **Meshes**: Contain the sequences of mesh operations that defines the computational meshes for the model. When there is only one mesh in the model, its **Mesh** node appears directly under the **Model** node.

Adding Nodes to the Model Builder

In the **Model Builder**, right-click any node to open its context menu. Once a node is highlighted, you can right-click anywhere in the **Model Builder** to open the context

menu, which lists all the features available to that particular node on that branch of the tree.

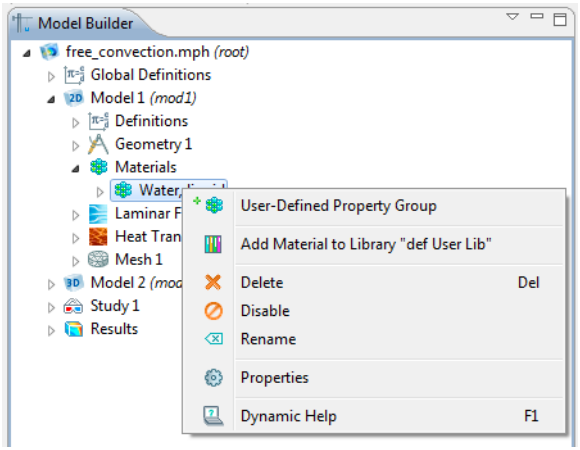


Figure 7-2: A context menu opens when you right-click any node in the Model Builder.

ABOUT THE SEQUENCE OF OPERATIONS

COMSOL Multiphysics evaluates most of the branch nodes in the **Model Builder** from the top-down as a *sequence of operations*. This makes it possible to, for example, to parameterize a model and rerun the simulation. COMSOL then re-evaluates each sequence, updating the geometry, mesh, and solution.

Some nodes under a physics branch can override other nodes higher up in the sequence. How COMSOL Multiphysics treats those nodes depends if they are contributing or exclusive nodes; see Physics Interface Exclusive and Contributing Feature Node Types.

DISABLING AND DELETING NODES

You can change the contents, and actions, of the sequences by disabling and deleting nodes. A disabled node does not take part in the evaluation of a sequence. See Figure 7-2. Not all nodes can be disabled and deleted. When this is the case, the context menu does not have these options available.

- To disable a node, right-click it and select **Disable** ( ). The node is grayed out, and the node name is in brackets. To enable the node, right-click and select **Enable** ( ).
- To delete a node, right-click the node and select **Delete** ( ) or press Del (the Delete key). Confirm the deletion of a node for it to take effect.

Model Administration

A variety of tasks can be done to organize and simplify the model building process. This section includes information about how to:

- Save models in different formats and create a model image associated to the file.
- Edit node properties, names, and identifiers.
- Open the Model Library and change the root folder location.
- Update the model libraries using the Model Library Update service.
- Update preferences including precision level, graphics rendering, the user name, GUI language, and what to display in the **Model Builder**.
- View a list of licenses and block as required.

Saving Models in Different Formats and Creating a Model Image

ABOUT COMSOL MODEL FILE FORMATS

There are three COMSOL Multiphysics model file formats—MPH, Model-M and Model-Java files.

COMSOL MPH Files

The default standard file with the extension `.mph`. The file contains both binary and text data. The mesh and solution data are stored as binary data, while all other information is stored as plain text.

You can quickly save and load MPH-files. All the models in the COMSOL Multiphysics Model Library and the model libraries in the add-on modules are MPH-files.

Model M-Files

Editable script file for MATLAB. Contains a sequence of COMSOL Multiphysics commands. Run the Model M-files in MATLAB or open them in COMSOL Multiphysics when you run it with MATLAB. You can edit the file in a text editor to add additional commands.

Note: Running files in the Model M-file format requires the COMSOL LiveLink for MATLAB.

SAVING A COMSOL MULTIPHYSICS MODEL

If this is the first time saving a model or you want to update the file and keep the current name and format:

1. Click the **Save** button (), press Ctrl+S, or from the **File** menu, select **Save**.
2. Navigate to a the location where you want to save the model and enter a **File name.**
3. Click **Save**. Also see Changing the Model Library Root Folder to learn how to build your own model library.

If the model has been saved before and you want to save it in a different format or create a copy:

1. From the **File** menu, select **Save As**.
2. In the **Save as type** list, select the file type that you want to use.
3. Navigate to a the location where you want to save the model and enter a **File name**.
4. Click **Save**. Also see Changing the Model Library Root Folder to learn how to build your own model library.

SAVING AS A MODEL M-FILE

1. From the **File** menu, select **Save As Model M-File**.
2. Navigate to a the location where you want to save the model and enter a **File name**.
3. Click **Save**. Also see Changing the Model Library Root Folder to learn how to build your own model library.

SAVING ON A SERVER COMPUTER

When running a COMSOL Multiphysics server on a remote computer you can save the Model MPH-file to that server.

1. From the **File** menu, select **Connect to Server**.
2. Enter your **Server** and **User** details and click **OK**.
3. In the window that opens, enter the full path to the file you want to save. Provide the path in the server computer's file system. Click **Save**.

SAVING A MODEL IMAGE

To illustrate the model you can save a model image that displays in the **Root Node Properties** and when opening a model in the **Model Library**. To save the current COMSOL Multiphysics plot as a model image, from the **File** main menu, select **Save Model Image**. The model image is a copy of the current plot.

REVERTING TO THE LAST SAVED MODEL FILE

1 From the **File** menu, select **Revert to Saved**.

2 Select **Yes** to continue, and **No** to cancel.

COMSOL opens the last saved version of the file and reinitializes the GUI.

Editing Node Properties, Names, and Identifiers

All nodes, except container nodes, have a common set of node properties, some of which can be changed and some system generated properties that cannot be edited. Also see Viewing Node Names, Identifiers, Types, Tags, and Equations to learn about how some of these properties can be viewed. The **Root** node has additional information that provides an overview of the complete model file.

EDITING A MODEL IDENTIFIER FOR USE WITH VARIABLES

Use a model **Identifier** to access variables throughout the model. The identifier is part of the full reference to variables (for example, when referring to variables in another model); see Variable Naming Convention and Scope for more information.

Note: To edit the model **Name**, see Editing Node Properties.

- To edit a model identifier, in the **Model Builder**, click a **Model** node. The **Model** page opens in the **Settings** window. Edit the default identifiers (**mod1**, **mod2**, and so on) as required in the **Identifier** field. See Figure 7-3 for an example.

RENAMING A NODE

- To rename a node, in the **Model Builder**, click a node and select **Rename** or press F2. Enter a **New name** and click **OK**. The **Name** is both updated in the **Model Builder** and in the **Node Properties** section.

EDITING NODE PROPERTIES

Opening the Properties Window

In the **Model Builder**, right-click a node and select **Properties** from the context menu (see Figure 7-2). The **Properties** window for that specific node replaces the **Settings** window.

Node Properties

This section contains these fields:

- The **Name** field defaults to a system name for the node. Edit as required.
- The **Tag** field has a unique system defined tag for the node, which cannot be changed.
- The **Author** field contains the name of the author (creator) of the node. In addition to editing the author name manually, you can make a change for all nodes that you add later on by selecting **Options>Preferences** from the main menu and then select **User** (see Editing Model Preferences Settings) and edit the username.
- The **Date created** field contains the node creation date and time. This field cannot be edited.
- The **Version** and **Comments** fields are empty by default. Enter version numbers or comments to track model changes or changes to specific node contents.

Used Products

Under **Used Products**, any COMSOL module that is used by that node is listed.

Returning to the Settings Window

When you are finished editing the properties, right-click the node again and select **Settings** to return to the **Settings** window.

ROOT NODE PROPERTIES

The **Settings** window for the root node includes a **Node Properties** section with additional information about the model file. It also incudes the **Used Products** section and an additional section, **Model Image**.

Opening the Root Node Properties Window

To open the root node's **Settings** window with the **Node Properties** section, click the top **Root** node in the **Model Builder**.

The following information displays in the **Settings** window:

Node Properties

- The **Name** field displays the file name as defined by the user. This field cannot be edited, but if the file name is changed, the new name displays here.
- The **Path** field displays the full path to the current MPH-file. When a new project is started, the field is empty until the MPH-file is saved for the first time. This field cannot be edited, but if the file path changes, the new path displays here.
- The **Program** field displays the name and build number for the version of COMSOL Multiphysics used. This field cannot be edited.
- The **Author** field contains the name of the author (creator) of the model (see above). Click the **Reset Author in Model** button to reset the author name for all nodes in the model to the name in the **Author** field for the root node.
- The **Tag** field has a unique system defined tag for the node. This field cannot be edited.
- The **Date created** field contains the node creation date and time. This field cannot be edited.
- The **Date modified** field contains the date and time for the last time when the MPH-file was saved. This field cannot be edited.
- The **Modified by** field contains the name of the user who most recently saved the MPH-file. Changing the author name in the **Author** field and then clicking the **Reset Author in Model** button also resets the name in the **Modified by** field.
- The **Version** and **Comments** fields are empty by default. Enter version numbers or comments to track model changes or changes to specific file contents.

Used Products

Under **Used Products**, any COMSOL module that is used by the file is listed.

Model Image

Under **Model Image**, and only after some results are plotted, an image displays representing the model. See Saving a Model Image for more information.

The Model Library

The Model Library contains sets of models that can be used for a variety of purposes. Each add-on module includes its own model library with models showing how to use the module within its application area. This section explains how to open the **Model Library**, search for a model, and how to change the model root directory location.

OPENING AND SEARCHING THE MODEL LIBRARY

1 From the **File** main menu, select **Open Model Library**.

The **Model Library** window.

2 Browse the available libraries or enter all or part of the model name and click **Search**.

Note: COMSOL models are named using an underscore between words, for example, *effective_diffusivity*. The underscore is required to form a valid filename so it is recommended that, if you are not sure of the full name, you enter only the first word in the **Search** field.

- If the search is successful, double-click the model file name or the **Open** button to open the file.
- If the search does not return any results, click the **Refresh** button () return to the root **Model Library** folder list.

CHANGING THE MODEL LIBRARY ROOT FOLDER

After developing and saving your own models, it may be useful to change the **Model Library** root folder. This redirects COMSOL to a different folder where customized models can be stored.

1 From the **File** menu, choose **Open Model Library**.

The **Model Library** window opens.

2 Click the **Set Model Library Root** button ().

3 In the **Model Library Root** dialog box, navigate to the new root folder location or click **Make New Folder**.

4 Click **OK** to save the changes, or **Cancel** to exit without saving.

Updating Model Libraries Using Model Library Update

The Model Library Update is a service that provides new and updated model for the model libraries. To use the Model Library Update, use the **Model Library Update** window (), which contains new and updated models for the model libraries of the COMSOL products that your license includes:

1 From the **Options** menu, choose **Model Library Update**.

2 In the **Model Library Update** window, browse the list of new and updated models that appear with a description and image.

3 Select the models that you want to download by selecting the check box next to the model image.

4 Click the **Download** button at the bottom of the **Model Library Update** window.

After downloading new and updated models, click the **Refresh** button ( ) in the **Model Library** window to update the lists of models in the model libraries.

Editing Model Preferences Settings

To open the model **Preferences** settings, select **Options>Preferences** from the Main menu. The following settings can be edited as required.

GENERAL

- Under **Precision**, edit the **Input** or **Table display precision** levels.
- Select a GUI **Language**—**Chinese (Simplified)**, **Chinese (Traditional)**, **English** (the default), **French**, **German**, **Italian**, **Japanese**, **Korean**, or **Spanish**. Click **OK** to exit and re-open COMSOL in order to display the GUI in the selected language.
- Under **Help**, select the **Preload help files** check box to preload the help files when launching COMSOL Multiphysics. By preloading the help files the context help opens faster when you use it for the first time.

USER

In the **User** section, edit the **Username**.

GRAPHICS

In the **Graphics** section, under **Rendering**:

- Select a **3D rendering** format—**OpenGL** (the default), **DirectX 9**, or **Software**.
- Select a **2D rendering** format—**Swing** (the default) or **SWT**.
- Select a **Detail**—**Normal** (the default), **Wireframe**, **Coarse**, or **Fine**. This controls the level of graphics rendering detail. Select **Wireframe** speed up rendering of complex models or to improve visual appearance.

In the **Graphics** section, under **Animation**, select a **Codec**—**MJPEG Compressor** (the default) or **DV Video Encoder**.

MODEL BUILDER

- Select the **Show equation view** check box to always display physics interface **Equation** nodes that display the definitions of the underlying equations and variables.
- Select the **Show more options** check box to always activate the **Show More Options** setting, which enables some additional options such as some more advances sections in the **Settings** windows and some additional nonstandard options in the physics interface's context menus.

Note: These settings also turn on the same items in the Model Builder's **View** menu. See Viewing Node Names, Identifiers, Types, Tags, and Equations for information.

RESULTS

- The **Update plot when selected** check box in the **Plot** area is selected by default. Click to clear the check box if required.
- The **Buffer size (rows)** in the **Table** area controls the size of the buffer for storing table data. The default value is 10,000.

When you are done editing, click **OK** to save the changes, or **Cancel** to exit without saving.

Checking and Controlling Products and Licenses Used

Open the **Licenses** window to list or block the products your license includes. Blocking a license may be useful for consultants who want to duplicate a client's environment while building a model or when collaborating with other users who do not have access to the same set of COMSOL products. You can also use these settings to prevent use of a module when sharing a floating-network license, for example.

1. To view a list of licenses or block the use of a product, from the main menu, select **Options>Licenses**. The **Licenses** window opens.

   By default the use of all products is active and the check boxes for all products are selected. The licenses currently in use are unavailable (you cannot block the use of products with functionality already in use).

2. Click to clear the check box next to a product to hide or block it from use. Click the **Select All** button to activate all products. Click the **Deselect All** button to block all products (except that ones that are already in use).

Note: The settings are local for each COMSOL Multiphysics session. The program does not store these settings for future modeling sessions.

3 Click **OK** to save the changes or **Cancel** do discard any changes and close the window.

Note: Under **Other products** is a list of COMSOL products the license does not include. Click the **Product Information** button to go to the product information pages on the COMSOL website, where you find information about all COMSOL products.

Viewing Node Names, Identifiers, Types, Tags, and Equations

Select the node labels to display in the **Model Builder**. **Names** and **Identifiers** can be changed by the user. The **Name** of a node can be changed for all levels, but the **Identifier** can only be changed for the top node. The unique **Tag** and **Type** is automatically assigned by the software and cannot be changed.

You can specify the name of a node in its **Properties** section (see Editing Node Properties, Names, and Identifiers) or by right-clicking on the node and choosing **Rename**, which opens a window where you can enter a **New name**. You can also specify the identifier names for nodes that include an identifier.

SELECTING THE LABELS TO VIEW IN THE MODEL BUILDER

1 In the **Model Builder**, click on the top node level where you want to display specific labels. Each node level can display different labels.

2 Click the small down arrow to the left of the **Minimize** button, the **View Menu** button, to select from the list: **Show Name Only**, **Show Name and Identifier**, **Show Name and Tag**, **Show Type and Identifier**, or **Show Type and Tag**. See Figure 7-3 for examples of the different label types. In addition, you can select **Show Equation View** and **Show More Options**.

The

- Select **Show Name Only** to only display the name.
- Select **Show Name and Identifier** to display the name with the identifier in parentheses using an italic font. The identifier appears only where it is defined, that is, for model

nodes, physics interface nodes, mathematical interface nodes, functions, model couplings, and geometry features. This is the default setting.

- Select **Show Name and Tag** to display each node's feature name with the predefined tag in curly braces using an italic font.
- Select **Show Type and Identifier** to display each node's feature type (predefined name) with the identifier in parentheses using an italic font.
- Select **Show Type and Tag** to display each node's feature type (predefined name) with the predefined tag in curly braces using an italic font.

The previous selections are mutually exclusive: only one of them can be active. The following two settings can be activated or turned off individually:

- Select **Show Equation View** to always display physics interface **Equation View** nodes. See Physics Interface Equation View Node.
- Select **Show More Options** to show some advanced sections in the **Settings** windows and some nonstandard options in the context menus for the physics interfaces. You normally do not need to use these additional options other than in some special cases. The documentation indicates when you need to select **Show More Options** to access some settings.

Note: These settings also turn on the same feature in the Model Preferences window. See Editing Model Preferences Settings.

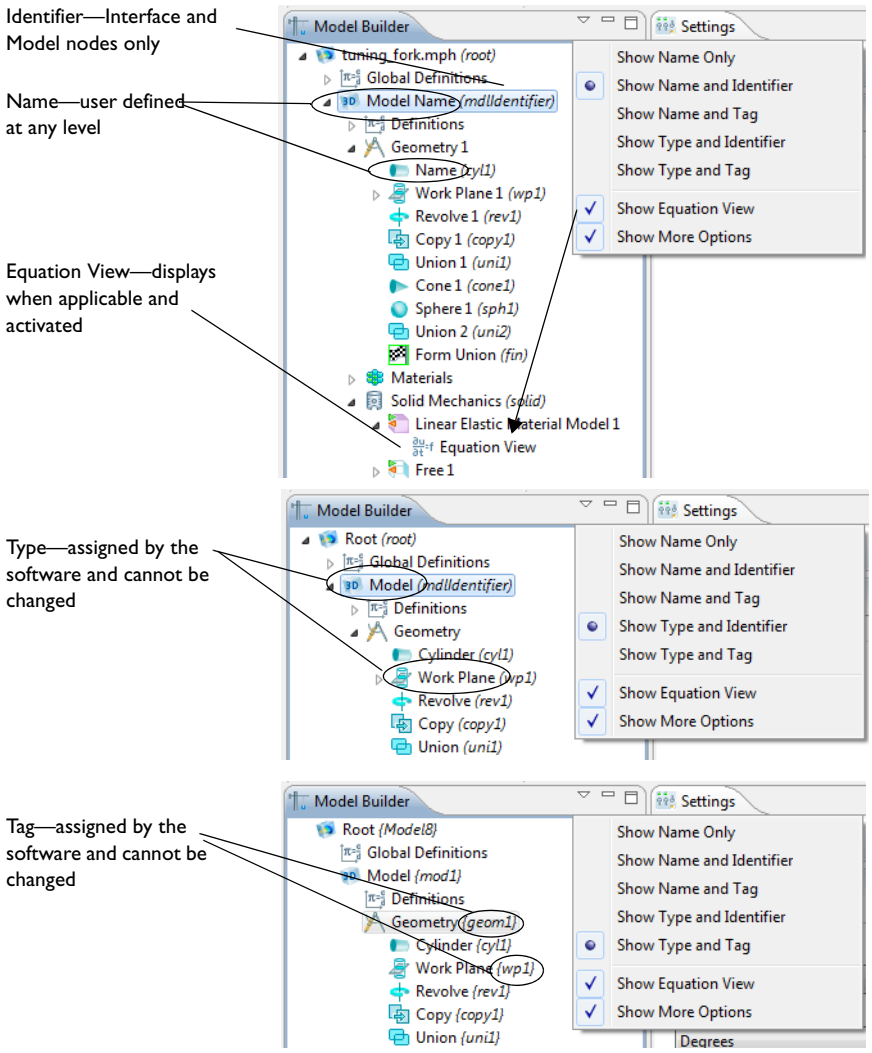


Figure 7-3: Examples of the available label combinations on the View menu.

Show More Options on the Context Menu: Equation Based Features

In the **Model Builder**, when you right-click any physics interface node and open the context menu, there are more features available if you select **Show More Options** from the **View** menu.

WEAK CONTRIBUTION

To display this option, select **Show More Options** from the **View** menu in the **Model Builder** and then select **More>Weak Contribution** or **Global>Weak Contribution** from the context menu. See The Weak Contribution Feature for details.

WEAK CONTRIBUTION ON MESH BOUNDARIES

To display this section, select **Show More Options** from the **View** menu in the **Model Builder** and then select **More>Weak Contribution on Mesh Boundaries** from the context menu.

This feature is similar to **Weak Contribution** but is active on mesh boundaries. The settings are the same as for the **Weak Contribution**. See The Weak Contribution Feature for details.

WEAK CONSTRAINT

To display this section, select **Show More Options** from the **View** menu in the **Model Builder** and then select **More>Weak Constraint** from the context menu. See Using Weak Constraints for details.

POINTWISE CONSTRAINT

To display this section, select **Show More Options** from the **View** menu in the **Model Builder** and then select **More>Pointwise Constraint** from the context menu. The settings in this section are similar to those for the weak constraints. See Using Weak Constraints for details.

GLOBAL EQUATIONS

To display this option, select **Show More Options** from the **View** menu in the **Model Builder** and then select **Global>Global Equations** from the context menu. See Global Equations for more information.

DISCRETIZATION

To display this section, select **Show More Options** from the **View** menu in the **Model Builder** and then select **Global>Discretization** from the context menu. A **Discretization** node is added to the **Model Builder** and you can select the **Element order**. This can be useful as an alternative discretization in, for example, a multigrid hierarchy.

Show More Options: Consistent and Inconsistent Stabilization

Numerical stabilization is available for physics interfaces that model transport such as fluid flow or convective heat transfer, where the fundamental governing equations are less stable than, for example, conduction-dominated models, solid mechanics models, and wave propagation in the frequency domain.

To display this section on a physics interface **Settings** window, select **Show More Options** from the **View** menu in the **Model Builder**.

Several interfaces have these settings available and below is the common information about these settings. Many interfaces have a couple of differences which are noted in the specific interface documentation.

For more information also see these sections in the *COMSOL Multiphysics Reference Guide*: Stabilization Techniques and for a general background on stabilization methods, Numerical Stabilization (or see Where Do I Access the Documentation and Model Library?).

CONSISTENT STABILIZATION

There are two consistent stabilization methods— **Streamline diffusion** and **Crosswind diffusion**. Usually, both check boxes are selected by default and should remain selected for optimal performance. These consistent stabilization methods do not perturb the original transport equation.

The crosswind diffusion method specifies the smallest allowable concentration change across an element. As the concentration gradient appears in the denominator in the equations describing crosswind diffusion, the gradient ensures that unreasonable values do not occur in regions with small to negligible concentration changes.

Crosswind Diffusion and Tuning Parameter

On most interfaces, when the **Crosswind diffusion** check box is selected, enter a **Tuning parameter** C_{k}. The default is most often 0.5. The **Tuning parameter** controls the amount of crosswind diffusion introduced. It recommended that it is kept in the order of 1 in order to not introduce excessive amounts of diffusion. The value used must also neither be space nor time dependent.

Crosswind Diffusion and Lower Gradient Limit

In some interfaces, if the **Crosswind diffusion** check box is selected, the **Lower gradient limit** g_{lim} (SI unit: K/m) field is available. This variable corresponds to the smallest concentration change across an element considered by the stabilization, and is used to

make sure that the crosswind diffusion expressions are valid also in regions with small to negligible concentration changes.

Residual

For some interfaces, and for both consistent stabilization methods, select a **Residual** (or **Equation Residual**). **Approximate residual** is the default setting and it means that derivatives of the diffusion tensor components are neglected. This setting is usually accurate enough and is faster to compute. If required, select **Full residual** instead.

INCONSISTENT STABILIZATION

There is usually just one inconsistent stabilization method— **Isotropic diffusion**. This method is equivalent to adding a term to the diffusion coefficient in order to dampen the effect of oscillations. Use of the inconsistent stabilization method should be minimized if possible because, by using it, you do not solve the original problem.

By default there is no isotropic diffusion selected. If required, select the **Isotropic diffusion** check box and enter a **Tuning parameter** δ_{id}. The default is 0.25. Increase or decrease this number to increase or decrease the amount of isotropic stabilization, and thus oscillation dampening.

Show More Options: Advanced Settings and Discretization

To display this section on a physics interface **Settings** window, select **Show More Options** from the **View** menu in the **Model Builder**.

ADVANCED SETTINGS

Normally these settings do not need to be changed.

Equation Form

Typically, the automatic equation form setting is appropriate. The equation formulation then complies with the analysis type. To specify the equation formulation manually select one of the available forms from the **Equation form** list.

Show All Model Inputs

Select the **Show all model inputs** check box to make all model inputs appear in the **Model Input** section of the settings windows for the main model equation features. Such normally hidden model inputs include field variables in predefined multiphysics interfaces and model inputs from user defined property groups.

Performance Index
Physics interfaces: Heat Transfer in Participating Media and Radiation in Participating Media

There is also a **Performance index** field where you can select a value between 0 and 1 that modifies the strategy used to define automatic solver settings. With small values, you can expect robust settings for the solver. With large values (up to 1), you can expect to need less memory to solve the model.

Circumferential Wave Number (1D and 2D Axisymmetric Models Only)
Physics interfaces: Pressure Acoustics, Boundary Mode Acoustics, Aeroacoustics, Transient Aeroacoustics, Boundary Mode Aeroacoustics.

The circumferential wave number m is an integer-valued number used in axisymmetry and is by default 0. You can change the value in the **Circumferential wave number** field.

Displacements Control Spatial Frame
Physics interfaces: Acoustics-Structure Interaction, Fluid-Structure Interaction, Solid Mechanics, Poroelasticity, Joule Heating and Thermal Expansion, Piezoelectric Devices.

The **Displacements control spatial frame** check box is selected by default. If required, click to clear the check box.

Convective Term
Physics interfaces: Transport of Diluted Species, Nernst-Planck Equations, Solute Transport, Species Transport in Porous Media, Laminar and Turbulent Two-Phase Flow, Level Set, Laminar and Turbulent Two-Phase Flow, Phase Field, Level Set, Phase Field.

Select a **Convective term**—**Non-conservative** or **Conservative**.

Regularization
Physics interface: Transport of Concentrated Species

From the **Regularization** list, select **On** or **Off**.

Out-of-plane Wave Number (Acoustics Module Only)
The out-of-plane wave number k_z is used in 2D and 1D axisymmetry and is by default 0. You can change the value in the **Out-of-plane wave number** field.

Heat Transfer in Biological Tissue
Physics interfaces: Non-Isothermal Flow/Conjugate Heat Transfer, Heat Transfer, Microwave Heating, Thermal Stress.

Select the **Heat Transfer in biological tissue** check box to enable the Biological Tissue feature. When enabled, Biological Tissue becomes available as a default model.

DISCRETIZATION

Specify the order of the shape functions. The default is to use second-order Lagrange elements (except for the Beam interface and all the Plasma interfaces which use first-order). In some interfaces, also select a **Frame type**—**Material** or **Spatial**.

The Physics Interface Feature Nodes

An important part of building a model is where you add **Physics** branches. These contain the nodes that define the material properties, equations, loads, initial values, boundary conditions, and other part of the physics that the model describes. This section describes some common behavior for these nodes:

- About exclusive and contributing feature nodes and how they impact operating sequences.
- Physics interface default feature nodes.
- Visual cues about the status of a physics interface node.
- About interior and exterior boundaries and axial symmetry.
- Common **Advanced** and **Discretization** settings for all physics interface nodes.
- About variables, shape functions, weak expressions, and constraints included in the **Equation** node.

Specifying Physics Settings

Each physics interface provide features for specifying all input data for a specific physics in a model:

- Material properties. Also see Materials.
- Boundary and interface conditions
- Equations (for equation-based modeling)
- Initial conditions

In addition, you can specify weak form contributions and element types for additional flexibility.

Specifically, the settings are available on the following parts of the geometry:

- Domains
- Boundaries
- Edges
- Points
- Additional properties that are independent of the geometry

Not all of these options are available for all geometry types and physics interfaces.

All **Settings** windows for the specification of the physics and equations accept parameters and variables as input data (see Global and Local Definitions for details).

Physics Interface Context Menu Layout

The context menu opens when you right-click on a physics interface node. This menu is divided into four sections for most physics interfaces: the first section contains domain settings, the second boundary settings, the third edge settings, and the fourth has point settings.

It is important to remember this division as sometimes there are menu items with the same name, but applied to different geometric entities.

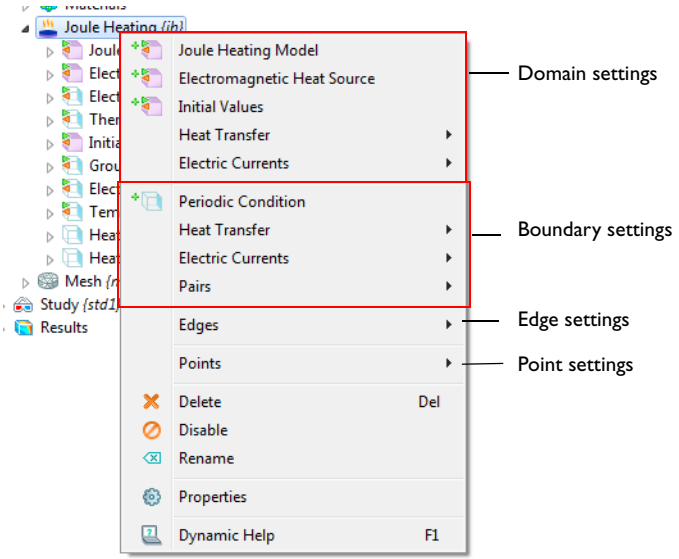


Figure 7-4: The physics interface context menu layout.

Physics Interface Exclusive and Contributing Feature Node Types

The nodes for the physics interfaces are in a sequence, which acts like a macro that the software runs in a top-down order. Depending on the selection for each node, a node can totally or partially overlap, or shadow, a node earlier in the sequence. How the

software treats these nodes depends on their relationship. There are two different types of nodes: exclusive ( ) and contributing ( ).

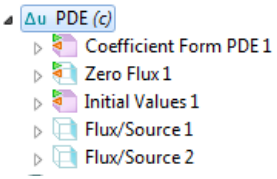


Figure 7-5: An example of exclusive and contributing nodes in a physics interface operating sequence. The Flux/Source nodes are contributing: COMSOL Multiphysics adds the contributions from each of them.

Note: The exclusive and contributing nodes maintain the described behavior only in relation to similar types of nodes within the same physics (for example, you can have a temperature constraint and a pressure constraint for the same boundary in the same model).

EXCLUSIVE NODES

The use of an *exclusive node* means that only one can be active for a given selection. That is, if you add another exclusive node (for example, an identical feature) with the same selection, the other exclusive node is overridden and thus has no effect.

Typical exclusive nodes include model equations, initial values, and boundary conditions that are constraints, such as prescribed values for displacements, temperatures, pressures, and so on. Also some boundary conditions that are not constraints but have a definitive meaning are exclusive nodes—for example, electric insulation, thermal insulation, and no-flow conditions. Any icon with green and red arrows: ( ) that display next to a node indicates that it is an exclusive node (on the boundary level). See Figure 7-5 for an example. Depending on the selections for each node, an exclusive node can shadow another node partially. Nodes are exclusive only within their specific physics.

CONTRIBUTING NODES

A *contributing node* means you can have several of these nodes with the same selection—the software adds these together when evaluating the model. Typical contribution nodes are loads, fluxes, and source terms, where you can have more than one of each type that is active on the same domain or boundary, for example.

COMSOL Multiphysics adds all contributing nodes to the model. Icons with no arrows () displaying next to a node indicate that it is a contributing node (on the boundary level). See Figure 7-5 for an example.

Physics Interface Default Feature Nodes

When you add a physics interface, it automatically adds a main physics interface branch, which typically includes a number of default feature nodes, including but not limited to:

- A model equation or material model node, typically on the domain level. This node defines the domain equations (except optional sources, loads, reactions, and similar contributing domain quantities) and the related material properties or coefficients.
- A boundary condition node. For multiphysics interfaces there is one boundary condition for each physics.
- For axisymmetric models, the symmetry axis has an **Axial Symmetry** boundary condition (see Physics Interface Axial Symmetry Feature Node).
- An **Initial Values** node.

The default nodes' initial selections include all domains or all boundaries (or all instances of another geometric entity level). Their selection is always every instance that has no selection defined by another node on the same geometric entity level that is not contributing to the default node.

For example, for a geometry with four boundaries, the default boundary condition's initial selection includes all four boundaries. If another exclusive boundary condition for Boundary 3 is added, that boundary becomes overridden (inactive) in the default boundary condition's selection. If you disable or remove that boundary condition (see Disabling and Deleting Nodes), the default boundary condition becomes active for Boundary 3 again. You cannot change a default node's selection.

Physics Interface Node Status

The status of a physics interface depends on if it is a default feature node (see Physics Interface Default Feature Nodes), the selection that it applies to, and other nodes in the same branch that can override nodes earlier in the sequence. You can change the order of nodes other than the default feature nodes by moving them up or down.

OVERRIDDEN SELECTIONS

A node can be partially or completely *overridden* by another node further down in the same branch of the model tree that is of a similar, exclusive type (see Exclusive Nodes). For example, if you specify a temperature boundary condition on boundary 1 and boundary 3, and then add another temperature boundary condition for boundary 3, the first temperature boundary condition is overridden on boundary 3. In the **Settings** window for the **Temperature** nodes that define the temperature boundary condition, the **Selection** list then shows **3 (overridden)** to indicate that the temperature boundary condition defined on this selection is overridden for boundary 3 but is still active on boundary 1. Deleting or disabling the other temperature boundary condition on boundary 3 reactivates the original temperature boundary condition, and then shows **3** (without the **(overridden)** indication).

SELECTIONS THAT ARE NOT APPLICABLE

For selections that are not applicable for a feature node (interior boundaries for an boundary condition that is only applicable for exterior boundaries, for example), the **Selection** list then shows **(not applicable)** next to the entries in the list that are, in this case, interior boundaries.

ENABLED/DISABLED NODES

By enabling or disabling physics interface nodes, you can activate and inactivate (shadow) other physics interface nodes that appear higher up in the physics inter face branches. See Disabling and Deleting Nodes.

Physics Interface Boundary Types

There are different types of boundaries for the physics interfaces, which all support different types of boundary conditions:

- Exterior boundaries, where most boundary conditions are applicable—see below.
- Interior boundaries, where special interface conditions can be applicable—see below.
- Axial symmetry boundaries, which are artificial boundaries representing the symmetry axis in axisymmetric models. See Physics Interface Axial Symmetry Feature Node.

If a selection for a boundary condition node, for example, contains boundaries of a type that is not applicable or supported, the **Selection** list has **(not applicable)** next to those boundary numbers.

INTERIOR AND EXTERIOR BOUNDARIES

When specifying boundary and interface conditions, COMSOL Multiphysics differentiates between exterior and interior boundaries:

- An *exterior boundary* is an outer boundary of the modeling domain.
- A *interior boundary* is a dividing interface between two domains in the modeling domain.

If an equation or physics interface is deactivated in one domain, the interior boundary between the active and inactive domain becomes an exterior boundary for its variables because it then borders on the outside of the active domain for those fields. The boundaries of the inactive domain are then void.

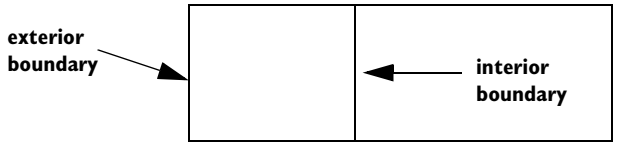


Figure 7-6: Examples of exterior and interior boundaries.

Continuity on Interior Boundaries

Unless you specify a specific boundary condition on interior boundaries (such as a contact resistance condition), COMSOL ensures continuity in the field variables across interior boundaries. For assembly geometries with identity pairs you can select a **Continuity** node on the **Pairs** menu in the boundary part of the context menu for most physics interfaces. The **Continuity** condition is only suitable for pairs where the boundaries match.

The **Continuity** form includes the following sections:

BOUNDARIES

The selection list in this section shows the boundaries for the selected pairs.

PAIR SELECTION

In this section you select the pairs where you want to impose continuity across the pair boundaries. Select the pairs from the **Pairs** list (Ctrl-click to deselect).

Physics Interface Axial Symmetry Feature Node

In axisymmetric models, boundaries on the symmetry axis are boundaries where only a condition for the axial symmetry exists. COMSOL Multiphysics adds a default **Axial Symmetry** feature node that is active on all boundaries on the symmetry axis. The condition on the symmetry axis is typically a zero Neumann or no-flux condition.

The **Axial Symmetry** form has this section:

BOUNDARIES

The selection list for boundaries is not available because this is a default boundary condition. In the list, boundaries that are not on the symmetry axis have **(not applicable)** added after the boundary number.

Physics Interface Common Settings for all the Main Nodes

The **Settings** windows for the main physics interface nodes (**Solid Mechanics**, for example) have two sections common to all physics interfaces: **Advanced Settings** and **Discretization**. To display these sections, select from the **View** menu in the **Model Builder**.

ADVANCED SETTINGS

The **Advanced Settings** section does not normally need to be changed, but if you do need to edit it, select **Show More Options** from the **View** menu in the **Model Builder**. In addition to physics-specific settings that are in some physics interfaces, these settings are always included:

- Select an option from the **Equation form** list to use an equation form that is different from the one that the study type sets up. **Automatic** is the default, which means that the equation follows the study type (for example, a time-dependent study uses equations that include time-derivatives, when applicable). To override the automatic selection, select an equation form for another study type (**Stationary**, for example). When the time-dependent solver runs, the solution for the physics where you have changed the equation form does not change with time.
- Select the **Show all model inputs** check box to make all *model inputs* appear in the equation model node's **Model Inputs** section. Model inputs are fields such as temperature and velocities that act as inputs for material models and model equations. This setting affects model inputs that are not part of the physics interface but that can appear if you define a material so that a material property becomes a function of the temperature, for example. The check box also affects field variables in predefined multiphysics interfaces. In those cases, the model inputs do not appear

in the equation model node's **Model Inputs** section if the **Show all model inputs** check box is not selected and there is a field that can serve as the model input. If no such field exists, the model input appears in the equation model node's **Model Inputs** section with a user defined input setting. When the **Show all model inputs** check box is not selected, the program automatically connects the model input to an existing field within the physics interface (but not to available fields in other physics interfaces). The **Show all model inputs** check box does not affect model inputs that are an integral part of a model equation, such as a velocity field for heat transfer in fluids.

DISCRETIZATION

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**.

The **Element order** (or, more precisely, the order of the shape function) directly affects the number of degrees of freedom in the solution and the accuracy of the solution. Increasing the order of the elements roughly corresponds to a uniform mesh refinement. Most physics interfaces uses Lagrange elements, which can be of order 1 to 4, with 2 being the default order. You can change the order using the **Element order** list. The software adapts the order of the numerical integration to the element orders for the physics interfaces in the model. Some physics interfaces use special element types or a reduced element order for some of the field variables.

Physics Interface Equation View Node

The **Equation View** node ($\frac{\partial u}{\partial t}$=f) is a subnode to the physics feature nodes. The **Equation View** node's **Settings** window contains information about the implementation of the physics feature: variables, shape functions, weak-form equation expressions, and constraints. You activate or disable the **Equation View** nodes by choosing **Show Equation View** from the View (down-arrow) menu to the left of the **Minimize** button at the top of the **Model Builder** window. See Viewing Node Names, Identifiers, Types, Tags, and Equations.

To update the values in the **Equation View** node's **Settings** window to reflect the latest changes in the physics feature, click the **Refresh Equations** button ( ) in the **Settings** window's toolbar.

Note: Editing the predefined expressions for variables, equations, and constraints means that you alter the equations that COMSOL Multiphysics solves or the values of variables.

You can edit the values of variables, weak-form expressions, and constraints in the corresponding tables. This makes it possible to introduce custom changes to the equations and variable definitions.

For a changed definition of a variable or a change to a weak form expression or constraint, a warning icon appears in the leftmost column. To restore only the change in the selected variable, weak-form expression, or constraint, click the **Reset Selected** button ( ) under the table in the **Variables**, **Weak Expression**, or **Constraints** section. To reset all changes in the equation view, click the **Reset All** button ( ) in the **Settings** window's toolbar.

The **Equation View** page contains the following sections:

VARIABLES

This section has a table with the variables that the physics feature defines has these columns:

- **Name**: the name of the variable.
- **Expression**: the expression, using COMSOL Multiphysics syntax, that defines the variables.
- **Unit**: the unit for the variable (in the active unit system)
- **Description**: a description of the variable.

SHAPE FUNCTIONS

This section has a table with the dependent variables (field variables) that the physics feature defines and their shape functions. This is primarily applicable to equation model features; for most physics features, the table is empty. The table consists of the following columns:

- **Name**: the name of the variable.
- **Shape function**: the type of shape function (element) for the variable (for example, **shlag** for Lagrange elements)

- **Unit**: the unit for the variable (in the active unit system)
- **Description**: a description of the variable.

WEAK EXPRESSIONS

This section has a table with the weak-formulation equation contributions that the physics feature generates. Each equation contribution appears on its own row under **Weak expression**, but the order is not significant.

Note: The PDE interfaces and the ODEs an DAEs interface do not display any weak expressions. They are either implemented using strong formulations, directly displays the weak formulation, or define equations discretized in the time domain only.

CONSTRAINTS

This section has a table with the constraints that the physics feature generates. This is typically the case for boundary conditions of constraint types, such as prescribed displacements, temperature, or velocities. Many other physics features do not generate any constraints, and the table is then empty. The table consists of the following columns:

- **Constraint**: the expression for the constraint.
- **Constraint force**: the expression that defines the associated constraint force, which is typically the test function of the constraint.
- **Shape function**: the type of shape function (element) for the constraint.

Specifying Model Equation Settings

The model equation settings appear in the main feature for a physics interface and describe the physics on the domains. You can use different values for each domain using settings of the following types:

- Coefficients that define the PDE on the domain. The PDE coefficients are available for the mathematics (equation-based) interfaces.
- Material properties that define the physics in the domain. These properties are available in the physics interfaces.

Specifying Initial Values

An **Initial Values** node is added by default to each physics interface.

In some types of analyses you must provide initial values:

- As the initial condition for a time-dependent analysis
- As an initial guess for the nonlinear stationary solver
- As a linearization (equilibrium) point when solving a linearized stationary model or when performing an eigenvalue study.

To enter initial values, in the **Model Builder**, click the **Initial Values** node under a physics interface node. In the **Settings** window, enter the **Initial Values** for all dependent variables (fields) in the physics interface. The default initial values are zero.

For some physics interfaces you can also enter initial values for the first time derivative of the dependent variables. These are used when solving time-dependent problems containing second time derivatives (wave-type applications). Like other default settings, these initial values apply to all domains where you have not specified any other values.

If you want to use different initial values in different domains, right-click the main physics interface node and select **Initial Values** to create additional nodes as needed.

For more information about using initial conditions, see the section Dependent Variables in the *COMSOL Multiphysics Reference Guide* (or see Where Do I Access the Documentation and Model Library?).

Modeling Anisotropic Materials

The c coefficient in the coefficient form PDE corresponds to physical properties that can be anisotropic such as:

- Diffusion coefficient
- Permittivity
- Thermal conductivity
- Electric conductivity

For an anisotropic material, the c coefficient is a tensor with at most four components in 2D and at most nine components in 3D:

$$c = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

When specify and of the c coefficient components—as well as for certain material properties in some of the other physics interfaces—you can choose from the following types from the list underneath the edit fields:

- **Isotropic** (the default)—enter only one value.
- **Diagonal**—enter the diagonal components for an anisotropic material with the main axes aligned with the model's coordinate system.
- **Symmetric**—enter a symmetric matrix using the diagonal components and the upper off-diagonal components.
- **Anisotropic**—enter the full 2-by-2 (2D) or 3-by-3 (3D) matrix for an anisotropic material.

Periodic Boundary Conditions

Using Periodic Boundary Conditions

Use *periodic boundary conditions* to make the solution equal on two different (but usually equally shaped) boundaries.

To add a periodic boundary condition, in the **Model Builder**, right-click a physics interface node and select **Periodic Condition**. The periodic boundary condition typically implements standard periodicity (that is, the value of the solution is the same on the periodic boundaries), but in most cases you can also choose antiperiodicity so that the solutions have opposing signs. For fluid flow interfaces, the **Periodic Flow Condition** provides a similar periodic boundary condition but without a selection of periodicity.

For some physics interfaces you can choose the direction in which you want a periodic boundary condition. For a description of the standard periodic boundary condition, which most physics interfaces use, see Periodic Condition. See the next section for an example of a periodic boundary condition.

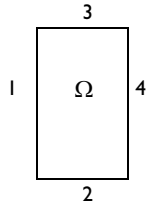
Periodic Boundary Condition Example

This example demonstrates how to implement a simple periodic boundary condition.

MODEL DEFINITION

Consider an eigenvalue equation on a rectangle of dimension $1\times\sqrt{\pi}$.

$-\Delta u = \lambda u$	on Ω
$u(x_1, y) = u(x_4, y)$	on Boundaries 1 and 4
$u(x, y_2) = u(x, y_3)$	on Boundaries 2 and 3



RESULTS AND DISCUSSION

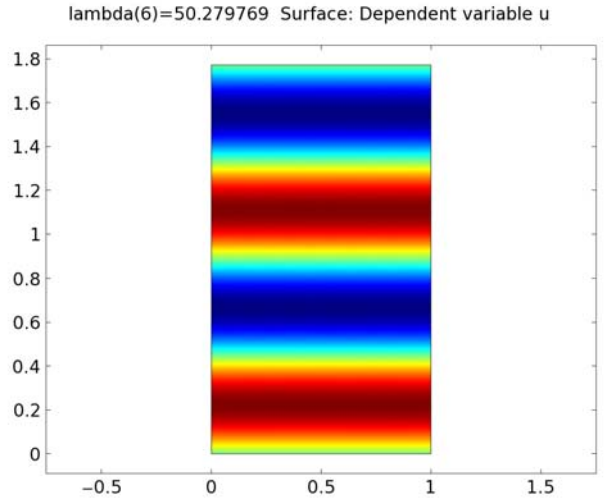


Figure 7-7: The periodic pattern in the solution (6th eigenmode).

The wavelength of the sinusoidal solutions is the rectangle side length divided by a series of integers, each of which corresponds to an eigensolution. The locations of the maxima are random because the model does not constrain the absolute value of the solution anywhere.

MODELING INSTRUCTIONS

Model Wizard

1 Start COMSOL Multiphysics.

2 In the **Model Wizard**, select **2D** from the **Space dimension** list. Click the **Next** button ( ).

3 From the list of physics interfaces on the **Add Physics** page, select **Mathematics>PDE Interfaces>Coefficient Form PDE (c)**. Click the **Next** button ( ).

4 On the **Select Study Page**, select **Preset Studies>Eigenvalue**.

5 Click the **Finish** button ( ).

Geometry Modeling

1 Right-click **Geometry 1** and select **Rectangle** ( ).

2 Type `sqrt(pi)` in the **Height** edit field.

3 Click the **Build All** button ( ) in the **Settings** window toolbar.

PDE Settings

The default coefficient values are correct.

Periodic Boundary Conditions

1. Right-click **PDE** and select **Periodic Condition**.
2. Select boundaries 1 and 4 in the **Graphics** window and click the **Add to Selection** button ( ).
3. Right-click **PDE** and select **Periodic Condition**.
4. Select boundaries 2 and 3in the **Graphics** window and click the **Add to Selection** button ( ).

In this case, COMSOL Multiphysics automatically identifies one boundary as the source and the other as the destination.

Mesh Generation

1. Right-click **Mesh 1** and select **Free Triangular** ( ).
2. Click the **Build All** button ( ) in the **Settings** window toolbar.

Computing the Solution

The eigenvalue algorithm does not work reliably when the parameter that you specify in the **Search for eigenvalues about** edit field is equal to an eigenvalue. This particular problem has a zero eigenvalue, so you must change the value of this parameter.

1. Select **Study 1>Eigenvalue 1** in the Model Tree.
2. In the **Settings** window, locate the **Study Settings** section.
3. Type 10 in the **Search for eigenvalues around** edit field.
4. Right-click **Study 1** and select **Solve** to compute the solution.

Postprocessing and Visualization

By default COMSOL Multiphysics plots the first eigenmode, which in this case is a flat solution associated with a zero eigenvalue.

1. Select **2D Plot Group 1** under **Results** in the Model Tree.
2. In the **Settings** window, locate the **Data** section.
3. Look at the solution for different eigenvalues by making a corresponding selection in the **Eigenvalue** list. Start with the last eigenvalue.
4. Click the **Plot** button ( ) in the **Settings** window toolbar to plot each eigenvalue.

Computing Accurate Fluxes

COMSOL Multiphysics provides two ways of computing accurate reaction forces and fluxes:

- The first approach involves the reaction force operator (`reacf`) that makes it possible to compute integrals of reaction forces or fluxes during postprocessing. Use the reaction force operator when computing integrals of reaction forces or fluxes during postprocessing. See The Reacf Operator for details.
- The second, more general approach for computing reaction forces and fluxes is to use weak constrains. Use this approach when you need reaction forces or fluxes in other contexts than computing integrals of reaction forces or fluxes.

Find further information on the use of weak constraints in the section Using Weak Constraints.

When using weak constraints, the Lagrange multipliers are additional dependent variables in physics interfaces using weak constraints. When using the reaction force operator, the reaction force operator of a certain dependent variable corresponds to the Lagrange multiplier of that dependent variable. The Lagrange multipliers correspond to the following quantities in the physics interfaces:

TABLE 7-1: PHYSICS INTERFACE INTERPRETATION OF LAGRANGE MULTIPLIERS

PHYSICS INTERFACE	QUANTITY
Electrostatics	Surface charge density
Magnetic Fields	Surface current
Electric Currents	Current flux
Heat Transfer	Heat flux
Transport of Diluted Species	Flux
Solid Mechanics	Force
Laminar Flow	Total force per area

COMSOL Multiphysics computes only the part of the boundary flux captured by the Lagrange multiplier. You might have additional flux coming from boundary sources or nonidentity constraint matrices. This should not happen in the physics interfaces, though.

Flux Computation Example

Consider computing the flux over the constrained boundary of a rectangle. The PDE of the rectangle is Poisson's equation, $\Delta u = 1$, with $u = 0$ on the boundary. This example computes the flux using an expression, a reaction force operator, and a Lagrange multiplier.

MODEL WIZARD

1. On the **Select Space Dimension** page, click the **2D** button. Then click the **Next** button.
2. In the list of physics interface, open **Mathematics>PDE Interfaces** and then select **Coefficient Form PDE (c)**. Then click the **Next** button.
3. From the **Studies** list, select **Preset Studies>Stationary**.
4. Click **Finish**.

GEOMETRY MODELING

Draw a single rectangle of any size.

COEFFICIENT FORM PDE

The default equation is Poisson' equation, but the default boundary condition is a zero flux (homogeneous Neumann boundary condition). Instead, add a homogeneous Dirichlet boundary condition: $u = 0$:

1. In the **Model Builder** window, right-click **PDE (c)** node and select **Dirichlet Boundary Condition**.
2. In the **Dirichlet Boundary Condition** node's **Settings** window, select **All boundaries** from the **Selection** list in the **Boundaries** section.

COMPUTING THE SOLUTION

In the **Model Builder** window, right-click **Study 1** and choose **Compute**.

RESULTS—FLUX EXPRESSION

1. In the **Model Builder** window, choose **Results>Derived Values>Surface Integration**.
2. In the **Surface Integration** node's **Settings** window, type `1` in the **Expression** edit field.
3. Select Domain 1 and click the **Evaluate** button ( ) at the top of the **Settings** window. The rectangle's area appears in the **Results** section.
4. Choose **Results>Derived Values>Line Integration**.
5. In the **Line Integration** node's **Settings** window, type `nx*ux+ny*uy` in the **Expression** edit field.

6 Select all boundaries and click the **Evaluate** button ( ) at the top of the **Settings** window. COMSOL Multiphysics computes the flux over the boundaries as the normal component of the gradient of u.

The influx is approximately equal to the integral of the negative source term in Poisson's equation.

RESULTS—REACTION FORCE OPERATOR

1 In the **Line Integration** node's **Settings** window, type `reacf(u)` in the **Expression** edit field.

2 Select all boundaries and click the **Evaluate** button ( ) at the top of the **Settings** window. COMSOL Multiphysics computes the flux over the boundaries using the reaction force operator for the solution u. The computed value is much closer to the expected value. To see the value as a double floating-point number with full precision, click the **Full Precision** button in the **Results** window's toolbar.

COEFFICIENT FORM PDE—LAGRANGE MULTIPLIER

1 Under **Physics**, select **Dirichlet Boundary Condition 1**. Clear the **Prescribed value of u** check box to remove this constraint.

2 Right-click **PDE (c)** and choose **More>Weak Constraint** on the boundary level.

3 In the **Weak Constraint** node's **Settings** window, select **All boundaries** from the **Selection** list.

4 In the **Weak Constraint** section, type `u` in the **Constraint expression** edit field to constrain u to 0 at the boundary.

5 Expand the **Lagrange Multiplier** section and replace `lm` with `flux` in the **Lagrange multiplier variable** edit field.

COMPUTING THE SOLUTION—LAGRANGE MULTIPLIER

Right-click **Study 1** and choose **Compute**.

RESULTS—LAGRANGE MULTIPLIER

1 In the **Line Integration** node's **Settings** window, type `flux` in the **Expression** edit field.

2 Select all boundaries and click the **Evaluate** button ( ) at the top of the **Settings** window. COMSOL Multiphysics computes the flux over the boundaries using the Lagrange multiplies variable. The integral of the flux exactly equals the integral of the finite-element projection of the source term in Poisson's equation.

Using Units

COMSOL Multiphysics supports a number of consistent unit systems, including the SI unit system, which is the default unit system. The user interface displays the unit for the physical quantities entered in the selected unit system, but with the unit syntax (see Using Standard Unit Prefixes and Syntax) you can use any available and applicable unit or SI prefix to define your input quantities. In addition to SI units, many English units and units from the CGS (or cgs) system are also available, regardless of the unit system used in the model. All data in the material databases and Material Library product use SI units with declared units using the unit syntax (see Materials). This makes it possible to use the material data also in models with non-SI unit systems. Regardless of the selected unit system, you can always choose from a list of applicable units for plotting and results evaluation.

Unit Systems in COMSOL Multiphysics

COMSOL Multiphysics supports the following unit systems:

METRIC UNIT SYSTEMS

- SI units, the International System of Units (SI, Système International d'Unités). This is the default unit system (sometimes also called MKS). For a list of SI units in COMSOL Multiphysics, see SI Base, Derived, and Other Units.
- CGSA units. The CGS system uses centimeter, gram, and second as basic units of length, mass, and time, respectively. The remaining basic units are identical to the SI units. The CGS unit system gives nice values for small lengths, masses, forces, pressures, and energies when working on a microscale and with weak electromagnetic forces. The derived units of force, pressure, and energy have well-known and widely used names: dyne, barye, and erg, respectively. CGSA adds *ampere* as the basic unit for electric current. For a list of the CGSA units, see Special CGSA Units.
- Electromagnetic units (EMU). This system is based on Ampère's law, which defines the unit of electric current once you select an appropriate value for the constant C. When dealing exclusively with magnetic effects, it is convenient to set $C = 1$. If CGS units are used for the remaining basic dimensions, the current unit is called an *abampere*, and the corresponding coherent unit system is called electromagnetic units. Unique names for derived units have been introduced by prefixing the SI name with *ab-*. For a list of EMU units, see Special EMU Units.

- Electrostatic units (ESU). Based on Coulomb's law for the force between point charges, ESU uses a unit of charge called the *statcoulomb* with CGS units for length, mass, and time. From there, the *statampere*, or *franklin*, and other derived units of the electrostatic unit system follow. For a list of ESU units, see Special ESU Units.
- MPa units. For stationary structural mechanics, where the density does not appear in the equations, it can be convenient to use a system where the newton and megapascal (hence the name: the MPa system) are naturally derived units of force and pressure. Keeping the SI unit for time, the basic units of length and mass become millimeter and tonne. Except for the force and pressure units, other derived units are nameless. For a list of MPa units, see Special MPa Units.

ENGLISH UNIT SYSTEMS

- Foot-pound-second unit system (FPS units). The original foot-pound-second system seems to be the absolute system using the pound as a unit of mass. This version of the FPS system is in agreement with the IEEE standard (the pound is a unit of mass and not of force). The natural derived unit of force is the *poundal*. For a list of FPS units, see Special FPS Units.
- British engineering units. An alternative to the standard FPS system is the British engineering unit system (also called gravitational foot-pound-second system or foot-slug-second system). Here, the pound force is the natural unit of force, which causes the introduction of the mass unit *slug* such that a pound force is a slug-foot per second squared. For a list of British engineering units, see Special British Engineering Units.
- Inch-pound-second unit system (IPS units). It is possible to define varieties of the FPS and British engineering systems based on the inch instead of the foot as basic unit of length. This gives rise to two distinct inch-pound-second systems: the *absolute IPS system* (just called IPS) and the *gravitational IPS system*. For a list of IPS units, see Special IPS Units.
- Gravitational IPS units. This alternative IPS unit system considers the pound a unit of weight rather than a unit of mass. For a list of Gravitational IPS units, see Special Gravitational IPS Units.

Selecting a Unit System

SETTING THE UNIT SYSTEM ON THE GLOBAL LEVEL

To set the unit system for the entire MPH-file on a global level:

1 In the **Model Builder**, click the root node (the top node in the model tree). The root node's name is the name of the MPH-file or **Untitled.mph** before you have saved your work.

2 In the root node's **Settings** window, select the unit system from the list in the **Unit System** section.

SETTING THE UNIT SYSTEM FOR INDIVIDUAL MODELS

By default, all models in the MPH file use the same global unit system, but it is possible to use different unit systems in each model. To do so, follow these steps:

1 In the **Model Builder**, click the top node for a model branch (**Model 1**, for example).

2 In the model node's **Settings** window, locate the **Unit System** section.

3 Select the **Override global system** check box, and then select the unit system from the list of units system that becomes available.

Using Standard Unit Prefixes and Syntax

STANDARD UNIT PREFIXES

For SI units you can scale data using the standard prefixes for powers of 10—*kilo*, *mega*, *milli*, or *micro*, for example. Either the full prefix or the symbol can be used but you must use the same form for the prefix and the unit—that is, `[milliampere]` and `[mA]` are valid but not `[mampere]` or `[milliA]`). In the **Settings** windows for plotting and numerical results, the **Unit** list contains the SI unit for the quantity, including the most common prefixes. The lists also contain applicable non-SI units, which in a few cases also support these prefixes.

Use Table 7-2 as a guide for the format to enter.

TABLE 7-2: SI PREFIXES

FULL PREFIX	SYMBOL	FACTOR
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}

TABLE 7-2: SI PREFIXES

FULL PREFIX	SYMBOL	FACTOR
hekto	h	10^{2}
deca	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	u	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

STANDARD UNIT SYNTAX

You can use the unit syntax to specify a quantity with any applicable unit. To do so, append the unit to any constant or variable in a model using a syntax where you enclose the unit in brackets, for example, `200[ft]` and `3e6[kg/m^3]`.

You can use both the name and the symbol for a unit. For example, `2.4[ampere]` and `2.4[A]` are both valid to indicate an electric current in SI units (see SI Base, Derived, and Other Units). The SI units can also contain standard prefixes. Appending a unit means that you *multiply* the constant or variable to the left of the unit declaration with this unit. This multiplication takes precedence over other operators so, for example, `1/2[m]` evaluates to 0.5 m^{-1} (`0.5[1/m]`) whereas both `(1/2)[m]` and `1/2*1[m]` evaluate to 50 cm (`0.5[m]` or `50[cm]`). Also, if `L` is a variable defined as `2[m]`, `L[1/s]` evaluates to `2[m/s]`.

The following examples show how to apply the unit syntax:

- Adding two quantities of the same kind that use different units: `0.5[ft]+33[mm]`. COMSOL Multiphysics converts the result to the SI length unit (meters).
- Using multiplication with a unit to get consistent units for two quantities that you want to add, for example, `14[kg]+ht.rho[m^3]`, which works if `ht.rho` represents the density for a model.

Note: For unit names with spaces and hyphens, such as *British thermal unit* and *pound-force*, you can only use the symbols when declaring units.

It is possible to add constants (without units) to any quantity. COMSOL Multiphysics then assumes that this value has the unit of the SI unit for that quantity. You can also concatenate several units, for example, `3.6[N][m]`, which is equivalent to `3.6[N*m]` and evaluates to `3.6[J]`.

All data in the material databases and Material Library product use SI units and this unit syntax (see Materials).

DECLARING UNITS FOR PARAMETERS AND EXPRESSION VARIABLES

It is important to be aware of the following:

- If you use parameters or variables in the physics features (see Global and Local Definitions), it is good practice to use the unit syntax to define them. The **Settings** windows for parameters and variables display the resulting unit, in SI units, of user defined parameters and expressions. It is important to verify that the variables have the expected unit before using them in the physics settings. The unit of parameters and variables are otherwise undefined.
- Using properties with undefined units in a model does not affect the numerical results during the analysis, but undefined units are required in the results and visualization stages—expressions involving such parameters and variables are also unitless.
- To declare the SI unit for electric resistance, ohm, use `[ohm]`. COMSOL Multiphysics then displays this as `[Ω]`.
- If non-SI units or SI prefixes are used, the conversion to SI units also affects the value (quantity) using a scaling factor (and an offset in the case of temperature units). The **Value** column in a **Parameter** node's **Settings** window displays the quantity and unit in the base SI unit system so that you can see the result of the unit conversion. For example, if you define a parameter as `3[ft]`, the result in the **Value** column is **0.9144 m**. See Global and Local Definitions for more information.

SI Base, Derived, and Other Units

The SI units form an internationally accepted system with seven units for base quantities and a large number of derived units. You can use the symbols for these and

other units when declaring units in COMSOL Multiphysics (for example, `10[m/s]` for a velocity).

- Table 7-3 lists the SI units for the seven base quantities.
- Table 7-4 lists the SI derived units supported in COMSOL Multiphysics.
- Table 7-5 lists additional units available in COMSOL regardless of the unit system in the model. If more than one name or symbol is available, you can use any of them, except when names contain more than one word or a hyphen. See also the tables with special units for other unit systems than the SI system; special units that are not listed in Table 7-5 are only available when using such a non-SI unit system.
- Table 7-6 lists other SI derived units without special names or symbols

TABLE 7-3: BASE SI UNITS

BASE QUANTITY	NAME	SYMBOL
length	meter, metre*	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
temperature	kelvin**	K
amount of substance	mole	mol
luminous intensity	candela	cd

* See About Editing Geometry Length and Angular Units

**See About Temperature Units

TABLE 7-4: SI DERIVED UNITS IN COMSOL MULTIPHYSICS

BASE QUANTITY	NAME	SYMBOL
absorbed dose	gray	Gy
capacitance	farad	F
conductance	siemens	S
dose equivalent	sievert	Sv
electric charge	coulomb	C
electric resistance, impedance, reactance	ohm	Ω
electric potential difference, voltage	volt	V
energy, work, heat	joule	J
force, weight	newton	N
frequency	hertz	Hz

TABLE 7-4: SI DERIVED UNITS IN COMSOL MULTIPHYSICS

BASE QUANTITY	NAME	SYMBOL
illuminance	lux	lx
inductance	henry	H
luminous flux	lumen	lm
magnetic flux	weber	Wb
magnetic flux density, magnetic induction	tesla	T
plane angle	radian	rad
power	watt	W
pressure	pascal	Pa
radioactivity	becquerel	Bq

TABLE 7-5: ADDITIONAL UNITS IN COMSOL MULTIPHYSICS

DERIVED QUANTITY	NAME	SYMBOLS	VALUE
acceleration	galileo	Gal	0.01 m/s^2
dipole moment	debye	D	$3.33564095 \cdot 10^{-30}$ C·m
energy	British thermal unit*	BTU, Btu	1055.05585 J
energy	calorie*	cal	4.184 J
energy	electronvolt	eV	$1.6021765314 \cdot 10^{-19}$ J
energy	erg	erg	10^{-7} J
force	dyne	dyn	10^{-5} N
force	poundal	pdl	0.138254954376 N
force	pound-force	lbf	4.4482216152605 N
frequency	rpm	RPM	1/60 Hz
length	angstrom	Å	10^{-10} m
length	inch	in	0.0254 m
length	foot	ft	0.3048 m
length	mile*	mi	1609.344 m
length	microinch	uin	$0.0254 \cdot 10^{-6}$ m
length	milliinch	mil, thou	$0.0254 \cdot 10^{-3}$ m
length	nautical mile*, nautimile	nmi	1852 m
length	yard	yd	0.9144 m

TABLE 7-5: ADDITIONAL UNITS IN COMSOL MULTIPHYSICS

DERIVED QUANTITY	NAME	SYMBOLS	VALUE
magnetic field strength	oersted	Oe	$10^3/(4\cdot\pi)$ A/m
magnetic flux density	gauss	G	10^{-4} T
mass	atomic mass unit, dalton	u, amu, Da	$1.660538782\cdot10^{-27}$ kg
mass	gram	g	0,001 kg
mass	pound, pound-mass	lb, lbm	0.45359237 kg
mass	stone	st	6.35029318 kg
mass	slug	slug	approx. 14.5939 kg
mass	ton, tonne	t	1000 kg
permeability	millidarcy*	mD	$9.869233\cdot10^{-16}$ m^2
plane angle	degree	deg	$\pi/180$
pressure	atmosphere	atm	101325 Pa
pressure	bar	bar	100000 Pa
pressure	barye	ba	0.1 Pa
pressure	psi	psi	$6.894757\cdot10^3$ Pa
pressure	torr	Torr, mmHg	133.322 Pa
speed	mph, MPH	mph	0.44704 m/s
speed	knot*	knot	1852 km/h (approx. 0.614 m/s)
temperature	Celsius**	degC	T+273.15
temperature	Fahrenheit**	degF	5/9·T+459.67
temperature	Rankine**	R, Ra	5/9·T
time	year*	a, yr	31556952 s
time	day	d	86400 s
time	hour	h	3600 s
time	minute	min	60 s
volume	gallon*	gal	0.003785411784 m^3
volume	imperialgallon	impgal	0.00454609 m^3
volume	liter, litre	L, l	0.001 m^3
volume	pint*	pt	0.000473176473 m^3
volume	quart*	qt	0.000946352946 m^3

TABLE 7-5: ADDITIONAL UNITS IN COMSOL MULTIPHYSICS

DERIVED QUANTITY	NAME	SYMBOLS	VALUE
volume flow rate	cubic feet per minute	CFM, cfm	$4.719474 \cdot 10^{-4}$ m^3/s

* See the additional notes following this table.

** See About Temperature Units

ADDITIONAL NOTES ABOUT Table 7-5

UNIT NAME	NOTE
British thermal unit	An energy unit defined as the amount of heat required to raise the temperature of one pound (pound-mass) of water by one degree from 60° to 61° Fahrenheit at a constant pressure of one atmosphere. Refer to the British thermal unit using the symbol only (Btu or BTU): for example, `0.28[Btu/(h*in*degF)]` for a thermal conductivity.
calorie	Small calorie or gram calorie, which equals 4.184 J. A large calorie or kilogram calorie is 1000 calories (4.184 kJ). Use [kcal] for large calories.
millidarcy (mD)	Widely used for permeability in petroleum engineering. Typical values for the permeability of porous media are in the range of a few to a few hundred mD. The symbol D represents the debye, a unit for the magnetic dipole moment, and not the darcy unit.
mile	The international statute mile, which equals 1609.344 m.
nautimile	The nautical mile equals 1852 m.
knot	The same as nautical miles per hour.
year	A Gregorian year, which equals 365.2425 days.
gallon (gal)	This is the U.S. liquid gallon which equals 0.003785411784 m^3; the Imperial (UK) gallon (imperialgallon, impgal) is equal to 0.00454609 m^3.
pint and quart	The U.S. liquid pint and U.S. liquid quart, respectively.

TABLE 7-6: EXAMPLES OF SI DERIVED UNITS WITHOUT SPECIAL NAMES

QUANTITY	NAME	SYMBOL
acceleration	meter per second squared	m/s^2
amount-of-substance concentration	mole per cubic meter	mol/m^3
area	square meter	m^2
current density	ampere per square meter	A/m^2
heat capacity, specific heat	joule per kilogram kelvin	J/(kg·K)
magnetic field strength	ampere per meter	A/m

TABLE 7-6: EXAMPLES OF SI DERIVED UNITS WITHOUT SPECIAL NAMES

QUANTITY	NAME	SYMBOL
mass density	kilogram per cubic meter	kg/m^3
permeability	henry per meter	H/m
speed, velocity	meter per second	m/s
wave number	reciprocal meter	m^{-1}
volume	cubic meter	m^3

Special British Engineering Units

The base units in the British engineering unit system are identical to the SI units with the following exceptions:

TABLE 7-7: SPECIAL BASE UNITS IN THE BRITISH ENGINEERING UNIT SYSTEM

BASE QUANTITY	NAME	SYMBOL
length	foot	ft
mass	slug	N/A
temperature	Fahrenheit	degF

There is one derived unit that differs from the corresponding SI unit:

TABLE 7-8: DERIVED BRITISH ENGINEERING UNITS IN COMSOL MULTIPHYSICS

BASE QUANTITY	NAME	SYMBOL
force	pound-force	lbf

The *British thermal unit* is also available as Btu or BTU.

Note: If you specify the British engineering unit system as the base unit system, COMSOL Multiphysics constructs derived units from the applicable SI base units and the units listed in Table 7-7 and Table 7-8. This means, for example, that the unit for voltage displayed in the user interface is lbf·ft/As rather than V (volt). In an edit field that expects a voltage as input, you therefore need to use the unit syntax when entering a numerical value, for example, `10[V]`.

Special CGSA Units

The base units in the CGSA unit system are identical to the SI units with the following exceptions:

TABLE 7-9: SPECIAL BASE UNITS IN THE CGSA UNIT SYSTEM

BASE QUANTITY	NAME	SYMBOL
length	centimeter	cm
mass	gram	g

The CGSA unit system includes the following derived units that differ from the corresponding SI units:

TABLE 7-10: DERIVED CGSA UNITS IN COMSOL MULTIPHYSICS

BASE QUANTITY	NAME	SYMBOL
acceleration	galileo	Gal
energy	erg	N/A
force	dyne	dyn
pressure	barye	N/A
speed	kyne	N/A

Special EMU Units

The base units in the EMU unit system are identical to the SI units with the following exceptions:

TABLE 7-11: SPECIAL BASE UNITS IN THE EMU UNIT SYSTEM

BASE QUANTITY	NAME	SYMBOL
length	centimeter	cm
mass	gram	g
electric current	abampere, biot	N/A

The EMU unit system includes the following derived units that differ from the corresponding SI units:

TABLE 7-12: DERIVED EMU UNITS IN COMSOL MULTIPHYSICS

BASE QUANTITY	NAME	SYMBOL
acceleration	galileo	Gal
capacitance	abfarad	N/A
conductance	absiemens	N/A

TABLE 7-12: DERIVED EMU UNITS IN COMSOL MULTIPHYSICS

BASE QUANTITY	NAME	SYMBOL
electric charge	abcoulomb	N/A
electric resistance	abohm	N/A
electric potential difference, voltage	abvolt	N/A
energy	erg	N/A
force	dyne	dyn
inductance	abhenry	N/A
magnetic flux	abweber	N/A
magnetic flux density	abtesla	N/A
pressure	barye	N/A
speed	kyne	N/A

Special ESU Units

The base units in the ESU unit system are identical to the SI units with the following exceptions:

TABLE 7-13: SPECIAL BASE UNITS IN THE ESU UNIT SYSTEM

BASE QUANTITY	NAME	SYMBOL
length	centimeter	cm
mass	gram	g
electric current	statampere, franklin	N/A

The ESU unit system includes the following derived units that differ from the corresponding SI units:

TABLE 7-14: DERIVED ESU UNITS IN COMSOL MULTIPHYSICS

BASE QUANTITY	NAME	SYMBOL
acceleration	galileo	Gal
capacitance	statfarad	N/A
conductance	statsiemens	N/A
electric charge	statcoulomb	N/A
electric resistance	statohm	N/A
electric potential difference, voltage	statvolt	N/A
energy	erg	N/A
force	dyne	dyn

TABLE 7-14: DERIVED ESU UNITS IN COMSOL MULTIPHYSICS

BASE QUANTITY	NAME	SYMBOL
inductance	stathenry	N/A
magnetic flux	statweber	N/A
magnetic flux density	stattesla	N/A
pressure	barye	N/A
speed	kyne	N/A

Special FPS Units

The base units in the FPS unit system are identical to the SI units with the following exceptions:

TABLE 7-15: SPECIAL BASE UNITS IN THE FPS UNIT SYSTEM

BASE QUANTITY	NAME	SYMBOL
length	foot	ft
mass	pound	lb
temperature	Fahrenheit	degF

There is one derived unit that differs from the corresponding SI unit:

TABLE 7-16: DERIVED FPS UNITS IN COMSOL MULTIPHYSICS

BASE QUANTITY	NAME	SYMBOL
force	poundal	N/A

Special IPS Units

The base units in the IPS unit system are identical to the SI units with the following exceptions:

TABLE 7-17: SPECIAL BASE UNITS IN THE IPS UNIT SYSTEM

BASE QUANTITY	NAME	SYMBOL
length	inch	in
mass	pound	lb
temperature	Fahrenheit	degF

Special MPa Units

The base units in the MPa unit system are identical to the SI units with the following exceptions:

TABLE 7-18: SPECIAL BASE UNITS IN THE MPA UNIT SYSTEM

BASE QUANTITY	NAME	SYMBOL
length	millimeter	mm
mass	tonne, ton	t

There is one derived unit that differs from the corresponding SI unit:

TABLE 7-19: DERIVED MPA UNITS IN COMSOL MULTIPHYSICS

BASE QUANTITY	NAME	SYMBOL
pressure	megapascal	MPa

Special Gravitational IPS Units

The base units in the Gravitational IPS unit system are identical to the SI units with the following exceptions:

TABLE 7-20: SPECIAL BASE UNITS IN THE GRAVITATIONAL IPS UNIT SYSTEM

BASE QUANTITY	NAME	SYMBOL
length	inch	in
mass	GIPS_mass	N/A
temperature	Fahrenheit	degF

The following derived units differ from the corresponding SI units:

TABLE 7-21: DERIVED GRAVITATIONAL IPS UNITS IN COMSOL MULTIPHYSICS

BASE QUANTITY	NAME	SYMBOL
force	pound-force	lbf
pressure	psi	psi

Switching Unit System

If you switch the unit system during modeling or use a unit system other than SI, COMSOL Multiphysics does not convert the data in the model. All physical constants and data in the materials libraries are in SI units, but you can use them with any unit system because COMSOL Multiphysics converts these values to the corresponding values in the model's unit system. Note, however, that you must also declare the units

of other input data for the model using the unit syntax (or convert the numerical values to the new units system). If the length unit changes, you must also convert the geometry dimensions to keep the size of the geometry using another length unit.

About Temperature Units

The relationship between different temperature units involve an offset in addition to the usual scale factor. In many cases, the offset is not important to the physics equations because these equations are concerned only with temperature differences. There are, however, some cases where you must use an absolute or thermodynamic temperature measure. The most common example is the Stefan-Boltzmann law for black-body radiation used in radiation boundary conditions.

The SI unit system uses the kelvin, which is an absolute temperature, as the basic unit of temperature. English unit systems use degree Fahrenheit as the basic unit of temperature, which, because the Fahrenheit scale is not absolute, is fine for most purposes except radiation. For such purposes, the Rankine scale provides the corresponding absolute temperature unit. See Table 7-22 for a list of acceptable unit syntax.

TABLE 7-22: TEMPERATURE UNITS

SCALE	UNIT
Celsius	`[degC]`
Fahrenheit	`[degF]`
Kelvin	[K]
Rankine	[R] or [Ra]

DIFFERENTIAL VS. ABSOLUTE TEMPERATURE

If the dimension of an expression that includes a unit is temperature or 1/temperature, COMSOL Multiphysics interprets the dimension as an absolute temperature. If the dimension is something other than temperature but the unit expression includes temperature, the temperature is a differential temperature; that is, COMSOL Multiphysics uses no offset when converting between different temperature units.

The following examples show how the unit conversion works for different expressions that include temperature units:

- `100[degC]` is an expression that has temperature as the dimension. COMSOL Multiphysics interprets it as an absolute temperature and evaluates it as 373.15 K.

- `373.15[1/K]` is interpreted as an absolute temperature (but no conversion is necessary from kelvin to kelvin).
- `100[degC/K]` is dimensionless, and the temperature is therefore a differential temperature; that is, the result is 100 because the conversion uses no offset.
- To make COMSOL Multiphysics interpret `100[degC/K]` as an absolute temperature, split the expression using two separate expressions such as `100[degC]*1[1/K]`, which equals 373.15. This is also what occurs when you use a variable (`TC`, for example) defined as `100[degC]`. `TC[1/K]` is then also two expressions where both are interpreted as absolute temperature.

About Editing Geometry Length and Angular Units

The default units are meters for length, and degrees for angles. For many applications, an independent length unit for the geometry may be required. For example, if the model describes a MEMS device, the natural length unit might be µm, or the geometry imported from a CAD file might use a unit than meters. It may also be useful to specify the angular unit in radians instead.

Note: The length unit for the geometry does not affect the units that include length in the physics interfaces or any other part of COMSOL Multiphysics.

---

1. Create or open a model file.
2. In the **Model Builder**, under a **Model** node, click the **Geometry** node.
3. Under **Units**, select a **Length unit** from the list.
4. Select an **Angular unit**—**Degrees** or **Radians**.
5. Select the **Scale values when changing units** check box to automatically scale for dimensions in the existing geometry.
6. Enter a **Default relative repair tolerance** and select a **Geometry representation.** For details about the geometry settings, see The Geometry Node and Its Settings of the *COMSOL Multiphysics Reference Guide* (or see Where Do I Access the Documentation and Model Library?).

Indication of Unexpected or Unknown Unit

The unit display appears orange for the properties in the settings for the physics and materials that have an invalid unit or a different unit than expected. A tooltip displays a message at the corresponding edit field. In the case of a valid but unexpected unit, this message contains the deduced and expected units in the current unit system.

8

Overview of the Physics Interfaces

This section gives an overview of the available physics interfaces as well as some general guidelines for effective modeling.

In this section:

The COMSOL Multiphysics Physics Interfaces

Introduction

Solving PDEs generally means you must take the time to set up the underlying equations, material properties, and boundary conditions for a given problem. COMSOL Multiphysics, however, relieves you of much of this work. The package provides a number of *physics interfaces* that consist of predefined templates and user interfaces already set up with equations and variables for specific areas of physics. The physics interfaces consist of a number of features with associated windows on the COMSOL Desktop for the physics in domains and on boundaries, edges, and points along with predefined PDEs. A set of interface-dependent variables makes it easy to visualize and evaluate the important physical quantities using conventional terminology and notation.

Note: Suites of physics interfaces that are optimized for specific disciplines together with specialized model libraries are available in a group of optional products: the *AC/DC Module*, *Acoustics Module*, *Batteries and Fuel Cells Module*, *CFD Module*, *Chemical Reaction Engineering Module*, *Earth Science Module*, *Heat Transfer Module*, *MEMS Module*, *Plasma Module*, *RF Module*, and *Structural Mechanics Module*.

A complement to the physics interfaces, special interfaces for equation-based modeling simplify the setup of PDEs for modeling that does not explicitly refer to any particular application field. In addition a number of interfaces supplement the physics interfaces with special functionality such as the Sensitivity and Moving Mesh interfaces.

Physics Interfaces Groups in the Model Wizard

The **Add Physics** page in the **Model Wizard** contains the following main groups of physics interfaces and mathematics interfaces (some appear only if your license includes some of the add-on modules):

- The **Recently Used** branch ( ) contains the physics interfaces that are most recently used for easy access to the interfaces that you use the most.
- The **AC/DC** branch ( ) contains physics interface for low-frequency electromagnetics such as Electrostatics and Electric Currents.
- The **Acoustics** branch ( ) contains physics interfaces for acoustics.
- The **Chemical Species Transport** branch ( ) contains physics interface for chemical species transport by, for example, convection and diffusion, solving for the species concentrations.
- The **Electrochemistry** branch ( ) contains physics interfaces for electrochemistry and modeling of electrochemical components such as batteries and fuel cells. This branch is only available if your license includes the Batteries and Fuel Cells Module.
- The **Fluid Flow** branch ( ) contains physics interfaces for fluid flow such as laminar single-phase flow and, with add-on modules, multiphase flow and turbulent flow.
- The **Heat Transfer** branch ( ) contains physics interface for heat transfer in solids and fluids and thermal multiphysics applications such as Joule heating.
- The **Plasma** branch ( ) contains physics interfaces for plasma modeling. This branch is only available if your license includes the Plasma Module.
- The **Radio Frequency** branch ( ) contains physics interfaces for high-frequency electromagnetic field simulations solving the full Maxwell equations. This branch is only available if your license includes the RF Module.
- The **Structural Mechanics** branch ( ) contains physics interface for structural mechanics, studying displacements and stresses in solids, for example.
- The **Mathematics** branch (Δu) contains mathematics interfaces for solving PDEs, ODEs, and DAEs, for optimization (requires the Optimization Module) and sensitivity analysis, and for modeling moving meshes and parameterized geometry.

Physics Interfaces in COMSOL Multiphysics

The following table lists the physics interfaces in COMSOL Multiphysics and their availability for 1D, 1D axisymmetric, 2D, 2D axisymmetric, and 3D geometries:

PHYSICS INTERFACE	ICON	TAG	1D	1D AXI	2D	2D AXI	3D
AC/DC							
Electrostatics		es			√	√	√
Electric Currents		ec			√	√	√
Magnetic Fields		mf			√	√	
Acoustics							
Pressure Acoustics		acpr			√	√	√
Chemical Species Transport							
Transport of Diluted Species		chds	√	√	√	√	√
Fluid Flow							
Single-Phase Flow, Laminar Flow		spf			√	√	√
Heat Transfer							
Heat Transfer in Solids		ht	√	√	√	√	√
Heat Transfer in Fluids		ht	√	√	√	√	√
Joule Heating		jh	√	√	√	√	√
Structural Mechanics							
Solid Mechanics		solid			√	√	√
Mathematics							
Coefficient Form PDE		c	√		√	√	√
General Form PDE		g	√		√	√	√

PHYSICS INTERFACE	ICON	TAG	1D	1D AXI	2D	2D AXI	3D
Weak Form PDE/Boundary/ Edge/Point		w/ b, e, p	√	√	√	√	√
ODEs and DAEs		ge	√	√	√	√	√
Optimization*		opt	√	√	√	√	√
Sensitivity		sens	√	√	√	√	√
Helmholtz Equation		hzeq	√		√		√
Laplace Equation		lpeq	√		√		√
Poisson's Equation		poeq	√		√		√
Wave Equation		waeq	√		√		√
Deformed Geometry		dg	√	√	√	√	√
Moving Mesh		ale	√	√	√	√	√
Wall Distance		wd	√	√	√	√	√

* This physics interface is available only if your COMSOL installation includes a valid license for the Optimization Module.

Selecting a Physics Interface

When creating a model in COMSOL Multiphysics, you can select a single physics interface that describes one type of physics or select several physics interfaces for multiphysics modeling and coupled-field analyses.

MODELING USING A SINGLE PHYSICS INTERFACE

Most of the physics interfaces contain stationary, eigenvalue, and time-dependent (dynamic) study types. As already mentioned, these physics interfaces provide features and windows where you can create models using material properties, boundary

conditions, and initial conditions. Each physics interfaces comes with a template that automatically supplies the appropriate underlying PDEs.

If you cannot find a physics interface that matches a given problem, try one of the mathematics interfaces, which allow you to define a custom model in general mathematical terms. Indeed, COMSOL Multiphysics can model virtually any scientific phenomenon or engineering problem that originates from the laws of science.

MULTIPHYSICS MODELING USING MULTIPLE INTERFACES

When modeling real-world systems, you often need to include the interaction between different kinds of physic: multiphysics. For instance, an electric current produces heat, and the properties of an electronic component such as an inductor vary with temperature. To solve such a problem, combine two or several physics interfaces into a single model using the multiphysics capabilities of COMSOL Multiphysics. For the example just mentioned, you can use the predefined Joule Heating interface, which is a combination of the Electric Currents and Heat Transfer physics interfaces. In this way you create a system of two PDEs with two dependent variables: V for the electric potential and T for the temperature. There are many other predefined multiphysics couplings that provide a unified interface that combines two or more coupled physics interfaces for common multiphysics applications.

You can also combine physics interfaces and equation-based modeling for maximum flexibility.

To summarize the proposed strategy for modeling processes that involve several types of physics: Look for physics interfaces suitable for the phenomena of interest. If you find them among the physics interfaces, use them; if not, add one or more equation interfaces.

When coupling multiple physics interfaces in a multiphysics model (without using a predefined multiphysics interface), the couplings can occur in domains and on boundaries. COMSOL Multiphysics automatically identifies potential *model inputs* for quickly forming couplings between physics. For example, a velocity field from a fluid-flow interface can provide convection in a heat transfer interface.

Modeling Guidelines

To allow you to model large-scale problems and for successful modeling in general, COMSOL Multiphysics lets you tune solver settings and use symmetries and other model properties to reach a solution or—failing that—interrupt the solution process to retrieve a partial solution. This section has some tips and guidelines when modeling.

Using Symmetries

By using symmetries in a model you can reduce its size by one-half or more, making this an efficient tool for solving large problems. This applies to the cases where the geometries and modeling assumptions include symmetries. The most important types of symmetries are axial symmetry and symmetry and antisymmetry planes or lines:

- *Axial Symmetry* is common for cylindrical and similar 3D geometries. If the geometry is axisymmetric, there are variations in the radial (r) and vertical (z) direction only and not in the angular (θ) direction. You can then solve a 2D problem in the rz-plane instead of the full 3D model, which can save considerable memory and computation time. Many COMSOL Multiphysics physics interfaces are available in axisymmetric versions and take the axial symmetry into account.
- *Symmetry and Antisymmetry Planes or Lines* are common in both 2D and 3D models. *Symmetry* means that a model is identical on either side of a dividing line or plane. For a scalar field, the normal flux is zero across the symmetry line. In structural mechanics, the symmetry conditions are different. *Antisymmetry* means that the loading of a model is oppositely balanced on either side of a dividing line or plane. For a scalar field, the dependent variable is 0 along the antisymmetry plane or line. Structural mechanics applications have other antisymmetry conditions. Many physics interfaces have symmetry conditions directly available as features.

To take advantage of symmetry planes and symmetry lines, all of the geometry, material properties, and boundary conditions must be symmetric, and any loads or sources must be symmetric or antisymmetric. You can then build a model of the symmetric portion, which can be half, a quarter, or an eighth of the full geometry, and apply the appropriate symmetry (or antisymmetry) boundary conditions.

Effective Memory Management

Especially in 3D modeling, extensive memory usage dictates some extra precautions. First, check that you have selected an iterative linear system solver. Normally you do

not need to worry about which solver to use, because the physics interface makes an appropriate default choice. In some situations, though, it might be necessary to make additional changes to the solver settings and the model.

ESTIMATING THE MEMORY USE FOR A MODEL

Out-of-memory messages can occur when COMSOL Multiphysics tries to allocate an array that does not fit sequentially in memory. It is common that the amount of available memory seems large enough for an array, but there might not be a contiguous block of that size due to memory fragmentation.

In estimating how much memory it takes to solve a specific model, the following factors are the most important:

- The number of node points
- The number of dependent and independent variables
- The element order
- The sparsity pattern of the system matrices. The sparsity pattern, in turn, depends on the shape of the geometry and the mesh. For example, an extended ellipsoid gives sparser matrices than a sphere.

The MUMPS and PARDISO out-of-core solvers can make use of available disk space to solve large models that do not fit in the available memory.

CREATING A MEMORY-EFFICIENT GEOMETRY

A first step when dealing with large models is to try to reduce the model geometry as much as possible. Often you can find symmetry planes and reduce the model to a half, a quarter or even an eighth of the original size. Memory usage does not scale linearly but rather polynomially (Cn^k, $k > 1$), which means that the model needs less than half the memory if you find a symmetry plane and cut the geometry size by half. Other ways to create a more memory-efficient geometry include:

- Avoiding small geometry objects where not needed and using Bézier curves instead of polygon chains.
- Using linear elements if possible (this is the default setting in many physics interfaces). See Selecting an Element Type.
- Making sure that the mesh elements are of a high quality. Mesh quality is important for an iterative linear system solver. Convergence is faster and more robust if the element quality is high.
- Avoiding geometries with sharp, narrow corners. Mesh elements get thin when they approach sharp corners, leading to poor element quality in the adjacent regions.

Sharp corners are also unphysical and can lead to very large (even infinite, in theory) stress concentrations, for example.

Selecting an Element Type

As the default element type for most interfaces, COMSOL Multiphysics uses typically uses first-order or second-order Lagrange elements or shape functions. Second-order elements and other higher-order elements add additional degrees of freedom on midpoint and interior nodes in the mesh elements. These added degrees of freedom typically provide a more accurate solution but also require more memory due to the reduced sparsity of the discretized system. For many application areas, such as stress analysis in structural and solid mechanics, the increased accuracy of a second-order element is important. In fluid-flow modeling using the incompressible Navier-Stokes equations, a combination of element types using an element for the velocity components of a higher order than that for the pressure usually provides the best result. The default element for the Laminar Flow interface, for example, is the P2-P1 element using second-order elements for the velocity components and linear elements for the pressure. For other applications you can select a first-order instead of a second-order element, or reduce the element order in general, to reduce memory use.

Analyzing Model Convergence and Accuracy

It is important that the finite element model accurately captures local variations in the solution such as stress concentrations. In some cases you can compare your results to values from handbooks, measurements, or other sources of data. Many examples in the COMSOL Multiphysics Model Library include comparisons to established results or analytical solutions. Look for these ***benchmark models*** as a means of checking results.

If a model has not been verified by other means, a ***convergence test*** is useful for determining if the mesh density is sufficient. Here you refine the mesh and run the study again, and then you see if the solution is converging to a stable value as the mesh is refined. If the solution changes when you refine the mesh, the solution is mesh dependent, so the model requires a finer mesh. You can use adaptive mesh refinement, which adds mesh elements based on an error criterion to resolve those areas where the error is large. For convergence, it is important to avoid singularities in the geometry (see Avoiding Singularities and Degeneracies in the Geometry).

Achieving Convergence When Solving Nonlinear Equations

Nonlinear problems are often difficult to solve. In many cases, no unique solution exists. COMSOL Multiphysics uses a Newton-type iterative method to solve nonlinear systems of PDEs. This solution method can be sensitive to the initial estimate of the solution. If the initial conditions are too far from the desired solution, convergence might be impossible, even though it might be simple from a different starting value.

You can do several things to improve the chances for finding the relevant solutions to difficult nonlinear problems:

- Provide the best possible initial values.
- Solve sequentially and iterate between single-physics equations; finish by solving the fully coupled multiphysics problem when you have obtained better starting guesses.
- Ensure that the boundary conditions are consistent with the initial solution and that neighboring boundaries have compatible conditions that do not create singularities.
- Refine the mesh in regions of steep gradients.
- For convection-type problem, introduce artificial diffusion to improve the problem's numerical properties (see Stabilization Techniques in the *COMSOL Multiphysics Reference Guide* or Where Do I Access the Documentation and Model Library?). Most physics interfaces for modeling of fluid flow and chemical species transport provide artificial diffusion as part of the default settings.
- Scaling can be an issue when one solution component is zero. In those cases, the automatic scaling might not work.
- Turn a stationary nonlinear PDE into a time-dependent problem. Making the problem time dependent generally results in smoother convergence. By making sure to solve the time-dependent problem for a time span long enough for the solution to reach a steady state, you solve the original stationary problem.
- Use the parametric solver and vary a material property or a PDE coefficient starting from a value that makes the equations less nonlinear to the value at which you want to compute the solution. This way you solve a series of increasingly difficult nonlinear problems. The solution of a slightly nonlinear problem that is easy to solve serves as the initial value for a more difficult nonlinear problem.

Avoiding Strong Transients

If you start solving a time-dependent problem with initial conditions that are inconsistent, or if you use boundary conditions or sources that switch instantaneously

at a certain time, you induce strong transient signals in a system. The time-stepping algorithm then takes very small steps to resolve the transient, and the solution time might be very long, or the solution process might even stop. Stationary problems can run into mesh-resolution issues such as overshooting and undershooting of the solution due to infinite flux problems.

Unless you want to know the details of these transients, start with initial conditions that lead to a consistent solution to a stationary problem. Only then turn on the boundary values, sources, or driving fluxes over a time interval that is realistic for your model.

In most cases you should turn on your sources using a smoothed step over a finite time. What you might think of as a step function is, in real-life physics, often a little bit smoothed because of inertia. The step or switch does not happen instantaneously. Electrical switches take milliseconds, and solid-state switches take microseconds.

Physics-Related Checks and Guidelines

There are some important checks and guidelines that primarily apply to different areas of physics. Making these checks ensure that the model input is sufficient and increase the chances for successful modeling. Also see the modeling sections of the documentation for the physics interfaces and the modules for more information related to modeling different physics.

FLUID FLOW AND TRANSPORT PHENOMENA

The following checks and guidelines primarily apply to fluid-flow modeling but also to modeling of other transport phenomena:

- If none of the boundary condition includes the pressure (most outlet conditions do, however), then you should specify the pressure at some point in the fluid domain. Without a specified pressure, the problem is underconstrained and it is difficult to get convergence.
- Make sure that the mesh if sufficiently fine, so that it contains at least 4–6 mesh elements across the thickness of a channel, for example.
- Make sure that the boundary conditions and the initial conditions match for time-dependent problems. For example, instead of starting with a full velocity on the wall, compared to a zero velocity initial velocity field in the fluid, ramp up the

velocity with a smoothed step function or a ramp function. See also Avoiding Strong Transients.

- For fluid-flow models it is important to estimate the flow regime (laminar or turbulent) using the Reynolds number, for example. If the flow is in the turbulent regime, a turbulence model is typically required.

ACOUSTICAL AND ELECTROMAGNETIC WAVE PROPAGATION

For models that describe wave propagation, it is important to fully resolve the wave in both time and space. In practice that means using a maximum mesh element size that provides about 10 linear or five 2nd-order elements per wavelength and also, for transient simulations, a fixed time step that is small enough.

STRUCTURAL MECHANICS

The following checks and guidelines primarily apply to modeling of structural mechanics:

- Make sure that the model is fully constrained. At a minimum, you typically need to constrain the model to avoid all rigid-body movement, which for a 3D solid mechanics model means 6 constraints for three translations and three rotations. Otherwise the solution is not well defined and does not converge. It is not possible to add all 6 constraints in a single point, where you can constrain at most three translational degrees of freedom. For a 3D solid model you can use a 3-2-1 approach to constrain 3 degrees of freedom at one point (a fixed constraint), 2 at another point, and 1 at a third point. To do so, select three convenient points (vertices) that are well separated. Then fix the first point in all three directions. Constrain the second point in the two directions orthogonal (normal) to the vector from point one to point two making sure that there is no restriction to deformation along the line from point one to point two. Finally constrain the third point in a direction normal to the plane formed by the three points. To test this approach, the model should expand or contract under temperature loading and have small stresses throughout with no stress concentrations. The corresponding minimum constraints for a 2D model are a fixed constraint at one point for the 2 translational degrees of freedom and an additional constraint in one direction at another point to constrain the one rotational degree of freedom.
- Consider if you can assume that the material is linear elastic and that the deformations are small. If not, you should consider using a nonlinear material model.
- Avoid sharp corners in the geometry, which are unphysical and lead to unbounded stress concentrations.

9

Electromagnetics

This section explains the physics interfaces in COMSOL Multiphysics for modeling electromagnetics and how to use them for electromagnetic field simulations. Specifically, it takes a detailed look at the **Electrostatics** and **Electric Currents** interfaces, which are available in all space dimensions, as well as at the **Magnetic Fields** interface available for 2D axisymmetric and 2D geometries.

In this section:

The underlying theory for each interface is also discussed:

Note: The optional AC/DC Module contains specialized and extended modeling interfaces for electromagnetic simulations, for example, for computations of inductors and capacitors. The optional RF Module includes modeling interfaces for wave-propagation simulations that are especially useful in microwave engineering and photonics.

The Electrostatics Interface

Applications involving *electrostatics* include high-voltage apparatus, electronic devices, and capacitors. The term "statics" is not to be interpreted literally but rather that the observation time or time scale at which the applied excitation changes is short compared to the charge relaxation time and that the electromagnetic wavelength and skin depth are very large compared to the size of the domain of interest. If you are uncertain whether to use the **Electric Currents** interface or the **Electrostatics** interface, which both solve for the scalar electric potential V, or maybe consider using an explicit charge transport model please refer to the section on Charge Relaxation Theory.

The **Electrostatics** interface ( ) provides the equations, boundary conditions, and space charges for modeling electrostatic fields, solving for the electric potential.

For an introduction to the physics and equations implemented by this interface, see the Theory for the Electrostatics Interface.

The **Charge Conservation** feature is the main feature, which adds the equation for the electric potential and provides a settings window for defining the constitutive relation and its associated properties such as the relative permittivity. When you add an **Electrostatics** interface, it adds a default **Charge Conservation** node and a default boundary condition, a **Zero Charge** node. There is also a default **Initial Values** node. Right-click the **Electrostatics** node to add other nodes that implement, for example, boundary conditions and space charges. The following sections provide information about all feature nodes available in the Electrostatics interface.

INTERFACE IDENTIFIER

This is the physics interface identifier, which you use to reach the fields and variables in expressions, for example.

The identifier appears in the **Identifier** edit field, and you can change it to any unique string that is a valid identifier. The default identifier (for the first Electrostatics interface in the model) is `es`.

DOMAINS

Select the domains where you want to define the electric potential and the equations that describe the potential field for dielectrics. The default setting is to include all domains in the model.

OUT-OF-PLANE THICKNESS (2D MODELS ONLY)

Define the out-of-plane thickness d by entering a value or expression (SI unit: m) in the **Thickness** edit field. The default value of 1 m is typically not representative for a thin dielectric medium, for example. Instead it describes a unit thickness that makes the 2D equation identical to the equation used for 3D models.

ADVANCED SETTINGS AND DISCRETIZATION

Normally these settings do not need to be changed. See Show More Options: Advanced Settings and Discretization.

DEPENDENT VARIABLES

The dependent variable (field variable) is for the **Electric potential** V. You can change the name in the corresponding edit field, but the names of fields and dependent variables must be unique within a model.

Charge Conservation

The **Charge Conservation** feature adds the equations for charge conservation according to Gauss' law for the electric displacement field. The **Charge Conservation** form contains the following sections for defining the related material properties:

DOMAINS

Select the domains where you want to define the electric potential and the equation based on Gauss' law that describes the potential field.

MODEL INPUTS

This section contains field variables that appear as model inputs, if the current settings include such model inputs. By default, this section is empty.

ELECTRIC FIELD

Select a **Constitutive relation** to describe the macroscopic properties of the medium (relating the electric displacement $\mathbf{D}$ with the electric field $\mathbf{E}$) and the applicable material properties, such as the relative permittivity. Select:

- **Relative permittivity** to use the constitutive relation $\mathbf{D} = e_0 e_r \mathbf{E}$ (the default).
- **Polarization** to use the constitutive relation $\mathbf{D} = e_0 \mathbf{E} + \mathbf{P}$.
- **Remanent displacement** to use constitutive relation $\mathbf{D} = e_0 e_r \mathbf{E} + \mathbf{D}_r$, where $\mathbf{D}_r$ is the remanent displacement (the displacement when no electric field is present).

- If **Relative permittivity** is selected, the default is to take the **Relative permittivity** (e_r) values **From material**. If **User defined** is selected, select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** and enter values or expressions in the field or matrix.
- If **Polarization** is selected, enter components (3 in 3D, 2 in 2D) for the **Polarization** vector $\mathbf{P}$ (SI unit: C/m^2).
- If **Remanent displacement** is selected, the default is to take the **Relative permittivity** (e_r) values **From material**. If **User defined** is selected, select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** and enter values or expressions in the field or matrix. Then enter components (3 in 3D, 2 in 2D) for the **Remanent displacement** $\mathbf{D}_r$ (SI unit: C / m^2).

COORDINATE SYSTEM SELECTION

Select a **Coordinate system**. By default you use the **Global coordinate system**, and the list contains any additional coordinate systems that the model includes.

EQUATION

This section displays the equations that the interface is based on.

Space Charge Density

The **Space Charge Density** feature adds a space charge density ρ, which appears on the right-hand side of the equation that the Electrostatics interface defines.

DOMAINS

Select the domains where you want to define a current source.

SPACE CHARGE DENSITY

Enter a value or expression for the space charge density (SI unit: C/m^3) in the **Space charge density** edit field.

Initial Values

The **Initial Values** feature adds an initial value for the electric potential V that can serve as an initial condition for a transient simulation or as an initial guess for a nonlinear solver.

DOMAINS

Select the domains where you want to define an initial value.

INITIAL VALUES

Enter a value or expression for the initial value of the electric potential V in the **Electric potential** edit field. The default value is 0 V.

Boundary Conditions

The following boundary conditions are available on exterior boundaries:

- Ground (ground conditions are also available for edges (3D) and points (2D and 3D) as Ground (Edge) and Ground (Point), respectively)
- Electric Potential (also available for edges (3D) and points (2D and 3D))
- Surface Charge Density
- Zero Charge (the default boundary condition)
- Displacement Field
- Periodic Condition

The relevant interface condition at interfaces between different media is

$$\mathbf{n}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$$

In the absence of surface charges, this condition is fulfilled by the natural boundary condition

$$\mathbf{n} \cdot [(\varepsilon_0 \nabla V - \mathbf{P})_1 - (\varepsilon_0 \nabla V - \mathbf{P})_2] = -\mathbf{n} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = 0$$

In addition, the following boundary conditions are available on interior boundaries:

- Ground
- Electric Potential
- Surface Charge Density
- Thin Low Permittivity Gap

For axisymmetric models, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and automatically adds an **Axial Symmetry** feature to the model that is valid on the axial symmetry boundaries only.

Ground

The **Ground** feature implements ground as the boundary condition:

$$V = 0$$

Ground means that there is a zero potential on the boundary. This boundary condition is also applicable at symmetry boundaries where the potential is known to be antisymmetric with respect to the boundary.

Ground is the default boundary condition.

BOUNDARIES

Select the boundaries where you want to apply a ground (zero potential) boundary condition.

Similarly, additional **Ground** features provide ground as a condition on edges in 3D model and at points in 2D and 3D models. You then select the edges in the **Edges** section or the points in the **Points** section, respectively.

CONSTRAINT SETTINGS

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Select a **Constraint type**—**Bidirectional, symmetric** or **Unidirectional**. If required, select the **Use weak constraints** check box.

Electric Potential

The **Electric Potential** feature provides an electric potential V_0 as the boundary condition:

$$V = V_0$$

Because you are solving for the electric potential in this interface, you typically define the value of the potential at some part of the geometry.

BOUNDARIES

Select the boundaries where you want to apply an electric potential as the boundary condition.

ELECTRIC POTENTIAL

Enter the value or expression for the electric potential (SI unit: V) in the V_0 edit field.

Similarly, the **Electric Potential** feature provides the electric potential as a condition on edges in 3D models, and the **Electric Potential** feature provides electric potential as a

condition at points in 2D and 3D models. You then select the edges in the **Edges** section or the points in the **Points** section, respectively.

Surface Charge Density

The **Surface Charge Density** feature provides the following surface-charge boundary condition for exterior boundaries (left) and interior boundaries (right):

$$-\mathbf{n}\cdot\mathbf{D} = \rho_s, \qquad \mathbf{n}\cdot(\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$$

You specify the surface charge density ρ_s at an outer boundary or at an interior boundary between two nonconducting media.

BOUNDARIES

Select the boundaries where you want to apply a surface charge density.

SURFACE CHARGE DENSITY

Enter the value or expression for the surface charge density ρ_s (SI unit: C/m^2).

Displacement Field

The **Displacement Field** feature provides the following electric-displacement boundary condition:

$$\mathbf{n}\cdot\mathbf{D} = \mathbf{n}\cdot\mathbf{D}_0$$

It specifies the normal component of the electric displacement field at a boundary.

BOUNDARIES

Select the boundaries where you want to use the normal component of the displacement field as the boundary condition.

DISPLACEMENT FIELD

Enter the coordinates (3 in 3D, 2 in 2D) of the **Boundary displacement field** $\mathbf{D}_0$ (SI unit: C/m^2).

COORDINATE SYSTEM SELECTION

Select a **Coordinate system**. By default you use the **Global coordinate system**, and the list contains any additional coordinate systems that the model includes.

Periodic Condition

The **Periodic Condition** feature can be used to define periodicity or antiperiodicity between two boundaries. You can also activate the feature on more than two boundaries, in which case the feature tries to identify two separate surfaces which can each consist of several connected boundaries. For more complex geometries it may be necessary to use the **Destination Selection** subfeature. With this subfeature you can manually specify which boundaries constitute the source and destination surfaces. To add the subfeature, right-click on the **Periodic Condition** node and select **Destination Selection**.

BOUNDARIES

Select the boundaries where you want to apply a periodic condition.

PERIODIC CONDITION

Select a **Type of periodicity**—**Continuity** or **Antiperiodicity**.

Zero Charge

The **Zero Charge** feature adds the condition that there is zero charge on the boundary:

$$\mathbf{n} \cdot \mathbf{D} = 0$$

This boundary condition is also applicable at symmetry boundaries where the potential is known to be symmetric with respect to the boundary. This is the default boundary condition.

BOUNDARIES

Select the boundaries where you want to apply a zero charge condition.

Thin Low Permittivity Gap

Use the thin low permittivity gap condition

$$\mathbf{n} \cdot \mathbf{D}_1 = \frac{\varepsilon_0 \varepsilon_{rL}}{d_L}(V_1 - V_2)$$

$$\mathbf{n} \cdot \mathbf{D}_2 = \frac{\varepsilon_0 \varepsilon_{rL}}{d_L}(V_2 - V_1)$$

to model a thin gap of a material with a small permittivity compared to the adjacent domains. The layer has the thickness d_L and the relative permittivity ε_{rL}. The indices 1 and 2 refer to the two sides of the boundary.

BOUNDARIES

Select the boundaries where you want to apply a thin low permittivity gap condition.

THIN LOW PERMITTIVITY GAP

The default is to take the **Relative permittivity** (e_r) values **From material**. If **User defined** is selected, enter a value.

Enter a **Surface thickness** d (SI unit: m).

Continuity (Pair Feature)

Continuity is available as an option at interfaces between parts in an assembly. Also see Defining an Identity Pair.

BOUNDARIES

Select individual boundaries in an existing identity pair. This pair first has to be created.

PAIR SELECTION

Select the boundary pair where you want to define continuity. First an identity pair may have to be created.

Line Charge

In 3D specify line charges along the edges of a geometry. To add a Line Charge feature, right-click the **Electrostatics** node and choose **Line Charge** from the **Edges** menu. The **Line Charge** form contains the following sections:

EDGES

Select the edges where you want to add a line charge.

LINE CHARGE

Apply a line charge Q_j (SI unit: C/m) to edges. This source represents electric charge per unit length. You enter a value or expression for the line charge density in the Q_j edit field.

Point Charge

It is possible to add point charges to both 2D and 3D models. To add a Point Charge feature, right-click the **Electrostatics** node and choose **Point Charge** from the **Points** menu.

POINTS

Select the points where you want to add a point charge.

POINT CURRENT SOURCE

Apply a point charge Q_p (SI unit: C) to points. This source represents an electric displacement field flowing out of the point. You enter a value or expression for the point charge in the Q_p edit field.

The Electric Currents Interface

The **Electric Currents** interface ( ) provides the equations, boundary conditions, and current sources for modeling steady electric currents in conductive media, solving for the electric potential. The **Current Conservation** is the main feature, which adds the equation for the electric potential and provides a settings window for defining the electric conductivity as well as the constitutive relation and its associated material properties such as the relative permittivity.

For a more extensive introduction to the physics and equations implemented by this interface, see the Theory for the Electric Currents Interface.

When you add the **Electric Currents** interface, it adds a default **Current Conservation** node and a default boundary condition, the **Electric Insulation** node. There is also a default **Initial Values** node. Right-click the **Electric Currents** node to add other nodes that implement, for example, boundary conditions and current sources. The following sections provide information about all feature nodes available in the Electric Currents interface.

INTERFACE IDENTIFIER

This is the physics interface identifier, which you use to reach the fields and variables in expressions, for example.

The identifier appears in the **Identifier** edit field, and you can change it to any unique string that is a valid identifier. The default identifier (for the first Electric Currents interface in the model) is `ec`.

DOMAINS

Select the domains where you want to define the electric potential and the equations that describe the potential field for conductive media. The default setting is to include all domains in the model.

OUT OF PLANE THICKNESS (2D MODELS ONLY)

Define the out-of-plane thickness d (see Equation 9-3) by entering a value or expression (SI unit: m) in the **Thickness** edit field. The default value of 1 m is typically not representative for a thin grounding plate, for example. Instead it describes a unit thickness that makes the 2D equation identical to the equation used for 3D models.

DEPENDENT VARIABLES

The dependent variable (field variable) is for the **Electric potential** V. You can change the name in the corresponding edit field, but the names of fields and dependent variables must be unique within a model.

ADVANCED SETTINGS AND DISCRETIZATION

Normally these settings do not need to be changed. See Show More Options: Advanced Settings and Discretization.

Current Conservation

The **Current Conservation** feature adds the appropriate current conservation law, and the **Current Conservation** form contains the following sections for defining the related material properties.

DOMAINS

Select the domains where you want to define the electric potential and the continuity equation that describes the potential field.

MODEL INPUTS

This section contains field variables that appear as model inputs, if the current settings include such model inputs. By default, this section is empty. If you add a linear temperature relation for the conductivity, you can then define the source for the temperature T. From the **Temperature** list, select an existing temperature variable (from another physics interface) if available, or select **User defined** to define a value or expression for the temperature (SI unit: K) in the edit field that appears underneath the list.

CONDUCTION CURRENT

Select an **Electric conductivity** σ—**From material** (the default), **Linearized resistivity**, or **User defined**:

- If **User defined** is selected, enter values or expressions for an isotropic or anisotropic conductivity. Select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** depending on the properties of the conductive media. If you want to use another type of temperature dependence than a linear temperature relation (see below), you can enter any expression for the conductivity as a function of temperature.

- Select **Linear temperature relation** for a temperature-dependent conductivity (which occurs in, for example, Joule heating, which is also called resistive heating). The following equation then describes the conductivity:

$$\sigma = \frac{1}{\rho_0(1 + \alpha(T - T_0))}$$

where ρ_0 is the resistivity at the reference temperature T_0. α is the temperature coefficient of resistance, which describes how the resistivity varies with temperature. The default setting in the corresponding **Reference resistivity**, **Reference temperature**, and **Resistivity temperature coefficient** lists is **From material**, which means that the values are taken from the domain material. To specify another values for any of these properties, select **User defined** from the corresponding list and then enter a value or expression in the edit field that appears. T is the current temperature, which can be a value that you specify as a model input or the temperature from a heat transfer interface (in the Joule Heating multiphysics interface, this is the default setting). The definition of the temperature field appears in the **Model Inputs** section.

ELECTRIC FIELD

In this section you specify the constitutive relation that describe the macroscopic properties of the medium (relating the electric displacement $\mathbf{D}$ with the electric field $\mathbf{E}$) and the applicable material properties, such as the relative permittivity.

Select one of the following constitutive relations from the list under **Constitutive relation** (the corresponding equation appears underneath the list):

- Select **Relative permittivity** to use the constitutive relation $\mathbf{D} = \varepsilon_0\varepsilon_r\mathbf{E}$ (the default). You then specify ε_r, the relative permittivity (unitless). The default is to take its value from the material, but you can select **User defined** instead of **From material** from the list under **Relative permittivity** and then specify a value or expression for the relative permittivity in the edit field that appears.
- Select **Polarization** to use the constitutive relation $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$. You then specify $\mathbf{P}$, the polarization vector (SI unit: $\mathrm{C/m^2}$). Enter its components (3 in 3D, 2 in 2D) in the corresponding edit fields under **Polarization**.
- Select **Remanent displacement** to use the constitutive relation $\mathbf{D} = \varepsilon_0\varepsilon_r\mathbf{E} + \mathbf{D}_r$, where $\mathbf{D}_r$ (SI unit: $\mathrm{C/m^2}$) is the remanent displacement (the displacement when no electric field is present). In this case you specify ε_r, the relative permittivity, and $\mathbf{D}_r$, the remanent displacement. For the relative permittivity, use the list under **Relative permittivity**: Select **From material** (the default) to use the value from the material or select **User defined** to specify a value or expression for the relative permittivity in the

edit field that appears. For the remanent displacement, enter its components (3 in 3D, 2 in 2D) in the corresponding edit fields under **Remanent displacement**.

External Current Density

The **External Current Density** feature adds an externally generated current density $\mathbf{J}_e$ (SI unit: A/m^2), which appears in Ohm's law

$$\mathbf{J} = \sigma\mathbf{E} + \mathbf{J}_e$$

and in the equation that the Electric Currents interface defines.

DOMAINS

Select the domains where you want to define an external current density.

EXTERNAL CURRENT DENSITY

Enter the components (x, y, and z components in 3D, for example) of the external current density $\mathbf{J}_e$ in the corresponding edit fields under **External current density.**

COORDINATE SYSTEM SELECTION

Specify in what coordinate system the **External current density** is specified.

Current Source

The **Current Source** feature adds a distributed current source Q_j (SI unit: A/m^3) in the equation that the Electric Currents interface defines.

DOMAINS

Select the domains where you want to define a current source.

CURRENT SOURCE

Enter a value or expression for the current source Q_j (SI unit: A/m^3) in the **Current source** edit field.

Force Calculation

The **Force Calculation** feature makes a force variable available for postprocessing. The method used is integration of the Maxwell Stress tensor over the exterior surfaces of the set of domains. This feature also gives access to the normal component of the Maxwell Stress tensor on the external surfaces.

For the **Magnetic and Electric Fields** physics interface, the force calculation includes both electric and magnetic forces.

DOMAINS

Select the domains where you want to calculate a resultant force.

FORCE CALCULATION

Enter a **Force name** for the global variable.

Enter coordinates for the **Torque axis** $\mathbf{r}_{ax}$ and **Torque rotation point** $\mathbf{r}_0$. A torque calculation about a given point (**Torque rotation point**) is made, and the resulting torque component parallel to the given **Torque axis** is given as a global postprocessing variable.

Infinite Elements

The **Infinite Elements** feature imposes a coordinate transformation to the selected domain that effectively moves one or more sides of the domain to infinity. Infinite elements are used for the modeling of open boundary problems.

DOMAINS

Select the domains where you want to use infinite elements.

GEOMETRIC SETTINGS

Select the type of infinite element scaling to use. The options are **Cartesian**, **Cylindrical** and **Spherical** (may be less depending on the spatial dimensions modeled).

PARAMETERS

Tweak two parameters affecting the coordinate transformation. These are the **Physical width** and the **Pole distance**. The **Physical width** parameter sets the modeled width of the infinite element region, which typically is a large value. The parameter **Pole distance** is a tuning parameter that controls the nature of the coordinate transform.

Initial Values

The **Initial Values** feature adds an initial value for the electric potential V that can serve as an initial condition for a transient simulation or as an initial guess for a nonlinear solver. If you need to specify more than one set of initial values, you can add additional **Initial Values** features from the **Other** menu when you right-click the main feature for the physics interface.

DOMAINS

Select the domains where you want to define an initial value.

INITIAL VALUES

Enter a value or expression for the initial value of the electric potential V in the **Electric potential** edit field. The default value is 0 V.

Boundary Conditions

The relevant interface condition at interfaces between different media and interior boundaries is continuity; that is,

$$\mathbf{n}_2 \cdot (\mathbf{J}_1 - \mathbf{J}_2) = 0$$

which is the natural boundary condition.

The following boundary conditions are available on exterior boundaries:

- Ground (also available for edges (3D) and points (2D and 3D))
- Electric Potential (also available for edges (3D) and points (2D and 3D))
- Normal Current Density
- Electric Insulation (the default boundary condition)
- Electric Insulation (the default boundary condition)
- Periodic Condition

In addition, the following boundary conditions are available on interior boundaries:

- Boundary Current Source
- Ground
- Electric Potential
- Distributed Impedance

For axisymmetric models, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and automatically adds an **Axial Symmetry** feature to the model that is valid on the axial symmetry boundaries only.

Boundary Current Source

The **Boundary Current Source** feature adds a current source Q_j on the boundary.

$$\mathbf{n} \cdot (\mathbf{J}_1 - \mathbf{J}_2) = Q_j$$

It is applicable to interior boundaries that represent either a source or a sink of current. The **Boundary Current Source** form contains the following sections:

BOUNDARIES

Select the boundaries where you want to apply a current source.

BOUNDARY CURRENT SOURCE

Enter a value or expression for the current source Q_j (SI unit: A/m^2) in the **Surface current source** edit field.

Ground

The **Ground** feature implements ground as the boundary condition:

$$V = 0$$

Ground means that there is a zero potential on the boundary. This boundary condition is also applicable at symmetry boundaries where the potential is known to be antisymmetric with respect to the boundary.

BOUNDARIES

Select the boundaries where you want to apply a ground (zero potential) boundary condition.

Similarly, additional **Ground** features provide ground as a condition on edges in 3D models and ground as a condition at points in 2D and 3D models. You then select the edges in the **Edges** section or the points in the **Points** section, respectively. Beware that constraining the potential on edges or points in 3D or on points in 2D usually yields a current outflow that is mesh dependent.

Electric Potential

The **Electric Potential** feature provides an electric potential V_0 as the boundary condition:

$$V = V_0$$

Because you are solving for the electric potential in this interface, you typically define the value of the potential at some part of the geometry.

BOUNDARIES

Select the boundaries where you want to apply an electric potential as the boundary condition.

ELECTRIC POTENTIAL

Enter the value or expression for the electric potential in the edit field under **Electric potential**.

Similarly, **Electric Potential** features provide the electric potential as a condition on edges in 3D models and at points in 2D and 3D models. You then select the edges in the **Edges** section or the points in the **Points** section, respectively. Beware that constraining the potential on edges or points in 3D or on points in 2D usually yields a current outflow that is mesh dependent.

Normal Current Density

The **Normal Current Density** feature is applicable to exterior boundaries that represent either a source or a sink of current. It provides a condition for specifying the normal current density of an inward or outward current flow:

$$-\mathbf{n} \cdot \mathbf{J} = J_n$$

You then specify the normal current density using the inward current density J_n.

Alternatively, you can use the current density $\mathbf{J}_0$ to define the normal current density:

$$\mathbf{n} \cdot \mathbf{J} = \mathbf{n} \cdot \mathbf{J}_0$$

The normal current density is positive when the current flows inward toward the edge.

BOUNDARIES

Select the boundaries where you want to apply a current flow as the boundary condition using the normal current density.

NORMAL CURRENT DENSITY

From the **Type** list, choose **Inward current density** to enter a value or expression for the normal current density (SI unit: $\mathrm{A/m^2}$) in the J_n edit field. Use a positive value for an inward current flow or a negative value for an outward current flow. Choose **Current density** to enter values or expressions for the components of the current density in the $\mathbf{J}_0$ edit fields.

Distributed Impedance

The **Distributed Impedance** feature adds a distributed impedance boundary condition according to the following equation for exterior boundaries (setting $\mathbf{J}_2 = 0$) and interior boundaries assuming DC currents:

$$\mathbf{n} \cdot (\mathbf{J}_1 - \mathbf{J}_2) = \frac{\sigma_L}{d_L}(V - V_{\text{ref}})$$

$$\mathbf{n} \cdot (\mathbf{J}_1 - \mathbf{J}_2) = \frac{1}{Z_L}(V - V_{\text{ref}})$$

You can use the Distributed Impedance boundary condition to model a thin sheet of a resistive material. The sheet has the electric conductivity σ_L and the surface thickness d_L, or a general (resistive) layer impedance Z_{L} and it is connected to the reference potential V_{ref}.

$_L$The **Distributed Impedance** form contains the following sections:

BOUNDARIES

Select the boundaries where you want to apply a distributed impedance.

DISTRIBUTED IMPEDANCE

Enter the **Reference potential** V_{ref}, together with either a potentially complex valued **Layer impedance** Z_{L}, or the values or expressions for the **Electric conductivity** σ_s, **Relative permittivity** ε_{rs} and **Surface thickness** d. The default value for the surface thickness is 10^{-3} m (1 mm).

Electric Insulation

The **Electric Insulation** feature adds electric insulation as the boundary condition:

$$\mathbf{n} \cdot \mathbf{J} = 0$$

The Electric Insulation boundary condition means that no electric current flows into the boundary. This boundary condition is also applicable at symmetric boundaries where the potential is known to be symmetric with respect to the boundary.

Electric Insulation is the default boundary condition.

The **Electric Insulation** form contains the following section:

BOUNDARIES

Select the boundaries where you want to apply electric insulation.

Periodic Condition

The **Periodic Condition** feature can be used to define periodicity or antiperiodicity between two boundaries. You can also activate the feature on more than two boundaries, in which case the feature tries to identify two separate surfaces which can each consist of several connected boundaries.

For more complex geometries it may be necessary to use the **Destination Domains** Subfeature. This subfeature allows you to manually specify which boundaries constitute the source and destination surfaces. To add the subfeature, right click on the **Periodic Condition** feature and select **Add Destination Domains**.

Contact Resistance

You can use the contact resistance boundary condition

$$\mathbf{n} \cdot \mathbf{J}_1 = \frac{\sigma}{d_L}(V_1 - V_2)$$

$$\mathbf{n} \cdot \mathbf{J}_2 = \frac{\sigma}{d_L}(V_2 - V_1)$$

$$\mathbf{n} \cdot \mathbf{J}_1 = \frac{1}{Z_L}(V_1 - V_2)$$

$$\mathbf{n} \cdot \mathbf{J}_2 = \frac{1}{Z_L}(V_2 - V_1)$$

to model a thin layer of resistive material. The layer has the thickness d and the conductivity σ or a general (resistive) layer impedance Z_L. The indices 1 and 2 refer to the two sides of the boundary.

BOUNDARIES

Select the boundaries where you want to apply a contact resistance.

CONTACT RESISTANCE

Enter the **Layer impedance** Z_L, or the values or expressions for the **Electric conductivity** σ_L, **Relative permittivity** $\varepsilon_{r\mathrm{L}}$ and **Surface thickness** d_L. The default value for the surface thickness is 10^{-3} m (1 mm).

Continuity (Pair Feature)

Continuity is available as an option at interfaces between parts in an assembly. Also see Defining an Identity Pair.

BOUNDARIES

Select individual boundaries in an existing identity pair. This pair first has to be created.

PAIR SELECTION

Select the boundary pair where you want to define continuity. First an identity pair may have to be created.

Sector Symmetry (Pair Feature)

Select sector symmetry at interfaces between rotating objects where sector symmetry is used. It is only available for assembly interfaces. Also see Defining an Identity Pair in the *COMSOL Multiphysics User's Guide*.

BOUNDARIES

Select individual boundaries in an existing identity pair. This pair first has to be created.

PAIR SELECTION

Select the boundary pair where you want to define sector symmetry. First an identity pair may have to be created.

SECTOR SETTINGS

Specify the **Number of sectors** (must be <50), **Type of periodicity** (**Continuity** or **Antiperiodicity)** and the **Axis of rotation**.

Electric Shielding

The **Electric Shielding** feature provides an electric shielding boundary condition. It describes a thin layer of a highly conductive medium that shields the electric field. The sheet has the electric conductivity σ_s and the surface thickness d. The condition is represented by the following equation for interior boundaries and (setting $\mathbf{J}_2$=0) exterior boundaries assuming DC currents

$$\mathbf{n} \cdot (\mathbf{J}_1 - \mathbf{J}_2) = -\nabla_t \cdot d(\sigma_s \nabla_t V)$$

For the frequency domain and time dependent study types, also displacement currents are accounted for via the bulk relative permittivity of the sheet; ε_{rs} and the conservation laws change to:

$$\mathbf{n} \cdot (\mathbf{J}_1 - \mathbf{J}_2) = -\nabla_t \cdot d((\sigma_s + j\omega\varepsilon_0\varepsilon_{rs})\nabla_t V)$$

$$\mathbf{n} \cdot (\mathbf{J}_1 - \mathbf{J}_2) = -\nabla_t \cdot d\left(\sigma_s \nabla_t V + \varepsilon_0\varepsilon_{rs}\frac{\partial}{\partial t}\nabla_t V\right)$$

You can use this boundary condition when approximating a thin domain with a boundary to reduce the number of mesh elements.

BOUNDARIES

Select the boundaries where you want to apply an electric shielding as the boundary condition.

CONDUCTION CURRENT

Specify the electric conductivity of the boundary. By default, the electric conductivity comes from the material defined on the domain (**From material** is selected in the list under **Electric conductivity**), but you can also enter a value or expression for the conductivity in the edit field that opens when you select **User-defined** from that list.

ELECTRIC FIELD

Specify the relative permittivity. By default, the relative permittivity comes from the material defined on the domain (**From material** is selected in the list under **Relative permittivity**), but you can also enter a value or expression for the relative permittivity in the edit field that opens when you select **User-defined** from that list.

THIN LAYER

Specify the thickness of the surface.

Line Current Source

In 3D you can specify line sources along the edges of a geometry. To add a Line Current Source feature, right-click the **Electric Currents** node and choose **Line Current Source** from the **Edges** menu.

EDGES

Select the edges where you want to add a current source.

LINE CURRENT SOURCE

Apply a line current source Q_j (SI unit: A/m) to edges. This source represents electric current per unit length. You enter a value or expression for the current source in the Q_j edit field.

Point Current Source

It is possible to add point sources to both 2D and 3D models. To add a Point Current Source feature, right-click the **Electric Currents** node and choose **Point Current Source** from the **Points** menu.

POINTS

Select the points where you want to add a current source.

POINT CURRENT SOURCE

Apply a point current source Q_j (SI unit: A) to points. This source represents an electric current flowing out of the point. You enter a value or expression for the current source in the Q_j edit field.

External Surface Charge Accumulation

The **External Surface Charge Accumulation** boundary condition adds the accumulation of surface charge density on boundaries due to a flux of charged particles.

BOUNDARIES

Select the boundaries where you want to apply a surface charge accumulation.

SURFACE CHARGE ACCUMULATION

Enter the values or expressions for the normal ion current density $\mathbf{n}$ (SI unit: A/m^2) and the normal electron current density $\mathbf{n}_e$ (SI unit: A/m^2).

Dielectric Shielding

The **Dielectric Shielding** feature provides a dielectric shielding boundary condition. It describes a thin layer with thickness d_s and a bulk relative permittivity; ε_{rs} that shields the electric field:

$$\mathbf{n} \cdot \mathbf{D} = -\nabla_t \cdot \varepsilon_0 \varepsilon_{rs} d_s \nabla_t V$$

You can use this boundary condition when approximating a thin domain with a boundary to reduce the number of mesh elements.

BOUNDARIES OR EDGES

Select the boundaries or edges (3D models) where you want to apply a dielectric shielding as the condition.

ELECTRIC FIELD

The default is to take the **Relative permittivity** (e_r) values **From material**. If **User-defined** is selected, select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** and enter values or expressions in the field or matrix.

THIN LAYER

Enter a **Surface thickness** d_s of the shielding (SI unit: m).

COORDINATE SYSTEM SELECTION

Select a **Coordinate system**. By default you use the **Global coordinate system**, and the list contains any additional coordinate systems that the model includes.

Terminal

The **Terminal** feature provides a boundary condition for connection to external circuits or with a specified voltage or charge. By specifying zero charge, a floating potential condition is obtained.

BOUNDARIES

Select the boundaries that you want to model as terminals connected to external circuits or an external charge or voltage.

TERMINAL

Specify the terminal's properties. To indicate which boundaries that belong to the same terminal, enter the same name in the **Terminal name** field. The **Terminal name** should be numeric for port sweeps to work properly.

Select a **Terminal type**—**Voltage**, **Charge**, or **Circuit**. Select:

- **Voltage** to enter a voltage V_0 (SI unit: V).
- **Charge** to enter a charge Q_0 (SI unit: C). The default is zero charge for an electrode at floating potential.
- **Circuit** to specify a terminal connected to an external circuit.

Floating Potential

The **Floating Potential** feature is used when modeling a metallic electrode at floating potential. For circuit connections use the Terminal feature instead.

BOUNDARIES

Select the boundaries where you want to define the floating electrode.

FLOATING POTENTIAL

Specify a an optionally non zero charge Q_0 in the **Terminal charge** edit field.

Distributed Capacitance

The **Distributed Capacitance** feature adds a distributed capacitance boundary condition according to the following equations for exterior boundaries (left) and interior boundaries (right):

$$-\mathbf{n}\cdot\mathbf{D} = \varepsilon_0\varepsilon_{rL}\frac{V_{ref}-V}{d_L} \qquad \mathbf{n}\cdot(\mathbf{D}_1-\mathbf{D}_2) = \varepsilon_0\varepsilon_{rL}\frac{V_{ref}-V}{d_L}$$

You can use the Distributed Capacitance boundary condition to model a thin sheet or film of a dielectric material. The sheet has the relative permittivity ε_{rL} and the surface thickness d_L, and it is connected to the reference potential V_{ref}.

BOUNDARIES

Select the boundaries where you want to apply a distributed capacitance.

DISTRIBUTED CAPACITANCE

Enter the values or expressions for **Relative permittivity** e_r, **Surface thickness** d_s (SI unit: m), and **Reference potential** V_{ref} (SI unit: V). The default value for the surface thickness is 10^{-3} m (1 mm).

The Magnetic Fields Interface

The **Magnetic Fields** interface ( ) provides the equations, boundary conditions, and external currents for modeling magnetic fields, solving for the magnetic vector potential. The main feature is the **Ampère's Law** feature, which adds the equation for the magnetic vector potential and provides an interface for defining the constitutive relation and its associated properties such as the relative permeability.

For a more thorough introduction to the equations solved by this physics interface, see the Theory for the Magnetic Fields Interface.

When you add a Magnetic Fields interface, it creates a **Magnetic Fields** node with a default **Ampère's Law** node added as well as a **Magnetic Insulation** node. Magnetic insulation is the default boundary condition. There is also a default **Initial Values** node. Right-click the **Magnetic Fields** node to add other nodes that implement, for example, boundary conditions and external currents. The following sections provide information about all feature nodes available in the Magnetic Fields interface.

DOMAINS

Select the domains where you want to define the magnetic vector potential and the equations that describe the potential field for magnetic fields. The default setting is to include all domains in the model.

INTERFACE IDENTIFIER

This is the physics interface identifier, which you use to reach the fields and variables in expressions, for example.

The identifier appears in the **Identifier** edit field, and you can change it to any unique string that is a valid identifier. The default identifier (for the first Magnetic Fields interface in the model) is `mf`.

OUT-OF-PLANE THICKNESS (2D MODELS ONLY)

Define the out-of-plane thickness d by entering a value or expression (SI unit: m) in the **Thickness** edit field. The default value of 1 m is typically not representative for a thin domain. Instead it describes a unit thickness that makes the 2D equation identical to the equation used for 3D models.

ADVANCED SETTINGS AND DISCRETIZATION

Normally these settings do not need to be changed. See Show More Options: Advanced Settings and Discretization.

Ampère's Law

The **Ampère's Law** form contains the following sections for defining the related material properties:

DOMAINS

Select the domains where you want to define the magnetic vector potential and the equation based on Ampère's law that defines the potential.

MODEL INPUTS

This section contains field variables that appear as model inputs, if the current settings include such model inputs. By default, this section is empty. If you add a linear temperature relation for the conductivity, you can then define the source for the temperature T. From the **Temperature** list, select an existing temperature variable (from another physics interface) if available, or select **User defined** to define a value or expression for the temperature (SI unit: K) in the edit field that appears underneath the list.

MAGNETIC FIELD

Select the **Constitutive relation** that describe the macroscopic properties of the medium (relating the magnetic flux density $\mathbf{B}$ and the magnetic field $\mathbf{H}$) and the applicable material properties, such as the relative permeability. The equation for the selected constitutive relation appears under the list.

- Select **Relative permeability** to use the constitutive relation $\mathbf{B} = \mu_0\mu_r\mathbf{H}$ (the default). The default is to use the value for μ_r **From material**. If **User defined** is selected, select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** and enter values or expressions in the field or matrix.
- Select **Magnetization** to use the constitutive relation $\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$. You then specify $\mathbf{M}$, the magnetization vector (SI unit: A/m). Enter its components (3 in 3D, 2 in 2D) in the edit fields in the table that appears under the constitutive equation.
- Select **Magnetic losses** to describe the relative permeability as a complex-valued quantity: $\mu_r = \mu' + i\mu''$, where μ' and μ'' are the real and imaginary parts, respectively. The default is to use the values **From material**. If **User defined** is selected for either variable, enter values or expressions for the real and imaginary parts.

- Select **HB curve** to use a curve that relates magnetic flux density $\mathbf{B}$ and the magnetic field $\mathbf{H}$ as $|\mathbf{H}| = f(|\mathbf{B}|)$. The default is to use the value $|\mathbf{H}|$, **From material**. If **User defined** is selected, enter a value or expression for the magnitude of the magnetic field.
- Select **Remanent flux density** to use the constitutive relation $\mathbf{B} = \mu_0\mu_r\mathbf{H} + \mathbf{B}_r$, where $\mathbf{B}_r$ is the remanent flux density (the flux density when no magnetic field is present). In this case you specify μ_r, the relative permeability (unitless), and $\mathbf{B}_r$, the remanent flux density (SI unit: T). For the relative permeability, use the μ_r list: Select **From material** (the default) to use the value from the material or select **User defined** to specify a value or expression for the relative permeability in the edit field that appears. You can select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** from the list under the edit field depending on the characteristics of the relative permeability. For the remanent displacement, enter its components (3 in 3D, 2 in 2D) in the $\mathbf{B}_r$ table.

CONDUCTION CURRENT

Specify the electric conductivity for the media. You can define the electric conductivity in three ways by selecting from the **Electric conductivity** list:

- By default, the electric conductivity comes from the material defined on the domain (select **From material**).
- Select **Linearized resistivity** to define the electric resistivity (and conductivity) as a linear function of temperature. The following equation then describes the conductivity:

$$\sigma = \frac{1}{\rho_0(1 + \alpha(T - T_0))}$$

  where ρ_0 is the resistivity at the reference temperature T_0. α is the temperature coefficient of resistance, which describes how the resistivity varies with temperature. The default setting in the corresponding **Reference resistivity**, **Reference temperature**, and **Resistivity temperature coefficient** lists is **From material**, which means that the values are taken from the boundary material. To specify another values for any of these properties, select **User defined** from the corresponding list and then enter a value or expression in the edit field that appears. T is the current temperature, which can be a value that you specify as a model input or the temperature from a heat transfer interface. The definition of the temperature field appears in the **Model Inputs** section.
- You can also enter a value or expression for the conductivity σ (SI unit: S/m) in the edit field that opens when you select **User defined** from that list. You can also choose

Isotropic, **Diagonal**, **Symmetric**, or **Anisotropic** from the list under the edit field depending on the characteristics of the electric conductivity.

ELECTRIC FIELD

Specify the relative permittivity for the media. By default, the relative permittivity comes from the material defined on the shell domain (**From material** is selected in the list under **Relative permittivity**), but you can also enter a value or expression for the relative permittivity ε_r (unitless) in the edit field that opens when you select **User defined** from that list. In addition, you can choose **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** from the list under the edit field depending on the characteristics of the permittivity.

External Current Density

The **External Current Density** feature adds an externally generated current density $\mathbf{J}^e$, which appears on the right-hand side of the equation that the Magnetic Fields interface defines.

DOMAINS

Select the domains where you want to define an external current density.

EXTERNAL CURRENT DENSITY

Enter a value or expression for each component of the external current density in the $\mathbf{J}^e$ table under **External current density**.

COORDINATE SYSTEM SELECTION

Specify in what coordinate system the **External current density** is specified.

Velocity (Lorentz Term)

The **Velocity (Lorentz term)** feature adds an external current density $\mathbf{J}^e$ induced by a velocity $\mathbf{v}$. The external current is equal to $\sigma\mathbf{v} \times \mathbf{B}$.

Initial Values

The **Initial Values** feature adds an initial value for the magnetic vector potential A that can serve as an initial value for a transient simulation or as an initial guess for a nonlinear solver.

BOUNDARIES

Select the domains where you want to define an initial value.

INITIAL VALUES

Enter a value or expression for the initial value of the magnetic vector potential A in the **Magnetic vector potential** edit field. The default value is 0 Wb/m.

Boundary Conditions

With no surface currents present the interface conditions

$$\mathbf{n}_2 \times (\mathbf{A}_1 - \mathbf{A}_2) = \mathbf{0}$$
$$\mathbf{n}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{0}$$

need to be fulfilled. As we are solving for $\mathbf{A}$, the tangential component of the magnetic potential is always continuous, and thus the first condition is automatically fulfilled. The second condition is equivalent to the natural boundary condition and is hence also fulfilled unless surface currents are explicitly introduced.

The following boundary conditions are available on exterior boundaries:

- Magnetic Insulation (the default boundary condition)
- Magnetic Field
- Surface Current
- Magnetic Potential
- Perfect Magnetic Conductor

In addition, the following boundary conditions are available on interior boundaries:

- Magnetic Insulation
- Magnetic Potential
- Surface Current

The following boundary conditions are only available on assembly boundary pairs:

- Continuity (Pair Feature)

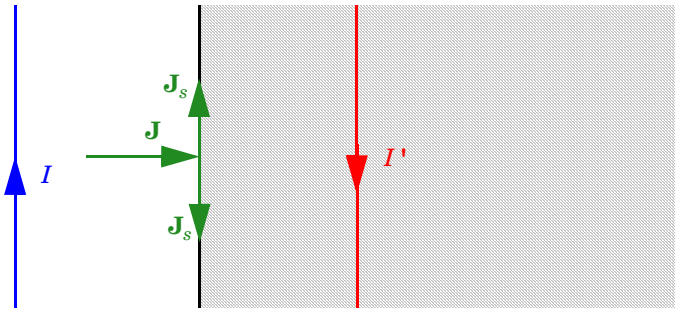
For axisymmetric models, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and automatically adds an **Axial Symmetry** feature to the model that is valid on the axial symmetry boundaries only.

Magnetic Insulation

The **Magnetic Insulation** feature adds a boundary condition that sets the tangential components of the magnetic potential to zero at the boundary:

$$\mathbf{n} \times \mathbf{A} = \mathbf{0}$$

It is a special case of the magnetic potential boundary condition that sets the tangential component of the magnetic potential to zero. It is used for the modeling of a lossless metallic surface, for example a ground plane or as a symmetry type boundary condition. It imposes symmetry for magnetic fields and "magnetic currents". In the transient and time harmonic formulations it also imposes antisymmetry for electric fields and electric currents. It supports induced electric surface currents and thus any prescribed or induced electric currents (volume, surface or edge currents) flowing into a perfect electric conductor boundary is automatically balanced by induced surface currents.



The magnetic insulation boundary condition is used on exterior and interior boundaries representing the surface of a lossless metallic conductor or (on exterior boundaries) representing a symmetry cut. The shaded (metallic) region is not part of the model but still carries effective mirror images of the sources. Note also that any current flowing into the boundary is perfectly balanced by induced surface currents. The tangential vector potential (and electric field) vanishes at the boundary.

The term *magnetic insulation* comes from the fact that this boundary condition makes the normal component of the magnetic field zero.

Magnetic insulation is the default boundary condition for the Magnetic Fields interface.

BOUNDARIES

Select the boundaries where you want to specify magnetic insulation.

Magnetic Field

The **Magnetic Field** feature adds a boundary condition for specifying the tangential component of the magnetic field at the boundary:

$$\mathbf{n} \times \mathbf{H} = \mathbf{n} \times \mathbf{H}_0$$

BOUNDARIES

Select the boundaries where you want to specify the magnetic field.

MAGNETIC FIELD

Enter the value or expression for the components of the magnetic field $\mathbf{H}_0$.

Surface Current

The **Surface Current** feature adds a boundary condition for a surface current density J_s:

$$-\mathbf{n} \times \mathbf{H} = \mathbf{J}_s$$
$$\mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$$

BOUNDARIES

Select the boundaries where you want to specify a surface current.

SURFACE CURRENT

Enter values or expressions for the components of the surface current density in the J_s edit fields.

Magnetic Potential

The **Magnetic Potential** feature adds a boundary condition for the magnetic vector potential:

$$\mathbf{n} \times \mathbf{A} = \mathbf{n} \times \mathbf{A}_0$$

BOUNDARIES

Select the boundaries where you want to specify the magnetic potential.

MAGNETIC POTENTIAL

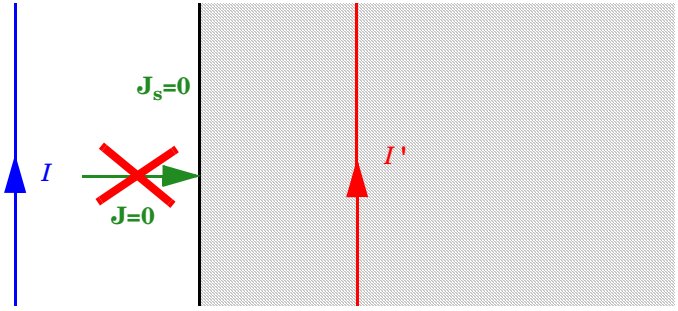
Enter a value or expression for the magnetic vector potential in the A_0 edit field.

Perfect Magnetic Conductor

The perfect magnetic conductor boundary condition

$$\mathbf{n} \times \mathbf{H} = \mathbf{0}$$

is a special case of the surface current boundary condition that sets the tangential component of the magnetic field and thus also the surface current density to zero. On external boundaries, this can be interpreted as a "high surface impedance" boundary condition or used as a symmetry type boundary condition. It imposes symmetry for electric fields and electric currents. Electric currents (volume, surface, or edge currents) are not allowed to flow into a perfect magnetic conductor boundary as that would violate current conservation.



The perfect magnetic conductor boundary condition is used on exterior boundaries representing the surface of a high impedance region or a symmetry cut. The shaded (high impedance) region is not part of the model but nevertheless carries effective mirror images of the sources. Note also that any electric current flowing into the boundary is forbidden as it cannot be balanced by induced electric surface currents. The tangential magnetic field vanishes at the boundary.

BOUNDARIES

Select the boundaries that you want to model as perfect magnetic conductors.

Thin Low Permeability Gap

You can use the thin low permeability gap boundary condition

$$\mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = \nabla_t \times \frac{d}{\mu_0 \mu_r} \nabla_t \times \mathbf{A}$$

to model gaps filled with a material with zero conductivity such as air. This boundary condition is only applicable on interior boundaries and pair boundaries.

BOUNDARIES

Select the boundaries where you want to model a thin low permeability gap.

THIN LOW PERMEABILITY GAP

Specify the relative permeability μ_r (unitless) for the gap in the **Relative permeability** edit field. Select **From material** (the default) to use a value taken from the material. To specify another value f select **User-defined** and then enter a value or expression for the relative permeability in the edit field that appears. You can select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** from the list under the edit field depending on the characteristics of the relative permeability.

You also specify the surface thickness d_s (SI unit: m) for the gap in the **Surface thickness** edit field.

Continuity (Pair Feature)

Continuity is available as an option at interfaces between parts in an assembly. Also see Defining an Identity Pair.

BOUNDARIES

Select individual boundaries in an existing identity pair. This pair first has to be created.

PAIR SELECTION

In this section you select the boundary pair where you want to define continuity. First an identity pair may have to be created.

Edge Current

The **Edge Current** feature allows you to specify a line current along one or more edges. Edges

Select the edges that you want to an edge current.

EDGE CURRENT

Specify the **Edge current**.

Line Current (Out of Plane)

The **Line Current (out of Plane)** feature allows you to specify a line current out of the modeling plane. In axially symmetric geometries this is the rotational direction, in 2D geometries this is the z-direction.

POINTS

Select the points where you want to add a line current.

LINE CURRENT (OUT OF PLANE)

Specify the **Out of plane current**.

Fundamentals of Electromagnetics

Maxwell's Equations

The problem of electromagnetic analysis on a macroscopic level is that of solving *Maxwell's equations* subject to certain boundary conditions. Maxwell's equations are a set of equations, written in differential or integral form, stating the relationships between the fundamental electromagnetic quantities. These quantities are:

- The *electric field intensity*, **E**
- The *electric displacement* or *electric flux density*, **D**
- The *magnetic field intensity*, **H**
- The *magnetic flux density*, **B**
- The *current density*, **J**
- The *electric charge density*, ρ

You can formulate the equations in differential or integral form. This discussion presents them in differential form because it leads to differential equations that the finite element method can handle.

For general time-varying fields, Maxwell's equations are

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

The first two equations are also referred to as *Maxwell-Ampère's law* and *Faraday's law*, respectively. The last two are forms of *Gauss' law* in the electric and magnetic form, respectively.

Another fundamental relationship is the *equation of continuity*:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

Out of these five equations only three are independent. The first two combined with either the electric form of Gauss' law or the equation of continuity form an independent system.

Constitutive Relationships

To obtain a closed system, you need the *constitutive relationships* describing the macroscopic properties of the medium. They are

$$\begin{aligned} \mathbf{D} &= \varepsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{B} &= \mu_0(\mathbf{H} + \mathbf{M}) \\ \mathbf{J} &= \sigma \mathbf{E} \end{aligned} \tag{9-1}$$

where ε_0 is the *permittivity of vacuum*, μ_0 is the *permeability of vacuum*, and σ is the *electric conductivity*. In the SI system the permeability of a vacuum is $4\pi \cdot 10^{-7}$ H/m. The velocity of an electromagnetic wave in a vacuum is given as c_0, and you can derive the permittivity of a vacuum from the relationship

$$\varepsilon_0 = \frac{1}{c_0^2 \mu_0} = 8.854 \cdot 10^{-12} \text{ F/m} \approx \frac{1}{36\pi} \cdot 10^{-9} \text{ F/m}$$

The electromagnetic constants ε_0, μ_0, and c_0 are available in COMSOL Multiphysics as predefined *physical constants*.

The *electric polarization vector* $\mathbf{P}$ describes how a material is polarized when an electric field $\mathbf{E}$ is present. It can be interpreted as the volume density of electric dipole moments. $\mathbf{P}$ is generally a function of $\mathbf{E}$. Some materials can have a nonzero $\mathbf{P}$ in the absence of an electric field.

The *magnetization vector* $\mathbf{M}$ similarly describes how a material is magnetized when a magnetic field $\mathbf{H}$ is present. It can be interpreted as the volume density of magnetic dipole moments. $\mathbf{M}$ is generally a function of $\mathbf{H}$. One use of the magnetization vector is to describe permanent magnets, which have a nonzero $\mathbf{M}$ when no magnetic field is present.

For linear materials the polarization is directly proportional to the electric field, $\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$, where χ_e is the *electric susceptibility*. Similarly, in linear materials the magnetization is directly proportional to the magnetic field, $\mathbf{M} = \chi_m \mathbf{H}$, where χ_m is the *magnetic susceptibility*. For such materials the constitutive relations are

$$\mathbf{D} = \varepsilon_0(1+\chi_e)\mathbf{E} = \varepsilon_0\varepsilon_r\mathbf{E} = \varepsilon\mathbf{E}$$
$$\mathbf{B} = \mu_0(1+\chi_m)\mathbf{H} = \mu_0\mu_r\mathbf{H} = \mu\mathbf{H}$$

where ε_r is the material's *relative permittivity*, and μ_r is its *relative permeability*. Usually these are scalar properties but can, in the general case, be 3-by-3 tensors when the material is anisotropic. The properties ε and μ (without subscripts) are the material's *permittivity* and *permeability*.

GENERALIZED CONSTITUTIVE RELATIONSHIPS

For nonlinear materials, a generalized form of the constitutive relationships is useful. The relationship used for electric fields is

$$\mathbf{D} = \varepsilon_0\varepsilon_r\mathbf{E} + \mathbf{D}_r$$

The field $\mathbf{D}_r$ is the *remanent displacement*, which is the displacement when no electric field is present.

Similarly, a generalized form of the constitutive relationship for the magnetic field is

$$\mathbf{B} = \mu_0\mu_r\mathbf{H} + \mathbf{B}_r$$

where $\mathbf{B}_r$ is the *remanent magnetic flux density*, which is the magnetic flux density when no magnetic field is present.

You can generalize the third line in Equation 9-1 by introducing an externally generated current $\mathbf{J}^e$. This relationship is then

$$\mathbf{J} = \sigma\mathbf{E} + \mathbf{J}^e$$

Potentials

Under certain circumstances it can be helpful to formulate a problem in terms of the *electric scalar potential* V and *magnetic vector potential* $\mathbf{A}$. They are given by the equalities

$$\mathbf{B} = \nabla \times \mathbf{A}$$
$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

which are direct consequences of the magnetic case of Gauss' law and Faraday's law, respectively.

Material Properties

This discussion has so far only formally introduced the constitutive relationships. These seemingly simple relationships can be quite complicated at times. In fact, these relationships require some special considerations when working with four main groups of materials:

- Inhomogeneous materials
- Anisotropic materials
- Nonlinear materials
- Dispersive materials

A material can belong to one or more of these groups.

INHOMOGENEOUS MATERIALS

Inhomogeneous materials are the least complicated. An inhomogeneous medium is one in which the constitutive parameters vary with the space coordinates so that different field properties prevail at different parts of the material structure.

ANISOTROPIC MATERIALS

For anisotropic materials the field relationships at any point differ for different directions of propagation. This means that a 3-by-3 tensor is necessary to properly define the constitutive relationships. If this tensor is symmetric, the material is often referred to as *reciprocal*. In such cases you can rotate the coordinate system such that a diagonal matrix results. If two of the diagonal entries are equal, the material is *uniaxially anisotropic*; if none of the elements have the same value, the material is *biaxially anisotropic* (Ref. 2). You need anisotropic parameters, for instance, to examine permittivity in crystals (Ref. 2) and when working with conductivity in solenoids.

NONLINEAR MATERIALS

In some nonlinear materials the permittivity or permeability depend on the intensity of the electromagnetic field. Nonlinearity also includes hysteresis effects where not only the existing field intensities influence a material's physical properties but the history of the field distribution also plays a role.

DISPERSIVE MATERIALS

Dispersion describes changes in a wave's velocity with wavelength. In the frequency domain you can express dispersion with a frequency dependence of the constitutive laws.

Boundary and Interface Conditions

To get a full description of an electromagnetics problem, you must also specify boundary conditions at material interfaces and physical boundaries. At interfaces between two media, you can mathematically express the boundary conditions as

$$\begin{aligned}
\mathbf{n}_2 \times (\mathbf{E}_1 - \mathbf{E}_2) &= \mathbf{0} \\
\mathbf{n}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) &= \rho_s \\
\mathbf{n}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) &= \mathbf{J}_s \\
\mathbf{n}_2 \cdot (\mathbf{B}_1 - \mathbf{B}_2) &= 0
\end{aligned}$$

where ρ_s and $\mathbf{J}_s$ denote the *surface charge density* and *surface current density*, respectively, and $\mathbf{n}_2$ is the outward normal from medium 2. Of these four equations, only two are independent. This is an overdetermined system of equations, so you need to reduce it. First select either equation one or equation four. Then select either equation two or equation three. Together these selections form a set of two independent conditions.

From these relationships, you can derive the interface condition for the current density,

$$\mathbf{n}_2 \cdot (\mathbf{J}_1 - \mathbf{J}_2) = -\frac{\partial \rho_s}{\partial t}$$

Interface Between a Dielectric and a Perfect Conductor

A perfect conductor has infinite electrical conductivity and as such has no internal electric field. Otherwise it would produce an infinite current density according to the third fundamental constitutive relationship. At an interface between a dielectric and a perfect conductor, the boundary conditions for the $\mathbf{E}$ and $\mathbf{D}$ fields are simplified. Assume that subscript 1 corresponds to a perfect conductor; then $\mathbf{D}_1 = \mathbf{0}$ and $\mathbf{E}_1 = \mathbf{0}$ in the relationships just given. If, in addition, you are dealing with a time-varying case, then $\mathbf{B}_1 = \mathbf{0}$ and $\mathbf{H}_1 = \mathbf{0}$, as well, as a consequence of Maxwell's equations. The result is the following set of boundary conditions for the fields in the dielectric medium for the time-varying case:

$$-\mathbf{n}_2 \times \mathbf{E}_2 = 0$$
$$-\mathbf{n}_2 \times \mathbf{H}_2 = \mathbf{J}_s$$
$$-\mathbf{n}_2 \cdot \mathbf{D}_2 = \rho_s$$
$$-\mathbf{n}_2 \cdot \mathbf{B}_2 = 0$$

Electromagnetic Forces

The Magnetic Field interface contains a predefined domain-level variable for computing the Lorentz force, which gives the force distribution exerted on a current-carrying conductor placed in magnetic flux density **B**. The Lorentz force is defined as

$$\mathbf{F} = \mathbf{J} \times \mathbf{B} \tag{9-2}$$

The Lorentz force gives very good accuracy for electromagnetic force calculations in conducting domains. For nonconducting domains you can use a more general method: integrating the Maxwell stress tensor variables over the boundaries of the object for which to calculate the total force. The Maxwell surface stress tensor is available as a boundary variable.

REFERENCES FOR THE ACDC INTERFACES

1. D.K. Cheng, *Field and Wave Electromagnetics*, Addison-Wesley, Reading, MA, 1989.

2. J. Jin, *The Finite Element Method in Electromagnetics*, John Wiley & Sons, New York, 1993.

3. B.D. Popovic, *Introductory Engineering Electromagnetics*, Addison-Wesley, Reading, MA, 1971.

Electrostatic Fields

Physics interfaces for the modeling of static electric fields and currents are offered in COMSOL Multiphysics. Physics interfaces for the modeling of dynamic, quasi-static (that is without including wave propagation effects) electric fields and currents are available in the AC/DC and MEMS Modules. What physics interface and study type to select for a particular modeling situation requires a basic understanding of the charge dynamics in conductors. This section is a brief introduction to that topic. After reading it, you should be more confident when deciding what physics interface and study type to use, depending on the material parameters and characteristic time scales involved.

Charge Relaxation Theory

The different physics interfaces involving only the scalar electric potential can be interpreted in terms of the charge relaxation process. The fundamental equations involved are *Ohm's law*

$$\mathbf{J} = \sigma\mathbf{E}$$

the *Equation of continuity*

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$$

and *Gauss' law*

$$\nabla \cdot (\varepsilon\mathbf{E}) = \rho$$

By combining these, one can deduce the following differential equation for the space charge density in a homogeneous medium

$$\frac{\partial \rho}{\partial t} + \frac{\sigma}{\varepsilon}\rho = 0$$

This equation has the solution

$$\rho(t) = \rho_0 e^{-t/\tau}$$

where

$$\tau = \frac{\varepsilon}{\sigma}$$

is called the charge relaxation time. For a good conductor like copper, τ is of the order of 10^{-19} s whereas for a good insulator like silica glass, it is of the order of 10^{3} s. For a pure insulator, it becomes infinite.

When modeling real world devices, there is not only the intrinsic time scale of charge relaxation time but also an external time scale t at which a device is energized or the observation time. It is the relation between the external time scale and the charge relaxation time that determines what physics interface and study type to use. The results are summarized in Table 9-1 below,

TABLE 9-1: SUITABLE PHYSICS INTERFACE AND STUDY TYPE FOR DIFFERENT TIME-SCALE REGIMES.

CASE	PHYSICS INTERFACE	STUDY TYPE
$\tau>>t$	Electrostatics	Stationary
$\tau<<t$	Electric Currents	Stationary
$\tau\sim t$	Electric Currents	Time Dependent or Frequency Domain (in AC/DC Module or MEMS Module)

First Case: $\tau >> t$

If the external time scale is short compared to the charge relaxation time, the charges do not have time to redistribute to any significant degree.Thus the charge distribution can be considered as given model input and the best approach is to solve the *Electrostatics* formulation using the electric potential *V*.

By combining the definition of the potential with Gauss' law, you can derive the classical Poisson's equation. Under static conditions, the electric potential *V* is defined by the equivalence

$$\mathbf{E} = -\nabla V$$

Using this together with the constitutive relation $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$ between $\mathbf{D}$ and $\mathbf{E}$, you can rewrite Gauss' law as a variant of Poisson's equation

$$-\nabla \cdot (\varepsilon_0 \nabla V - \mathbf{P}) = \rho$$

This equation is used in the Electrostatics interface. It is worth noting that Gauss' law does not require the charge distribution to be static. Thus, provided dynamics are slow enough that induced electric fields can be neglected and hence a scalar electric potential is justified, the formulation can be used also in the time dependent study type. That typically involves either prescribing the charge dynamics or coupling a

separate formulation for this. Such separate charge transport formulations can be found in the Plasma Module and the Chemical Reaction Engineering Module.

Second Case: τ <<t

If the external time scale is long compared to the charge relaxation time, the stationary solution to the equation of continuity has been reached. In a stationary coordinate system, a slightly more general form than above of *Ohm's law* states that

$$\mathbf{J} = \sigma\mathbf{E} + \mathbf{J}^{\mathrm{e}}$$

where $\mathbf{J}^{\mathrm{e}}$ is an externally generated current density. The static form of the equation of continuity then reads

$$\nabla \cdot \mathbf{J} = -\nabla \cdot (\sigma \nabla V - \mathbf{J}^{\mathrm{e}}) = 0$$

To handle current sources the equation can be generalized to

$$-\nabla \cdot (\sigma \nabla V - \mathbf{J}^{\mathrm{e}}) = Q_j$$

This equation is used in the static study type for the Electric Currents interface.

Theory for the Electrostatics Interface

The **Electrostatics** interface is available for 3D, 2D in-plane, and 2D axisymmetric models.

ELECTROSTATICS EQUATIONS

Under static conditions the electric potential, V, is defined by the relationship

$$\mathbf{E} = -\nabla V$$

Combining this equation with the constitutive relationship $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$ between the electric displacement $\mathbf{D}$ and the electric field $\mathbf{E}$, it is possible to represent Gauss' law as the following equation:

$$-\nabla \cdot (\varepsilon_0 \nabla V - \mathbf{P}) = \rho$$

In this equation, ε_0 is the permittivity of vacuum (a physical constant; SI unit: F/m); $\mathbf{P}$ is the electric polarization vector (SI unit: C/m^2); and ρ is a space charge density (SI unit: C/m^3).

This equation describes the electrostatic field in dielectric materials.

For in-plane 2D modeling, the Electrostatics interface assumes a symmetry where the electric potential varies only in the x and y directions and is constant in the z direction. This implies that the electric field, $\mathbf{E}$, is tangential to the xy-plane. Given this symmetry, you solve the same equation as in the 3D case. The Electrostatics interface solves the following equation where d is the thickness in the z direction:

$$-\nabla \cdot d(\varepsilon_0 \nabla V - \mathbf{P}) = \rho$$

The axisymmetric version of the **Electrostatics** interface considers the situation where the fields and geometry are axially symmetric. In this case the electric potential is constant in the φ direction, which implies that the electric field is tangential to the rz-plane.

Theory for the Electric Currents Interface

The **Electric Currents** interface solves a current conservation problem for the scalar electric potential V and is available for 3D, 2D in-plane, and 2D axisymmetric models. Electrolysis and the computation of resistances of grounding plates are examples that involve conductive media with electric conductivity σ and electric currents. If you are uncertain whether to use the **Electric Currents** interface or the **Electrostatics** interface which both solve for the scalar electric potential V, please refer to the preceding section on Charge Relaxation Theory.

ELECTRIC CURRENTS EQUATIONS IN STEADY STATE

When handling stationary electric currents in conductive media you must consider the stationary equation of continuity. In a stationary coordinate system, the point form of *Ohm's law* states that

$$\mathbf{J} = \sigma\mathbf{E} + \mathbf{J}_e$$

where σ is the electric conductivity (SI unit: S/m), and $\mathbf{J}_e$ is an *externally generated current density* (SI unit: A/m^2). The static form of the equation of continuity then states

$$\nabla \cdot \mathbf{J} = -\nabla \cdot (\sigma\nabla V - \mathbf{J}_e) = 0$$

To handle *current sources*, you can generalize the equation to

$$-\nabla \cdot (\sigma\nabla V - \mathbf{J}_e) = Q_j$$

In planar 2D the *Electric Currents* interface assumes that the model has a symmetry where the electric potential varies only in the x and y directions and is constant in the z direction. This implies that the electric field, $\mathbf{E}$, is tangential to the xy-plane. The Electric Currents interface then solves the following equation where d is the thickness in the z direction:

$$-\nabla \cdot d(\sigma\nabla V - \mathbf{J}_e) = dQ_j \qquad (9\text{-}3)$$

In 2D axisymmetry, the *Electric Currents* interface considers the situation where the fields and geometry are axially symmetric. In this case the electric potential is constant in the φ direction, which implies that the electric field is tangential to the rz-plane.

Magnetostatic and Quasistatic Fields

Quasi-static analysis of magnetic and electric fields is valid under the assumption that $\partial \mathbf{D}/\partial t = 0$.

Maxwell's Equations

This implies that it is possible to rewrite Maxwell's equations in the following manner:

$$\nabla \times \mathbf{H} = \mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \mathbf{J}_e$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{J} = 0$$

Here $\mathbf{J}_e$ is an externally generated current density, and $\mathbf{v}$ is the velocity of the conductor. The crucial criterion for the quasi-static approximation to be valid is that the currents and the electromagnetic fields vary slowly. This means that the dimensions of the structure in the problem need to be small compared to the wavelength.

Magnetic and Electric Potentials

Using the definitions of the potentials,

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

and the constitutive relation $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$, Ampère's law can be rewritten as

$$\sigma \frac{\partial \mathbf{A}}{\partial t} + \nabla \times (\mu_0^{-1} \nabla \times \mathbf{A} - \mathbf{M}) - \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) + \sigma \nabla V = \mathbf{J}_e \tag{9-4}$$

The equation of continuity, which is obtained by taking the divergence of the above equation, adds the following equation:

$$\nabla \cdot \left(-\sigma \frac{\partial \mathbf{A}}{\partial t} + \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) - \sigma \nabla V + \mathbf{J}_e \right) = 0 \tag{9-5}$$

Equation 9-4 and Equation 9-5 form a system of equations for the two potentials $\mathbf{A}$ and V.

Gauge Transformations

The electric and magnetic potentials are not uniquely defined from the electric and magnetic fields through

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla V$$
$$\mathbf{B} = \nabla \times \mathbf{A}$$

Introducing two new potentials

$$\tilde{\mathbf{A}} = \mathbf{A} + \nabla \Psi$$
$$\tilde{V} = V - \frac{\partial \Psi}{\partial t}$$

gives the same electric and magnetic fields:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla V = -\frac{\partial (\tilde{\mathbf{A}} - \nabla \Psi)}{\partial t} - \nabla\left(\tilde{V} + \frac{\partial \Psi}{\partial t}\right) = -\frac{\partial \tilde{\mathbf{A}}}{\partial t} - \nabla \tilde{V}$$
$$\mathbf{B} = \nabla \times \mathbf{A} = \nabla \times (\tilde{\mathbf{A}} - \nabla \Psi) = \nabla \times \tilde{\mathbf{A}}$$

The variable transformation of the potentials is called a *gauge transformation*. To obtain a unique solution you need to choose the gauge, that is, put constraints on Ψ that make the solution unique. Another way of expressing this additional condition is to put a constraint on $\nabla \cdot \mathbf{A}$. A vector field is uniquely defined up to a constant if both $\nabla \cdot \mathbf{A}$ and $\nabla \times \mathbf{A}$ are given. This is called *Helmholtz's theorem*.

One particular gauge is the *Coulomb gauge* given by the constraint:

$$\nabla \cdot \mathbf{A} = 0$$

Selecting a Particular Gauge

Important observations are that in the dynamic case $\mathbf{A}$ and V are coupled via the selected gauge. For a dynamic formulation, it is also possible to select a Ψ such that the scalar electric potential vanishes and only the magnetic vector potential has to be considered. The dynamic formulations (frequency domain and time dependent study types) of the Magnetic Fields interface are operated in this gauge as it involves only $\mathbf{A}$. In the static limit, $\mathbf{A}$ and V are not coupled via the gauge selection and thus any gauge can be chosen for $\mathbf{A}$ when performing magnetostatic modeling.

The Gauge and the Equation of Continuity for Dynamic Fields

After eliminating the electric potential by choosing the appropriate gauge and disregarding the velocity term. The equation of continuity obtained by taking the divergence of Ampère's law reads:

$$\nabla \cdot \left(-\sigma \frac{\partial \mathbf{A}}{\partial t} + \mathbf{J}_e\right) = 0$$

It is clear that unless the electrical conductivity is uniform, the particular gauge used to eliminate V cannot be the Coulomb gauge as that would violate the equation of continuity and would thereby also violate Ampère's law.

Time-Harmonic Magnetic Fields

In the time-harmonic case, there is no computational cost for including the displacement current in Ampère's law (then called Maxwell-Ampère's law):

$$\nabla \times \mathbf{H} = \mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + j\omega\mathbf{D} + \mathbf{J}^e$$

In the transient case the inclusion of this term would lead to a second-order equation in time, but in the harmonic case there are no such complications. Using the definition of the electric and magnetic potentials, the system of equations becomes

$$-\nabla \cdot ((j\omega\sigma - \omega^2\varepsilon_0)\mathbf{A} - \sigma\mathbf{v} \times (\nabla \times \mathbf{A}) + (\sigma + j\omega\varepsilon_0)\nabla V - (\mathbf{J}^e + j\omega\mathbf{P})) = 0$$

$$(j\omega\sigma - \omega^2\varepsilon_0)\mathbf{A} + \nabla \times (\mu_0^{-1}\nabla \times \mathbf{A} - \mathbf{M}) - \sigma\mathbf{v} \times (\nabla \times \mathbf{A}) + (\sigma + j\omega\varepsilon_0)\nabla V = \mathbf{J}^e + j\omega\mathbf{P}$$

The constitutive relation $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$ has been used for the electric field.

You obtain a particular gauge that reduces the system of equation by choosing $\Psi = -jV/\omega$ in the gauge transformation. This gives

$$\tilde{\mathbf{A}} = \mathbf{A} - \frac{j}{\omega}\nabla V \qquad \tilde{V} = 0$$

When $\tilde{V}$ vanishes from the equations, you only need the second one,

$$(j\omega\sigma - \omega^2\varepsilon_0)\tilde{\mathbf{A}} + \nabla \times (\mu_0^{-1}\nabla \times \tilde{\mathbf{A}} - \mathbf{M}) - \sigma\mathbf{v} \times (\nabla \times \tilde{\mathbf{A}}) = \mathbf{J}^e + j\omega\mathbf{P}$$

Working with $\tilde{\mathbf{A}}$ is often the best option when it is possible to specify all source currents as external currents $\mathbf{J}^e$ or as surface currents on boundaries. This leads to the Magnetic Fields interface described in the next section.

Theory for the Magnetic Fields Interface

Simulation of magnetic fields is of interest when studying magnets, motors, transformers, and conductors carrying static or alternating currents.

You can use the *Magnetic Fields* interface for 3D, 2D in-plane, and 2D axisymmetric models. Unless you have a license for the AC/DC Module, only 2D modeling involving out-of-plane currents and axisymmetric modeling involving azimuthal currents are supported. For a deeper theoretical background to the magnetic vector potential used below, please refer to the preceding section starting with Maxwell's Equations.

MAGNETOSTATICS EQUATION

To derive the magnetostatic equation, start with Ampère's law for static cases,

$$\nabla \times \mathbf{H} = \mathbf{J}$$

The current is

$$\mathbf{J} = \sigma \mathbf{v} \times \mathbf{B} + \mathbf{J}^{\mathrm{e}}$$

where $\mathbf{J}^{\mathrm{e}}$ is an externally generated current density, and $\mathbf{v}$ is the velocity of the conductor.

Using the definitions of magnetic potential,

$$\mathbf{B} = \nabla \times \mathbf{A}$$

and the constitutive relationship, $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$, you can rewrite Ampère's law as

$$\nabla \times (\mu_0^{-1} \nabla \times \mathbf{A} - \mathbf{M}) - \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) = \mathbf{J}^{\mathrm{e}}$$

which is the equation used in magnetostatics.

FREQUENCY DOMAIN EQUATION

To derive the time harmonic equation this physics interface solves, start with Ampère's law including displacement currents (then called Maxwell-Ampère's law) as these do not involve any extra computational cost in the frequency domain,

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = \sigma \mathbf{E} + \sigma \mathbf{v} \times \mathbf{B} + \mathbf{J}^e + \frac{\partial \mathbf{D}}{\partial t}.$$

Now assume time-harmonic fields and use the definitions of the fields,

$$\mathbf{B} = \nabla \times \mathbf{A}$$
$$\mathbf{E} = -j\omega \mathbf{A}$$

and combine them with the constitutive relationships $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$ and $\mathbf{D} = \varepsilon_0 \mathbf{E}$ to rewrite Ampère's law as

$$(j\omega\sigma - \omega^2 \varepsilon_0)\mathbf{A} + \nabla \times (\mu_0^{-1} \nabla \times \mathbf{A} - \mathbf{M}) - \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) = \mathbf{J}^e$$

TRANSIENT EQUATION

The transient equation this physics interface solves is Ampère's law, here illustrated with the constitutive relation $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$:

$$\sigma \frac{\partial \mathbf{A}}{\partial t} + \nabla \times (\mu_0^{-1} \nabla \times \mathbf{A} - \mathbf{M}) - \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) = \mathbf{J}_e$$

10

Acoustics

This section describes how to use the Pressure Acoustics interface for modeling and simulation of acoustics and vibrations.

In this section:

The Pressure Acoustics Interface

The **Pressure Acoustics** interface ( ) has the equations, boundary conditions, and sources for modeling acoustics, solving for the sound pressure. For more information also see Acoustics Theory and Theory for the Pressure Acoustics Interface.

These default nodes are added when using this interface— **Pressure Acoustics Model**, **Sound Hard Boundary (Wall)**, and **Initial Values**.

INTERFACE IDENTIFIER

The unique physics interface identifier used to reach the fields and variables in expressions, for example. The default **Identifier** (for the first Pressure Acoustics interface in the model) is `acpr`. You can edit this to be any valid unique string.

DOMAINS

Select the domains where you want to define a sound pressure field and the associated acoustics equation. The default setting includes all domains in the model.

SOUND PRESSURE LEVEL SETTINGS

The zero level on the dB scale varies with the type of fluid. That value is a reference pressure that corresponds to 0 dB. This variable occurs in calculations of the sound pressure level based on the root mean square (rms) pressure,

$$\bar{p} = \sqrt{\frac{1}{2} p \, \mathrm{conj}(p)}$$

an expression valid for the case of harmonically time-varying acoustic pressure, p.

From the **Reference pressure for the sound pressure level** list, three reference pressures are available for the sound pressure level depending on the fluid. Select:

- **Use reference pressure for air** to use a reference pressure of 0.02 mPa ($20 \cdot 10^{-6}$ Pa).
- **Use reference pressure for water** to use a reference pressure of 1 µPa ($1 \cdot 10^{-6}$ Pa).
- **User defined reference pressure** to enter a reference pressure p_{ref} (SI unit: Pa) in the $P_{\mathrm{ref,\,SPL}}$ edit field. The default value is the same as for air, 0.02 mPa.

DEPENDENT VARIABLES

This interface defines one dependent variable (field), the **Pressure** p. If required, edit the name, but dependent variables must be unique within a model.

ADVANCED SETTINGS AND DISCRETIZATION

Normally these settings do not need to be changed. See Show More Options: Advanced Settings and Discretization. In addition, the Pressure Acoustics interface has the following settings:

Circumferential Wave Number (Axisymmetric Models Only)

The circumferential wave number m is an integer-valued number used in axisymmetry and is by default 0. You can change the value in the **Circumferential wave number** field.

Out-of-plane Wave Number (2D and 1D Axisymmetric Models)

The out-of-plane wave number k_z is used in 2D and 1D axisymmetry and is by default 0. You can change the value in the **Out-of-plane wave number** field.

Pressure Acoustics Model

The **Pressure Acoustics Model** feature adds the equations for time-harmonic and eigenfrequency acoustics modeling. In the **Settings** window you define the properties for the acoustics model and model inputs including temperature.

DOMAINS

Select the domains where you want to compute the acoustic pressure field and the equation that defines it. The default setting includes all domains in the model.

PRESSURE ACOUSTICS MODEL

The **Fluid Model** for pressure acoustics is a linear elastic fluid where you define the density and the speed of sound. By default the values for the density ρ (SI unit: kg/m^3) and the speed of sound c (SI unit: m/s) come from the material. Select **User defined** to enter other values for these properties. Select **Impedance and wave number** to specify those properties as complex-valued data. By default the characteristic acoustic impedance Z (SI unit: Pa·s/m) is the value from the fluid material. The wave number k is a user defined value.

$$\rho_c = \frac{\rho c^2}{c_c^2}$$

Monopole Source

Use the **Monopole Source** feature to radiate a uniform sound field in all directions.

DOMAINS

Select the domains where you want to define the monopole source.

MONOPOLE SOURCE

Enter a **Monopole source**, Q (SI unit: $1/s^2$). The default is 0.

Dipole Source

Use the **Dipole Source** feature to radiate a sound field that is typically stronger in two opposite directions.

DOMAINS

Select the domains where you want to define the dipole source.

DIPOLE SOURCE

Enter values in the **Dipole source** fields. These contain the individual components of the dipole source vector q (SI unit: N/m^3), one for each space dimension. The default is 0.

Initial Values

The **Initial Values** feature node adds initial values for the sound **Pressure** p and the **Pressure time derivative** dp/dt that can serve as an initial guess for a nonlinear solver. If you need to specify more than one initial value, right-click **Pressure Acoustics** to add more **Initial Values** nodes.

DOMAINS

Select the domains where you want to define an initial value.

INITIAL VALUES

Enter a value or expression for the initial values.

- **Pressure** p (SI unit: Pa). The default is 0.
- **Pressure time derivative**, dp/dt (SI unit: Pa/s). The default is 0.

Pressure Acoustics Boundary Conditions

The following boundary conditions are available:

- "Sound Hard Boundary (Wall)"
- Normal Acceleration

- Sound Soft Boundary
- Pressure
- Impedance
- Plane Wave Radiation
- Spherical Wave Radiation
- Periodic Condition
- Cylindrical Wave Radiation
- Interior Sound Hard Boundary (Wall)

For axisymmetric models, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and automatically adds an **Axial Symmetry** feature node to the model that is valid on the axial symmetry boundaries only.

Continuity in pressure is the default condition on interior boundaries.

On pairs the following conditions are available:

- Continuity (Pair Feature)
- Sound Soft Boundary (Pair Feature)
- Pressure (Pair Feature)

Sound Hard Boundary (Wall)

The **Sound Hard Boundary (Wall)** creates a boundary condition for a *sound hard boundary* or wall, which is a boundary at which the normal component of the acceleration is zero:

$$-\mathbf{n}\cdot\left(-\frac{1}{\rho_0}(\nabla p - \mathbf{q})\right) = 0$$

For zero dipole charge and constant fluid density, this means that the normal derivative of the pressure is zero at the boundary:

$$\frac{\partial p}{\partial n} = 0$$

Sound-hard boundaries are available for all study types.

BOUNDARIES

Select the boundaries that you want to define as sound hard boundaries (wall).

Normal Acceleration

The **Normal Acceleration** adds an inward normal acceleration a_n:

$$-\mathbf{n} \cdot \left(-\frac{1}{\rho_0}(\nabla p - \mathbf{q})\right) = a_n$$

Alternatively, specify the acceleration a of the boundary. The part in the normal direction is used to define the boundary condition:

$$\mathbf{n} \cdot \left(-\frac{1}{\rho_0}(\nabla p - \mathbf{q})\right) = \mathbf{n} \cdot \mathbf{a}_0$$

This feature represents an external source term. It can also be used to couple an acoustics model with a structural analysis.

BOUNDARIES

Select the boundaries where you want to specify a normal acceleration boundary condition.

NORMAL ACCELERATION

From the **Type** list, select **Inward Acceleration** to enter the value of the inward normal acceleration a_n (SI unit: m/s^2). Use a positive value for inward acceleration or a negative value for outward acceleration. Select **Acceleration** to enter values for the components of the acceleration a_0.

Sound Soft Boundary

The **Sound Soft Boundary** creates a boundary condition for a *sound soft boundary*, where the acoustic pressure vanishes:

$$p = 0$$

This boundary condition is an appropriate approximation for a liquid-gas interface and in some cases for external waveguide ports.

BOUNDARIES

Select the boundaries that you want to define as sound soft boundaries.

CONSTRAINT SETTINGS

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Select a **Constraint type**—**Bidirectional, symmetric,** or **Unidirectional**. If required, select the **Use weak constraints** check box.

Pressure

The **Pressure** node creates a boundary condition that acts as a pressure source at the boundary, which means you specify a constant acoustic pressure p_0 to be maintained at the boundary:

$$p = p_0$$

In the frequency domain, p_0 is the amplitude of a harmonic pressure source.

BOUNDARIES

Select the boundaries where you want to specify a Pressure boundary condition.

PRESSURE

Enter the value of the **Pressure** p_0 (SI unit: Pa) at the boundary.

CONSTRAINT SETTINGS

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Select a **Constraint type**—**Bidirectional, symmetric,** or **Unidirectional**. If required, select the **Use weak constraints** check box.

Impedance

The **Impedance** feature adds an impedance boundary condition, which is a generalization of the sound-hard and sound-soft boundary conditions:

$$\mathbf{n} \cdot \left(-\frac{1}{\rho_0}(\nabla p - \mathbf{q})\right) - \frac{i\omega p}{Z} = 0$$

Here Z is the acoustic input impedance of the external domain. From a physical point of view, the acoustic input impedance is the ratio between pressure and normal particle velocity.

The **Impedance** boundary condition is a good approximation for a locally reacting surface—a surface for which the normal velocity at any point depends only on the pressure at that exact point.

Note that in the two opposite limits $Z \to \infty$ and $Z \to 0$, this boundary condition is identical to the **Sound Hard** boundary condition and the **Sound Soft** boundary condition, respectively.

BOUNDARIES

Select the boundaries where you want to specify an Impedance boundary condition.

IMPEDANCE

Enter the value of the input **impedance** Z_i (SI unit: Pa·s/m). The default value is $1.2[\mathrm{kg/m^3}]\cdot 343[\mathrm{m/s}]$.

Plane, Spherical, and Cylindrical Radiation Boundary Conditions

Specify a **Plane**, **Spherical**, or **Cylindrical Radiation** boundary condition to allow an outgoing wave to leave the modeling domain with minimal reflections. The condition can be adapted to the geometry of the modeling domain. The **Plane** wave type is suitable for both far-field boundaries and ports. Because many waveguide structures are only interesting in the plane-wave region, it is particularly relevant for ports.

Radiation boundary conditions are available for all types of studies. For the Frequency domain study, Givoli and Neta's reformulation of the Higdon conditions (Ref. 1) for plane waves has been implemented to the second order. For **Cylindrical** and **Spherical** waves COMSOL uses the corresponding 2nd-order expressions from Bayliss, Gunzburger, and Turkel (Ref. 2). The Eigenfrequency study implement the same expansions to the first order.

The first order radiation boundary conditions in the frequency domain read

$$\mathbf{n}\cdot\left(\frac{1}{\rho_0}(\nabla p - \mathbf{q})\right) + (ik + \kappa(r))\frac{p}{\rho_0} = (ik + \kappa(r))\frac{p_\mathrm{i}}{\rho_0} + \mathbf{n}\cdot\left(\frac{\nabla p_\mathrm{i}}{\rho_0}\right)$$

where k is the wave number (a predefined variable; see Table 6-21 in the *COMSOL Multiphysics Reference Guide* or see Where Do I Access the Documentation and Model Library?) and $\kappa(r)$ is a function whose form depends on the wave type:

- Plane wave: $\kappa(r) = 0$ (see Plane Wave Radiation)
- Cylindrical wave: $\kappa(r) = 1/(2r)$ (see Cylindrical Wave Radiation)
- Spherical wave: $\kappa(r) = 1/r$ (see Spherical Wave Radiation)

In the latter two cases, r is the shortest distance from the point $\mathbf{r} = (x, y, z)$ on the boundary to the source. The right-hand side of the equation represents an optional incoming pressure field $\mathrm{p_i}$ (see Incident Pressure Field).

The second order radiation boundary conditions in the frequency domain are defined below. In these equations, Δ_T at a given point on the boundary denotes the Laplace operator in the tangent plane at that particular point.

- **Plane wave**:

$$\mathbf{n} \cdot \left(\frac{1}{\rho_0}(\nabla p - \mathbf{q})\right) + i\frac{k}{\rho_0}p + \frac{i}{2k\rho_0}\Delta_\mathrm{T} p = \frac{i}{2k\rho_0}\Delta_\mathrm{T} p_i + i\frac{k}{\rho_0}p_i + \mathbf{n} \cdot \frac{1}{\rho_0}\nabla p_i$$

  In the notation of Givoli and Neta (Ref. 1), the above expressions correspond to the parameter choices $C_0 = C_1 = C_2 = \omega/k$. For normally incident waves this gives a vanishing reflection coefficient.

- **Cylindrical wave**:

  This boundary condition is based on a series expansion of the outgoing wave in cylindrical coordinates (Ref. 2), and it assumes the field is independent of the axial coordinate. You specify the axis of this coordinate system by giving an orientation (n_x, n_y, n_z) and a point (x_0, y_0, z_0) on the axis. In axisymmetric geometries the symmetry axis is the natural and only choice.

- **Spherical wave**:

$$\mathbf{n} \cdot \left(\frac{1}{\rho_0}(\nabla p - \mathbf{q})\right) + \left(ik + \frac{1}{r}\right)\frac{p}{\rho_0} - \frac{r\Delta_\mathrm{T} p}{2\rho_0(ikr+1)}$$
$$= -\frac{r\Delta_\mathrm{T} p_\mathrm{i}}{2\rho_0(ikr+1)} + \left(ik + \frac{1}{r}\right)\frac{p_\mathrm{i}}{\rho_0} + \mathbf{n} \cdot \frac{1}{\rho_0}\nabla p_\mathrm{i}$$

  Use this option to allow a radiated or scattered wave—emanating from an object centered at the point (x_0, y_0, z_0) that you specify—to leave the modeling domain without reflections. The boundary condition is based on an expansion in spherical coordinates from Bayliss, Gunzburger, and Turkel (Ref. 2), implemented to the second order.

Plane Wave Radiation

The **Plane Wave Radiation** feature adds a radiation boundary condition for a plane wave. If required, right-click the main node to add an **Incident Pressure Field** (see Incident Pressure Field).

BOUNDARIES

Select the boundaries where you want to use a plane wave radiation condition.

Spherical Wave Radiation

The **Spherical Wave Radiation** feature adds a radiation boundary condition for a spherical wave. If required, right-click the main node to add an **Incident Pressure Field** (see Incident Pressure Field).

BOUNDARIES

Select the boundaries where you want to use a spherical wave radiation condition.

SPHERICAL WAVE RADIATION

Define the **Source location** r_0.

Cylindrical Wave Radiation

The **Cylindrical Wave Radiation** feature adds a radiation boundary condition for a cylindrical wave. If required, right-click the main node to add an **Incident Pressure Field** (see Incident Pressure Field).

BOUNDARIES

Select the boundaries where you want to use a cylindrical wave radiation condition.

CYLINDRICAL WAVE RADIATION

Define the **Source location** r_0, and the **Source axis** direction r_{axis}.

Incident Pressure Field

The **Incident Pressure Field** feature is a subfeature to all non-reflecting boundary conditions. Add an incident pressure field by right-clicking the boundary condition and selecting **Incident Pressure Field**.

BOUNDARIES

Select the boundaries where you want to include an incident pressure field in the boundary condition. By default, this feature node inherits the selection from its parent node, and you can only use a selection that is a subset of the parent node's selection.

INCIDENT PRESSURE FIELD

From the **Incident pressure field type** list, select **Plane wave** to define a background pressure field of plane wave type. Enter a **Pressure amplitude** p_0 (SI unit: Pa) and **Wave direction** *dir* (SI unit: m).

Select **User defined** to enter the expression for p_i (SI unit: Pa) as a function of space and in the **Incident pressure field** p_i. The default is 0.

Periodic Condition

The **Periodic Condition** feature adds a periodic boundary condition. Right-click the main node to add a Destination Selection.

BOUNDARIES

Select the boundaries where you want to apply a periodic condition.

PERIODIC CONDITION

Select a **Type of periodicity**—**Continuity** or **Antiperiodicity**.

Interior Sound Hard Boundary (Wall)

This feature creates a boundary condition for the **Interior Sound Hard Boundary (Wall)**. This boundary condition is only available on interior boundaries. A *sound-hard boundary* is a boundary at which the normal component of the acceleration is zero:

$$\mathbf{n}\cdot\left(\frac{1}{\rho_0}(\nabla p-\mathbf{q})\right)_1=0 \qquad \mathbf{n}\cdot\left(\frac{1}{\rho_0}(\nabla p-\mathbf{q})\right)_2=0$$

For zero dipole charge and constant fluid density, this means that the normal derivative of the pressure is zero at the boundary.

BOUNDARIES

Select the interior boundaries you want to define as sound hard boundaries (wall).

$$-\mathbf{n}\cdot\left(-\frac{1}{\rho_0}(\nabla p-\mathbf{q})\right)_1=\mathbf{n}\cdot\mathbf{a}_0 \qquad -\mathbf{n}\cdot\left(-\frac{1}{\rho_0}(\nabla p-\mathbf{q})\right)_2=\mathbf{n}\cdot\mathbf{a}_0$$

$$-\mathbf{n}\cdot\left(-\frac{1}{\rho_0}(\nabla p-\mathbf{q})\right)_1=a_n \qquad -\mathbf{n}\cdot\left(-\frac{1}{\rho_0}(\nabla p-\mathbf{q})\right)_2=a_n$$

Axial Symmetry

The **Axial symmetry** node is a default node added for all 1D and 2D axisymmetric models. The boundary condition is active on all boundaries on the symmetry axis.

BOUNDARIES

The boundaries section shows on which boundaries the feature is active. All boundaries on the symmetry axis are automatically selected.

Continuity (Pair Feature)

Continuity is available as an option at interfaces between parts in an assembly. Also see Defining an Identity Pair.

This condition gives continuity in the normal acceleration over the pair (subscripts 1 and 2 in the equation refers to the two sides in the pair):

$$\mathbf{n} \cdot \left[\left(\frac{1}{\rho_0} (\nabla p - \mathbf{q}) \right)_1 - \left(\frac{1}{\rho_0} (\nabla p - \mathbf{q}) \right)_2 \right] = 0$$

BOUNDARIES

This list cannot be edited. It shows the boundaries in the selected pairs.

PAIR SELECTION

Select the boundary pairs where you want to define continuity. First the pairs have to be defined.

CONSTRAINT SETTINGS

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Select a **Constraint type**—**Bidirectional, symmetric,** or **Unidirectional**. If required, select the **Use weak constraints** check box.

Sound Soft Boundary (Pair Feature)

The **Sound Soft Boundary** is available as an option at interfaces between parts in an assembly. Also see Defining an Identity Pair. It creates a boundary condition for a *sound soft boundary*, where the acoustic pressure vanishes:

$$p = 0$$

BOUNDARIES

This list cannot be edited. It shows the boundaries in the selected pairs.

PAIR SELECTION

Select the boundary pairs that you want to define as sound soft boundaries. First the pairs have to be defined.

CONSTRAINT SETTINGS

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Select a **Constraint type**—**Bidirectional, symmetric,** or **Unidirectional**. If required, select the **Use weak constraints** check box.

Pressure (Pair Feature)

The **Pressure** pair feature is available as an option at interfaces between parts in an assembly. Also see Defining an Identity Pair. It creates a boundary condition that acts as a pressure source at the boundary, which means you specify a constant acoustic pressure p_0 to be maintained at the boundary:

$$p = p_0$$

In the frequency domain, p_0 is the amplitude of a harmonic pressure source.

BOUNDARIES

This list cannot be edited. It shows the boundaries in the selected pairs.

PAIR SELECTION

Select the boundary pairs where you want to specify a pressure boundary condition. First the pairs have to be defined.

PRESSURE

Enter the value of the **Pressure** p_0 (SI unit: Pa) at the boundary.

CONSTRAINT SETTINGS

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Select a **Constraint type**—**Bidirectional, symmetric,** or **Unidirectional**. If required, select the **Use weak constraints** check box.

Acoustics Theory

What is Acoustics?

Acoustics is the physics of ***sound***. Sound is the sensation, as detected by the ear, of very small rapid changes in the air pressure above and below a static value. This static value is atmospheric pressure (about 100,000 pascals), which varies slowly. Associated with a sound pressure wave is a flow of energy. Physically, sound in air is a longitudinal wave where the wave motion is in the direction of the energy flow. The wave crests are the pressure maxima, while the troughs represent the pressure minima.

Sound results when the air is disturbed by some source. An example is a vibrating object, such as a speaker cone in a hi-fi system. It is possible to see the movement of a bass speaker cone when it generates sound at a very low frequency. As the cone moves forward, it compresses the air in front of it, causing an increase in air pressure. Then it moves back past its resting position and causes a reduction in air pressure. This process continues, radiating a wave of alternating high and low pressure at the speed of sound.

Five Standard Acoustics Problems

Five standard problems or scenarios occur frequently when analyzing acoustics:

- The radiation problem—A vibrating structure (a speaker, for example) radiates sound into the surrounding space. A far-away boundary condition is necessary to model the unbounded domain.
- The scattering problem—An incident wave impinges on a body and creates a scattered wave. A far-away radiation boundary condition is necessary.
- The sound field in an interior space (such as a room)—The acoustic waves stay in a finite volume so no radiation condition is necessary.
- Coupled fluid-elastic structure interaction (structural acoustics)—If the radiating or scattering structure consists of an elastic material, then you must consider the interaction between the body and the surrounding fluid. In the multiphysics coupling, the acoustic analysis provides a load (the sound pressure) to the structural analysis, and the structural analysis provides accelerations to the acoustic analysis.
- The transmission problem—The incident sound wave propagates into a body, which can have different acoustic properties. Pressure and acceleration are continuous on the boundary.

Sound waves in a lossless medium are governed by the following equation for the (differential) pressure p (with SI unit $\mathrm{N/m^2}$):

$$\frac{1}{\rho_0 {c_s}^2}\frac{\partial^2 p}{\partial t^2} + \nabla \cdot \left(-\frac{1}{\rho_0}(\nabla p - \mathbf{q})\right) = Q$$

Here ρ_0 ($\mathrm{kg/m^3}$) refers to the density and c_s (m/s) denotes the speed of sound. The *dipole source* $\mathbf{q}$ ($\mathrm{N/m^3}$) and the *monopole source* Q ($\mathrm{1/s^2}$) are both optional additional sources. The combination $\rho_0 {c_s}^2$ is called the *bulk modulus*, commonly denoted β ($\mathrm{N/m^2}$).

A special case is a time-harmonic wave, for which the pressure varies with time as

$$p(\mathbf{x}, t) = p(\mathbf{x})e^{i\omega t}$$

where $\omega = 2\pi f$ (rad/s) is the angular frequency, f (Hz) as usual denoting the frequency. Assuming the same harmonic time-dependence for the source terms, the wave equation for acoustic waves reduces to an inhomogeneous Helmholtz equation:

$$\nabla \cdot \left(-\frac{1}{\rho_0}(\nabla p - \mathbf{q})\right) - \frac{\omega^2 p}{\rho_0 {c_s}^2} = Q \tag{10-1}$$

With the source terms removed, you can also treat this equation as an eigenvalue PDE to solve for eigenmodes and eigenfrequencies.

Typical boundary conditions are:

- Sound-hard boundaries (walls)
- Sound-soft boundaries
- Impedance boundary conditions
- Radiation boundary conditions

These are described in more detail below.

In lossy media, an additional term of first order in the time derivative needs to be introduced to model attenuation of the sound waves:

$$\frac{1}{\rho_0 {c_s}^2}\frac{\partial^2 p}{\partial t^2} - d_a\frac{\partial p}{\partial t} + \nabla \cdot \left(-\frac{1}{\rho_0}(\nabla p - \mathbf{q})\right) = Q$$

Note also that even when the sound waves propagate in a lossless medium, attenuation frequently occurs by interaction with the surroundings at the boundaries of the system.

Acoustics Quantities and Their SI Units

The following table collects the names SI units for the most important physical quantities in the Pressure Acoustics interface:

QUANTITY	SYMBOL	SI UNIT	ABBREVIATION
Pressure	p	pascal	Pa
Density	ρ	kilogram/meter^3	kg/m^3
Frequency	f	hertz	Hz
Wave number	k	1/meter	1/m
Dipole source	$\mathbf{q}$	newton/meter^3	N/m^3
Monopole source	Q	1/second^2	1/s^2
Speed of sound	c_s	meter/second	m/s
Acoustic impedance	Z	pascal-second/meter	Pa·s/m
Normal acceleration	a_n	meter/second^2	m/s^2
Source location	r_0	meter	m
Wave direction	$\mathbf{n}_k$	(dimensionless)	1

Theory for the Pressure Acoustics Interface

The **Pressure Acoustics** interface ( ) is designed for the analysis of various types of pressure acoustics problems in the frequency domain, all concerning pressure waves in a fluid. An acoustics model can be part of a larger multiphysics model that describes, for example, the interactions between structures and acoustic waves. This interface is suitable for modeling acoustics phenomena that do not involve fluid flow.

This interface solves for the acoustic pressure, p. It is available for 3D, 2D, and 1D Cartesian geometries as well as for 2D and 1D axisymmetric geometries.

FREQUENCY DOMAIN STUDY

The frequency-domain—or time-harmonic—formulation uses the following inhomogeneous Helmholtz equation:

$$\nabla \cdot \left(-\frac{1}{\rho_0}(\nabla p - \mathbf{q})\right) - \frac{\omega^2 p}{\rho_0 c^2} = Q \qquad (10\text{-}2)$$

In this equation, $p = p(\mathbf{x}, \omega)$ (the dependence on ω is henceforth not explicitly indicated). With this formulation you can compute the frequency response with a parametric sweep over a frequency range using a harmonic load.

When there is damping, ρ_0 and c are complex quantities. The available damping models and how to apply them is described in Damping Models.

Equation 10-2 is the equation that the software solves for 3D geometries. In lower-dimensional and axisymmetric cases, restrictions on the coordinate dependence mean that the equations differ from case to case. Here is a brief summary of the situation.

2D

In 2D, the pressure is of the form $p(\mathbf{r}) = p(x, y)e^{-ik_z z}$, which inserted in Equation 10-2 gives

$$\nabla \cdot \left(-\frac{1}{\rho_0}(\nabla p - \mathbf{q})\right) - \frac{1}{\rho_0}\left(\frac{\omega^2}{c^2} - k_z^2\right)p = Q \qquad (10\text{-}3)$$

The *out-of-plane wave number*, k_z, can be set on the **Pressure Acoustics** page (see Out-of-plane Wave Number (2D and 1D Axisymmetric Models). By default its value is zero. In the Mode analysis type, $-ik_z$ is used as the eigenvalue.

2D Axisymmetry

For 2D axisymmetric geometries the independent variables are the radial coordinate, r, and the axial coordinate, z. The only dependence allowed on the azimuthal coordinate, φ, is through a phase factor,

$$p(r, \phi, z) = p(r, z)e^{-im\varphi} \tag{10-4}$$

where m denotes the *circumferential wave number*. Because the azimuthal coordinate is periodic, m must be an integer. Just like k_z in the 2D case, m can be set on the **Pressure Acoustics** page.

As a result of Equation 10-4, the equation to solve for the acoustic pressure in 2D axisymmetric geometries becomes

$$\frac{\partial}{\partial r}\left[-\frac{r}{\rho_0}\left(\frac{\partial p}{\partial r} - q_r\right)\right] + r\frac{\partial}{\partial z}\left[-\frac{1}{\rho_0}\left(\frac{\partial p}{\partial z} - q_z\right)\right] - \left[\left(\frac{\omega}{c}\right)^2 - \left(\frac{m}{r}\right)^2\right]\frac{rp}{\rho_0} = rQ\,.$$

1D Axisymmetry

In 1D axisymmetric geometries, $p(r, \phi, z) = p(r)e^{-i(k_z z + m\varphi)}$, leading to the radial equation

$$\frac{\partial}{\partial r}\left[-\frac{r}{\rho_0}\left(\frac{\partial p}{\partial r} - q_r\right)\right] - \left[\left(\frac{\omega}{c}\right)^2 - \left(\frac{m}{r}\right)^2 - k_z^2\right]\frac{rp}{\rho_0} = rQ$$

where both the circumferential wave number, m, and the *axial wave number*, k_z appear as parameters.

1D

The equation for the 1D case is obtained by taking the pressure to depend on a single Cartesian coordinate, x:

$$\frac{d}{dx}\left(-\frac{1}{\rho_0}\left(\frac{dp}{dx} - q\right)\right) - \frac{\omega^2}{\rho_0 c^2}p = Q$$

EIGENFREQUENCY STUDY

In the eigenfrequency formulation the source terms are absent, and you solve for the eigenmodes and eigenfrequencies:

$$\nabla \cdot \left(-\frac{1}{\rho_0}\nabla p\right) + \frac{\lambda^2 p}{\rho_0 c^2} = 0 \qquad (10\text{-}5)$$

The eigenvalue λ introduced in this equation is related to the eigenfrequency, f, and the angular frequency, ω, through $\lambda = i2\pi f = i\omega$. Because they are independent of the pressure, the solver ignores any dipole and monopole sources unless you are solving a coupled eigenvalue problem.

Equation 10-5 applies to the 3D case. The equations solved in eigenfrequency studies in lower dimensions and for axisymmetric geometries are obtained from their time-harmonic counterparts, given in the previous subsection, by the substitution $\omega^2 \rightarrow -\lambda^2$.

You can switch between specifying the eigenvalues, the eigenfrequencies, or the angular frequencies by selecting from the **Eigenvalue transformation** list in the solver sequence's **Eigenvalue** feature node's **Settings** window.

REFERENCES FOR THE PRESSURE ACOUSTICS INTERFACE

1. D. Givoli and B. Neta, "High-order non-reflecting boundary scheme for time-dependent waves", *J. Comput. Phys.*, vol. 186, pp. 24–46, 2004.

2. A. Bayliss, M. Gunzburger, and E. Turkel, "Boundary Conditions for the Numerical Solution of Elliptic Equations in Exterior Regions," *SIAM J. Appl. Math.*, vol. 42, no. 2, pp. 430–451, 1982.

3. A.B. Bauer, "Impedance theory and measurements on porous acoustic liners," *J. Aircr.*, vol. 14, pp. 720–728, 1977.

4. S. Temkin, *Elements of Acoustics*, Acoustical Society of America, 2001.

11

The Chemical Species Transport Interfaces

This section explains how to use the **Transport of Diluted Species** interface to model and simulate mass transfer (chemical species transport) by diffusion and convection based on Fick's law of diffusion.

In this section:

The Transport of Diluted Species Interface

Mass transfer is an important part of chemical engineering because this is the field that considers the conversion of one type of substance into another. A lot of this occurs through chemical reactions, although separation and other unit operations are an important part. You can use the Transport of Diluted Species interface to model transport of a diluted species in chemical systems by convection and diffusion.

In the Transport of Diluted Species interface, Fick's law describes the diffusive transport in the flux vector. Fick's law is adequate when the diffusing species is dilute with respect to a solvent. Assuming a binary mixture of solute A in solvent B, concentrations of up to 10 mol% of A can be considered dilute.

Note: The optional Chemical Reaction Engineering Module has an extension of this physics interface for modeling multicomponent convection, diffusion, and migration (electrokinetic flow).

The **Transport of Diluted Species** interface ( ) has the equations, boundary conditions, and rate expression terms for modeling mass transport of diluted species in mixtures, solutions and solids, solving for the species concentration.

The interface supports simulation of transport by convection and diffusion in 1D, 2D, and 3D as well as for axisymmetric models in 1D and 2D. The dependent variable is the mass concentration, c.

The following default nodes are added when using this interface—**Convection and Diffusion**, **No Flux**, and **Initial Values**. Right-click the **Transport of Diluted Species** node to add other nodes with appropriate boundary conditions and rate expression terms.

INTERFACE IDENTIFIER

This is the physics interface identifier, which you use to reach the fields and variables in expressions, for example.

The identifier appears in the **Identifier** edit field, and you can change it to any unique string that is a valid identifier. The default identifier (for the first Transport of Diluted Species interface in the model) is `chds`.

DOMAINS

Select the domains where you want to implement the physics boundary conditions. By default, all domains are included. There may be a domain in your model that is not described by mass transfer, such as a reactor's solid wall, and you may want to deselect this domain.

TRANSPORT MECHANISMS

Control which of the following transport mechanism are to be included in the model:

- Diffusion is always included.

DEPENDENT VARIABLES

Note: The species are dependent variables, and their names must be unique with respect to all other dependent variables in the model.

ADVANCED SETTINGS AND DISCRETIZATION

Normally these settings do not need to be changed. See Show More Options: Advanced Settings and Discretization. In addition this section is available with this interface:

Convective Term

Select a **Convective term**—**Non-conservative** or **Conservative**.

CONSISTENT STABILIZATION

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. See Show More Options: Consistent and Inconsistent Stabilization for information about these settings. Any settings unique to this interface are listed below.

- When the **Crosswind diffusion** check box is selected, the **Lower gradient limit** g_{lim} (SI unit: mol/m^4) field defaults to `0.1[mol/m^3)/chds.helem`, where `chds.helem` is the local element size.
- For both consistent stabilization methods, select an **Equation residual. Approximate residual** is the default setting and it means that derivatives of the diffusion tensor

components are neglected. This setting is usually accurate enough and is faster to compute. If required, select **Full residual** instead.

INCONSISTENT STABILIZATION

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. See Show More Options: Consistent and Inconsistent Stabilization for information about these settings.

Transport Feature Node

This node is dynamic in that it is dependent on the transport mechanisms that have been chosen in the **Transport of Diluted Species** physics interface (convection, diffusion and migration), and includes only the input fields required by the activated transport mechanisms. It has all the equations defining the transport of diluted species as well as inputs for the material properties. The name of the node also changes the activated transport mechanisms, and can be one of the following:

- Diffusion
- Convection and Diffusion

DOMAINS

Select the domains where you want to define material properties and other parameters that govern the transport equations. As you may have more than one type of domain, with subsequent material properties, you may want to deselect some of the domains. These would then be defined in a subsequent **Convection and Diffusion** node.

MODEL INPUTS

If transport by convection is active the velocity field of the solvent needs to be specified as a model input.

Select the source of the **Velocity field** $\boldsymbol{u}$:

- Select **User defined** to enter values or expressions for the velocity components (SI unit: m/s) in the edit fields or table that appears below the drop-down menu. This input option is always available.
- Select the velocity field solved by a fluid flow physics interface that has also been added to the model. These physics interfaces have their own tags or Interface Identifier, and they are available to choose in the Velocity field drop-down menu, if they are also active in the domains being defined here. This lists the variable names

related to the fluid flow physics interface in the table underneath the drop-down menu.

DIFFUSION

Enter the **Diffusion coefficient** D_c (SI unit: m^2/s), which can be a scalar value for isotropic diffusion or a tensor describing anisotropic diffusion. Select the appropriate tensor type that describes the diffusion transport, and then enter the values in the corresponding field.

Reactions

In order to account for the consumption or production of species, right-click on the **Transport of Diluted Species** node to add the **Reactions** node. This node contains an edit field to define the rate expression which in turn appear on the right hand side of the species transport equation.

DOMAINS

Select the domains where you want to define rate expression that govern the source term in the transport equations. As you may have more than one type of domain, with subsequent and different reactions occurring within them, you may want to deselect some of the domains. These would then be defined in a subsequent **Reactions** node.

REACTIONS

Add a rate expression, (SI unit: $mol/(m^3 \cdot s)$), for the species to be solved for. Type a value or expression in the corresponding field.

Initial Values

The **Initial Values** feature allows the initial value or guess for the variables and species, nominally the concentration, to be solved for to be specified. This can serve as the an initial condition for a transient simulation.

DOMAINS

Select the domains where you want to define an initial value. As you may have more than one type of domain, with subsequent and different initial values occurring within them, you may want to deselect some of the domains. These would then be defined in a subsequent **Initial Value** node.

INITIAL VALUES

Enter a value or expression for the initial value of the concentration c_i, in the **Concentration** edit field. The default value is 0 $\mathrm{mol/m^3}$.

Boundary Conditions

The following boundary conditions are available for the exterior boundaries:

- Concentration
- Flux
- Inflow
- No Flux (the default boundary condition)
- Outflow
- Symmetry

For interior boundaries, continuity in the species concentration is the default boundary condition. In addition, the following boundary condition is available on interior boundaries:

- Flux Discontinuity

For axisymmetric models, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and automatically adds an **Axial Symmetry** feature to the model that is valid on the this boundary.

Concentration

The **Concentration** boundary node adds a boundary condition for the species concentration; for example the following condition specifies the concentration of species c:

$$c = c_0$$

BOUNDARIES

Select the boundaries where you want to apply the concentration boundary condition.

CONCENTRATION

You can specify the concentration for each species individually. Select the check box for the species for which you want to specify the concentration, and then enter a value or expression in the corresponding edit field. Conversely, if you want to use another

boundary condition for a specific species, clear the check box for that species' mass fraction.

Flux

Note: This boundary condition is not available for models where only diffusion occurs.

The **Flux** feature can be used to specify the total species flux across a boundary. The total flux of species c is defined accordingly:

$$-\mathbf{n} \cdot (c\mathbf{u} - D\nabla c) = N_0$$

where N_0 is an arbitrary user-specified flux expression (SI unit: mol/(m^2·s)). For example, N_0 can represent a flux into a much larger surrounding environment, a phase change, or a flux due to chemical reactions. N_0 can also be a function of the concentration and the potential (if the mass transport includes migration of ionic species).

When diffusion is the only transport mechanism present, the flux condition is extended to include a mass transfer term to describe flux into a surrounding environment:

$$-\mathbf{n} \cdot (-D\nabla c) = N_0 + k_c(c_b - c)$$

where k_c is a mass transfer coefficient (SI unit: m/s), and c_b is the concentration (SI unit mol/m^3) in the surroundings of the modeled system (the bulk concentration). The mass transfer coefficient is to be specified, often given by boundary-layer theory.

BOUNDARIES

Select the boundaries where you want to apply a flux boundary condition.

INWARD FLUX

You can specify the flux of each species individually. Select the check box for the species for which you want to specify the flux, and enter a value or expression in the corresponding edit field. Conversely, if you want to use another boundary condition for a specific species, clear the check box for that species' mass fraction. Note that if you are specifying a flux leaving the system, then you have to use a minus sign appropriately.

Inflow

The **Inflow** feature adds a boundary condition for an inflow boundary, where you specify the concentration of all species:

$$c = c_0$$

This condition is similar to that of the **Concentration** feature, except that you must specify the concentrations of all species.

BOUNDARIES

Select the boundaries where you want to apply an inflow boundary condition.

CONCENTRATION

Enter a value or expression for the concentration of each species.

No Flux

The **No Flux** node, which is the default boundary condition on exterior boundaries, represents boundaries where no mass flows in or out of this boundary, such that the total flux is zero:

$$-\mathbf{n} \cdot (c\mathbf{u} - D\nabla c) = N_0$$

BOUNDARIES

Select the boundaries where you want to apply a no-flux boundary condition.

NO FLUX

Select **Apply for all species** to from the drop-down menu list to specify that the boundary is completely impervious for all species. Select **Apply for...** and select the check boxes for the species for which you want to specify the No-flux condition.

Outflow

Note: This boundary condition is not available for models where only diffusion occurs.

The **Outflow** feature is the preferred boundary condition at outlets where the species is transported out of the model domain by a fluid flow. It is useful in mass transport models where you can assume that convection is the dominating effect which causes the mass flow through the outflow boundary, and therefore useful to ignore a diffusive effect from this boundary, such that:

$$\mathbf{n} \cdot (-D\nabla c) = 0$$

This is also a useful boundary condition, particularly in convection-dominated mass balances where the outlet concentration is unknown.

BOUNDARIES

Select the boundaries where you want to apply an outflow boundary condition.

Symmetry

The **Symmetry** feature can be used to represents boundaries where the concentration of species is symmetric; that is, there is no mass flux in the normal direction across the boundary.

This boundary condition is identical to that of the No Flux node, but applies to all species and cannot be applied to individual species.

BOUNDARIES

Select the boundaries where you want to apply a symmetry condition.

Flux Discontinuity

The **Flux Discontinuity** feature represents a discontinuity in the mass flux across an interior boundary:

$$-\mathbf{n} \cdot (\mathbf{N}_\mathrm{d} - \mathbf{N}_\mathrm{u}) = N_0 \qquad \mathbf{N} = (c\mathbf{u} - D\nabla c)$$

where the value N_0 (SI unit: mol/(m^2·s)) specifies the jump in flux evaluated the boundary. This interior boundary could represent a membrane or thin-film that adds its own resistance to mass transfer in a very small volume, and where you do not want to have to specify this volume. This boundary condition is only available on interior boundaries.

BOUNDARIES

Select the interior boundaries where you want to apply a flux discontinuity boundary condition.

FLUX DISCONTINUITY

Specify the jump in the species flux, using a positive value for increasing flux when going from the down to the up side of the boundary. The normal direction `(nx,ny,nz)` (`(nr,nz)` in an axisymmetric model) points in the direction from down side towards the up side of an internal boundary and can be plotted for visualization.

Open Boundary

Use the **Open Boundary** feature to define the exterior concentration to boundaries.

BOUNDARIES

Select the boundaries where you want to apply the open boundary condition.

EXTERIOR CONCENTRATION

Enter a value or expression for the **Exterior concentration** $c_{0,c}$ (SI unit: mol/m^3).

Electrode-Electrolyte Interface Coupling

Use the **Electrode-Electrolyte Interface Coupling** feature to add a pair coupling to a model.

BOUNDARIES

Select the boundaries where you want to apply the interface coupling.

PAIR SELECTION

Select the boundary pair where you want to define the coupling. First an identity pair may have to be created.

Theory for the Transport of Diluted Species Interface

The **Transport of Diluted Species** interface ( ) provides a predefined modeling environment for studying the evolution of chemical species transported by diffusion, convection and migration due to an electric field. The interface assumes that all species present are dilute, that their concentration is small compared to a solvent fluid or solid. As a rule of thumb, a mixture containing several species can be considered dilute when the concentration of the solvent is more than 90 mol%. Due to the dilution, mixture properties such as density and viscosity, can be assumed to correspond to those of the solvent.

Fick's law governs the diffusion of the solutes dilute mixtures or solutions, while the phenomenon of ionic migration is sometimes referred to as electrokinetic flow. The **Transport of Diluted Species** physics interface supports the simulation of chemical species transport by convection, diffusion, and migration in 1D, 2D, and 3D as well as for axisymmetric models in 1D and 2D.

MASS BALANCE EQUATION

The default node attributed to the **Transport of Diluted Species** interface assumes chemical species transport through diffusion and convection (a button to activate migration is readily available) and implements the mass balance equation:

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = \nabla \cdot (D \nabla c) + R \qquad (11\text{-}1)$$

Equation 11-1 includes these quantities (with the SI unit in parentheses):

- c is the concentration of the species (mol/m^3)
- D denotes the diffusion coefficient (m^2/s)
- R is a reaction rate expression for the species (mol/(m^3·s))
- $\mathbf{u}$ is the velocity vector (m/s)

The first term on the left-hand side of Equation 11-1 corresponds to the accumulation (or indeed consumption) of the species.

The second term accounts for the convective transport due to a velocity field $\mathbf{u}$. This field can be expressed analytically or be obtained from coupling this physics interface

to one that describes fluid flow (momentum balance). To include convection in the mass balance equation, an expression that includes the spatial and time variables, or else the velocity vector component variable names from the fluid flow physics interface can be entered into the appropriate edit field.

On the right-hand side of the mass balance equation (Equation 11-1), the first term describes the diffusion transport, accounting for interaction between the dilute species and the solvent. An edit field for the **Diffusion coefficient** is available in the interface and any equation that relates to another variable, such as temperature, can be entered there. Furthermore, the node also gives you access to a matrix to describe the diffusion coefficient, if it is vectorized or is a tensor. Anisotropic diffusion can therefore be simulated here.

Finally, the second term on the right hand side of Equation 11-1 represents a source or sink term, typically due to a chemical reaction. In order for the chemical reaction to be specified, another node must be added to the **Transport of Diluted Species** interface—the **Reaction** node—which contains an edit field for you to specify a reaction equation using the variable names of all participating species. If you have a license for the **Chemical Reaction Engineering Module** and have added the **Reaction Engineering** interface to set up the mass balance, then the reaction equations show up in this node.

CONVECTIVE TERM FORMULATION

The default node attributed to the **Transport of Diluted Species** interface assumes chemical species transport through diffusion and convection (a button to activate migration is readily available) and implements the mass balance equation:

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = \nabla \cdot (D \nabla c) + R \qquad (11\text{-}2)$$

Equation 11-2 includes these quantities (with the SI unit in parentheses):

- c is the concentration of the species ($\mathrm{mol/m^3}$)
- D denotes the diffusion coefficient ($\mathrm{m^2/s}$)
- R is a reaction rate expression for the species ($\mathrm{mol/(m^3 \cdot s)}$)
- $\mathbf{u}$ is the velocity vector (m/s)

There are two different ways to present a mass balance where chemical species transport occurs through diffusion and convection. These are the non-conservative and conservative formulations of the convective term:

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = \nabla \cdot (D\nabla c) + R \quad \text{non-conservative} \tag{11-3}$$

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\mathbf{u}) = \nabla \cdot (D\nabla c) + R \quad \text{conservative} \tag{11-4}$$

and each is treated slightly differently by the solver algorithms. In these equations D is the diffusion coefficient (SI unit: m^2/s), R is a production or consumption rate expression (SI unit: $\mathrm{mol}/(\mathrm{m}^3 \cdot \mathrm{s})$), and $\mathbf{u}$ is the solvent velocity field (SI unit: m/s). The diffusion process can be anisotropic, in which case D is a tensor.

If you were to expand the conservative formulation using the chain rule, then one of the terms from the convection part, $c\nabla \cdot \mathbf{u}$, would equal zero for an incompressible fluid and would result in the non-conservative formulation above. This is in fact the default formulation in the **Transport of Diluted Species** interface and ensures that non-physical source terms cannot come from the solution of a flow field. to switch between the two formulations, you have to activate the **Show More Options** for this interface.

SOLVING A DIFFUSION EQUATION ONLY

You can remove the convection term from the above equations by clearing the **Convection** button under the **Transport Mechanisms** section in the **Transport of Diluted Species** feature. The equation then becomes

$$\frac{\partial c}{\partial t} = \nabla \cdot (D\nabla c) + R \tag{11-5}$$

12

The Fluid Flow Interface

This section explains how to use the **Laminar Flow** interface for the modeling and simulation of fluid mechanics for laminar, incompressible fluids. Note that the engineering community often uses the term *CFD*, *computational fluid dynamics*, to refer to the numerical simulation of fluids.

In this section:

The Single-Phase Flow, Laminar Flow Interface

The **Laminar Flow** interface ( ) has the equations, boundary conditions, and volume forces for modeling freely moving fluids using the Navier-Stokes equations, solving for the velocity field and the pressure.

The main node is **Fluid Properties**, which adds the Navier-Stokes equations and provides an interface for defining the fluid material and its properties.

The following default nodes are added when using this interface—**Fluid Properties**, **Wall** (the default boundary condition is No slip) and **Initial Values** nodes.

Right-click the **Laminar Flow** node to add other feature nodes that implement, for example, boundary conditions and volume forces.

INTERFACE IDENTIFIER

The interface identifier is a text string that can be used to reference the respective physics interface if appropriate. Such situations could occur when coupling this interface to another physics interface, or when trying to identify and use variables defined by this physics interface, which you use to reach the fields and variables in expressions, for example. You can change it to any unique string through the **Identifier** edit field.

The default identifier (for the first Laminar Flow or other single-phase flow interface in the model) is `spf`.

DOMAINS

Select the domains where you want to define the fluid pressure and velocity and the Navier-Stokes equations that describe those fields. The default setting is to include all domains in the model.

PHYSICAL MODEL

Control the properties of the Laminar Flow interface, which control the overall type of fluid-flow model:

Compressibility

By default the interface uses the **Compressible flow (Ma<0.3)** formulation of the Navier-Stokes equations. Select **Incompressible** to use the incompressible (constant density) formulation.

CONSISTENT STABILIZATION

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. See Show More Options: Consistent and Inconsistent Stabilization for information about these settings.

INCONSISTENT STABILIZATION

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. See Show More Options: Consistent and Inconsistent Stabilization for information about these settings.

ADVANCED SETTINGS AND DISCRETIZATION

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Normally these settings do not need to be changed.

The check box **Use pseudo time stepping for stationary equation form** adds pseudo time derivatives to the equation when the Stationary equation form is used. When selected, a **Local CFL number** list with the options

- **Automatic**
- **User defined**

The **Automatic** option calculates the local CFL number from a built-in expression while **User defined** makes it possible to manually define the local CFL number. See also Pseudo Time Stepping.

More information on advanced settings can be found in Show More Options: Advanced Settings and Discretization.

DEPENDENT VARIABLES

This interface defines these dependent variables (fields):

- **Velocity field** u (SI unit: m/s)
- **Pressure** p (SI unit: Pa)

If required, edit the name, but dependent variables must be unique within a model.

Fluid Properties

The **Fluid Properties** feature adds the momentum equations solved by the interface, except for volume forces which are added by a separate feature (Volume Force), and provides an interface for defining the material properties of the fluid.

DOMAINS

Select the domains where you want to apply the fluid properties defined in the **Fluid Properties** feature.

MODEL INPUTS

In this section input variables to the momentum equations can be edited. For fluid flow, these are typically introduced when a material requiring inputs has been applied.

FLUID PROPERTIES

Specify the density and the dynamic viscosity of the fluid.

Density

The default **Density** ρ (SI unit: kg/m^3) uses the value **From material**. Select **User defined** to enter a different value or expression.

Dynamic Viscosity

The dynamic viscosity describes the relationship between the shear rate and the shear stresses in a fluid. Intuitively, water and air have a low viscosity, and substances often described as thick, such as oil, have a higher viscosity.

For the **Dynamic viscosity** μ (SI unit: Pa·s), select from these options to model Newtonian fluids:

- The default uses the value **From material**. The value of the viscosity is then the value defined for the material that you select in the **Material** section.
- Select **User defined** to define a value or expression for the **Dynamic viscosity**. Using a built-in variable for the shear rate magnitude, `sr_spf`, makes it possible to define arbitrary expressions of the dynamics viscosity as a function of the shear rate.

Absolute Pressure

This section controls both the variable as well as any property value (reference pressures) used when solving for pressure. There are usually two ways for including a variable in expressions describing fluid flow, and mass and heat transfer. You can solve for the absolute pressure or a pressure (often denoted gauge pressure) that relates back to the absolute pressure through a reference pressure.

Using one or the other usually depends on the system and the equations you are solving for. For example, in a straight incompressible flow problem, the pressure drop over the modeled domain will probably be many orders of magnitude less than atmospheric pressure, which, if included, reduces the chances for stability and convergence during the solving process for this variable. In an other case, the absolute pressure may be required to be solved for, such as where pressure is a part of an expression for gas volume or diffusion coefficients.

The pressure as a variable input is controlled by both a drop-down list and a check box within this section. The **User defined** option requires that the equations will solve for the variable itself (the absolute pressure), which describes your system. You need to maintain control over any external pressures that may affect your system, if indeed there are any.

The default option, which is denoted **Pressure nitf/fluid**, also activates a check box for defining the reference pressure, where `1 [atm]` has been automatically included. This will allow you to use a system-based (gauge) pressure as the pressure variable, while automatically including the reference pressure in places where it is required, such as for gas flow governed by the gas law. While this check box maintains control over the pressure variable and instances where absolute pressure is required within this respective physics interface, it may not with interfaces that you are coupling to, such as mass and heat transfer physics interfaces. You should check between the two that are solving for the same variable.

Volume Force

The **Volume Force** feature specifies the volume force $\mathbf{F}$ on the right-hand side of Equation 12-16. Use it, for example, to incorporate the effects of gravity in your model.

DOMAINS

Select the domains where the volume force acts on the fluid.

VOLUME FORCE

Enter the components of the volume force $\mathbf{F}$ (SI unit: $\mathrm{N/m^3}$).

Initial Values

The **Initial Values** feature adds initial values for the velocity field $\mathbf{u}$ and the pressure P that can serve as an initial condition for a transient simulation or as an initial guess for a nonlinear solver.

DOMAINS

Select the domains where you want to define initial values.

INITIAL VALUES

Enter values or expressions for the initial value of the **Velocity field u** (SI unit: m/s) and for the **Pressure** p (SI unit: Pa). The default values are 0.

Boundary Conditions

The following six features contain all boundary conditions for the single-phase flow interfaces and represent the major types of boundaries in a fluid-flow model:

- Wall (the default boundary condition feature).
- Inlet
- Outlet
- Symmetry
- Open Boundary
- Boundary Stress
- Periodic Flow Condition

For axisymmetric models, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and automatically adds an **Axial Symmetry** feature to the model that is valid on the axial symmetry boundaries only.

You find the theory of most boundary conditions in Gresho and Sani ().

Wall

The **Wall** feature includes a set of boundary conditions describing the fluid flow condition at a wall. The following boundary conditions are available:

In laminar flow interfaces the following wall boundary conditions are available:

- Slip

- No Slip (Default)
- Moving Wall
- Leaking Wall
- Sliding Wall

BOUNDARIES

Select the boundaries that represent solid walls.

BOUNDARY CONDITION

Select a **Boundary condition** for the wall:

No Slip

No slip is the standard and default boundary condition for a stationary solid wall. The condition prescribes

$$\mathbf{u} = \mathbf{0}$$

that is, that the fluid at the wall is not moving.

Slip

The **Slip** condition assumes that there are no viscous effects at the slip wall and hence, no boundary layer develops. From a modeling point of view, this may be a reasonable approximation if the important effect of the wall is to prevent fluid from leaving the domain. Mathematically, the constraint can be formulated as:

$$\mathbf{u} \cdot \mathbf{n} = 0, \qquad \mathbf{t} \cdot (-p\mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathrm{T}}))\mathbf{n} = 0$$

where $\mathbf{t}$ is a tangential vector to the boundary.

Sliding Wall

The **Sliding wall** boundary condition is appropriate if the wall behaves like a conveyor belt; that is, the surface is sliding in its tangential direction. The wall does not have to actually move in the coordinate system.

In 2D, the tangential direction is unambiguously defined by the direction of the boundary, but the situation becomes more complicated in 3D. For this reason, this boundary condition has slightly different definitions in the different space dimensions.

2D and Axial Symmetry The velocity is given as a scalar U_{w} and the condition prescribes

$$\mathbf{u} \cdot \mathbf{n} = 0, \qquad \mathbf{u} \cdot \mathbf{t} = U_{\mathrm{w}}$$

where $\mathbf{t} = (n_y, -n_x)$ for 2D and $\mathbf{t} = (n_z, -n_r)$ for axial symmetry.

Enter the components of the **Velocity of the tangentially moving wall** U_w (SI unit: m/s).

3D The velocity is set equal to a given vector $\mathbf{u}_w$ projected onto the boundary plane:

$$\mathbf{u} = \frac{\mathbf{u}_w - (\mathbf{n} \cdot \mathbf{u}_w)\mathbf{n}}{1 - \frac{\mathbf{n} \cdot \mathbf{u}_w}{|\mathbf{u}_w|}}$$

The denominator makes $\mathbf{u}$ have the same magnitude as $\mathbf{u}_w$ even if $\mathbf{u}_w$ is not exactly parallel to the wall.

Enter the components of the **Velocity of the sliding wall** $\mathbf{u}_w$ (SI unit: m/s).

Moving Wall

If the wall moves, so must the fluid. Hence, this boundary condition prescribes

$$\mathbf{u} = \mathbf{u}_w$$

Note: Specifying this boundary condition does not automatically cause the associated wall to move.

Enter the components of the **Velocity of moving wall** $\mathbf{u}_w$ (SI unit: m/s).

Leaking Wall

Use this boundary condition to simulate a wall where fluid is leaking into or leaving through a perforated wall:

$$\mathbf{u} = \mathbf{u}_l$$

Enter the components of the **Fluid velocity** $\mathbf{u}_l$ (SI unit: m/s).

Inlet

The **Inlet** feature includes a set of boundary conditions describing the fluid flow condition at an inlet. The following boundary condition are available:

- Velocity
- Pressure, No Viscous Stress

- Laminar Inflow
- Normal Stress

The Velocity boundary condition is the default boundary condition for inlet boundaries.

Note: The formulations contained in this boundary type all appear, some of them slightly modified, in the Outflow type as well. This means that there is nothing in the mathematical formulations that prevents a fluid from leaving the domain through boundaries where you have specified the Inlet type.

BOUNDARIES

Select the boundaries that represent inlets.

BOUNDARY CONDITION

Select a **Boundary condition** for the inlet:

Velocity

This boundary condition offers two ways to specify an inlet velocity:

- Click **Normal inflow velocity** to specify a normal inflow velocity magnitude:

$$\mathbf{u} = -\mathbf{n}U_0$$

  where $\mathbf{n}$ is the boundary normal pointing out of the domain.

  Enter the velocity magnitude in the U_0 field (SI unit: m/s).

- Click **Velocity field** to set the velocity equal to a given velocity vector $\mathbf{u}_0$:

$$\mathbf{u} = \mathbf{u}_0$$

  Enter the velocity components in the $\mathbf{u}_0$ fields (SI unit: m/s).

Pressure, No Viscous Stress

This boundary condition specifies vanishing viscous stress along with a Dirichlet condition on the pressure:

$$\left(\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - \frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I}\right)\mathbf{n} = \mathbf{0}, \qquad p = p_0$$

$$\mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)\mathbf{n} = \mathbf{0}, \qquad p = p_0$$

using the compressible and the incompressible formulation respectively.

Enter the **Pressure** p_0 (SI unit: Pa) at the boundary.

This boundary condition is physically equivalent to a boundary that is adjacent to a large container. It is numerically stable and admits total control of the pressure level along the entire inlet boundary; however, it can give artifacts on the inlet boundary if the viscous stresses just downstream the inlet are not zero. In such situations there are two choices. Either move the boundary farther away to a location where the artifacts do not matter or use a another stress type boundary condition present in the Boundary Stress feature.

Note: This condition is identical to the Pressure, no viscous stress condition in the Outflow feature. Depending on the pressure field in the rest of the domain, a boundary with this condition can become an outflow boundary.

Laminar Inflow

This boundary prescribes a laminar velocity profiles using the implementation described in Laminar Inflow Condition.This condition is not valid for turbulent flows.

Use the following settings to specify a Laminar inflow boundary condition:

First select which flow quantity to specify for the inlet:

- Click **Average velocity** to enter an average velocity U_{av} (SI unit: m/s).
- Click **Flow rate** to enter the flow rate V_0 (SI unit: m^3/s).
- Click **Entrance pressure** to enter the entrance pressure p_{entr} (SI unit: Pa) at the end of the inlet.

Second, you need to specify the following parameters and options:

- Enter the **Entrance length** L_{entr} (SI unit: m). This defines the length of the inlet channel outside the model domain. This value must be large enough so that the flow can reach a laminar profile. The default is 1.
- For 2D and 2D axisymmetric models, select the **Constrain endpoints to zero** check box to force the laminar profile to go to zero at the outer sides of the inlet channel. Otherwise the velocity is defined by the boundary condition of the adjacent boundary in the model. For example, if one end of a boundary with a Laminar

inflow condition connects to a Slip boundary condition, then the laminar profile has a maximum at that end.

- For 3D models, select the **Constrain outer edges to zero** check box, which has the same effect as described for 2D models.

Normal Stress

The total stress on the boundary is set equal to a stress vector of magnitude f_0, oriented in the negative normal direction:

$$\left(-p\mathbf{I}+\left(\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)-\frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I}\right)\right)\mathbf{n} = -\mathrm{f}_0\mathbf{n}$$

$$(-p\mathbf{I}+\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T))\mathbf{n} = -\mathrm{f}_0\mathbf{n}$$

using the compressible and the incompressible formulation respectively.

Enter the magnitude of **Normal stress** f_0 (SI unit: $\mathrm{N/m^2}$).

This implies that the total stress in the tangential direction is zero. This boundary condition implicitly sets a constraint on the pressure that for 2D flows is

$$p = 2\mu\frac{\partial u_n}{\partial n}+\mathrm{f}_0 \qquad (12\text{-}1)$$

If $\partial u_n/\partial n$ is small, Equation 12-1 states that $p \approx \mathrm{f}_0$.

Outlet

The **Outlet** feature includes a set of boundary conditions describing fluid flow conditions at an outlet. The following boundary condition are available:

- Velocity
- Pressure, No Viscous Stress
- Pressure
- No Viscous Stress
- Normal Stress
- Laminar Outflow

The Pressure, no viscous stress boundary condition is the default boundary condition for outlet boundaries.

Selecting appropriate outlet conditions for the Navier-Stokes equations is not a trivial task. A general rule of thumb, however, is that if there is something interesting happening at an outflow boundary, extend the computational domain to include this phenomenon.

Note: All of the formulations for the Outlet type are also available, possibly slightly modified, in other boundary types as well. This means that there is nothing in the mathematical formulations that prevents a fluid from entering the domain through boundaries where you have specified the Outflow Boundary type.

BOUNDARIES

Select the boundaries that represent outlets.

BOUNDARY CONDITION

Select a **Boundary condition** for the outlet:

Velocity

This boundary condition offers two ways to specify an inlet velocity:

- Click **Normal outflow velocity** to specify a normal outflow velocity magnitude:

$$\mathbf{u} = -\mathbf{n}U_0$$

  where $\mathbf{n}$ is the boundary normal pointing out of the domain.

  Enter the magnitude of the **Normal outflow velocity** U_0 (SI unit: m/s).

- Click **Velocity field** to set the velocity equal to a given velocity vector $\mathbf{u}_0$:

$$\mathbf{u} = \mathbf{u}_0$$

  Enter the components of the **Velocity field** $\mathbf{u}_0$ (SI unit: m/s).

Pressure, No Viscous Stress

This boundary condition specifies vanishing viscous stress along with a Dirichlet condition on the pressure:

$$\left(\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - \frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I}\right)\mathbf{n} = \mathbf{0}, \qquad p = p_0$$

$$\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)\mathbf{n} = \mathbf{0}, \qquad p = p_0$$

for the compressible and the incompressible formulation respectively.

Enter the **Pressure** p_0 (SI unit: Pa) at the boundary.

This boundary condition is physically equivalent to the flow exiting into a large container. It is numerically stable and admits total control of the pressure level along the entire inlet boundary; however, it can give artifacts on the outlet boundary if the viscous stresses just inside the outlet are not zero. In such situations there are two choices. Either move the boundary farther away to a location where the artifacts do not matter or use a another stress type boundary condition present in the Boundary Stress feature.

Note: This condition is identical to the **Pressure, no viscous stress** condition in the Inflow feature. Depending on the pressure field in the rest of the domain, a boundary with this condition can become an inflow boundary.

Pressure

This boundary condition prescribes only a Dirichlet condition for the pressure:

$$p = p_0$$

Enter the **Pressure** p_0 (SI unit: Pa) at the boundary.

While this boundary condition is flexible and seldom give any artifacts on the boundary (compare to Pressure, No Viscous Stress), it can be numerically unstable. Theoretically, the stability is guaranteed by the streamline diffusion and a high enough cell Reynolds number $\mathrm{Re}^c = \rho|\mathbf{u}|h/(2\mu) >> 1$ (h is the local mesh element size). It does however work well in most other situations as well.

No Viscous Stress

This boundary condition prescribes vanishing viscous stress:

$$\left(\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - \frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I}\right)\mathbf{n} = \mathbf{0}$$

$$\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)\mathbf{n} = \mathbf{0}$$

using the compressible and the incompressible formulation respectively.

This condition can be useful in some situations because it does not impose any constraint on the pressure. A typical example is a model with volume forces that give

rise to pressure gradients that are hard to prescribe in advance. To make the model numerically stable, you need to combine this boundary condition with a point constraint on the pressure (see Pressure Point Constraint).

Normal Stress

The total stress on the boundary is set equal to a stress vector of magnitude f_0, oriented in the negative normal direction:

$$\left(-p\mathbf{I}+\left(\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)-\frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I}\right)\right)\mathbf{n} = -f_0\mathbf{n}$$

$$(-p\mathbf{I}+\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T))\mathbf{n} = -f_0\mathbf{n}$$

using the compressible and the incompressible formulation respectively.

Enter the magnitude of the **Normal stress** f_0 (SI unit: N/m^2).

This condition implies that the total stress in the tangential direction is zero. It also implicitly sets a constraint on the pressure that for 2D flows is

$$p = 2\mu\frac{\partial u_n}{\partial n}+f_0 \tag{12-2}$$

If $\partial u_n/\partial n$ is small, Equation 12-2 states that $p \approx f_0$.

Laminar Outflow

This boundary prescribes a laminar velocity profiles using the implementation described in Laminar Outflow Condition. This condition is not valid for turbulent flows.

Use the following settings to specify a Laminar outflow boundary condition:

First select which flow quantity to specify for the outlet:

- Click **Average velocity** to enter an average velocity U_{av} (SI unit: m/s) through the outlet.
- Click **Flow rate** to enter the flow rate V_0 (SI unit: m^3/s) through the outlet.
- Click **Exit pressure** to specify the entrance pressure p_{exit} (SI unit: Pa) at the end of the inlet.

Second, specify the following parameters and options:

- Enter the **Exit length** L_{exit} (SI unit: m). This defines the length of the inlet channel outside the model domain. This value must be large enough so that the flow can reach a laminar profile. The default is 1.
- For 2D and 2D axisymmetric models, select the **Constrain endpoints to zero** check box to force the laminar profile to go to zero at the outer sides of the outlet channel. Otherwise the velocity is defined by the boundary condition of the adjacent boundary in the model. For example, if one end of a boundary with a Laminar inflow condition connects to a Slip boundary condition, then the laminar profile has a maximum at that end.
- For 3D models, select the **Constrain outer edges to zero** check box, which has the same effect as described for 2D models.

Symmetry

The **Symmetry** feature adds a boundary conditions that describe symmetry boundaries in a fluid-flow simulation. The boundary condition for symmetry boundaries prescribes no penetration and vanishing shear stresses:

$$\mathbf{u}\cdot\mathbf{n} = 0, \qquad \mathbf{t}\cdot\left(-p\mathbf{I}+\left(\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^{T})-\frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I}\right)\right)\mathbf{n} = 0$$

$$\mathbf{u}\cdot\mathbf{n} = 0, \qquad \mathbf{t}\cdot(-p\mathbf{I}+\eta(\nabla\mathbf{u}+(\nabla\mathbf{u})^{\mathrm{T}}))\mathbf{n} = 0$$

for the compressible and the incompressible formulation respectively.

BOUNDARIES

Select the boundaries that are symmetry boundaries.

Note: You do not need to specify a boundary condition for axial symmetry. For the symmetry axis at $r = 0$, the program automatically provides a condition that prescribes $u_r = 0$ and vanishing stresses in the z direction and adds an **Axial Symmetry** feature that implements this condition on the axial symmetry boundaries only.

Open Boundary

The **Open Boundary** feature adds boundary conditions that describe boundaries that are open to large volumes of fluid. Fluid can both enter and leave the domain on

boundaries with this type of condition. The following boundary condition are available:

- Normal Stress (the default)
- No Viscous Stress

BOUNDARIES

Select the boundaries that are open boundaries.

BOUNDARY CONDITIONS

Select a **Boundary condition** for the open boundaries:

Normal Stress

The total stress on the boundary is set equal to a stress vector of magnitude f_0, oriented in the negative normal direction:

$$\left(-p\mathbf{I}+\left(\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)-\frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I}\right)\right)\mathbf{n} = -\mathrm{f}_0\mathbf{n}$$

$$(-p\mathbf{I}+\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T))\mathbf{n} = -\mathrm{f}_0\mathbf{n}$$

using the compressible and the incompressible formulation respectively.

Enter the magnitude of the **Normal stress** f_0 (SI unit: $\mathrm{N/m^2}$).

This implies that the total stress in the tangential direction is zero. This boundary condition implicitly sets a constraint on the pressure that for 2D flows is

$$p = 2\mu\frac{\partial u_n}{\partial n}+\mathrm{f}_0 \tag{12-3}$$

If $\partial u_n/\partial n$ is small, Equation 12-2 states that $p \approx \mathrm{f}_0$.

No Viscous Stress

This boundary condition prescribes vanishing viscous stress:

$$\left(\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)-\frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I}\right)\mathbf{n} = \mathbf{0}$$

$$\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)\mathbf{n} = \mathbf{0}$$

using the compressible and the incompressible formulation respectively.

This condition can be useful in some situations because it does not impose any constraint on the pressure. A typical example is a model with volume forces that give rise to pressure gradients that are hard to prescribe in advance. To make the model numerically stable, you need to combine this boundary condition with a point constraint on the pressure (see Pressure Point Constraint).

Boundary Stress

The **Boundary Stress** feature adds a boundary conditions that represent a very general class of conditions also known as *traction boundary conditions*. The following boundary condition are available:

- General Stress (the default)
- Normal Stress
- Normal Stress, Normal Flow

BOUNDARIES

Select the boundaries where you have boundary stress.

BOUNDARY CONDITION

Select a **Boundary condition** for the boundary stress:

General Stress

The total stress on the boundary is set equal to a given stress $\boldsymbol{F}$:

$$\left(-p\mathbf{I}+\left(\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)-\frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I}\right)\right)\mathbf{n} = \mathbf{F}$$

$$(-p\mathbf{I}+\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T))\mathbf{n} = \mathbf{F}$$

using the compressible and the incompressible formulation respectively.

Enter the components of the **Stress** $\boldsymbol{F}$ (SI unit: $\mathrm{N/m^2}$).

This boundary condition implicitly sets a constraint on the pressure that for 2D flows is

$$p = 2\mu\frac{\partial u_n}{\partial n}-\mathbf{n}\cdot\mathbf{F} \tag{12-4}$$

If $\partial u_n/\partial n$ is small, Equation 12-4 states that $p \approx -\mathbf{n}\cdot\mathbf{F}$.

Normal Stress

The total stress on the boundary is set equal to a stress vector of magnitude f_0, oriented in the negative normal direction:

$$\left(-p\mathbf{I}+\left(\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)-\frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I}\right)\right)\mathbf{n} = -f_0\mathbf{n}$$

$$(-p\mathbf{I}+\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T))\mathbf{n} = -f_0\mathbf{n}$$

using the compressible and the incompressible formulation respectively.

Enter the magnitude of the **Normal stress** f_0 (SI unit: N/m^2).

This implies that the total stress in the tangential direction is zero. This boundary condition implicitly sets a constraint on the pressure that for 2D flows is

$$p = 2\mu\frac{\partial u_n}{\partial n}+f_0 \tag{12-5}$$

If $\partial u_n/\partial n$ is small, Equation 12-2 states that $p \approx f_0$.

Normal Stress, Normal Flow

In addition to the stress condition set in the Normal stress condition, this condition also prescribes that there must be no tangential velocities on the boundary:

$$\left(-p\mathbf{I}+\left(\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)-\frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I}\right)\right)\mathbf{n} = -f_0\mathbf{n}, \qquad \mathbf{t}\cdot\mathbf{u} = 0$$

$$(-p\mathbf{I}+\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T))\mathbf{n} = -f_0\mathbf{n}, \qquad \mathbf{t}\cdot\mathbf{u} = 0$$

using the compressible and the incompressible formulation respectively.

Enter the magnitude of the **Normal stress** f_0 (SI unit: N/m^2).

This boundary condition also implicitly sets a constraint on the pressure that for 2D flows is

$$p = 2\eta\frac{\partial u_n}{\partial n}+f_0 \tag{12-6}$$

If $\partial u_n/\partial n$ is small, Equation 12-6 states that $p \approx f_0$.

Periodic Flow Condition

The **Periodic Flow Condition** splits its selection in two groups—a source and a destination group. Fluid that leaves the domain through one of the destination boundaries enters the domain over the corresponding source boundary. This corresponds to a situation that the geometry is a periodic part of a larger geometry. If the boundaries are not parallel to each other, the velocity vector is automatically transformed. If the boundaries are curved, it is recommended to only include two boundaries in this feature.

The **Periodic Flow Condition** has no input when the interface property **Compressibility** is set to **Compressible flow (Ma<0.3)**. When **Compressibility** is set to **Incompressible flow**, it contains an input field for a **Pressure difference**, $p_{\mathrm{src}}-p_{\mathrm{dst}}$. This can be used to drive the flow in a fully developed channel flow. To control which sides that becomes the destination sides, add a **Destination Domains** subfeature to the **Periodic Flow Condition** feature. All destination sides must be connected.

Flow Continuity

The **Flow Continuity** node can be added to assembly pairs (see Pairs). It prescribes that the flow field is continuous across the pair. The feature is only suitable for pairs where the boundaries match.

BOUNDARIES

This section shows the boundaries for the selected pairs.

PAIR SELECTION

Select the **Pairs** where you want to impose continuity across the pair boundaries (Ctrl-click to deselect).

Pressure Point Constraint

The **Pressure Point Constraint** feature adds a pressure constraint at a point. If it is not possible to specify the pressure level using a boundary condition, you must set the pressure in some other way, for example, by specifying a fixed pressure at a point.

POINTS

Select the points where you want to use a pressure constraint.

PRESSURE CONSTRAINT

Enter a point constraint for the **Pressure** p_0 (SI unit: Pa).

Theory for the Single-Phase, Laminar Flow Interface

Each of the **Single-Phase Fluid Flow** interfaces has underlying theory that is discussed in this section.

- Theory for all the Single-Phase Flow Interfaces
- Theory for the Laminar Flow Interface

Theory for all the Single-Phase Flow Interfaces

The single-phase fluid-flow interfaces are based on the Navier-Stokes equations, which in their most general form read

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{12-7}$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla \cdot [-p\mathbf{I} + \tau] + \mathbf{F} \tag{12-8}$$

$$\rho C_p \left(\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla)T \right) = -(\nabla \cdot \mathbf{q}) + \tau : \mathbf{S} - \frac{T}{\rho} \frac{\partial \rho}{\partial T}\bigg|_p \left(\frac{\partial p}{\partial t} + (\mathbf{u} \cdot \nabla)p \right) + Q \tag{12-9}$$

where

- ρ is the density (SI unit: kg/m^3)
- $\mathbf{u}$ is the velocity vector (SI unit: m/s)
- p is pressure (SI unit: Pa)
- τ is the viscous stress tensor (SI unit: Pa)
- $\mathbf{F}$ is the volume force vector (SI unit: N/m^3)
- C_p is the specific heat capacity at constant pressure (SI unit: J/(kg·K))
- T is the absolute temperature (SI unit: K)
- $\mathbf{q}$ is the heat flux vector (SI unit: W/m^2)
- Q contains the heat sources (SI unit: W/m^3)

$\mathbf{S}$ is the strain rate tensor:

$$\mathbf{S} = \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$$

The operation ":" denotes a contraction between tensors defined by

$$\mathbf{a}:\mathbf{b} = \sum_n \sum_m a_{nm} b_{nm} \tag{12-10}$$

Equation 12-7 is the continuity equation and represents the conservation of mass. Equation 12-8 is a vector equation and represents the conservation of momentum. Equation 12-9 describes the conservation of energy, formulated in terms of temperature. This is an intuitive formulation that facilities specification of boundary conditions.

To close the equation system 12-7 through 12-9, some constitutive relations are needed. A common relation is derived by assuming that the fluid is Newtonian. Together with Stokes' assumption, the viscous stress tensor becomes:

$$\tau = 2\mu\mathbf{S} - \frac{2}{3}\eta(\nabla \cdot \mathbf{u})\mathbf{I} \tag{12-11}$$

The dynamic viscosity μ (SI unit: Pa·s) is allowed to depend on the thermodynamic state but not on the velocity field. All gases and many liquids can be considered Newtonian. Examples of non-Newtonian fluids are honey, mud, blood, liquid metals, and most polymer solutions.

Other commonly used constitutive relations are Fourier's law of conduction and the ideal gas law.

There are several books where you find derivations of the Navier-Stokes equations and detailed explanations of concepts such as Newtonian fluids and the Stokes assumption. See, for example, the classical text by Batchelor (Ref. 1) and the more recent work by Panton (Ref. 2).

Many applications describe isothermal flows where Equation 12-9 is decoupled from Equation 12-7 and Equation 12-8.

Theory for the Laminar Flow Interface

Fluid mechanics deals with studies of gases and liquids either in motion (*fluid dynamics*) or at rest (*fluid statics*). When studying liquid flows, it is often safe to assume that the material's density is constant or almost constant. You then have an

incompressible fluid flow. Using the Incompressible Navier-Stokes interface you can solve transient and steady-state models of incompressible fluid dynamics.

In theory, the same equations describe laminar as well as turbulent flows. In practice, however, the mesh resolution required to simulate turbulence with the Laminar Flow interface makes such an approach impractical.

COMPRESSIBLE FLOW

The Navier-Stokes equations solved by default in all single-phase flow interfaces are the compressible formulation of the continuity:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{12-12}$$

and the momentum equations:

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla \cdot \left(\mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I} \right) + \mathbf{F} \tag{12-13}$$

These equations holds for incompressible as well as compressible flows where the density varies.

THE MACH NUMBER LIMIT

An important dimensionless number in fluid dynamics is the Mach number, Ma, defined by

$$\mathrm{Ma} = \frac{|\mathbf{u}|}{a} \tag{12-14}$$

where a is the speed of sound. A flow is formally incompressible when $\mathrm{Ma} = 0$. This is theoretically achieved by letting the speed of sound tend to infinity. The Navier-Stokes equations will then have the numerical property that a disturbance anywhere in the computational domain will instantaneously spread to the entire domain. This results in a parabolic equation system.

The fully compressible Navier-Stokes equations, Equation 12-7 through Equation 12-9, have a finite speed of sound and hence a Mach number larger than zero. This will have no numerical significance as long as the Mach number is well below one. However, when the Mach number approaches unity, the equations turn from parabolic to hyperbolic. When this happens, the numerical properties of the equation change, one of several implications being that the boundary conditions used for incompressible Navier-Stokes equations become invalid. The compressible

formulation of the laminar and turbulent interfaces uses the same boundary conditions as the incompressible interfaces, which implies that the compressible interfaces cannot be used for flows with Mach number larger than or equal to one.

The practical Mach number limit is lower than one, however. The first reason for this is that the sound wave transport term that has been neglected in the heat equation. This term becomes important already at moderate Mach numbers. The second reason is that already at moderate Mach number, the fully compressible Navier-Stokes equations start to display very sharp gradients. To handle these gradient, special numerical techniques are needed. It is impossible to give an exact limit where the low Mach number regime ends and the moderate Mach number regime begins, but a rule of thumb is that the Mach number effects start to appear at $\mathrm{Ma} = 0.3$. For this reason the compressible formulation is referred to as **Compressible flow (Ma<0.3)** in the COMSOL GUI.

INCOMPRESSIBLE FLOW

When the temperature variations in a flow are small, a single-phase fluid can often be assumed incompressible; that is, ρ is constant or nearly constant. This is the case for all fluids under normal conditions and also for gases at low velocities. For constant ρ, Equation 12-12 reduces to

$$\nabla \cdot \mathbf{u} = 0 \tag{12-15}$$

and Equation 12-13 becomes

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla \cdot [-p\mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] + \mathbf{F} \tag{12-16}$$

THE REYNOLDS NUMBER

Fundamental to the analysis of fluid flow is the Reynolds number:

$$\mathrm{Re} = \frac{\rho UL}{\mu} \tag{12-17}$$

where U denotes a velocity scale, and L denotes a representative length. The Reynolds number represents the ratio between inertial and viscous forces. At low Reynolds numbers, viscous forces dominate and tend to damp out all disturbances, which leads to laminar flow. At high Reynolds numbers, the damping in the system is very low giving small disturbances the possibility to grow by nonlinear interactions. If the Reynolds number is high enough, the fluid flow field eventually ends up in a chaotic state called turbulence. The Navier-Stokes interface automatically calculates the local

cell Reynolds number $\mathrm{Re}^c = \rho|\mathbf{u}|h/(2\mu)$ using the element length h for L and the magnitude of the velocity vector u for the velocity scale U. The cell Reynolds number is a predefined quantity available for visualization and evaluation.

NUMERICAL STABILITY—STABILIZATION TECHNIQUES

The momentum equations (Equation 12-13 and Equation 12-16) are (nonlinear) convection-diffusion equations. As described in Stabilization Techniques in the *COMSOL Multiphysics Reference Guide* such equations are unstable if discretized using the Galerkin finite element method. Stabilized finite element methods are therefore necessary in order to obtain physical solutions. You find the stabilization settings in the main fluid-flow features. Also see Where Do I Access the Documentation and Model Library?.

There are three types of stabilization methods available for Navier-Stokes. Two of them are consistent stabilization methods:

- Streamline diffusion (GLS)
- Crosswind diffusion

There is also an inconsistent stabilization method: Isotropic diffusion

For optimal functionality, the exact weak formulations and constants of GLS and crosswind diffusion depend on the order of the basis functions (elements). The values of constants of GLS and crosswind diffusion follow Ref. 4 and Ref. 5.

Streamline Diffusion

For strongly coupled systems of equations, the streamline diffusion must be applied to the whole system of equations, not only to each equation separately. These ideas were first explored by Hughes and Mallet (Ref. 6) and were later extended to Galerkin least-squares (GLS) applied to the Navier-Stokes equations (Ref. 7), which is the form that COMSOL Multiphysics supports. The time-scale tensor is the diagonal tensor presented in Ref. 8.

Streamline diffusion is active per default because it is necessary when convection is the dominating part of the flow.

The unstabilized incompressible Navier-Stokes equations are subject the Babuska-Brezzi condition, which states that the basis functions for the pressure must be of lower order than the basis functions for the velocity. If the incompressible Navier-Stokes equations are stabilized by GLS, it is possible to use equal-order interpolation. Hence, streamline diffusion is necessary when using first order elements. This applies also if the model is solved using geometric multigrid (either as a solver or

as a preconditioner) and at least one multigrid hierarchy level uses linear Lagrange elements.

Crosswind Diffusion

Crosswind diffusion can also be formulated for systems of equations, and when applied to the Navier-Stokes equations it becomes a shock-capturing operator. COMSOL supports the formulation in Ref. 7 with shock capturing viscosity taken from Ref. 9.

Incompressible flows do not contain shock waves, but crosswind diffusion is still useful for introducing extra diffusion in sharp boundary layers and shear layers that otherwise would require a very dense mesh to resolve.

The tuning parameter, C_k, controls the amount of crosswind diffusion introduced in the model. The recommended range for low Mach number flows and incompressible flows is $0 < C_k < 0.5$ ($C_k = 0$ means no diffusion at all). The value must be neither space dependent nor time dependent.

Crosswind diffusion is active per default because it makes it easier to obtain a solution even if the mesh is not perfect.

Crosswind diffusion also has the effect that iterative solvers can use inexpensive presmoothers such as SSOR (see Preconditioners for the Iterative Solvers). If you deactivate crosswind diffusion, you must also change all applications of SSOR to Vanka (see Preconditioners for the Iterative Solvers in the *COMSOL Multiphysics Reference Guide* or Where Do I Access the Documentation and Model Library?).

Isotropic Diffusion

Isotropic diffusion adds diffusion to the momentum equations of the Navier-Stokes equations in the same way as described in Isotropic Diffusion of the *COMSOL Multiphysics Reference Guide* (or see Where Do I Access the Documentation and Model Library?). The continuity equation gains no extra stability.

PSEUDO TIME STEPPING

A stationary formulation has per definition no time derivatives. Hence, Equation 12-16 reduces to:

$$\rho(\mathbf{u}\cdot\nabla)\mathbf{u} = \nabla\cdot[-p\mathbf{I}+\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)]+\mathbf{F} \qquad (12\text{-}18)$$

Solving Equation 12-18 requires a starting guess that is close enough to the final solution. If no such guess is at hand, the fully transient problem can be solved instead. This is however a rather costly approach in terms of computational time. An intermediate approach is to add a fictive time derivative to Equation 12-18:

$$\rho\frac{\mathbf{u}-\text{nojac}(\mathbf{u})}{\tilde{\Delta t}}+\rho(\mathbf{u}\cdot\nabla)\mathbf{u}=\nabla\cdot[-p\mathbf{I}+\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)]+\mathbf{F} \tag{12-19}$$

where $\tilde{\Delta t}$ is a *pseudo time step*. Since $\mathbf{u}-\texttt{nojac}(\mathbf{u})$ is always zero, this term does not affect the final solution. It does however affect the discrete equation system and will effectively transform a non-linear iteration into a time step of size $\tilde{\Delta t}$.

The pseudo time step $\tilde{\Delta t}$ can be chosen individually for each element based on the local CFL number:

$$\tilde{\Delta t}=\text{CFL}_{\text{loc}}\frac{h}{|\mathbf{u}|} \tag{12-20}$$

where h is the mesh cell size. A small CFL number means a small time step. It is hence practical to start with a small CFL number and gradually increase it as the solution approaches steady state. If the automatic expression for CFL_{loc} is

$$\begin{gathered}1.5^{\min(\text{niterCMP},\,12)}+\\ \text{if}(\text{niterCMP}>15,\,10\cdot 1.5^{\min(\text{niterCMP}-15,\,12)},\,0)+\\ \text{if}(\text{niterCMP}>30,\,100\cdot 1.5^{\min(\text{niterCMP}-30,\,12)},\,0)\end{gathered} \tag{12-21}$$

The variable `niterCMP` is an integer that starts at one. CFL_{loc} starts at 1.5 and increases with 50% each iteration until it reaches $1.5^{12}\approx 130$. It remains there until iteration 16 where it starts to increase unit it reaches approximately 1430. A final increase then takes it to 14400.

The continuity equation has a time derivative only in the incompressible case and it can then be written as

$$\frac{\partial\rho}{\partial p}\frac{\partial p}{\partial t} \tag{12-22}$$

A pseudo time derivative can hence be written as

$$\frac{\partial\rho}{\partial p}\frac{p-\text{nojac}(p)}{\tilde{\Delta t}} \tag{12-23}$$

For incompressible flow, the following approximation is used:

$$\frac{\partial\rho}{\partial p}\approx\frac{0.1\rho}{1[\text{atm}]} \tag{12-24}$$

while for compressible flows

$$\frac{\partial \rho}{\partial p} \approx \text{if}\left(\frac{\partial \rho}{\partial p} = 0, \frac{0.1\rho}{1[\text{atm}]}, \frac{\partial \rho}{\partial p}\right) \tag{12-25}$$

Pseudo time stepping is active per default in 3D but can be activated also for 2D models.

Equation 12-21 can for some advanced flows increase CFL_{loc} too quickly. CFL_{loc} can then be prescribed manually.

THE BOUSSINESQ APPROXIMATION

The Boussinesq approximation is a way to treat some simple cases of buoyant flows without having to use a compressible formulation of the Navier-Stokes equations.

The Boussinesq approximation assumes that variations in density have no effect on the flow field except that they give rise to buoyant forces. The density is taken to be a reference value, ρ_0, except in the body force term, which is set to

$$\mathbf{F} = (\rho_0 + \Delta\rho)\mathbf{g} \tag{12-26}$$

where $\mathbf{g}$ is the gravity vector. You can enter an expression for Equation 12-26 in the **Volume force** edit fields in the **Volume Force** feature; however, further simplifications are often possible. Because $\mathbf{g}$ can be written in terms of a potential, Φ, it is possible to write Equation 12-26 as:

$$\mathbf{F} = -\nabla(\rho_0\Phi) + \Delta\rho\mathbf{g}$$

The first part can be canceled out by splitting the true pressure, p, as a sum of a hydrodynamic component, P, and a hydrostatic component, $-\rho_0\Phi$. Then you can write Equations 12-15 and 12-16 in terms of the hydrodynamic pressure $P = p + \rho_0\Phi$:

$$\nabla \cdot \mathbf{u} = 0 \tag{12-27}$$

$$\rho_0\frac{\partial \mathbf{u}}{\partial t} + (\rho_0\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla P + \nabla \cdot (\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}})) + \mathbf{g}\Delta\rho \tag{12-28}$$

To obtain the Boussinesq approximation on this form, you only have to enter the expression for $\mathbf{g}\Delta\rho$ in the **Volume force** edit fields.

In practice, the shift from p to P can be ignored except where the pressure appears in boundary conditions. The pressure that you specify at boundaries is the hydrodynamic pressure in this case. For example, on a vertical outflow or inflow boundary, the hydrodynamic pressure is typically a constant, while the true pressure is a function of the vertical coordinate.

The system that Equation 12-27 and Equation 12-28 form has its limitations. The main assumption is that the density fluctuations must be small; that is, $\Delta\rho/\rho_0 << 1$. There are also some more subtle constraints that, for example, makes the Boussinesq approximation unsuitable for systems of very large dimensions. An excellent discussion of the Boussinesq approximation and its limitations appears in Chapter 14 of Ref. 10.

LAMINAR INFLOW CONDITION

In order to prescribe an inlet velocity profile, this boundary conditions adds a weak form contribution corresponding to a one-dimensional Navier-Stokes equations projected on the boundary. The applied condition corresponds to the situation shown in Figure 12-1: a fictitious domain of length L_{entr} is assumed to be attached to the inlet of the computational domain. This boundary condition uses the assumption that flow in this fictitious domain is a laminar plug flow. If you select the option that constrains outer edges or endpoints to zero, the assumption is instead that the flow in the fictitious domain is fully developed laminar channel flow (in 2D) or fully developed laminar internal flow (in 3D). This does not affect the boundary condition in the real domain, Ω, where the boundary conditions are always fulfilled.

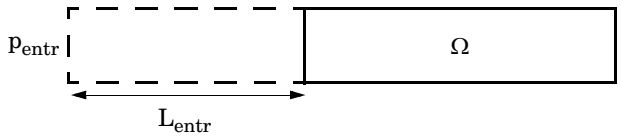


Figure 12-1: Sketch of the physical situation simulated when using the Laminar inflow boundary condition. Ω is the actual computational domain while the dashed domain is a fictitious domain.

If you specify an average inlet velocity or inlet volume flow instead of the pressure, COMSOL Multiphysics adds an ODE that calculates a pressure, p_{entr}, such that the desired inlet velocity or volume flow is obtained.

LAMINAR OUTFLOW CONDITION

In order to prescribe an outlet velocity profile, this boundary conditions adds a weak form contribution corresponding to a one-dimensional Navier-Stokes equations projected on the boundary. The applied condition corresponds to the situation shown in Figure 12-2: assume that a fictitious domain of length L_{exit} is attached to the outlet of the computational domain. This boundary condition uses the assumption that the flow in this fictitious domain is laminar plug flow. If you select the option that constrains outer edges or endpoints to zero, the assumption is instead that the flow in the fictitious domain is fully developed laminar channel flow (in 2D) or fully developed

laminar internal flow (in 3D). This does not affect the boundary condition in the real domain, Ω, where the boundary conditions are always fulfilled.

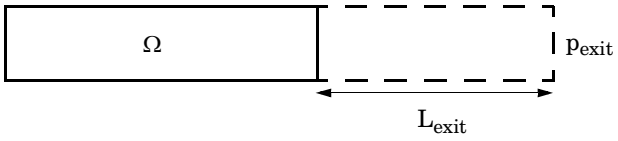


Figure 12-2: Sketch of the physical situation simulated when using Laminar outflow boundary condition. Ω is the actual computational domain while the dashed domain is a fictitious domain.

If you specify an average outlet velocity or outlet volume flow instead of the pressure, the software adds an ODE that calculates p_{exit} such that the desired outlet velocity or volume flow is obtained.

REFERENCES FOR THE SINGLE-PHASE FLOW INTERFACES

1. G.K. Batchelor, *An Introduction To Fluid Dynamics*, Cambridge University Press, 1967.

2. R.L. Panton, *Incompressible Flow*, second edition, John Wiley & Sons, Inc., 1996.

3. P.M. Gresho and R.L. Sani, *Incompressible Flow and the Finite Element Method, Volume 2: Isothermal Laminar Flow*, John Wiley and Sons, LTD, 2000.

4. I. Harari and T.J.R. Hughes, "What are *C* and *h*? Inequalities for the analysis and design of finite element methods," *Comp. Meth. Appl. Mech. Engrg*, vol. 97, pp. 157–192, 1992.

5. Y. Bazilevs, V.M. Calo, T.E. Tezduyar, and T.J.R. Hughes, "*YZ*β discontinuity capturing for advection-dominated processes with application to arterial drug delivery," *Int.J.Num. Meth. Fluids*, vol. 54, pp. 593–608, 2007.

6. T.J.R. Hughes and M. Mallet, "A new finite element formulation for computational fluid dynamics: III. The Generalized Streamline Operator for Multidimensional Advective-Diffusive System," *Comp. Meth. Appl. Mech. Engrg*, vol. 58, pp. 305–328, 1986.

7. G. Hauke and T.J.R. Hughes, "A unified approach to compressible and incompressible flows," *Comp. Meth. Appl. Mech. Engrg*, vol. 113, pp. 389–395, 1994.

8. G. Hauke, “Simple stabilizing matrices for the computation of compressible flows in primitive variables,” *Comp. Meth. Appl. Mech. Engrg*, vol. 190, pp. 6881–6893, 2001.

9. A. Soulaïmani and M. Fortin, “Finite element solution of compressible viscous flows using conservative variables,” *Comp. Meth. Appl. Mech. Engrg*, vol. 118, pp. 319–350, 2001.

10. D.J. Tritton, *Physical Fluid Dynamics*, second edition, Oxford University Press, 1988.

11. J.M. Coulson and J.F. Richardson, “Particle Technology and Separation Processes,” *Chemical Engineering*, vol. 2, Butterworth-Heinemann.

12. P.M. Gresho and R.L. Sani, *Incompressible Flow and the Finite Element Method, Volume 2: Isothermal Laminar Flow*, John Wiley and Sons, LTD, 2000.

13

Heat Transfer Interfaces

This section covers the **Heat Transfer** interface. It starts with some background on heat transfer. It then reviews the specifics of this physics interface. It also contains information about the predefined multiphysics interface for Joule heating.

In this section:

- See The Heat Transfer Interface for information about **Heat Transfer in Solids (ht)** () and **Heat Transfer in Fluids (ht)** ()
- The Joule Heating Interface 

The underlying theory for these interface is also discussed:

- Theory for the Heat Transfer Interfaces

The Heat Transfer Interface

The **Heat Transfer** interfaces model heat transfer by conduction and convection. You can also includes surface-to-ambient radiation effects around edges and boundaries. The interfaces are suitable for modeling heat transfer in solids and fluids. The interfaces are available in 1D, 2D, and 3D and or axisymmetric models with cylindrical coordinates in 1D and 2D. The default dependent variable is the temperature, T.

Accessing the Heat Transfer Interfaces via the Model Wizard

There are **Heat Transfer** interfaces displayed in the **Model Builder** with the same name but with different icons and default models. After selecting a **Heat Transfer** interface in the **Model Wizard**, default settings are added under the main node. For example, if you select **Heat Transfer in Solids** (), a Heat Transfer node is added with a default Heat Transfer in Solids model. If you select **Heat Transfer in Fluids** (), a Heat Transfer in Fluids model is added instead, but the parent nodes are both called Heat Transfer. All the interfaces that are based on the main Heat Transfer feature has the suffix **ht**.

Select:

- **Heat Transfer in Solids (ht)** () to model mainly heat transfer in solid materials. A default **Heat Transfer in Solids** model is added, but all functionality for including fluid domains is also available.
- **Heat Transfer in Fluids (ht)** () to model mainly heat transfer in fluid materials. A default **Heat Transfer in Fluids** model is added, but all functionality for including solid domains is also available.

Select the other available physics interfaces as required. Select:

- **Joule Heating (jh)** () to combine all features from the Electric Currents interface with the Heat Transfer interface for modeling Joule heating (resistive heating or ohmic heating). See The Joule Heating Interface.

The Heat Transfer Interface

The **Heat Transfer (ht)** interface provides the equations, boundary conditions, and sources for modeling conductive and convective heat transfer, solving for the temperature.

This interface adds these default nodes based on your selection in the **Model Wizard**—**Heat Transfer in Solids** or **Heat Transfer in Fluids**, **Thermal Insulation** (the default boundary condition) and **Initial Values**.

Right-click the **Heat Transfer** node to add a **Heat Transfer in Solids** or **Heat Transfer in Fluids** node or other feature that implements boundary conditions and sources.

The following sections provide information about all features available in the interface.

INTERFACE IDENTIFIER

This is the physics interface identifier, which you use to reach the fields and variables in expressions, for example.

The identifier appears in the **Identifier** edit field, and you can change it to any unique string that is a valid identifier. The default identifier (for the first Heat Transfer interface in the model) is `ht`.

DOMAINS

Select the domains where you want to define heat transfer and a temperature field. The default setting is to include all domains in the model.

PHYSICAL MODEL

From the **Default model** list, select **Heat transfer in solids** if you want the Heat Transfer in Solids feature for heat conduction in solids as the default property group, or select **Heat transfer in fluids** if you want the Heat Transfer in Fluids feature for heat convection and conduction in fluids as the default property group.

DEPENDENT VARIABLES

The Heat Transfer interface includes a dependent variable for the temperature T. You can change the default name **T** in the **Temperature** edit field. The surface radiosity J is not used.

ADVANCED SETTINGS AND DISCRETIZATION

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Normally these settings do not need to be changed. More information on advanced settings can be found in Show More Options: Advanced Settings and Discretization.

CONSISTENT STABILIZATION

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Also see Show More Options: Consistent and Inconsistent Stabilization for information about these settings.

- The consistent stabilization methods take effect for fluids and for solids with translational motion (see Translational Motion). A stabilization method is active when the corresponding check box is selected. The Streamline diffusion check box is selected by default and should remain active for optimal performance for heat transfer in fluids or other applications that include a convective or translational term.
- The crosswind diffusion provides extra diffusion in the region of sharp gradients. The added diffusion is orthogonal to the streamline diffusion, so you can use streamline diffusion and crosswind diffusion simultaneously.
- If **Crosswind diffusion** is selected, enter a **Lower gradient limit** $\mathbf{g}_{\text{lim}}$ (SI unit: K/m). The default is `0.01[K]/ht.helem`. The variable $\mathbf{g}_{\text{lim}}$ is needed because both Equation 13-1 and Equation 13-2 contain terms of the form $1/|\nabla T|$, which become singular if $\nabla T = 0$. Hence, all occurrences of $1/|\nabla T|$ are replaced by $1/\max(|\nabla T|, g_{\text{lim}})$ where g_{lim} is a measure of a small gradient.

The method in the Heat Transfer interfaces adds the following contribution to the weak formulation (see Codina in Ref. 2):

$$-\sum_{e=1}^{N_{\text{el}}}\int_{\Omega^e}\frac{1}{2}\max\left(0, C^e - \frac{2k}{h|\beta|}\right)\frac{h|R|}{|\nabla T|}\nabla\hat{T}\left(I - \frac{\mathbf{u}\otimes\mathbf{u}}{|\mathbf{u}|^2}\right)\nabla T\,d\Omega \qquad (13\text{-}1)$$

where R is the PDE residual, $\hat{T}$ is the test function for T, h is the element size, and β is defined as

$$\beta = \begin{cases} \dfrac{\rho C_p(\mathbf{u}\cdot\nabla T)}{|\nabla T|^2}\nabla T & \text{if } \nabla T \neq 0 \\ 0 & \text{if } \nabla T = 0 \end{cases} \qquad (13\text{-}2)$$

INCONSISTENT STABILIZATION

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Also see Show More Options: Consistent and Inconsistent Stabilization for information about these settings.

Heat Transfer in Solids

The **Heat Transfer in Solids** model uses the following version of the *heat equation* as the mathematical model for heat transfer in solids.

$$\rho C_p \frac{\partial T}{\partial t} - \nabla \cdot (k \nabla T) = Q \qquad (13\text{-}3)$$

with the following material properties:

- ρ is the *density*
- C_p is the *heat capacity*
- k is the *thermal conductivity* (a scalar or a tensor if the thermal conductivity is anisotropic)
- Q is the heat source (or sink); one or more heat sources can be added separately

For a steady-state problem the temperature does not change with time and the first term disappears.

When parts of the model (for example, a heat source) is moving, you can add a **Translational Motion** subfeature to the **Heat Transfer in Solids** node to take this into account. See Translational Motion.

The **Heat Transfer in Solids** page contains the following sections:

DOMAINS

Select the domains where you want to define the heat transfer.

MODEL INPUTS

This section contain fields and values that are inputs to expressions that define material properties. If you have added such user defined property groups, their model inputs appear here. Initially, this section is empty.

COORDINATE SYSTEM SELECTION

Select the **Coordinate system** to use. The list has all available coordinate systems (except boundary coordinate systems). The default is to use the Global Cartesian coordinate system, which always exists. The coordinate system is used for interpreting directions of orthotropic and anisotropic thermal conductivity.

HEAT CONDUCTION

The default setting is to use the **Thermal conductivity** k (SI unit: W/(m·K)) **From material**. Select **User defined** to enter another value or expression instead. Then, depending on the characteristics of the thermal conductivity, select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** and enter values. The thermal conductivity describes the relationship between the heat flux vector $\mathbf{q}$ and the temperature gradient ∇T as in

$$\mathbf{q} = -k\nabla T$$

which is *Fourier's law of heat conduction*. Enter this quantity as power per length and temperature.

THERMODYNAMICS

The default **Heat capacity at constant pressure** C_p (SI unit: J/(kg·K)) and **Density** ρ (SI unit: kg/m^3) use values **From material**. Select **User defined** to enter other values. The heat capacity at constant pressure describes the amount of heat energy required to produce a unit temperature change in a unit mass.

Translational Motion

The **Translational Motion** feature provides movement by translation for modeling of heat transfer in solid. It adds the following contribution to the right hand side of Equation 13-3:

$$-\rho C_p \mathbf{u} \cdot \nabla T$$

The contribution describes the effect of a moving coordinate system that is required to model, for example, a moving heat source.

Observe that special care must be taken at boundaries where $\mathbf{n} \cdot \mathbf{u} \neq 0$. The Heat Flux boundary condition does not, for example, work at boundaries where $\mathbf{n} \cdot \mathbf{u} < 0$.

DOMAINS

Select the domains where you want to prescribe a translational. By default, the selection is the same as for the **Heat Transfer in Solids** node that it is attached to, but it is possible to use more than one **Heat Translation** subfeature, each covering a subset of the **Heat Transfer in Solids** node's selection.

TRANSLATIONAL MOTION

Enter component values for x, y, and z (in 3D) for the **Velocity field** u (SI unit: m/s).

Heat Transfer in Fluids

The **Heat Transfer in Fluids** feature uses the following version of the *heat equation* as the mathematical model for heat transfer in fluids:

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p \mathbf{u} \cdot \nabla T = \nabla \cdot (k \nabla T) + Q \qquad (13\text{-}4)$$

with the following material properties:

- ρ is the *density*
- C_p is the fluid *heat capacity* at constant pressure and describes the amount of heat energy required to produce a unit temperature change in a unit mass
- k is the fluid *thermal conductivity* (a scalar or a tensor if the thermal conductivity is anisotropic)
- $\mathbf{u}$ is the fluid velocity field, which can be an analytic expression or a velocity field from a fluid-flow interface
- Q is the heat source (or sink); one or more heat sources can be added separately

For a steady-state problem the temperature does not change with time and the first term disappears.

Also, the *ratio of specific heats* γ is defined on this page. It is the ratio of heat capacity at constant pressure, C_p, to heat capacity at constant volume, C_v. When using the ideal gas law to describe a fluid, specifying γ is enough to evaluate C_p. For common diatomic gases such as air, $\gamma = 1.4$ is the standard value. Most liquids have $\gamma = 1.1$ while water has $\gamma = 1.0$. γ is used in the streamline stabilization and in the postprocessing variables for heat fluxes and total energy fluxes.

The **Heat Transfer in Fluids** form contain the following sections:

DOMAINS

Select the domains where you want to define the heat transfer.

MODEL INPUTS

This section has fields and values that are inputs to expressions that define material properties. If you have added such user defined property groups, their model inputs appear here.

There are also two standard model inputs—**Absolute pressure** and **Velocity field**. The absolute pressure is used in some postprocessing quantities that include the enthalpy (the energy flux, for example).

Enter the **Absolute pressure** p_a (SI unit: Pa). The default is atmosphere pressure (101,325 Pa).

From the **Velocity field** list, select an existing velocity field in the model (for example, **Velocity field (spf/fp1)** from a Laminar Flow interface) or select **User defined** to enter values or expressions for the components of the **Velocity field** (SI unit: m/s).

COORDINATE SYSTEM SELECTION

Select the **Coordinate system** to use. The list has all available coordinate systems (except boundary coordinate systems). The default is to use the Global Cartesian coordinate system, which always exists. The coordinate system is used for interpreting directions of orthotropic and anisotropic thermal conductivity.

HEAT CONDUCTION

The default is to use the **Thermal conductivity** k (SI unit: W/(m·K)) **From material**. Select **User defined** to enter another value. Then, depending on the characteristics of the thermal conductivity, select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic**. The thermal conductivity describes the relationship between the heat flux vector $\mathbf{q}$ and the temperature gradient ∇T as in

$$\mathbf{q} = -k\nabla T$$

which is *Fourier's law of heat conduction*. Enter this quantity as power per length and temperature.

THERMODYNAMICS

The defaults in this section take values **From material** for the **Density** (SI unit: kg/m^3), **Heat capacity at constant pressure** C_P (SI unit: J/(kg·K)), and the **Ratio of specific heats** γ (unitless) for a general gas or liquid. Select **User defined** to enter other values.

Heat Source

Add one or more **Heat Source** nodes. The heat source describes heat generation within the domain. Express heating and cooling with positive and negative values, respectively.

DOMAINS

Select the domains where you want to add the heat source.

HEAT SOURCE

Select either the **General source** or **Linear source** button.

If **General source** is selected, enter a value for Q (SI unit: W/m^3).

If **Linear source** (Q=qs·T) is selected, enter the **Production/absorption coefficient**, q_s (SI unit: $W/(m^3 \cdot K)$).

Note: The advantage in writing the source in this form is that it can be stabilized by the streamline diffusion (see Stabilization Techniques in the *COMSOL Multiphysics Reference Guide* or Where Do I Access the Documentation and Model Library?). The theory covers q_s that is independent of the temperature, but some stability can be gained as long as q_s is only weakly dependent on the temperature.

Electromagnetic Heat Source

The **Electromagnetic Heat Source** node is added as a default node in the Joule Heating and Microwave Heating predefined multiphysics interfaces. Microwave Heating requires the RF Module.

The resistive heating (ohmic heating) due to the electric current is proportional to $I^2 \cdot R$, where I is the electric current and R is the resistance.

Initial Values

The **Initial Values** node adds an initial value for the temperature T that can serve as an initial condition for a transient simulation or as an initial guess for a nonlinear solver. If you need to specify more than one set of initial values, you can add additional **Initial Values** features.

DOMAINS

Select the domains where you want to define an initial value.

INITIAL VALUES

Enter a value or expression for the initial value of the temperature T in the **Temperature** edit field. The default value is approximately room temperature, 293.15 K (20° C).

Boundary Conditions

The following boundary conditions are available:

- Temperature
- Thermal Insulation (the default boundary condition)
- Outflow
- Symmetry
- Heat Flux
- Surface-to-Ambient Radiation
- Surface-to-Ambient Radiation
- Heat Continuity

In addition, it is possible to specify boundary heat sources (see Boundary Heat Source). In 3D and 2D it is also possible to add point heat sources (see Point Heat Source). 3D furthermore supports line heat sources (Line Heat Source).

For axisymmetric models, COMSOL Multiphysics takes the axial symmetry boundaries into account and automatically adds an **Axial Symmetry** node to the model that is valid on the axial symmetry boundaries only.

Temperature

The **Temperature** feature prescribes a temperature on a boundary.

BOUNDARIES

Select the boundaries to which you want to apply a prescribed temperature.

TEMPERATURE

The equation for this condition is

$$T = T_0$$

where T_0 is the prescribed temperature (SI unit: K) on the boundary. Enter the value or expression for the **Temperature** T_0. The default is 293.15 K.

Thermal Insulation

The **Thermal Insulation** feature is the default boundary condition for all heat transfer interfaces. This boundary condition means that there is no heat flux across the boundary:

$$\mathbf{n} \cdot (k \nabla T) = 0$$

This condition specifies where the domain is well insulated. Intuitively this equation says that the temperature gradient across the boundary must be zero. For this to be true, the temperature on one side of the boundary must equal the temperature on the other side. Because there is no temperature difference across the boundary, heat cannot transfer across it.

An interesting numerical check for convergence is the numerical evaluation of Thermal Insulation condition along the boundary. Another check is to plot the temperature field as a contour plot. Ideally the contour lines are perpendicular to any insulated boundary.

The **Thermal Insulation** feature does not require any user input.

Outflow

The **Outflow** feature provides a suitable boundary condition for convection-dominated heat transfer at outlet boundaries. In a model with convective heat transfer, this condition states that the only heat transfer over a boundary is by convection. The temperature gradient in the normal direction is zero, and there is no radiation. This is usually a good approximation of the conditions at an outlet boundary in a heat transfer model with fluid flow.

The **Outflow** node does not usually require any user input.

BOUNDARIES

Select the boundaries that are convection-dominated outlet boundaries.

Heat Flux

Use the **Heat Flux** feature to add heat flux across boundaries. A positive heat flux adds heat to the domain. This feature is not applicable to inlet boundaries.

BOUNDARIES

Select the boundaries where you want to add the heat flux contribution.

HEAT FLUX

Select either the **General inward heat flux** or **Inward heat flux** buttons.

If **General inward heat flux** q_0 (SI unit: $\mathrm{W/m^2}$) is selected, it adds to the total flux across the selected boundaries. Enter a value for q_0 to represent a heat flux that enters the domain. For example, any electric heater is well represented by this condition, and you can omit its geometry.

If **Inward heat flux** is selected (in the form $q_0 = h \cdot (T_{\mathrm{ext}} - T)$), enter the **Heat transfer coefficient** h (SI unit: $\mathrm{W/(m^2 \cdot K)}$. The default is 0.

Enter an **External temperature** T_{ext} (SI unit: K). The default is 293.15 K. The value depends on the geometry and the ambient flow conditions. For a thorough introduction on how to calculate heat transfer coefficients, see Incropera and DeWitt in Ref. 1.

Surface-to-Ambient Radiation

Use the **Surface-to-Ambient Radiation** boundary condition to add surface-to-ambient radiation to boundaries. The net inward heat flux from surface-to-ambient radiation is

$$q = \varepsilon\sigma(T_{\mathrm{amb}}^4 - T^4)$$

where ε is the surface emissivity, σ is the Stefan–Boltzmann constant (a predefined physical constant), and T_{amb} is the ambient temperature.

BOUNDARIES

Select the boundaries where you want to add surface-to-ambient radiation contribution.

SURFACE-TO-AMBIENT RADIATION

The default **Surface emissivity** e (a dimensionless number between 0 and 1) is taken **From material**. An emissivity of 0 means that the surface emits no radiation at all and an emissivity of 1 means that it is a perfect blackbody.

Enter an **Ambient temperature** T_{amb} (SI unit: K). The default is 293.15 K.

Periodic Heat Condition

Use the **Periodic Heat Condition** feature to add a periodic heat condition to boundaries.

BOUNDARIES

Select the boundaries where you want to add a periodic heat condition.

Heat Continuity

The **Heat Continuity** node can be added to assembly pairs (see Pairs in the *COMSOL Multiphysics Reference Guide* or Where Do I Access the Documentation and Model Library?). It prescribes that the temperature field is continuous across the pair. The **Heat Continuity** is only suitable for pairs where the boundaries match.

BOUNDARIES

The selection list in this section shows the boundaries for the selected pairs.

PAIR SELECTION

Select the pairs where you want to impose continuity across the pair boundaries. Select the pairs from the **Pairs** list (Ctrl-click to deselect).

Symmetry

The **Symmetry** node provides a boundary condition for symmetry boundaries. This boundary condition is similar to an insulation condition, and it means that there is no heat flux across the boundary. In most cases, the **Symmetry** node does not require any user input.

BOUNDARIES

Define the symmetry boundaries as required.

Boundary Heat Source

The **Boundary Heat Source** node models a heat source (or heat sink) that is embedded in the boundary.

BOUNDARIES

Select the boundaries where you want to apply the heat source.

BOUNDARY HEAT SOURCE

Enter the quantity Q_b (SI unit: W/m^2). A positive Q_b means heating while a negative Q_b means cooling. The default is 0.

Point Heat Source

The **Point Heat Source** node models a heat source (or sink) that is so small that it can be considered to have no spatial extension. It is available in 2D and 3D. It is not available in 1D since points are boundaries (possibly internal boundaries) there.

In theory, the temperature in a point source in 2D or 3D is plus or minus infinity (to compensate for the fact that the heat source does not have a spatial extension). The finite element discretization used in COMSOL returns a finite value, but that value must be interpreted in a weak sense.

POINTS

Select the points where you want to apply the heat source.

POINT HEAT SOURCE

Enter the quantity Q_p in unit power (W in SI units). Positive Q_p means heating while a negative Q_p means cooling.

Line Heat Source

The **Line Heat Source** node models a heat source (or sink) that is so thin that it has no thickness. It is available in 3D only since a line in 2D is a boundary and it is a domain in 1D.

In theory, the temperature in a line source in 3D is plus or minus infinity (to compensate for the fact that the heat source does not have any volume). The finite element discretization used in COMSOL returns a finite temperature distribution along the line, but that distribution must be interpreted in a weak sense.

EDGES

Select the edges where you want to apply the heat source.

LINE HEAT SOURCE

Enter the quantity Q_l in unit power per unit length (W/m in SI units). Positive Q_l means heating while a negative Q_l means cooling.

Pair Thin Thermally Resistive Layer

Use the **Pair Thin Thermally Resisitive Layer** feature to define the layer thickness and thermal conductivity on pair boundaries. It can be added to assembly pairs (see Pairs

in the *COMSOL Multiphysics Reference Guide* or Where Do I Access the Documentation and Model Library?)

BOUNDARIES

The selection list in this section shows the boundaries for the selected pairs.

PAIR SELECTION

Select the pairs where you want to impose a thermally resisitive layer across the pair boundaries. Select the pairs from the **Pairs** list (Ctrl-click to deselect).

PAIR THIN THERMALLY RESISTIVE LAYER

Enter a value or expression for the **Layer thickness** d_s (SI unit: m).

The default is to use the **Thermal conductivity** k_s (SI unit: W/(m·K)) **From material**. Select **User-defined** to enter another value or expression.

The Joule Heating Interface

The **Joule Heating** predefined multiphysics interface ( ) combines all feature from the Electric Currents interface with the Heat Transfer interface for modeling of *Joule heating* (*resistive heating* or *ohmic heating*). The interaction is coupled in both directions:

- The resistive heating appears as a heat source in the default **Electromagnetic Heat Source** node.
- The default setting is to use the value for the electric conductivity σ from the material. By selecting **Linearized resistivity** from the **Electric conductivity** list, the following temperature-dependent expression describes the electric conductivity: $1/(\rho_0(1+\alpha(T-T_{ref})))$, where T is the dependent variable for temperature from the heat transfer part, which automatically appears as a model input. By default, the values for ρ_0 (resistivity at reference temperature), α (temperature coefficient), and T_{ref} (reference temperature) are taken from the material. These settings are built into the Joule Heating Model node, which is the central feature in the Joule Heating interface.

The Joule Heating interface is available in 2D, axisymmetric 2D, and 3D. The default dependent variables is the temperature, T, and the electric potential, V.

The Joule Heating Interface

The **Joule Heating** interface provides the equations, boundary conditions, and sources for modeling Joule heating solving for the temperature and the electric potential. The main component is the Joule Heating Model, which adds the equation and interface for modeling of Joule heating. The Joule Heating interface adds a default **Joule Heating Model** node, a default **Electromagnetic Heat Source** node, and a default boundary conditions: the **Thermal Insulation** node. There is also a default **Initial Values** node. Right-click the **Joule Heating** node to add a **Heat Transfer in Solids** or **Heat Transfer in Fluids** node or other feature nodes that implements boundary conditions and sources The following sections provide information about the feature nodes that are available in the Joule Heating interface but not in the Heat Transfer or Electric Currents interfaces.

The **Joule Heating** form contains the following sections:

DOMAINS

Select the domains where you want to define Joule heating. The default setting is to include all domains in the model.

DEPENDENT VARIABLES

The interface includes a dependent variable for the **Temperature** T and the **Electric potential** V. Depending on the license, there may also be a variable for the **Surface radiosity** J.

ADVANCED SETTINGS AND DISCRETIZATION

Normally these settings do not need to be changed. See Show More Options: Advanced Settings and Discretization.

CONSISTENT STABILIZATION

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Also see Show More Options: Consistent and Inconsistent Stabilization for information about these settings. Any settings unique to this interface are listed below.

- The consistent stabilization methods take effect for fluids and for solids with translational motion (see Translational Motion). A stabilization method is active when the corresponding check box is selected. The Streamline diffusion check box is selected by default and should remain active for optimal performance for heat transfer in fluids or other applications that include a convective or translational term.
- The crosswind diffusion provides extra diffusion in the region of sharp gradients. The added diffusion is orthogonal to the streamline diffusion, so you can use streamline diffusion and crosswind diffusion simultaneously.
- If **Crosswind diffusion** is selected, enter a **Lower gradient limit** $\mathbf{g}_{\text{lim}}$ (SI unit: K/m). The default is `0.01[K]/jh.helem`. The variable $\mathbf{g}_{\text{lim}}$ is needed because both Equation 13-5 and Equation 13-6 contain terms of the form $1/|\nabla T|$, which become singular if $\nabla T = 0$. Hence, all occurrences of $1/|\nabla T|$ are replaced by $1/\max(|\nabla T|, g_{\text{lim}})$ where g_{lim} is a measure of a small gradient.

The method in the Heat Transfer interfaces adds the following contribution to the weak formulation (Ref. 2):

$$-\sum_{e=1}^{N_{\text{el}}}\int_{\Omega^e}\frac{1}{2}\max\left(0, C^e-\frac{2k}{h|\beta|}\right)\frac{h|R|}{|\nabla T|}\nabla\hat{T}\left(I-\frac{\mathbf{u}\otimes\mathbf{u}}{|\mathbf{u}|^2}\right)\nabla T d\Omega \qquad (13\text{-}5)$$

where R is the PDE residual, $\hat{T}$ is the test function for T, h is the element size, and β is defined as

$$\beta = \begin{cases} \dfrac{\rho C_p(\mathbf{u}\cdot\nabla T)}{|\nabla T|^2}\nabla T & \text{if } \nabla T \neq 0 \\ 0 & \text{if } \nabla T = 0 \end{cases} \tag{13-6}$$

INCONSISTENT STABILIZATION

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Also see Show More Options: Consistent and Inconsistent Stabilization for information about these settings.

Joule Heating Model

The **Joule Heating Model** node uses the following version of the *heat equation* as the mathematical model for heat transfer in solids:

$$\rho C_p \frac{\partial T}{\partial t} - \nabla\cdot(k\nabla T) = Q \tag{13-7}$$

with the following material properties:

- ρ is the *density*.
- C_p is the *heat capacity*.
- k is the *thermal conductivity* (a scalar or a tensor if the thermal conductivity is anisotropic).
- Q is the heat source (or sink). For Joule heating, it comes from the electric current and is added in the **Electromagnetic Heat Source** node.

For a steady-state problem the temperature does not change with time and the first term disappears. In addition, an equation for the electric current is also added.

DOMAINS

Select the domains where you want to define the Joule heating.

MODEL INPUTS

The **Model Inputs** section contain fields and values that are inputs to expressions that define material properties. If you have added such user defined property groups, their model inputs appear here. Initially, this section is empty.

CONDUCTION CURRENT

In this section you define the electric conductivity σ (SI unit: S/m) from the **Electric conductivity** list:

- Select **From materia**l (the default) to take the electric conductivity from the material.
- Select **Linearized resistivity** to use the following temperature-dependent expression to describe the electric conductivity: $1/(\rho_0(1+\alpha(T-T_{ref})))$, where T is the dependent variable for temperature. By default, the values for ρ_0 (resistivity at reference temperature), α (temperature coefficient), and T_{ref} (reference temperature) are taken from the material but you can select **User defined** to type in values or expressions for these properties.
- Select **User defined** to type in a value or expression for the electric conductivity in the associated edit field.

ELECTRIC FIELD

In this section you specify the constitutive relation that describe the macroscopic properties of the medium (relating the electric displacement $\mathbf{D}$ with the electric field $\mathbf{E}$) and the applicable material properties, such as the relative permittivity.

Select one of the following constitutive relations from the list under **Constitutive relation** (the corresponding equation appears underneath the list):

- Select **Relative permittivity** to use the constitutive relation $\mathbf{D} = \varepsilon_0\varepsilon_r\mathbf{E}$ (the default). You then specify ε_r, the relative permittivity (unitless). The default is to take its value from the material, but you can select **User defined** instead of **From material** from the list under **Relative permittivity** and then specify a value or expression for the relative permittivity in the edit field that appears.
- Select **Polarization** to use the constitutive relation $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$. You then specify $\mathbf{P}$, the polarization vector (SI unit: C/m^2). Enter its components (3 in 3D, 2 in 2D) in the corresponding edit fields under **Polarization**.
- Select **Remanent displacement** to use the constitutive relation $\mathbf{D} = \varepsilon_0\varepsilon_r\mathbf{E} + \mathbf{D}_r$, where $\mathbf{D}_r$ (SI unit: C/m^2) is the remanent displacement (the displacement when no electric field is present). In this case you specify ε_r, the relative permittivity, and $\mathbf{D}_r$, the remanent displacement. For the relative permittivity, use the list under **Relative permittivity**: Select **From material** (the default) to use the value from the material or select **User defined** to specify a value or expression for the relative permittivity in the edit field that appears. For the remanent displacement, enter its components (3 in 3D, 2 in 2D) in the corresponding edit fields under **Remanent displacement**.

HEAT CONDUCTION

The default setting is to use the thermal conductivity of the material. Select **User defined** to enter another value or expression for the thermal conductivity (SI unit: W/(m·K)). You can select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** from the associated list depending on the characteristics of the thermal conductivity. The thermal conductivity k describes the relationship between the heat flux vector $\mathbf{q}$ and the temperature gradient ∇T as in

$$\mathbf{q} = -k\nabla T$$

which is *Fourier's law of heat conduction*. Enter this quantity as power per length and temperature.

THERMODYNAMICS

The default uses the **Heat capacity at constant pressure** C_p (SI unit: J/(kg·K)) and **Density** ρ (SI unit: kg/m^3) values **From material**. Select **User defined** to enter other values or expressions for one or both variables.

Electromagnetic Heat Source

The **Electromagnetic Heat Source** feature is only available in the Joule Heating predefined multiphysics interface. It is added as a default node. The resistive heating (ohmic heating) due to the electric current is proportional to $I^2 \cdot R$, where I is the electric current and R is the resistance.

Initial Values

The **Initial Values** node adds initial values for the temperature T and the electric potential V that can serve as an initial condition for a transient simulation or as an initial guess for a nonlinear solver. If you need to specify more than one set of initial values, you can add additional **Initial Values** features.

DOMAINS

Select the domains where you want to define initial values.

INITIAL VALUES

Enter values or expressions for the initial values of the **Temperature** T and the **Electric potential** V. The default value for the temperature is approximately room temperature, 293.15 K (20° C) and for the electric potential it is 0 V.

Theory for the Heat Transfer Interfaces

Also see Ref. 1 and Ref. 3 For more detailed discussions.

What is Heat Transfer?

Heat transfer is defined as the movement of energy due to a difference in temperature. It is characterized by the following mechanisms:

- *Conduction*—Heat conduction takes place through different mechanisms in different media. Theoretically it takes place in a gas through collisions of the molecules; in a fluid through oscillations of each molecule in a "cage" formed by its nearest neighbors; in metals mainly by electrons carrying heat and in other solids by molecular motion which in crystals take the form of lattice vibrations known as phonons. Typical for heat conduction is that the heat flux is proportional to the temperature gradient.
- *Convection*—Heat convection (sometimes called heat advection) takes place through the net displacement of a fluid, which transports the heat content in a fluid through the fluid's own velocity. The term convection (especially convective cooling and convective heating) also refers to the heat dissipation from a solid surface to a fluid, typically described by a heat transfer coefficient.
- *Radiation*—Heat transfer by radiation takes place through the transport of photons, which opaque surfaces can absorb or reflect. Surface-to-ambient radiation treats the ambient surroundings as a black body with known temperature.

The Heat Equation

The fundamental law governing all heat transfer is the first law of thermodynamics, commonly referred to as the principle of conservation of energy. However, internal energy, U, is a rather inconvenient quantity to measure and use in simulations. Therefore, the basic law is usually rewritten in terms of temperature, T. For a fluid, the resulting *heat equation* is:

$$\rho C_p\left(\frac{\partial T}{\partial t}+(\mathbf{u}\cdot\nabla)T\right)=-(\nabla\cdot\mathbf{q})+\tau:\mathbf{S}-\frac{T}{\rho}\frac{\partial\rho}{\partial T}\bigg|_p\left(\frac{\partial p}{\partial t}+(\mathbf{u}\cdot\nabla)p\right)+Q \qquad (13\text{-}8)$$

where

- ρ is the density (SI unit: kg/m^3)
- C_p is the specific heat capacity at constant pressure (SI unit: J/(kg·K))
- T is absolute temperature (SI unit: K)
- $\mathbf{u}$ is the velocity vector (SI unit: m/s)
- $\mathbf{q}$ is the heat flux by conduction (SI unit: W/m^2)
- p is pressure (SI unit: Pa)
- τ is the viscous stress tensor (SI unit: Pa)
- $\mathbf{S}$ is the strain rate tensor (SI unit: 1/s):

$$\mathbf{S} = \frac{1}{2}(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)$$

- Q contains heat sources other than viscous heating (SI unit: W/m^3)

Note: Specific heat capacity at constant pressure is the amount of energy required to raise one unit of mass of a substance by one degree while maintained at constant pressure. This quantity is also commonly referred to as *specific heat* or *specific heat capacity*.

In deriving Equation 13-8, a number of thermodynamic relations have been used. The equation also assumes that mass is always conserved, which means that density and velocity must be related through:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0$$

The heat transfer interfaces use Fourier's law of conduction, which states that the conductive heat flux, $\mathbf{q}$, is proportional to the temperature gradient:

$$q_i = -k\frac{\partial T}{\partial x_i} \tag{13-9}$$

where k is the thermal conductivity (SI unit: W/(m·K)). In a solid, the thermal conductivity can be anisotropic (that is, it has different values in different directions). Then k becomes a tensor

$$k = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}$$

and the conductive heat flux is given by

$$q_i = -\sum_j k_{ij} \frac{\partial T}{\partial x_j}$$

The second term on the right of Equation 13-8represents viscous heating of a fluid. An analogous term arises from the internal viscous damping of a solid. The operation ":" is a contraction and can in this case be written on the following form:

$$\mathbf{a}:\mathbf{b} = \sum_n \sum_m a_{nm} b_{nm} \tag{13-10}$$

The third term represents pressure work and is responsible for the heating of a fluid under adiabatic compression and for some thermo-acoustic effects. It is generally small for low Mach number flows. A similar term can be included to account for thermo-elastic effects in solids.

Inserting Equation 13-9 into Equation 13-8, reordering the terms and ignoring viscous heating and pressure work puts the heat equation on a perhaps more familiar form:

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p \mathbf{u} \cdot \nabla T = \nabla \cdot (k \nabla T) + Q \tag{13-11}$$

The Heat Transfer interface with the **Heat Transfer in Fluids** feature solves this equation for the temperature, T. If the velocity is set to zero, you get the equation governing pure conductive heat transfer:

$$\rho C_p \frac{\partial T}{\partial t} + \nabla \cdot (-k \nabla T) = Q$$

A Note on Heat Flux

The concept of heat flux is not as simple as it might first appear. The reason is that heat is not a conserved property. The conserved property is instead the total energy. There is hence heat flux and energy flux which are similar, but not identical.

This section briefly describes the theory for the postprocessing variables for *Total heat flux* and *Total energy flux*. The approximations made do not affect the computational results, only postprocessing variables.

TOTAL ENERGY FLUX

The total energy flux is equal to (Ref. 4, chapter 3.5)

$$\rho\mathbf{u}(H_0 + \Psi) - k\nabla T + \tau \cdot \mathbf{u} + q_r \tag{13-12}$$

Above, H_0 is the total enthalpy

$$H_0 = H + \frac{1}{2}(\mathbf{u} \cdot \mathbf{u}) = U + \frac{p}{\rho} + \frac{1}{2}(\mathbf{u} \cdot \mathbf{u}) \tag{13-13}$$

where U is the internal energy. In Equation 13-12 τ is the viscous stress tensor and q_r is the radiative heat flux.

The enthalpy, H, has the general form (Ref. 5)

$$H = H_{\text{ref}} + \int_{T_{\text{ref}}}^{T} C_p dT + \int_{p_{\text{ref}}}^{p} \frac{1}{\rho}\left(1 + \frac{T}{\rho}\left(\frac{\partial\rho}{\partial T}\right)\Big|_p\right) dp \tag{13-14}$$

This expression is not really useful since the reference enthalpy, H_{ref}, is not commonly available. There are some common approximations:

- For a calorically perfect gas, $(T/\rho)/(\partial\rho/\partial T) = -1$ and C_p is constant. By taking $H_{\text{ref}} = T_{\text{ref}} = 0$, it follows from Equation 13-14 that

  $$H = C_p T \tag{13-15}$$

  Using the ideal gas law, Equation 13-15 can be rewritten as

  $$H = \frac{C_p}{\gamma} T + \frac{p}{\rho} \tag{13-16}$$

  where γ is the ratio of specific heats, $\gamma = C_p/C_v$. C_v is the specific heat capacity at constant volume.

- It is also true that $H = U + p/\rho$. For an incompressible substance,

  $$U = U_{\text{ref}} + \int_{T_{\text{ref}}}^{T} C_v dT \tag{13-17}$$

  Selecting $U_{\text{ref}} = T_{\text{ref}} = 0$, it follows from Equation 13-17 that

$$H = \frac{C_p}{\gamma}T + \frac{p}{\rho} \tag{13-18}$$

Notice that this identical to Equation 13-16.

- For most solid materials, $C_p = C_v$, that is, $\gamma = 1$. Furthermore, the first term on the right hand side of Equation 13-18 is commonly much larger than the second term. Hence, for a solid material, Equation 13-18 can to a good approximation be reduced to

$$H = C_p T \tag{13-19}$$

From above points, it follows that H is calculated from Equation 13-19 for Solid domains and Biological Tissue domains. For Fluid domain with ideal gas, H is calculated from Equation 13-15, otherwise from Equation 13-18.

Ψ in Equation 13-12 is the potential energy, or more accurately the force potential. It can be formulated in some special cases, for example, for gravitational effects (Chapter 1.4 in Ref. 4), but it is in general rather difficult to derive. Potential energy is therefore often excluded and the total energy flux is approximated by

$$\rho\mathbf{u}\left(H + \frac{1}{2}(\mathbf{u}\cdot\mathbf{u})\right) - k\nabla T + \tau\cdot\mathbf{u} + q_r \tag{13-20}$$

HEAT FLUX

The total heat flux vector is defined as (Ref. 6):

$$\rho\mathbf{u}U - k\nabla T + q_r \tag{13-21}$$

What is the difference between Equation 13-20 and Equation 13-21? An example is a channel with fully developed incompressible flow. The walls are assumed to be insulated. Along the channel there is a pressure drop that drives the flow. Start by neglecting the viscous heating, so that the flow is isothermal. Equation 13-20 states that the energy flow in is larger than the energy flow out. The reason is that the work done by the pressure should have turned into heat (by viscous heating), but instead it just disappeared. Equation 13-21, however, says that the heat flux in equals the heat flux out. If viscous heat is included, Equation 13-20 states that the energy in equals energy out while Equation 13-21 says that the heat flux out is larger than the heat flux in.

Boundary Conditions

The heat equation accepts two basic types of boundary conditions: *specified temperature* and *specified heat flux*. The former is of a constraint type and prescribes the temperature at a boundary:

$$T = T_0 \qquad \text{on } \partial\Omega$$

while the latter specifies the inward heat flux

$$-\mathbf{n} \cdot \mathbf{q} = q_0 \qquad \text{on } \partial\Omega$$

where

- $\mathbf{q}$ is the conductive heat flux vector (SI unit: $\mathrm{W/m^2}$)

$$\mathbf{q} = -k\nabla T$$

- $\mathbf{n}$ is the normal vector of the boundary
- q_0 is inward heat flux (SI unit: $\mathrm{W/m^2}$), normal to the boundary

The inward heat flux, q_0, is often a sum of contributions from different heat transfer processes, for example radiation and convection. The special case $q_0 = 0$ is called *thermal insulation*.

A common type of heat flux boundary conditions are those where $q_0 = h \cdot (T_{\text{inf}} - T)$, where T_{inf} is the temperature far away from the modeled domain and the heat transfer coefficient, h, represents all the physics occurring between the boundary and "far away." It can include almost anything, but the most common situation is that h represents the effect of an exterior fluid cooling or heating the surface of solid, a phenomenon often referred to as convective cooling or heating.

Note: Heat flux on inlet boundaries are not covered by the arguments above.

Radiative Heat Transfer in Transparent Media

This discussion so far has considered heat transfer by means of conduction and convection. The third mechanism for heat transfer is radiation. Let's consider an environment with fully transparent or fully opaque objects. Thermal radiation denotes the stream of electromagnetic waves emitted from a body at a certain temperature.

DERIVING THE RADIATIVE HEAT FLUX

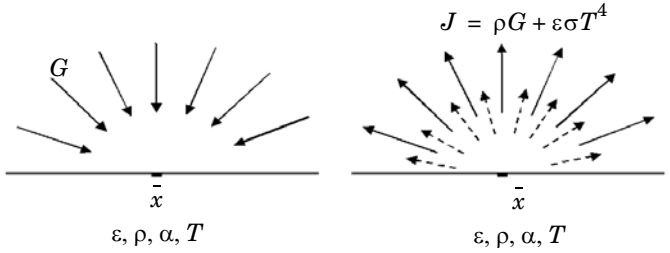


Figure 13-1: Arriving irradiation (left), leaving radiosity (right).

Consider Figure 13-1. A point $\bar{x}$ is located on a surface that has an emissivity ε, reflectivity ρ, absorptivity α, and temperature T. Assume the body is opaque, which means that no radiation is transmitted through the body. This is true for most solid bodies.

The total *arriving* radiative flux at $\bar{x}$ is named the irradiation, G. The total *outgoing* radiative flux $\bar{x}$ is named the radiosity, J. The radiosity is the sum of the reflected radiation and the emitted radiation:

$$J = \rho G + \varepsilon\sigma T^4 \tag{13-22}$$

The *net inward* radiative heat flux, q, is then given the difference between the irradiation and the radiosity:

$$q = G - J \tag{13-23}$$

Using Equation 13-22 and Equation 13-23 you can eliminate J and obtain a general expression for the net inward heat flux into the opaque body based on G and T.

$$q = (1-\rho)G - \varepsilon\sigma T^4 \tag{13-24}$$

Most opaque bodies also behave as ideal gray bodies, meaning that the absorptivity and emissivity are equal, and the reflectivity is therefore given from the following relation:

$$\alpha = \varepsilon = 1 - \rho \tag{13-25}$$

Thus, for ideal gray bodies, q is given by:

$$q = \varepsilon(G - \sigma T^4) \tag{13-26}$$

This is the equation used as a radiation boundary condition.

RADIATION TYPES

It is common to differentiate between two types of radiative heat transfer: *surface-to-ambient* radiation and *surface-to-surface* radiation. Equation 13-26 holds for both radiation types, but the irradiation term, G, is different for each of them. The Heat Transfer interface supports radiation.

SURFACE-TO-AMBIENT RADIATION

Surface-to-ambient radiation assumes the following:

- The ambient surroundings in view of the surface have a constant temperature, T_{amb}.
- The ambient surroundings behave as a black body. This means that the emissivity and absorptivity are equal to 1, and zero reflectivity.

These assumptions allow you to explicitly express the irradiation as

$$G = \sigma T_{amb}^4 \tag{13-27}$$

Inserting Equation 13-27 into Equation 13-26 results in the net inward heat flux for surface-to-ambient radiation

$$q = \varepsilon\sigma(T_{amb}^4 - T^4) \tag{13-28}$$

For boundaries where you have specified surface-to-ambient radiation, COMSOL Multiphysics adds this term to the right-hand side of Equation 13-28.

REFERENCES FOR THE HEAT TRANSFER INTERFACES

1. F.P. Incropera and D.P. DeWitt, *Fundamentals of Heat and Mass Transfer*, 4th ed., John Wiley & Sons, 1996.

2. R. Codina, "Comparison of some finite element methods for solving the diffusion-convection-reaction equation," *Comp. Meth.Appl. Mech. Engrg*, vol. 156, pp. 185–210, 1998.

3. A. Bejan, *Heat Transfer*, Wiley, 1993.

4. G.K. Batchelor, *An Introduction to Fluid Dynamics*, Cambridge University Press, 2000.

5. R.L. Panton, *Incompressible Flow*, Second edition, John Wiley and Sons, inc., 1996.

6. M. Kaviany, *Principles of Convective Heat Transfer*, Second edition, Springer, 2001.

14

The Structural Mechanics Interface

This section explains how to use the Solid Mechanics physics interface, found under the Structural Mechanics branch in the Model Wizard, to simulate and analyze applications involving solid mechanics. The following sections describe these cases in detail and provide a theory background. The rest of the section describes the Solid Mechanics interface.

Note: The optional **Structural Mechanics Module** contains physics interfaces and models that allow for extended, specialized analyses of structural and solid mechanics problems.

In this section:

Solid Mechanics Geometry

The Solid Mechanics interface is used for stress analysis and general solid mechanics simulation. It supports the following geometry types:

- 3D solid
- 2D plane stress and plane stress
- 2D axial symmetry

The last cases are 2D simplifications of the full 3D equations; these simplifications are valid under certain assumptions.

3D Geometry

The degrees of freedom (dependent variables) in 3D are the global displacements u, v, and w in the global x, y, and z directions, respectively, and the pressure help variable (used only if a nearly incompressible material is selected).

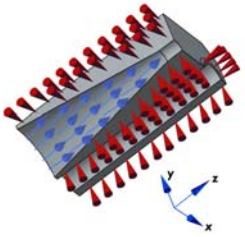


Figure 14-1: Loads and constraints applied to a 3D solid using the Solid Mechanics interface.

PLANE STRESS

The plane stress variant of the 2D interface is useful for analyzing thin in-plane loaded plates. For a state of plane stress, the out-of-plane components of the stress tensor are zero.

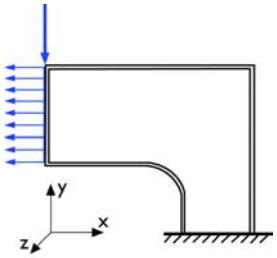


Figure 14-2: Plane stress models plates where the loads are only in the plane; it does not include any out-of-plane stress components.

The 2D interface for plane stress allows loads in the x and y directions, and it assumes that these are constant throughout the material's thickness, which can vary with x and y. The plane stress condition prevails in a thin flat plate in the xy-plane loaded only in its own plane and without any z direction restraint.

PLANE STRAIN

The plane strain variant of the 2D interface that assumes that all out-of-plane strain components of the total strain ε_z, ε_{yz}, and ε_{xz} are zero.

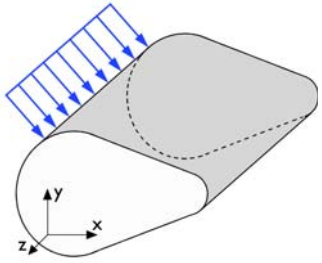


Figure 14-3: A geometry suitable for plane strain analysis.

Loads in the x and y directions are allowed. The loads are assumed to be constant throughout the thickness of the material, but the thickness can vary with x and y. The

plane strain condition prevails in geometries, whose extent is large in the z direction compared to in the x and y directions, or when the z displacement is in some way restricted. One example is a long tunnel along the z-axis where it is sufficient to study a unit-depth slice in the xy-plane.

Axisymmetric Geometry

The axisymmetric variant of the **Solid Mechanics** interface uses cylindrical coordinates r, φ (`phi`), and z. Loads are independent of φ, and the axisymmetric variant of the interface allows loads only in the r and z directions.

You can view the 2D axisymmetric geometry as the intersection between the original axially symmetric 3D solid and the half plane $\varphi = 0$, $r \geq 0$. You therefore draw the geometry only in the half plane $r \geq 0$ and recover the original 3D solid by rotating the 2D geometry about the z-axis.

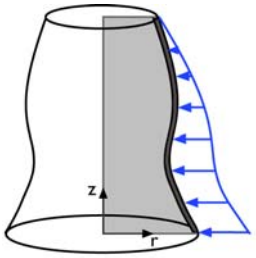


Figure 14-4: Rotating a 2D geometry to recover a 3D solid.

The Solid Mechanics Interface

The **Solid Mechanics** interface ( ) has the equations and features for stress analysis and general linear solid mechanics, solving for the displacements. The **Linear Elastic Material Model** is the default material model, which adds a linear elastic equation for the displacements and has a **Settings** window to define the elastic material properties.

The following default nodes are added when using this interface—**Linear Elastic Material Model**, **Free** (a boundary condition where boundaries are free, with no loads or constraints), and **Initial Values**.

Right-click the **Solid Mechanics** node to add other nodes.

INTERFACE IDENTIFIER

This is the physics interface identifier, which you use to reach the fields and variables in expressions, for example.

The identifier appears in the **Identifier** field, and you can change it to any unique string that is a valid identifier. The default identifier (for the first Solid Mechanics interface in the model) is `solid`.

DOMAINS

Select the domains where you want to define the displacements and the equations that describe the solid mechanics. The default setting is to include all domains in the model.

2D APPROXIMATION (2D MODELS ONLY)

From the **2D approximation** list select **Plane stress** or **Plane strain**. For more information about these 2D plane conditions, see the Theory for the Solid Mechanics Interface. When you model using plane stress, the Solid Mechanics interface solves for the out-of-plane strain, e_z, in addition to the displacement field $\mathbf{u}$.

THICKNESS (2D MODELS ONLY)

Define the **Thickness** d by entering a value or expression (SI unit: m). The default value of 1 m is suitable for plane strain models, where it represents a a unit-depth slice, for example. For plane stress models, enter the actual thickness, which should be small compared to the size of the plate for the plane stress assumption to be valid. In rare cases, you may want to change thickness in parts of the geometry; you can then use the Change Thickness feature (see Change Thickness).

REFERENCE POINT FOR MOMENT COMPUTATION

Enter the coordinates for the **Reference point for moment computation** ref_{pnt} (SI unit: m). All moments are then computed relative to this reference point.

DEPENDENT VARIABLES

The Solid Mechanics interface includes a field variable for the **Displacement field** u. You can change the names in the corresponding edit fields, but the names of fields and dependent variables must be unique within a model.

ADVANCED SETTINGS AND DISCRETIZATION

Normally these settings do not need to be changed. See Show More Options: Advanced Settings and Discretization. In addition, there is this section for this interface:

Displacements Control Spatial Frame

The **Displacements control spatial frame** check box is selected by default. If required, click to clear the check box.

Linear Elastic Material Model

The **Linear Elastic Material Model** node adds the equations for a linear elastic solid, and the page contains the following sections for defining the related material properties:

DOMAINS

Select the domains where you want to define a linear elastic solid and compute the displacements, stresses, and strains.

MODEL INPUTS

Define model inputs, for example, the temperature field if the material model uses a temperature-dependent material property. If no model inputs are required, this section is empty.

COORDINATE SYSTEM SELECTION

Select the **Coordinate system** to use. The list has all available coordinate systems (except boundary coordinate systems). The default is to use the Global Cartesian coordinate system, which always exists. The coordinate system is used when stresses or strains are presented in a local system.

LINEAR ELASTIC MATERIAL MODEL

Define the solid model and the linear elastic material properties.

Solid Model

The solid model is linear isotropic elastic.

Specification of Elastic Properties for Isotropic Materials

If **Isotropic** is selected, from the **Specify** list, select a pair of elastic properties for an isotropic material. Select:

- **Young's modulus and Poisson's ratio** to specify Young's modulus (elastic modulus) E (SI unit: Pa) and Poisson's ratio ν (dimensionless). For an isotropic material Young's modulus is the spring stiffness in Hooke's law, which in 1D form is

  $$\sigma = E\varepsilon$$

  where σ is the stress and ε is the strain. Poisson's ratio defines the normal strain in the perpendicular direction, generated from a normal strain in the other direction and follows the equation

  $$\varepsilon_{\perp} = -\upsilon\varepsilon_{\parallel}$$

- **Bulk modulus and shear modulus** to specify the bulk modulus K (SI unit: Pa) and the shear modulus G (SI unit: Pa). The bulk modulus is a measure of the solid's resistance to volume changes. The shear modulus is a measure of the solid's resistance to shear deformations.
- **Lamé constants** to specify the Lamé constants λ (SI unit: Pa) and μ (SI unit: Pa).
- **Pressure-wave and shear-wave speeds** to specify the pressure-wave speed (longitudinal wave speed) c_p (SI unit: m/s) and the shear-wave speed (transverse wave speed) c_s (SI unit: m/s).

For each pair of properties, select from the applicable list to use the value **From material** or enter a **User defined** value or expression.

Each of these pairs define the elastic properties and it is possible to convert from one set of properties to another (see Table 14-1).

Density

Define the material **Density** ρ (SI unit: kg/m^3). Select **From material** to use the value from the material or select **User defined** and enter a value.

Damping

In time-dependent, eigenfrequency, and frequency domain studies you can model undamped or damped problems. The **Damping** node adds Rayleigh damping to **Linear Elastic Material Model** solid mechanics models.

Right-click the **Linear Elastic Material Model** node to add a **Damping** node.

Note: The time-stepping algorithms also add numerical damping, which is independent of the explicit damping that you add. For the Generalized alpha time-stepping algorithm, it is possible to control the amount of numerical damping that it adds.

Rayleigh Damping

In the Rayleigh damping model, the damping parameter ξ is expressed in terms of the mass m and the stiffness k as

$$\xi = \alpha_{dM} m + \beta_{dK} k$$

That is, Rayleigh damping is proportional to a linear combination of the stiffness and mass, There is no direct physical interpretation of the mass damping parameter α_{dM} (SI unit: $1/s$) and the stiffness damping parameter β_{dM} (SI unit: s).

The following sections are defined in the **Damping** node's **Settings** window:

DOMAINS

Select the domains where you want to add damping. By default, this feature node inherits the selection from its parent node, and you can only use a selection that is a subset of the parent node's selection.

DAMPING SETTINGS

The only available **Damping type** is **Rayleigh damping**.

For **Rayleigh** damping, enter the **Mass damping parameter** α_{dM} and the **Stiffness damping parameter** β_{dK} in the corresponding fields. The default values are 0, which is no damping.

Change Thickness

The **Change Thickness** feature models domains with a thickness other than the overall thickness defined in the interface **Thickness** section.

For the Solid Mechanics interface, this is available in 2D only.

DOMAINS (OR BOUNDARIES)

Select the domains (or boundaries with Shell models) and where you want use a different thickness.

CHANGE THICKNESS

Enter a value for the **Thickness** d (SI unit: m). This value replaces the overall thickness for the geometric entity selected above.

Initial Values

The **Initial Values** node adds an initial value for the displacement field $\mathbf{u}$ (the displacement components u, v, and w in 3D) that can serve as an initial condition for a transient simulation or as an initial guess for a nonlinear analysis. It also adds a **Velocity field** $\partial u/\partial t$. If you need to specify more than one set of initial values, you can add additional **Initial Values** nodes.

DOMAINS

Select the domains where you want to define an initial value.

INITIAL VALUES

Enter values or expressions for the initial values of the **Displacement field** (SI unit: m) and **Velocity field** (SI unit: m/s). The defaults are 0.

Body, Boundary, Edge, and Point Loads

Add force loads acting on all levels of the geometry. Add a:

- Body Loadto domains (gravity effects, for example).
- Boundary Loadto boundaries (a pressure acting on a boundary, for example).
- Edge Loadto edges in 3D (a force distributed along an edge, for example).
- Point Load to points (concentrated forces at points).

Body Load

Add a **Body Load** to domains for gravity effects, for example.

DOMAINS

Select the domains where you want to define a body load.

COORDINATE SYSTEM SELECTION

Select a **Coordinate system**. By default you use the **Global coordinate system**, and the list contains any additional coordinate systems that the model includes.

FORCE

Select a **Load type**.

- (3D models) **Load defined as force per unit volume** (the default) (SI unit: N/m^3).
- (2D models) **Load defined as force per unit area** (SI unit: N/m^2). The body load as force per unit volume is then the value of $\boldsymbol{F}$ divided by the thickness.

  The **Load** list normally only contains **User defined**. Enter values or expressions for the components of the body load $\boldsymbol{F}$ in the **Load** fields.
- **Total force** (SI unit: N). Enter values or expressions for the components of the **Total force** $\boldsymbol{F}$. COMSOL then divides the total force by the volume of the domains where the body load is active.

Boundary Load

Add a **Boundary Load** to boundaries for a pressure acting on a boundary, for example.

BOUNDARIES

Select the boundaries where you want to define a load.

COORDINATE SYSTEM SELECTION

Specify the coordinate system to use for specifying the load. From the **Coordinate system** list select from:

- **Global coordinate system** (the default)
- **Boundary System** (a predefined normal-tangential coordinate system)
- Any additional user defined coordinate systems

FORCE

Select a **Load type**.

- **Load defined as force per unit area** (SI unit: N/m^2). The **Load** list normally only contains **User defined**. Enter values or expressions for the components of the **Load** F. When combining the Solid Mechanics interface with, for example, film damping, it is also possible to choose a predefined load from this list.
- *(2D models)* **Load defined as force per unit length** (SI unit: N/m). Enter values or expressions for the components of the **Load** F.
- **Total force** (SI unit: N). Enter values or expressions for the components of the **Total force** F. COMSOL then divides the total force by the area of the surfaces where the body load is active.
- **Follower pressure** (SI unit: Pa). Enter a value or expression for the follower **Pressure** P. The pressure is positive when directed in towards the solid.

Edge Load

Add an **Edge Load** to edges in 3D for a force distributed along an edge, for example.

EDGES

Select the edges (in 3D models) where you want to define an edge load.

COORDINATE SYSTEM SELECTION

Select a **Coordinate system**. By default you use the **Global coordinate system** and the list contains any additional coordinate systems that the model includes.

FORCE

Select a **Load type**.

- **Load defined as force per unit length** (SI unit: N/m). The **Load** list normally only contains **User defined**. Enter values or expressions for the components of the **Load** F. When combining the Solid Mechanics interface with, for example, film damping, it is also possible to choose a predefined load from this list.
- **Total force** (SI unit: N). Enter values or expressions for the components of the **Total force** F. COMSOL then divides the total force by the area of the surfaces where the body load is active.

Point Load

Add a **Point Load** to points for concentrated forces at points.

POINTS

Select the points where you want to define a point load.

COORDINATE SYSTEM SELECTION

Select a **Coordinate system**. By default you use the **Global coordinate system** and the list contains any additional coordinate systems that the model includes.

FORCE

The list under **Point load** normally only contains **User defined**. Enter values or expressions for the components of the **Point load** F (SI unit: N).

Boundary Conditions

The following constraints are available on exterior boundaries, and in some cases on domains, edges or points:

- Free
- Fixed Constraint (available on domains, boundaries, edges, and points)
- Prescribed Displacement (available on domains, boundaries, edges, and points)
- Roller

If there are subsequent boundary conditions specified on the same geometrical entity, the last one will take precedence.

For axisymmetric models, COMSOL Multiphysics takes the axial symmetry boundaries (at $r = 0$) into account and automatically adds an **Axial Symmetry** feature to the model that is valid on the axial symmetry boundaries only.

Free

The **Free** feature is the default boundary condition. It means that there are no constraints and no loads acting on the boundary.

BOUNDARIES

Select the boundaries that are free.

Fixed Constraint

The **Fixed Constraint** feature adds a condition that makes the geometric entity (domain, boundary, edge, or point) fixed (fully constrained); that is, the displacements are zero in all directions.

DOMAINS, BOUNDARIES, EDGES, OR POINTS

Select the geometric entity (domains, boundaries, edges, or points) that are fixed.

CONSTRAINT SETTINGS

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Select a **Constraint type**—**Bidirectional, symmetric** or **Unidirectional**. If required, select the **Use weak constraints** check box. See Using Weak Constraints.

Prescribed Displacement

The **Prescribed Displacement** feature adds a condition where you can prescribe the displacements in one or more directions to the geometric entity (domain, boundary, edge, or point).

If you prescribe a displacement in one direction, this leaves the solid free to deform in the other directions. You can also define more general displacements as a linear combination of the displacements in each direction.

- If a prescribed displacement is not activated in any direction, this is the same as a **Free** constraint.
- If a zero displacement is applied in all directions, this is the same as a **Fixed Constraint**.

DOMAINS, BOUNDARIES, EDGES, OR POINTS

Select the geometric entity (domains, boundaries, edges, or points) on which you want to prescribe a displacement.

COORDINATE SYSTEM SELECTION

Select a **Coordinate system**. By default you use the **Global coordinate system** and the list contains any additional coordinate systems that the model includes.

PRESCRIBED DISPLACEMENT

Define the prescribed displacements using a **Standard notation** or a **General notation**.

Standard Notation

To define the displacements individually, click the **Standard notation** button (the default).

2D and 3D Models: To define a prescribed displacement for each space direction (x, y, and z for 3D), select one or all of the **Prescribed in X, Y,** and **Z direction** check boxes. Then enter a value or expression for the prescribed displacements U_0, V_0, or W_0 (SI unit: m).

2D Axisymmetric Models: To define a prescribed displacement for each space direction (R and Z), select one or both of the **Prescribed in R,** and **Z direction** check boxes. Then enter a value or expression for the prescribed displacements U_0, or W_0 (SI unit: m).

General Notation

To specify the displacements using a **General notation** that includes any linear combination of displacement components, click the **General notation Hu=R** button. For example, for 2D models, use the relationship

$$H\begin{bmatrix} u \\ v \end{bmatrix} = R$$

Enter values in the **H matrix** and **R vector** fields. For the H matrix, also select an **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** matrix and enter values as required. For example, to achieve the condition $u = v$, use the settings

$$H = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \qquad R = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

which force the domain to move only diagonally in the xy-plane.

CONSTRAINT SETTINGS

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Select a **Constraint type**—**Bidirectional, symmetric** or **Unidirectional**. If required, select the **Use weak constraints** check box. See Using Weak Constraints.

Roller

The **Roller** node adds a roller constraint as the boundary condition; that is, the displacement is zero in the direction perpendicular (normal) to the boundary, but the boundary is free to move in the tangential direction.

BOUNDARIES

Select the boundaries that have roller constraints.

CONSTRAINT SETTINGS

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Select a **Constraint type**—**Bidirectional, symmetric** or **Unidirectional**. If required, select the **Use weak constraints** check box. See Using Weak Constraints.

Periodic Condition

The **Periodic Condition** feature adds a *periodic boundary condition*. This periodicity make $u_i(x_0) = u_i(x_1)$ for a displacement u_i. You can control which of the directions that the periodic condition applies to. Right-click the **Periodic Condition** node to add a **Destination Selection** boundary condition. If the source and destination boundaries are rotated with respect to each other, this transformation is automatically performed, so that corresponding displacement components are connected.

BOUNDARIES

Select the boundaries where you want to define a periodic boundary condition. The software automatically identifies the boundaries as either source boundaries or destination boundaries. This works fine for cases like opposing parallel boundaries. In other cases you can use the Destination Selection subfeature to control the destination. By default it contains the selection that COMSOL Multiphysics has identified.

PERIODIC CONDITION

By default, there is no periodic condition is active in any of the directions.

2D and 3D Models: For each space direction (x, y, and z for 3D), select one or all of the **Periodic in X, Y,** and **Z direction** check boxes. Then select a **Type of periodicity**—**Continuity** or **Antiperiodicity**.

2D Axisymmetric Models: For each space direction (R and Z), select one or both of the **Periodic in R direction** and **Periodic in Z direction** check boxes. Then select a **Type of periodicity**—**Continuity** or **Antiperiodicity**.

Destination Selection

The **Destination Selection** is a subfeature of the **Periodic Condition** and is used to control the destination. To add this to the **Model Builder**, right-click **Periodic Condition** and select **Destination Selection**.

BOUNDARIES

Select the boundaries where you want to define a periodic boundary condition. The software automatically identifies the boundaries as either source boundaries or destination boundaries. By default it contains the selection that COMSOL Multiphysics has identified.

Pairs

In the Identity and Contact Pairs section you can find general information about the usage of pairs in COMSOL. There are a several possible selections under **Pairs**. In the following, only **Contact** is described. All other loads, boundary conditions, and continuity conditions follow the general behavior of pairs. The loads and boundary conditions have the same data as described above.

Symmetry

The **Symmetry** feature adds a boundary condition, which must exist both in the geometry and in the loads. A symmetry condition is free in the plane and fixed in the out-of-plane direction.

BOUNDARIES

Select the boundaries that are symmetry boundaries.

CONSTRAINT SETTINGS

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Select a **Constraint type**—**Bidirectional, symmetric** or **Unidirectional**. If required, select the **Use weak constraints** check box.

Antisymmetry

The **Antisymmetry** feature adds a boundary condition for an antisymmetry boundary, which must exist in both the geometry and in the loads. An antisymmetry condition is fixed in the plane and free in the out-of-plane direction.

In a geometrically non-linear analysis large rotations must not occur at the antisymmetry plane, since this will cause artificial straining.

BOUNDARIES

Select the boundaries that are antisymmetry boundaries.

CONSTRAINT SETTINGS

To display this section, select **Show More Options** from the **View** menu in the **Model Builder**. Select a **Constraint type**—**Bidirectional, symmetric** or **Unidirectional**. If required, select the **Use weak constraints** check box.

Rigid Connector

The **Rigid Connector** is a boundary condition for modeling rigid regions and kinematic constraints such as prescribed rigid rotations. The rigid connector always takes finite rotations into account.

Forces and moments can be applied to the rigid connector.

BOUNDARIES

Select the boundaries to which you want to connect this rigid connector.

COORDINATE SYSTEM SELECTION

Select a **Coordinate system**. By default you use the **Global coordinate system**. In addition, the **Coordinate system** list contains any additional coordinate systems that the model includes. Prescribed displacements or rotations are specified along the axes of this coordinate system.

CENTER OF ROTATION

Select a **Center of rotation**—**Automatic** or **User defined**. Enter X, Y, and Z coordinates in the table. The default is **Automatic**, and the center of rotation is then at the geometrical centre of the selected boundaries. Any loads or constraints are applied at the centre of rotation.

PRESCRIBED DISPLACEMENT

To define a prescribed displacement for each space direction (x, y, and z), select one or all of the **Prescribed in X, Y,** and **Z direction** check boxes. Then enter a value or expression for the prescribed displacements U_0, V_0, or W_0 (SI unit: m).

PRESCRIBED ROTATION

Select an option from the **By** list—**Free**, **Constrained rotation**, or **Prescribed rotation**. The default is **Free**.

If **Constrained rotation** is selected, select one or more of the **Constrain rotation about X, Y,** and **Z axis** check boxes in order to enforce zero rotation around the corresponding axis in the selected coordinate system.

If **Prescribed rotation** is selected, enter an **Axis of rotation** Ω and an **Angle of rotation** ϕ. The axis of rotation is given in the selected coordinate system.

Theory for the Solid Mechanics Interface

Frames and Coordinate Systems

The formulation used for structural analysis in COMSOL Multiphysics for both small and finite deformations is totally Lagrangian. The dependent variables are the displacement vector components, which are defined in the initial (or reference) configuration, for which the coordinate system on the material frame is always chosen to coincide with the coordinate system on the spatial frame.

The displacement gradients are computed on the material frame. All the material properties are also defined on the material frame.

It is possible to define any number of coordinate systems on the material frame. By default, in every domain the coordinate system is the same and defined by the material frame. In this document, this coordinate system is denoted by (G). In addition, the property groups can be applied in the local coordinate system that you can choose for each domain. In this document, it is denoted by (L). Loads on domains and boundaries can also be applied in their own local coordinate systems.

Linear Elastic Material

The total strain tensor is written in terms of the displacement gradient

$$\varepsilon = \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^T)$$

or in components as

$$\varepsilon_{mn} = \frac{1}{2}\left(\frac{\partial u_m}{\partial x_n} + \frac{\partial u_n}{\partial x_m}\right) \tag{14-1}$$

The Duhamel-Hooke's law relates the stress tensor to the strain tensor and temperature:

$$s = s_0 + \mathsf{C} : (\varepsilon - \varepsilon_0 - \alpha\theta)$$

where C is the 4th order *elasticity tensor*, ":" stands for the double-dot tensor product (or double contraction), s_0 and ε_0 are initial stresses and strains, $\theta = T - T_{\text{ref}}$ and α is the thermal expansion tensor.

The elastic energy is

$$W_s = \frac{1}{2}(\varepsilon - \varepsilon_0 - \alpha\theta) : \mathsf{C} : (\varepsilon - \varepsilon_0 - \alpha\theta) \tag{14-2}$$

or using the tensor components:

$$W_s = \sum_{i,j,m,n} \frac{1}{2}\mathsf{C}^{ijmn}(\varepsilon_{ij} - \varepsilon_{ij}^0 - \alpha_{ij}\theta)(\varepsilon_{mn} - \varepsilon_{mn}^0 - \alpha_{mn}\theta)$$

TENSOR VS. MATRIX FORMULATIONS

Because of the symmetry, the strain tensor can be written as the following matrix:

$$\begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_z \end{bmatrix}$$

Similar representation applies to the stress and the thermal expansion tensors:

$$\begin{bmatrix} s_x & s_{xy} & s_{xz} \\ s_{xy} & s_y & s_{yz} \\ s_{xz} & s_{yz} & s_z \end{bmatrix}, \begin{bmatrix} \alpha_x & \alpha_{xy} & \alpha_{xz} \\ \alpha_{xy} & \alpha_y & \alpha_{yz} \\ \alpha_{xz} & \alpha_{yz} & \alpha_z \end{bmatrix}$$

Due to the symmetry, the elasticity tensor can be completely represented by a symmetric 6-by-6 matrix as:

$$D = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{12} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{13} & D_{23} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{14} & D_{24} & D_{34} & D_{44} & D_{45} & D_{46} \\ D_{15} & D_{25} & D_{35} & D_{45} & D_{55} & D_{56} \\ D_{16} & D_{26} & D_{36} & D_{46} & D_{56} & D_{66} \end{bmatrix} = \begin{bmatrix} \mathsf{C}^{1111} & \mathsf{C}^{1122} & \mathsf{C}^{1133} & \mathsf{C}^{1112} & \mathsf{C}^{1123} & \mathsf{C}^{1113} \\ \mathsf{C}^{1122} & \mathsf{C}^{2222} & \mathsf{C}^{2233} & \mathsf{C}^{2212} & \mathsf{C}^{2223} & \mathsf{C}^{2213} \\ \mathsf{C}^{1133} & \mathsf{C}^{2233} & \mathsf{C}^{3333} & \mathsf{C}^{3312} & \mathsf{C}^{3323} & \mathsf{C}^{3313} \\ \mathsf{C}^{1112} & \mathsf{C}^{2212} & \mathsf{C}^{3312} & \mathsf{C}^{1212} & \mathsf{C}^{1223} & \mathsf{C}^{1213} \\ \mathsf{C}^{1123} & \mathsf{C}^{2223} & \mathsf{C}^{3323} & \mathsf{C}^{1223} & \mathsf{C}^{2323} & \mathsf{C}^{2313} \\ \mathsf{C}^{1113} & \mathsf{C}^{2213} & \mathsf{C}^{3313} & \mathsf{C}^{1213} & \mathsf{C}^{2313} & \mathsf{C}^{1313} \end{bmatrix}$$

which is the *elasticity matrix*.

The Hooke's law can be presented then in the form involving the elasticity matrix and the following vectors:

$$\begin{bmatrix} s_x \\ s_y \\ s_z \\ s_{xy} \\ s_{yz} \\ s_{xz} \end{bmatrix} = \begin{bmatrix} s_x \\ s_y \\ s_z \\ s_{xy} \\ s_{yz} \\ s_{xz} \end{bmatrix}_0 + D\left(\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ 2\varepsilon_{xy} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{xz} \end{bmatrix} - \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ 2\varepsilon_{xy} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{xz} \end{bmatrix}_0 - \theta \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \\ 2\alpha_{xy} \\ 2\alpha_{yz} \\ 2\alpha_{xz} \end{bmatrix} \right)$$

Thus, the general conversion rule for indices is:

$$\begin{bmatrix} 11 \\ 22 \\ 33 \\ 12, 21 \\ 23, 32 \\ 13, 31 \end{bmatrix} \leftrightarrow \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \leftrightarrow \begin{bmatrix} x \\ y \\ z \\ xy \\ yz \\ xz \end{bmatrix}$$

COMSOL Multiphysics uses the complete tensor representation internally to perform the coordinate system transformations correctly.

ANISOTROPIC MATERIAL

In the most general case of fully anisotropic material, you provide explicitly 21 components of the symmetric elasticity matrix D and 6 components of the symmetric thermal expansion matrix.

ISOTROPIC MATERIAL AND ELASTIC MODULI

In this case, the elasticity matrix becomes

$$D = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

and the thermal expansion matrix is:

$$\begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix}$$

You can use different pairs of elastic moduli, and as long as two of them are defined, the others can be computed according to Table 14-1.

TABLE 14-1: EXPRESSIONS FOR THE ELASTIC MODULI.

DESCRIPTION	VARIABLE	$D(E,\nu)$	$D(K,G)$	$D(\lambda,\mu)$
Young's modulus	E		$\frac{9KG}{3K+G}$	$\mu\frac{3\lambda+2\mu}{\lambda+\mu}$
Poisson's ratio	ν		$\frac{1}{2}\left(1-\frac{3G}{3K+G}\right)$	$\frac{\lambda}{2(\lambda+\mu)}$
Bulk modulus	K	$\frac{E}{3(1-2\nu)}$		$\lambda+\frac{2\mu}{3}$
Shear modulus	G	$\frac{E}{2(1+\nu)}$		μ
Lamé constant	λ	$\frac{E\nu}{(1+\nu)(1-2\nu)}$	$K-\frac{2G}{3}$	
Lamé constant	μ	$\frac{E}{2(1+\nu)}$	G	
Pressure-wave speed	c_p		$\sqrt{\frac{K+4G/3}{\rho}}$	
Shear-wave speed	c_s		$\sqrt{G/\rho}$	

ORTHOTROPIC MATERIAL

The elasticity matrix for orthotropic material has the following structure:

$$D = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ D_{12} & D_{22} & D_{23} & 0 & 0 & 0 \\ D_{13} & D_{23} & D_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix}$$

where the components are as follows:

$$D_{11} = \frac{E_x^2(E_z\nu_{yz}^2 - E_y)}{D_{\text{denom}}}, \qquad D_{12} = -\frac{E_xE_y(E_z\nu_{yz}\nu_{xz} + E_y\nu_{xy})}{D_{\text{denom}}}$$

$$D_{13} = -\frac{E_xE_yE_y(\nu_{xy}\nu_{yz} + \nu_{xz})}{D_{\text{denom}}}, \qquad D_{22} = \frac{E_y^2(E_z\nu_{xz}^2 - E_x)}{D_{\text{denom}}}$$

$$D_{23} = -\frac{E_yE_z(E_y\nu_{xy}\nu_{xz} + E_x\nu_{yz})}{D_{\text{denom}}}, \qquad D_{33} = \frac{E_yE_z(E_y\nu_{xy}^2 - E_x)}{D_{\text{denom}}}$$

$$D_{44} = G_{xy}, \quad D_{55} = G_{yz}, \text{ and } D_{66} = G_{xz}$$

where

$$D_{\text{denom}} = E_yE_z\nu_{xz}^2 - E_xE_y + 2\nu_{xy}\nu_{yz}\nu_{xz}E_yE_z + E_xE_z\nu_{yz}^2 + E_y^2\nu_{xy}^2$$

where you supply the values of E_x, E_y, E_z, ν_{xy}, ν_{yz}, ν_{xz}, G_{xy}, G_{yz}, and G_{xz} in designated edit fields in the user interface. COMSOL deduces the remaining components—ν_{yx}, ν_{zx}, and ν_{zy}—using the fact that the matrices D and D^{-1} are symmetric.

The thermal expansion matrix is diagonal:

$$\begin{bmatrix} \alpha_x & 0 & 0 \\ 0 & \alpha_y & 0 \\ 0 & 0 & \alpha_z \end{bmatrix}$$

ENTROPY AND THERMOELASTICITY

The free energy for the linear thermoelastic material can be written as:

$$F = \rho f_0(T) + W_s(\varepsilon, T)$$

where $W_s(\varepsilon, T)$ is given by Equation 14-2. Hence, the stress can be found as:

$$s = \left(\frac{\partial F}{\partial \varepsilon}\right)_T = \left(\frac{\partial W}{\partial \varepsilon}\right)_T = \mathsf{C} : (\varepsilon - \varepsilon_0 - \alpha\theta)$$

and the entropy per unit volume can be calculated as:

$$-\left(\frac{\partial F}{\partial T}\right)_\varepsilon = \rho C_p \log(T/T_0) + S_{\text{elast}}$$

where T_0 is a reference temperature, the volumetric heat capacity ρC_P can be assumed independent of the temperature (Dulong-Petit law), and

$$S_{\text{elast}} = \alpha : s$$

For an isotropic material, it simplifies into:

$$S_{\text{elast}} = \alpha(s_x + s_y + s_z)$$

The heat balance equation can be written as:

$$\rho C_p \frac{\partial T}{\partial t} + T\frac{\partial}{\partial t} S_{elast} = \nabla \cdot (k\nabla T) + Q_h$$

where k are the thermal conductivity matrix, and

$$Q_h = \tau : \dot{\varepsilon}$$

where $\dot{\varepsilon}$ is the strain-rate tensor, tensor τ presents all possible inelastic stresses (for example, a viscous stress).

Using the tensor components, the heat balance can be rewritten as:

$$\rho C_p \frac{\partial T}{\partial t} + \sum_{m,n} T\alpha_{mn} \frac{\partial}{\partial t} s_{mn} = \nabla \cdot (k\nabla T) + Q_h$$

In many cases, you can neglect the second term in the left-hand side of the above equation because all $T\alpha_{mn}$ are small. The resulting approximation is often called uncoupled thermoelasticity.

NEARLY INCOMPRESSIBLE MATERIALS

Nearly incompressible materials can cause numerical problems if only displacements are used in the interpolating functions. Small errors in the evaluation of volumetric strain, due to the finite resolution of the discrete model, are exaggerated by the high

bulk modulus. This leads to an unstable representation of stresses, and in general to underestimation of the deformation since spurious volumetric stresses may balance also applied shear and bending loads.

In such cases, you may choose to use a mixed formulation which represents the pressure as a dependent variable in addition to the displacement components. This removes the effect of volumetric strain from the original stress tensor and replaces it with an interpolated pressure, p_w. A separate equation constrains the interpolated pressure to equal (in a finite-element sense) the original pressure calculated from the strains.

For an isotropic linear elastic material, the second Piola-Kirchoff stress tensor S, computed directly from the strains, is replaced by a modified version:

$$\tilde{s} = s + pI - p_w I$$

where I is the unit tensor and the pressure p is calculated from the strains and material data as

$$p = -\frac{1}{3} trace(s)$$

The auxiliary dependent variable p_w is set equal to p using the equation

$$\frac{p_w}{\kappa} - \frac{p}{\kappa} = 0 \qquad (14\text{-}3)$$

where κ is the bulk modulus.

The modified stress tensor $\tilde{s}$ is used then in calculations of the energy variation.

For orthotropic and anisotropic materials, the auxiliary pressure equation is scaled so as to result in a symmetric stiffness matrix. Note, however, that the stiffness matrix in this formulation is not positive definite and even contains a zero block on the diagonal in the incompressible limit. This limits the possible choices of direct and iterative linear solver.

Plane Strain and Plane Stress Cases

For linear Elastic material in case of plane stress, COMSOL solves three equations $s_{i3} = 0$ for ε_{i3} with $i = 1, 2, 3$, and uses the solution instead of Equation 14-1 for these

three strain components. Thus, three components ε_{i3} are treated as extra degrees of freedom. The remaining three strain components are computed as in 3D case according to Equation 14-1. Note that for an isotropic material, only the normal out-of-plane component ε_{33} needs to be solved for.

In case of plane strain, set $\varepsilon_{i3} = 0$ for $i = 1, 2, 3$. The out-of-plane stress components s_{i3} are postprocessing variables.

Axial Symmetry

The axially symmetric geometry uses a cylindrical coordinate system. Such a coordinate system is orthogonal but curvilinear, and one can choose between a covariant basis $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ and a contravariant basis $\mathbf{e}^1$, $\mathbf{e}^2$, $\mathbf{e}^3$.

The metric tensor is

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

in the coordinate system given by $\mathbf{e}^1$, $\mathbf{e}^2$, $\mathbf{e}^3$, and

$$\left[g^{ij} \right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^{-2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

in $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$.

The metric tensor plays the role of a unit tensor for a curvilinear coordinate system.

For any vector or tensor A, the metric tensor can be used for conversion between covariant, contravariant and mixed components:

$$A_i^{\ j} = \sum_m (A_{im} g^{mj})$$

$$A^{ij} = \sum_{m,n} (A_{nm} g^{ni} g^{mj})$$

In both covariant and contravariant basis, the base vector in the azimuthal direction has a non-unit length. To cope with this issue, the so called physical basis vectors of unit length are introduced. These are:

$$\mathbf{e}_r = \mathbf{e}_1 = \mathbf{e}^1, \mathbf{e}_\varphi = \frac{1}{r}\mathbf{e}_2 = r\mathbf{e}_2, \mathbf{e}_z = \mathbf{e}_3 = \mathbf{e}^3$$

The corresponding components for any vector or tensor are called physical.

For any tensor, the physical components are defined as:

$$A_{ij}^{\text{phys}} = \sqrt{g_{ii}}\sqrt{g_{jj}}A^{ij}$$

where no summation is done over repeated indices.

Mixed Components and Principal Invariants

The mixed strain components:

$$A_j^i = \sum_m (g^{im}A_{mj})$$

The principal invariants are:

$$I_1(A) = \text{trace}[A_i^i] = \sum_i A_i^i = A_{11} + A_{22}\frac{1}{r^2} + A_{33}$$

$$I_2(A) = \frac{1}{2}\left((I_1(A))^2 - \sum_{i,j} A_j^i A_i^j\right)$$

$$I_3(A) = \det[A_i^i]$$

Displacements and Axial Symmetry assumptions

The axial symmetry implementation in COMSOL Multiphysics assumes independence of the angle, and also that the torsional component of the displacement is identically zero. The physical components of the radial and axial displacement, u and w, are used as dependent variables for the axially symmetric geometry.

For the Linear Elastic material, the stress components in coordinate system (L) are:

$$:s^{ij} = s_0^{ij} + \mathsf{C}^{ijkl}(\varepsilon_{kl} - \alpha_{kl}\theta - \varepsilon_{0kl})$$

where $\theta = T - T_{\text{ref}}$.

For anisotropic and orthotropic materials, the 4th-order elasticity tensor is defined from D matrix according to:

$$\begin{bmatrix} s_r \\ s_\varphi \\ s_{z\varphi} \\ s_{r\varphi} \\ s_{\varphi z} \\ s_{rz} \end{bmatrix} = \begin{bmatrix} s_r \\ s_\varphi \\ s_{z\varphi} \\ s_{r\varphi} \\ s_{\varphi z} \\ s_{rz} \end{bmatrix}_0 + D\left(\begin{bmatrix} \varepsilon_r \\ \varepsilon_\varphi \\ \varepsilon_{z\varphi} \\ 2\varepsilon_{r\varphi} \\ 2\varepsilon_{\varphi z} \\ 2\varepsilon_{rz} \end{bmatrix} - \begin{bmatrix} \varepsilon_r \\ \varepsilon_\varphi \\ \varepsilon_{z\varphi} \\ 2\varepsilon_{r\varphi} \\ 2\varepsilon_{\varphi z} \\ 2\varepsilon_{rz} \end{bmatrix}_0 - \theta \begin{bmatrix} \alpha_r \\ \alpha_\varphi \\ \alpha_{z\varphi} \\ 2\alpha_{r\varphi} \\ 2\alpha_{\varphi z} \\ 2\alpha_{rz} \end{bmatrix} \right)$$

Note that the user input D matrix always contains the physical components of the elasticity tensor C_{ijkl}^{phys}, and the corresponding tensor components are computed internally according to:

$$C^{ijkl} = \frac{C_{ijkl}^{\text{phys}}}{\sqrt{g_{ii}}\sqrt{g_{jj}}\sqrt{g_{kk}}\sqrt{g_{ll}}}$$

For an isotropic material:

$$C^{ijkl} = \lambda g^{ij} g^{kl} + \mu(g^{ik} g^{jl} + g^{il} g^{jk})$$

where λ and μ are Lamé elastic constants.

Initial Stress and Strain

Initial stress refers to the stress before the system applies any loads, displacements, or initial strains. The initial strain is the one before the system has applied any loads, displacements, or initial stresses.

Both the initial stress and strains are tensor variables defined via components on the local coordinate system (L) for each subdomain. You input them as the following matrices:

$$\begin{bmatrix} \varepsilon_{0x} & \varepsilon_{0xy} & \varepsilon_{0xz} \\ \varepsilon_{0xy} & \varepsilon_{0y} & \varepsilon_{0yz} \\ \varepsilon_{0xz} & \varepsilon_{0yz} & \varepsilon_{0z} \end{bmatrix}, \begin{bmatrix} s_{0x} & s_{0xy} & s_{0xz} \\ s_{0xy} & s_{0y} & s_{0yz} \\ s_{0xz} & s_{0yz} & s_{0z} \end{bmatrix}$$

Note that in case of nearly incompressible material (mixed formulation), you still input the components of the total initial stress (i.e without volumetric-deviatoric split). The initial pressure in the equation for the pressure help variable p_w is computed as

Axial Symmetry

User inputs the physical components of ε_0 and s_0:

$$\begin{bmatrix} \varepsilon_{0r} & \varepsilon_{0r\varphi} & \varepsilon_{0rz} \\ \varepsilon_{0r\varphi} & \varepsilon_{0\varphi} & \varepsilon_{0\varphi z} \\ \varepsilon_{0rz} & \varepsilon_{0\varphi z} & \varepsilon_{0z} \end{bmatrix}, \begin{bmatrix} s_{0r} & s_{0r\varphi} & s_{0rz} \\ s_{0r\varphi} & s_{0\varphi} & s_{0\varphi z} \\ s_{0rz} & s_{0\varphi z} & s_{0z} \end{bmatrix}$$

Other Possible Uses of Initial Strains and Stresses

Many inelastic effects in solids and structure (such as creep, plasticity, damping, viscoelasticity, poroelasticity, and so on) are additive contributions to either the total strain or total stress. Then, the initial value input fields can be used for coupling the elastic equations (solid physics) to the constitutive equations (usually General Form PDEs) modeling such extra effects.

Loads

You can specify loads as

- Distributed loads. The load is a distributed force in a volume, on a face, or along an edge.
- Total force. The specification of the load is as the total force. The software then divides this value with the area or the volume where the force acts.
-

For 2D plane stress and plane strain models you can choose how to specify the distributed boundary load as a load defined as force per unit area or a load defined as force per unit length acting on boundaries. In the same way you can choose between defining the load as force per unit volume or force per unit area for body loads acting in a domain.

The following table shows how to define distributed loads on different geometric entity levels; the entries show the SI unit in parentheses.

GEOMETRIC ENTITY	POINT	EDGE	FACE	DOMAIN
Plane stress, plane strain	force (N)	force/area (N/m^2) or force/length (N/m)		force/volume (N/m^3) or force/area (N/m^2)
Axial symmetry	total force along the circumferential (N)	force/area (N/m^2)		force/volume (N/m^3)
3D Solid	force (N)	force/length (N/m)	force/area (N/m^2)	force/volume (N/m^3)

For plane stress, plane strain, and axisymmetric models, the boundary loads apply on edges (boundaries). For 3D solids, the boundary loads apply on faces (boundaries).

Equation Implementation

The COMSOL Multiphysics implementation of the equations in the Solid Mechanics interface is based on the principle of virtual work.

The principle of virtual work states that the sum of virtual work from internal strains is equal to work from external loads.

The total stored energy, W, for a linear material from external and internal strains and loads equals:

$$W = \int_V (-\varepsilon : s + \mathbf{u} \cdot \mathbf{F}_V) dv$$

$$+ \int_S (\mathbf{u} \cdot \mathbf{F}_S) ds + \int_L (\mathbf{u} \cdot \mathbf{F}_L) dl + \sum_p (\mathbf{U}^t \cdot \mathbf{F}_p)$$

The principle of virtual work states that

$$\delta W = 0$$

which leads to

$$\int_V (-\varepsilon_{\text{test}} : s + \mathbf{u}_{\text{test}} \cdot \mathbf{F}_V - \rho \mathbf{u}_{\text{test}} \cdot \mathbf{u}_{tt}) dv$$

$$+ \int_S (\mathbf{u}_{\text{test}} \cdot \mathbf{F}_S) ds + \int_L (\mathbf{u}_{\text{test}} \cdot \mathbf{F}_L) dl + \sum_p (\mathbf{U}_{\text{test}}^t \cdot \mathbf{F}_p)$$

Setting up Equations for Different Studies

The Solid Mechanics interface supports stationary (static), eigenfrequency, time-dependent (transient), and frequency domain study types.

Stationary Studies

COMSOL Multiphysics uses an implementation based on the stress and strain variables. The normal and shear strain variables depend on the displacement derivatives.

Using the tensor strain, stress, and displacement variables, you can express the principle of virtual work as:

$$\delta W = \int_V (-\varepsilon_{\text{test}} : s + \mathbf{u}_{\text{test}} \cdot \mathbf{F}_V) dv$$

$$+ \int_S (\mathbf{u}_{\text{test}} \cdot \mathbf{F}_S) ds + \int_L (\mathbf{u}_{\text{test}} \cdot \mathbf{F}_L) dl + \sum_p (\mathbf{U}_{\text{test}}^t \cdot \mathbf{F}_p)$$

Time-Dependent Studies

$$\int_V (-\varepsilon_{\text{test}} : (s + \beta_{dM} s_t) + \mathbf{u}_{\text{test}} \cdot \mathbf{F}_V - \rho \mathbf{u}_{\text{test}} \cdot \mathbf{u}_{tt} - \alpha_{dM} \rho \mathbf{u}_{\text{test}} \cdot \mathbf{u}_t) dv$$

$$+ \int_S (\mathbf{u}_{\text{test}} \cdot \mathbf{F}_S) ds + \int_L (\mathbf{u}_{\text{test}} \cdot \mathbf{F}_L) dl + \sum_p (\mathbf{U}_{\text{test}}^t \cdot \mathbf{F}_p)$$

To model viscous damping, COMSOL Multiphysics uses *Rayleigh damping*, where you specify two damping coefficients. As an example, consider a system with a single degree of freedom. The equation of motion for such a system with viscous damping is

$$m \frac{d^2 u}{dt^2} + \xi \frac{du}{dt} + ku = f(t) .$$

In the Rayleigh damping model, you express the damping parameter ξ in terms of the mass m and the stiffness k as

$$\xi = \alpha_{dM} m + \beta_{dK} k$$

The Rayleigh damping proportional to mass and stiffness is added to the static weak term. The default value for both α_{dM} and β_{dK} is zero; that is, the default settings add no damping.

Frequency Domain Studies

In the frequency domain you can study the frequency response when applying harmonic loads. You specify harmonic loads using two components:

- The amplitude value, F_x
- The phase, $F_{x\mathrm{Ph}}$

To derive the equations for the steady-state response from harmonic excitation loads

$$F_{x\mathrm{freq}} = F_x(f)\cos\left(\omega t + F_{x\mathrm{Ph}}(f)\frac{\pi}{180}\right)$$

$$\mathbf{F}_{\mathrm{freq}} = \begin{bmatrix} F_{x\mathrm{freq}} \\ F_{y\mathrm{freq}} \\ F_{z\mathrm{freq}} \end{bmatrix}$$

assume a harmonic response with the same angular frequency as the excitation load

$$u = u_{\mathrm{amp}}\cos(\omega t + \phi_u)$$

$$\mathbf{u} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

You can also describe this relationship using complex notation

$$u = \mathrm{Re}(u_{\mathrm{amp}} e^{j\phi_u} e^{j\omega t}) = \mathrm{Re}(\tilde{u} e^{j\omega t}) \text{ where } \tilde{u} = u_{\mathrm{amp}} e^{j\phi_u}$$

$$\mathbf{u} = \mathrm{Re}(\tilde{\mathbf{u}} e^{j\omega t})$$

$$F_{x\text{freq}} = \text{Re}\left(F_x(\omega)e^{jF_{xPh}(f)\frac{\pi}{180}}e^{j\omega t}\right) = \text{Re}(\tilde{F}_x e^{j\omega t})$$

where

$$\tilde{F}_x = F_x(f)e^{jF_{xPh}(f)\frac{\pi}{180}}$$

$$\tilde{\mathbf{F}} = \begin{bmatrix} \tilde{F}_x \\ \tilde{F}_y \\ \tilde{F}_z \end{bmatrix}$$

Eigenfrequency Studies

The eigenfrequency equations are derived by assuming a harmonic displacement field, similar as for the frequency response formulation. The difference is that this study type uses a new variable $j\omega$ explicitly expressed in the eigenvalue.

$$j\omega = -\lambda$$

The eigenfrequency f is then derived from $j\omega$ as

$$f = \left|\frac{\text{Im}(j\omega)}{2\pi}\right|$$

You can study damped eigenfrequencies by adding viscous damping terms to the equation. In addition to the eigenfrequency you can also look at the quality factor, Q, and decay factor, δ, for the model:

$$Q = \frac{\text{Im}(\lambda)}{2\text{Re}(\lambda)}$$

$$\delta = \text{Re}(\lambda)$$

Damping Models

The Solid Mechanics interface offers two predefined damping models: Rayleigh damping and loss factor damping.

Rayleigh Damping

To model damping effects within the material, COMSOL Multiphysics uses *Rayleigh damping*, where you specify two damping coefficients. As an example, consider a system with a single degree of freedom. The equation of motion for such a system with viscous damping is

$$m\frac{d^2u}{dt^2} + \xi\frac{du}{dt} + ku = f(t)$$

In the Rayleigh damping model you express the damping parameter ξ in terms of the mass m and the stiffness k as

$$\xi = \alpha_{dM}m + \beta_{dK}k$$

The Rayleigh damping proportional to mass and stiffness is added to the static weak term.

A complication with the Rayleigh damping model is to obtain good values for the damping parameters. A more physical damping measure is the damping ratio, the ratio between actual and critical damping, often expressed as a damping factor in percentage of the critical damping. You can find commonly used values of damping factors in the literature.

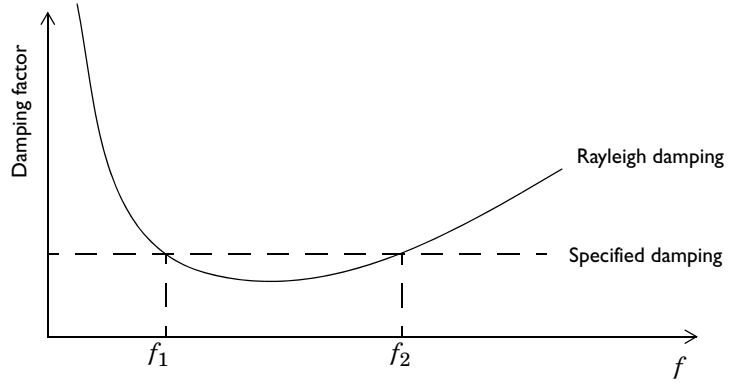
It is possible to transform damping factors to Rayleigh damping parameters. The damping factor, ξ, for a specified pairs of Rayleigh parameters, α_{dM} and β_{dK}, at a frequency, f, is

$$\xi = \frac{1}{2}\left(\frac{\alpha_{dM}}{2\pi f} + \beta_{dK}2\pi f\right)$$

Using this relationship at two frequencies, f_1 and f_2, with different damping factors, ξ_1 and ξ_2, results in an equation system that can be solved for α_{dM} and β_{dK}:

$$\begin{bmatrix} \frac{1}{4\pi f_1} & \pi f_1 \\ \frac{1}{4\pi f_2} & \pi f_2 \end{bmatrix} \begin{bmatrix} \alpha_{dM} \\ \beta_{dK} \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$$

Using the same damping factors, $\xi_1 = \xi_2$, does not result in a constant damping factor inside the interval $f_1 < f < f_2$. It can be shown that the damping factor is lower inside the interval, as the following figure shows.



Loss Factor Damping

Loss factor damping (sometimes referred to as material or structural damping) can be applied in the frequency domain.

In COMSOL Multiphysics, the loss information appears as a multiplier of the total strain in the stress-strain relationship:

$$s = s_0 + \mathsf{C} : [(1 + j\eta_s)\varepsilon - \varepsilon_0 - \alpha\theta]$$

where η_s is the loss factor, and j is the imaginary unit.

If you model the damping in the structural analysis via the loss factor, use the following definition for the elastic part of the entropy:

$$S_{\text{elast}} = \alpha : (s - j\eta_s(\mathsf{C} : \varepsilon))$$

This is because the entropy is a function of state and thus independent of the strain rate, while the damping represents the rate-dependent effects in the material, for example viscous or viscoelastic. The internal work of such inelastic forces averaged over the time period $2\pi/\omega$ can be computed as:

$$Q_h = \frac{1}{2}\omega\eta_s \text{Real}(\varepsilon : \text{Conj}(\mathsf{C} : \varepsilon))$$

You can use the above expression as a heat source for modeling of the heat generation in vibrating structures, when coupled with the frequency domain analysis for the stresses and strains.

15

Equation-Based Modeling

This section describes the use of the mathematical interfaces for equation-based modeling. With those interfaces you can solve various types of PDEs using different formulations. You can also solve ODEs and other global equations.

In this section:

The Mathematical Interfaces

Overview of the Mathematical Interfaces

The *mathematical interfaces* are physics interfaces for equation-based modeling. They support several PDE formulation as well as general ways of adding ODEs, algebraic equations, and other global (space-independent) equations. You find the following mathematical interfaces in the Model Wizard's **Mathematics** section:

- *PDE Interfaces*. These are interfaces for solving PDEs in three different forms:
  - *Coefficient form* for linear or almost linear PDEs, explained in detail in the section Coefficient Form PDE Interface (c).
  - *Weak form* using the weak formulation of the PDE for maximum flexibility. For more information about the weak form, see Weak Form Modeling.
- *ODEs and DAEs*. Use this interface to solve space-independent equations that include ordinary differential equations (ODEs), differential-algebraic equations (DAEs), algebraic equations, and transcendental equations. For more information about global equations and ODEs, see Solving ODEs and DAEs.
- *Optimization and Sensitivity*. These are interfaces for optimization (requires the Optimization Module) and sensitivity analysis. See Sensitivity Analysis and Optimization.
- *Classical PDEs*. These specialized instances of the coefficient form PDE provide interfaces for a number of classical PDEs. See Classical PDEs for more information.
- *Moving Interface*. The Level Set interface and the Phase Field interface provide the possibility to track fluid-fluid interfaces or other moving interfaces for multiphase applications, for example. See the MEMS Module or the Chemical Reaction Engineering Module documentation as moving interfaces require these modules.
- *Deformed Mesh*. The Deformed Geometry interface makes it possible to model prescribed deformation of the mesh that represents the model domain. The Moving Mesh interface is vital for modeling moving meshes using the ALE (arbitrary Lagrangian-Eulerian) technique in applications such as fluid-structure interaction (FSI).
- *Wall Distance*. This interface, which computes the distance to the boundary (wall), is primarily intended for use in connection to turbulence modeling for fluid flow. See The Wall Distance Interface for details.

Table 15-1 lists the available interfaces for PDEs, ODEs, and DAEs with their default interface identifiers:

TABLE 15-1: DEFAULT INTERFACE IDENTIFIERS FOR THE MATHEMATICAL INTERFACES

MATHEMATICAL INTERFACE	INTERFACE IDENTIFIER
PDE INTERFACES	
Coefficient Form PDE	c
General Form PDE	g
Weak Form PDE	w
Weak Form Boundary PDE	wb
Weak Form Edge PDE	we
Weak Form Point PDE	wp
CLASSICAL PDES	
Helmholtz Equation	hzeq
Laplace's Equation	lpeq
Poisson's Equation	poeq
Wave Equation	waeq
ODES AND DAES	ge

PDE Interfaces

Starting a Model Using a PDE Interface

To create a new model using one of the equation interfaces:

1. In the **Model Wizard**, select a space dimension on the **Select Space Dimension** page. Click the **Next** button ( ).
2. Expand the **Mathematics>PDE Interfaces** node in the list of physics interfaces and select one of the PDE interface in the list.
3. Double-click the interface or click the **Add Selected** button ( ) underneath the list to add the selected PDE interface to the model. The interface is added to **Selected physics**.
4. Click the **Next** button ( ) in the upper-right corner of the **Model Wizard** window.
5. Optionally, choose a **Study Type**.
6. Click the **Finish** button ( ) in the upper-right corner of the **Model Wizard** window.

SPECIFYING A SYSTEM OF EQUATIONS

COMSOL Multiphysics allows the creation of equations with more than one dependent variable. To do so, after Step 4 above, under **Dependent variables** type the number of dependent variables in the **Number of dependent variables** edit field. COMSOL Multiphysics then automatically assigns variable names, typically `u1`, `u2`, `u3`, and so on. You can also type the name of your choice (as long as they are valid and unique) in **Variable names** edit fields.

You can also couple several scalar PDEs using a multiphysics approach.

Note: Regardless if you start with a PDE interface in coefficient form or general form, you can add additional equation nodes in coefficient form, general form, or weak form to the same PDE interface.

Coefficient Form PDE Interface (c)

The **PDE** interface (Δu) in coefficient form covers many well-known PDEs. This section covers the formulation and settings pertaining to the coefficient form, as well as the general PDE terminology used in COMSOL Multiphysics.

The following default nodes are added when using this interface—**Coefficient Form PDE**, **Zero Flux**, and **Initial Values**. Right-click the **PDE** node to add other nodes that implement other boundary conditions, for example.

The following sections provides information about all features available in the Coefficient Form PDE interface.

THE SCALAR COEFFICIENT FORM EQUATION

A single dependent variable u is an unknown function on the computational domain. COMSOL Multiphysics determines it by solving the PDE problem that you specify. In coefficient form, the PDE problem reads

$$\begin{cases} e_a\frac{\partial^2 u}{\partial t^2} + d_a\frac{\partial u}{\partial t} + \nabla\cdot(-c\nabla u - \alpha u + \gamma) + \beta\cdot\nabla u + au = f & \text{in } \Omega \\ \mathbf{n}\cdot(c\nabla u + \alpha u - \gamma) + qu = g - h^T\mu & \text{on } \partial\Omega \\ u = r & \text{on } \partial\Omega \end{cases} \tag{15-1}$$

where

- Ω is the computational domain—the union of all domains
- $\partial\Omega$ is the domain boundary
- $\mathbf{n}$ is the outward unit normal vector on $\partial\Omega$

The first equation in the list above is the PDE, which must be satisfied in Ω. The second and third equations are the boundary conditions, which must hold on $\partial\Omega$. The second equation is a *Neumann* boundary condition, whereas the third equation is a *Dirichlet* boundary condition. In finite element terminology, Neumann boundary conditions are called *natural boundary conditions* because they do not occur explicitly in the weak form of the PDE problem. Dirichlet conditions are called *essential boundary conditions* because they restrict the trial space. Dirichlet boundary conditions often represent *constraints*.

This manual uses the following conventions:

- The symbol ∇ is the vector differential operator (gradient), defined as

$$\nabla = \left(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}\right)$$

  The space coordinates are denoted $x_1, \ldots, x_n$, where n represents the number of space dimensions.

- The symbol Δ is the Laplace operator

$$\frac{\partial^2}{\partial x_1^2} + \ldots + \frac{\partial^2}{\partial x_n^2}$$

- $\nabla \cdot (c\nabla u)$ means

$$\frac{\partial}{\partial x_1}\left(c\frac{\partial u}{\partial x_1}\right) + \ldots + \frac{\partial}{\partial x_n}\left(c\frac{\partial u}{\partial x_n}\right)$$

- $\beta \cdot \nabla u$ means

$$\beta_1\frac{\partial u}{\partial x_1} + \ldots + \beta_n\frac{\partial u}{\partial x_n}$$

where $\beta_1, \ldots, \beta_n$ are the components of the vector β.

Within COMSOL Multiphysics, you specify the coefficients c, α, γ, β, a, and h, and the terms f, g, and r. They can all be functions of the spatial coordinates.

- A PDE is *linear* when the coefficients depend only on the spatial coordinates (or are constants).
- A PDE is *nonlinear* if the coefficients depend on u or its derivatives (e.g. the components of ∇u).
- All the coefficients in the above equation are scalars except α, β, and γ, which are vectors with n components. The coefficient c can alternatively be an n-by-n matrix to model anisotropic materials. For more information see Modeling Anisotropic Materials.

The e_a coefficient in Equation 14-1 is a scalar or a matrix for time-dependent systems called the *mass matrix* (or mass coefficient). The d_a coefficient represents a damping term (however, if $e_a = 0$, then d_a is often called the mass coefficient). See Solving Time-Dependent Problems for more information.

INTERFACE IDENTIFIER

This is the identifier for the interface, which you use to reach the fields and variables in expressions, for example.

The identifier appears in the **Identifier** edit field, and you can change it to any unique string that is a valid identifier. The default identifier (for the first Coefficient Form PDE interface in the model) is `c`.

DOMAINS

Select the domains where you want to define the PDE. The default setting is to include all domains in the model.

DEPENDENT VARIABLES

Define the number of the dependent variables and the variable names. The default name for a scalar PDE variable is u.

ADVANCED SETTINGS AND DISCRETIZATION

Select **Show More Options** from the **View** menu in the **Model Builder**. Normally these settings do not need to be changed. See Show More Options: Advanced Settings and Discretization for more information.

Coefficient Form PDE

This is the default equation for a Coefficient Form **PDE** interface. Here you specify the coefficients for a coefficient form PDE (see Equation 14-1), with the following equation coefficients (see also Interpreting Boundary Conditions):

$$e_a\frac{\partial^2 u}{\partial t^2} + d_a\frac{\partial u}{\partial t} + \nabla\cdot(-c\nabla u - \alpha u + \gamma) + \beta\cdot\nabla u + au = f$$

- e_a is the *mass coefficient*.
- d_a is a *damping coefficient* or a *mass coefficient*.
- c is the *diffusion coefficient*.
- α is the *conservative flux convection coefficient*.
- β is the *convection coefficient*.
- a is the *absorption coefficient*.
- γ is the *conservative flux source* term.
- f is the *source term*.

The **Coefficient Form PDE** page has these sections:

DOMAINS

Select the domains where you want to define the coefficient form PDE. The default setting is to include all domains in the model.

EQUATION

This section displays the coefficient form PDE.

DIFFUSION COEFFICIENT

Enter a value or expression for the diffusion coefficient c, which is a tensor. Select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** and enter a c coefficient on various forms in 2D and 3D. If there are multiple dependent variables, there is a matrix of c component inputs.

ABSORPTION COEFFICIENT

Enter a value or expression for the absorption coefficient α. If there are multiple dependent variables, there is a matrix of α component inputs.

SOURCE TERM

Enter a value or expression for the source term f. If there are multiple dependent variables, there is a vector of f component inputs.

MASS COEFFICIENT

Enter a value or expression for the mass coefficient e_a. If there are multiple dependent variables, there is a matrix of e_a component inputs.

DAMPING OR MASS COEFFICIENT

Enter a value or expression for the damping (or mass) coefficient d_a. If there are multiple dependent variables, there is a matrix of d_a component inputs.

CONSERVATIVE FLUX CONVECTION COEFFICIENT

Enter values or expressions for the conservative flux convection coefficient α vector's components. If there are multiple dependent variables, there is a matrix of α vector component inputs.

CONVECTION COEFFICIENT

Enter values or expressions for the convection coefficient β vector's components. If there are multiple dependent variables, there is a matrix of β vector component inputs.

CONSERVATIVE FLUX SOURCE

Enter values or expressions for the conservative flux source term γ vector's components. If there are multiple dependent variables, there is a vector of γ vector component inputs.

Using the General Form PDE Interface (g)

The **PDE** interface (Δu) in general form provides a flexible way of specifying PDEs. This section covers the formulation and settings pertaining to the general form.

The following default nodes are added when using this interface—**General Form PDE**, **Zero Flux**, and **Initial Values**. Right-click **General Form PDE** to add other nodes that implement other boundary conditions, for example.

The following sections provides information about all feature nodes available in the General Form PDE interface.

THE SCALAR GENERAL FORM EQUATION

Use the *general form* for nonlinear PDEs. Assuming that you are working with a single dependent variable u, then a stationary problem in general form reads

$$\begin{cases} e_a\dfrac{\partial^2 u}{\partial t^2} + d_a\dfrac{\partial u}{\partial t} + \nabla\cdot\Gamma = F & \text{in } \Omega \\ -\mathbf{n}\cdot\Gamma = G + \left(\dfrac{\partial R}{\partial u}\right)^T \mu & \text{on } \partial\Omega \\ 0 = R & \text{on } \partial\Omega \end{cases}$$

The first equation is the PDE. The second and third equations are the Neumann and Dirichlet boundary conditions, respectively. For information on the time-dependent general form equation, see Solving Time-Dependent Problems.

The terms Γ, F, G, and R are coefficients. They can be functions of the spatial coordinates, the solution u, and the space derivatives of u. The coefficients F, G, and R are scalar, whereas Γ is the *flux vector*. The superscript "T" in the Neumann boundary condition denotes the transpose. The variable μ is the Lagrange multiplier.

The **General Form PDE** page has these sections:

INTERFACE IDENTIFIER

This is the identifier for the interface, which you use to reach the fields and variables in expressions, for example.

The identifier appears in the **Identifier** edit field, and you can change it to any unique string that is a valid identifier. The default identifier (for the first General Form PDE interface in the model) is `g`.

DOMAINS

Select the domains where you want to define the PDE. The default setting is to include all domains in the model.

DEPENDENT VARIABLES

Define the number of the dependent variables and the variable names. The default name for a scalar PDE variable is u.

ADVANCED SETTINGS AND DISCRETIZATION

Select **Show More Options** from the **View** menu in the **Model Builder**. Normally these settings do not need to be changed. See Show More Options: Advanced Settings and Discretization for more information.

General Form PDE

This is the default equation for a General Form PDE interface. Here you specify the coefficients for a general form PDE, with the following equation coefficients:

$$e_a\frac{\partial^2 u}{\partial t^2} + d_a\frac{\partial u}{\partial t} + \nabla \cdot \Gamma = F$$

- e_a is the *mass coefficient*.
- d_a is a *damping coefficient* or a *mass coefficient*.
- Γ is the conservative *flux vector*.
- F is the *source term*.

The **General Form PDE** form has these sections:

DOMAINS

Select the domains where you want to define the general form PDE. The default setting is to include all domains in the model.

EQUATION

This section displays the general form PDE.

CONSERVATIVE FLUX

Enter a values or expressions for the components of the conservative flux vector Γ. The default values `-ux`, `-uy`, and `-uz` (in 3D) represent the negative gradient of u and makes the left-hand side equal to the Laplace operator. If there are multiple dependent variables, there is one Γ vector for each variable.

ABSORPTION COEFFICIENT

Enter a value or expression for the absorption coefficient α. If there are multiple dependent variables, there is a matrix of α component inputs.

SOURCE TERM

Enter a value or expression for the source term f. If there are multiple dependent variables, there is a vector of f component inputs.

DAMPING OR MASS COEFFICIENT

Enter a value or expression for the damping (or mass) coefficient d_a. If there are multiple dependent variables, there is a matrix of d_a component inputs.

MASS COEFFICIENT

Enter a value or expression for the mass coefficient e_a. If there are multiple dependent variables, there is a matrix of e_a component inputs.

Initial Values

The **Initial Values** feature adds initial values for the dependent variables that can server as an initial condition for a transient simulation or as an initial guess for a nonlinear solver. If you need to specify more than one set of initial values, you can add additional **Initial Values** features.

DOMAINS

Select the domains where you want to define an initial value.

INITIAL VALUES

Enter a value or expression for the initial value of the dependent variables.

Adding Extra Source Terms

You can add additional source term nodes on different geometry levels: **Source** on domains, **Edge Source** on edges in 3D models and **Point Source** at points.

DOMAINS/EDGES/POINTS

Select the geometric entities where you want to define a source.

SOURCE TERM

Enter a value or expression for the source term f.

Boundary Conditions

The PDE interfaces include the following boundary condition types:

- Zero Flux
- Dirichlet Boundary Condition
- Constraint
- Flux/Source
- No Flux
- Periodic Condition

Dirichlet Boundary Condition

The **Dirichlet Boundary Condition** specifies a value of u on the boundary of the domain: $u = r$. This is the default boundary condition. It is a unidirectional constraint (see Bidirectional and Unidirectional Constraints).

BOUNDARIES

Select the boundaries where you want to define a Dirichlet boundary condition.

UNIDIRECTIONAL DIRICHLET BOUNDARY CONDITION

Enter a value or expression for the dependent variables on the boundary. To activate the Dirichlet boundary condition for a dependent variable (for example, u_2), select the corresponding check box (**Prescribed values for u2**, for example). Then enter a value or expression for the prescribed value in the associated field.

Constraint

The **Constraint** boundary condition specifies a value of R in the constraint $R = 0$. This is a bidirectional constraint (see Bidirectional and Unidirectional Constraints). You can use an expression for the constraint that, for example, implements a *Robin boundary condition* by including a term on the form qu, where u is the dependent variable.

BOUNDARIES

Select the boundaries where you want to define a constraint.

BIDIRECTIONAL CONSTRAINT, R = 0

Enter a value or expression for the value of R in the constraint $R = 0$. For example, to constrain u to 2, type `2-u` in the edit field for R.

Flux/Source

The **Flux/Source** boundary condition adds a flux or source g on the boundary:

$$\mathbf{n} \cdot (c\nabla u + \alpha u - \gamma) = g + qu$$

BOUNDARIES

Select the boundaries where you want to define a flux or source.

BOUNDARY FLUX/SOURCE

Enter a value or expression for the value of the boundary flux or source g in the corresponding field. The default value is 0.

BOUNDARY ABSORPTION/IMPEDANCE TERM

Enter a value or expression for the value of the coefficient q in the corresponding field. The default value is 0. It adds a term qu to the boundary flux or source, which can represent absorption or impedance at the boundary-

Zero Flux

The **Zero Flux** boundary condition is the default boundary condition and prescribes a zero flux (insulation) across the boundary:

$$\mathbf{n} \cdot (c\nabla u + \alpha u - \gamma) = 0$$

BOUNDARIES

Select the boundaries where you want to define a flux or source.

EQUATION

This section displays the equation for the zero flux.

Periodic Condition

The **Periodic Condition** feature adds a *periodic boundary condition*. This periodicity can be continuous (the default) so that $u(x_0) = u(x_1)$ or anti-periodic so that $u(x_0) = -u(x_1)$. You can control which of the dependent variables that the periodic condition applies to.

BOUNDARIES

Select the boundaries where you want to define a periodic boundary condition. The software automatically identifies the boundaries as either source boundaries or destination boundaries. This works fine for cases like opposing parallel boundaries. In other cases you can use the Destination Selection subfeature (see below) to control the destination (right-click **Periodic Condition** in the Model Tree and choose **Destination Selection**). By default it contains the selection that COMSOL Multiphysics has identified.

PERIODIC CONDITION

Select a **Type of periodicity**—**Continuity** (the default) to make the dependent variables equal, or **Antiperiodicity** to make them antiperiodic: $u(x_0) = -u(x_1)$.

For each dependent variable in the PDE you can choose to apply the periodic condition by selecting, for example, the **Apply condition on variable u1** check box. By default, the periodic condition applies to all dependent variables.

Destination Selection

Using the **Destination Selection** feature to change the selection for the destination. The selection that COMSOL Multiphysics makes appears as the default selection in the **Selection** list (as **Selection 1**, for example).

BOUNDARIES

Select the boundaries where you want to define a the destination for the periodic boundary condition. By default it contains the selection that COMSOL Multiphysics has identified.

Interpreting Boundary Conditions

The formulation of the boundary conditions imposes both Dirichlet and Neumann conditions. This combination is possible because of a new dependent variable μ, which is defined only on the boundary. The unknown variable μ is called a *Lagrange multiplier*. Often you can reformulate boundary conditions without Lagrange multipliers. In structural mechanics problems the Lagrange multiplier equals the reaction forces on the boundary. The factor h^T in the Neumann boundary condition is the transpose of h. If h is a scalar, then $h^T = h$.

How do the Lagrange multipliers relate to the conventional formulation with either a Dirichlet or a Neumann boundary condition?

- First, assume that $h = 1$. Then the Dirichlet condition is $u = r$. The Neumann condition becomes:

  $$\mathbf{n} \cdot (c\nabla u + \alpha u - \gamma) + qu = g - \mu$$

  The Lagrange multiplier, μ, adjusts so as to satisfy the requested Dirichlet condition. Specifying a nonzero g changes the value of the Lagrange multiplier but does not affect the actual solution u. Therefore, this equation can usually be ignored, leaving a pure Dirichlet condition.
- Second, assume that $h = 0$ and $r = 0$. Then the Dirichlet condition reads $0 = 0$, and the Neumann condition is

  $$\mathbf{n} \cdot (c\nabla u + \alpha u - \gamma) + qu = g$$

  This is the generalized Neumann condition without a Lagrange multiplier. For more on the Lagrange multiplier formulation, see System for Two Variables in the General Form.

The vector $\Gamma = -c\nabla u - \alpha u + \gamma$ is the *flux vector*. In transport equations its first term describes diffusion, the second term describes convection with a velocity $-\alpha$, and the third term γ is a source term.

In certain applications Γ can be discontinuous across interior boundaries. There can be a jump in the normal component of Γ across such a boundary. For instance, denote the

two adjacent subdomains as 1 and 2, and let Γ_i and $\mathbf{n}_i$ be the values of Γ and $\mathbf{n}$ from the two subdomains. Then you can state the jump condition as the Neumann condition

$$-\mathbf{n}_1 \cdot \Gamma_1 - \mathbf{n}_2 \cdot \Gamma_2 + qu = g - h^T \mu$$

where $\mathbf{n}_1$ is the outward normal from Subdomain 1, and $\mathbf{n}_2$ is the outward normal from Subdomain 2 so that $\mathbf{n}_1 = -\mathbf{n}_2$. At the same time there is a Dirichlet condition $u = r$.

The rest of this section handles stationary problems where $\partial u/\partial t = 0$ and transient effects have vanished.

Note: The c, α, and γ coefficients in the equations for the boundary conditions come from the PDE specification.

Multiple Dependent Variables—Equation Systems

THE COEFFICIENT FORM EQUATION SYSTEM

With two dependent variables u_1 and u_2, the stationary PDE problem in coefficient form results in the following equation system:

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} - \nabla \cdot (c\nabla u + \alpha u - \gamma) + \beta \cdot \nabla u + au = f$$

where $u = (u_1, u_2)$. The mass term is defined as

$$e_a \frac{\partial^2 u}{\partial t^2} = \begin{bmatrix} e_{a11} & e_{a12} \\ e_{a21} & e_{a22} \end{bmatrix} \begin{bmatrix} \frac{\partial^2 u_1}{\partial t^2} \\ \frac{\partial^2 u_2}{\partial t^2} \end{bmatrix} = \begin{bmatrix} e_{a11}\frac{\partial^2 u_1}{\partial t^2} + e_{a12}\frac{\partial^2 u_2}{\partial t^2} \\ e_{a21}\frac{\partial^2 u_1}{\partial t^2} + e_{a22}\frac{\partial^2 u_2}{\partial t^2} \end{bmatrix}$$

Similarly, the damping term is

$$d_a\frac{\partial u}{\partial t} = \begin{bmatrix} d_{a11} & d_{a12} \\ d_{a21} & d_{a22} \end{bmatrix}\begin{bmatrix} \frac{\partial u_1}{\partial t} \\ \frac{\partial u_2}{\partial t} \end{bmatrix} = \begin{bmatrix} d_{a11}\frac{\partial u_1}{\partial t} + d_{a12}\frac{\partial u_2}{\partial t} \\ d_{a21}\frac{\partial u_1}{\partial t} + d_{a22}\frac{\partial u_2}{\partial t} \end{bmatrix}$$

However, if $e_a = 0$, then d_a is often called the mass coefficient.

The diffusive flux is defined as

$$c\nabla u = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\nabla\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} \nabla u_1 \\ \nabla u_2 \end{bmatrix} = \begin{bmatrix} c_{11}\nabla u_1 + c_{12}\nabla u_2 \\ c_{21}\nabla u_1 + c_{22}\nabla u_2 \end{bmatrix}$$

$$= \begin{bmatrix} c_{11}\begin{bmatrix} \frac{\partial u_1}{\partial x} \\ \frac{\partial u_1}{\partial y} \end{bmatrix} + c_{12}\begin{bmatrix} \frac{\partial u_2}{\partial x} \\ \frac{\partial u_2}{\partial y} \end{bmatrix} \\ c_{21}\begin{bmatrix} \frac{\partial u_1}{\partial x} \\ \frac{\partial u_1}{\partial y} \end{bmatrix} + c_{22}\begin{bmatrix} \frac{\partial u_2}{\partial x} \\ \frac{\partial u_2}{\partial y} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} c_{11}\frac{\partial u_1}{\partial x} + c_{12}\frac{\partial u_2}{\partial x} \\ c_{11}\frac{\partial u_1}{\partial y} + c_{12}\frac{\partial u_2}{\partial y} \end{bmatrix} \\ \begin{bmatrix} c_{21}\frac{\partial u_1}{\partial x} + c_{22}\frac{\partial u_2}{\partial x} \\ c_{21}\frac{\partial u_1}{\partial y} + c_{22}\frac{\partial u_2}{\partial y} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \texttt{cu1x} \\ \texttt{cu1y} \end{bmatrix} \\ \begin{bmatrix} \texttt{cu2x} \\ \texttt{cu2y} \end{bmatrix} \end{bmatrix}$$

where ∇u_1 and ∇u_2 are column vectors. The flux matrix or flux tensor is a column vector in this presentation. For anisotropic materials, the components c_{11}, c_{12}, c_{21}, and c_{22} can be matrices as described above for the one-variable coefficient form PDE. In this case, the diffusive flux reads

$$c\nabla u = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \nabla u_1 \\ \nabla u_2 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} c_{1111} & c_{1112} \\ c_{1121} & c_{1122} \end{bmatrix} & \begin{bmatrix} c_{1211} & c_{1212} \\ c_{1221} & c_{1222} \end{bmatrix} \\ \begin{bmatrix} c_{2111} & c_{2112} \\ c_{2121} & c_{2122} \end{bmatrix} & \begin{bmatrix} c_{2211} & c_{2212} \\ c_{2221} & c_{2222} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \nabla u_1 \\ \nabla u_2 \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} c_{1111} & c_{1112} \\ c_{1121} & c_{1122} \end{bmatrix} \nabla u_1 + \begin{bmatrix} c_{1211} & c_{1212} \\ c_{1221} & c_{1222} \end{bmatrix} \nabla u_2 \\ \begin{bmatrix} c_{2111} & c_{2112} \\ c_{2121} & c_{2122} \end{bmatrix} \nabla u_1 + \begin{bmatrix} c_{2211} & c_{2212} \\ c_{2221} & c_{2222} \end{bmatrix} \nabla u_2 \end{bmatrix}$$

The conservative convective flux is defined as

$$\alpha u = \alpha \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \alpha_{111} \\ \alpha_{112} \end{bmatrix} & \begin{bmatrix} \alpha_{121} \\ \alpha_{122} \end{bmatrix} \\ \begin{bmatrix} \alpha_{211} \\ \alpha_{212} \end{bmatrix} & \begin{bmatrix} \alpha_{221} \\ \alpha_{222} \end{bmatrix} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \alpha_{111} \\ \alpha_{112} \end{bmatrix} u_1 + \begin{bmatrix} \alpha_{121} \\ \alpha_{122} \end{bmatrix} u_2 \\ \begin{bmatrix} \alpha_{211} \\ \alpha_{212} \end{bmatrix} u_1 + \begin{bmatrix} \alpha_{221} \\ \alpha_{222} \end{bmatrix} u_2 \end{bmatrix}$$

Here the third index, k, of α_{ijk} corresponds to the space coordinate suffixes x and y.

The conservative flux source is defined as

$$\gamma = \begin{bmatrix} \begin{bmatrix} \gamma_{11} \\ \gamma_{12} \end{bmatrix} \\ \begin{bmatrix} \gamma_{21} \\ \gamma_{22} \end{bmatrix} \end{bmatrix}$$

Here the second index, j, of γ_{ij} denotes the space coordinate suffixes for x and y.

For the flux terms the divergence operator works on each row separately. To illustrate this, consider the divergence of the conservative flux source

$$\nabla \cdot \gamma = \nabla \cdot \begin{bmatrix} \begin{bmatrix} \gamma_{11} \\ \gamma_{12} \end{bmatrix} \\ \begin{bmatrix} \gamma_{21} \\ \gamma_{22} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \nabla \cdot \begin{bmatrix} \gamma_{11} \\ \gamma_{12} \end{bmatrix} \\ \nabla \cdot \begin{bmatrix} \gamma_{21} \\ \gamma_{22} \end{bmatrix} \end{bmatrix}$$

The convection term is defined as

$$\beta \cdot \nabla u = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \cdot \begin{bmatrix} \nabla u_1 \\ \nabla u_2 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \beta_{111} \\ \beta_{112} \end{bmatrix} & \begin{bmatrix} \beta_{121} \\ \beta_{122} \end{bmatrix} \\ \begin{bmatrix} \beta_{211} \\ \beta_{212} \end{bmatrix} & \begin{bmatrix} \beta_{221} \\ \beta_{222} \end{bmatrix} \end{bmatrix} \cdot \begin{bmatrix} \nabla u_1 \\ \nabla u_2 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \beta_{111} \\ \beta_{112} \end{bmatrix} \cdot \nabla u_1 + \begin{bmatrix} \beta_{121} \\ \beta_{122} \end{bmatrix} \cdot \nabla u_2 \\ \begin{bmatrix} \beta_{211} \\ \beta_{212} \end{bmatrix} \cdot \nabla u_1 + \begin{bmatrix} \beta_{221} \\ \beta_{222} \end{bmatrix} \cdot \nabla u_2 \end{bmatrix}$$

The variable names for these components are `beu1` and `beu2`.

The absorption term is defined as

$$au = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} a_{11}u_1 + a_{12}u_2 \\ a_{21}u_1 + a_{22}u_2 \end{bmatrix}$$

The variable names for these components are `au1` and `au2`.

The source term is defined as

$$f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

The variable names for these components are `f1` and `f2`.

The Boundary Condition Terms

The Dirichlet boundary condition, in expanded form, reads

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

If you choose the Dirichlet condition, you also get the generalized Neumann boundary condition, which reads

$$\mathbf{n} \cdot (c\nabla u + \alpha u - \gamma) + qu = g - h^T \mu$$

The normal vector $\mathbf{n} = (n_x, n_y)$ operates on the flux vector in the same way as the divergence operator as explained earlier. If h has full rank (as in the default identity matrix, for example) only the constraints from the Dirichlet condition are active.

If you choose the Neumann condition, you get only the boundary condition

$$\mathbf{n} \cdot (c\nabla u + \alpha u - \gamma) + qu = g$$

The normal component of the diffusive flux is defined as

$$\mathbf{n} \cdot c\nabla u = \mathbf{n} \cdot \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \nabla u_1 \\ \nabla u_2 \end{bmatrix} = \begin{bmatrix} \mathbf{n} \cdot (c_{11}\nabla u_1 + c_{12}\nabla u_2) \\ \mathbf{n} \cdot (c_{21}\nabla u_1 + c_{22}\nabla u_2) \end{bmatrix}$$

The normal component of the conservative convective flux is defined as

$$\mathbf{n} \cdot \alpha u = \mathbf{n} \cdot \begin{bmatrix} \begin{bmatrix} \alpha_{111} \\ \alpha_{112} \end{bmatrix} u_1 + \begin{bmatrix} \alpha_{121} \\ \alpha_{122} \end{bmatrix} u_2 \\ \begin{bmatrix} \alpha_{211} \\ \alpha_{212} \end{bmatrix} u_1 + \begin{bmatrix} \alpha_{221} \\ \alpha_{222} \end{bmatrix} u_2 \end{bmatrix} = \begin{bmatrix} (n_x, n_y) \cdot \left(\begin{bmatrix} \alpha_{111} \\ \alpha_{112} \end{bmatrix} u_1 + \begin{bmatrix} \alpha_{121} \\ \alpha_{122} \end{bmatrix} u_2 \right) \\ (n_x, n_y) \cdot \left(\begin{bmatrix} \alpha_{211} \\ \alpha_{212} \end{bmatrix} u_1 + \begin{bmatrix} \alpha_{221} \\ \alpha_{222} \end{bmatrix} u_2 \right) \end{bmatrix}$$

The normal component of the conservative flux source is defined as

$$\mathbf{n} \cdot \gamma = (n_x, n_y) \cdot \begin{bmatrix} \begin{bmatrix} \gamma_{11} \\ \gamma_{12} \end{bmatrix} \\ \begin{bmatrix} \gamma_{21} \\ \gamma_{22} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} (n_x, n_y) \cdot \begin{bmatrix} \gamma_{11} \\ \gamma_{12} \end{bmatrix} \\ (n_x, n_y) \cdot \begin{bmatrix} \gamma_{21} \\ \gamma_{22} \end{bmatrix} \end{bmatrix}$$

The boundary absorption term is defined as

$$qu = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} q_{11}u_1 + q_{12}u_2 \\ q_{21}u_1 + q_{22}u_2 \end{bmatrix}$$

The boundary source term is defined as

$$g = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

THE GENERAL FORM EQUATION SYSTEM

In the case of several dependent variables u_1, u_2, …, u_N, the following system of equations represents a stationary problem in the general form:

$$\begin{cases} \nabla \cdot \Gamma_l = F_l & \text{in } \Omega \\ -\mathbf{n} \cdot \Gamma_l = G_l + \dfrac{\partial R_m}{\partial u_l}\mu_m & \text{on } \partial\Omega \\ 0 = R_m & \text{on } \partial\Omega \end{cases}$$

The equation index l ranges from 1 to N, while the constraint index m ranges from 1 to M. This discussion uses the summation convention. F_l, G_l, and R_m are scalars, whereas Γ_l is a vector. In this case there are several Lagrange multipliers: μ_1, μ_2,…, μ_M.

For a more compact form, let u be a vector with components u_k, let Γ be a vector with components Γ_l, and so on. Then the system of equations takes on the same form as given above for a single dependent variable.

It is possible to rewrite the system to introduce the components Γ_{lj} of the vector Γ_l and the components n_j of the normal vector $\mathbf{n}$. Then the system of equations becomes

$$\begin{cases} \dfrac{\partial \Gamma_{lj}}{\partial x_j} = F_l & \text{in } \Omega \\ -n_j\Gamma_{lj} = G_l + \dfrac{\partial R_m}{\partial u_l}\mu_m & \text{on } \partial\Omega \\ 0 = R_m & \text{on } \partial\Omega \end{cases}$$

System for Two Variables in the General Form

The following example of a PDE in the general form is a stationary system for $N = 2$ solution components in $n = 2$ space dimensions with $M = 2$ constraints:

$$\begin{cases} \nabla \cdot \Gamma_1 = F_1 & \text{in } \Omega \\ \nabla \cdot \Gamma_2 = F_2 & \text{in } \Omega \end{cases}$$

with the Neumann boundary conditions

$$\begin{cases} -\mathbf{n} \cdot \Gamma_1 = G_1 + \dfrac{\partial R_1}{\partial u_1}\mu_1 + \dfrac{\partial R_2}{\partial u_1}\mu_2 & \text{on } \partial\Omega \\ -\mathbf{n} \cdot \Gamma_2 = G_2 + \dfrac{\partial R_1}{\partial u_2}\mu_1 + \dfrac{\partial R_2}{\partial u_2}\mu_2 & \text{on } \partial\Omega \end{cases}$$

and the Dirichlet boundary conditions

$$\begin{cases} 0 = R_1 & \text{on } \partial\Omega \\ 0 = R_2 & \text{on } \partial\Omega \end{cases}$$

Specifying and Interpreting Boundary Conditions

You have access to the same set of boundary conditions in all PDE interfaces. To illustrate the flexibility of the Constraint boundary condition $R = 0$, consider five cases:

1. Let $R_1 = R_2 = 0$. Then the Dirichlet boundary conditions give $0 = 0$. In addition, the terms containing the Lagrange multipliers disappear from the Neumann boundary condition. Thus you have only the Neumann boundary conditions

$$\begin{cases} -\mathbf{n} \cdot \Gamma_1 = G_1 & \text{on } \partial\Omega \\ -\mathbf{n} \cdot \Gamma_2 = G_2 & \text{on } \partial\Omega \end{cases}$$

2. Let $R_1 = r_1 - u_1$ and $R_2 = r_2 - u_2$. Then the Dirichlet conditions are the usual $u_1 = r_1$ and $u_2 = r_2$. The Neumann boundary conditions become

$$\begin{cases} -\mathbf{n} \cdot \Gamma_1 = G_1 - \mu_1 & \text{on } \partial\Omega \\ -\mathbf{n} \cdot \Gamma_2 = G_2 - \mu_2 & \text{on } \partial\Omega \end{cases}$$

These equations impose no restrictions on u_1 or u_2, because the Lagrange multipliers μ_1 and μ_2 always adjust so as to fulfill the Dirichlet conditions. In this case, you can therefore ignore the Neumann boundary conditions.

3 Let $R_1 = r_1 - u_1$ and $R_2 = 0$. Then the Dirichlet conditions are

$$\begin{cases} 0 = r_1 - u_1 & \text{on } \partial\Omega \\ 0 = 0 & \text{on } \partial\Omega \end{cases}$$

and the Neumann conditions are

$$\begin{cases} -\mathbf{n} \cdot \Gamma_1 = G_1 - \mu_1 & \text{on } \partial\Omega \\ -\mathbf{n} \cdot \Gamma_2 = G_2 & \text{on } \partial\Omega \end{cases}$$

The first Neumann condition can be ignored because it imposes no restriction on u_1 or u_2. You effectively have only the Dirichlet condition on u_1 together with the second Neumann condition.

4 The same as Case 3 above but with the two PDEs interchanged (Γ_1 and Γ_2 as well as F_1 and F_2). Then the PDEs are

$$\begin{cases} \nabla \cdot \Gamma_2 = F_2 & \text{in } \Omega \\ \nabla \cdot \Gamma_1 = F_1 & \text{in } \Omega \end{cases}$$

The Dirichlet condition is similar to that in Case 3: $u_1 = r_1$. The Neumann conditions then become

$$\begin{cases} -\mathbf{n} \cdot \Gamma_2 = G_2 - \mu_1 & \text{on } \partial\Omega \\ -\mathbf{n} \cdot \Gamma_1 = G_1 & \text{on } \partial\Omega \end{cases}$$

Effectively, you have only the Neumann condition $-\mathbf{n} \cdot \Gamma_1 = G_1$. In comparison with Case 3 above, the PDEs and the Dirichlet conditions are identical, while the Neumann conditions are different. Thus, when mixing Dirichlet and Neumann

conditions, the ordering of the equations and the dependent variables are important. However, the ordering of the Dirichlet conditions does not matter since the different Lagrange multipliers are for all practical purposes indistinguishable from each other.

5 Finally, let $R_1 = u_2 - u_1$ and $R_2 = 0$. Then the Dirichlet conditions are

$$\begin{cases} 0 = u_2 - u_1 & \text{on } \partial\Omega \\ 0 = 0 & \text{on } \partial\Omega \end{cases}$$

and the Neumann conditions are

$$\begin{cases} -\mathbf{n} \cdot \Gamma_1 = G_1 - \mu_1 & \text{on } \partial\Omega \\ -\mathbf{n} \cdot \Gamma_2 = G_2 + \mu_1 & \text{on } \partial\Omega \end{cases}$$

Note that the same Lagrange multiplier now appears in both Neumann conditions, which can have different definitions of Γ and G. Therefore, contrary to Cases 2 and 3 above, the Neumann conditions cannot be ignored. Instead, adding the two conditions, it becomes apparent that the solution and flux on the boundary must fulfill

$$\begin{cases} 0 = u_2 - u_1 & \text{on } \partial\Omega \\ -\mathbf{n} \cdot \Gamma_2 - \mathbf{n} \cdot \Gamma_2 = G_1 + G_2 & \text{on } \partial\Omega \end{cases}$$

In these examples, the values of the Lagrange multipliers do not matter. However, they often have a physical significance. In structural mechanics, the term

$$\left(\frac{\partial R}{\partial u}\right)^T \mu$$

in the Neumann condition is the reaction force necessary to satisfy the kinematic constraints described by the Dirichlet conditions. This term has a special form because of the variational principles that give rise to it.

COEFFICIENT FORM VS. GENERAL FORM

The following substitutions show that the coefficient form and the general form are equivalent:

$$\Gamma = -c\nabla u - \alpha u + \gamma, \quad F = f - \beta\nabla u - au, \quad G = g, \quad R = r - u$$

This duality lets you choose the representation that best suits a particular PDE.

Solving Time-Dependent Problems

To get the equation for a time-dependent PDE in COMSOL Multiphysics, add terms containing time derivatives to the left-hand side of the stationary equation. The time derivatives must appear linearly, and the Dirichlet conditions must be linear. A time-dependent problem in the coefficient form reads

$$\begin{cases} e_a\dfrac{\partial^2 u}{\partial t^2} + d_a\dfrac{\partial u}{\partial t} + \nabla\cdot(-c\nabla u - \alpha u + \gamma) + \beta\cdot\nabla u + au = f & \text{in } \Omega \\ \mathbf{n}\cdot(c\nabla u + \alpha u - \gamma) + qu = g - h^T\mu & \text{on } \partial\Omega \\ u = r & \text{on } \partial\Omega \end{cases}$$

If u is a vector of dependent variables then the *mass coefficient* e_a is a matrix. All coefficients can depend on time. The name *mass matrix* or mass coefficient stems from the fact that in many physics applications e_a contains the mass density. The d_a coefficient represents damping for wave-type equations. However, if $e_a = 0$, then d_a is often called the mass coefficient. The default settings are $e_a = 0$ and $d_a = 1$, representing a time-dependent PDE such as the heat equation. Using $e_a = 1$ and $d_a = 0$ represents an undamped wave equation.

For a stationary model, COMSOL Multiphysics ignores any values or expressions that you enter in the edit field for the d_a and e_a coefficient. To activate the d_a and e_a coefficients and convert the model into a time-dependent model, select a time-dependent study.

The time-dependent problem in the general form is

$$\begin{cases} e_a\dfrac{\partial^2 u}{\partial t^2} + d_a\dfrac{\partial u}{\partial t} + \nabla\cdot\Gamma = F & \text{in } \Omega \\ -\mathbf{n}\cdot\Gamma = G + \left(\dfrac{\partial R}{\partial u}\right)^T\mu & \text{on } \partial\Omega \\ 0 = R & \text{on } \partial\Omega \end{cases}$$

The flux vector Γ and the scalar coefficients F, G, and R can be functions of the spatial coordinates, the solution u, and the space and time derivatives of u. The superscript T

in the Neumann boundary condition denotes the transpose. The variable μ is the Lagrange multiplier.

For the weak form, the time-dependent problem (in 2D) looks like

$$\int_\Omega W^{(2t)}dA + \int_B W^{(1t)}ds + \sum_P W^{(0t)} = \int_\Omega W^{(2)}dA + \int_B W^{(1)}ds + \sum_P W^{(0)} +$$
$$+ \int_\Omega v_l \frac{\partial R_m^{(2)}}{\partial u_l}\,\mu_m^{(2)}dA + \int_B v_l \frac{\partial R_m^{(1)}}{\partial u_l}\,\mu_m^{(1)}ds + \sum_P v_l \frac{\partial R_m^{(0)}}{\partial u_l}\,\mu_m^{(0)}$$
$$0 = R^{(2)} \quad \text{on } \Omega$$
$$0 = R^{(1)} \quad \text{on B}$$
$$0 = R^{(0)} \quad \text{on P}$$

where the integrands $W^{(it)}$ depend bilinearly on the test functions v_l and on the first and second time derivatives of the dependent variables and their space derivatives. For example,

$$W^{(2t)} = v_1 \frac{\partial u_2}{\partial t} - \frac{\partial v_1}{\partial x}\cos(3u_1)\frac{\partial u_1}{\partial t} + v_2 \frac{\partial^2 u_2}{\partial t \partial y}$$

Time-Dependent Systems

For time-dependent systems of PDEs the d_a and e_a coefficients are matrices. For example, for the system

$$e_{11}\frac{\partial^2 u}{\partial t^2} + e_{12}\frac{\partial^2 u_2}{\partial t^2} + d_{11}\frac{\partial u_1}{\partial t} + d_{12}\frac{\partial u_2}{\partial t} + \nabla \cdot (-c_{11}\nabla u_1 - c_{12}\nabla u_2) + a_{11}u_1 + a_{12}u_2 = f_1$$
$$e_{21}\frac{\partial^2 u}{\partial t^2} + e_{22}\frac{\partial^2 u_2}{\partial t^2} + d_{21}\frac{\partial u_1}{\partial t} + d_{22}\frac{\partial u_2}{\partial t} + \nabla \cdot (-c_{21}\nabla u_1 - c_{22}\nabla u_2) + a_{21}u_1 + a_{22}u_2 = f_2$$

the coefficient matrices are

$$e_a = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} \qquad d_a = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$
$$c = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \qquad a = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

(where each element c_{ij} can be an n-by-n matrix). Many interesting problems have a singular d_a matrix (with $e_a = 0$), or a singular nonzero e_a matrix. Such problems are called *differential-algebraic equation* (DAE) systems. The COMSOL Multiphysics solvers for time-dependent problems handle DAEs.

Solving Eigenvalue Problems

COMSOL Multiphysics handles scalar eigenvalue problems for all PDE forms. These eigenvalue problems are related to time-dependent problems via the correspondence $\partial/\partial t \leftrightarrow -\lambda$, linking the time derivative to the eigenvalue λ (with the eigenvalue name `lambda`). An eigenvalue problem in the coefficient form reads

$$\begin{cases} (\lambda-\lambda_0)^2 e_a u - (\lambda-\lambda_0) d_a u + \nabla\cdot(-c\nabla u - \alpha u) + \beta\cdot\nabla u + a u = f & \text{in } \Omega \\ \mathbf{n}\cdot(c\nabla u + \alpha u) + q u = -h^T \mu & \text{on } \partial\Omega \\ u = r & \text{on } \partial\Omega \end{cases}$$

where λ_0 is the linearization point for the eigenvalue. The source terms are included if they depend on the solution components. If the coefficients depend on u or the eigenvalue λ, COMSOL Multiphysics performs a linearization of the problem about the linearization point $u = u_0$, $\lambda = \lambda_0$. See Solving for information about linearization.

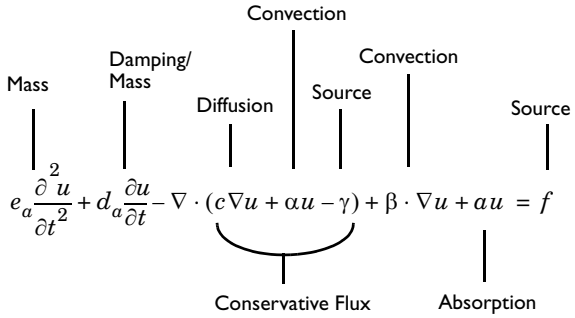
To specify the linearization point u_0 and the linearization point λ_0, use the settings in the **Linearization Point** section in the **Eigenvalue** feature.

Interpreting PDE Coefficients

The COMSOL PDE formulations can model a variety of problems, but note that this documentation uses coefficient names that fall within the realm of continuum mechanics and mass transfer. For the coefficient form PDE:

- e_a is the *mass coefficient*.
- d_a is a *damping coefficient* or a *mass coefficient*.
- c is the *diffusion coefficient*.
- α is the *conservative flux convection coefficient*.
- β is the *convection coefficient*.
- a is the *absorption coefficient*.

- γ is the *conservative flux source term*.
- f is the *source term*.



There are many interesting PDE problems to which this interpretation does not apply. For instance, a time-harmonic PDE such as the Helmholtz equation represents a time-dependent phenomenon transformed into the frequency domain.

For the Neumann boundary condition of the coefficient form

$$\mathbf{n}\cdot(c\nabla u + \alpha u - \gamma) + qu = g - h^T\mu$$

- q is the b*oundary absorption coefficient*.
- g is the *boundary source term*.

Classical PDEs

Many classical PDEs are instances of the coefficient form PDE. The classical PDEs in this section have their own interfaces. To find them, open the **Model Wizard**. In the **Physics interfaces** tree, open **Equation-Based Modeling** and then **Classical PDEs** to select one of the classical PDEs. Table 14-2 shows the available classical PDEs using two

notations: the compact notation of vector analysis (used in this documentation) and an expanded mathematical notation.

TABLE 15-2: CLASSICAL PDES IN COMPACT AND STANDARD NOTATION

EQUATION	COMPACT NOTATION	STANDARD NOTATION (2D)
Laplace's equation	$-\nabla \cdot (\nabla u) = 0$	$-\frac{\partial}{\partial x}\frac{\partial u}{\partial x} - \frac{\partial}{\partial y}\frac{\partial u}{\partial y} = 0$
Poisson's equation	$-\nabla \cdot (c\nabla u) = f$	$-\frac{\partial}{\partial x}\left(c\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial y}\left(c\frac{\partial u}{\partial y}\right) = f$
Helmholtz equation	$-\nabla \cdot (c\nabla u) + au = f$	$-\frac{\partial}{\partial x}\left(c\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial y}\left(c\frac{\partial u}{\partial y}\right) + au = f$
Wave equation	$e_a\frac{\partial^2 u}{\partial t^2} - \nabla \cdot (c\nabla u) = f$	$e_a\frac{\partial^2 u}{\partial t^2} - \frac{\partial}{\partial x}\left(c\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial y}\left(c\frac{\partial u}{\partial y}\right) = f$

The default values are 1 for f and c and -1 for a, so the default Helmholtz equation, for example, is $-\Delta u - u = 1$.

The Classical PDEs have the same boundary conditions and other settings as the Coefficient Form PDE (see Coefficient Form PDE Interface (c)).

Bidirectional and Unidirectional Constraints

Constraints formulated through the coefficients h and r in the Coefficient Form PDE interface and through the coefficient R in the General Form PDE interface give rise to constraints called *bidirectional*. A bidirectional constraint dictates exactly how the flux conditions (or Neumann boundary conditions) are influenced by the constraint force. For the coefficient form, the flux condition is

$$\mathbf{n} \cdot (c\nabla u + \alpha u) + qu = g - h^T \mu$$

and for the general form, the flux condition is

$$-\mathbf{n} \cdot \Gamma = G + \left(\frac{\partial R}{\partial u}\right)^T \mu$$

The last term on the right-hand side in both expressions is the bidirectional constraint force. Thus, with bidirectional constraints you cannot enforce a flux condition independently of the constraints. In mathematics as well as in multiphysics modeling it is often necessary to enforce Neumann conditions and Dirichlet conditions more

freely than what is possible through bidirectional constraints. Assume that you want to enforce the boundary conditions

$$\begin{cases} 0 = r_1 - u_1 & \text{on } \partial\Omega \\ -\mathbf{n} \cdot \Gamma_2 = G_2 & \text{on } \partial\Omega \end{cases}.$$

If $r_1 = r_1(u_2)$, the first condition is fulfilled but not the second. This is because the bidirectional constraint force is not zero:

$$-\mathbf{n} \cdot \Gamma_2 = G_2 + \frac{\partial R_1}{\partial u_2}\mu_1 + \frac{\partial R_2}{\partial u_2}\mu_2 = G_2 + \frac{\partial r_1}{\partial u_2}\mu_1 \neq G_2 .$$

To remedy this limitation with ideal constraints you can use *unidirectional constraints*, by which you can specify the constraint force independently of the constraints. The term unidirectional refers to the fact that the constraint force differs from $-h^T\mu$ for coefficient form and from $\left(\frac{\partial R}{\partial u}\right)^T \mu$ for general form. In multiphysics modeling, unidirectional constraints are, for example, necessary for the following boundary conditions:

- Normal-direction constraints on a moving mesh, where the mesh motion is part of the problem. These conditions are of the type $\mathbf{n} \cdot \mathbf{u} - r = 0$ where $\mathbf{n} = \mathbf{n}(\mathbf{x})$ is the boundary normal, $\mathbf{u}$ is a vector field (displacements or velocity), and $\mathbf{x}$ is the mesh coordinate vector. Bidirectional constraints give constraint forces not only on the equations for $\mathbf{u}$ but also on the equations for $\mathbf{x}$, which typically are not wanted.
- Wall boundary conditions for turbulent fluid flow. For the k-ε turbulence model this condition is of the type $k - r(\varepsilon)$, $-\mathbf{n} \cdot \nabla\varepsilon$, where r is a given function. Bidirectional constraints for the first relation imply that the second relation cannot hold.

Unidirectional constraints can be enforced in COMSOL Multiphysics both in a pointwise and in a weak sense. For descriptions of how to use pointwise and weak non-ideal constraints see Constraint Forces for PDEs and Weak Constraint, respectively.

PDE Interface Variables

The following list shows the variables that are available for results evaluation and visualization in addition to the other variables for PDEs:

NAME	TYPE	DESCRIPTION	EXPRESSION
`ui`	S, B	The solution variable (dependent variable)	u_i
`uixj`	S, B	The derivative of the solution variable u_i with respect to the space coordinate x_j, for example, uy	$\frac{\partial u_i}{\partial x_i}$
`uixjxk`	S, B	The second derivative of the solution variable u_i with respect to the space coordinates x_j and x_k, for example, uxx, uxy	$\frac{\partial^2 u_i}{\partial x_j \partial x_k}$
`uit`	S, B	The derivative of the solution variable u_i with respect to time	$\frac{\partial u_i}{\partial t}$
`uitt`	S, B	The second derivative of the solution variable u_i with respect to time	$\frac{\partial^2 u_i}{\partial t^2}$
`uixjt`	S, B	The mixed derivative of the solution variable u_i with respect to time and the space coordinate x_j	$\frac{\partial^2 u_i}{\partial x_j \partial t}$

PDE Coefficients and Boundary Conditions with Time Derivatives

The d coefficients relate to 1st-order time derivatives of the solution u. The only directly available d coefficient is d_a, the coefficient in front of $\partial u/\partial t$; the subscript a is used because d_a is similar to the a coefficient in the absorption term except that it multiplies $\partial u/\partial t$ instead of u. In contrast, the coefficients d_c, $\mathbf{d}_\alpha$, and $\mathbf{d}_\beta$ for $\partial u/\partial t$—analogous to the coefficients c, α, and β for u, respectively—in the extended PDE formulation

$$-\nabla\cdot\left(d_c\nabla\frac{\partial u}{\partial t}+\mathbf{d}_\alpha\frac{\partial u}{\partial t}\right)+\mathbf{d}_\beta\cdot\nabla\frac{\partial u}{\partial t}+d_a\frac{\partial u}{\partial t}+\nabla\cdot(-c\nabla u+\ldots)$$

are not directly available in the general or coefficient form PDE models. Instead, you can enter them in the existing γ and f terms:

- In 1D, add `-d_c*uxt-d_al*ut` to the γ term, and add `-d_be*uxt` to the f term.
- In 2D, add `-d_c*uxt-d_al1*ut` to the first γ component, and add `-d_c*uyt-d_al2*ut` to the second γ component. Add `-d_be1*uxt-d_be2*uyt` to the f term.

Replace the variables `d_c`, `d_al`, `d_be`, `d_al1`, `d_al2`, `d_be1`, and `d_be2` with appropriate expressions.

You enter 2nd-order time derivative terms in an analogous manner.

To specify a boundary condition containing time-derivative terms as in

$$\mathbf{n} \cdot (c\nabla u + \ldots) + e_q \frac{\partial^2 u}{\partial t^2} + d_q \frac{\partial u}{\partial t} + qu = g - h^T \mu ,$$

add the terms `-e_q*utt-d_q*ut` to the g term, and provide appropriate values or expressions for the coefficients e_q and d_q in, for instance, a **Global Expressions** node's **Settings** window.

Weak Form Modeling

Introduction

Do not be misled by the term "weak;" the weak form is very powerful and flexible. The term *weak form* is borrowed from mathematics, but in this context it has a slightly different meaning; this implementation incorporates features in addition to those defined in the mathematical weak form. Moreover, knowledge of the mathematical weak form is not a prerequisite to using the COMSOL Multiphysics implementation.

The distinguishing features of the weak form in COMSOL Multiphysics are that it makes it possible to:

- Enter certain equations that can be derived from an energy principle in a very compact and convenient form. Such equations, for example, arise in structural mechanics.
- Add and modify nonstandard constraints, such as various contact and friction models.
- Build models with extra equations on boundaries, edges, and points.
- Use the *test operator* to conveniently work with problems in variational calculus and parametric optimization.

COMSOL Multiphysics provides the possibility to add weak form contributions to any physics interface in the model. In addition, you can add *weak constraints*, which, for example, provide accurate fluxes and reaction forces.

By default, COMSOL Multiphysics converts all models to the *weak form* before solving. Although this feature is a part of the solution technique rather than part of the modeling process, it nonetheless belongs in this discussion because it is based on the weak form implementation.

The discussion in this section reviews the ways to work with the weak form by adding weak form contributions.

The Weak Form Interfaces

In the **Model Wizard**, you can add weak form interfaces on all domain levels: **Weak Form PDE** on domains, **Weak Form Boundary PDE** on boundaries, **Weak Form Edge PDE** on edges (in 3D models), and **Weak Form Point** for points. In all of them you can add weak

expressions, which COMSOL Multiphysics adds to the overall equation. Adding one of these interfaces creates a **PDE** node (∫du) for PDE modeling using a weak formulation. You can also add a Weak Form PDE on the domain level to any other PDE interface.

The following default nodes are added when using these interfaces—**Weak Form PDE**, **Zero Flux** (for a Weak Form PDE on the domain level only), and **Initial Values**. Right-click the main **PDE** node to add other nodes that implement other boundary conditions, for example. On the domain level, edge, and boundary levels you can use the same boundary conditions as for the Coefficient Form PDE and General Form PDE.

The following sections provides information about the specific feature nodes available in the Weak Form PDE interface.

INTERFACE IDENTIFIER

This is the identifier for the interface, which you use to reach the fields and variables in expressions, for example.

The identifier appears in the **Identifier** edit field, and you can change it to any unique string that is a valid identifier. The default identifier (for the first Weak Form PDE interface in the model) is `w`, `wb`, `we`, or `wp`, depending on the geometry level).

DOMAINS, BOUNDARIES, EDGES, OR POINTS

Select the domains (boundaries, edges, points) where you want to define the weak-form PDE. The default setting is to include all domains (boundaries, edges, points) in the model.

DEPENDENT VARIABLES

Define the number of the dependent variables and the variable names. The default name for a scalar PDE variable is u.

ADVANCED SETTINGS AND DISCRETIZATION

Select **Show More Options** from the **View** menu in the **Model Builder**. Normally these settings do not need to be changed. See Show More Options: Advanced Settings and Discretization for more information.

Weak Form PDE

Use **Weak Form PDE** to specify the PDE using a weak formulation.

DOMAINS/BOUNDARIES/EDGES/POINTS

Select the geometric entities where you want to define the weak form PDE. The default setting is to include all selected geometric entities in the model.

EQUATION

This section displays the weak form PDE.

WEAK EXPRESSION

Enter the weak expression that COMSOL Multiphysics (together with any other weak expressions on the same domain) sets equal to 0 in the **weak** field. On the domain level, the default expression is `-test(ux)*ux-test(uy)*uy+test(u)`, which is the weak formulation of Poisson's equation with $f = 1$. On other domain levels, the default weak expression is 0.

The Weak Contribution Feature

The **Weak Contribution** feature is available in all physics interfaces and for all geometry levels, including the global level.

To add a **Weak Contribution** feature to the main PDE, in the **Model Builder**, right-click the main PDE node. Depending on the geometric entity level, select **More>Weak Contribution**, **Edges>Weak Contribution** or **Points>Weak Contribution** from the context menu. Or select **Global>Weak Contribution** option to apply it globally. To create an independent weak form equation rather than a weak form contribution, you need to add extra states (dependent variables) using the **Auxiliary Dependent Variables** subfeature.

The **Weak Contribution** page has these sections:

DOMAINS, BOUNDARIES, EDGES, OR POINTS

Select the geometric entity where you want to add a weak contribution—to domains, boundaries, edges in 3D models, and points in 2D and 3D models.

WEAK CONTRIBUTION

Enter the weak form contribution in the **Weak expression** field. For example, `-ux*test(ux)-uy*test(uy)+1*test(u)` is the weak formulation of a Poisson's equation with u as the dependent variable 1 as the source term on the right-hand side. To add a time derivative as in the time-dependent coefficient form PDE, add `-ut*test(u)` (notice the sign and the syntax `ut` for the time derivative of u).

QUADRATURE SETTINGS

These settings affect the numerical integration, and you do normally not need to change them.

Integration Order

The integration order is typically twice the order of the shape function. For example, the default integration order for second-order Lagrange elements is 4. The integration order is a positive integer that you enter in the **Integration order** edit field.

Integrate on Frame

This setting determines which frame to base the integration on: the spatial frame, the material frame, or the mesh frame. The default is to use the default frame for the physics interface. You select the frame from the **Integrate on frame** list.

VARIATIONAL CALCULUS

You can use the `test` operator to work with problems in variational calculus and optimization theory.

For more details about the Auxiliary Dependent Variable feature, see below.

Weak Contributions on Mesh Boundaries

To display this option, select **Show More Options** from the **View** menu in the **Model Builder**. To add a **Weak Contribution on Mesh Boundaries** feature to the main PDE, in the **Model Builder**, right-click the main PDE node and select **More>Weak Contribution on Mesh Boundaries** at the domain level.

This feature is similar to the **Weak Contribution** feature but is active on mesh boundaries. The settings are the same as for the **Weak Contribution** (see The Weak Contribution Feature).

The Auxiliary Dependent Variable Feature

To display this option, select **Show More Options** from the **View** menu in the **Model Builder**.

If you specify a complete equation on weak form in a **Weak Contribution** feature and use a new dependent variable that is not part of the physics interface, right-click the **Weak Contribution** node to add an **Auxiliary Dependent Variable**.

DOMAINS, BOUNDARIES, EDGES, OR POINTS

Select the geometric entity where you want to add the auxiliary dependent variable. The default geometric entity is the same as for the parent **Weak Contribution** feature (domains, boundaries, edges in 3D models, or points in 2D and 3D models). In rare cases, you can use more than one **Auxiliary Dependent Variable** feature on subsets of the **Weak Contribution** feature's selection (to use different initial values, for example).

AUXILIARY DEPENDENT VARIABLE

Define the name and properties of the auxiliary dependent variable.

Field Variable Name

Enter the **Field variable name** (the dependent variable).

Initial Value

Enter the **Initial value** for the dependent variable. The default is 0.

Shape Function

From the **Shape function** list, select from first-order to third-order Lagrange elements and form constant to second-order discontinuous elements. Quadratic Lagrange elements are the default elements for most physics interfaces.

Define Derivatives With Respect to Frame

Select an option from the **Define derivatives with respect to frame** list—**Spatial**, **Material**, or **Mesh**. The default is to use the default frame for the physics interface.

Modeling with PDEs on Boundaries, Edges, and Points

The weak form makes it possible to do equation modeling on boundaries, edges, and points, thus opening up a wider class of models than some of the physics interfaces cover. To add such equations, you can add auxiliary dependent variables to a **Weak Contribution** feature. You can use such weak form equations as a way of handling thin layers; COMSOL Multiphysics then solves the problem by modeling rather than meshing. This approach reduces the solution time.

The model "Transport and Adsorption" in the COMSOL Multiphysics Model Library shows how to use the weak form boundary mode to model a thin adsorbtion layer with diffusion as a PDE on the boundary of a convection-diffusion problem. Another model using tangential derivative variables is "Shell Diffusion" in the COMSOL Multiphysics Model Library.

Specifying PDEs in Weak Form

In addition to shape function variables, you can use the test operator in the **Weak expression** edit fields in the **Weak Contribution** features and the **Constraint expression** edit fields in the **Weak Constraint** features. The test operator can operate on the shape function variables: for example, `test(u1)`. See Special Operators for more information about the test operator and other operators.

The test functions always occur linearly in the weak terms. You can use these weak terms either alone to define the problem (in the Weak Form PDE interface or by adding an auxiliary dependent variable), or in addition to the other equations defined by the physics interface.

Using Weak Constraints

The *weak constraints* feature in COMSOL Multiphysics implements constraints by using finite elements on the constrained domain for the Lagrange multipliers, and by solving for the Lagrange multipliers along with the original problem. The weak constraints have a number of distinct advantages. They can:

- Provide very accurate flux computations—the Lagrange multipliers along the constraints optimally balance the finite element projection of the applied loads (reaction forces). See Computing Accurate Fluxes for more information and an example. For computing integrals of reaction forces or fluxes, the reaction force operator (`reacf`) is a better alternative than weak constraints. See The Reacf Operator. The reaction force operator does not have the drawbacks discussed below.
- Handle nonlinear constraints—the nonlinear solver does not store the Lagrange multipliers arising from standard constraints from one step to the next, which affects the convergence when constraints are nonlinear. Weak constraints can handle general nonlinearities because the Lagrange multipliers are updated as a part of the solution vector and give correct contributions to the stiffness matrix.
- Implement constraints including derivatives—constraints only on the tangential component of the derivative work when using standard constraints, whereas you must use weak constraint to be able to handle nontangential constraints. Note that boundary conditions involving the normal component of the derivative can almost always be reformulated as a Neumann condition, which is usually preferable.

Weak constraints also come with the following drawbacks:

- Because extra unknowns are introduced for the Lagrange multipliers the problem size increases compared to when eliminating the constraints.
- The formulation normally implies that a saddle-point problem is introduced with zeros on the main diagonal of the Jacobian of the discretized equations. This class of problems is often more difficult to solve. Because iterative methods for linear systems are sensitive to the eigenvalue distribution of the system matrix, a weak constraint formulation can be much more difficult to solve than the corresponding pointwise constraint formulation.
- Discontinuous constraints result in (theoretically) infinite Lagrange multipliers. In practice you get large oscillations.

Weak Constraint

To add a **Weak Constraint** feature on the domain, boundary, edge, point, or global level, select **Show More Options** from the **View** menu in the **Model Builder**. Depending on the geometric entity level, select **More>Weak Constraint**, **Edges>Weak Constraint** or **Points>Weak Constraint** from the context menu. There is no global weak constraint option.

DOMAINS, BOUNDARIES, EDGES, OR POINTS

Select the geometric entity where you want to add a weak constraint—to domains, boundaries, edges in 3D models, and points in 2D and 3D models.

The default setting is to include all geometric entities in the model.

WEAK CONSTRAINT

Select a **Constraint type**—**Bidirectional, symmetric** or **User defined**. Enter a **Constraint expression**. See Bidirectional and Unidirectional Constraints for more information. If **User defined** is selected, enter a **Constraint force expression**.

QUADRATURE SETTINGS

The **Use automatic quadrature settings** check box is selected by default. If required, click to clear the check box.

LAGRANGE MULTIPLIER

Enter a **Lagrange multiplier variable** and an **Initial value**. The variable name using weak constraints, such as `lm`, correspond to the Lagrange multipliers. Change the name if required. The weak constraints supersede the original constraints and implement them using a weak formulation.

Select a **Shape function**—**Linear Lagrange**, **Quadratic Lagrange** (the default), **Cubic Lagrange**, **Constant discontinuous**, **Linear discontinuous**, or **Quadratic discontinuous**.

From the **Define derivatives with respect to frame** list, select **Spatial**, **Material**, or **Mesh**.

Pointwise Constraint

To add a **Pointwise Constraint** feature on the domain, boundary, edge, or point level, select **Show More Options** from the **View** menu in the **Model Builder**. Depending on the geometric entity level, select **More>Weak Constraint** at the domain or boundary level, **Edges>Weak Constraint** or **Points>Weak Constraint** from the context menu. There is no global pointwise constraint option.

You can also add pointwise (strong) constraints, which do not create Lagrange multipliers. The settings in the **Pointwise Constraint** section are similar to those for the weak constraints.

DOMAINS, BOUNDARIES, EDGES, OR POINTS

Select the geometric entity where you want to add to a pointwise constraint. The default setting is to include all selected geometric entities in the model.

POINTWISE CONSTRAINT

Select a **Constraint type**—**Bidirectional, symmetric** or **User defined**. Enter a **Constraint expression**. If **User defined** is selected, enter a **Constraint force expression**. See Bidirectional and Unidirectional Constraints for more information.

SHAPE FUNCTION

Select a **Shape function**—**Linear Lagrange**, **Quadratic Lagrange** (the default), **Cubic Lagrange**, **Constant discontinuous**, **Linear discontinuous**, or **Quadratic discontinuous**.

From the **Define derivatives with respect to frame** list, select **Spatial**, **Material**, or **Mesh**.

Bidirectional and Unidirectional Constraints

The default constraint type is *bidirectional*, which means that COMSOL Multiphysics derives the constraint force directly from the constraint at a boundary, edge, or point. You can also use the *unidirectional* constraints, which for constraints written as $0 = R$ on $\partial\Omega$, changes the constraint force according to

$$\left(\frac{\partial R}{\partial u}\right)^T \mu \rightarrow -I_{\partial\Omega}\mu$$

where $I_{\partial\Omega}$ is an identity operator for points on the boundary $\partial\Omega$. For example, using two variables u_1 and u_2 and the constraints

$$\begin{cases} 0 = R_1(u_1, u_2) & \text{on } \partial\Omega \\ 0 = R_2(u_1, u_2) & \text{on } \partial\Omega \end{cases},$$

then the non-ideal constraint force is $-\mu$ acting on both equations for the boundary $\partial\Omega$.

BIDIRECTIONAL VS. UNIDIRECTIONAL WEAK CONSTRAINTS

Bidirectional weak constraints in theory implement the same boundary conditions as the standard, pointwise, constraints. In some cases, the result may differ slightly, but this is only due to the different discretization. Using unidirectional constraints, on the other hand, modifies the way boundary conditions are interpreted. Switching from pointwise constraints to unidirectional weak constraints therefore can modify the *physics* of the model, while using bidirectional weak constraints only affects the *numerics*.

The difference between bidirectional and unidirectional constraints is the way the Lagrange multipliers—which can be interpreted as generalized reaction forces—are applied. In a bidirectional constraint, the Lagrange multipliers are applied symmetrically on all dependent variables involved in the constraint so as to keep symmetric problems symmetric. In a unidirectional constraint, the reaction forces are applied only on the physics interface's dependent variable at the boundary where the constraint is specified.

Imagine a model with two physics interfaces A and B, with dependent variables u and v, respectively. If physics interface A specifies a standard constraint as $R = 2u - 3v = 0$, COMSOL Multiphysics implements the ideal weak boundary constraint by adding the weak term

$$\int_B \hat{\mu} \cdot (2u - 3v) ds + \int_B \mu \cdot (2\hat{u} - 3\hat{v}) ds$$

where μ and $\hat{\mu}$ are the Lagrange multiplier and corresponding test function, and $\hat{u}$ and $\hat{v}$ are the test functions corresponding to u and v, respectively. Because μ multiplies both test functions in the second integral, both u and v are affected by the constraint, which is therefore *bidirectional*.

The corresponding non-ideal weak constraint adds the weak form contribution

$$\int_B \hat{\mu} \cdot (2u - 3v) ds + \int_B \mu \cdot \hat{u} ds$$

Here, the Lagrange multiplier μ only multiplies the test function $\hat{u}$. All reaction forces therefore apply only to u, while v is left unaffected by the constraint. This makes the non-ideal constraint *unidirectional*. Note also that the non-ideal constraint does not consider the factor 2 in front of u when applying the forces. Even if the constraint would contain spatial derivatives, the Lagrange multiplier would still be applied only directly on the dependent variable itself.

Constraint Forces for PDEs

In the **Weak Constraint** features you can also specify which constraint type to use. The default constraint type is *bidirectional*, which means that the Coefficient Form PDE and General Form PDE interfaces derive the constraint force directly from the constraint that you specify in the **Constraint expression** edit field.

If you want to use the *unidirectional* constraints, the constraint force from a constraint written as **constr** = $R = 0$ on $\partial\Omega$, changes according to

$$\left(\frac{\partial R}{\partial u}\right)^T \mu \rightarrow -I_{\partial\Omega}\mu$$

where $I_{\partial\Omega}$ is an identity operator for points on the boundary $\partial\Omega$. If, for example, a two-variable problem for variables u_1 and u_2 specifies the constraints (using the **Constraint expression** edit field)

$$\begin{cases} 0 = R_1(u_1, u_2) & \text{on } \partial\Omega \\ 0 = R_2(u_1, u_2) & \text{on } \partial\Omega \end{cases},$$

then the default unidirectional constraint force is $-\mu_1$ acting on u_1 and $-\mu_2$ acting on u_2. The corresponding bidirectional constraint would give reaction forces

$$\left(\frac{\partial R_1}{\partial u_1}\right)^T \mu_1 + \left(\frac{\partial R_2}{\partial u_1}\right)^T \mu_2$$

acting on u_1 and

$$\left(\frac{\partial R_1}{\partial u_2}\right)^T \mu_1 + \left(\frac{\partial R_2}{\partial u_2}\right)^T \mu_2$$

acting on u_1.

If you want a unidirectional constraint or a constraint force different from any of these two choices, you must specify a separate expression for the constraint force. Select **User defined** from the **Constraint type** list and enter the expression in the **Constraint force expression** edit field. In this edit field you specify the constraint force Jacobian using the `test` operator. For example, the expression corresponding to the default unidirectional constraint in the example above is

$$\begin{cases} -\text{test}(u1) \\ -\text{test}(u2) \end{cases}$$

Note: The bidirectional constraint type normally generates a symmetric constraint Jacobian matrix and constraint force Jacobian matrix coupling for the discretized equations, while unidirectional constraints generate an unsymmetric coupling. See also Solving.

Limitations of Weak Constraints

Weak constraints are more difficult to use than conventional pointwise constraints. Notes these tips when using them:

- Pointwise and weak constraints on the same set of variables on adjacent boundaries (boundaries that share common node points in the mesh) do not work. This means that if you must constrain all boundaries on a solid and want to use a weak constraint on one boundary segment (one face), you must use the weak constraint on the entire boundary of the solid (if the boundary is connected).
- You must always have a nontrivial constraint (that is, a Dirichlet condition) on the boundaries where you enable the weak constraint for the constraint to take effect.
- Problems with the scaling of linear systems sometimes arise in conjunction with weak constraints. The Lagrange multiplier variables always have a different unit than the main system variables, and can therefore be of a completely different order of magnitude. Usually the automatic variable scaling in the solvers is sufficient, but there are cases when you might want to consider manual scaling.
- Always use the same shape function type for the weak constraint as for the variables that you constrain, possibly with lower-order elements for the weak constraint. In some cases (for example, when constraining derivatives) the system of equations can become singular. The reason is usually that there are redundant Lagrange multiplier degrees of freedom in the model. Try to lower the order of the Lagrange multiplier variables or use constraints on the Lagrange multiplier to remove some degrees of freedom.
- If you want to use iterative methods for the linear system and if the weak constraint formulation gives zeros on the diagonal of the system matrix, the SOR class of preconditioners and smoothers do not work. Instead try use the Vanka algorithm with the Lagrange multipliers as the Vanka variables or use the incomplete LU factorization algorithm.

Solving ODEs and DAEs

Adding ODEs, DAEs, and Other Global Equations

When working on complex models, you might need to introduce single named degrees of freedom, or *states*, which are not logically related to any particular point in space, and their corresponding equations. In particular, such situations arise when modeling physics in interaction with an external system, for example, a controller or an electrical circuit built from standard components. It is often possible to describe such external systems by a system of ordinary differential equations, ODEs, with a limited number of degrees of freedom.

The Global Equations and ODEs interface provides a Global Equations feature that is designed for implementing this type of external equation. Such equations are often tightly coupled to a model in a physical domain. The Global Equations feature is also available from any of the physics interfaces. You access it by right-clicking the main feature and then choosing **Global>Global Equations**. You can use it for ODEs, differential algebraic equations, purely algebraic equations and conditions, and transcendental equations, or to add degrees of freedom to a model using the introduced states. Possible uses include:

- Controllers
- Rigid-body mechanics
- Nonlinear eigenvalue problems
- Continuation
- Integral constraints
- Augmented or generalized equations

The ODEs and DAEs Interface

The ODEs and DAEs interface provides the possibility to add ODEs or other global equations to a model. The main feature is the **Global Equations** feature, where you define the global equations including the names of the variables (states), the required initial values, and an optional description. When you add the **Global Equations and ODEs** interface, it adds a default **Global Equations** feature. Right-click the **Global Equations and ODEs** node to add additional **Global Equations** feature or a **Weak Contribution** feature.

The following sections provide information about the **Global Equations** and **Weak Contributions** features.

Global Equations

To add a global (space-independent) equations such as ODEs, you add them in the **Global Equations** form, which contains the following section:

GLOBAL EQUATIONS

The global equations that you can solve have the following form:

$$f(u, \dot{u}, \ddot{u}, t) = 0$$

with the initial conditions $u(t_0) = u_0$ and $u_t(t_0) = u_{t,0}$ (where the subscript t indicates the time derivative). You can add several equations, and the equation can be coupled.

The first time derivative of u is specified as `ut`, and the second time derivative of u is `utt`. With time derivatives, this equation is an ODE (ordinary differential equation). With no time derivatives, the equation is an algebraic equation or a transcendental equation. The initial conditions are only necessary for ODEs.

In the table, each row corresponds to a named state, that is, a single degree of freedom and one equation. Fill in the name of the state variable in the **Name** column. This also defines time-derivative variables. If a state variable is called `u`, its first and second time derivatives are `ut` and `utt`, respectively. These variables become available in all geometries. Therefore the names must be unique.

Use the **Equation f(u,ut,utt,t)** column to specify the left side of the equation that is to be set equal to zero. The software then adds this global equation to the system of equations. When you solve the model, the value of the state variable `u` is adapted in such a way that the associated global equation is satisfied. You can use all state variables and their time derivatives as well as any parameters, global expression variables, and coupling operators with a scalar output and global destination in the **f(u,ut,utt,t)** column. Setting an equation for a state is optional. The default value of `0` means that the software does not add any additional condition to the model.

If the time derivative of a state variable appears somewhere in the model during a time-dependent solution, the state variable needs an initial condition. Models that contain second time derivatives also require an initial value for the first time derivatives of the state variables. You set these conditions in the third (**Initial value (u)**) and fourth

(**Initial value (ut)**) columns. The last column, **Description**, gives you the option to enter comments about the state or the equation.

To add another equation, click the **Add** button. To remove an equation, select some part of that equation's row in the table and click the **Remove** button.

Weak Contribution (ODEs and DAEs)

Another option is to enter equations in the weak form using the **Weak Contribution** feature. This can be convenient in advanced modeling because it gives you control over the test variables multiplying the equations. Wherever a test function of a state variable appears (in the **Global Equations** feature or elsewhere in the model), whatever it multiplies ends up in the same equation in the discrete system. Note that you can have zero or more weak expressions, regardless of the number of states.

WEAK CONTRIBUTION

You type the expression that contains the weak formulation in the **Weak expression** edit field. See Adding an ODE to a Boundary for an example of how to write an ODE using a weak formulation.

Discretization

By right-clicking the main physics interface node and choosing **Global>Discretization** you can add extra **Discretization** nodes for use in, for example, multigrid hierarchy. You can select which discretization to use in the **Multigrid Level** subnodes for the study steps.

The **Discretization** form contains a single **Discretization** section, which is identical to the **Discretization** section in the main physics interface node's **Settings** window.

Presenting Results for Global Equations

The state variables are scalar values and they are available globally. To view the results for an ODE, you can use the Line Graph, Point Graph, and Global plot types and Global Evaluation for displaying the numerical solution.

Solving ODEs

As an example of ODEs, the following equations are the *Lotka-Volterra equations*:

$$\dot{r} = ar - brf$$
$$\dot{f} = -cf + drf$$

where r is the rabbit population, and f is the population of foxes. This is an example of a system of two coupled ODEs.

You enter these equations as `rt-a*r+b*r*f` and `ft+c*f+d*r*f`, where a, b, c, and d are scalar values defined using the **Parameters** feature. For this first-order ODE, you need to specify initial values for r and f.

Solving Algebraic and Transcendental Equations

As an example of an algebraic equation, consider the equation $f(u) = 0$ for

$$f(u) = u^3 + u - 2$$

This equation has a single root at $u = 1$. To enter it into the **Global Equations** window, type `u` in the **Name** column and `u^3+u-2` in the **Equation f(u,ut,utt,t)** column (both entries on the same row). Then solve this using a stationary solver. To display the solution value, right-click **Numerical Results** in the **Results** branch and select **Global Evaluation**. In the **Settings** window, select **u** from the list of predefined quantities (click the down arrow in the **Expression** section and select **ODEs and DAEs>(u)**).Click the Evaluate button, and the value of the root appears in the **Results** section.

As an example of a transcendental equation, consider the equation $f(u) = 0$ for

$$f(u) = e^{-u} - u$$

A root to this equation is approximately 0.56714. To enter it into the **Global Equations** window, type `u` in the **Name** column and `exp(-u)-u` in the **Equation f(u,ut,utt,t)** column (both entries on the same row). Then compute the solution. To display the solution value, right-click **Numerical Results** in the **Results** branch and select **Global Evaluation**. In the **Settings** window, select **u** from the list of predefined quantities (click the down arrow in the **Expression** section and select **ODEs and DAEs>(u)**). Click the **Evaluate** button, and the value of the root (rounded to 0.567) appears in the **Results** section.

Adding an ODE to a Boundary

Sometimes a boundary equation is coupled to a PDE in an adjacent subdomain. For example, adding an ODE for v on a boundary such that

$$\frac{\partial v}{\partial t} = u$$

where u is the dependent variable in an adjacent subdomain. To implement an ODE on a boundary, do the following:

1. In the physics interface where u is defined, right-click the main feature and select **More>Weak Contribution** from the boundary feature section (you might need to select **Show More Options** from the **View** menu in the **Model Builder**).
2. Select the boundary where you want to add the ODE.
3. Type `(u-vt)*test(v)` in the **Weak expression** field. This is the weak formulation of the ODE.
4. Right-click the **Weak Contribution** node and select **Auxiliary Dependent Variable**.
5. In the **Auxiliary Dependent Variable** section, type `v` in the **Field variable name** edit field. You can also specify an initial value for the variable (the default initial value is 0). You do not need to make a selection of boundaries for this variable; by default, it uses the same selection as for the weak contribution.
6. When done, solve the model.

Also keep in mind that there must be no constraint (Dirichlet condition) for u on the boundaries where you defined the ODE.

The Wall Distance Interface

The **Wall Distance** interface ( ) provides the equations and boundary conditions for computing the distance to walls in fluid-flow simulation using a modified Eikonal equation, solving for a dependent variable G that is related to the wall distance. The main feature is the **Distance Equation** node, which adds the distance equation (modified Eikonal equation) and provides an interface for defining the reference length scale.

When you add the Wall Distance interface, it creates the **Wall Distance** node with a default **Distance Equation** node added as well as default boundary conditions, which is a homogeneous Neumann condition that does not appear in the Model Tree. Right-click the **Wall Distance** node to add boundary conditions for walls. There is also a default **Initial Values** node. The following sections provide information about all features available in the Wall Distance interface.

DOMAINS

Select the domains where you want to define the distance equation. The default setting is to include all domains in the model.

PHYSICAL MODEL

Enter a **Smoothing parameter** σ_w. in Equation 4-71. The default value is 0.1.

DEPENDENT VARIABLES

The dependent variable (field variable) is for the **Reciprocal wall distance** G. You can change the name in the corresponding edit field, but the names of fields and dependent variables must be unique within a model.

ADVANCED SETTINGS AND DISCRETIZATION

Normally these settings do not need to be changed. See Show More Options: Advanced Settings and Discretization.

Distance Equation

The **Distance Equation** node adds to Equation 4-71, and the **Distance Equation** form contains the following sections for defining the length scale:

DOMAINS

Select the domains where you want to use the distance equation with the length scale defined in this **Distance Equation** node.

DISTANCE EQUATION

Specify the length scale using the reference length l_{ref}. From the **Length scale specification** you can choose from two methods to compute the reference length:

- Select **Automatic** to use Equation to compute l_{ref}. This is the default setting.
- Select **Manual** to specify the reference length (SI unit: m) manually in the l_{ref} edit field. The default value is 1.

Initial Values

The **Initial Values** node adds an initial value for the reciprocal wall distance that can serve as initial guess for a nonlinear solver.

DOMAINS

Select the domains where you want to define an initial value.

INITIAL VALUES

Enter a value or expression for the initial value of the reciprocal wall distance G (SI unit: $1/\text{m}$) in the **Reciprocal wall distance** edit field. The default value is $G_0 = 2/l_{\text{ref}}$.

Boundary Conditions

The following boundary conditions are available:

- The default boundary condition is a homogeneous Neumann condition, which applies to all boundaries that are not walls:

$$\nabla G \cdot \mathbf{n} = 0$$

  This boundary condition does appear as a node in the model tree.
- The Wall boundary condition (see below) for boundaries that are walls.

For axisymmetric models, COMSOL Multiphysics automatically takes the axial symmetry boundaries (at $r = 0$) into account, and prohibits the wall boundary feature from being defined here.

Wall

The **Wall** node implements the following boundary condition for walls:

$$G = G_0 = \frac{2}{l_{\text{ref}}}$$

BOUNDARIES

Select the boundaries that represent walls.

Theory for the Wall Distance Interface

Turbulence models often use the distance to the closest wall to approximate the mixing length or for regularization purposes. One way to compute the wall distance is to solve the *Eikonal equation*:

$$|\nabla D| = 1 \tag{15-2}$$

with $D = 0$ on solid walls and $\nabla D \cdot \mathbf{n} = 0$ on other boundaries. The Eikonal equation can be tricky to solve, and produces the exact distance to the closest wall. The modified equation described below is less computationally expensive to solve. It also uses a reference length to put more emphasis on solid objects larger than the reference length and reduce the emphasis on smaller objects. The introduction of a reference length produces a more relevant wall distance in the following instances:

- In convex regions of small dimensions, the wall distance is reduced to reflect the close proximity of several walls.
- When a small object, such as a thin wire for example, is present in free flow, the wall distance is affected only in a very small region around it.

MODIFIED EIKONAL EQUATION

COMSOL Multiphysics uses a modified Eikonal equation based on the approach in Ref. . This modification changes the dependent variable from D to $G = 1/D$. Equation 15-2 then transforms to

$$\nabla G \cdot \nabla G = G^4 \tag{15-3}$$

Additionally, the modification adds some diffusion and multiplies G^4 by a factor to compensate for the diffusion. The result is the following equation, which the Wall Distance interface uses:

$$\nabla G \cdot \nabla G + \sigma_w G(\nabla \cdot \nabla G) = (1 + 2\sigma_w)G^4 \tag{15-4}$$

where σ is a small constant. If σ_w is less than 0.5, the maximum error falls of exponentially when σ_w tends to zero. The default value of 0.1 is a good choice for both linear and quadratic elements.

The boundary conditions for Equation 15-4 is $G = G_0 = C/l_{ref}$ on solid walls and homogeneous Neumann conditions on other boundaries. The effect of C is that the solution becomes less smeared the higher the value of C. The error tends asymptotically to $0.2l_{ref}$ as C tends to infinity, but making it very large destabilizes Equation 15-4. C is 2 in the Wall Distance interface.

For a channel, l_{ref} should typically be set to the channel width or there about. l_{ref} has a lower bound in that it must be larger than all cells adjacent to any boundary where the boundary condition $G = G_0$ is applied; otherwise, the solution displays oscillations. l_{ref} is the only parameter in the model, and the default value is half the shortest side of the geometry bounding box. If the geometry consists of several very slender entities, or if the geometry contains very fine details, this measure can be too large. You can then define l_{ref} manually.

The initial value is by default defined as $G_0 = 2/l_{ref}$, in correspondence with the boundary conditions.

The wall distance $D_w = 1/G - 1/G_0$ is a predefined variable that you can use for postprocessing. You have also access to a vector-valued variable that represents the direction toward the nearest wall, which is defined as

$$\frac{\nabla G}{\sqrt{\max(\nabla G^2, \text{eps})}} \tag{15-5}$$

REFERENCE FOR THE WALL DISTANCE INTERFACE

E. Fares and W. Schröder, “A differential equation for approximate wall distance,” *International Journal for Numerical Methods in Fluids*, vol. 39, pp. 743–762, 2002.

16

Sensitivity Analysis

This section describes how to perform sensitivity analysis using the Sensitivity interface. In this section:

About Sensitivity Analysis

Introduction

The Sensitivity interface is different from most other physics interfaces in that it does not contain any physics of its own. Instead, it is a tool that makes it possible to evaluate the sensitivity of a model with respect to almost any variable.

Simulation is a powerful tool in science and engineering for predicting the behavior of physical systems, particularly those that are governed by partial differential equations. However, often a single simulation (or just a few) is not enough to provide sufficient understanding of system. Hence, a more exploratory process might be needed. *Sensitivity analysis* is an example of such an exploratory process. Here one is interested in the sensitivity of a specific quantity with respect to variations in certain parameters included in the model. Such an analysis can for example be used for estimating modeling errors caused by uncertainties in material properties, or for predicting the effect of a geometrical change.

Many times it is possible to reformulate problems of the above type as the problem of calculating derivatives, so differentiation plays a central role in solving such problems. In general, you have a variable, the *sensitivity variable*, and an *objective function*. The task is then to evaluate the derivative of the objective function with respect to the sensitivity variable at some point. The sensitivity variable can be a scalar, a vector, or even an element in some infinite-dimensional function space.

Furthermore, in many cases the objective function only depends implicitly on the scientific variable, say by means of a multiphysics model. The Sensitivity interface provides a framework for performing sensitivity analysis in this aforementioned setting.

Sensitivity Problem Formulation

Because the Sensitivity interface does not contain any physics, it is not intended for use on its own. When you add the Sensitivity interface to a multiphysics model, no new equations are introduced, and the set of solution variables remains the same. Instead, you have the possibility to specify an objective function and a sensitivity variable. To this end, the physics interface lets you perform three distinct tasks:

- Select a sensitivity variable and set its value

- Define a scalar objective function
- Compute the sensitivities efficiently using the sensitivity solver

---

Note: Note that the sensitivity variable is on one hand an independent variable whose value is not affected by the solution process; on the other hand, it has degrees of freedom (DOFs) stored in the solution vector. When defining a sensitivity variable, you must therefore supply its *initial value*, which the software uses to initialize the sensitivity variable DOFs; these remain fixed during the solution step.

Sensitivity Analysis in the General Case

Analysis of the sensitivity of an objective function with respect to some sensitivity variable at a specific point can be rephrased as the problem of calculating the derivative $Q(\xi_0)$ where ξ_0 denotes the value of sensitivity variable ξ and Q is the scalar-valued objective function.

Often Q is *not* given solely in terms of explicit expressions in the sensitivity variable ξ. Such sensitivity analysis problems arise especially when the response of the system against variations in the sensitivity variable is modeled by (partial) differential equations, thereby precluding the possibility to use explicit closed-form expression for the objective function and constraints. In such cases it is valuable to single out the parts of the objective function Q that are given implicitly from those that are explicit in ξ. To do this, introduce an *auxiliary variable* u, which again can be scalar or vector-valued, encoding the implicit relation to ξ. Now, the objective function Q is explicitly expressible in terms of u and ξ and the resulting sensitivity analysis boils down to evaluating the derivative of $Q(u, \xi)$ with respect to u and ξ.

In many cases, the implicit relation of u to ξ is given as by a system of equations $L(u, \xi) = 0$. Furthermore, often this system of equations is given as a system of differential equations which in turn are defined through the general multiphysics model used for simulating the physical system in question. Thus, to perform sensitivity analysis in this setting one is required to calculate the derivative of $Q(u(\xi), \xi)$ with respect to ξ.

With the Sensitivity interface you can specify the objective function in terms of expressions that are explicit in ξ and u. The relation between u and ξ, which is a system of equations written here compactly as $L(u, \xi) = 0$, is given by the multiphysics model that you define. Furthermore, you can use the Sensitivity interface to let each

component of the vectors ξ and u be either globally defined or defined only on a specific domain.

Specification of the Objective Function

Each equation in the system $L(u, \xi) = 0$ can be defined either globally or over a specific domain, which in turn is specified during the multiphysics modeling phase. It therefore makes sense to group all equations and expressions according to the dimension of their corresponding domains.

This previously mentioned principle yields the following representation for objective function Q:

$$Q(u, \xi) = Q_{\text{global}}(u, \xi) + Q_{\text{probe}}(u, \xi) + \sum_{k=0}^{n} Q_{\text{int},k}(u, \xi)$$

where n is the space dimension of the multiphysics model and the various terms in the sum above are defined as follows:

- Q_{global} is the *global contribution* to the objective function Q. It is given as a closed form expression in those components of ξ and u that are global.
- Q_{probe} is a *probe contribution* to the objective function Q. It is a *probe objective* so its definition is restricted to a domain (that is, an n-dimensional set) and it is given as a point evaluation of a closed form expression in those components of ξ and u that are either are either global or available in the domain in question. The *probe point* used for the point evaluation is a point given by the user and has to be contained in the domain.
- $Q_{int,k}$ is an *integral contribution* to the objective function Q. It is an *integral objective* so its definition is restricted to a specific k-dimensional domain where k is between 0 and n. It is given as the integral of a closed form expression in those components of ξ and u that are either global or available in the domain in question. For the case $k = 0$, the integration reduces to a summation.

The user can define several global, probe, and integral contributions. In such case, the total global, probe, and integral contribution is given as the sum of the aforementioned global, probe, and integral contributions that you actively select in the solver feature settings for the optimization.

Methods for Performing Sensitivity Analysis

To perform a sensitivity analysis requires that it is possible to evaluate derivatives of $Q(u(\xi), \xi)$ with respect to ξ at a specific point ξ_0 and under the constraint $L(u, \xi) = 0$. The central difficulty is in the calculation of the derivative of the auxiliary variable u with respect to the sensitivity variable ξ. This is the general form of the sensitivity analysis considered in the Sensitivity interface.

To evaluate the above derivative, an auxiliary linear problem is solved, in addition to the original equation $L(u, \xi) = 0$. Here you can choose in-between two methods:

- Select the *forward sensitivity* method to evaluate the derivatives of all solution variables and an optional objective function.
- Select the *adjoint sensitivity* method to look only at derivatives of a scalar objective function.

Both methods have their benefits as are explained below.

FORWARD SENSITIVITY

Use the forward sensitivity method to solve for the derivatives of all dependent variables, plus an optional scalar objective function, with respect to a small number of sensitivity variables. The forward method requires one extra linear system solution for each sensitivity variable.

The linear system that must be solved is the same as the last linearization needed for solving the forward model. Thus, when using a direct solver (for example, PARDISO) the extra work amounts only to one back-substitution per sensitivity variable DOF. The iterative linear and segregated solvers can reuse preconditioners and other data but must otherwise perform a complete solution each time.

ADJOINT SENSITIVITY

The adjoint method solves for the derivatives of a single scalar objective function with respect to any number of sensitivity variables, requiring only one single additional linear system solution. In addition to the objective function gradient, the discrete adjoint solution is computed. This quantity represents the sensitivity of the objective function with respect to an additional generalized force applied as a nodal force to the corresponding solution component.

The auxiliary linear system is in this case the transpose of the last linearization needed for solving the forward model. The UMFPACK linear solver can always solve the transposed problem at the cost of a back-substitution, while remaining direct solvers

need to do a new factorization if the problem is not symmetric or Hermitian. The iterative solvers can reuse most preconditioning information as can the segregated solver, which, however, loops over the segregated steps in reversed order.

Note: Sensitivity Analysis can be used together with all stationary and parametric standard solvers. The available solver settings are described in Solving. For technical details about the solution procedure, see the *COMSOL Multiphysics Reference Guide* (or see Where Do I Access the Documentation and Model Library?).

Issues to Consider Regarding the Sensitivity Variable

THE EFFECT OF DISCRETIZATION

The sensitivity analysis is always performed on the discretized system of equations. As already mentioned, the sensitivity variable can be a scalar/vector or an element in some infinite-dimensional function space. In the latter case it is represented on the finite element mesh, just like the solution variables, or global scalar quantities. When using a sensitivity variable field represented on the finite element mesh, the sensitivities are therefore associated with individual sensitivity variable degrees of freedom rather than with the field value at each point. This makes it difficult to interpret the result. For example, if you set up a subdomain sensitivity variable using a 1st-order Lagrange shape function representation to control the material density in your model, your solution contains the sensitivity of the objective function with respect to the discrete density value at each node point in the mesh. Because each node influences the density in a small surrounding region, the size of which varies from node to node, the individual sensitivities are not directly comparable to each other.

Displaying such subdomain sensitivity variables results in a plot that is not smooth due to the varying element size. It must therefore not be used to draw any conclusions about the physics and the effect of changing the physical field represented by the sensitivity variable. Some insight may, however, be gained by looking at the sensitivities divided by the mesh volume scale factor `dvol`. This makes the sensitivities in the plot comparable between different parts of the surface, but still not mathematically well defined. In particular, using discontinuous constant shape functions together with the division by `dvol` gives you a plot which is proportional to the true pointwise sensitivity.

Note: If you intend to use the sensitivities in an automatic optimization procedure, as is done through the Optimization interface available with the optional COMSOL Optimization Module, the discrete nature of the sensitivities causes no additional complication. The optimization solver searches for optimum values of the discrete sensitivity variables using the discrete gradient provided by the sensitivity analysis.

GEOMETRICAL SENSITIVITY

You can use the sensitivity variables directly to parameterize any aspect of the physics which is controlled by an expression. This applies for example to material properties, boundary conditions, loads and sources. However, the shape, size, and position of parts of the geometry cannot be changed as easily at solution time, and therefore require special attention.

Sensitivity variables cannot be used directly in the geometry description. Instead, you must set up your model using *ALE* and tie all physics to an *ALE frame* controlled by a Deformed Geometry physics interface (in 2D only) or a Moving Mesh (ALE) physics interface. You can then use sensitivity variables to control the mesh movement, effectively parameterizing the geometry.

See Deformed Meshes for details on the Deformed Geometry and Moving Mesh (ALE) physics interfaces as well as on ALE in general.

Issues to Consider Regarding the Objective Function

THE PRINCIPLE OF VIRTUAL WORK

Potential energy has a special status among scalar objective functions, because its derivatives with respect to scalar sensitivity variables can in many cases be interpreted as (true or generalized) forces.

COMPLEX-VALUED OBJECTIVE FUNCTIONS

Sensitivity analysis can be applied only when the objective function is a real-valued differentiable function of the sensitivity variables. This is usually not a very severe constraint, even for frequency-domain models where the solution variables are complex valued. The reason is that physical quantities of interest to the analyst are always real valued, and if complex-valued sensitivity variables are required, it is possible to treat the real and imaginary parts separately.

Many common quantities of interest are time averages which can be written in the form $Q = \text{real}(a \cdot \text{conj}(b))$, where a and b are complex-valued linear functions of the solution variables and therefore implicit functions of the sensitivity variables. The problem with this expression is that while Q is indeed a real-valued differentiable function of the sensitivity variables, it is not an analytical function of a and b. This complicates matters slightly because the sensitivity solver relies on partial differentiation and the chain rule.

While the partial derivatives of Q with respect to a and b are, strictly speaking, undefined, it can be proven that if they are chosen such that

$$Q(a + \delta a, b + \delta b) \approx Q(a, b) + \text{real}\left(\frac{\partial Q}{\partial a}\delta a + \frac{\partial Q}{\partial b}\delta b\right) \quad (16\text{-}1)$$

for any small complex increments δa and δb, the final sensitivities are evaluated correctly. The special function `realdot(a,b)` is identical to `real(a*conj(b))` when evaluated but implements partial derivatives according to Equation 16-1. For that reason, use it in the definition of any time-average quantity set as objective function in a sensitivity analysis.

Adding Sensitivity

You can add a **Sensitivity** interface ( ) when creating a new model or at any time during modeling. For a new model, you select physics interfaces as the second step in the **Model Wizard** (after specifying the space dimension). In an active model, right-click the model node in the Model Tree and choose **Add Physics**. In both cases, the **Add Physics** page appears with a list of physics interfaces and mathematical interfaces. Take the following steps to add a Sensitivity interface to the model:

1. Expand the **Mathematics>Optimization and Sensitivity** node in the list of physics interfaces.
2. Select **Sensitivity**.
3. Click the **Add Selected** button ( ) underneath the list to add the selected physics interface to the model. The physics interface then appears in the list under **Selected physics**.
4. When you are ready click the **Next** button ( ) in the upper-right corner of the **Model Wizard** window.
5. Optionally, choose an study type for the sensitivity analysis on the **Select Study Type** page.

6 Click the **Finish** button ( ) in the upper-right corner of the **Model Wizard** window.

The Sensitivity Interface

The **Sensitivity** interface ( ) provides the tools for defining and solving sensitivity problems. For a more extensive introduction to the mathematics implemented by this interface, see Sensitivity Analysis in the General Case.

You can define objective functions and constraints in terms of components of the control and auxiliary variable (the latter is given as the solution to the differential equations defined by the multiphysics model) and restrict these to specific domains or make them globally available. This flexibility is reflected in the user interface by grouping these settings according to the dimension of the domain to which they apply. In such group of settings you have the possibility to specify the following settings, to each of which corresponds a separate feature and its **Settings** window:

- Integral Objective
- Probe Objective
- Control Variable Field

INTERFACE IDENTIFIER

This is the identifier for the physics interface, which you use to reach the fields and variables in expressions, for example.

The identifier appears in the **Identifier** edit field, and you can change it to any unique string that is a valid identifier. The default identifier (for the first Sensitivity interface in the model) is `sens`.

DOMAINS

In this section you select the domains containing the control variables and expressions that enter into the formulation of the optimization problem. The default setting is to include all domains in the model. For information about selecting domains, see Selecting and Deselecting Geometric Entities.

Integral Objective

An integral objective is defined as the integral of a closed form expression of those components of the control and auxiliary variable (the latter is given as the solution to the differential equations defined by the multiphysics model) that are either global or available in the domain in question. Hence, its definition is restricted to a specific

k-dimensional domain where k is between 0 and n (n denoting the dimension of the multiphysics model). For the case $k = 0$, the integration reduces to a summation.

DOMAINS

In this section you select the domain used in the integration for the integral objective.

OBJECTIVE

In this section you specify the expression that is integrated over the domain in the integral objective.

QUADRATURE SETTINGS

In this section you can specify the settings for the quadrature used to numerically evaluate the integral in the integral objective.

Probe Objective

A probe objective is defined as a point evaluation of a closed form expression of the components of the control and auxiliary variable (the latter is given as the solution to the differential equations defined by the multiphysics model) that are either global or available in the domain in question. The point used for the point evaluation has to be contained in the domain.

DOMAINS

In this section you select the domain containing the point used for the point evaluation.

OBJECTIVE

In this section you specify the expression that is evaluated at the point in the domain.

PROBE COORDINATES

Here you specify the probe coordinates, that is the coordinates for the point in the domain where the expression for the objective is evaluated.

After specifying the probe coordinates you select the frame in which the evaluations take place from the **Evaluate in frame** list. The default setting **Spatial**.

Control Variable Field

Here you specify the control variables specific for the domain in question.

DOMAINS

In this section you select the domain where the control variable is defined.

CONTROL VARIABLE

In this section you start out by providing the name and initial value of the control variable.

Next, you can specify the order of the shape functions used to discretize the control variable. The default is to use second-order Lagrange elements.

After specifying the order of the shape functions you select the frame in which the derivatives are to be evaluated from the **Define derivatives with respect to frame** list. The default setting **Spatial**.

Global Objective

Here you specify the global contribution to the objective function.

OBJECTIVE

In this section you specify the expression that defines the contribution to the objective function. It can be an expression of those components of the control and auxiliary variable (the latter is given as the solution to the differential equations defined by the multiphysics model) that are globally available.

Global Control Variables

Here you specify those components of the control variable that are globally available.

CONTROL VARIABLES

In the table that follows you list the names, initial values, upper bounds and lower bounds of those components of the control variable that are globally available. To specify equality constraints, simply make sure the upper and lower bounds have the same value.

17

Optimization

With a license for the Optimization Module you get access to the Optimization interface in COMSOL Multiphysics. The Optimization interface is designed to facilitate setting up and solving optimization problems.

In this section:

- Overview of Optimization provides a quick introduction to the theory and terminology of optimization.
- The Optimization Interface describes all available features and their components in the COMSOL Desktop.

Note: The Optimization interface requires the Optimization Module.

Overview of Optimization

Introduction

Simulation is a powerful tool in science and engineering for predicting the behavior of physical systems, particularly those governed by partial differential equations. In many cases a single or a few simulations are not enough to provide sufficient understanding of a system. Two important classes of problems whose resolution relies on a more systematic exploratory process are:

- *Design problems* with a single objective. Here, the problem is to find the values of some decision variables that yield the best performance of the output of a simulation model when the latter is quantified by means of a single function. Problems of this kind arise in structural optimization, antenna design, and process optimization.
- *Inverse problems*, and in particular *parameter estimation in PDEs*. Here, the problem is to reliably determine the values of a set of parameters that provide simulated data which best matches measured data. Such problems arise in geophysical imaging, nondestructive testing, biomedical imaging, and weather data assimilations.

It is often possible to reformulate problems of the above type as *optimization problems*. The Optimization interface in COMSOL Multiphysics is useful for solving design problems as well as inverse problems and parameter estimation.

Basic Optimization Concepts

In general there are two fundamental parts of an optimization problem:

- The *control variable*
- The *objective function*

The task is now to find the value of the control variable that minimizes (or maximizes) the objective function subject to a number of constraints. The constraints collectively define a set, *feasible set*, of permissible values for the control variable.

The control variable can be a scalar, a vector, or even an element in some infinite-dimensional function space. In the mathematical and engineering literature, the control variable is sometimes also referred to as the *optimization variable* or *decision variable*. Alternative, and frequently occurring, terminology used for the objective function is *cost function*, *goal function*, or *quantity of interest*.

For design problems, the objective function is the aforementioned function quantifying the performance and the control variable is for such problems referred to as the decision variables. For parameter estimation in PDEs, the objective function defines a robust quantifier for how well simulated data matches measured data and the optimization variables are the parameters that are to be determined. The control variable is the unknown parameter in the parameter estimation problem that is to be recovered.

The Optimization interface provides a framework for specifying and solving optimization problems. The objective function and constraints can depend implicitly on the control variable by means of multiphysics models. For more information about the Optimization interfaces and its features, see The Optimization Interface.

Optimization Problem Formulation

The Optimization interface is built around a general formulation of a minimization problem (to perform maximization, simply minimize the negative of the objective function). You specify the objective function that is to be minimized and the constraints that apply on the control variable.

THE GENERAL OPTIMIZATION PROBLEM

The general formulation of an optimization problem can be written as

$$\begin{cases} \min_{\xi} Q(\xi) \\ \xi \in C \end{cases} \tag{17-1}$$

Here, the control variable is denoted by ξ, the scalar valued objective function by Q, and the feasible set, denoted by C, is assumed to be given by means of inequality constraints

$$C = \{\xi : \mathrm{lb} \le G(\xi) \le \mathrm{ub}\}$$

where G is a vector-valued function (G is scalar-valued in case of a single constraint).

Note: For vectorial quantities, the inequality defining C is to be interpreted component-wise and lb and ub are the corresponding vectors containing the upper and lower bounds.

CLASSICAL OPTIMIZATION

In *classical optimization*, Q and G are given explicitly as closed-form expressions of the control variable ξ. However, design and parameter estimation problems mentioned before often result in objective functions Q and constraints G that are not explicitly expressible by closed-form expressions of the control variable ξ.

SIMULATION OPTIMIZATION

Simulation optimization problems refer to the case where Q or G are *not* given solely in terms of explicit expressions in the control variable ξ. Such optimization problems arise especially when the response of the system against variations in the control variable is modeled by (partial) differential equations, thereby precluding the possibility to use explicit closed-form expression for the objective function and constraints. In such cases it is valuable to single out the parts of the objective function Q and constraints G that are given implicitly from those that are explicit in ξ. To do this, introduce an *auxiliary variable* u, which again can be scalar or vector-valued, encoding the implicit relation to ξ. Now, the objective function Q and constraints G are explicitly expressible in terms of u and ξ and the simulation optimization problem can be rephrased as classical optimization problem where the control variable is (u, ξ). In many cases, the implicit relation of u to ξ is given as by a system of equations $L(u, \xi) = 0$.

PDE-CONSTRAINED OPTIMIZATION

The system of equations $L(u, \xi) = 0$ is often given as a system of differential equations, which in turn are defined through the general multiphysics model used for simulating the physical system in question. In such case, the simulation optimization problem is often referred to as *PDE-constrained optimization*.

The Optimization interface provides tools to define and solve general PDE-constrained optimization problems that are given as

$$\begin{cases} \min\limits_{\xi} \; Q(u(\xi), \xi) \\ L(u(\xi), \xi) = 0 \\ \mathrm{lb} \le G(u(\xi), \xi) \le \mathrm{ub} \end{cases} \tag{17-2}$$

It is advantageous to single out those constraints given by G that are defined as explicit expressions of ξ only, and those that mix u and ξ. Hence, the general constraint formulation $\mathrm{lb} \le G(u(\xi), \xi) \le \mathrm{ub}$ above is replaced by two classes of constraints:

$$\mathrm{lb}_P \le P(\xi, u) \le \mathrm{ub}_P$$
$$\mathrm{lb}_\Psi \le \Psi(\xi) \le \mathrm{ub}_\Psi$$

and the optimization problem in Equation 17-2 can be written as

$$\begin{cases} \min\limits_{\xi} \quad Q(u(\xi), \xi) \\ L(u(\xi), \xi) = 0 \\ \mathrm{lb}_P \le P(u(\xi), \xi) \le \mathrm{ub}_P \\ \mathrm{lb}_\Psi \le \Psi(\xi) \le \mathrm{ub}_\Psi \end{cases} \tag{17-3}$$

This is the general form of the optimization problem considered in the Optimization interface. With the Optimization interface you can specify the objective function and the constraints in terms of expressions that are explicit in ξ and u. The relation between u and ξ, which is a system of equations written here compactly as $L(u, \xi) = 0$, is given by the multiphysics model that you define. Furthermore, you can use the Optimization interface to let each component of the vectors ξ and u be either globally defined or defined only on a specific domain.

SPECIFICATION OF THE OBJECTIVE FUNCTION

Each equation in the system $L(u, \xi) = 0$ can be defined either globally or over a specific domain, which in turn is specified during the multiphysics modeling phase. It therefore makes sense to group all equations and expressions according to the dimension of their corresponding domains.

This previously mentioned principle yields the following representation for objective function Q:

$$Q(u, \xi) = Q_{\mathrm{global}}(u, \xi) + Q_{\mathrm{probe}}(u, \xi) + \sum_{k=0}^{n} Q_{\mathrm{int},\,k}(u, \xi)$$

where n is the space dimension of the multiphysics model and the various terms in the sum above are defined as follows:

- Q_{global} is the *global contribution* to the objective function Q. It is given as a closed-form expression in those components of ξ and u that are global.
- Q_{probe} is a *probe contribution* to the objective function Q. It is a *probe objective* so its definition is restricted to a domain (that is, an n-dimensional set) and it is given as a point evaluation of a closed-form expression in those components of ξ and u that are either are either global or available in the domain in question. The *probe*

point used for the point evaluation is a point given by the user and has to be contained in the domain.

- $Q_{\text{int},k}$ is an *integral contribution* to the objective function Q. It is an *integral objective* so its definition is restricted to a specific k-dimensional domain where k is between 0 and n. It is given as the integral of a closed-form expression in those components of ξ and u that are either global or available in the domain in question. For the case $k = 0$, the integration reduces to a summation.

It is possible to define several global, probe, and integral contributions. In such case, the total global, probe, and integral contribution is the sum of the aforementioned global, probe, and integral contributions that you actively select in the solver settings for the optimization.

SPECIFICATION OF THE CONSTRAINTS

As already mentioned, the constraints $\text{lb} \le G(u(\xi), \xi) \le \text{ub}$ in the general PDE-constrained optimization problem in Equation 17-2 are written as

$$\begin{aligned} \text{lb}_P &\le P(\xi, u) \le \text{ub}_P \\ \text{lb}_\Psi &\le \Psi(\xi) \le \text{ub}_\Psi \end{aligned}$$

The first row above constitutes the *implicit constraints*, which are given in terms of expressions involving both the auxiliary variable u and control variable ξ. The second row constitutes the *explicit constraints* that are those constraints given by explicit expressions only in the control variable ξ.

Note: The motivation for this subdivision is computational. To properly account for implicit constraints within an optimization scheme is computationally expensive whereas the explicit constraints are much easier. As an example, in gradient-based optimization methods the successive iterates in the optimization depend on the sensitivities of the auxiliary variable u with respect to the control variable ξ. To calculate this sensitivity is computationally demanding, see Methods for Performing Sensitivity Analysis.

Furthermore, the Optimization interface differs between thee following constraints (in the description that follows, n denotes the dimension of the multiphysics model):

bound constraints, *pointwise inequality constraints*, and *integral inequality constraints*, each of which are described below

- *Bound constraints*, also called *control variable bounds*, are constraints defined for a specific k-dimensional domain where k is between 0 and n. A bound constraint is a restriction to the values of the control variable at all points in the k-dimensional domain. Hence, bound constraints correspond to constraints of the form $\mathrm{lb} \leq \xi \leq \mathrm{ub}$ where the constraint is to be interpreted component-wise for vectorial quantities.
- *Pointwise inequality constraints* are restrictions to the values of a closed-form expression at *all* points in a specific k-dimensional domain where k is between 0 and n. The expression is a closed-form expression in those components of ξ and u that are either global or available in the k-dimensional domain in question.
- *Integral inequality constraints* are restrictions to the values of the integral of a closed form expression taken over a specific k-dimensional domain where k is between 0 and n. The expression is a closed form expression in those components of ξ and u that are either global or available in the domain in question. For the case $k = 0$, the integration reduces to a summation.

Note: The pointwise inequality constraint is actually one constraint for each point in the k-dimensional domain. The exact finite ensemble of points used to represent the continuum of such points in the k-dimensional domain is given by the node points. In almost all cases of interest there are a large number of node points, so a pointwise inequality constraint results in a large number of inequality constraints. Implicit constraints are computationally very expensive to account for, so even though the Optimization interface makes it possible to specify pointwise inequality constraints that are implicit, and even though such constraints do make sense from a mathematical (and perhaps also physics) point of view, in the actual computations *ignore such constraints.*

Methods for Solving Classical Optimization Problems

In the previous section the focus was on specifying *which* optimization problem to solve rather than *how* to solve it. This section describes the methods for how to solve the optimization problem.

THE GENERAL PDE-CONSTRAINED OPTIMIZATION PROBLEM

The approach taken by the Optimization interface in solving a general PDE-constrained optimization problem can be subdivided into two main steps.

1. Recast the PDE-constrained optimization problem into a classical optimization problem.
2. Solve the classical optimization problem.

The general PDE-constrained optimization problem that is to be solved is reformulated as a classical optimization problem, which in general form can be formulated as in Equation 17-3. The focus here is on how to perform (and configure) the second step.

GRADIENT-BASED METHODS

Solving the equation $L(u, \xi) = 0$ in the optimization problem Equation 17-3, which relates the auxiliary variable u to the control variable ξ, can be quite costly, which in turn implies that evaluation of the objective and implicit constraints can be costly. Therefore, it is advantageous to select an algorithm for solving that minimizes the number of such evaluations.

It is well-known that *gradient-based methods* are the preferred methods in cases when the objective and constraints are expensive to evaluate because such methods often require a minimum number of evaluations of the objective function and constraints. However, usage of gradient-based methods requires that the objective and constraints have certain smoothness properties that are stated below:

- The equation $L(u, \xi) = 0$ has a unique solution for all u in the relevant domain.
- The objective and constraints are twice continuously differentiable with respect to both ξ and u on an open set.
- The inverse $\xi \rightarrow \nabla_u L(u, \xi)$ exists for all (u, ξ) in an open set.

A central part in all gradient-based methods is the accurate and efficient calculation of the derivative of the auxiliary variable u with respect to the control variable ξ when they are related to each other by a system of differential equations $L(u, \xi) = 0$. This highly technical and nontrivial issue is automatically taken care of by the Optimization interface and is described in Methods for Solving Classical Optimization Problems.

The Optimization interface calls an external optimization software module, SNOPT (Sparse Nonlinear OPTimizer), for solving Equation 17-3. Below is a brief description of how SNOPT works.

SQP METHODS AND SNOPT

Sequential quadratic programming (SQP) methods have proved highly effective for solving constrained classical optimization problems with smooth nonlinear functions in the objective and constraints.

The idea of SQP methods is to solve the nonlinearly constrained problem, such as the one in Equation 17-3, using a sequence of quadratic programming (QP) subproblems. The constraints of each QP subproblem are linearizations of the constraints in the original problem, and the objective function of the subproblem is a quadratic approximation to the Lagrangian function of Equation 17-3:

$$\Lambda(\xi, \lambda, \mu) = Q(u(\xi), \xi) + \lambda \cdot [P(u(\xi), \xi), \psi(\xi)] + \mu \cdot L(u(\xi), \xi)$$

Note that the dimension of the vector λ equals the number of user defined constraints. The dimension of the vector μ equals the dimension of the solution vector u to the system of differential equations $L(u, \xi) = 0$ given by the Multiphysics model.

SNOPT is the implementation of a particular SQP algorithm that exploits sparsity in the constraint Jacobian and maintains a limited-memory quasi-Newton approximation to the Hessian of the above Lagrangian. A new method is used to update Hessian in the presence of negative curvature which is the case whenever Equation 17-3 is nonconvex. The QP subproblems are solved using an inertia-controlling reduced-Hessian active-set method (SQOPT) that allows for variables appearing linearly in the objective and constraint functions (in which case the limited-memory Hessian is semi-definite). Other features include the treatment of infeasible nonlinear constraints using elastic programming, use of a well-conditioned nonorthogonal basis for the null-space of the QP working set (assisted by sparse rank-revealing LU factors), early termination of the QP subproblems, and whenever necessary, finite-difference estimates of missing gradients. The method used by the QP solver SQOPT is based on solving a sequence of linear systems involving a reduced Hessian based on a sparse LU factorization. See References for a more detailed description of SNOPT.

REFERENCES

1. P.E. Gill, W. Murray, and M.A. Saunders, *User's Guide for SNOPT Version 7:* Software for Large-Scale Nonlinear Programming, Systems Optimization Laboratory (SOL), Stanford University, 2006.

2. P.E. Gill, W. Murray, and M.A. Saunders, *SNOPT: An SQP Algorithm for Large-Scale Constrained Optimization*, SIAM Review, vol. 47, no. 1, pp. 99–13, 2005.

The Optimization Interface

Adding Optimization to a Model

You can add a **Optimization** interface when creating a new model or at any time during modeling. For a new model, you select physics interfaces as the second step in the **Model Wizard** (after specifying the space dimension). In an active model, right-click the **Physics** node in the Model Tree and choose **Add Physics**. In both cases, the **Physics Interfaces** list appears. Take the following steps to add an Optimization interface to the model:

1. Expand the **Mathematics>Optimization and Sensitivity** node in the list of physics interfaces.
2. Select **Optimization**.
3. Click the **Add Selected** button ( ) underneath the list to add the selected physics interface to the model. The physics interface then appears in the list under **Selected physics**.
4. When you are ready click the **Next** button ( ) in the upper-right corner of the **Model Wizard** window.
5. Optionally, choose an study type for the optimization on the **Select Study Type** page.
6. Click the **Finish** button ( ) in the upper-right corner of the **Model Wizard** window.

About the Optimization Interface

The **Optimization** interface ( ) provides the tools for defining and solving optimization problems. The main purpose of the interface is its ability solve PDE-constrained optimization problems. For a more extensive introduction to the mathematics implemented by this interface, see Optimization Problem Formulation.

You can define objective functions and constraints in terms of components of the control and auxiliary variable (the latter is given as the solution to the differential equations defined by the multiphysics model) and restrict these to specific domains or make them globally available. This flexibility is reflected in the user interface by grouping these settings according to the dimension of the domain to which they apply. In such group of settings you have the possibility to specify the following settings, to each of which corresponds a separate feature and its **Settings** window:

- Integral Objective

- Probe Objective
- Integral Inequality Constraint
- Pointwise Inequality Constraint
- Control Variable Field (which includes the settings for the associated bound constraints)

INTERFACE IDENTIFIER

This is the identifier for the physics interface, which you use to reach the fields and variables in expressions, for example.

The identifier appears in the **Identifier** edit field, and you can change it to any unique string that is a valid identifier. The default identifier (for the first Optimization interface in the model) is `opt`.

DOMAINS

In this section you select the domains containing the control variables and expressions that enter into the formulation of the optimization problem. The default setting is to include all domains in the model.

Integral Objective

An integral objective is defined as the integral of a closed form expression of those components of the control and auxiliary variable (the latter is given as the solution to the differential equations defined by the multiphysics model) that are either global or available in the domain in question. Hence, its definition is restricted to a specific k-dimensional domain where k is between 0 and n (n denoting the dimension of the multiphysics model). For the case $k = 0$, the integration reduces to a summation.

DOMAINS

Selection

In this section you select the domain used in the integration for the integral objective.

OBJECTIVE

Objective Expression

In this section you specify the expression that is integrated over the domain in the integral objective.

QUADRATURE SETTINGS

In this section you can specify the settings for the quadrature used to numerically evaluate the integral in the integral objective.

Probe Objective

A probe objective is defined as a point evaluation of a closed form expression of the components of the control and auxiliary variable (the latter is given as the solution to the differential equations defined by the multiphysics model) that are either global or available in the domain in question. The point used for the point evaluation has to be contained in the domain.

DOMAINS

Selection

Select the domain containing the point used for the point evaluation.

OBJECTIVE

Objective Expression

Specify the expression that is evaluated at the point in the domain.

PROBE COORDINATES

Probe Coordinates

Specify the probe coordinates, that is the coordinates for the point in the domain where the expression for the objective is evaluated.

Evaluate in Frame

After specifying the probe coordinates you select the frame in which the evaluations take place from the **Evaluate in frame** list. The default setting **Spatial**.

Integral Inequality Constraints

Integral inequality constraints are given as restrictions to the values of the integral of a closed form expression taken over a specific k-dimensional domain where k is between 0 and n (n denoting the dimension of the multiphysics model). The expression is a closed form expression of those components of the control and auxiliary variable (the latter is given as the solution to the differential equations defined by the multiphysics model) that are either global or available in the domain in question. For the case $k = 0$, the integration reduces to a summation.

DOMAINS

Selection
In this section you select the domain used in the integration for the integral objective.

CONSTRAINT

Constraint expression
In the **Constraint expression** edit field you specify the expression that is integrated over the domain in the integral inequality constraint.

QUADRATURE SETTINGS

In this section you can specify the settings for the quadrature used to numerically evaluate the integral in the integral objective.

BOUNDS

In this section you can specify the upper and lower bounds for the integral inequality constraint. To specify equality constraints, simply make sure the upper and lower bounds have the same value.

Lower bound
Select the **Lower bound** check box to activate and provide the lower bound.

Upper bound
Select the **Upper bound** check box to activate and provide the upper bound.

Pointwise Inequality Constraints

A pointwise inequality constraint is given as a restriction to the values of a closed form expression at *all* points in a specific k-dimensional domain where k is between 0 and n. Due to computational issues, the expression has to be a closed form expression of *only* the control variable. Furthermore, only those components of the control variable that are either global or available in the domain in question are usable.

DOMAINS

Selection
In this section you select the domain over which the point inequality constraint is defined.

CONSTRAINT

Constraint Expression
In this section you specify the expression whose values at all points in the domain are to be constrained.

SHAPE FUNCTION

Shape Function
In this section you can specify the order of the shape functions used to discretize the control variable. The default is to use second-order Lagrange elements.

Define Derivatives With Respect To Frame
Here you select the frame in which the derivatives are to be evaluated from the **Define derivatives with respect to frame** list. The default setting **Spatial**.

BOUNDS

In this section you can specify the upper and lower bounds for the pointwise inequality constraint. To specify equality constraints, simply make sure the upper and lower bounds have the same value.

Lower Bound
Select the **Lower bound** check box to activate and provide the lower bound.

Upper Bound
Select the **Upper bound** check box to activate and provide the upper bound.

Control Variable Field

Here you specify the control variables specific for the domain in question.

DOMAINS

Selection
In this section you select the domain where the control variable is defined.

CONTROL VARIABLE

In this section you specify the control variables.

Control Variable Name
Provide the name of the control variable.

Initial Value
Provide the initial value of the control variable.

Shape Function
Specify the order of the shape functions used to discretize the control variable. The default is to use second-order Lagrange elements.

Define Derivatives with Respect to Frame
Select the frame in which the derivatives are to be evaluated from the **Define derivatives with respect to frame** list. The default setting is **Spatial**.

Control Variable Bounds

This feature is an attribute of the control variable feature and allows the user to specify the bounds for the bound constraint applied to the control variable in question.

DOMAINS

Selection
In this section you can further restrict the domain where the control variable is defined.

BOUNDS
In this section you can specify the upper and lower bounds for the bound constraint applied to the control variable in question. To specify equality constraints, simply make sure the upper and lower bounds have the same value.

Lower bound
Select the **Lower bound** check box to activate and provide the lower bound.

Upper bound
Select the **Upper bound** check box to activate and provide the upper bound.

Global Objective

Here you specify the global contribution to the objective function.

OBJECTIVE

Objective Expression
In this section you specify the expression that defines the contribution to the objective function. It can be an expression of those components of the control and auxiliary

variable (the latter is given as the solution to the differential equations defined by the multiphysics model) that are globally available.

Global Inequality Constraint

Here you specify the pointwise inequality constraint that applies globally. Due to computational issues, the expression has to be a closed form expression of *only* the control variable. Furthermore, only those components of the control variable that are global are usable.

CONSTRAINT

Constraint expression

In the **Constraint expression** edit field you specify the expression whose values at all points in all domains are to be constrained.

BOUNDS

In this section you can specify the upper and lower bounds for the pointwise constraint applied to the expression in question. To specify equality constraints, simply make sure the upper and lower bounds have the same value.

Lower bound

Select the **Lower bound** check box to activate and provide the lower bound.

Upper bound

Select the **Upper bound** check box to activate and provide the upper bound.

Global Control Variables

Here you specify those components of the control variable that are globally available.

CONTROL VARIABLES

In the table that appears in this section you list the variable names, initial values, lower bounds, and upper bounds of those components of the control variables that are globally available. To specify equality constraints, simply make sure the upper and lower bounds have the same value.

To add a control variable to the table, click the **Add** button (). To remove a control variable and its values from the table, click the **Delete** button ().

18

Meshing

This section summarizes how to create and control your mesh for 1D, 2D, and 3D geometries in COMSOL Multiphysics. It also explains the possibilities for importing and exporting meshes in different formats.

In this section:

Creating Meshes

A mesh is a discretization of the geometry model into small units of simple shapes, referred to as *mesh elements*.

Mesh Elements

ELEMENTS FOR 1D GEOMETRIES

The mesh generator discretized the domains (intervals) into smaller intervals (or mesh elements). The endpoints of the mesh elements are called *mesh vertices*.

The boundaries (or vertices) defined in the geometry are represented in the mesh by *boundary elements* (or *vertex elements*).

ELEMENTS FOR 2D GEOMETRIES

The mesh generator discretizes the domains into *triangular* or *quadrilateral* mesh elements. If the boundary is curved, these elements represent only an approximation of the original geometry. The sides of the triangles and quadrilaterals are called *mesh edges*, and their corners are *mesh vertices*. A mesh edge must not contain mesh vertices in its interior.

The boundaries defined in the geometry are discretized (approximately) into mesh edges, referred to as boundary elements (or *edge elements*), which must conform with the mesh elements of the adjacent domains.

The geometry vertices are represented by vertex elements.

ELEMENTS FOR 3D GEOMETRIES

The mesh generator discretizes the domains into *tetrahedral*, *hexahedral*, *prism*, or *pyramid* mesh elements whose faces, edges, and corners are called *mesh faces*, *mesh edges*, and *mesh vertices*, respectively.

The boundaries in the geometry are discretized into triangular or quadrilateral boundary elements.

The geometry edges are discretized into edge elements.

Similar to 2D, the geometry vertices are represented by vertex elements.

Pyramid Elements

Pyramid elements appear in the mesh in the following situations.

- If you import a NASTRAN file containing pyramid elements.
- If you create a swept mesh where the source and destination faces share an edge and the source face contains a triangular mesh.
- If you convert a quad mesh to a triangular mesh for a face adjacent to a domain that contains a mesh.
- If you create a boundary layer mesh in 3D for a geometry with sharp edges.

Meshing Techniques

The following meshing techniques for creating a mesh are available in COMSOL Multiphysics:

- Free meshing
- Mapped meshing
- Boundary layer meshing
- Swept meshing

The mesh generator used for free meshing, also referred to as the *free mesher*, is the only mesher that can be used on all types of geometry objects.

2D AND 3D MESHING OPTIONS

In a 2D geometry, choose between:

- Free meshing generating an unstructured mesh with triangular elements (see Free Meshing in 2D)
- Free meshing generating an unstructured mesh with quadrilateral elements (see Free Meshing in 2D)
- Mapped meshing generating a structured mesh with quadrilateral elements (see Mapped Meshing in 2D)

Compared to an unstructured mesh the interior mesh vertices in a structured mesh are adjacent to the same number of elements.

If you want to use mapped meshing on a geometry, you must build the geometry so that the domains are reasonably “regular” in shape and do not contain holes.

Note: You can create a structured triangular mesh by using the convert operation, which introduces a diagonal edge to each quadrilateral element.

In a 3D geometry, choose between:

- Free meshing generating an unstructured mesh with tetrahedral elements (see Free Tetrahedral Meshing in 3D)
- Swept meshing generating a structured mesh (at least in the direction of the sweep) with prism or hexahedral elements (see Creating 3D Swept Meshes)
- Boundary layer meshing generating structured layers of elements along specific boundaries integrating into an existing structured or unstructured mesh (see Creating Boundary Layer Meshes)

Note: Platforms handle floating-point operations differently, which sometimes results in slight differences between identical model files that are generated on two different computers.

ABOUT FREE MESHING

The free mesher is available in all dimensions, and you can use it for all types of geometries regardless of their topology or shape. If you have not defined or generated a mesh, the free mesher automatically creates an unstructured mesh and adds a corresponding node to the **Model Builder** window when you compute a study.

When you use the free mesher:

- The number of mesh elements is determined by the shape of the geometry and various mesh parameters.
- You control mesh parameters for the free mesher by **Size** and **Distribution** feature nodes in the meshing sequences.

You can also control the size of the mesh generated by a specific **Free Triangular**, **Free Quadrilateral**, or **Free Tetrahedral** node by adding a **Size** or **Distribution** subnode.

ABOUT 2D MAPPED MESHING

Geometry Requirements

For the 2D mapped meshing technique to work properly, the geometry must be reasonably regular. The following conditions must be satisfied:

- Each domain must be bounded by at least four boundary segments.
- Each domain must be bounded by only one connected boundary component (that is, no holes are allowed).
- The domains must not contain isolated or embedded vertices or boundary segments.
- The shape of each domain must not differ significantly from a rectangle.

For a geometry model that does not initially meet these criteria, it is usually possible to modify it so that a mapped mesh is generated, usually this is done by splitting it into simpler domains.

ABOUT 2D AND 3D BOUNDARY LAYER MESHES

A boundary layer mesh is a mesh with dense element distribution in the normal direction along specific boundaries. This type of mesh is typically used for fluid flow problems in order to resolve the thin boundary layers along the no-slip boundaries.

- In 2D, a layered quadrilateral mesh is used along the specified no-slip boundaries.
- In 3D, the boundary layer mesh is a layered prism mesh or a hexahedral mesh, depending on whether the corresponding boundary-layer boundaries contain a triangular or quadrilateral mesh.

Boundary layer meshes can also be used to resolve large temperature gradients close to heated surfaces subjected to sudden changes over time. In the heat sink model, you can see the introduction of a boundary layer mesh at the surfaces of the inner half-circle arc.

ABOUT 3D SWEPT MESHES

The swept mesher operates on a 3D domain by meshing a source face and then sweeping the resulting face mesh along the domain to an opposite target face. A swept mesh is structured in the sweep direction and can be either structured or unstructured orthogonally to the sweep direction.

For straight and circular sweep paths, you can use several connected faces as source faces. Each face about a domain that is to be operated on by the swept mesher is classified as either a source face, a target face, or a boundary face. The boundary faces

are the faces linking the source and target face (see Figure 18-1). The swept mesher can handle domains with multiple boundary faces in the sweep direction.

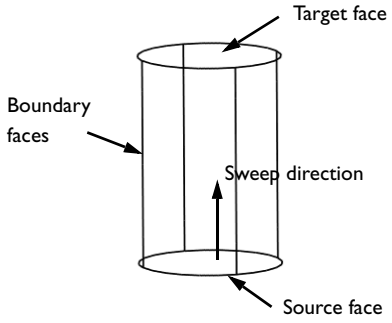


Figure 18-1: Classification of the boundaries about a domain used for swept meshing.

Free Meshing in 2D

Use the free mesher in 2D to create an unstructured mesh with triangular elements or quadrilateral elements. You can combine triangular and quadrilateral meshes by adding domains to the **Domain** list in the corresponding mesh operation's settings. From here, you can define specific meshing operations to each domain in your model. Figure 18-2 shows the resulting mesh when combining a **Free Triangular** node with a **Free Quad** node and the corresponding meshing sequence that generates the mesh. The model represents a cross section of a heat sink for high power applications.

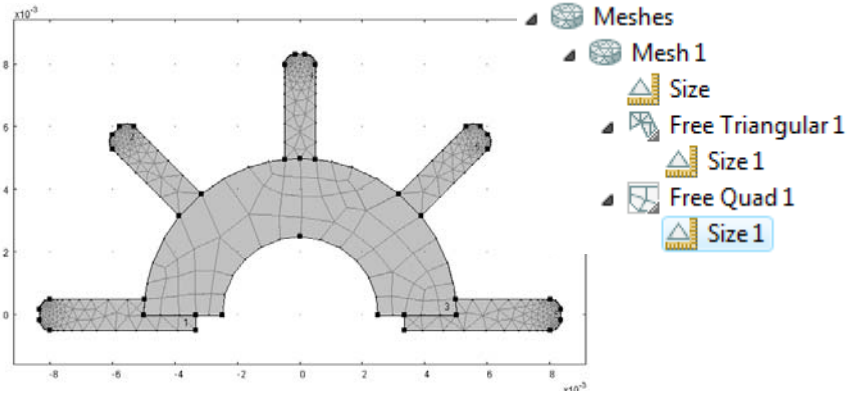


Figure 18-2: A combined triangular and quadrilateral mesh.

Adding a Free Triangular or a Free Quad node to a 2D Model

1 Open a 2D file or create a 2D object.

2 In the **Model Builder**, right-click **Mesh 1** and select **Free Triangular** or **Free Quad**.

3 In the **Settings** window, under the **Domains** section, select **Domain** from the **Geometric entity level** list.

4 In the **Graphics** window, left-click and then right-click any or all of the domains to add to the **Selection** list.

5 If the geometry requires it, you can now control the number of mesh elements along an edge.

Controlling the Number of Elements on Edges

In this example, Figure 18-2 shows that the two cooling fins at 45 degrees to the horizontal are only meshed with one mesh element at their attachment to the base. Use the **Edge** operation to fine tune the mesh and to give you control over the number of elements along the edge separating the fins and the base.

1 Insert an **Edge** operation before the **Free Triangular 1** operation to control the mesh at the interface between the base and the fin.

   a Click the first **Size** node.

   b Right-click and select **Build Selected**.

   c Right-click **Mesh 1** and select **Edge** from the **More Operations** submenu.

Note: When inserting a note to the sequence it appears after the current feature. The current feature is indicated by a quadratic frame around its icon. To make a feature become the current feature, select it, right-click, and select **Build Selected**. You can also move mesh nodes up and down in the sequence by right-clicking and selection **Move Up** or **Move Down** (or using the equivalent keyboard shortcuts).

---

2 In the **Graphics** window, left-click and then right-click the edges to add to the **Selection** list.

3 Right-click the Edge node and select **Distribution**. Note that the **Distribution** subnode inherits the selection from the Edge node.

4 Click **Build Selected**. In this example, three mesh elements are generated along the selected edges (Figure 18-3).

5 To copy an edge mesh from one edge to another:

a Right-click **Mesh 1** and select **Copy Edge** from the **More Operations** submenu.

b In the **Settings** window, select the source and destination edge respectively. You can use the **Copy Edge** mesh operation to define periodic boundary conditions, where the solution on the two periodic boundaries has to be identical.

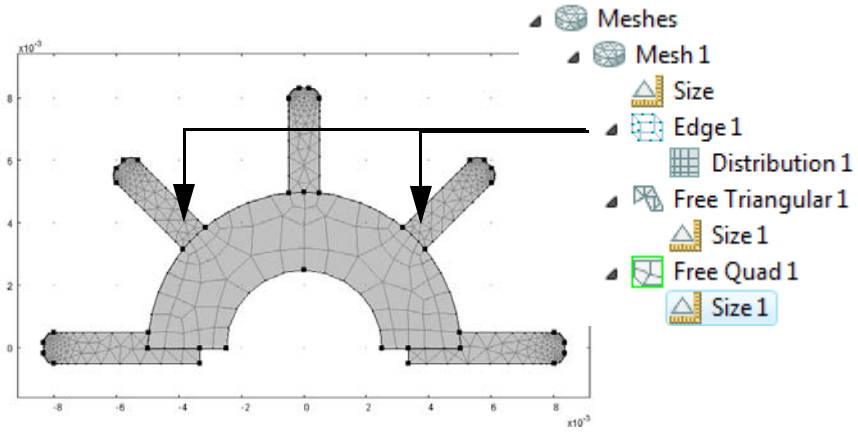


Figure 18-3: Adding a distribution attribute to an edge.

Mapped Meshing in 2D

Figure 18-4 is an example of the combination of a mapped mesh for the base of a heat sink cross section, and a triangular mesh for the fins.

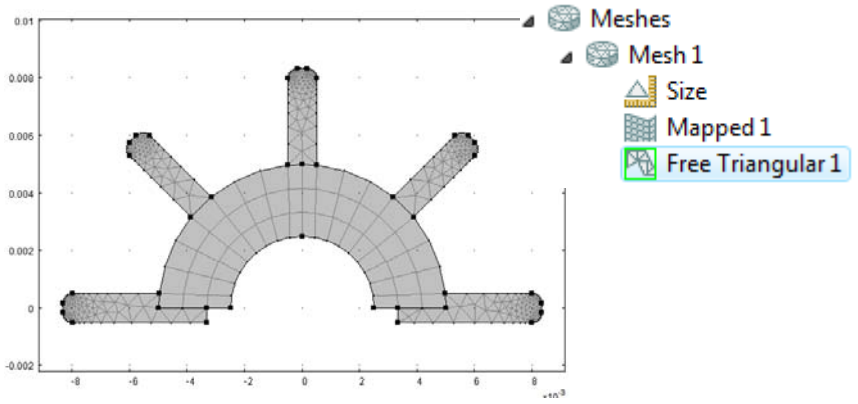


Figure 18-4: Mesh resulting from a combination of free triangular meshing and mapped quad meshing.

- To create a structured quad mesh on the base, right-click the Mesh node and select **Mapped**. In the **Settings** window, select the domain corresponding to the base and click the **Build Selected** button.

The quad mesh is generated using a mapping technique. For each domain, the mapping algorithm defines a regular grid on a logical unit square and then maps it onto the real geometry using transfinite interpolation.

Converting Meshes in 2D

You can create a structured triangular mesh by using the convert operation, which introduces a diagonal edge to each quadrilateral element.

- In the **Model Builder** window, right-click the Mesh node and select **Convert** from the **More Operations** submenu.

This operation should be placed in the sequence after the **Mapped** node as in Figure 18-5.

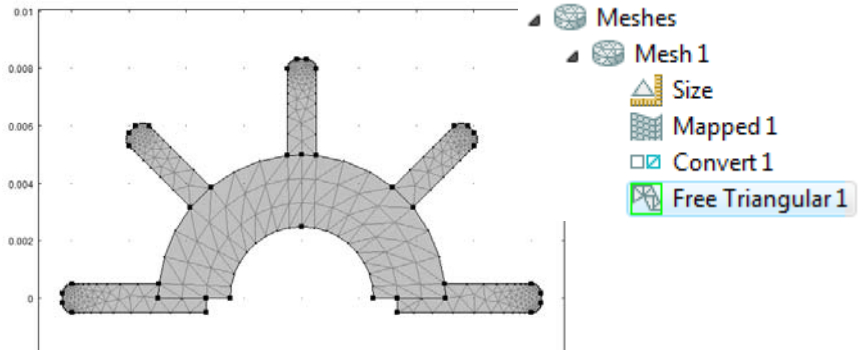


Figure 18-5: Example of a structured triangular mesh converted from a structured quad mesh.

Creating Boundary Layer Meshes

1. In the **Model Builder** window, right-click the Mesh node and select **Boundary Layers**.
2. Under the **Boundary Layers** node a **Boundary Layer Properties** node appears where you can select the boundaries where boundary layers should be applied. See Figure 18-6.

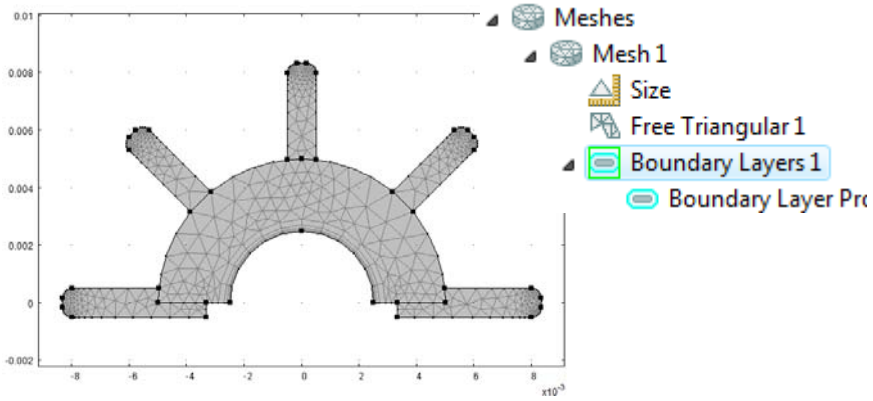


Figure 18-6: Adding a boundary layer elements.

ADDING A FREE TETRAHEDRAL FEATURE

1. Open a 3D model or create a 3D model.
2. In the **Model Builder**, right-click the Mesh node and select **Free Tetrahedral**.
3. In the **Settings** window, under the **Domains** section, select **Domain** from the **Geometric entity level** list.
4. In the **Graphics** window, left-click and then right-click any or all of the domains to add to the **Selection** list.

SPECIFYING THE MESH SIZE

To adapt the mesh for the selected domains, add a **Size** feature as a subnode to the Free Tetrahedral node created. Otherwise the **Size** node directly following the main **Mesh** node governs the mesh-element size for all **Free Tetrahedral** nodes.

Figure 18-7 shows a meshing sequence consisting of two Free Tetrahedral mesh operations, one for each meshed domain that correspond to pillars in a heat sink geometry. The mesh sizes are assigned as follows:

- The first **Size** node (under **Mesh 1**) is defined as **Extra Coarse**, which is selected from the **Predefined Element Size** list. In this example, the left pillar is selected to have the extra coarse mesh size and this is generated by the first **Free Tetrahedral 1** operation.
- The mesh size of the second pillar is defined under the **Free Tetrahedral 2** operation, it has its own **Size 1** attribute (**Coarser**), a finer mesh that is also selected from the **Predefined Element Size** list.

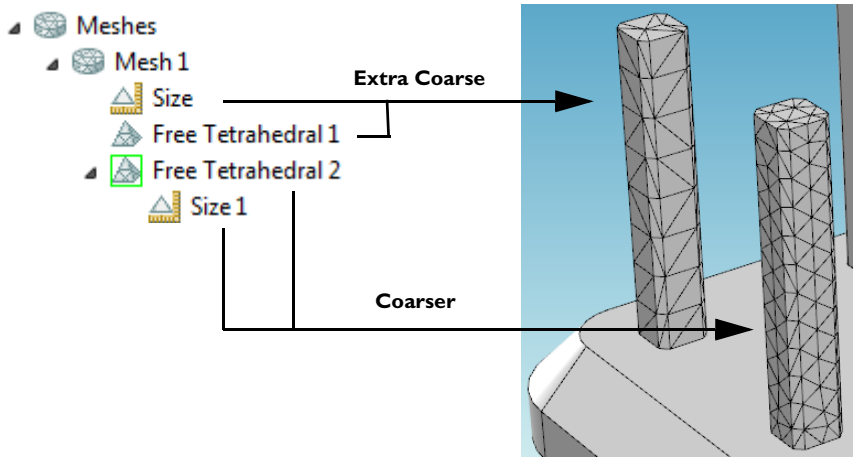


Figure 18-7: An example of two free tetrahedral meshing sizes.

The difference between the two meshes is based on the following.

To Specify the Size

1 In the **Model Builder** window, click the **Size** node to display the **Settings** window.

2 Select **Custom** in the **Element Size** section. In the **Element Size Parameters** section, define the mesh further by entering values in the following fields if required. Also see Figure 18-9.

- **Maximum element size**: specify the maximum allowed element size.
- **Minimum element size**: specify the minimum allowed element size. You can use this value to, for example, prevent the generation of many elements around small curved parts of the geometry.
- **Maximum element growth rate**: determine the maximum rate at which the element size can grow from a region with small elements to a region with larger elements. The value must be greater or equal to one. For example, with a maximum element growth rate of 1.5, the element size can grow by at most 50% (approximately) from one element to another. Figure 18-8 shows the base of the heat sink meshed using different two different element growth rates for otherwise identical settings.
- **Resolution of curvature**: determine the size of boundary elements compared to the curvature of the geometric boundary. The curvature radius multiplied by the resolution of curvature, which must be a positive scalar, gives the maximum

allowed element size along the boundary. A lower value gives a finer mesh along curved boundaries.

- **Resolution of narrow regions**: control the number of layers of elements that are created in narrow regions (approximately). The value must be a non-negative scalar. If the value of this parameter is less than one, the mesh generator might create elements that are anisotropic in size in narrow regions.

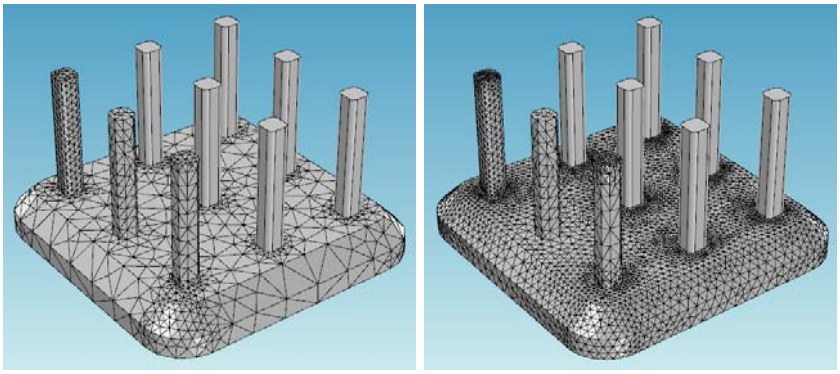


Figure 18-8: A comparison of mesh element growth rates in a heat sink.

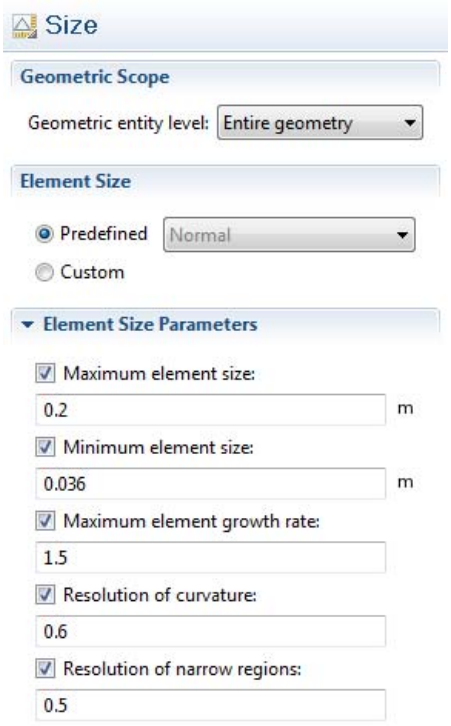


Figure 18-9: An example of custom element mesh sizes.

SPECIFYING MESH DISTRIBUTION

In addition to customizing the mesh size, you can also add a **Distribution** attribute to a Free Tetrahedral feature. In the settings for this Distribution, constrain the number of elements for the selected edges. Figure 18-10 shows three heat sink pillars with different mesh sizes and distribution.

Note: The settings in the **Distribution** attribute overrule the settings in the **Size** node.

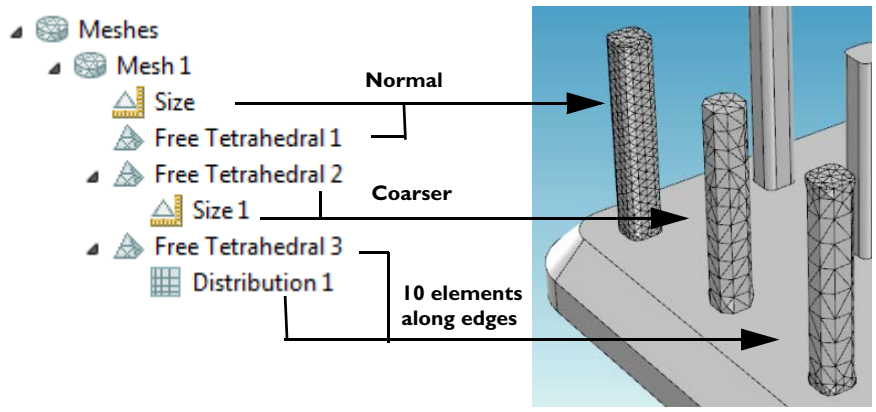


Figure 18-10: Specifying element distribution in heat sink pillars.

Creating 3D Swept Meshes

Figure 18-11 shows the 3D swept mesh for a simple geometry but with a layered structure typical for printed circuit boards or MEMS geometries. In such cases, the swept mesh generation presents an alternative to using a free tetrahedral meshing.

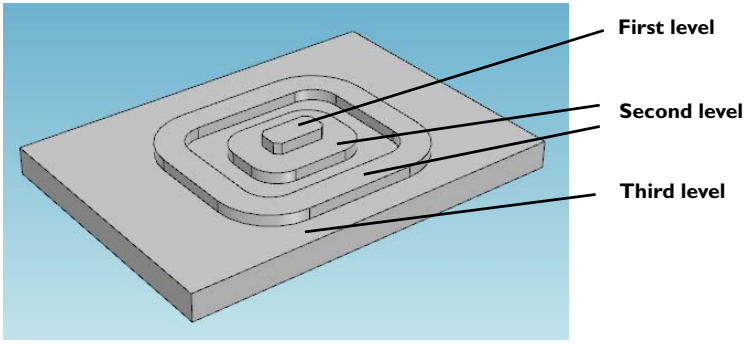
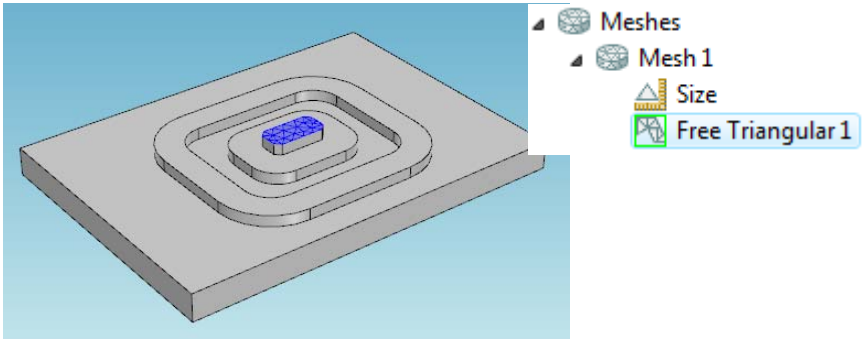


Figure 18-11: An example of the layered geometry used for creating a swept mesh.

1. In the **Model Builder** window, right-click the Mesh node and select **Free Triangular** from the **More Operations** submenu to operate at the boundary level.
2. Add the first level boundary to the selection list (see Figure 18-11 for an example of a suitable geometry).

3 Click **Build Selected**. The mesh below displays.



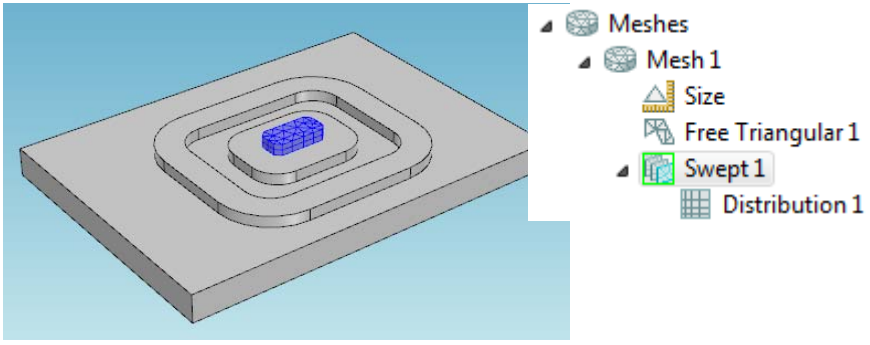
4 Right-click the Mesh node and select **Swept**.

5 Select the domain in the first level.

6 In the **Model Builder** window, right-click **Swept 1** and select **Distribution**.

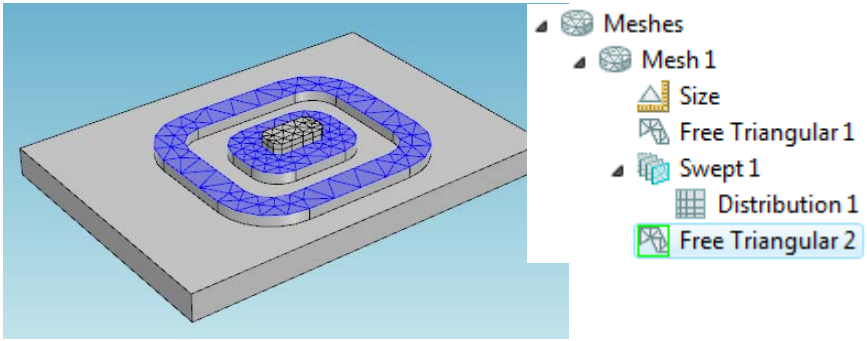
7 Enter the **Number of elements** in the corresponding field (for example 2).

8 Click **Build Selected**.

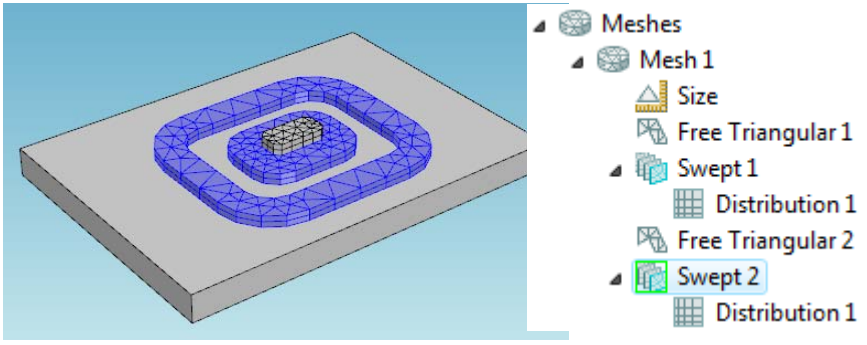


9 Right-click the Mesh node and select **Free Triangular** from the **More Operations** submenu.

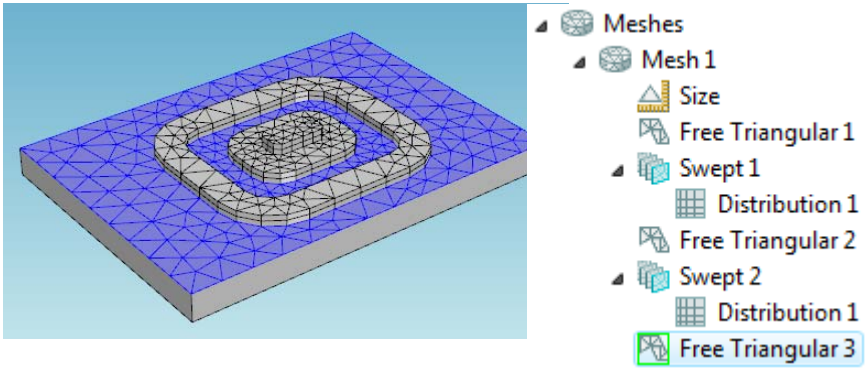
10 Select the boundaries at the second level and click the **Build Selected** button.



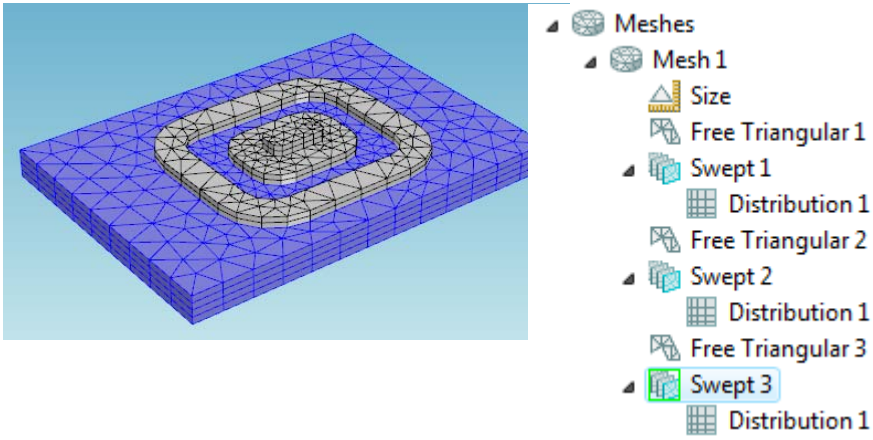
11 Repeat the same swept operations for the first level domains but now for the second level.



12 Mesh the third level boundaries using the **Free Triangular** mesh operation.



13 Mesh the third level domain. Use the **Swept** mesh operation and enter 4 for the **Number of elements** in the corresponding **Distribution** attribute.



14 The meshing sequence displayed in the **Model Builder** window enables you to return to your attribute settings and change mesh sizes and distributions. After making any changes, click the **Build All** button to update the entire meshing sequence.

19

Multiphysics Modeling

This section covers the use of COMSOL Multiphysics for multiphysics modeling and coupled-field analyses. It describes various ways of building multiphysics models and provides an overview of the available predefined multiphysics interfaces in COMSOL Multiphysics and the add-on modules.

In this section:

Creating Multiphysics Models

Multiphysics Modeling Approaches

The ability to create multiphysics models—those with more than one type of physics or equation such as coupled-field problems—is one of COMSOL Multiphysics' most powerful capabilities. In such a model, the software can solve all the equations, taken from various areas of physics, as one fully coupled system.

Within COMSOL Multiphysics you can choose from several ways to approach multiphysics modeling and coupled-field analysis.

Using Predefined Multiphysics Interfaces

COMSOL Multiphysics and the add-on modules provide a number of predefined multiphysics interfaces. They appear in the **Model Wizard**'s list of physics interfaces, in folders with the names in the Category/Folder column in Table 19-1. The multiphysics interfaces includes the functionality of two or more physics interfaces with additional features for the multiphysics couplings. Default settings provide the typical field couplings for the multiphysics application. The available multiphysics couplings vary depending on the installed modules (see Table 19-1 for details).

TABLE 19-1: PREDEFINED MULTIPHYSICS COUPLINGS

CATEGORY/ FOLDER	NAME	SPACE DIMENSION	REQUIRED MODULES	OPTIONAL MODULES
Heat Transfer	Joule Heating	2D, 2D Axi, 3D	None	AC/DC, Heat Transfer, MEMS
Heat Transfer	Induction Heating	2D, 2D Axi, 3D	AC/DC	Heat Transfer
Heat Transfer	Microwave Heating	2D, 2D Axi, 3D	RF	Heat Transfer
Fluid Flow	Fluid-Structure Interaction	2D, 2D Axi, 3D	Structural Mechanics or MEMS	CFD
Fluid Flow	Non-Isothermal Flow/Conjugate Heat Transfer	2D, 2D Axi, 3D	CFD or Heat Transfer	
Structural Mechanics	Thermal Stress	2D, 2D Axi, 3D	Structural Mechanics or MEMS	Heat Transfer

TABLE 19-1: PREDEFINED MULTIPHYSICS COUPLINGS

CATEGORY/ FOLDER	NAME	SPACE DIMENSION	REQUIRED MODULES	OPTIONAL MODULES
Structural Mechanics	Joule Heating and Thermal Expansion	2D, 2D Axi, 3D	Structural Mechanics or MEMS	Heat Transfer, AC/DC
Structural Mechanics	Piezoelectric Devices	2D, 2D Axi, 3D	Structural Mechanics, MEMS, or Acoustics	
Acoustics	Acoustic-Structure Interaction	2D, 2D Axi, 3D	Acoustics	Structural Mechanics, MEMS

The predefined multiphysics interfaces include all features in the optional modules if your license includes them; otherwise they include the corresponding but more limited physics interface in COMSOL Multiphysics or the required module. The couplings are identical in both cases, but the optional modules typically offer additional functionality in other areas. These predefined multiphysics interfaces are quick entry points for common multiphysics applications. It is possible to create the same couplings using any of the other methods for multiphysics modeling, and you can continue to add, modify, disable, and remove physics interfaces in a model that you start using one of the predefined multiphysics interfaces. If you want to add additional physics interfaces in the **Model Builder**, right-click the model node, choose **Add Physics** and then use the **Model Wizard** that appears.

Heat Transfer—Electromagnetic Heating Interfaces

The multiphysics interfaces in the Electromagnetic Heating section couple electromagnetic fields and heat transfer:

- Joule Heating (part of the COMSOL Multiphysics base package)
- Induction Heating (requires the AC/DC Module)
- Microwave Heating (requires the RF Module)

They appear in the Heat Transfer branch.

JOULE HEATING

The **Joule Heating** predefined multiphysics interface () combines all feature from the Electric Currents interface with the Heat Transfer interface for modeling of *Joule*

heating (*resistive heating* or *ohmic heating*). The interaction is coupled in both directions:

- The resistive heating appears as a heat source in the default **Electromagnetic Heat Source** node.
- The default setting is to use the value for the electric conductivity σ from the material. By selecting **Linearized resistivity** from the σ list, the following temperature-dependent expression describes the electric conductivity: $1/(\rho_0(1+\alpha(T-T_{\text{ref}})))$, where T is the dependent variable for temperature from the heat transfer part, which automatically appears as a model input. By default, the values for ρ_0 (resistivity at reference temperature), α (temperature coefficient), and T_{ref} (reference temperature) are taken from the material. These settings are built into the Joule Heating Model node, which is the central feature in the Joule Heating interface.

You can use a stationary or time-dependent study.

For an example of a model using the Joule Heating predefined multiphysics coupling, see the `thermal_actuator_jh` model in the Multiphysics folder of the COMSOL Multiphysics Model Library.

INDUCTION HEATING

The **Induction Heating** predefined multiphysics interface ( ) combines the features of the Induction Currents interface with those of the Heat Transfer interface. The predefined interaction adds the resistive heating from the induction currents as a heat source.

For more information, see the *AC/DC Module User's Guide*.

MICROWAVE HEATING

The **Microwave Heating** predefined multiphysics interface ( ) combines the features of an Electromagnetic Waves interface from the RF Module with those of the Heat Transfer interface. The predefined interaction adds the resistive heating from the electromagnetic waves as a heat source.

For more information, see the *RF Module User's Guide*.

Structural Mechanics

The Structural Mechanics branch contains three multiphysics interfaces: Thermal Stress, Joule Heating and Thermal Expansion, and Piezoelectric Devices. All of these interfaces require additional modules.

THERMAL STRESS

The **Thermal Stress** multiphysics interface () combines solid mechanics with heat transfer for modeling of thermal expansion. It combines the features of a Solid Mechanics interface with those of a Heat Transfer interface.

The interaction is a one-way coupling using the temperature from the heat transfer interface to define the strain temperature.

You can use a steady-state or time-dependent study. For the time-dependent study, the software by default treats the structural part as static (quasi-static transient behavior).

By default, COMSOL Multiphysics solves for the temperature and displacements simultaneously. For large problems, you can take advantage of the one-way coupling and solve the problem sequentially (unless there are thermal properties that depend on the displacements): first solve for temperature and then perform the stress-strain analysis using the computed temperature field from the heat transfer equation using a segregated solution scheme.

JOULE HEATING AND THERMAL EXPANSION

The **Joule Heating and Thermal Expansion** multiphysics interface () combines Joule heating and thermal expansion for modeling, for example, thermoelectromechanical (TEM) effects. This interface requires the Structural Mechanics Module or the MEMS Module. It combines the features of a Heat Transfer interface and a Electric Currents interface with those of a Solid Mechanics interface from the Structural Mechanics Module or the MEMS Module.

The interactions include Joule heating, which is fully coupled in both directions (see Joule Heating), and thermal expansion, which is a one-way coupling (see Thermal Stress).

You can select a steady-state or transient analysis (where the conductive media and solid mechanics equations are stationary).

By default, COMSOL Multiphysics solves for the temperature, electric potential, and displacements simultaneously.

PIEZOELECTRIC DEVICES

The **Piezoelectric Devices** multiphysics interface () combines Solid Mechanics and Electrostatics for modeling on piezoelectric devices where all or some of the domains contain a piezoelectric material. The piezoelectric coupling can be on stress-charge or stress-charge form. You also have access to all solid mechanics and electrostatics functionality.

Acoustics

The Acoustics branch contains multiphysics interfaces for acoustic-structure interaction.

ACOUSTIC-STRUCTURE INTERACTION

The **Acoustic-Structure Interaction** multiphysics interface () for acoustic-structural interaction is available with the Acoustics Module. This multiphysics interface combines Pressure Acoustics and Solid Mechanics and connects the acoustics pressure in a fluid domain with the structural deformation in a solid domain. Special interface conditions define the fluid loads on the solid domain and the structural acceleration's effect on the fluid.

For more information, see the Acoustics Module documentation.

Fluid Flow

The following multiphysics interfaces appear in the Fluid Flow branch:

FLUID-STRUCTURE INTERACTION (FSI)

The **Fluid-Structure Interaction** multiphysics interface () for fluid-structure interaction (FSI) is available in the MEMS Module and the Structural Mechanics Module. FSI combines fluid flow with structural mechanics, using a moving mesh to capture the fluid movement. The interface provides a predefined interface condition for the solid-fluid boundary. For more information and example models, see the Structural Mechanics Module and MEMS Module documentation.

FLOW WITH VARIABLE DENSITY—NON-ISOTHERMAL FLOW

The CFD Module and the Heat Transfer Module include the laminar and turbulent Non-Isothermal Flow/Conjugate Heat Transfer predefined multiphysics interface. In the CFD Module you select both laminar and turbulent non-isothermal flow in the **Non-Isothermal Flow** folder (); in the Heat Transfer Module, you select a laminar or

turbulent non-isothermal flow in the **Conjugate Heat Transfer** folder ( ). For more information about these predefined couplings, see the CFD Module and Heat Transfer Module documentation.

POROELASTICITY

The **Poroelasticity** multiphysics interface ( ) is available in the Earth Science Module and combines the Darcy's Law physics interface from that module with a structural mechanics physics interface from the Structural Mechanics Module, which is required in addition to the Earth Science Module. The Poroelasticity predefined multiphysics coupling is useful for modeling poroelastic phenomena such as diffusion in elastic solids or elasticity of fluid-infiltrated porous solids. For more information, see the Earth Science Module documentation.

Adding Physics Sequentially

With this approach you can verify that each type of physics or equation gives the expected results before adding more complexity to the model by adding another physics or coupling fields.

Use these steps to add one physics at the time:

1. Start a new model using the **Model Wizard**.
2. Draw the geometry.
3. Define the physics settings.
4. Solve and visualize the results.

Then add another physics interface:

5. Right-click the model node in the **Model Builder** and choose **Add Physics**.
6. Add one or more new physics interface using the **Model Wizard**.
7. Add and modify feature nodes in the physics interfaces, including couplings between the physics.

Building a Multiphysics Model Directly

Another approach is to include multiple physics interfaces when you start creating the model in the Model Wizard:

1. On the **Add Physics** page, select an physics interface for the model.

2 Click the **Add Selected** button ( ) underneath the list of available physics interfaces. The physics interface then appears in the list under **Selected Physics**.

3 If needed, you can change the names of the dependent variables. For some physics interfaces and mathematical interfaces you can also adjust the number of dependent variables.

4 Continue selecting physics interfaces and adding them to the list under **Selected Physics** by clicking the **Add Selected** button.

5 When done, continue in the Model Wizard by clicking the **Next** button ( ) at the top of the **Add Physics** page.

Adding Multiphysics Couplings

With more than one physics interface in the model, coupling of the fields is easy: all applicable fields that can serve as inputs in another physics interface automatically appears in the other physics interface's **Settings** window's **Model Inputs** section. For example, with a Heat Transfer (ht) interface for heat transfer in fluids and a Laminar Flow (spf) interface, you can select **Velocity field (spf/fp1)**, which the **Fluid Properties (fp1)** node in the **Laminar Flow** branch defines, from the **Velocity field** list.

Removing Physics Interfaces

To remove a physics interface from a model:

1 Select the main feature node for the physics interface under the model node in the **Model Builder**.

2 Right-click and choose **Delete** from the context menu or press the Delete key.

3 Click **OK**.

20

Deformed Meshes

This section explains how to use the modeling interfaces that control mesh deformation: the Moving Mesh interface and the Deformed Geometry interface.

In this section:

Deformed Mesh Fundamentals

A deformed mesh can be useful if the boundaries of your computational domain are moving in time or deform as a function of some parameter. The point is that a new mesh need not be generated for each configuration of the boundaries—instead, the software simply perturbs the mesh nodes so they conform with the moved boundaries.

In COMSOL Multiphysics, you can control the movement of the interior nodes in two ways:

- By propagating the moving boundary displacement throughout the domain to obtain a smooth mesh deformation everywhere — this is done by solving PDEs for the mesh displacements (a Laplace or Winslow smoothing PDE) with boundary conditions given by the movement of the boundaries.
- By specifying an explicit formula for the mesh deformation. The formula can make use of other dependent variables, such as the displacement components of structural mechanics.

Arbitrary Lagrangian-Eulerian Formulation (ALE)

The partial differential equations of physics are usually formulated either in a *spatial* coordinate system, with coordinate axes fixed in space, or in a *material* coordinate system, fixed in the material in its reference configuration and following the material as it deforms. The former is often referred to as an *Eulerian* formulation, while the latter is *Lagrangian*.

Structural mechanics and other fields of physics dealing with a possibly anisotropic, normally solid, material are most conveniently simulated using material coordinates. The Lagrangian formulation makes the anisotropic material properties independent of the current spatial orientation of the material.

If, on the other hand, the focus is on simulating the physical state at fixed points in space, an Eulerian formulation is usually more convenient. In particular, when liquids and gases are involved, it is often unreasonable to follow the state of individual material particles. Rather, the quantities of interest are pressure, temperature, concentration, etc., at fixed positions in space.

An inherent problem with the pure Eulerian formulation is that it cannot handle moving domain boundaries, since physical quantities are referred to fixed points in space, while the set of spatial points currently inside the domain boundaries changes

with time. Therefore, to allow moving boundaries, the Eulerian equations must be rewritten so as to describe all physical quantities as functions of *mesh* coordinates instead of spatial coordinates. In the mesh coordinate system, the domain is fixed, and there is a one-to-one map from the mesh coordinates to the current spatial configuration of the domain. Otherwise, the mesh coordinate system can be defined freely and separately from both the spatial and material systems.

Rewriting physics equations in this way, on a freely moving mesh, results in an *arbitrary Lagrangian-Eulerian* (ALE) method. In the special case when the map from mesh coordinates to spatial coordinates follows the material deformation, a *Lagrangian* method is recovered. Similarly, when the map is an identity map, the ALE method becomes entirely Eulerian.

The ALE method is therefore an intermediate between the Lagrangian and Eulerian methods, and it combines the best features of both—it allows moving boundaries without the need for the mesh movement to follow the material.

Frames

COMSOL Multiphysics refers to the spatial, material/reference and mesh coordinate systems described above as *Spatial frame*, *Material frame*, and *Mesh frame*, respectively. Physics can be formulated on the Spatial frame or on the Material frame, depending on whether it is more convenient to interpret the equations as Eulerian or Lagrangian, respectively. The Mesh frame and its associated coordinates cannot be used to formulate physics since it is neither connected to the material, nor to the true Euclidean space.

Conceptually, all three frames always exist, but all or some of them may point to the same actual coordinate system. It is the actual coordinate system which decides the names of the independent variables (the coordinate names like `x`, `y`, `z` or `r`, `phi`, `z`). Before you add any Moving Mesh or Deformed Geometry interface to your model, all three frames coincide and use the spatial coordinate names. Also all interfaces based on solid mechanics include moving mesh functionality and by default behave much in the same way as a Moving Mesh interface.

When you add a Moving Mesh or Solid Mechanics interface, the Spatial frame is separated from the Material frame, which is given a new set of independent variable names (by default capital `X`, `Y`, `Z` or `R`, `PHI`, `Z`). From this point, Eulerian and Lagrangian formulations behave differently since they, among other things, define derivatives with respect to different sets of independent variables.

The Mesh and Material frames share coordinate system until you add a Deformed Geometry interface. At that point, a new mesh coordinate system is created and given a new set of independent variable names (by default `Xm`, `Ym`, `Zm` or `Rm`, `PHIm`, `Zm`). The new Mesh frame refers to the geometry as it is represented by the Geometry Sequence. By inserting a nontrivial transformation from mesh coordinates to material coordinates, you can effectively change the shape of the geometry without having to create a new mesh. This can in some situations be useful as a means of parameterizing the geometry before performing optimization or sensitivity analysis.

Note that using Deformed Geometry affects both Eulerian and Lagrangian physics in the same way. The reason is that the Deformed Geometry interface controls the Material frame in relation to the Mesh frame. Unless there is also a Moving Mesh or Solid Mechanics interface present, the Material and Spatial frames still refer to the same coordinate system. The three frames refer to three different sets of coordinates only when there is both a Deformed Geometry and some Moving Mesh interface active in the model.

Mathematical Description of the Mesh Movement

Though moving meshes are also possible in 3D, consider a 2D geometry for simplicity, where the spatial and material frame coordinates are called (x, y) and (X, Y), respectively. Let (X_0, Y_0) be the spatial coordinates of a mesh node in the initial material configuration. The spatial coordinates (x_0, y_0) of the same mesh node at some other time, t, are then given by the functions

$$x_0 = x(X_0, Y_0, t), \qquad y_0 = y(X_0, Y_0, t) \tag{20-1}$$

These functions can be either explicit transformations (expressions) or be the solution to a mesh smoothing equation. The mesh node's material coordinates (X_0, Y_0) may in turn bee seen as functions of an underlying system of mesh coordinates $(X_\mathrm{m}, Y_\mathrm{m})$ and a parameter, p, such that

$$X_0 = X(X_m, Y_m, p), \qquad Y_0 = Y(X_m, Y_m, p) \tag{20-2}$$

with similar options for the transformations. Obviously, the transformations can also be chained such that (x_0, y_0) are seen as functions of $(X_\mathrm{m}, Y_\mathrm{m})$, t and p.

In addition to the different sets of coordinate variables, some other geometric variables that the software defines are available for both the spatial and the material frames (see Geometric Variables).

To avoid confusion, note that:

- The *spatial frame* is the usual, fixed, global, Euclidean coordinate system with the *spatial coordinates* (x, y). In the ALE context, the spatial coordinate system as such is fixed while the spatial coordinates (x,y) of each material point and mesh node may be functions of time. Therefore, it is correct to refer to the model as having a *moving mesh*.
- The *material frame* is a coordinate system which identifies material points by their spatial coordinates (X, Y) in some—actual or imagined—reference configuration. You can think of the material coordinate system as having been printed on the material in the reference configuration such that it follows it during deformation. It is therefore in general curvilinear and cannot be used directly to measure true distances and angles. See also Figure 20-1 and Figure 20-2
- The *mesh frame* is a coordinate system which identifies mesh points by their spatial coordinates (X_m, Y_m) in the original mesh created from the geometry. It is often natural to use the original geometry also as reference state to define material coordinates. Therefore, the mesh frame and material frame usually coincide. The only exception is when you use a Deformed Geometry interface to deform or parameterize the original geometry.

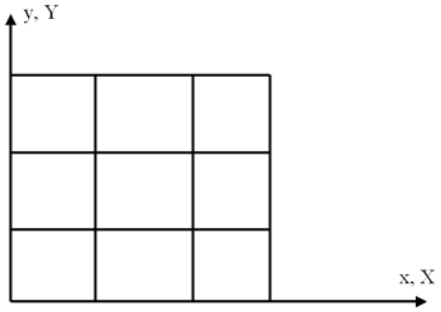


Figure 20-1: An undeformed mesh. In the reference configuration, which may be the actual configuration at a reference time or a hypothetical state, the spatial frame (x, y) and the material frame (X, Y) coincide.

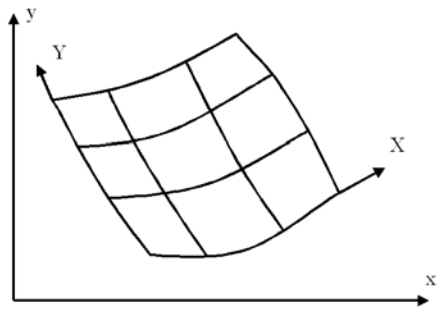


Figure 20-2: After deformation of the material, the spatial frame (x, y) remains the same, while the material coordinate system (X, Y) has been deformed, following the material. Meanwhile, the material coordinates of each material point remains the same, but its spatial coordinates have changed.

Derivatives of Dependent Variables

When solving for some physical quantity, u, COMSOL Multiphysics always stores the solution for a fixed set of mesh nodes. That is, the dependent variable u is treated internally as a function of the mesh coordinates, $u\ (X_m, Y_m, t)$. The essence of the ALE system is that it allows treating the physical quantities as functions of the material or spatial coordinates, $u\ (X, Y, t)$ or $u(x, y, t)$, instead. This transformation is possible only if the mappings given by Equation 20-1 and Equation 20-2 are invertible.

SPATIAL DERIVATIVES

With respect to spatial differentiation, each dependent variable is treated as a function of one of the frames present in the model. The choice of frame for each variable is either fixed by the physics interface, or coincides with the **Frame type** setting found under **Discretization** in the Physics interface which defines the variable. For a dependent variable `u`, there is typically two possibilities:

- The variable is defined on the spatial frame and its derivatives with respect to the spatial coordinates are denoted `ux` and `uy` in the software.
- The variable is defined on the material frame and its derivatives with respect to the material coordinates are denoted `uX` and `uY` in the software.

In COMSOL Multiphysics, it is only possible to use one of these types of derivatives of each dependent variable.

TIME DERIVATIVES

When using ALE, the software defines two kinds of time derivatives:

- The common *frame time derivative*, valid for a fixed point in the frame on which the variable is defined. This derivative is always denoted `ut` in the software. For example, for `u` defined on the spatial frame:

$$u_t(x_0, y_0) = \left.\frac{\partial u}{\partial t}\right|_{x_0, y_0}$$

- The *mesh time derivative*, which is taken for a fixed point in the mesh:

$$u_{\mathrm{TIME}}(X_m, Y_m) = \left.\frac{\partial u}{\partial t}\right|_{X_m, Y_m}$$

  This derivative is denoted `uTIME` in the software. Since internally, everything is formulated on the mesh frame, the mesh time derivative is the one computed by the solvers and stored in the solution vector.

The two derivatives are related by the chain rule:

$$u_t = u_{\mathrm{TIME}} - u_x x_{\mathrm{TIME}} - u_y y_{\mathrm{TIME}}$$

where $(x_{\mathrm{TIME}}, y_{\mathrm{TIME}})$ is the mesh velocity. The mesh time derivative is often less important from the user point of view because its value depends on the mesh movement, which in itself has no physical significance. However, for the special case

when the mesh follows the material's motion, the mesh time derivative is physically significant and is also called the *material time derivative*.

Smoothing Methods

In the domains with free displacement, the Moving Mesh interface solves an equation for the mesh displacement. This equation smoothly deforms the mesh given the constraints you place on the boundaries. You can choose between *Laplace smoothing* and *Winslow smoothing*. To specify the smoothing methods, use the **Mesh smoothing type** list in the **Free Deformation Settings** section of the **Moving Mesh** or **Deformed Geometry** node.

To see how these smoothing methods differ, let x and y be the spatial coordinates of the spatial frame, and let X and Y be the reference coordinates of the material frame. If you choose Laplace smoothing, the software introduces deformed mesh positions x and y as degrees of freedom in the model. In the static case, it solves the equation

$$\frac{\partial^2 x}{\partial X^2} + \frac{\partial^2 x}{\partial Y^2} = 0$$

and in the transient case it solves the equation

$$\frac{\partial^2}{\partial X^2}\frac{\partial x}{\partial t} + \frac{\partial^2}{\partial Y^2}\frac{\partial x}{\partial t} = 0$$

Similar equations hold for the y coordinate.

If you choose Winslow smoothing, the software solves the equation

$$\frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 X}{\partial y^2} = 0$$

and does the same for Y. Equivalently, X and Y satisfy Laplace equations as functions of the x and y coordinates.

Limitations of the ALE Method

The following limitations apply to the ALE method in general and therefore to the Moving Mesh and Deformed Geometry interfaces:

- The connectivity of the mesh remains unchanged during the mesh deformation, which means that topological changes in the geometry are not allowed.
- When the mesh deformation becomes large, the quality of the mesh created by the smoothing equations can deteriorate. If this happens, the solver runs into convergence problems. Sometimes you see the warning **Inverted mesh element** in the Progress window for the solver, which means that a mesh element has (partially) warped inside-out. Sometimes, introducing extra boundaries with explicit deformation inside the domains can help.
- When using **Geometry shape order** larger than 1 in the Moving Mesh and Deformed Geometry interfaces, the mesh moving techniques often produce elements with distorted shapes. If you get warnings about inverted mesh elements, you may consider reducing the geometry shape order to 1. This, however, makes the geometry representation polygonal, which may affect accuracy. Note also that the measure of mesh quality does not capture these distorted shapes because it is computed from the positions of the corners of the mesh element (ignoring midside nodes, for instance).

The Moving Mesh Interface

With the **Moving Mesh** interface () you can create models where the geometry (actually the mesh) changes shape due to some physics in the model. You can study both stationary states and time-dependent deformations where the geometry changes its shape due to the dynamics of the problem. It contains the following features:

- "Moving Mesh"
- "Free Deformation"
- "Prescribed Deformation"
- "Fixed Mesh"
- "Prescribed Mesh Displacement"
- "Prescribed Mesh Velocity"

Moving Mesh

The **Moving Mesh** feature lets you specify the domains and properties of the moving mesh interface.

INTERFACE IDENTIFIER

This is the identifier for the physics interface, which you use to reach the fields and variables in expressions, for example.

The identifier appears in the **Identifier** edit field, and you can change it to any unique string that is a valid identifier. The default identifier (for the first Moving Mesh interface in the model) is `ale`.

DOMAINS

Specify the domains (extent) for the interface.

Note: The Moving Mesh interface requires a domain selection that covers all domains in which some physics is active.

FRAME SETTINGS

Specify the space coordinates of the base frame for the interface; the material frame in the **Material frame coordinates** edit fields. The default is the coordinates of the spatial frame in uppercase letters.

The **Geometry shape order** setting controls the order of polynomials used for representing the geometry shape in the spatial frame. The same order is used for Lagrange shape functions defining the mesh position in domains where **Free displacement** has been activated.

FREE DEFORMATION SETTINGS

Select the smoothing type for freely deformed domains from the **Mesh smoothing type** list. Choose between Lagrange and Winslow smoothing. For details, see Smoothing Methods.The default is Laplace smoothing.

ADVANCED SETTINGS

Normally these settings do not need to be changed. See Advanced Settings.

Free Deformation

The **Free Deformation** feature constrains the mesh displacement only by the boundary conditions on the surrounding boundaries. The displacement in the domain is obtained by solving a PDE (see Smoothing Methods).

DOMAINS

Specify the domains where you want free displacement.

INITIAL DEFORMATION

Give initial values for mesh displacements, relative to the material frame, in the **Initial mesh displacement** edit fields.

Prescribed Deformation

Use the **Prescribed Deformation** feature when you want to define the deformation explicitly using expressions, or want the spatial coordinates to follow a deformation computed by, for example, a Solid Mechanics interface.

DOMAINS

Specify the domains where you want prescribed displacement.

PRESCRIBED MESH DISPLACEMENT

Specify expressions that define the deformation in the **Prescribed mesh displacement** fields. Use one expression per space coordinate.

Fixed Mesh

Use the **Fixed Mesh** feature to specify that the selected domains remain at their reference material shape, and do not move. This is the default.

DOMAINS

Specify the domains where you want no displacement.

Prescribed Mesh Displacement

Use the **Prescribed Mesh Displacement** feature on the boundary of domains with free deformation. The spatial frame in the adjacent domain moves in accordance with the displacement.

COORDINATE SYSTEM SELECTION

Specify the coordinate system to use for specifying the mesh displacement. From the **Coordinate system** list select from:

- **Global coordinate system** (the default)
- **Boundary System** (a predefined normal-tangential coordinate system)
- Any additional user defined coordinate systems

BOUNDARIES

Specify the boundaries where you want prescribed displacement.

PRESCRIBED MESH DISPLACEMENT

Select the check box for each coordinate direction where you want to prescribe the displacement. The default settings provide a fixed boundary (zero displacements in all directions).

CONSTRAINT SETTINGS

Select the **Use weak constraints** check box to use weak constraints instead of the default strong constraints.

Prescribed Mesh Velocity

On the boundary of domains with free displacement, you can specify the velocity of the boundary. Right-click the **Free Deformation** node to add a **Prescribed Mesh Velocity** node. The spatial frame in the adjacent domains moves in accordance with the velocity.

BOUNDARIES

Specify the boundaries where you want a prescribed velocity.

COORDINATE SYSTEM SELECTION

Specify the coordinate system to use for specifying the mesh displacement. From the **Coordinate system** list select from:

- **Global coordinate system** (the default)
- **Boundary System** (a predefined normal-tangential coordinate system)
- Any additional user defined coordinate systems

PRESCRIBED MESH VELOCITY

Select the check box for each coordinate directory where you want to prescribe a velocity. The default settings provide zero velocities in all directions.

CONSTRAINT SETTINGS

Select the **Use weak constraints** check box to use weak constraints instead of the default strong constraints.

The Deformed Geometry Interface

Using the **Deformed Geometry** interface ( ) you can study how the physics changes when the geometry changes as a function of a parameter. The only real difference between the Deformed Geometry interface and the Moving Mesh interface is that the former defines a deformation of the material frame relative to the mesh frame, while the latter defines a displacement of the spatial frame relative to the material frame.

See also Selecting and Deselecting Geometric Entities for detailed information about selecting geometric entities (domains, boundaries, edges, and points), and The Moving Mesh Interface for more information about deformed meshes. The section about smoothing methods also applies to the Deformed Geometry interface.

This section contains the following features:

- "Deformed Geometry"
- "Free Deformation"
- "Free Deformation"
- "Prescribed Deformation"
- "Fixed Mesh"
- "Prescribed Mesh Displacement"
- "Prescribed Mesh Velocity"

Deformed Geometry

The **Deformed Geometry** feature lets you specify the domains and properties of the deformed geometry interface.

INTERFACE IDENTIFIER

This is the identifier for the physics interface, which you use to reach the fields and variables in expressions, for example.

The identifier appears in the **Identifier** edit field, and you can change it to any unique string that is a valid identifier. The default identifier (for the first Deformed Geometry interface in the model) is `dg`.

DOMAINS

Specify the domains (extent) for the interface.

Note: The Deformed Geometry interface requires a domain selection which covers all domains in which some physics is active.

FRAME SETTINGS

Specify the space coordinates of the base frame for the interface: the mesh frame. The default is upper case versions of the spatial coordinates followed by a lowercase m (for example, **Ym**).

The **Geometry shape order** setting controls the order of polynomials used for representing the geometry shape in the material frame. The same order is used for Lagrange shape functions defining the mesh position in domains where **Free displacement** has been activated.

FREE DEFORMATION SETTINGS

Select the smoothing type for freely deformed domains. Choose between Lagrange and Winslow smoothing. For details, see Smoothing Methods.

ADVANCED SETTINGS

Normally these settings do not need to be changed. See Advanced Settings.

Free Deformation

The **Free Deformation** feature constrains the mesh displacement only by the boundary conditions on the surrounding boundaries. The material frame displacement in the domain is obtained by solving a PDE (see Smoothing Methods).

DOMAINS

Specify the domains where you want free displacement.

INITIAL DEFORMATION

Give initial values for mesh displacements, relative to the material frame, in the **Initial mesh displacement** edit fields.

Prescribed Deformation

Use the **Prescribed Deformation** feature when you want to define the deformation of the material frame explicitly using expressions.

DOMAINS

Specify the domains where you want prescribed displacement.

PRESCRIBED MESH DISPLACEMENT

Specify expressions that define the deformation in the **Prescribed mesh displacement** edit fields. Select the check box to enable the prescribed mesh displacement in the directions where you want to use such a displacement condition. Use one expression per space coordinate.

Fixed Mesh

Use the **Fixed** feature to specify that the selected domains retain their original shape as defined by the geometry and original mesh. This is the default.

DOMAINS

Specify the domains where you want no displacement.

Prescribed Mesh Displacement

Use the **Prescribed Mesh Displacement** feature on the boundary of domains with free deformation. The material frame in the adjacent domain moves in accordance with the displacement.

BOUNDARIES

Specify the boundaries where you want prescribed mesh displacement.

COORDINATE SYSTEM SELECTION

Specify the coordinate system to use for specifying the mesh displacement. From the **Coordinate system** list select from:

- **Global coordinate system** (the default)
- **Boundary System** (a predefined normal-tangential coordinate system)
- Any additional user defined coordinate systems

PRESCRIBED MESH DISPLACEMENT

Select the check box for each coordinate direction where you want to prescribe the displacement. The default settings provide a fixed boundary (zero displacements in all directions).

CONSTRAINT SETTINGS

Select the **Use weak constraints** check box to use weak constraints instead of the default strong constraints.

Prescribed Mesh Velocity

Use the **Prescribed Mesh Velocity** feature on the boundary of domains with free displacement. The material frame in the adjacent domains moves in accordance with the velocity.

BOUNDARIES

Specify the boundaries where you want a prescribed velocity.

COORDINATE SYSTEM SELECTION

Specify the coordinate system to use for specifying the mesh displacement. From the **Coordinate system** list select from:

- **Global coordinate system** (the default)
- **Boundary System** (a predefined normal-tangential coordinate system)
- Any additional user defined coordinate systems

PRESCRIBED VELOCITY

Select the check box for each coordinate directory where you want to prescribe a velocity. The default settings provide zero velocities in all directions.

CONSTRAINT SETTINGS

Select the **Use weak constraints** check box to use weak constraints instead of the default strong constraints.

21

Solving

This section lists the various study types in COMSOL Multiphysics and explains the study steps and solver configurations. It also describes the major solvers and settings as well as batch jobs, parametric sweeps, and cluster computing. Also see the *COMSOL Multiphysics Reference Guide* for other supplementary information.

In this section:

Solver Studies and Study Types

Introduction

The process of solving a problem in COMSOL Multiphysics is divided into a hierarchy of varying details. On the coarsest level, containing the least amount of details, is the **Study** node. Selecting any of the predefined *study types* creates a **Study** node in the **Model Builder**'s model tree and also a corresponding *study step* and, in some cases, additional study steps. The study steps represent the next level of details. Most study steps are used to control the form of the equations, which physics interfaces are included and which meshes are used. These study steps correspond to a part of a solver sequence, which is the next level of details (there are also study steps that are used for, for example, cluster computing which correspond to a part of a job sequence). A solver sequence contains nodes that define variables to solve for, the solvers and their settings, and additional sequence nodes for storing the solution, for example. The solvers also contain nodes which allow you to control the solver settings at varying level of detail.

Solving a problem can amount to simply selecting a predefined study type and selecting **Compute**, which generates the corresponding study steps and the default solver sequence and compute the solution. But you may also control the settings at any level of detail. There might not be any predefined study type which corresponds to the simulations you are interested in doing, in which case you can add individual study steps. Also, by changing the settings in the solver sequence, you may for example control the desired tolerance for the error in the solution or which time-integration method or linear solver to use.

Adding a Study

There are two ways to add a study to a model:

- You can add a study when creating a model using the **Model Wizard**. The **Model Wizard** includes a **Select Study Type** page (it is the last page), where you can select from preset studies for the selected physics interfaces and other, custom studies.
- You can also add a study by right-clicking the root node in the **Model Builder** and choose  **Add Study**. This opens the **Select Study Type** page in the **Model Wizard**. The list of preset studies is then based on the physics interfaces in the model.

You can have more than one study (each generating one or more solutions) for different scenarios using the same geometry and physics.

Available Study Types

The following study types are available in the **Preset Studies** branch of the **Studies** list on the **Select Study Type** page (however, some of them are only available with certain add-on modules):

- **Stationary** (): For a stationary or steady-state situation where you can use a stationary solver and an equation formulation where all time derivatives vanish. This study type is also used for optimization problems which are constrained with a stationary PDE problem. Adds a **Stationary** study step to the **Study** node.
- **Time Dependent** (): For a time-dependent or transient simulation using a time-dependent solver for computing the solution over time. Adds a **Time Dependent** study step to the **Study** node.
- **Eigenvalue** (): This study uses a formulation to compute eigenvalues and eigenmodes using an eigenvalue solver. Adds an **Eigenvalue** study step to the **Study** node.
- **Eigenfrequency** (): Similar to an **Eigenvalue** study but computes the eigenfrequencies instead of the eigenvalues. Adds an **Eigenfrequency** study step to the **Study** node.
- **Frequency Domain** (): For a study in the frequency domain such as solutions to a wave equation or frequency response. Adds a **Frequency Domain** study step to the **Study** node.
- **Transient with Initialization** (): For time-dependent two-phase flow models that require an initialization of a level set function or phase field function. Adds a **Transient Initialization** study step followed by a **Time Dependent** study step to the **Study** node.
- **Time-Dependent Modal** (): For analyzing time-dependent wave problems using a modal solver, the **Time-Dependent Modal** study adds an **Eigenfrequency** study step followed by a **Time-Dependent Modal** study step to the **Study** node.
- **Frequency Domain Modal** (): For analyzing wave problems in the frequency domain using a modal solver, the **Frequency Domain Modal** study adds an **Eigenfrequency** study step followed by a **Frequency Domain Modal** study step to the **Study** node.
- **Mode Analysis** (): A mode analysis computes the modes for an acoustics or electromagnetic wave using an eigenvalue solver. Adds a **Mode Analysis** study step to the **Study** node.

- **Boundary Mode Analysis** (): A boundary mode analysis combines a mode analysis on a port (boundary) (which can represent, for example, a cross section of a waveguide) with a frequency domain study for the full geometry. Adds a **Boundary Mode Analysis** study step followed by a **Frequency Domain** study step to the **Study** node.
- **Linear Buckling** ( ): A linear buckling study for a structural model includes a **Stationary** study step followed by a **Linear Buckling** study step, which solves for the critical load factor using an eigenvalue solver.
- **Empty Study** (): An empty study that initially does not contain any study steps.

Adding Study Steps

The predefined study types correspond to the most commonly performed simulations for the different physics interfaces. However, sometimes you might want to do other investigations. For example, you may first want to solve a stationary problem for a physical quantity and use that solution as input to a time-dependent simulation for another physical quantity. You can do this by adding (by right-clicking on the study node) a **Stationary** study step followed by a **Time Dependent** study step and selecting in each study step which physics interfaces to include.

Most study steps correspond to a part of a solver sequence which includes a solver for the specific problem:

- **Stationary**: Generates equations without time derivatives. Corresponds to a **Stationary** solver or, if the check box **Optimization** is selected, an **Optimization** solver.
- **Time dependent:** Generates equations for transient simulations. Corresponds to an **Time** solver.
- **Eigenvalue**: Generates equations formulated for computing eigenvalues and eigenfunctions. Corresponds to an **Eigenvalue** solver.
- **Eigenfrequency**: Similar to an **Eigenvalue** study step. Corresponds to an **Eigenvalue** solver which is set to transform eigenvalues to eigenfrequencies.
- **Frequency Domain**: Generates stationary equations which are used for a frequency sweep. Corresponds to a **Stationary** parametric solver which is preset to linearize the equations. By selecting the check box **Use asymptotic waveform evaluation**, this study step corresponds to an **AWE** solver.

- **Time-Dependent Modal:** Generates equations for time-dependent modal analysis. Corresponds to a **Modal** solver with **Study type** set to **Time dependent**.
- **Frequency Domain Modal**: Generates equations for modal analysis in the frequency domain. Corresponds to a **Modal** solver with **Study type** set to **Frequency domain**.

There are some study steps that do not generate equations and which can only be used in combination with other study steps. These study steps do not correspond to any part of a solver sequence. Instead, they correspond to a part of a job sequence. The available study steps of this type are:

- **Parametric Sweep** (): Used to formulate a sequence of problems that arise when you vary some parameters. The problem at a fixed parameter value is defined by the rest of the study steps in the study. Generates a **Parametric** job sequence, unless the problem and parameters are such that the parametric sweep can be realized through a **Stationary** solver with a **Parametric** node, in which case such a solver is generated in the solver sequence.
- **Batch** (): Creates a job which can be run without the graphical user interface and which stores the solution on disk. Generates a **Batch** job sequence.
- **Cluster Computing** (): Used to solve the problem on a distributed-memory computer architecture. Generates a **Cluster Computing** job sequence and a **Batch** job sequence.

Generating Solver- and Job Configurations

When you have added the desired study steps to the study, a solver sequence (and possibly a job sequence) is generated by right-clicking on the study and selecting **Compute**. The generated solver sequence is placed under the **Solver Configurations** node and if a job sequence was generated, it is placed under the **Job Configurations** node.

It is possible to have several solver sequences under a **Solver Configuration** node. The particular sequence that is enabled and will be run when selecting **Compute** has a green border around its icon. An enabled sequence can be disabled by right-clicking it and selecting **Disable**. If no sequence is enabled when the study attempts to generate a sequence, a new sequence with default settings will be generated. Only one sequence per study can be enabled.

Computing a Solution

Computing a solution can be done in a few different ways. As mentioned the most straightforward way is to select **Compute** ( ) from the **Study** node. This always runs the enabled solver sequence with a green border around its icon. However, you can also run a disabled solver sequence by right-clicking it and selecting **Compute** ( ).

Sometimes, the default settings in the study steps are not sufficient to specify the details of how to obtain a solution. For example, you may want to change a tolerance or use a different time-stepping method. This is done in the attached solver sequences. If you want to run such a solver sequence, which has been manually altered, right-click on the solver sequence and select **Compute** ( ). Alternatively, enable the solver sequence, right-click on the study, and select **Compute** ( ).

If your solver sequence consists of several solvers you can run a particular solver by right-clicking it and select **Compute to Selected** ( ). Choosing instead the other option **Compute All** ( ) will run all solvers in the solver sequence.

Updating the Model

Right-clicking a solver sequence and selecting **Solution>Update** updates the model. This is useful in the following situations:

- You have added or edited variables and want to use these during postprocessing without having to solve the model again.
- You have edited the equations, material properties, or boundary conditions and want to use these without having to solve the model again.
- You have changed the element order and want to interpolate the solution onto the new elements for postprocessing purposes.
- You have remeshed or modified a geometry and want to interpolate the solution onto the new geometry for postprocessing purposes.

In all these cases the software passes, or interpolates, the solution to postprocessing, but does not recompute it to reflect any changes in variables, equations, mesh or geometry.

Computing the Initial Values

In some situations you may want to evaluate and plot values, expressions, or functions that need not be solved for, such as initial values. To make the initial values available

for results evaluation and visualization, generate a solver sequence, right-click the **Dependent Variables** node, and select **Compute to Selected** (). By default this plots the initial values of the variables solved for as specified by the **Field** subnode. (Note, however, that this will run everything above the **Dependent Variables** node, including any solvers, which might be computationally expensive.) More precisely, choosing **Initial values** from the **Keep solution** list makes the variables solved for available for results evaluation and visualization. To evaluate the initial values the settings of the corresponding **Method** list is used. The method **Initial expression** uses the settings of the **Initial Values** node, whereas **Solution** uses a solution object.

Also variables not solved for (for example, a solution from a previous time-dependent or parametric analysis) can be made available for results evaluation and visualization. To do so, select **Variables not solved for** from the **Keep solution** list and edit the corresponding **Method** list accordingly.

22

Results

To help you analyze results COMSOL Multiphysics provides numerous result-evaluation and visualization tools, including advanced graphics, data display, and export functions. The following section gives details about how to use these tools.

Processing and Analyzing Results

DATA SETS

Data sets contain the source of data for plotting, for example, by indicating a solution and geometry or by transforming another data set.

DATA SET	DESCRIPTION AND PLOT USE	GO TO
Average or Integral Evaluation	Creates a data set that computes the average of another data set, for example if you want to plot the average.	page 643
Contour 2D	To postprocess on 2D contour lines. Use this data set for 2D arrow plots, 2D line plots, and 1D global plots.	page 644
Cut Line 2D or 3D	To create lines through 2D and 3D geometry to visualize along the line. Use this data set to create 2D or 3D cross-sections line plots.	page 644
Cut Plane 3D	Plots on cut planes are made on 3D data sets and can be visualized in either 2D or 3D plot groups.	page 645
Cut Point 1D, 2D or 3D	Plot and evaluate a value in a certain point along time or along a parametric solution. Use this data set to create 1D, 2D or 3D cross-sections point plots.	page 646
Function 1D, 2D, and 3D	Create a data set that can evaluate functions on a domain.	page 647
Isosurface 3D	To visualize isosurfaces in 3D. Use this data set to create arrow surface plots, surface plots, and contour plots. In addition, the contour data set can be applied to the isosurface data set.	page 647
Maximum or Minimum Evaluation	Creates a data set that computes the maximum or minimum of another data set.	page 648
Mesh	To make a mesh available for visualization and results analysis. Use a Mesh feature in a plot group to get the actual plot.	page 648
Mirror 2D	To extend a solution defined on one side of an axis to the other side of the axis. Useful for visualization of a solution to an axisymmetric problem.	page 649
Parameterized Curve 2D or 3D	To visualize data along a general parameterized curve in 2D or 3D. Use this data set to create a line plot in its original dimension and as a line graph plot in 1D.	page 649

DATA SET	DESCRIPTION AND PLOT USE	GO TO
Parameterized Surface	To visualize data on a general parameterized surface. Use this data set to with a surface plot in its original dimension and as any plot type in 2D.	page 650
Parametric Extrusion 1D or 2D	To extend another data set by using a parameter, such as time, as a dimension.	page 650
Revolution 1D and 2D	Use a revolution data set to visualize a 2D or 1D axisymmetric solution in 3D and 2D, respectively.	page 651
Solution	To make solutions available for visualization and results analysis. Solvers create Solution data sets automatically.	page 652

PLOTS AND PLOT GROUPS

A plot group is a collection of plot features to display simultaneously in the **Graphics** window. You can enable or disable plots in a plot group to determine the most applicable final image for your model or project. Plot Groups and Plotting and Creating 1D, 2D, and 3D Plots. Use a combination of data sets and plot groups to create cross-section plots. Creating Cross-Section Plots.

PLOT NAME	1D, 2D OR 3D	DESCRIPTION	GO TO
1D Plot Group	1D	Use a 1D Plot Group to plot Global, Point, Line or Table graphs.	page 658
2D or 3D Plot Group	2D 3D	Use a 2D or 3D Plot Group to combine several plots into one to visualize all the plots simultaneously.	page 660
Arrow Line	2D 3D	Plot a vector quantity as arrows on lines or edges (3D). Add deformations or color expressions.	page 667
Arrow Surface	2D 3D	Visualize a vector quantity in arrows. Add deformations or color expressions.	page 669
Arrow Volume	3D	Visualize a vector quantity as arrows in a volume. Add deformations or color expressions.	page 671
Color Expression	2D 3D	To add coloring to the shapes defined by a plot. Add attributes to these 2D and 3D plots—arrow, contour, isosurface, particle tracing, and streamline.	page 665
Contour	2D 3D	Visualize a scalar quantity as a contour plot. Add deformations or color expressions.	page 673

PLOT NAME	1D, 2D OR 3D	DESCRIPTION	GO TO
Coordinate System Volume	3D	To plot coordinate systems for 3D models.	page 675
Coordinate System Surface	2D, 3D	To plot coordinate systems for 2D and 3D models.	page 675
Deformation	2D 3D	Deform the plot according to a vector quantity, for example, the displacement field in structural mechanics. Add this to most 2D and 3D plots—arrow, contour, isosurface, line, slice, streamline, surface, and volume.	page 663
Filter	2D 3D	Filter the element selection for a plot. Add this to 2D and 3D plots—arrow, contour, isosurface, line plot, slice, and volume.	page 667
Global	1D	Plot a global scalar quantity as a function of time or a parameter.	page 676
Height Expression	2D	Height attributes introduce a 3D height on a 2D surface plot. Only 2D surface plots support height attributes.	page 716
Isosurface	3D	Plot a scalar quantity as an isosurface plot. Add deformations or color expressions.	page 678
Line Plot	2D 3D	Plot a quantity on lines, boundaries (2D), or edges (3D). Add deformations.	page 683
Line Graph	1D	Plot a scalar quantity along a geometric line. The line can be a edge in the geometry, a parameterized curve, or a cut line.	page 680
Max/Min Volume	3D	To plot the maximum and minimum of an expression and the points there they are attained within the geometry.	page 686
Max/Min Surface Max/Min Line	2D 3D		
Mesh Plot	2D 3D	To display a mesh.	page 687
Particle Tracing	2D 3D	Visualize the trajectory of a massless particle subject to a flow field. Add a color expression.	page 688
Particle Tracing with Mass	2D 3D	Make a particle tracing plot considering the particle's mass. Add a color expression.	page 692
Point Graph	1D	Visualize the value in a point along time or a parameter value. The point can be a point in the geometry or a cut point.	page 699

PLOT NAME	1D, 2D OR 3D	DESCRIPTION	GO TO
Principal Stress Volume	3D	Plot principal stress and strain in structural mechanics models.	page 701
Principal Stress Surface	2D 3D	Plot principal stress and strain in structural mechanics models.	page 701
Scatter Surface	2D	Visualize a scalar quantity as scattered spheres. The radius of the spheres is proportional to the value of the quantity.	page 702
Scatter Volume	3D	Scatter plots can be used as alternatives to arrow plots or when you can to get a feeling for how entities correlate.	page 702
Slice	3D	Display a scalar quantity on slices inside a 3D domain.	page 705
Streamline	2D 3D	Plot a vector field as a streamline plot. Add deformations or color expressions.	page 708
Surface	2D 3D	Display a quantity on a 2D domain or a 3D boundary. Add Deformations. Add Height Expressions for 2D plots.	page 714
Table	1D	Table plots can be added to 1D plot groups and display data from a table with one line per output column.	page 718
Volume	3D	To display a quantity inside a 3D domain. Add a deformation.	page 718
*Cross-section point plot	1D 2D 3D	View an expression at an arbitrary set of spatial coordinates.	page 728
*Cross-section line plot	2D 3D	Create lines through 2D and 3D geometries to visualize along the line. Also explains how to use the interactive cross-section toolbar buttons.	page 730
*Cross-section surface plot	3D	Create planes through a 3D model in a 2D geometry to visualize on the plane. The cut plane corresponds to an orthogonal 2D coordinate system embedded in 3D. Also explains how to use the interactive cross-section toolbar buttons.	page 735

* Cross-section plots are not selected from this menu. Instead, you create it in two steps using data sets and plot groups to generate the plot or use interactive cross-section toolbar buttons.

DERIVED VALUES AND TABLES

Derived values define the evaluation of values of integrals, variables in points, or global variables. The evaluation results are stored in **Tables** and displayed in the **Results** window. Defining Derived Values and Tables.

DERIVED VALUE	DESCRIPTION	GO TO
Volume Integration	To evaluate an integral over a set of domains in 3D models.	page 738
Surface Integration	To evaluate an integral over a set of domains in 2D, 2D axisymmetric, or boundaries in 3D.	page 738
Line Integration	To evaluate an integral over a set of domains in 1D, boundaries in 2D, or edges in 3D. Integrate over any data set of the right dimension. For example, make a volume integration of a 2D revolve, or a surface integration of a cut plane.	page 738
Point Evaluation	To evaluate expressions or variables defined in a point.	page 741
Global Evaluation	To evaluate the numerical value of a global variable.	page 742

REPORTS

Create a variety of reports using data, images, or animations from plot groups. Creating Reports and Exporting Data.

REPORT	DESCRIPTION	GO TO
Animation	To define and export a movie or series of images based on a plot group. Play the animation in a web browser or use it in presentations or on a web site.	page 745
Data	Exports numerical data to file. Data export operates directly on data sets.	page 748
Image	To export plot images from 1D, 2D and 3D Plot Groups	page 750
Player	To create interactive animations.	page 752
Plot Data	To export a plot from a 1D, 2D, or 3D Plot Group.	page 751

VIEWS

For some plots, the **View** node displays under **Results**. For example, when 2D axisymmetric revolved plots or 2D cut plane plots for a 3D models are created.

For the majority of plots, go to the **Definitions** node to add views. See User-Defined Views for details.

Defining Data Sets

Data sets contain or refer to the source of data for plotting purposes. This can be the solution itself, a mesh, or some transformation or cut plane applied to other data sets—that is, create new data sets from other data sets.

All plots refer to data sets; the solutions are always available as default data sets. Characteristics of a data set include:

- Visualization mesh
- Mapping to a previous data set (except for solutions and meshes)
- Ability to refer the evaluation to the previous data set.

The base data sets map to a solution and geometry or some other source of data. An example of a transformation data set is Revolution 2D, which sweeps a 2D data set into 3D.

Note: **Cut Point**, **Cut Line**, and **Cut Plane** data sets are used in combination with **Plot Groups** and **Line**, **Point**, and **Surface** graphs to create cross-section plots. This is done differently than in previous versions of COMSOL Multiphysics.

- In the **Model Builder**, under **Results**, right-click **Data Sets**. Select an option from the list and continue defining each data set as described in Table 22-1.

TABLE 22-1: DATA SET TYPES

DATA SET	DESCRIPTION AND PLOT USE	GO TO
Average or Integral Evaluation	Creates a data set that computes the average of another data set, for example if you want to plot the average.	page 643
Contour 2D	To postprocess on 2D contour lines. Use this data set for 2D arrow plots, 2D line plots, and 1D global plots.	page 644
Cut Line 2D or 3D	To create lines through 2D and 3D geometry to visualize along the line. Use this data set to create 2D or 3D cross-sections line plots.	page 644
Cut Plane 3D	Plots on cut planes are made on 3D data sets and can be visualized in either 2D or 3D plot groups.	page 645

TABLE 22-1: DATA SET TYPES

DATA SET	DESCRIPTION AND PLOT USE	GO TO
Cut Point 1D, 2D, or 3D	Plot and evaluate a value in a certain point along time or along a parametric solution. Use this data set to create 1D, 2D or 3D cross-sections point plots.	page 646
Function 1D, 2D, or 3D	Create a data set that can evaluate functions on a domain.	page 647
Isosurface 3D	To visualize isosurfaces in 3D. Use this data set to create arrow surface plots, surface plots, and contour plots. In addition, the contour data set can be applied to the isosurface data set.	page 647
Maximum or Minimum Evaluation	Creates a data set that computes the maximum or minimum of another data set.	page 648
Mesh	To make a mesh available for visualization and results analysis. Use a Mesh feature in a plot group to get the actual plot.	page 648
Mirror 2D	To extend a solution defined on one side of an axis to the other side of the axis. Useful for visualization of a solution to an axisymmetric problem.	page 649
Parameterized Curve 2D or 3D	To visualize data along a general parameterized curve in 2D or 3D. Use this data set to create a line plot in its original dimension and as a line graph plot in 1D.	page 649
Parameterized Surface	To visualize data on a general parameterized surface. Use this data set to with a surface plot in its original dimension and as any plot type in 2D.	page 650
Parametric Extrusion 1D or 2D	To extend another data set by using a parameter, such as time, as a dimension.	page 650
Revolution 1D and 2D	Use a revolution data set to visualize a 2D or 1D axisymmetric solution in 3D and 2D, respectively.	page 651
Solution	To make solutions available for visualization and results analysis. Solvers create Solution data sets automatically. All Model Library files have examples.	page 652

Defining an Average or Integral Evaluation for a Data Set

Use an **Average** or **Integral Evaluation** data set to compute the average or integral of another data set.

1 In the **Model Builder**, under **Results**, right-click **Data Sets>Evaluation** and select **Average** ( ) or **Integral** ( ).

The **Average** or **Integral** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Data** select a **Data set**.

Note: If you want to integrate on a **Solution** data set, use a **Selection** to define the geometric entity (point, boundary, edge or domain) to integrate over. Right click the data set and select **Add Selection**.

---

3 Under **Settings**:

a Select a **Method**—**Auto**, **Integration**, or **Summation**.

- **Auto**—the default and computes the integral for fields by numerical integration and for reaction forces by summation. Predefined quantities for reaction forces use summation instead of integration. Also, if you specify an expression for the integrand using the `reacf` operator, the automatic setting chooses the summation method.
- **Integration**—the standard numerical integration method (quadrature).
- **Summation**—a summation method useful for computing reaction forces. The summation method finds all nodes on the boundary, evaluates the expression in the nodes and sum up the values. Reaction force variables are predefined in the structural mechanics interfaces.

b For **Integration order**, the check box is selected by default. COMSOL automatically determines an appropriate integration order for the expression. Clear the check box to make a different entry in the field. COMSOL then performs the integration elementwise using numeric quadrature of the selected order.

c Select a **Geometry level**—**Data set highest**, **Volume**, **Surface**, **Line**, or **Point**.

4 Click the **Plot** button ( ).

Defining a Contour Data Set (2D)

Use a **Contour** data set to postprocess on 2D contour lines. Contour lines cannot be parameterized in general, so only 2D arrow plots, 2D line plots, and 1D global plots can be used to visualize the data set.

1 In the **Model Builder**, under **Results**, right-click **Data Sets** and select **Contour** ( ).

The **Contour** page opens in the **Settings** window. A blank plot opens in the **Graphics** window.

2 Under **Data** select a **Data set**. Only 2D data sets can be used.

3 Under **Expression**:

a Click the **Replace Expression** ( ) or **Replace Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c Select the **Description** check box to enter a description (or edit the default).

4 Enter a **Level** for the data set. Only one contour level is supported in the data set. The contour plot supports multiple levels.

5 Click the **Plot** button ( ).

Defining a Cut Line Data Set (2D or 3D)

Use a **Cut Line** data set to create lines through 2D or 3D geometries to visualize along the line. All plots and results analysis features available in 1D are available for cut line data sets as well as 3D plots and results analysis features for edges. These data sets are used to create cross-section line plots.

1 In the **Model Builder**, under **Results**, right-click **Data Sets** and select **Cut Line 2D** ( ) or **Cut Line 3D** ( ).

The **Cut Line 2D** or **Cut Line 3D** page opens in the **Settings** window. A blank plot opens in the **Graphics** window.

2 Under **Data** select a **Data set**.

3 To specify the cut line, under **Line Data**, select **Two Points** or **Point and direction** from the **Line entry method** list.

4 If **Two Points** is selected:

   a Enter coordinates **x**, **y**, and **z** coordinates as required for the space dimension in the **Point 1** and **Point 2** fields.

   b Select the **Bounded by points** check box to constrain the line between the defined points.

   c *2D only:* Select the **Additional parallel lines** check box to define multiple lines for plotting or evaluation, for example. Enter **Distances** from the original line in the field.

5 If **Point and direction** is selected:

   a Enter **Point** and **Direction** values for the **x**, **y**, and **z** coordinates as required for the space dimension.

   b *2D only:* Select the **Additional parallel lines** check box to define multiple lines for plotting or evaluation, for example. Enter **Distances** from the original line in the field.

6 Click the **Plot** button ().

Defining a Cut Plane Data Set (3D)

Use a **Cut Plane** data set to visualize plots on cut planes using 2D or 3D plot groups. All plots and results analysis features available in 2D are available for cut plane data sets as well as for surfaces in 3D. The cut plane corresponds to an orthogonal 2D coordinate system embedded in 3D. This data set is used to create 3D cross-section surface plots.

1 In the **Model Builder**, under **Results**, right-click **Data Sets** and select **Cut plane** ().

   The **Cut Plane** page opens in the **Settings** window. A blank plot opens in the **Graphics** window.

2 Under **Data** select a **Data set**.

3 Under **Plane Data**, select **Quick** or **General** from the **Plane type** list.

   The **Plane Type** consists of the sets of planes orthogonal to the coordinate axes applicable for the model geometry—for example, **xy-planes**, **yz-planes**, and **zx-planes** in 3D.

4 If **Quick** is selected, select a set of cut planes orthogonal to a coordinate axis and specify the transverse coordinate by entering the location along the transverse coordinate axis:

   a Select **xy-planes**, **yz-planes**, or **zx-planes** from the **Plane** list.

   b Enter the **x-**, **y-**, or **z-coordinate** in the field based on the **Plane** selection.

5 If **General** is selected, select a **Plane entry method**—**Three points** or **Point and normal**.

   - If **Three points** is selected, enter **Point 1**, **Point 2**, and **Point 3** in the **x-**, **y-**, and **z-**coordinate fields.
   - If **Point and normal** is selected, enter **Point** and **Normal** data in the **x-**, **y-**, and **z-**coordinate fields.

6 Click the **Plot** button ( ).

Defining a Cut Point Data Set (1D, 2D, or 3D)

Use a **Cut Point** data set to visualize data on points in 1D, 2D, or 3D. You can plot and evaluate a value in a certain point along time or along a parametric solution, use the data set in its original dimension, or as cross-section data in 1D.

1 In the **Model Builder**, under **Results**, right-click **Data Sets** and select **Cut Point 1D** ( ), **Cut Point 2D** ( ), or **Cut Point 3D** ( ) to make a quantity at a point available for visualization and results analysis.

   The choice of 1D, 2D, or 3D only controls the type of input the data set accepts. For example, a 1D cut point can only be added to 1D data sets, 2D can only be added to 2D data sets and so forth. Either of these can be used to make a point graph plot along time and so forth.

   - Select **Cut Point 1D** for one point as cross-section data.
   - Select **Cut Point 2D** or **3D**, for example, to plot the value in a certain point along time or along a parametric solution and use the data set in its original dimension.

   The **Cut Point 1D, 2D**, or **3D** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Data** select a **Data set**.

3 Under **Point Data**, enter **x**, **y**, and **z** coordinates as required for the space dimension.

4 Select the **Snap to closest boundary** check box to snap the selected points to the grid.

5 Click the **Plot** button ( ).

Defining Function 1D, 2D, and 3D Data Sets

Use the **Function 1D**, **Function 2D**, or **Function 3D** features to create a data set that can evaluate functions on a domain. All functions in the same list as the selected function can be evaluated. The domain is an interval for Function 1D, a rectangle for Function 2D, and a block for Function 3D. The domain does not need to have the same dimension as the number of arguments to the function.

1 In the **Model Builder**, under **Results**, right-click **Data Sets** and select **Function>Function 1D**, **Function 2D**, or **Function 3D**.

The **Function 1D, 2D**, or **3D** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Select a **Function** from the list.

3 Under **Parameter Bounds** enter the following. Available fields are based on the function dimension:

a Enter a **Name**. The **First, Second**, and **Third Parameter** default names are `s`, `t`, and `u` respectively.

b For each parameter, enter a **Minimum** lower bound for the first, second, and third dimension of the domain.

c For each parameter, enter a **Maximum** upper bound for the first, second, and third dimension of the domain.

4 Enter a **Resolution**. This is the number of points into which each dimension is discretized. The defaults are 1000 (1D), 100 (2D), and 30 (3D).

5 Click the **Plot** button ().

Defining an Isosurface Data Set (3D)

Use an **Isosurface** data set to visualize isosurfaces in 3D. Isosurfaces cannot be parameterized in general so only arrow surface plots, surface plots, and contour plots can be used to visualize the data set. The contour data set can be applied to the isosurface data set. Only one isosurface level is supported in the isosurface data set. The isosurface plot supports multiple levels.

1 In the **Model Builder**, under **Results**, right-click **Data Sets** and select **Isosurface** ().

The **Isosurface** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Data** select a **Data set**.

3 Under **Expression**:

   a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression** in the field.

   b Select a **Unit**.

   c Select the **Description** check box to enter a description (or edit the default).

4 Enter a **Level** for the data set. Only one isosurface level is supported in the data set. The isosurface plot supports multiple levels.

5 Click the **Plot** button ( ).

Defining a Maximum or Minimum Evaluation for a Data Set

The **Maximum** or **Minimum** data set creates a data set that computes the maximum or minimum of another data set.

1 In the **Model Builder**, under **Results**, right-click **Data Sets>Evaluation** and select **Maximum** ( ) or **Minimum** ( ).

   The **Maximum** or **Minimum** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Data** select a **Data set**.

3 Under **Settings**, select a **Geometry level**—**Data set highest**, **Volume**, **Surface**, **Line**, or **Point**.

4 Click the **Plot** button ( ).

Defining a Mesh Data Set

Use a **Mesh** data set to make a mesh available for visualization. Use a **Mesh** feature in a plot group to get the actual plot.

1 In the **Model Builder**, under **Results**, right-click **Data Sets** and select **Mesh** ( ).

   The **Mesh** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Select a **Mesh** from the list.

3 Click the **Plot** button ( ).

Defining a Mirror 2D Data Set

Use a **Mirror 2D** data set to extend a solution defined on one side of an axis to the other side of the axis. This can be useful for visualization of a solution to an axisymmetric problem.

1 In the **Model Builder**, under **Results**, right-click **Data Sets** and select **Mirror 2D** ().

The **Mirror 2D** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Data** select a **Data set**.

3 Under **Axis Data**, select an **Axis entry method**—**Two points** or **Point and direction**. Select **Two points** to enter the mirror axis by specifying two points or **Point and direction** to specify the mirror axis by specifying one point and a direction vector. Then:

- If **Two points** is selected, enter coordinates in the **Point 1** and **Point 2** fields for **x** and **y** coordinates.
- If **Point and Direction** is selected, enter **Point** and **Direction** vector for **x** and **y** coordinates.

4 Click the **Plot** button ().

Defining a Parameterized Curve Data Set (2D or 3D)

Use a **Parameterized Curve** data set to visualize data along a general parameterized curve in 2D or 3D. The parameterized curve is visualized as a line plot in its original dimension and as a line graph plot in 1D.

1 In the **Model Builder**, under **Results**, right-click **Data Sets** and select **Parameterized Curve 2D** () or **Parameterized Curve 3D** ().

The **Parameterized Curve 2D** or **Parameterized Curve 3D** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Data** select a **Data set**.

3 Under **Parameter**, enter a **Name** and the **Minimum** and **Maximum** range of the parameter curve.

4 Under **Expressions**, enter functions for the **x**, **y**, and **z** coordinates as required for the space dimension.

5 In the **Resolution** field, enter the number of subdivisions of the parameter range. The default is 1000.

6 Click the **Plot** button ( ).

Defining a Parameterized Surface Data Set

Use a **Parameterized Surface** data sets to visualize data on a general parameterized surface. You can visualize the parameterized surface as a surface plot in its original dimension and as any plot type in 2D.

1 In the **Model Builder**, under **Results**, right-click **Data Sets** and select **Parameterized Surface** ( ).

The **Parameterized Surface** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Data** select a **Data set**.

3 Under **Parameters**, enter the following for both the **First parameter** and **Second parameter** fields.

a Enter a **Name**.

b Enter a range of the parameter in the **Minimum** and **Maximum** fields.

4 Under **Expressions**, enter functions for coordinates of the two parameters in the **x**, **y**, and **z** fields.

5 In the **Resolution** field, enter the number of subdivisions of the parameter ranges. the default is 200.

6 Click the **Plot** button ( ).

Defining a Parametric Extrusion Data Set (1D or 2D)

A **Parametric Extrusion** data set extends another data set by using a parameter, such as time, as a dimension.

1 In the **Model Builder**, under **Results**, right-click **Data Sets** and select **Parametric Extrusion 1D** or **2D** ( ).

The **Parametric Extrusion 1D** or **Parametric Extrusion 2D** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Data** select a **Data set**.

3 Under **Settings**:

a Select the **Level scale factor** check box to enter a value or edit the field.

b The **Separate levels** check box is selected by default. Click to clear the check box.

4 Click the **Plot** button ( ).

Defining a Revolution Data Set (1D or 2D)

Use a **Revolution** data set to visualize a 2D or 1D axisymmetric solution in 3D and 2D, respectively. All plot group features in 3D or 2D are available for visualization through the revolution data set.

1 In the **Model Builder**, under **Results**, right-click **Data Sets** and select **Revolution 1D** ( ) or **Revolution 2D** ( ).

The **Revolution 1D** or **Revolution 2D** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Data** select a **Data set**.

3 Under **Axis Data**, for **1D**, enter a value in the **x** field to specify the revolution point. For **2D,** from the **Axis entry method** list, select **Two points** to enter the revolution axis by specifying two points or **Point and direction** to specify the revolution axis by specifying one point and a direction vector. Then:

- If **Two points** is selected, enter coordinates in the **Point 1** and **Point 2** fields for **x** and **y** coordinates.
- If **Point and Direction** is selected, enter **Point** and **Direction** vector for **x** and **y** coordinates.

4 For both 1D and 2D, under **Revolution Layers**:

a In the **Layers** field, enter the number of subdivisions about the revolution axis.

b In the **Revolution angle** field, enter the angle to revolve the model to see into the geometry.

5 For both 1D and 2D, under **Advanced**:

a In the **Space variables** section, enter or edit the variable names for the revolved coordinate system. For 1D, enter **x** and **y** values. for 2D, enter **x**, **y**, and **z** values.

b Enter or edit the **phi** value. Phi is the name of the angle variable in the revolved coordinate system.

6 Click the **Plot** button ( ).

Defining a Solution Data Set

The **Solution** data sets make solutions available for visualization and results analysis. Solvers automatically create **Solution** data sets.

1 In the **Model Builder**, under **Results**, right-click **Data Sets** and select **Solution** ().

The **Solution** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Select a **Solution** to make available for visualization and results analysis.

3 Select the **Geometry** to perform visualization and results analysis on—**Mesh**, **Material**, or **Spatial**.

4 Select the **Frame** to evaluate the coordinates in. This list is only visible when there are several frames in the model.

5 Enter a value for the **Solution at angle (phase)**.

Use the **Solution at angle (phase)** option to integrate complex-valued expressions by multiplying the solution by a factor of `exp(i*pi*phase/180)` prior to expression evaluation.

6 Click the **Plot** button ().

Plot Groups and Plotting

A plot group contains one or more plots (for example, combining a surface plot and a streamline plot) using the same data set, such as a solution. You can define plot groups for 1D, 2D, and 3D plots. You then create individual or a series of plots in a plot group. Information in the form of data and images can be used to generate a report, or exported.

Before you begin creating plot groups and plots, this section provides you with basic plotting information about color tables, adding expressions, accurate derivative recovery, and defining vector inputs.

About Color Tables

For many plot types you can select the color table to use for coloring the surfaces, boundaries, contours, streamlines, slices, and so on. These color tables use 1024 colors each.

Note: The best way to compare the color tables is to open a model and experiment with the options.

RAINBOW AND RAINBOW LIGHT

Rainbow is the default for plots that support color tables. The color ordering corresponds to the wavelengths of the visible part of the electromagnetic spectrum. It starts at the small-wavelength ends with dark blue. The colors range through shades of blue, cyan, green, yellow, and red. The disadvantage of this color table is that people with color vision deficiencies (affecting up to 10% of technical audiences) cannot see distinctions between reds and greens.

RainbowLight is similar but uses lighter colors.

THERMAL AND THERMALEQUIDISTANT

Thermal colors range from black through red and yellow to white, corresponding to the colors iron takes as it heats up.

ThermalEquidistant is similar but uses equal distances from black to red, yellow, and white, which means that the black and red regions become larger compared to the Thermal color table.

CYCLIC

The **Cyclic** color table is useful for displaying periodic functions because it has a sharp color gradient —it varies the hue component of the hue-saturation-value color model, keeping the saturation and value constant (equal to 1). The colors begin with red, then pass through yellow, green, cyan, blue, magenta, and finally return to red.

WAVE AND WAVELIGHT

The **Wave** color table is useful for data that naturally have positive and negative attributes in addition to a magnitude. An example of a double-ended scheme, it ranges linearly from blue to light gray, and then linearly from white to red. When the range of the visualized quantity is symmetric around zero, the color red or blue indicates whether the value is positive or negative, and the saturation indicates the magnitude.

People with color vision deficiencies can interpret the **Wave** color table because it does not use red-green-gray distinctions, making it efficient for 99.98% of the population.

WaveLight is similar and ranges linearly from a lighter blue to white (instead of light gray) and then linearly from white to a lighter red.

TRAFFIC AND TRAFFICLIGHT

The **Traffic** color table spans from green through yellow to red. **TrafficLight** is similar but uses lighter colors.

DISCO AND DISCOLIGHT

The **Disco** color table spans from red through magenta and cyan to blue. **DiscoLight** is similar but uses lighter colors.

GRAYSCALE

The **Grayscale** color table uses the linear gray scale from black to white—the easiest palette to understand and order.

These plots are often easier to use for publication in journal articles. People can also better perceive structural detail in a gray scale than with color. Use this plot to increase the probability that a plot is interpreted correctly by people with color vision deficiencies.

GRAYPRINT

Grayprint varies linearly from dark gray (0.95, 0.95, 0.95) to light gray (0.05, 0.05, 0.05). Choose this to overcome two difficulties that the **GrayScale** has when used for printing on paper—it gives the impression of being dominated by dark colors, and white is indistinguishable from the background.

CUSTOM COLOR TABLES

You can also add your own continuous and discrete color scales as files with RGB data. See Color Tables in the *COMSOL Multiphysics Reference Guide* for more information (or see Where Do I Access the Documentation and Model Library?).

Expressions and Predefined Quantities

When plotting and evaluating results, COMSOL Multiphysics provides a large number of predefined quantities that are specific to the physics interfaces in the model as well as general quantities for the geometry, coordinate systems, and mesh.

COMSOL Multiphysics does not limit the results calculations to predefined quantities; you can plot and evaluate any function by entering the corresponding expression. You can combine user defined variables, physics interface variables, standard variables, and mathematical and logical functions and numbers. The **Expression** edit field or list is available for most plot types as well as for integration and data display and evaluation. You can type any expression directly in the edit field or insert variables from a list of predefined quantities that you open by pressing Ctrl+Space or by clicking the **Insert Expression** () button.

In the **Expression** sections in the **Settings** windows for plot nodes you can:

- Click the **Replace Expression** () button to select a predefined quantity and replace the contents of the **Expression** edit field with the corresponding variable
- Click the **Insert Expression** () button to insert the corresponding variable at the current position in the **Expression** edit field.

EVALUATION OF UNDEFINED QUANTITIES

During the evaluation of expressions, by default COMSOL Multiphysics does not report partially undefined quantities, and the program plots a quantity where it is defined. If a results quantity is undefined everywhere, an error occurs for all plot types.

ACCESSING OTHER SOLUTIONS THAN THE SELECTED SOLUTION

When you use the names of the dependent variables in a results expression, COMSOL Multiphysics uses the solution associated with the selected parameter value, eigenvalue, or time for a parametric analysis, eigenvalue analysis, or time-dependent analysis, respectively. To access other solutions in the model, use the `with` operator.

Accurate Derivative Recovery

Plotting and evaluating stresses or fluxes boils down to evaluating space derivatives of the dependent variables. By default, postprocessing of a derivative like `ux` or `uxx` (first and second derivatives of u with respect to x) is done by evaluating the derivative of the shape functions used in the finite element approximation. These values have poorer accuracy than the solution `u` itself. For example, `uxx=0` if `u` has linear elements. COMSOL Multiphysics then evaluates the derivatives (and `u` itself) using a polynomial-preserving recovery technique by Z. Zhang (see Ref. 1). The recovery is only applied on variables that are discretized using Lagrange shape functions.

The polynomial-preserving recovery is a variant of the superconvergent patch recovery by Zienkiewicz and Zhu that forms a higher-order approximation of the solution on a patch of mesh elements around each mesh vertex. For regular meshes, the convergence rate of the recovered gradient is $O(h^{p+1})$—the same as for the solution itself. However, near boundaries, the accuracy is not as good, and it might even be worse than without recovery. Postprocessing is about 2–5 times slower when using accurate derivative recovery. For this reason, prefer using recovery of stresses or fluxes for surface plots, contour plots, slice plots, and isosurface plots. Use recovery to a lesser extent for domain plots, boundary plots, and edge plots.

By default, the accurate derivative recovery smoothes the derivatives within each group of domains with equal settings. Thus, there is no smoothing across material discontinuities. You find the setting for accurate derivative recovery in the plot node's **Settings** windows' **Quality** section. Due to performance reason, the default value for **Recover** list is **Off** (that is, no accurate derivative recovery). Select **Within domains** to smooth the derivatives within each group of domains with equal settings. Select **Everywhere** to smooth the derivatives across the entire geometry.

Reference

1. A. Naga and Z. Zhang, "The polynomial-preserving recovery for higher order finite element methods in 2D and 3D," *Discrete and continuous dynamical systems—series B*, vol. 5, pp. 769–798, 2005.

Vector Inputs and Parametric Sweep Studies

For transient time dependent problems, and when the parameter values are selected via **Interpolated times**, you can use the **Vector Input** window to enter expressions for the **Times**. See Entering Ranges and Vector-Valued Expressions for detailed information.

When setting parameters for parametric sweep studies in the Results node, the available settings depend on the problem type. A time-dependent problem, for example, allows you to select both time steps and parameter values. Similarly, an eigenvalue problem contains both eigenvalue and parameter settings. In results features, the time and eigenvalue settings are referred to as the inner solutions. Thus, in a graph plot for a parametric eigenvalue solution, for example the "Solution" setting in X-Axis Data controls whether you want the inner (that is to say, eigenvalue) or outer (that is to say, parametric) solutions on the x-axis.

Creating 1D, 2D, and 3D Plots

This section describes how to:

- Define 1D, 2D, and 3D plot groups
- Add and define the plots in each plot group
- Add deformations and color expressions to a plot
- Combine plots in a plot group.

Note: At any time during plot creation, you can click the **Plot** button () to visualize a data set or plot.

Note: The time related settings only display on the interfaces for time dependent models.

Defining a 1D Plot Group and Adding Plots

Note: It is not possible to create plots in a higher dimension than the data set you want to visualize. For example, you cannot make a 2D plot group to visualize a solution for a 1D model. 1D plot groups can be used for all models.

---

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **1D Plot Group** ().

The **1D Plot Group** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data,** select a **Data set**.

a For **Parametric Sweep** studies (see Solver Studies and Study Types) also select an option from the **Select via** list —**Stored output times** or **Interpolated times**. If

Stored output times is selected, the **Times** section is auto-filled with information from the **Data set** selected.

b If **Interpolated times** is selected, enter **Times** or click the **Vector Input** button ( ) to select and define specific times. See Entering Ranges and Vector-Valued Expressions for details.

4 Under **Plot Settings**:

a The **Title** check box is selected by default. Click to clear if required.

b Enter **x-axis** and **y-axis labels**.

5 Under **Axis and Grid Settings**:

a In the **Axis settings** section, select the **Manual axis limits** check box to edit the **x**- and **y minimum** and **maximum** limits already assigned based on the **Data Set**.

b As required, select one or all of these check boxes: **Preserve aspect ratio**, **x-axis log scale**, **y-axis log scale**.

c In the **Grid setting** section, select the **Manual spacing** check box to edit the predefined **x spacing** and **y spacing** fields.

d Enter values as required in the **Extra x** and **Extra y** fields.

6 Under **Plot in Window Settings**:

a Select a **Plot window**. **New window** is the default.

b Select the **Window title** check box to enter a custom title, which is then available in the **Plot window** list for all models. For example, if you add a window title, `Global plot`, when you right-click the **2D** or **3D Plot Group** and select **Plot in Window**, a new window with this name displays in the **Graphics** window.

7 To add 1D plots, right-click the **1D Plot Group** ( ) node and continue defining each type of plot as required. See Table 22-2.

TABLE 22-2: 1D PLOT TYPES

PLOT NAME	DESCRIPTION	GO TO
Global	Plot a global scalar quantity as a function of time or a parameter.	page 676
Line Graph	Plot a scalar quantity along a geometric line. The line can be a edge in the geometry, a parameterized curve, or a cut line.	page 680
Point Graph	Visualize the value in a point along time or a parameter value. The point can be a point in the geometry or a cut point.	page 699

TABLE 22-2: 1D PLOT TYPES

PLOT NAME	DESCRIPTION	GO TO
Table Plot	Table plots can be added to 1D plot groups and display data from a table with one line per output column.	page 718
*Cross-section point plot	A point cross-section plot makes it easy to view an expression at an arbitrary set of spatial coordinates and results in a line plot.	page 728

* Cross-section plots are not selected from this menu. Instead, you create it in two steps using data sets and plot groups to generate the plot.

Defining a 2D or 3D Plot Group and Adding Plots

Note: It is not possible to create plots in a higher dimension than the data set you want to visualize. For example, you cannot make a 3D plot group to visualize a solution for a 3D model. For 3D models, all plot groups are available. Conversely, some data sets add dimensions to their parent, for example, Revolve 2D. In this case, the solutions is 2D, but the data set is 3D, which can be used for 3D plot groups.

Note: Similarly, 2D plot groups can be used for 3D solutions when there are e.g. cut planes or other data sets that remove a dimension.

Note: The time related settings only display on the interfaces for time dependent models.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and select **2D Plot Group** ( ) or **3D Plot Group** ( ).

The **2D Plot Group** or **3D Plot Group** page opens in the **Settings** window and a node is added to the **Model Builder**.

Under **Data** select a **Data set**, for **Parametric Solver** and **Parametric Sweep** studies, select **Parameter values** as required (see Solver Studies and Study Types). For time-dependent, Parametric Sweep problems, select a **Time**.

3 Under **Plot Settings**:

   a Select a **View**. The default is **Automatic**. Other **Views** are defined under **Definitions**.

   b Select the check box to enter a **Title**.

   c If required, select the **Plot data set edges** check box.

   d Select a **Color** or **Custom** to choose a different color.

4 Under **Plot in Window Settings**:

   a Select a **Plot window**. **New window** is the default.

   b Select the **Window title** check box to enter a custom title, which is then available in the **Plot window** list for all models. For example, if you add a window title, `Contour plot with Color Expression`, when you right-click the **2D** or **3D Plot Group** and select **Plot in Window**, a second tab with this name displays in the **Graphics** window.

5 To add plots to a group, right-click the **2D Plot Group** ( ) or **3D Plot Group** ( ) nodes to select as many as required. Each plot group can have several plots combined to create a meaningful representation of the data.

6 Continue defining each plot as described. See Table 22-3 for a list of plot types.

TABLE 22-3: 2D AND 3D PLOT TYPES

PLOT NAME	2D OR 3D	DESCRIPTION	GO TO
Arrow Line	2D 3D	Plot a vector quantity as arrows on lines or edges (3D). Add deformations or color expressions.	page 667
Arrow Surface	2D 3D	Visualize a vector quantity in arrows. Add deformations or color expressions.	page 669
Arrow Volume	3D	Visualize a vector quantity as arrows in a volume. Add deformations or color expressions.	page 671
Color Expression	2D 3D	To add coloring to the shapes defined by a plot. Add attributes to these 2D and 3D plots—arrow, contour, isosurface, particle tracing, and streamline.	page 665
Contour	2D 3D	Visualize a scalar quantity as a contour plot. Add deformations or color expressions.	page 673
Coordinate System Volume	3D	To plot coordinate systems for 3D models.	page 675
Coordinate System Surface	2D, 3D	To plot coordinate systems for 2D and 3D models.	page 675

TABLE 22-3: 2D AND 3D PLOT TYPES

PLOT NAME	2D OR 3D	DESCRIPTION	GO TO
Deformation	2D 3D	Deform the plot according to a vector quantity, for example, the displacement field in structural mechanics. Add this to most 2D and 3D plots—arrow, contour, isosurface, line, slice, streamline, surface, and volume.	page 663
Filter	2D 3D	Filter the element selection for a plot. Add this to 2D and 3D plots—arrow, contour, isosurface, line plot, slice, and volume.	page 667
Height Expression	2D	Height attributes introduce a 3D height on a 2D surface plot. Only 2D surface plots support height attributes.	page 716
Isosurface	3D	Plot a scalar quantity as an isosurface plot. Add deformations or color expressions.	page 678
Line	2D 3D	Plot a quantity on lines, boundaries (2D), or edges (3D). Add deformations.	page 683
Max/Min Volume	3D	To plot the maximum and minimum of an expression and the points there they are attained within the geometry.	page 686
Max/Min Surface Max/Min Line	2D 3D		
Mesh	2D 3D	To display a mesh.	page 687
Particle Tracing	2D 3D	Visualize the trajectory of a massless particle subject to a flow field. Add a color expression.	page 688
Particle Tracing with Mass	2D 3D	Make a particle tracing plot considering the particle's mass. Add a color expression.	page 692
Principal Stress Volume	3D	Plot principal stress and strain in structural mechanics models.	page 701
Principal Stress Surface	2D, 3D	Plot principal stress and strain in structural mechanics models.	page 701
Scatter Surface	2D	Visualize a scalar quantity as scattered spheres. The radius of the spheres is proportional to the value of the quantity.	page 702
Scatter Volume	3D	Scatter plots can be used as alternatives to arrow plots or when you can to get a feeling for how entities correlate.	page 702
Slice	3D	Display a scalar quantity on slices inside a 3D domain.	page 705

TABLE 22-3: 2D AND 3D PLOT TYPES

PLOT NAME	2D OR 3D	DESCRIPTION	GO TO
Streamline	2D 3D	Plot a vector field as a streamline plot. Add deformations or color expressions.	page 708
Surface	2D 3D	Display a quantity on a 2D domain or a 3D boundary. Add deformations or a filter. Add height expressions for 2D plots.	page 714
Volume	3D	To display a quantity inside a 3D domain. Add a deformation.	page 718
*Cross-section point plot	1D 2D 3D	View an expression at an arbitrary set of spatial coordinates.	page 728
*Cross-section line plot	2D 3D	Create lines through 2D and 3D geometries to visualize along the line. Also explains how to use the interactive cross-section toolbar buttons.	page 730
*Cross-section surface plot	3D	Create planes through a 3D model in a 2D geometry to visualize on the plane. The cut plane corresponds to an orthogonal 2D coordinate system embedded in 3D. Also explains how to use the interactive cross-section toolbar buttons.	page 735

* Cross-section plots are not selected from this menu. Instead, you create it in two steps using data sets and plot groups to generate the plot or use interactive cross-section toolbar buttons.

Adding Deformations, Color Expressions, and Filters to Plots

Add attributes to a plot to modify the plot's behavior—**Deformation** ( ) attributes deform a plot, **Color Expression** ( ) attributes modify the color of a plot, and element selection is selected using the **Filter** attribute. Select an appropriate **Color Table** for your plot based on your audience and what you plan to do with the final analysis.

ADDING DEFORMATIONS TO 2D AND 3D PLOTS

Add a **Deformation** ( ) to deform the plot according to a vector quantity, for example, the displacement field in structural mechanics. Add attributes to most 2D and 3D plots—arrow, contour, isosurface, line, particle tracing, slice, streamline, surface, and volume. By default, COMSOL scales the deformation to 10% of the geometry. This is useful for models that include structural mechanics and where displacements are computed. See Figure 22-1 and Figure 22-2 for examples from the Earth Science Module Model Library.

1 In the **Model Builder**, add and define a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node (for example, **Arrow Surface**) and select **Deformation** ( ).

The **Deformation** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Define the **Expression**:

a Enter or select **x**, **y**, and **z-components** as required for the space dimension. Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model.

b To enter a **Description** (or edit the default), select the check box.

3 Under **Scale**, select the **Scale factor** check box to edit the default. Enter a value.

4 Click the **Plot** button ( ), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

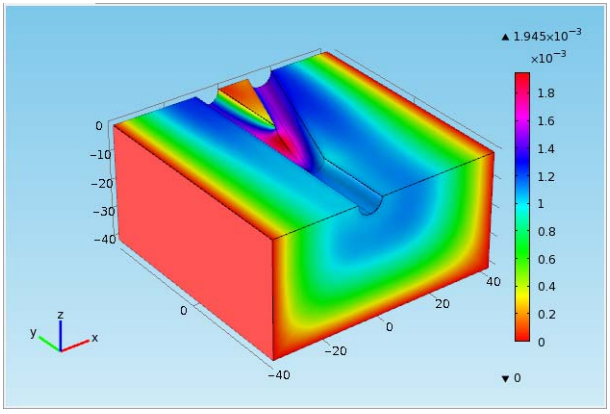


Figure 22-1: A 3D surface plot without deformation and using a cyclic color table. From the Model Library: Poroelastic Flow and Deformation: Open-Hole Multilateral Well.

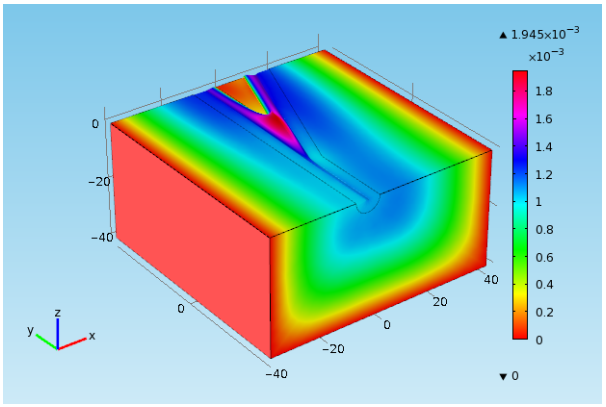


Figure 22-2: A 3D surface plot with deformation. From the Model Library: Poroelastic Flow and Deformation: Open-Hole Multilateral Well.

ADDING COLOR EXPRESSIONS TO 2D AND 3D PLOTS

Use **Color Expressions** ( ) to add coloring to the shapes defined by a plot. Add attributes to these 2D and 3D plots—arrow, contour, isosurface, particle tracing, and streamline. See Figure 22-3 and Figure 22-4 for examples from the Earth Science Module Model Library

1 In the **Model Builder**, add and define a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node (for example, **Streamline**) and select **Color Expression** ( ).

   The **Color Expression** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Expression**:

   a Click the **Replace Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

   b Select a **Unit**.

   c To enter a **Description** (or edit the default), select the check box.

3 Under **Range**, manually override the **Minimum** and **Maximum color range** or **data range**. Click to select the check box, then specify a range using the fields or use the sliders to control values.

4 Under **Coloring and Style**:

a Select a **Color Table**. If the default (**Rainbow**) is not suitable for the plot, try other options.

b The **Color legend** is selected by default. Click to clear the check box if required. The legend displays to the right of the plot.

5 Click the **Plot** button ( ), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

.

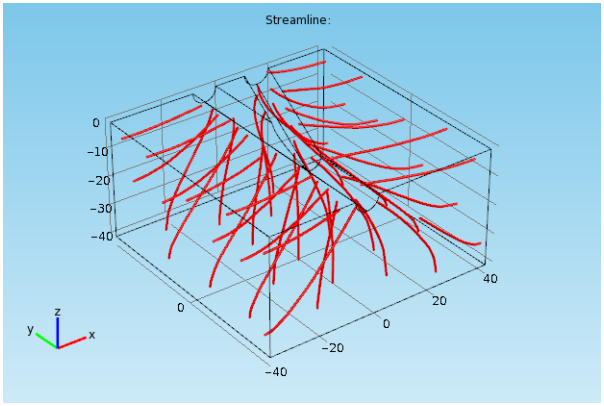


Figure 22-3: A 3D streamline plot with no color expression. From the Model Library: Poroelastic Flow and Deformation: Open-Hole Multilateral Well.

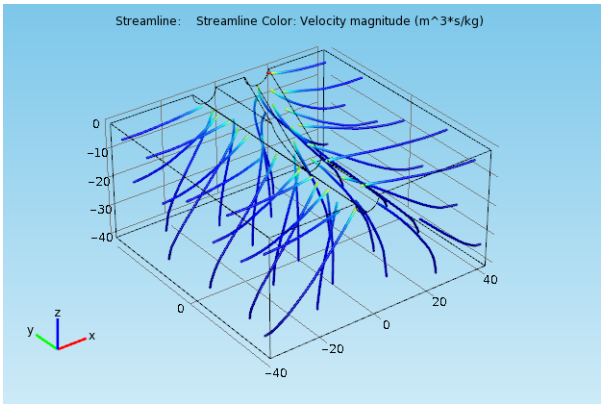


Figure 22-4: A 3D streamline plot with color expression. From the Model Library: Poroelastic Flow and Deformation: Open-Hole Multilateral Well.

ADDING FILTERS TO 2D AND 3D PLOTS

Add a **Filter** ( ) to 2D and 3D plots—arrow, contour, isosurface, line plot, slice, and volume.

1. In the **Model Builder**, add and define a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node (for example, **2D Surface**) and select **Filter** ( ).

   The **Filter** page opens in the **Settings** window and a **Filter** node is added to the **Model Builder**.
2. Under **Element Selection**, enter a **Logical expression for inclusion**.
3. Select the **Element nodes to fulfill expression**—**All**, **At least one**, or **At least one but not all**.
4. Click the **Plot** button ( ), or right-click the node and select **Plot**.

   The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

Creating a 2D or 3D Arrow Line Plot

Plot a vector quantity as arrows on lines (2D) or lines and edges (3D). See Figure 22-5, and Figure 22-6 for examples from the AC/DC Module Model Library. You can use a contour data set.

1. Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node and select **Arrow Line** ( ).

The **Arrow Line** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data**, select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Expression**:

a Enter or select **x**, **y**, and **z** coordinates as required for the space dimension. Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model.

b To enter a **Description** (or edit the default), select the check box.

5 Under **Coloring and Style**:

a Select an **Arrow length**—select **Proportional** to use arrows with lengths proportional to the magnitude of the field, or **Normalized** for equal sized arrows.

b Enter a **Scale factor** or use the sliders to select a value.

c Select an arrow **Color** or **Custom** to choose a different color.

6 If required, add a **Deformation**, **Color Expression** and/or **Filter**.

7 Click the **Plot** button ( ), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

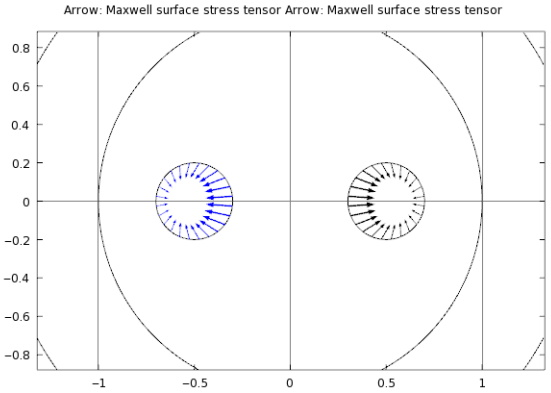


Figure 22-5: A 2D arrow line plot. From the Model Library: Electromagnetic Forces on Parallel Current-Carrying Wires.

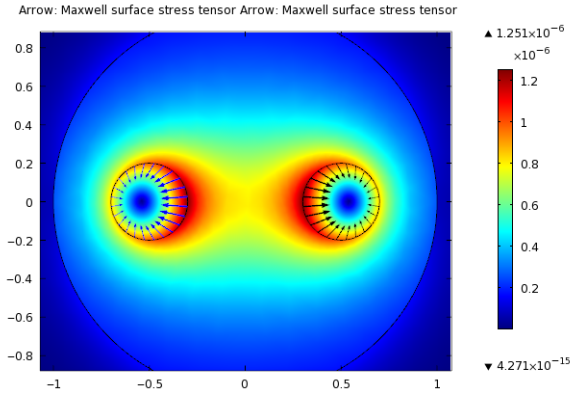


Figure 22-6: A 2D arrow line plot combined with a surface plot. From the Model Library: Electromagnetic Forces on Parallel Current-Carrying Wires.

Creating a 2D or 3D Arrow Surface Plot

Use a **2D** or **3D Arrow Surface** to plot a vector quantity as arrows on a surface. See Figure 22-7, and Figure 22-8 and for examples from the COMSOL Multiphysics and Chemical Reaction Engineering or Earth Science Model Libraries. Use an isosurface data set.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node and select **Arrow Surface** ( ).

The **Arrow Surface** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Expression**:

a Enter or select **x**, **y**, and **z-components** as required for the space dimension. Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model.

b To enter a **Description** (or edit the default), select the check box.

Note: Arrow positioning is available when the plot dimension is the same as the highest dimension available, so always for 3D arrow volume plots. Arrow positioning is also available for 2D arrow surface plots in 2D plot groups but NOT for 2D arrow surface plots in 3D plot groups.

5 Under **Arrow Positioning**, in the **x grid points** and **y grid points** fields select an **Entry method**:

- If **Number of Points** is selected, enter **Points**.
- If **Coordinates** is selected, enter **Coordinates**.

6 Under **Coloring and Style**:

a Select an **Arrow length**—select **Proportional** to use arrows with lengths proportional to the magnitude of the field, or **Normalized** for equal sized arrows.

b Enter a **Scale factor** or use the sliders to select a value.

c Select an arrow **Color** or **Custom** to choose a different color.

7 If required, add a **Deformation**, **Color Expression** and/or **Filter**.

8 Click the **Plot** button ( ), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

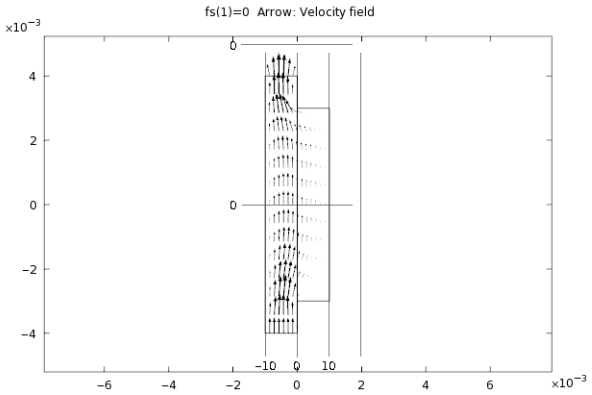


Figure 22-7: A 2D arrow surface plot. From the Earth Science Model Library: Forchheimer Flow.

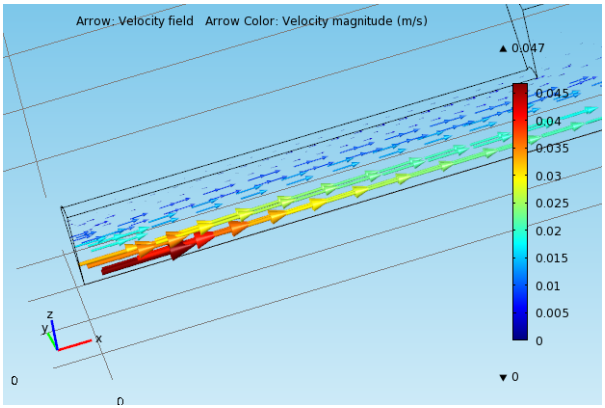


Figure 22-8: A 3D arrow surface plot with color expression.

Creating a 3D Arrow Volume Plot

Use a 3D **Arrow Volume** plot visualize a vector quantity as arrows in a volume. See Figure 22-9 and Figure 22-10 for examples from the Earth Science Module Model Library.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **3D Plot Group** ( ). Right-click the plot node and select **Arrow Volume** ( ).

The **Arrow Volume** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Expression**:

a Enter or select **x**-, **y-**, and **z-components**. Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model.

b To enter a **Description** (or edit the default), select the check box.

5 Under **Arrow Positioning**, in the **x grid points**, **y grid points**, and **z grid points** fields select an **Entry method**:

- If **Number of Points** is selected, enter **Points**.
- If **Coordinates** is selected, enter **Coordinates**.

6 Under **Coloring and Style**:

   a Select an **Arrow length**—select **Proportional** to use arrows with lengths proportional to the magnitude of the field, or **Normalized** for equal sized arrows.

   b Enter a **Scale factor** or use the sliders to select a value.

   c Select an arrow **Color** or **Custom** to choose a different color.

7 If required, add a **Deformation**, **Color Expression** and/or **Filter**.

8 Click the **Plot** button (), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

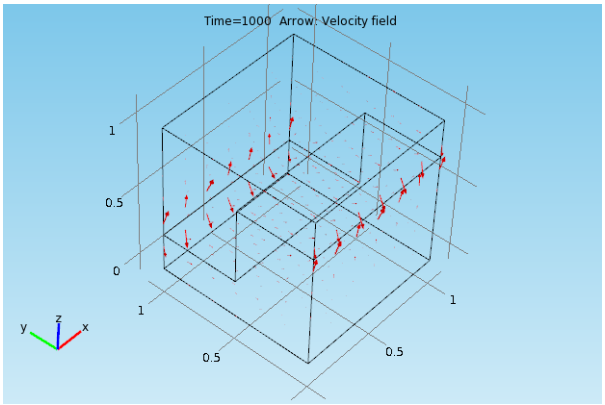


Figure 22-9: A 3D arrow volume plot. From the Model Library: Discrete Fracture.

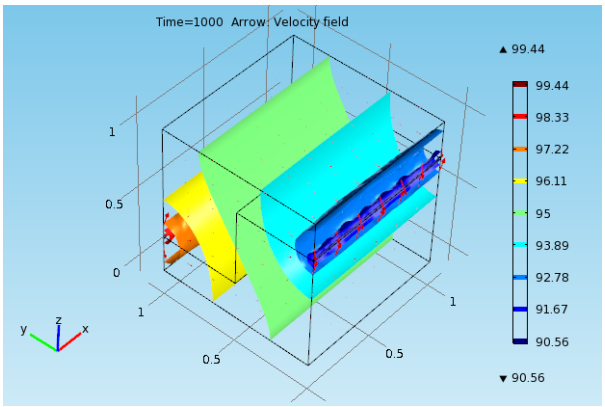


Figure 22-10: A 3D combined arrow volume and isosurface plot. From the Model Library: Discrete Fracture.

Creating a 2D or 3D Contour Plot

Visualize a scalar quantity as a **Contour** plot in 2D or 3D. A contour plot displays a quantity as a set of colored contour lines. The selected quantity has a constant value on these contour lines, optionally with a 3D height. See Figure 22-11 for an example from the Earth Science Module Model Library.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node and select **Contour** ( ).

The **Contour** page opens in the **Settings** window and a **Contour** node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Expression**, click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

a Select a **Unit**.

b To enter a **Description** (or edit the default), select the check box.

5 Select the number of contour **Levels**. From the **Entry method** list select **Number of Levels** or **Levels**.

- If **Number of Levels** is selected, enter the **Total levels** (the number of bands of constant color contours) required.
- If **Levels** is selected, enter the number of contour **Levels**.

6 Under **Coloring and Style**:

a Select a **Contour type**—**Lines** or **Filled**.

b Select a **Coloring**—**Color Table** (default) or **Uniform**. If **Uniform** is selected, select a **Color** or **Custom** to choose a different color.

c Select a **Color Table**. If the default (**Rainbow**) is not suitable for the plot, try other options.

d The **Color legend** is selected by default. Click to clear the check box if required.

The legend displays to the right of the plot. In plots where each line represents a certain time value, eigenvalue, or parameter value, these values also display.

7 Under **Quality**, select a plot **Resolution**—**Finer**, **Fine**, **Normal**, **Coarse**, or **No refinement**. A higher resolution means that elements are split into smaller patches during rendering.

8 To enforce continuity on discontinuous data, under **Quality**, from the **Smoothing** list, select:

- **None**: to plot elements independently.
- **Internal**: to smooth the quantity inside the geometry but no smoothing takes place across borders between domains with different settings.
- **Everywhere**: to apply smoothing to the entire geometry.

9 Under **Quality**, the **Recover** default is **Off** because recovery takes processing time. See Accurate Derivative Recovery for more information.

To edit the default and use polynomial-preserving recovery and recover fields with derivatives such as stresses or fluxes with a higher theoretical convergence than smoothing, from the **Recover** list select:

- **Within domains**: to perform recovery inside domains.
- **Everywhere**: to apply recovery to all domain boundaries.

10 If required, add a **Deformation**, **Color Expression** and/or **Filter**.

11 Click the **Plot** button (), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

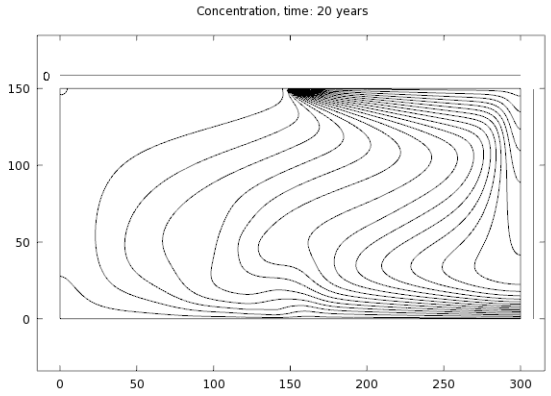


Figure 22-11: A 2D contour plot. From the Model Library: Buoyancy Flow with Darcy's Law: The Elder Problem.

Creating Coordinate System Volume, Surface, and Line Plots

Use the **Coordinate System Volume** (3D), **Coordinate System Surface** (2D and 3D), and **Coordinate System Line** (3D) plots to plot coordinate systems.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node and select **More Plots>Coordinate System Volume**, **Coordinate System Surface**, or **Coordinate System Line** ( ).

The page with the same name opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

a For **Parametric Solver** and **Parametric Sweep** studies, select an option from the **Parameter values** list. This section lists the associated parameter values.

b For **Parametric Sweep** studies (see Solver Studies and Study Types) also select a **Time**.

4 Select a **Coordinate system** to plot. By default the Global coordinate system is selected and the list contains any additional coordinate systems that the model includes.

5 For **3D Plot Groups** - **Coordinate System Volume** plots or **2D Plot Groups - Coordinate System Surface** plots:
Under **Positioning**, in the **x grid points**, **y grid points**, and **z grid points** (3D only) fields select an **Entry method**:

- If **Number of Points** is selected, enter **Points**.
- If **Coordinates** is selected, enter **Coordinates** (SI units: m).

6 Under **Coloring and Style**:

a Select an **Arrow length**—select **Proportional** to use arrows with lengths proportional to the magnitude of the field, or **Normalized** for equal sized arrows.

b Enter a **Scale factor** or use the sliders to select a value.

7 If required, add a **Deformation**.

8 Click the **Plot** button ( ), right-click the node and select **Plot**, or press F8.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

Creating a 1D Global Plot

Use a **Global** graph to plot a scalar quantity as a function of time or a parameter. See Figure 22-12 for an example from the COMSOL Multiphysics Model Library. You can use a contour data set.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **1D Plot Group** ( ). Right-click the plot node and select **Global** ( ).

The **Global** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From Parent** means that the plot uses the same data set as the plot group it belongs to.

For **Parametric Solver** and **Parametric Sweep** studies, the **Parameter values** section lists the associated parameter values.

a For time dependent **Parametric Sweep** studies (see Solver Studies and Study Types) also select an option from the **Select via** list —**Stored output times** or **Interpolated times** (time dependent models only).

The time steps cannot be selected directly because the different parametric solutions could have different time steps. Selecting **Stored output times** plots all

of them, or select **Interpolated times** to get the same interpolated times for every parameter.

b If **Interpolated times** is selected, enter **Times** or click the **Vector Input** button ( ) to select and define specific times. See Entering Ranges and Vector-Valued Expressions for details.

4 Under **Expressions**, in the table or field, enter an **Expression** and **Description**. Click the **Replace Expression** ( ) or **Add Expression** ( ) buttons to select predefined expressions based on the physics of the model. Use the **Move Up** ( ), **Move Down** ( ), and **Delete** ( ) buttons as required to move rows in the table.

5 Under **X-Axis Data**:

a For **Parametric Sweep** studies, and when there are multiple inner solutions, select an option from the **Solutions** list—**Inner** or **Outer**. An example would be a time dependent problem with a geometric parametric sweep. The time steps are the inner solutions, the parameter sweep the outer solutions.

- If **Inner** is selected, and for time dependent studies, the **Times** steps are plotted on the x-axis and one line per parameter is included in the graph (as listed in the **Data>Parameter values** section on this page).
- If **Outer** is selected, one line in the graph is plotted for each inner solution and the **Parameter values** are plotted on the x-axis.

b From the **Parameter** list, select a **Parameter value** to use the x-axis data stored in the solution or **Expression** to specify an expression for the x-axis data.

c Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model.

d If **Expression** is selected, select a **Unit** and select the check box to enter a **Description** (or edit the default).

6 Under **Coloring and Style**, in the **Line style** section:

a Select a **Line**—**Cycle**, **Solid**, **Dotted**, **Dashed**, or **Dash-dot**. If **Cycle** is selected, it cycles through all the options.

b Select a **Color** or **Custom** to define another color.

c Enter a line **Width** or use the slider to select.

7 Under **Coloring and Style**, in the **Line markers** section:

a Select a **Marker** type—**None**, **Cycle**, **Asterisk**, **Circle**, or **Diamond**.

b Enter the **Number** of markers to display or use the slider to select.

8 Under **Legends**, the **Show legends** check box is selected by default to display to the right of the plot. In plots where each line represents a certain time value, eigenvalue, or parameter value, these values also display. For point plots, the legend displays the coordinate (or vertex number).

- If **Manual** is selected from the **Legends** list, enter your own legend text into the table. Use the **Move Up** ( ), **Move Down** ( ), and **Delete** ( ) buttons as required to move rows in the table.

9 Click the **Plot** button ( ), right-click the node and select **Plot**, or press F8.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

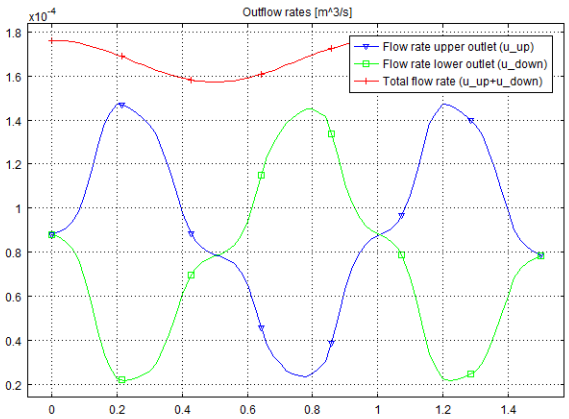


Figure 22-12: A 1D global plot. From the Model Library: Fluid Valve.

Creating a 3D Isosurface Plot

Plot a scalar quantity as an **Isosurface** plot in 3D. An isosurface plot displays a quantity as a colored set of isosurfaces on which the result has constant value. The plot can also color isosurfaces based on an independent quantity. See Figure 22-13 for an example from the Acoustics Module Model Library.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **3D Plot Group** ( ). Right-click the plot group and select **Isosurface** ( ).

The **Isosurface** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Expression**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c To enter a **Description** (or edit the default), select the check box.

5 Specify the **Levels** for the isosurfaces. From the **Entry method** list, select **Number of Levels** or **Levels**.

- If **Number of Levels** is selected, enter the **Total levels** of isosurfaces (the default is 5).
- If **Levels** is selected, enter the number of isosurface **Levels**.

6 Under **Coloring and Style**:

a Select a **Coloring**—**Color Table** (default) or **Uniform**. If **Uniform** is selected, select a **Color** or **Custom** to choose a different color.

b Select a **Color Table**. If the default (**Rainbow**) is not suitable for the plot, try other options.

c The **Color legend** is selected by default. Click to clear the check box if required.

The legend displays to the right of the plot. In plots where each line represents a certain time value, eigenvalue, or parameter value, these values also display.

7 Under **Quality**, select a plot **Resolution**—**Finer**, **Fine**, **Normal**, **Coarse**, or **No refinement**. A higher resolution means that elements are split into smaller patches during rendering.

8 To enforce continuity on discontinuous data, under **Quality**, from the **Smoothing** list, select:

- **None**: to plot elements independently.
- **Internal**: to smooth the quantity inside the geometry but no smoothing takes place across borders between domains with different settings.
- **Everywhere**: to apply smoothing to the entire geometry.

9 Under **Quality**, the **Recover** default is **Off** because recovery takes processing time. See Accurate Derivative Recovery for more information.

To edit the default and use polynomial-preserving recovery and recover fields with derivatives such as stresses or fluxes with a higher theoretical convergence than smoothing, from the **Recover** list select:

- **Within domains**: to perform recovery inside domains.
- **Everywhere**: to apply recovery to all domain boundaries.

10 By default the **Use derivatives** check box is selected. To increase the speed of computation, click to clear the check box. However, this results in less smooth surfaces.

11 If required, add a **Deformation**, **Color Expression** and/or **Filter**.

12 Click the **Plot** button ( ), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

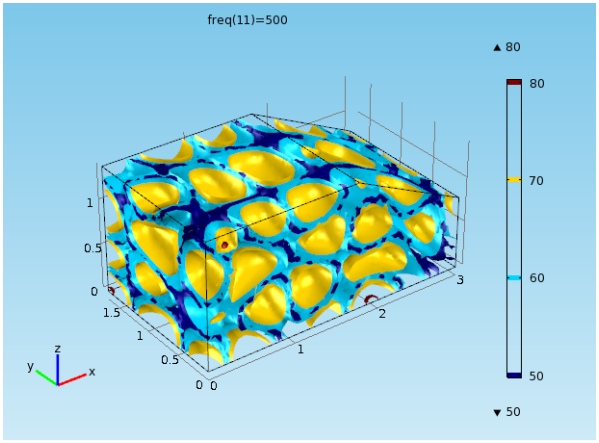


Figure 22-13: A 3D isosurface plot. From the Model Library: Car Interior.

Creating a 1D Line Graph

Use a **Line Graph** to plot a scalar quantity along a geometric line. The line can be a edge in the geometry, a parameterized curve, or a cut line. Make a graph plot of a quantity versus another quantity (for example, time). See Figure 22-14 for an example from the COMSOL Multiphysics Model Library. You can use a parameterized curve data set.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **1D Plot Group** ( ). Right-click the plot node and select **Line Graph** ( ).

The **Line Graph** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. Select:

- **From Parent** to use the same data set as the plot group it belongs to.
- a **Solution** data set to visualize a quantity along a geometric edge.
- a **Cut Line** data set to visualize a quantity along the cut line (a cross-section).
- a **Parameterized Curve** data set to visualize a quantity along the parameterized curve.

4 If a **Solution** data set is selected, under **Selection**:

a Select **All** or **Manual**.

b If **Manual** is selected choose geometry directly from the **Graphics** window. Select **All** to add the applicable geometry or any other predefined grouping. Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required in the **Settings** window.

5 Under **Y-Axis Data**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions available based on the physics of the model. Or enter an **Expression** in the field.

b Select a **Unit**.

c To enter a **Description** (or edit the default), select the check box.

6 Under **X-Axis Data**, from the **Parameter** list, select **Arc length** to visualize along arc length, or **Expression** to visualize along, for example, a coordinate expression.

Note: For **Parametric Sweep** studies, for each pair of outer solution or inner solutions, one line is plotted on the graph. For example, if there are 10 outer solutions and each outer solution has five inner solutions, then 50 lines are drawn. The number of inner solutions can vary between outer solutions.

If **Expression** is selected:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c To enter a **Description** (or edit the default), select the check box.

7 Under **Coloring and Style**:

a Select a **Line** type—**Cycle**, **Solid**, **Dotted**, **Dashed**, or **Dash-dot**. If **Cycle** is selected, it cycles through all the options.

b Select a **Color**—**Cycle**, **Black**, **Blue**, **Cyan**, or **Green**.

c Enter a line **Width** or use the slider to select.

d In the **Line markers** section, select a **Marker** type—**None**, **Cycle**, **Asterisk**, **Circle**, or **Diamond**. Enter the **Number** of markers to display or use the slider to select.

8 If required, under **Legends**, select the **Show legends** check box to display a legend to the right of the plot. In plots where each line represents a certain time value, eigenvalue, or parameter value, these values also display. For point plots, the legend displays the coordinate (or vertex number).

- If **Manual** is selected from the **Legends** list, enter your own legend text into the table. Use the **Move Up** ( ), **Move Down** ( ), and **Delete** ( ) buttons as required to move rows in the table.

9 Under **Quality**, select a plot **Resolution**—**Finer**, **Fine**, **Normal**, **Coarse**, or **No refinement**. A higher resolution means that elements are split into smaller patches during rendering.

10 Under **Quality**, the **Recover** default is **Off** because recovery takes processing time. See Accurate Derivative Recovery for more information.

To edit the default and use polynomial-preserving recovery and recover fields with derivatives such as stresses or fluxes with a higher theoretical convergence than smoothing, from the **Recover** list select:

- **Within domains**: to perform recovery inside domains.
- **Everywhere**: to apply recovery to all domain boundaries.

11 Click the **Plot** button (), right-click the node and select **Plot**, or press F8.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

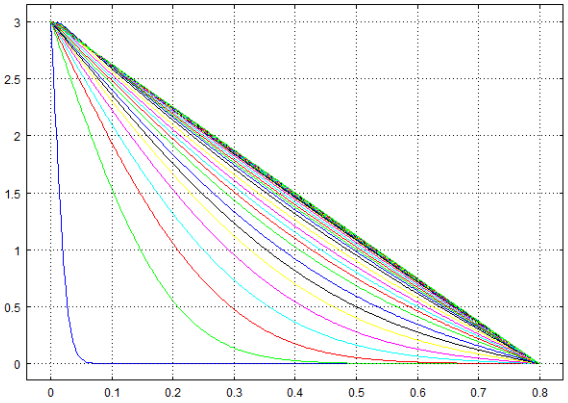


Figure 22-14: A 1D line graph. From the Model Library: Effective Diffusivity in Porous Materials.

Creating a 2D or 3D Line Plot

Use a **Line** plot to display a quantity on lines, that is, boundaries in 2D or edges in 3D. See Figure 22-15, and Figure 22-16 for examples from the Heat Transfer Module and Structural Mechanics Module model libraries. Use a parameterized curve data set or a contour data set.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node and select **Line** ( ).

The **Line** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Expression**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model.

b Select a **Unit**.

c To enter a **Description** (or edit the default), select the check box.

5 Under **Range**, manually override the **Minimum** and **Maximum color range** or **data range**. Click to clear the check box, then specify a range using the fields or use the sliders to control values.

6 Under **Coloring and Style**, select a **Line type**—**Line** or **Tube**. If **Tube** is selected:

a Enter a **Tube radius expression** (the radius of the tube).

b To edit the **Radius scale factor** select the check box and enter a number.

7 Under **Coloring and Style**:

a Select a **Coloring**—**Color Table** (default) or **Uniform**. If **Uniform** is selected, select a **Color** or **Custom** to choose a different color.

b Select a **Color Table**. If the default (**Rainbow**) is not suitable for the plot, try other options.

c The **Color legend** is selected by default. Click to clear the check box if required.

The legend displays to the right of the plot. In plots where each line represents a certain time value, eigenvalue, or parameter value, these values also display.

8 Under **Quality**, select a plot **Resolution**—**Finer**, **Fine**, **Normal**, **Coarse**, or **No refinement**. A higher resolution means that elements are split into smaller patches during rendering.

9 To enforce continuity on discontinuous data, under **Quality**, from the **Smoothing** list, select:

- **None**: to plot elements independently.
- **Internal**: to smooth the quantity inside the geometry but no smoothing takes place across borders between domains with different settings.
- **Everywhere**: to apply smoothing to the entire geometry.

10 Under **Quality**, the **Recover** default is **Off** because recovery takes processing time. See Accurate Derivative Recovery for more information.

To edit the default and use polynomial-preserving recovery and recover fields with derivatives such as stresses or fluxes with a higher theoretical convergence than smoothing, from the **Recover** list select:

- **Within domains**: to perform recovery inside domains.
- **Everywhere**: to apply recovery to all domain boundaries.

11 If required, add a **Deformation** and/or **Filter**.

12 Click the **Plot** button (), right-click the node and select **Plot**, or press F8.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

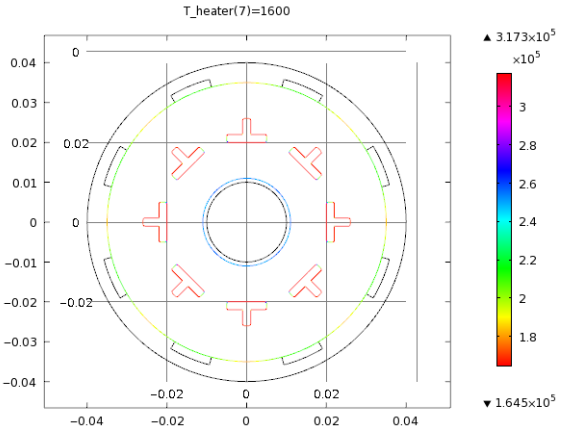


Figure 22-15: A 2D line plot. From the Heat Transfer Model Library: Thermo-Photo-Voltaic Cell.

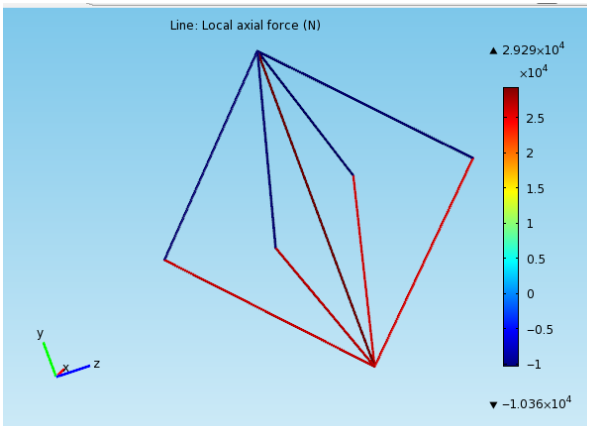


Figure 22-16: A 3D line plot. From the Structural Mechanics Model Library: In-Plane and Space Truss.

Creating Max/Min Volume, Surface, and Line Plots

Use the **Max/Min Volume Max/Min Surface** or **Max/Min Line** plots to plot the maximum and minimum of an expression and the points there they are attained within the geometry.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node and select **More Plots>Max/Min Volume**, **Max/Min Surface**, or **Max/Min Line**.

The **Max/Min Volume**, **Max/Min Surface**, or **Max/Min Line** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Expression**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select the predefined expression you want to compute the maximum and minimum for. Or enter an **Expression**.

b Select a **Unit** to plot.

c To enter a **Description** (or edit the default), select the check box.

5 Enter a **Precision** for the number of decimals displayed in the labels.

6 Under **Advanced**:

a The **Recover** default is **Off** because recovery takes processing time. To edit the default and use polynomial-preserving recovery and recover fields with derivatives such as stresses or fluxes with a higher theoretical convergence than smoothing, from the **Recover** list select:

- **Within domains**: to perform recovery inside domains.
- **Everywhere**: to apply recovery to all domain boundaries.

b Define the number of refinements of each mesh element when computing the maximum and minimum by entering a value in the **Element refinement** field.

7 Click the **Plot** button (), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

Creating a 2D or 3D Mesh Plot

Use mesh plots to display a mesh in 2D or 3D.

1 Define one or more **Data Sets**.

Note: You can visualize the mesh without solving a model using a mesh data set.

---

2 In the **Model Builder**, right-click **Results** and add a **2D Plot Group** () or **3D Plot Group** (). Right-click the plot node and select **Mesh**.

The **Mesh** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Level**, select a **Level to** display the mesh—**All**, **Volume** (3D only), **Surface**, **Line**, or **Point**.

- If **Surface** is selected, select the base **Element type** to visualize—**All**, **Triangle**, or **Quad**.
- *3D only*: If **Volume** is selected, select the base **Element type** to visualize—**All**, **Tetrahedron**, **Prism**, or **Hex**.

5 Under **Color**:

a From the **Element color** list, select:

- **Quality** to get an element quality plot.
- **Custom** to select a different color.
- **None** to plot with no color.

b Select a **Color table** for the element quality. If the default (**Rainbow**) is not suitable for the plot, try other options.

c From the **Wireframe color** list, select:

- **Quality** to get an element quality plot.
- **Custom** to select a different color.
- **None** to plot with no color.

6 Under **Element Filter**, specify the elements to display:

a Select the **Enable filter** check box.

b Select a **Criterion**—**Random**, **Worst Quality**, **Best Quality**, **Size**, **Expression**, or **Logical expression**.

c If **Expression** or **Logical expression** is selected, enter an **Expression** in the field.

d Specify the **Fraction** of elements to show or use the slider to choose.

7 To scale elements in the mesh plot, under **Shrink Elements**, enter an **Element scale factor** between 0 and 1.

8 Click the **Plot** button (), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

Creating a 2D or 3D Particle Tracing Plot

Use a **Particle Tracing** plot to visualize the trajectory of a massless particle subject to a flow field. With this plot type, you can visualize *pathlines*, that is, trajectories of particles released in a flow field, which can be time-dependent or static. For

time-dependent flows you can also use a snapshot in time of the flow field as a static field. The motion of the particles does not affect the flow field.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node and select **Particle Tracing** ( ).

The **Particle Tracing** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Expression**:

a Enter or select **x**-and **y-components** for 2D, **x**-, **y-**, and **z-components** for 3D. Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model.

b To enter a **Description** (or edit the default), select the check box.

5 Under **Particle Positioning**, for *3D* only, enter the initial position of particles in the **x**, **y**, and **z** fields.

For *2D*:

a From the **Positioning** list, select **Start point controlled** or **Boundary coordinates**. Boundary coordinates are useful, for example, for flow models with one or more inflow boundaries.

- If **Start point controlled** is selected, enter the initial position of particles in the **x** and **y** fields and go to the next step.

b If **Boundary coordinates** is selected, select an item from the **Named selection** list.

c Select an **Entry method**—**Number of points** or **Boundary parameters**.

- If **Number of Points** is selected, enter the number of grid **Points**.
- If **Boundary parameters** is selected, enter the **Relative coordinates**.

6 Under **Release**, specify when to **Release particles**:

- Select **Once** to release particles once at the first available time, typically at time 0 (zero). To delay the release, select the **Start time** check box and enter a time.
- Select **At intervals** to release particles at regular intervals starting at the first available time, typically at time 0 (zero). To delay the release, select the **Start time** check box and enter a time. Enter a **Time between releases**.
- Select **At times** to release particles at an arbitrary time point; enter multiple **Times** to release particles.

7 Under **Coloring and Style**, select a **Line type**—**Line**, **Tube**, or **None**. If **Tube** is selected:

   a Enter a **Tube radius expression** (the radius of the tube).

   b To edit the **Radius scale factor** select the check box and enter a number.

   c Select a line **Color** or **Custom** to choose a different color.

8 Under **Coloring and Style**, select a **Point style**—**Point** or **None**.

9 If **Point** is selected:

   a Enter a **Point radius**.

   b To edit the **Radius scale factor** select the check box and enter a number.

   c Select a point **Color** or **Custom** to choose a different color.

10 Under **Coloring and Style**, select a **Point motion** to specify what should happen as particles leave the domain—**Stick to Boundary** (to plot the points on the boundary at the exit point) or **Disappear** (to not render these points at all).

   For static fields, specify the **End time** in the **Advanced** section. It is possible that all particles have left the domain at the selected time. In that case, all points appear at the outflow boundary if **Stick to boundary** is selected, and no points appear if **Disappear** is selected. To make the points appear, specify an earlier end time.

11 If you are planning to create an **Animation** report (see Defining and Exporting Plot Animations), and if required, select the **Plot along lines when animating** check box.

12 Under **Quality**, select a plot **Resolution**—**Finer**, **Fine**, **Normal**, **Coarse**, or **No refinement**. A higher resolution means that elements are split into smaller patches during rendering.

13 Under **Quality**, the **Recover** default is **Off** because recovery takes processing time. See Accurate Derivative Recovery for more information if you want to change the setting.

14 In the **ODE solver settings** section, enter a **Relative tolerance** for the ODE solver. The default is 0.001.

- When solving the 2nd-order ODE $m\ddot{x} = F(t, x, \dot{x})$ for x, the solver first rewrites it as two coupled 1st-order ODEs: one for position x and one for velocity $\dot{x}$, each with two components in 2D and three components in 3D.
- The **Relative tolerance** value is the relative error tolerance that the ODE solver uses. It applies to all components of the particle's position and velocity. The solver

controls the step size so that the estimated error e in each integration step satisfies

$$\begin{cases} e < \max(\text{atolpos}, \text{rtol} \cdot |x_i|) & \text{(for all components } x_i \text{ of } x) \\ e < \max(\text{atolvel}, \text{rtol} \cdot |\dot{x}_i|) & \text{(for all components } \dot{x}_i \text{ of } \dot{x}) \end{cases}$$

where rtol is the relative tolerance that you specify, atolpos is the absolute tolerance for the particle's position components, and atolvel equals the absolute tolerance for the particle's velocity components.

15 In the **ODE solver settings** section, specify the solver's absolute tolerance. The default is **Automatic**. To enter different values, select **Manual** from the **Absolute tolerance** list and enter a **Position**.

- The **Position** field can contain a single value—it applies to all components of the position and is the absolute tolerance.

16 In the **ODE solver settings** section, specify the solver **Step size.** The default is **Automatic**—COMSOL uses the initial value of the acceleration (force divided by mass) and the relative and absolute tolerances to determine the initial time step.

- The automatic maximum step size is 10% of the total simulation time for time-dependent flows as well as for static flow fields where you can specify the end time manually in the **Advanced** section (in the **Plot static flow field even when time dependent>End Time** field). For static flow fields where you do not set the end time manually, there is no upper limit of the step size. However, in this case, the initial time step is less than or equal to 0.1.

To edit the settings, select **Manual** from the **Step size** list and enter values in the **Initial time step** and **Maximum time step** fields.

- The **Maximum time step** is the longest time step the solver takes. It has higher priority than the **Initial time step**; that is, if you specify an initial step size larger than the maximum step size, the solver lowers the initial step size to the maximum step size.

Note: The initial step size, whether entered manually or computed automatically, is not necessarily the first step the solver takes but is a first try. If this step leads to an error such that the tolerances are not met, COMSOL Multiphysics lowers it.

Note: The **Advanced** section contains settings that do not normally need to be adjusted.

17 Under **Advanced**, the **Termination** section contains settings that determine when to end the particle tracing simulation.

   a To specify an upper limit of the number of time steps, click to select the **Maximum number of steps** check box and edit the default (1000). The particle simulation ends after this number of steps.

   b To specify how close to the geometry boundary the path lines are cut when they exit the geometry, edit the **Edge tolerance** default (0.001). This is an absolute tolerance controlling how close to the geometry boundary the pathlines are cut when they exit the geometry. A lower value cuts the line closer to the geometry boundary.

18 Under **Advanced**, in the **Instantaneous flow field** section:

   a To specify if you want to plot an instantaneous even if the solution is time dependent, select the **Plot static flow field even when time dependent** check box. This freezes the time selected previously— for example, from a **Plot Group** page in the **Data>Time** list—to the value specified and considers this a static flow field.

   b Edit the **Time variable** default (`partt`) if required. Normally it is not necessary to change the default name but you can use the name in expressions as well as for the color when coloring the pathlines according to an expression.

   c If required, select the **End time** check box and enter a value.

19 If required, add a **Deformation** and/or **Color Expression**.

20 Click the **Plot** button (), right-click the node and select **Plot**, or press F8.

   The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

Creating a 2D or 3D Particle Tracing with Mass Plot

Use a **Particle Tracing with Mass** plot to visualize the trajectory of a particle with mass and subject to a flow field. See Figure 22-17 and Figure 22-18 for examples from the COMSOL Multiphysics Model Library.

BACKGROUND INFORMATION

For particles with mass, COMSOL generates the pathlines by solving the fundamental equation of motion:

$$m\ddot{x} = F(t, x, \dot{x})$$

for the pathline $x(t)$. Here, m is the particle's mass, F equals the force acting upon the particle, and t is time. This is a system of ODEs for x, which COMSOL Multiphysics solves using a pair of Runge-Kutta methods of orders four and five. The solver advances the algorithm with the solution of order five and uses the difference between the order-five and order-four solutions to obtain the local error estimate.

For massless particles, the equation of motion is:

$$\dot{x} = v(t, x)$$

Note: The true formulation of Newton's second law of motion is $\frac{d}{dt}(m\dot{x}) = F(t, x, \dot{x})$, that is, you must consider the time derivative of the mass. The particle-tracing algorithm does not solve this equation. Thus, if you specify an expression for the particle mass that depends on time, you will not get correct results.

Axisymmetric Models

For 2D axisymmetric models, three components for the force are available for particles with mass. When specifying all three, the algorithm solves for a line in 3D in cylindrical coordinates, but the plot only shows the projection on the axisymmetry plane. In this case, the centripetal force is considered; that is, the algorithm solves the equation

$$\ddot{r} = \frac{F_r}{m} + r\dot{\varphi}^2 \qquad \ddot{\varphi} = \frac{F_\varphi}{rm} - \frac{2\dot{r}\dot{\varphi}}{r} \qquad \ddot{z} = \frac{F_z}{m},$$

where m is the particle mass and (r, φ, z) are the cylindrical coordinates. Note that the variable corresponding to the velocity component in the φ direction (whose default name is `partv`) has the dimension length/time, and equals $r\dot{\varphi}$ as $\dot{\varphi}$ has the dimension radians/time.

DEFINING A PARTICLE TRACING WITH MASS PLOT

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node and select **Particle Tracing with Mass** ( ).

The **Particle Tracing with Mass** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Specify the force acting on the particles under **Equation of Motion**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**—**Fx**, **Fy**, and **Fz** components of the force as required for the space dimension.

b Enter a **Description** (or edit the default).

c When some predefined forces are added, there are additional **Parameters** with a **Value** to enter into a table.

5 Under **Mass and Velocity**:

a Enter the particle's **Mass**.

b Enter values in the **Initial velocity** section—for 2D enter **x** and **y components**; for 3D enter **x**, **y**, and **z components**.

6 Under **Particle Positioning**, for *3D* only, enter the initial position of particles in the **x**, **y**, and **z** fields.

a From the **Positioning** list, select **Start point controlled** or **Boundary coordinates**. Boundary coordinates are useful, for example, for flow models with one or more inflow boundaries.

- If **Start point controlled** is selected, enter the initial position of particles in the **x** and **y** fields and go to the next step. Click the **Range** button ( ) to define a range using the **Range** window. See Entering Ranges and Vector-Valued Expressions for details.
- If **Boundary coordinates** is selected, select an item from the **Named selection** list.

b Select an **Entry method**—**Number of points** or **Boundary parameters**.

- If **Number of Points** is selected, enter the number of grid **Points**.
- If **Boundary parameters** is selected, enter the **Relative coordinates**.

7 Under **Release**, specify when to **Release particles**—**Once**, **At intervals**, or **At times.** Select:

- **Once** to release particles once at the first available time, typically at time 0 (zero). To delay the release, select the **Start time** check box and enter a time.
- **At intervals** to release particles at regular intervals starting at the first available time, typically at time 0 (zero). To delay the release, select the **Start time** check box and enter a time. Enter a **Time between releases**.
- **At times** to release particles at an arbitrary time point; enter multiple **Times** to release particles.

8 Under **Coloring and Style**, select a **Line type**—**Line**, **Tube**, or **None**. If **Tube** is selected:

a Enter a **Tube radius expression** (the radius of the tube).

b To edit the **Radius scale factor** select the check box and enter a number.

c Select a line **Color** or **Custom** to choose a different color.

9 Under **Coloring and Style**, select a **Point style**—**Point** or **None**. If **Point** is selected also enter a **Point radius**.

a To edit the **Radius scale factor** select the check box and enter a number.

b Select a point **Color** or **Custom** to choose a different color.

10 Under **Coloring and Style**, select a **Point motion** to specify what should happen as particles leave the domain—**Stick to Boundary** (to plot the points on the boundary at the exit point) or **Disappear** (to not render these points at all).

For static fields, specify the **End time** in the **Advanced** section. It is possible that all particles have left the domain at the selected time. In that case, all points appear at the outflow boundary if **Stick to boundary** is selected, and no points appear if **Disappear** is selected. To make the points appear, specify an earlier end time.

11 If you are planning to create an **Animation** (see Defining and Exporting Plot Animations), and if required, select the **Plot along lines when animating** check box.

12 Under **Quality,** to increase the number of output points by adding points between each time step taken in the ODE solver, using a 4th-order interpolation, and to produce a smoother output select a plot **Resolution**—**Finer**, **Fine**, **Normal**, **Coarse**, or **No refinement**. A higher resolution means that elements are split into smaller patches during rendering.

13 Under **Quality**, the **Recover** default is **Off** because recovery takes processing time. See Accurate Derivative Recovery for more information.

I4 In the **ODE solver settings** section, enter a **Relative tolerance** for the ODE solver. The default is 0.001.

- When solving the 2nd-order ODE $m\ddot{x} = F(t, x, \dot{x})$ for x, the solver first rewrites it as two coupled 1st-order ODEs: one for position x and one for velocity $\dot{x}$, each with two components in 2D and three components in 3D.
- The **Relative tolerance** value is the relative error tolerance that the ODE solver uses. It applies to all components of the particle's position and velocity. The solver controls the step size so that the estimated error e in each integration step satisfies

$$\begin{cases} e < \max(\text{atolpos}, \text{rtol} \cdot |x_i|) & \text{(for all components } x_i \text{ of } x) \\ e < \max(\text{atolvel}, \text{rtol} \cdot |\dot{x}_i|) & \text{(for all components } \dot{x}_i \text{ of } \dot{x}) \end{cases}$$

where rtol is the relative tolerance that you specify, atolpos is the absolute tolerance for the particle's position components, and atolvel equals the absolute tolerance for the particle's velocity components.

I5 In the **ODE solver settings** section, specify the absolute tolerance. The default is **Automatic**. To enter different values, select **Manual** from the **Absolute tolerance** list and enter a **Position** or **Velocity**.

- The **Position** or **Velocity** field can contain a single value—it applies to all components of the position and is the absolute tolerance.

I6 In the **ODE solver settings** section, specify the **Step size.** The default is **Automatic**—COMSOL uses the initial value of the acceleration (force divided by mass) and the relative and absolute tolerances to determine the initial time step.

- The automatic maximum step size is 10% of the total simulation time for time-dependent flows as well as for static flow fields where you can specify the end time manually in the **Advanced** section (in the **Plot static flow field even when time dependent>End Time** field). For static flow fields where you do not set the end time manually, there is no upper limit of the step size. However, in this case, the initial time step is less than or equal to 0.1.

To edit the settings, select **Manual** from the **Step size** list and enter values in the **Initial time step** and **Maximum time step** fields.

- The **Maximum time step** is the longest time step the solver takes. It has higher priority than the **Initial time step**; that is, if you specify an initial step size larger than the maximum step size, the solver lowers the initial step size to the maximum step size.

Note: The initial step size, whether entered manually or computed automatically, is not necessarily the first step the solver takes but is a first try. If this step leads to an error such that the tolerances are not met, COMSOL Multiphysics lowers it.

Note: The **Advanced** section contains settings that do not normally need to be adjusted.

17 Under **Advanced**, the **Termination** section contains settings that determine when to end the particle tracing simulation.

- **a** To specify an upper limit of the number of time steps, select the **Maximum number of steps** check box and edit the default (1000). The particle simulation ends after this number of steps.
- **b** To specify how close to the geometry boundary the path lines are cut when they exit the geometry, edit the **Edge tolerance** default (0.001). This is an absolute tolerance controlling how close to the geometry boundary the pathlines are cut when they exit the geometry. A lower value cuts the line closer to the geometry boundary.

18 Under **Advanced**, in the **Instantaneous flow field** section:

- **a** To specify if you want to plot an instantaneous even if the solution is time dependent, select the **Plot static flow field even when time dependent** check box. This freezes the time selected previously— for example, from a **Plot Group** page in the **Data>Time** list—to the value specified and considers this a static flow field.
- **b** Edit the **Time variable** default (`partt`) if required. Normally it is not necessary to change the default name but you can use the name in expressions as well as for the color when coloring the pathlines according to an expression.
- **c** If required, select the **End time** check box and enter a value.

19 Under **Advanced**, in the **Particle velocity variables** section, edit the default variable component names for each particle's velocity. The default names are `partu` (x-component), `partv` (y-component), and `partw` (z-component).

20 If required, add a **Deformation** and/or **Color Expression**.

21 Click the **Plot** button ( ), right-click the node and select **Plot**, or press F8.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

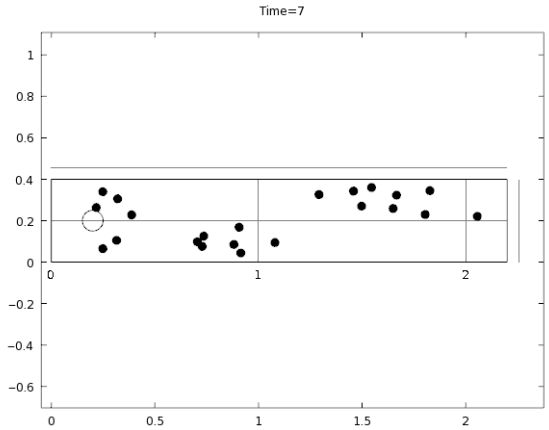


Figure 22-17: A 2D particle tracing with mass plot. From the Model Library: Flow Past a Cylinder.

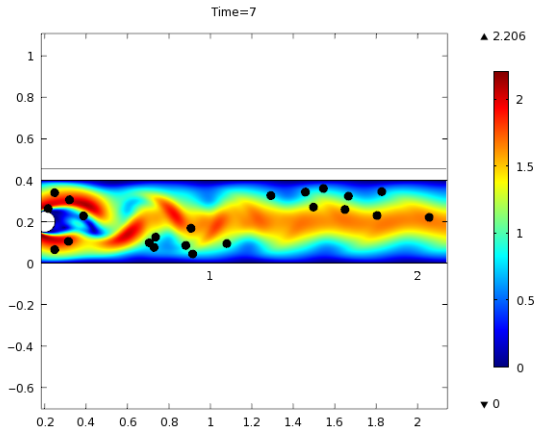


Figure 22-18: A 2D particle tracing with mass combined with surface plot. From the Model Library: Flow Past a Cylinder.

Creating a 1D Point Graph

Use a **Point Graph** plot to visualize the value in a point along time or a parameter value. The point can be a point in the geometry or a cut point. See Figure 22-19 for an example from the COMSOL Multiphysics Model Library.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **1D Plot Group** ( ). Right-click the plot node and select **Point Graph** ( ).

The **Point Graph** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. Select:

- **From Parent** to use the same data set as the plot group it belongs to.
- a **Solution** data set to visualize a quantity along a geometric point.
- a **Cut Point** data set to visualize a quantity in the cut point.
- If a **Parametric Solver** or **Parametric Sweep** data set is selected, the **Parameter values** section lists the associated parameter values.

a For **Parametric Sweep** studies (see Solver Studies and Study Types) select an option from the **Select via** list —**Stored output times** or **Interpolated times** (time dependent models only).

The time steps cannot be selected directly because the different parametric solutions could have different time steps. Selecting **Stored output times** plots all of them, or select **Interpolated times** to get the same interpolated times for every parameter.

b If **Interpolated times** is selected, enter **Times** or click the **Vector Input** button ( ) to select and define specific times. See Entering Ranges and Vector-Valued Expressions for details.

4 Under **X-axis Data**, for **Parametric Sweep** studies, and when there are multiple inner solutions,

a Select an option from the **Solutions** list—**Inner** or **Outer**.

- If **Inner** is selected, the **Times** steps are plotted on the x-axis and one line per parameter is included in the graph (as listed in the **Data>Parameter values** section on this page).
- If **Outer** is selected, one line in the graph is plotted for each inner solution and the **Parameter values** are plotted on the x-axis.

b From the **Parameter** list, select **Parameter value** to use the x-axis data stored in the solution or **Expression** to specify an expression for the x-axis data.

5 If a **Solution** data set is selected, under **Selection**:

a Select **All** or **Manual**.

b If **Manual** is selected choose geometry directly from the **Graphics** window. Select **All** to add the applicable geometry or any other predefined grouping. Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required in the **Settings** window.

6 Under **Expression**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c To enter a **Description** (or edit the default), select the check box.

7 Under **X-Axis Data**, from the **Parameter** list, select **Parameter value** to use the x-axis data stored in the solution or **Expression** to specify an expression for the x-axis data.

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c To enter a **Description** (or edit the default), select the check box.

8 Under **Coloring and Style**, in the **Line style** section:

a Select a **Line**—**None, Cycle**, **Solid**, **Dotted**, **Dashed**, or **Dash-dot**. If **Cycle** is selected, it cycles through all the options.

b Select a line **Color** or **Custom** to define another color.

c Enter a line **Width** or use the slider to select.

9 Under **Coloring and Style**, in the **Line markers** section:

a Select a **Marker** type—**None**, **Cycle**, **Asterisk**, **Circle**, or **Diamond**.

b Enter the **Number** of markers to display or use the slider to select.

10 If required, under **Legends**, select the **Show legends** check box to display a legend to the right of the plot. In plots where each line represents a certain time value, eigenvalue, or parameter value, these values also display. For point plots, the legend displays the coordinate (or vertex number).

- If **Manual** is selected from the **Legends** list, enter your own legend text into the table. Use the **Move Up** ( ), **Move Down** ( ), and **Delete** ( ) buttons as required to move rows in the table.

11 Click the **Plot** button ( ), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

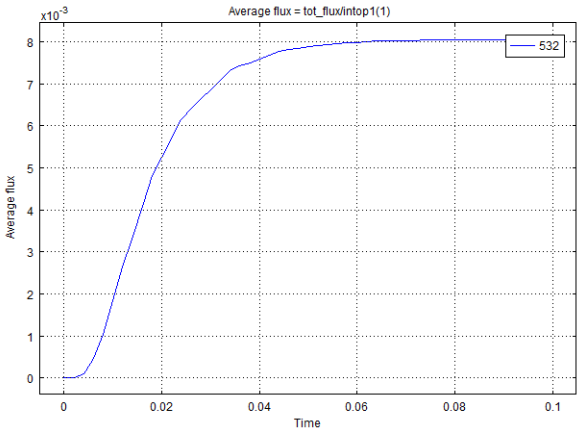


Figure 22-19: A 1D point graph. From the Model Library: Effective Diffusivity in Porous Materials.

Creating Principal Stress Volume or Principal Stress Surface Plots

Use the **Principal Stress Volume** (3D) and **Principal Stress Surface** (2D and 3D) to plot principal stress and strain in structural mechanics models.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node and select **More Plots>Principal Stress Volume** ( ) or **Principal Stress Surface** ( ).

The page with the same name opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Principal Components**:

a Select a **Type**—**Principal stress** or **Principal strain**.

b Under **Principal values**, enter **First**, **Second**, and **Third** in the **Value** fields.

c Under **Principal directions**, enter **First**, **Second**, and **Third** in the **x, y**, and **z** (for 3D) coordinate fields.

5 For transient problems, enter a **Time**.

6 For **3D Plot Groups** - **Principal Stress Volume** plots or **2D Plot Groups - Principal Stress Surface** plots:

Under **Positioning**, in the **x grid points**, **y grid points**, and **z grid points** fields select an **Entry method**:

- If **Number of Points** is selected, enter **Points**.
- If **Coordinates** is selected, enter **Coordinates** (SI units: m).

7 Under **Coloring and Style**:

a Select an **Arrow length**—select **Proportional** to use arrows with lengths proportional to the magnitude of the field, or **Normalized** for equal sized arrows.

b Enter a **Scale factor** or use the sliders to select a value.

c Select an arrow **Color** or **Custom** to choose a different color.

8 If required, add a **Deformation** and/or **Color Expression**.

9 Click the **Plot** button ( ), right-click the node and select **Plot**, or press F8.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

Creating a 2D Scatter Surface or 3D Scatter Volume Plot

Use a **2D Scatter Surface** plot to visualize a scalar quantity as scattered spheres on a 2D surface or on a **3D Scatter Volume** plot. Scatter plots can be used as alternatives to arrow plots or when you can to get a feeling for how entities correlate. The radius and color

can be functions of independent quantities. See Figure 22-20 and Figure 22-21 for examples from the Earth Science Module Model Library.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **2D Plot Group** ( ) (Scatter Surface) or **3D Plot Group** ( ) (Scatter Volume). Right-click the plot node and select **More Plots>Scatter Surface** ( ) or **Scatter Volume** ( ).

The **Scatter Surface** (2D) or **Scatter Volume** (3D) page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Expression**:

a Enter or select **x**-and **y-components** for 2D, **x**-, **y-**, and **z-components** for 3D. Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model.

b To enter a **Description** (or edit the default), select the check box.

5 Under **Evaluation Points**, select an **Entry method** for the **x-** and **y-grid points** (2D) or for the **x-**, **y-**, and **z-grid points** (3D). The evaluation points are located in a block-shaped grid.

- If **Number of Points** is selected, enter **Points**.
- If **Coordinates** is selected, enter **Coordinates** for each axis.

6 Under **Radius**:

a The default **Expression** is 1. But you can enter any **Expression** in the field.

b COMSOL automatically scales the radius. To edit the **Radius scale factor**, select the check box and enter a number.

7 Under **Color**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c To enter a **Description** (or edit the default), select the check box.

8 Under **Coloring and Style**:

a Select a **Coloring**—**Color Table** (default) or **Uniform**. If **Uniform** is selected, select a **Color** or **Custom** to choose a different color.

b Select a **Color Table**. If the default (**Rainbow**) is not suitable for the plot, try other options.

c The **Color legend** is selected by default. Click to clear the check box if required.

The legend displays to the right of the plot. In plots where each line represents a certain time value, eigenvalue, or parameter value, these values also display.

9 Click the **Plot** button (), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

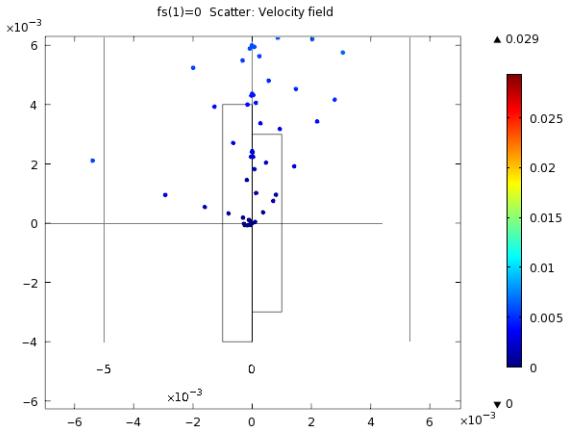


Figure 22-20: A 2D scatter surface plot. From the Model Library: Forchheimer Flow.

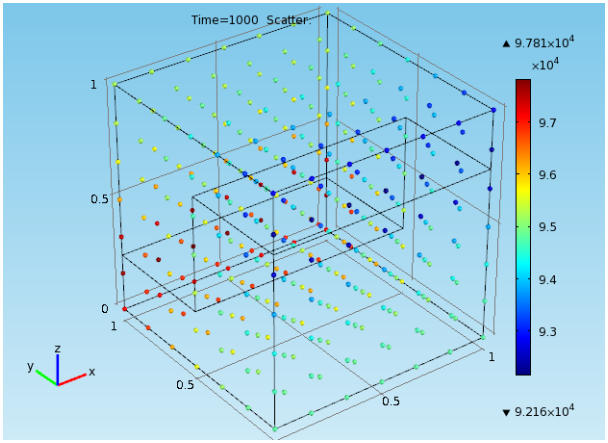


Figure 22-21: A 3D scatter volume plot. From the Model Library: Discrete Fracture.

Creating a 3D Slice Plot

Use **Slice** plots to display a scalar quantity on slices inside a 3D domain. See Figure 22-22 for an example from the Acoustic Module Model Library.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **3D Plot Group** ( ). Right-click the plot group and select **Slice** ( ).

The **Slice** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Expression**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c To enter a **Description** (or edit the default), select the check box.

5 Under **Plane Data**, specify the slices from the **Plane Type** list. **Quick** is the default; it allows you to specify planes orthogonal to the coordinate axes.

If **Quick** is selected:

a From the **Plane** list, select **xy-planes**, **yz-planes**, or **zx-planes** as the set of planes orthogonal to the coordinate axes applicable for the model geometry.

b Select an **Entry method**.

- If **Number of Planes** is selected, enter **Planes**.
- If **Coordinates** is selected, enter the applicable (x, y, or z) grid **Coordinates**. Choose a set of cut plane slices to a coordinate axis, specify the transverse coordinate by entering the location along the transverse coordinate axis in the **Coordinates** field.

Select **General** to specify general planes. If **General** is selected:

a Select an option from the **Plane entry method** list—**Three points** or **Point and normal**.

- If **Three points** is selected, enter **x**, **y**, or **z** coordinates in the **Point 1**, **Point 2** and **Point 3** fields.
- If **Point and normal** is selected, enter **x**, **y**, or **z** coordinates in both the **Point** and **Normal** sections.

b If required, select the **Additional parallel planes** check box and select an **Entry method** from the specified plane— **Number of planes** or **Distances**.

- If **Number of Planes** is selected, enter the number of grid **Planes**.
- If **Distances** is selected, enter the **Distances**.

6 Under **Range**, manually override the **Minimum** and **Maximum color range** or **data range**. Click to clear the check box, then specify a range using the fields or use the sliders to control values.

7 Under **Coloring and Style**:

a Select a **Coloring**—**Color Table** (default) or **Uniform**. If **Uniform** is selected, select a **Color** or **Custom** to choose a different color.

b Select a **Color Table**. If the default (**Rainbow**) is not suitable for the plot, try other options.

c The **Color legend** is selected by default. Click to clear the check box if required.

The legend displays to the right of the plot. In plots where each line represents a certain time value, eigenvalue, or parameter value, these values also display.

8 Under **Quality**, select a plot **Resolution**—**Finer**, **Fine**, **Normal**, **Coarse**, or **No refinement**. A higher resolution means that elements are split into smaller patches during rendering.

9 To enforce continuity on discontinuous data, under **Quality**, from the **Smoothing** list, select:

- **None**: to plot elements independently.
- **Internal**: to smooth the quantity inside the geometry but no smoothing takes place across borders between domains with different settings.
- **Everywhere**: to apply smoothing to the entire geometry.

10 Under **Quality**, the **Recover** default is **Off** because recovery takes processing time. See Accurate Derivative Recovery for more information.

To edit the default and use polynomial-preserving recovery and recover fields with derivatives such as stresses or fluxes with a higher theoretical convergence than smoothing, from the **Recover** list select:

- **Within domains**: to perform recovery inside domains.
- **Everywhere**: to apply recovery to all domain boundaries.

11 If required, add a **Deformation** and/or **Filter**.

12 Click the **Plot** button (), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

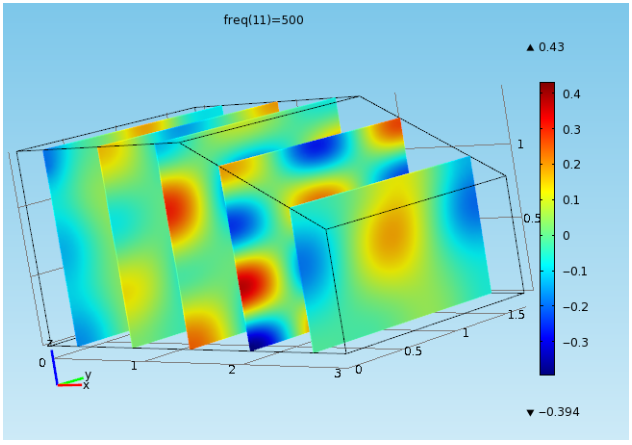


Figure 22-22: A 3D slice plot. From the Model Library: Car Interior.

Creating a 2D or 3D Streamline Plot

Streamlines visualize a vector quantity. A streamline is a curve everywhere tangent to an instantaneous vector field. The streamline plot in 3D is analogous to the 2D case except that there is no height data setting and the start point selection is different. See Figure 22-23 and Figure 22-24 for examples from the Earth Science Module and AC/DC Module Model Libraries.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node and select **Streamline** ( ).

The **Streamline** page opens in the **Settings** window and a **Streamline** node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Expression**:

a Enter or select **x**, **y**, and **z** coordinates as required for the space dimension. Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model.

b To enter a **Description** (or edit the default), select the check box.

5 Under **Streamline Positioning**, select one of these options from the **Positioning** list—**Start point controlled**, **Uniform density**, or **Magnitude controlled**. Then follow one of the methods described in Streamline Positioning Section (Continued):

- Method 1: Specifying Points by Entering Coordinates
- Method 2: Selecting the Specified Number of Start Points in the Geometry
- Method 3: Creating Streamlines with Uniform Density
- Method 4: Creating Streamlines with Variable Density and Magnitude Controlled

6 Under **Coloring and Style**, select a **Line type**—**Line** or **Tube**.

If **Tube** is selected:

a Enter a **Tube radius expression** (the radius of the tube).

b To edit the **Radius scale factor** select the check box and enter a number.

7 Under **Coloring and Style**, select a **Color** or select **Custom** to choose a different color.

8 Under **Quality**, select a plot **Resolution**—**Finer**, **Fine**, **Normal**, **Coarse**, or **No refinement**. A higher resolution means that elements are split into smaller patches during rendering.

9 Under **Quality**, the **Recover** default is **Off** because recovery takes processing time. See Accurate Derivative Recovery for more information.

To edit the default and use polynomial-preserving recovery and recover fields with derivatives such as stresses or fluxes with a higher theoretical convergence than smoothing, from the **Recover** list select:

- **Within domains**: to perform recovery inside domains.
- **Everywhere**: to apply recovery to all domain boundaries.

10 Under **Advanced** set these general settings. Also see Advanced Section Setting Effects for more information.

a The **Integration tolerance** field default is 0.01 for 3D and 0.001 for 2D. Edit to specify how accurately streamlines are computed.

b The **Maximum number of integration steps** field makes sure that the integration does not continue indefinitely. Edit the default (5000) to control when the computation stops.

c The **Maximum integration time** field sets an upper time limit for the integration. The default is infinity (inf).

d The **Stationary point stop tolerance** can be adjusted to make sure the integration stops near a stationary point in the field. The default is 0.01.

e The **Loop tolerance** field default is 0.01. This is a fraction of the mean of the lengths of the bounding box of the geometry. If a streamline gets closer to its start point than this distance, the streamline snaps to its start point and is plotted as a connected loop. Also see Method 4: Creating Streamlines with Variable Density and Magnitude Controlled.

f Select the **Allow backward time integration check box** to integrate points from the starting points both in the direction of the vector field and in the opposite direction.

g Select the **Normalize vector field check box** if required.

11 If required, add a **Deformation** and/or **Color Expression**.

12 Click the **Plot** button (), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

STREAMLINE POSITIONING SECTION (CONTINUED)

Method 1: Specifying Points by Entering Coordinates

1. From the **Positioning** list, select **Start point controlled**.
2. Select **Coordinates** from the **Entry method** list.
3. Enter **x** and **y** coordinates. You can also use a scalar value to represent a fixed value for some of the coordinates.

Method 2: Selecting the Specified Number of Start Points in the Geometry

1. From the **Positioning** list, select **Start point controlled**.
2. Select **Number of points** from the **Entry method** list.
3. Enter the number of **Lines**.

Method 3: Creating Streamlines with Uniform Density

The algorithm saturates the entire domain with evenly spaced streamlines.

1. From the **Positioning** list, select **Uniform density**.
2. Enter the **Separating distance** between the streamlines.

   The value for the separating distance is a fraction of the mean of the lengths of the bounding box of the geometry. In this case, a streamline stops whenever it gets too close to another streamline or itself (or if any of the general termination criteria specified in the **Advanced** section is fulfilled).
3. If required, under **Streamline Positioning**, from the **Advanced parameters** list, select **Manual** to set advanced parameters as follows.

   a. Edit the **Boundary element refinement** field if streamlines do not behave as expected near boundaries on a coarse mesh—try increasing this number. It is a measurement of the density of points on the boundaries used to set up the structure and is used to measure distances between streamlines. Refining the mesh in the problematic area can also resolve the problem.

   b. Edit the **Fraction of streamline length to ignore** field when a streamline is close to itself, typically for spiraling streamlines. This number controls how big part of the

streamline, starting from its start point, that the streamline itself is allowed to get close to.

c The value in the **Starting distance factor** field is a factor multiplied with the distance specified in the **Separating distance** field, which sets the minimum distance between streamlines and the start point for the next streamline.

When the domain is close to be saturated with streamlines, new start points tend to be positioned where the streamline has nowhere to go before it gets too close other streamlines, resulting in short streamlines. The higher the value of this factor disqualifies the start point and thus reduces the number of short streamlines.

d The value in the **Terminating distance factor** field is a factor multiplied with the distance specified in the **Separating distance** field. It sets the minimum distance between any pair of streamlines. Thus, this distance is the minimal distance under which the integration of a streamline stops.

e By default the **First start point list** is the start point for the first streamline. It is chosen in the element where the highest value of the velocity of the specified vector field occurs. If required, select **Manual** from the list to override the default. then enter **x** and **y** coordinates.

Method 4: Creating Streamlines with Variable Density and Magnitude Controlled

To create streamlines with a variable density according to the magnitude of the specified vector field.

1 From the **Positioning** list, select **Magnitude controlled**.

2 The **Magnitude controlled** setting gives proper streamline plots only for incompressible flow fields. In this case, the algorithm places the streamlines so that the flow between each pair of adjacent streamlines is the same throughout the domain, giving streamlines that are more dense where the magnitude of the field is high. The next step is depends if you are working on a 2D or 3D model.

- *2D*: Enter a **Density**. This value is roughly the number of streamlines. Prior to streamline generation, the software computes a rough estimate of the total flow of the flow field in the model, divides this value with the specified **Density** setting, and uses the resulting value as the flow between each pair of adjacent streamlines.
- *3D*: Enter the **Min** (Minimum) **distance** and **Max** (Maximum) **distance** between streamlines. These distances are specified as fractions of the mean of the lengths of the bounding box of the geometry. The minimum velocity in the model is mapped to the minimum distance and the maximum velocity to the maximum distance. Thus every point on a streamline and on the boundary has a separating

distance associated with it. Given a set of streamlines, the start point for the next streamline is selected using these separating distances.

A streamline stops only if it exits the domain or gets too close to its own start point, using the **Loop tolerance** option in the **Advanced** section (or if any of the general termination criteria specified in the **Advanced** section is fulfilled).

3 If required, under **Streamline Positioning**, from the **Advanced parameters** list, select **Manual** to set advanced parameters as described in Method 3: Creating Streamlines with Uniform Density.

ADVANCED SECTION SETTING EFFECTS

The **Advanced** settings have the following effects:

- When calculating streamlines, the software selects a set of starting points (controlled by the streamline start points and the number of start points)
- The algorithm then finds the vectors of the given vector field at these points by interpolation. It normalizes the vector field if that option is selected.
- The algorithm integrates the points along the direction of the vector using the integration tolerance using a second-order Runge-Kutta algorithm.
- At the new positions, the algorithm finds vector values by interpolation and performs another integration.

This process stops if:

- It reaches a predetermined number of integration steps (controlled by the maximum number of integration steps entry).
- The points end up outside the geometry.
- The points reach a "stationary point" where the vector field is zero. You control the meaning of "zero" with the stationary point stop tolerance.
- It has used a predetermined amount of "time" for integrating (you control this parameter with the **Maximum integration time** field).

Finally, the software connects the calculated points for each streamline consecutively with straight lines.

Note: When integrating, the software uses a pseudotime that has nothing to do with the time in time-dependent problems. Use the massless particle tracing tool to integrate in time-varying fields and to control the real time in stationary fields.

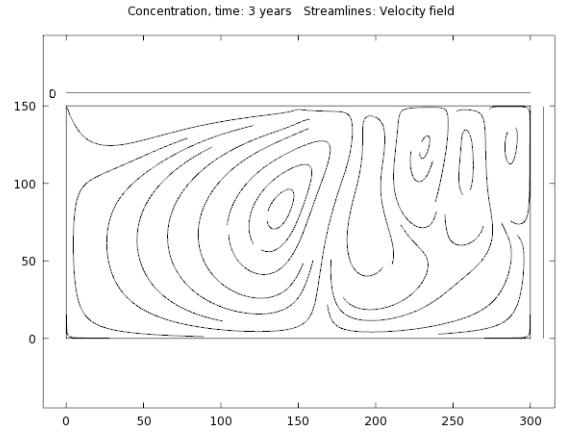


Figure 22-23: A 2D streamline plot. From the Earth Science Model Library: Buoyancy Flow with Darcy's Law: The Elder Problem.

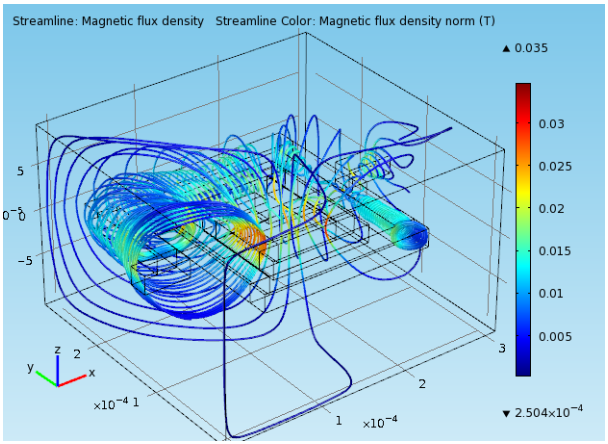


Figure 22-24: A 3D streamline plot with color expression. From the AC/DC Model Library: Integrated Square-Shaped Spiral Inductor.

Creating a 2D or 3D Surface Plot

Use **Surface** plots to display a quantity on a domain in 2D or on a boundary in 3D. See Figure 22-25, and Figure 22-26 and for examples from the COMSOL Multiphysics Model Library using the default **Rainbow** color table. Use an isosurface data set or a parameterized surface data set.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **2D Plot Group** ( ) or **3D Plot Group** ( ). Right-click the plot node and select **Surface** ( ).

The **Surface** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Expression**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c To enter a **Description** (or edit the default), select the check box.

5 Under **Range**, as required, manually override the **Minimum** and **Maximum color range** and **data range**. Click to clear the check box, then specify a range using the fields or use the sliders to control values.

6 Under **Coloring and Style**:

a Select a **Coloring**—**Color Table** (default) or **Uniform**. If **Uniform** is selected, select a **Color** or **Custom** to choose a different color.

b Select a **Color Table**. If the default (**Rainbow**) is not suitable for the plot, try other options.

c The **Color legend** is selected by default. Click to clear the check box if required.

The legend displays to the right of the plot. In plots where each line represents a certain time value, eigenvalue, or parameter value, these values also display.

d To plot only on the visualization mesh, select the **Wireframe** check box. This displays the surface as a triangular grid.

7 Under **Quality**, select a plot **Resolution**—**Finer**, **Fine**, **Normal**, **Coarse**, or **No refinement**. A higher resolution means that elements are split into smaller patches during rendering.

8 To enforce continuity on discontinuous data, under **Quality**, from the **Smoothing** list, select:

- **None**: to plot elements independently.
- **Internal**: to smooth the quantity inside the geometry but no smoothing takes place across borders between domains with different settings.
- **Everywhere**: to apply smoothing to the entire geometry.

9 Under **Quality**, the **Recover** default is **Off** because recovery takes processing time. See Accurate Derivative Recovery for more information.

To edit the default and use polynomial-preserving recovery and recover fields with derivatives such as stresses or fluxes with a higher theoretical convergence than smoothing, from the **Recover** list select:

- **Within domains**: to perform recovery inside domains.
- **Everywhere**: to apply recovery to all domain boundaries.

10 If required, add a **Deformation** and/or **Filter**.

11 *2D only*: If required, add a **Height Expression**.

12 Click the **Plot** button (), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

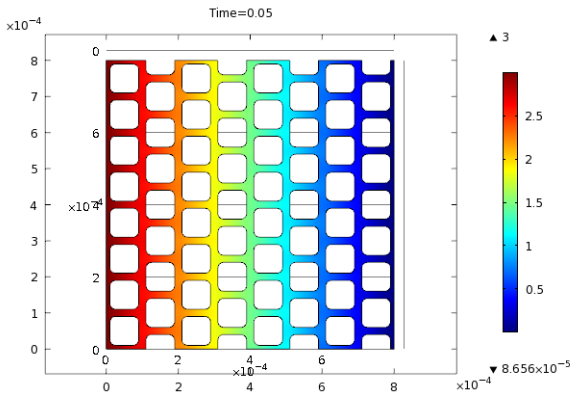


Figure 22-25: A 2D surface plot. From the Model Library: Effective Diffusivity in Porous Materials.

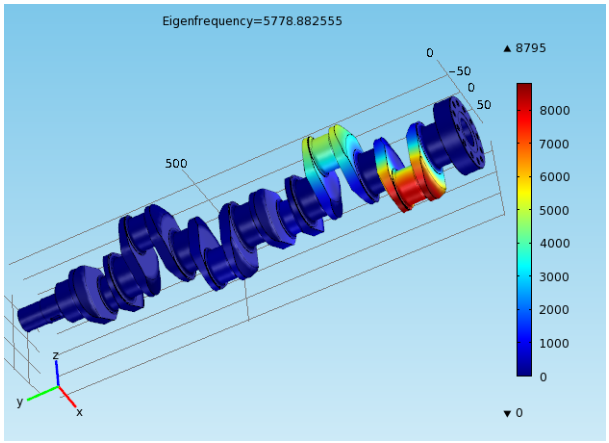


Figure 22-26: A 3D surface plot. From the Model Library: Eigenvalue Analysis of a Crankshaft.

ADDING A HEIGHT EXPRESSION TO A 2D SURFACE PLOT

Height attributes introduce a 3D height on a 2D surface plot. Add a **Height Expression** so the height of the surface represents a scalar quantity. Only 2D surface plots support height attributes. See Figure 22-27 and Figure 22-28 for an example from the Acoustics Module Model Library.

1 In the **Model Builder**, add and define a 2D **Surface** plot ( ). Right-click the plot node and select **Height Expression** ( ).

   The **Height Expression** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Expression**, the **Height data** defaults to **From parent** to use the same data set as the plot group it belongs to. If **Expression** is selected instead:

   a Enter an **Expression** or the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model.

   b Select a **Unit**.

   c To enter a **Description** (or edit the default), select the check box.

3 Under **Scale**:

   a Enter a **Scale factor** or use the sliders to control height values.

   b Enter an **Offset** or use the sliders to control values.

4 Click the **Plot** button (), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

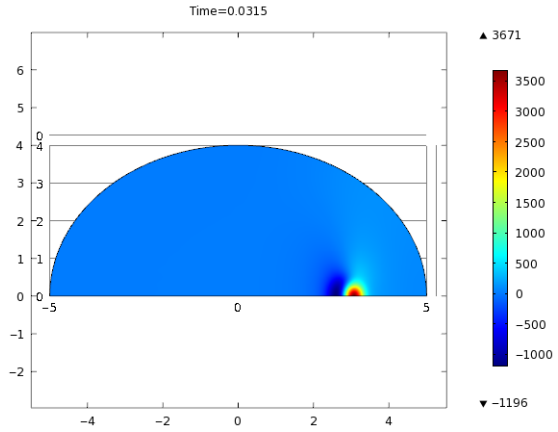


Figure 22-27: A 2D surface plot before a height expression is added. From the Model Library: Transient Gaussian Explosion.

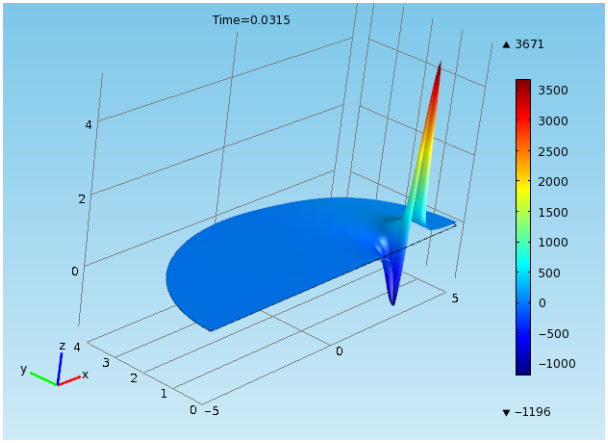


Figure 22-28: A 2D surface plot with height expression. From the Model Library: Transient Gaussian Explosion.

Creating a 1D Table Plot

Table plots can be added to 1D plot groups and display data from a table with one line per output column.

1 Define a table to plot. See Adding and Evaluating Tables for a Derived Value.

2 In the **Model Builder**, right-click **Results** and add a **1D Plot Group** ( ). Right-click the plot node select **Table Plot** ( ).

The **Table Plot** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Table**.

4 Under **Coloring and Style**, in the **Line style** section:

a Select a **Line**—**Cycle**, **Solid**, **Dotted**, **Dashed**, or **Dash-dot**. If **Cycle** is selected, it cycles through all the options.

b Select a **Color** or **Custom** to define another color.

c Enter a line **Width** or use the slider to select.

5 Under **Coloring and Style**, in the **Line markers** section:

a Select a **Marker** type—**None**, **Cycle**, **Asterisk**, **Circle**, or **Diamond**.

b Enter the **Number** of markers to display or use the slider to select.

6 Under **Legends**, the **Show legends** check box is selected by default to display to the right of the plot. In plots where each line represents a certain time value, eigenvalue, or parameter value, these values also display. For point plots, the legend displays the coordinate (or vertex number).

- If **Manual** is selected from the **Legends** list, enter your own legend text into the table. Use the **Move Up** ( ), **Move Down** ( ), and **Delete** ( ) buttons as required to move rows in the table.

7 Click the **Plot** button ( ), right-click the node and select **Plot**, or press F8.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

Creating a 3D Volume Plot

Use **Volume** plots display a quantity inside a domain in 3D. See Figure 22-30 and Figure 22-29 for examples from the COMSOL Multiphysics Model Library.

1 Define one or more **Data Sets**.

2 In the **Model Builder**, right-click **Results** and add a **3D Plot Group** ( ). Right-click the plot group and select **Volume** ( ).

The **Volume** page opens in the **Settings** window and a node is added to the **Model Builder**.

3 Under **Data** select a **Data set**. **From parent** means that the plot uses the same data set as the plot group it belongs to.

4 Under **Expression**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c To enter a **Description** (or edit the default), select the check box.

5 Under **Range**, as required, manually override the **Minimum** and **Maximum color range** and **data range**. Click to clear the check box, then specify a range using the fields or use the sliders to control values.

6 Under **Coloring and Style**:

a Select a **Coloring**—**Color Table** (default) or **Uniform**. If **Uniform** is selected, select a **Color** or **Custom** to choose a different color.

b Select a **Color Table**. If the default (**Rainbow**) is not suitable for the plot, try other options.

c The **Color legend** is selected by default. Click to clear the check box if required.

The legend displays to the right of the plot. In plots where each line represents a certain time value, eigenvalue, or parameter value, these values also display.

d To plot only on the visualization mesh, select the **Wireframe** check box. This displays the surface as a triangular grid. See Figure 22-29 for an example.

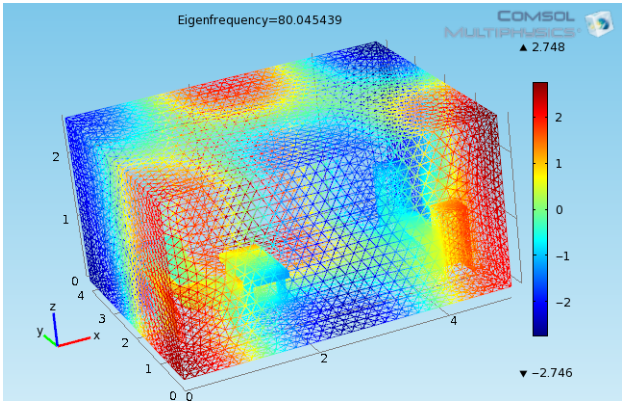


Figure 22-29: A 3D volume plot with the wireframe check box selected. From the Model Library: Eigenmodes of a Room.

7 Under **Quality**, select a plot **Resolution**—**Finer**, **Fine**, **Normal**, **Coarse**, or **No refinement**. A higher resolution means that elements are split into smaller patches during rendering.

8 To enforce continuity on discontinuous data, under **Quality**, from the **Smoothing** list, select:

- **None**: to plot elements independently.
- **Internal**: to smooth the quantity inside the geometry but no smoothing takes place across borders between domains with different settings.
- **Everywhere**: to apply smoothing to the entire geometry.

9 Under **Quality**, the **Recover** default is **Off** because recovery takes processing time. See Accurate Derivative Recovery for more information.

To edit the default and use polynomial-preserving recovery and recover fields with derivatives such as stresses or fluxes with a higher theoretical convergence than smoothing, from the **Recover** list select:

- **Within domains**: to perform recovery inside domains.
- **Everywhere**: to apply recovery to all domain boundaries.

10 Under **Element Filter**, specify the elements to display:

- **a** Select the **Enable filter** check box.
- **b** From the **Criterion** list, select **Random**, **Expression**, or **Logical expression**.
- **c** If **Expression** or **Logical expression** is selected, enter an **Expression** in the field.
- **d** If **Random** or **Expression** is selected, specify the **Fraction** of elements to show or use the slider to choose.

11 To scale elements in the mesh plot, under **Shrink Elements**, enter an **Element scale factor** between 0 and 1.

12 If required, add a **Deformation** and/or **Filter**.

13 Click the **Plot** button ( ), right-click the node and select **Plot**, or press F8.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

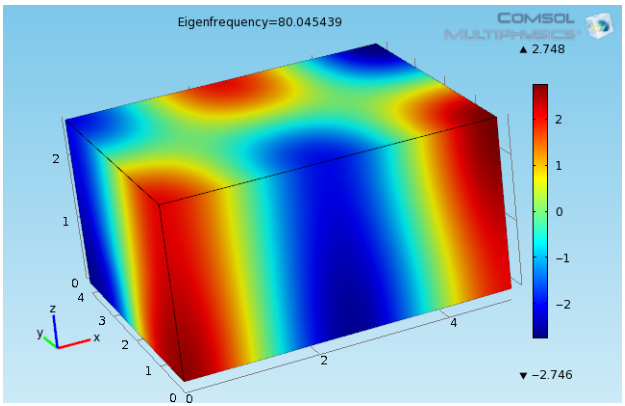


Figure 22-30: A 3D volume plot. From the Model Library: Eigenmodes of a Room.

Combining Plots

1 Define one or more **Data Sets**.

2 In the **Model Builder,** add one or more **1D** ( ), **2D** ( ), or **3D Plot Groups** ( ).

3 Add individual plots to the plot group. Experiment with how different combinations of plots look by adding a plot, then right-clicking and selecting **Disable**.

The revised plot displays in the **Graphics** window. Right-click the plot node again to **Enable** the plot in the plot group. Or select **Delete** to remove it.

4 Click the **Plot** button (), or right-click individual plot nodes and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

EXAMPLES OF COMBINED PLOTS

The examples below are taken from the **Model Library** as indicated. A variety of color tables are used in these examples.

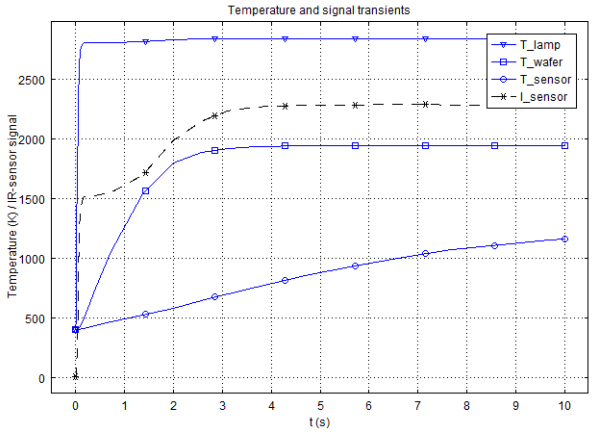


Figure 22-31: A 1D combined point and global graph. From the Heat Transfer Module Model Library: Rapid Thermal Annealing.

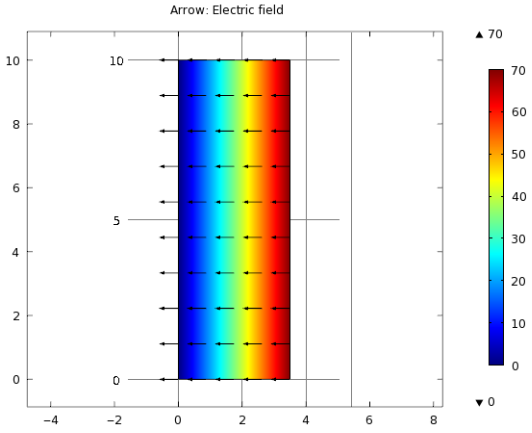


Figure 22-32: A 2D combined surface and arrow surface plot with a rainbow color table. From the Chemical Reaction Engineering Module Model Library: Isoelectric Separation.

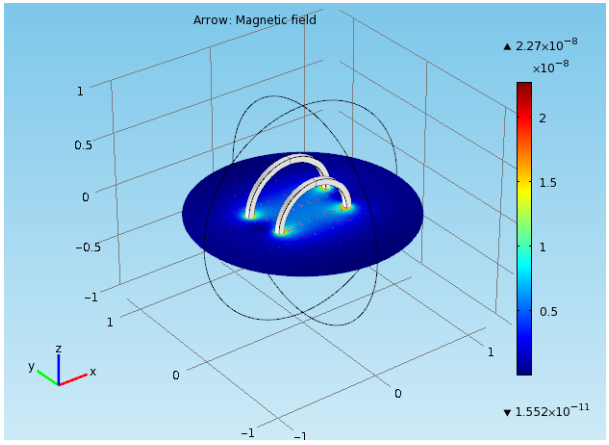


Figure 22-33: A 3D combined slice, arrow, and surface plot. From the AC/DC Module Model Library: Magnetic Field of a Helmholtz Coil.

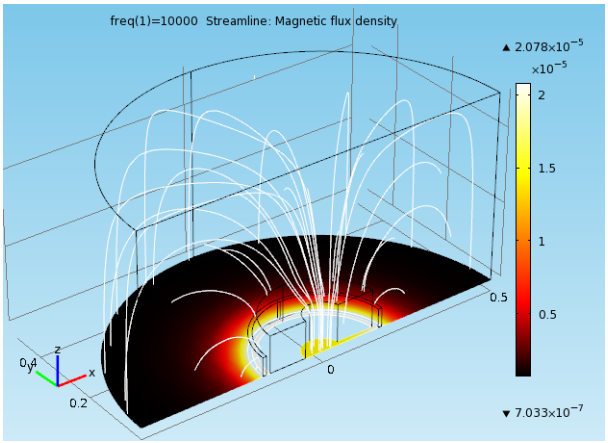


Figure 22-34: A 3D combined streamline and slice plot with a thermal color table. From the AC/DC Module Model Library: Cold Crucible.

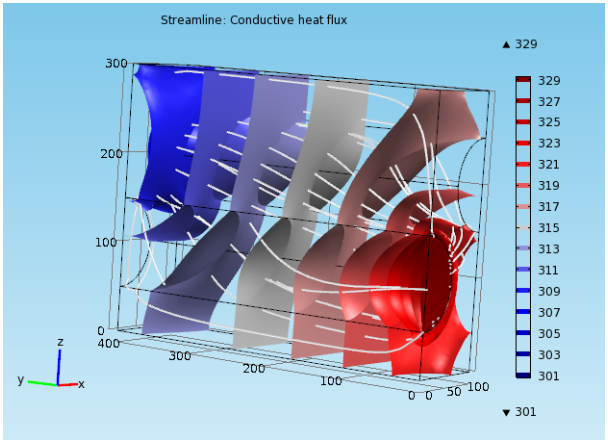


Figure 22-35: A 3D combined isosurface and streamline plot with a wave color table. From the Heat Transfer Module Model Library: Isothermal Heat Exchanger.

Creating Cross-Section Plots

Cross-section plots are created using a combination of data sets and plot groups. Cross-section plots show the values over time, along a parametric solution, or for several eigenvalues. Cross-section plots visualize a quantity as a family of plots on:

- An arbitrary set of points (in 1D, 2D, or 3D)

  A point cross-section plot makes it easy to view an expression at an arbitrary set of spatial coordinates and results in a line plot. Note that expressions and variables that include derivatives of the dependent variables (for example, stresses in a structural analysis) are not available at isolated geometry vertices (points). See 1D, 2D, and 3D Cross-Section Point Plots.

- An arbitrary line (in 2D or 3D). See 2D Cross-Section Line Plots and 3D Cross-Section Line Plots.

  Use cut line data sets to create lines through 2D or 3D geometries to visualize along the line. All plots and results analysis features available in 1D are available for cut line data sets as well as 3D plots and results analysis features for edges.

- Arbitrary planes (in 3D) using a surface plot and cut plane data set. See 3D Cross-Section Surface Plot.

  Use cut plane data sets to create planes through a 3D in a 2D geometry to visualize on the plane. All plots and results analysis features available in 2D are available for cut plane data sets as well as for surfaces in 3D. The cut plane corresponds to an orthogonal 2D coordinate system embedded in 3D.

INTERACTIVE CROSS-SECTION LINE AND SURFACE PLOTS

You can also interactively create cross-section line and surface plots using a combination of cross-section toolbar buttons and clicking on the geometry. When you use the cross-section toolbar, plot groups and data sets are automatically added and updated in the Model Builder whenever any line or plane is changed. As soon as a line or plane is completed, a rendering mesh snaps the entry and exit points to the faces of the domain containing the data and to computes the line. See Creating Interactive 2D Cross-Section Line Plots, Creating Interactive 3D Cross-Section Line Plots and Creating Interactive 3D Cross-Section Surface Plots.

Plotting and Cross-Section Interactive Toolbar

On the main toolbar for 2D and 3D plots, interactive buttons are available based on the plot type. Use these buttons during the creation of cross-section plots or just in general while creating plots.

TABLE 22-4: PLOTTING AND CROSS-SECTION TOOLBAR

BUTTON	NAME	USE AND RESULT
	Select First Point for Cut Line	Available for 2D and 3D plot groups to create a cross-section line plot. Adds a **Cut Line** data set and a **1D Plot Group** with a **Line Graph** that uses this data set. Click these buttons to plot a cross-section of data between two points.
	Select Second Point for Cut Line	
	Select Cut Line Direction	Available with 3D plot groups to create a cross-section line plot. Adds a **Cut Line** data set and a **1D Plot Group** with a **Line Graph** that uses this data set. Click this button to plot a line perpendicular to a point selected in the Graphics window.
	Select Cut Line Surface Normal	Available with 3D plot groups to create a cross-section line plot. Adds a **Cut Line** data set and a **1D Plot Group** with a **Line Graph** that uses this data set. Click this button to plot a line in the same way as a domain point probe, with point and direction.
	Select First Point for Cut Plane Normal	Available with 3D plot groups to create a cross-section surface plot. Adds a **Cut Plane 3D** data set and a **2D Plot Group** with a **Surface** plot that uses this data set. Click these buttons to plot a cross-section of data between the two points along the plane.
	Select Second Point for Cut Plane Normal	
	Select Cut Plane Normal	Available with 3D plot groups to create a cross-section surface plot. Adds a **Cut Plane 3D** data set and a **2D Plot Group** with a **Surface** plot that uses this data set. Click this button to plot a plane perpendicular to a point selected in the Graphics window.
	Select Cut Plane Normal from Surface	Available with 3D plot groups to create a cross-section surface plot. Adds a **Cut Plane 3D** data set and a **2D Plot Group** with a **Surface** plot that uses this data set. Click this button to plot a plane.
	Surface	Click this button to add a Surface plot to a 2D or 3D Plot Group.

TABLE 22-4: PLOTTING AND CROSS-SECTION TOOLBAR

BUTTON	NAME	USE AND RESULT
	Surface with Height	Click this button to add a surface plot with a height attribute to a 2D Plot Group.
	Arrow Surface	Click this button to add an Arrow Surface plot to a 2D or 3D Plot Group.
	Streamline	Click this button to add a Streamline plot to a 2D or 3D Plot Group.
	Line	Click this button to add a Line plot to a 2D or 3D Plot Group.
	Slice	Click this button to add a Slice plot to a 3D Plot Group.
	Isosurface	Click this button to add an Isosurface plot to a 3D Plot Group.
	Volume	Click this button to add a Volume plot to a 3D Plot Group.
	Arrow Volume	Click this button to add an Arrow Volume plot to a 3D Plot Group.

NOTES ABOUT USING THE CROSS-SECTION INTERACTIVE TOOLBAR

The first time any of the buttons are clicked on the cross-section toolbar, a data set and a plot group containing either a line graph or surface plot are added to the **Model Builder**. No new data set or plot group is created unless the generated data set or plot groups are deleted or disabled. See below for exceptions. COMSOL also chooses default coordinates as a vertical line intersecting the data in the middle.

It is also important to ensure the areas of the geometry selected contain data when defining the line or plane. When lines or planes are changed, the coordinates and calculations are automatically updated in the data set and in the final plot.

Deleting and Disabling Data Sets and Plot Groups

The following exceptions apply to the data sets and plot groups that are automatically added using the cross-section toolbar.

- If a plot group is *disabled*, no new data set or plot group is created even if you click one of the interactive buttons. You need to enable the plot group to regenerate the cross-section plot.
- If a plot group is *deleted*, click one of the interactive buttons to regenerate the plot group using the cut plane or cut line data set.

- If a data set is *deleted*, and it is used with a plot group, the plot group is also deleted at the same time. However, if the plot group is using another data set it is not deleted.
- If a data set is *disabled*, the associated plot group is not disabled. However, if you want to plot another cross-section, click one of the interactive buttons to create a new data set to use with the plot group.

1D, 2D, and 3D Cross-Section Point Plots

CREATING A 1D CROSS-SECTION PLOT USING A CUT POINT DATA SET

A 1D cross-section point plot visualizes a quantity in one or several points in time, along a parameter range, or for several different eigenvalues.

1 Create a **Cut Point 1D** data set.
   - Under **Point Data** enter the **x**-coordinates for the plot. Enter one or several space-separated values or a vector of coordinates, for example, `range(0,10,100)`.
2 Click the **Plot** button ( ).
3 Add a **1D Plot Group** ( ). Select **Cut Point 1D** as the **Data set**. If required, right-click and **Rename** the plot group, for example, **Cross-Section-1D Point**.
4 Add a **Point Graph** to this 1D plot group and select **Cut Point 1D** as the **Data set** or **From parent** to use the same data set as the plot group it belongs to.
5 Continue to define the **Point Graph** as required.
6 Click the **Plot** button ( ), right-click the node and select **Plot**, or press F8.

   The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

CREATING A 2D CROSS-SECTION PLOT USING A CUT POINT DATA SET

The 2D point cross-section plot visualizes a quantity in one or several points in time, along a parameter range, or for several eigenvalues. This example uses the Stresses in a Pulley model from the COMSOL Multiphysics Model Library.

1 Create a **Cut Point 2D** data set.
   - Under **Point Data** enter the x- and y-coordinate values for the plot. Enter the same number of space-separated values in the **x** and **y** fields. Alternatively, enter a vector of coordinates, for example, `range(0,10,100)`.
2 Click the **Plot** button ( ).

3 Add a **1D Plot Group** ( ). Select **Cut Point 2D** as the **Data set**. If required, right-click and **Rename** the plot group, for example, **Cross-Section-2D Point**.

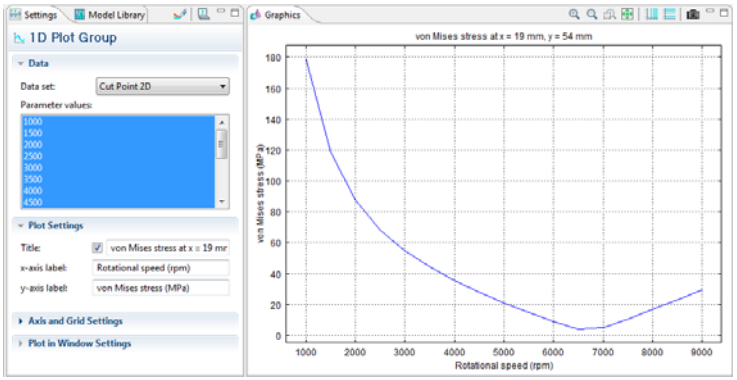
4 Add a **Point Graph** and select **Cut Point 2D** as the **Data set** or **From parent** to use the same data set as the plot group it belongs to.

- The x-axis corresponds to time, parameter values, or the eigenvalue number.
- The settings in the **y-axis data** area determine the quantity on the y-axis. Select from predefined quantities or enter an expression that contains variables.

5 Continue to define the **Point Graph** as required.

6 Click the **Plot** button ( ) right-click the node and select **Plot**, or press F8.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.



CREATING A 3D CROSS-SECTION PLOT USING A CUT POINT DATA SET

A 3D point cross-section plot visualizes a quantity in one or several points in time, along a parameter range, or for several eigenvalues. This example uses the Star-Shaped Microchannel Chipmodel from the MEMS Module Model Library.

1 Create a **Cut Point 3D** data set.

- Under **Point Data** enter the x-, y-, and z-coordinate values for the plot. Enter the same number of space-separated values in the **x**, **y**, and **z** fields. Alternatively, enter a vector of coordinates, for example, `range(0,10,100)`.

2 Click the **Plot** button ( ).

3 Add a **1D Plot Group** ( ). Select **Cut Point 3D** as the **Data set**. If required, right-click and **Rename** the plot group, for example, **Cross-Section-3D Point**.

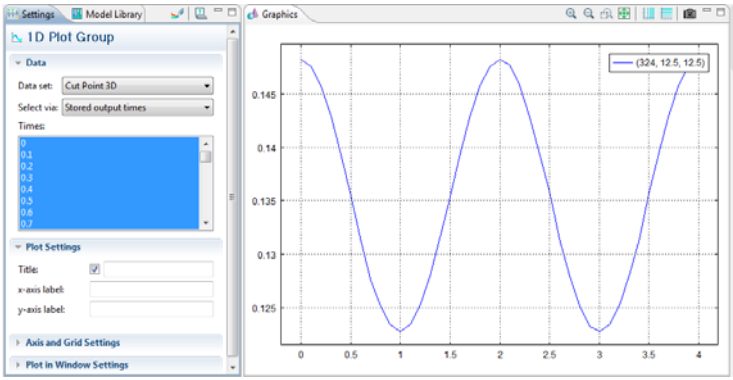
4 Add a **Point Graph** and select **Cut Point 3D** as the **Data set** or **From parent** to use the same data set as the plot group it belongs to.

- The *x*-axis corresponds to time, parameter values, or the eigenvalue number.
- The settings in the **y-axis data** area determine the quantity on the *y*-axis. Select from predefined quantities or enter an expression that contains variables.

5 Continue to define the **Point Graph** as required.

6 Click the **Plot** button (), right-click the node and select **Plot**, or press F8.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.



2D Cross-Section Line Plots

CREATING A 2D CROSS-SECTION PLOT USING A CUT LINE DATA SET

The 2D line cross-section plot visualizes a quantity in one or several lines in time, along a parameter range, or for several eigenvalues.

1 Create a **Cut Line 2D** data set.

- Enter the 2D coordinates for the plot. Set the start and end point of the line under x and y, for Point 1 and Point 2.
- Select the **Additional parallel lines** check box to visualize on a set of parallel lines. Enter the **Distances** from the line as space-separated values.

2 Click the **Plot** button ().

3 Add a **1D Plot Group** ( ). Right-click the plot node and select **Cut Line 2D** as the **Data set**. If required, right-click and **Rename** the plot group, for example, **Cross-Section-2D Line**.

4 Add a **Line Graph** and select **Cut Line 2D** as the **Data set** or **From parent** to use the same data set as the plot group it belongs to.

- Settings under **Y-Axis Data** determine the quantity to display on that axis. Either select from predefined **Expressions** or enter an **Expression** containing variables.
- Settings under **X-Axis Data** determine the quantity on that axis. Select **Arc length** or **Expression** as required. Enter or select an **Expression**, **Unit** and **Description**.

5 Continue to define the **Line Graph** as required.

6 Click the **Plot** button ( ), right-click the node and select **Plot**, or press F8.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.

CREATING INTERACTIVE 2D CROSS-SECTION LINE PLOTS

1 In the **Model Builder**, click a **2D Plot Group** node to display the buttons available on the main toolbar.

2 On the main toolbar, click the **Select First Point for Cut Line** button ( ). Click a start point on the geometry. COMSOL chooses default coordinates as a vertical line intersecting the data in the middle.

3 Click the **Select Second Point for Cut Line** button ( ). Click an end point on the geometry.

A line connecting the two points is displayed in the **Graphics** window. The first time the cross-section toolbar buttons are clicked, a **Cut Line 2D** data set and a **1D Plot Group** with a **Line Graph** are added to the **Model Builder**.

4 Adjust the cut line as required by clicking the buttons, then clicking on the geometry to change where the first and second point start and end (respectively). The coordinates are updated automatically in the data set and plot group. Click the **1D Plot Group** node to view the updates to the line graph.

5 Continue adjusting the cut line until the line graph representing the points plots the data as required.

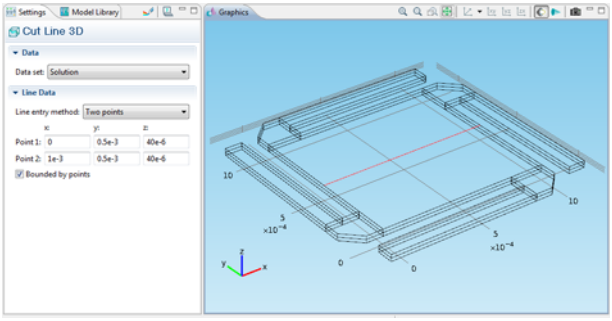
CREATING A 3D CROSS-SECTION PLOT USING A CUT LINE DATA SET

A 3D line/extrusion cross-section plot visualizes a quantity in one or several lines in time, along a parameter range, or for several eigenvalues. This example uses the Prestressed Micromirrormodel from the MEMS Module Model Library.

1 Create a **Cut Line 3D** data set.

- Enter the 3D coordinates for the plot. Set the start and end point of the line under **x**, **y**, and **z** for **Point 1** and **Point 2**.

2 Click the **Plot** button ( ).



3 Add a **1D Plot Group** ( ). Right-click the plot node and select **Cut Line 3D** as the **Data set**. If required, right-click and **Rename** the plot group, for example, **Cross-Section-3D Line**.

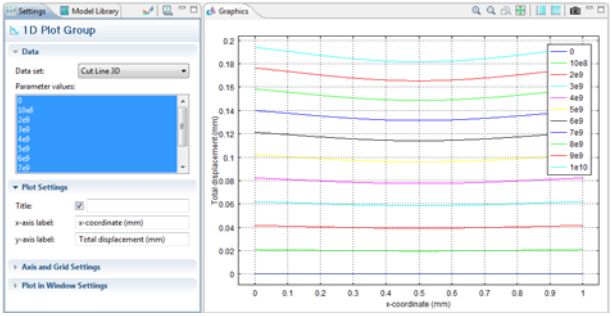
4 Add a **Line Graph** and select **Cut Line 3D** as the **Data set** or **From parent** to use the same data set as the plot group it belongs to.

- Settings under **Y-Axis Data** determine the quantity to display on the vertical axis. Either select from predefined **Expressions** or enter an **Expression** containing variables.
- Settings under **X-Axis Data** determine the quantity on the horizontal axis of the cross-section plot. Select **Arc length** or **Expression** as required. Enter or select an **Expression**, **Unit** and **Description**.

5 Continue to define the **Line Graph** as required (see Creating a 1D Line Graph).

6 Click the **Plot** button ( ) right-click the node and select **Plot**, or press F8.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.



CREATING INTERACTIVE 3D CROSS-SECTION LINE PLOTS

1 In the **Model Builder**, click a **3D Plot Group** node to display the buttons available on the main toolbar. On the main toolbar, click the cross-section buttons as required.

The first time the cross-section toolbar buttons are clicked, a **Cut Line 3D** data set and a **1D Plot Group** with a **Line Graph** are added to the **Model Builder**.

Defining a Cut Line:

a On the main toolbar, click the **Select First Point for Cut Line** button ( ). Click a start point on the geometry. COMSOL chooses default coordinates as a vertical line intersecting the data in the middle.

b Click the **Select Second Point for Cut Line** button ( ). Click an end point on the geometry. A line connecting the two points is created in the **Graphics** window.

c Click either of the buttons and then on the geometry to change the start and end points respectively.

d Click the **1D Plot Group** to view the **Line Graph** based on the selected points.

Defining a Cut Line - Direction:

a On the main toolbar, click the **Select Cut Line Direction** button ( ).

b Click on the geometry to add a line perpendicular to where you clicked. COMSOL chooses default coordinates as a vertical line intersecting the data in the middle.

c Click the **1D Plot Group** node to view the **Line Graph** based on the selected points.

Defining a Cut Line - Surface Normal:

a On the main toolbar, click the **Select Cut Line Surface Normal** button ( ).

b Click on the geometry to add a line with a point and direction. COMSOL chooses default coordinates as a vertical line intersecting the data in the middle.

c Click the **1D Plot Group** node to view the **Line Graph** based on the selected points.

2 Adjust the cut line as required by clicking the buttons, then clicking on the geometry to change its coordinates. The data set and plot group are updated automatically with the cut line data. Click the **1D Plot Group** node to view the updates to the line graph.

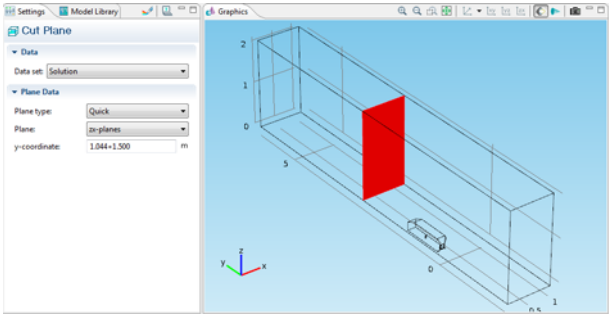
3 Continue adjusting the cut line until the line graph representing the points plots the data as required.

3D CROSS-SECTION SURFACE PLOT USING A CUT PLANE DATA SET

A 3D surface cross-section plot visualizes a quantity in one or several planes in time, along a parameter range, or for several eigenvalues. This example uses the Airflow Over an Ahmed Bodyfile from the CFD Module Model Library.

1 Create a **Cut Plane 3D** data set.

2 Click the **Plot** button (), or right-click the node and select **Plot**.



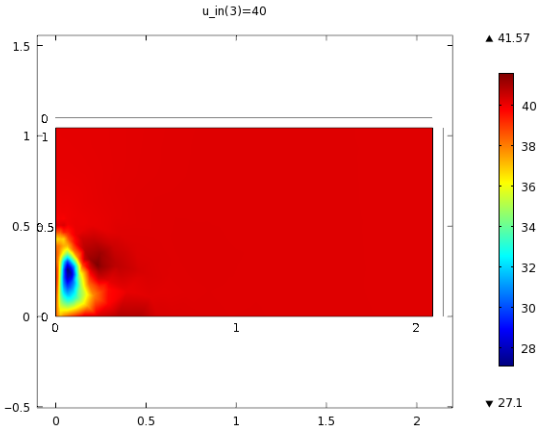
3 Add a **2D Plot Group** ( ). Right-click the plot node and select **Cut Plane 3D** as the **Data set**. If required, right-click and **Rename** the plot group, for example, **Cross-Section-3D Surface**.

4 Add a **Surface** plot and select **Cut Plane 3D** as the **Data set** or **From parent** to use the same data set as the plot group it belongs to.

5 Continue to define the **Surface** plot as required.

6 Click the **Plot** button (), or right-click the node and select **Plot**.

The plot displays in the **Graphics** window. To plot results in separate windows, right-click the plot node and select **Plot in Window**.



CREATING INTERACTIVE 3D CROSS-SECTION SURFACE PLOTS

1 In the **Model Builder**, click a **3D Plot Group** node to display the buttons available on the main toolbar. On the main toolbar, click the cross-section buttons as required.

The first time the cross-section toolbar buttons are clicked, a **Cut Plane 3D** data set and a **2D Plot Group** with a **Surface** plot are added to the **Model Builder**.

To Define a Cut Plane:

a Click the **Select First Point for Cut Plane Normal** button ( ). Click a start point on the geometry. COMSOL chooses default coordinates as a vertical line intersecting the data in the middle. The green highlighted areas show you what the cut plane will look like if this first click point is chosen for the surface plot.

b Click the **Select Second Point for Cut Plane Normal** button ( ). Click an end point on the geometry. The green highlighted areas show you what the cut plane will look like if this second click point is chosen for the surface plot.

c Click either of the buttons and then on the geometry to change the start and end points respectively.

d Click the **2D Plot Group** to view the **Surface** plot based on the selected points.

To Define a Normal Cut Plane:

a Click the **Select Cut Plane Normal** button ( ).

b Click on the geometry to add a plane perpendicular to the click location. COMSOL chooses default coordinates as a vertical line intersecting the data in

the middle. The green highlighted areas show you what the plot will look like if this normal cut plane is chosen for the surface plot.

c Click the **2D Plot Group** to view the **Surface** plot based on the selected points.

To Define a Normal Cut Plane from Surface:

a Click the **Select Cut Plane Normal from Surface** button ().

b Click on the geometry to add a line with a point and direction starting at the click location. COMSOL chooses default coordinates as a vertical line intersecting the data in the middle. The green highlighted areas show you what the plot will look like if this normal from surface cut plane is chosen for the surface plot.

c Click the **2D Plot Group** to view the **Surface** plot based on the selected points.

2 Adjust the cut plane as required by clicking the buttons, then clicking on the geometry to change the coordinates. The data set and plot group are updated automatically with the cut plane data. Click the **2D Plot Group** node to view the updates to the surface plot.

3 Continue adjusting the cut plane until the surface plot representing the points plots the data as required.

Defining Derived Values and Tables

You can integrate any postprocessing quantity to compute quantities such as total flux, charges, inductances, reaction forces, and average values. Use **Derived Values** to define evaluations of numerical results.

ADDING DERIVED VALUE NODE

1 In the **Model Builder**, under **Results**, right-click **Derived Values**.

2 Select an option from the list and continue defining each derived value as listed in Table 22-5:

TABLE 22-5: DERIVED VALUE TYPES

DERIVED VALUE	DESCRIPTION	GO TO
Volume Integration	To evaluate an integral over a set of domains in 3D models.	page 738
Surface Integration	To evaluate an integral over a set of domains in 2D, 2D axisymmetric, or boundaries in 3D.	page 738
Line Integration	To evaluate an integral over a set of domains in 1D, boundaries in 2D, or edges in 3D. Integrate over any data set of the right dimension. For example, make a volume integration of a 2D revolve, or a surface integration of a cut plane.	page 738
Point Evaluation	To evaluate expressions or variables defined in a point.	page 741
Global Evaluation	To evaluate the numerical value of a global variable.	page 742

Defining a Volume, Surface, or Line Integration Derived Value

1 In the **Model Builder**, under **Results**, right-click **Derived Values** and select:

- **Volume Integration** ($\iiint$) to evaluate an integral over a set of domains in 3D models.
- **Surface Integration** ($\iint$) to evaluate an integral over a set of domains in 2D, 2D axisymmetric, or boundaries in 3D.
- **Line Integration** ($\int$) to evaluate an integral over a set of domains in 1D, boundaries in 2D, or edges in 3D. Integrate over any data set of the right

dimension. For example, make a volume integration of a 2D revolve, or a surface integration of a cut plane.

The **Volume**, **Surface**, or **Line Integration** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Data**, select a **Data set**. The result is computed by integrating over the data set selected.

Note: If **None** is selected from the **Data set** list, skip the next step.

For **Parametric Solver** and **Parametric Sweep** studies, the **Parameter values** section lists the associated parameter values.

a For **Parametric Sweep** studies (see Solver Studies and Study Types) select an option from the **Select via** list —**Stored output times** or **Interpolated times** (time dependent models only).

The time steps cannot be selected directly because the different parametric solutions could have different time steps. Selecting **Stored output times** plots all of them, or select **Interpolated times** to get the same interpolated times for every parameter.

b If **Interpolated times** is selected, enter **Times** or click the **Vector Input** button ( ) to select and define specific times. See Entering Ranges and Vector-Valued Expressions for details.

c When available, from the **Table rows** list, select **Inner solutions** or **Outer solutions**. These options are available when there is a Parametric Sweep problem with dynamic inner solutions (that is to say, time, eigenvalue, or parametric solutions).

- If **Inner solutions** is selected, when you click the **Evaluate** button ( ) the results table displays the *dynamic value* (for example, time, eigenvalue, or parametric) solutions in rows.
- If **Outer solutions** is selected, when you click the **Evaluate** button ( ) the results table displays the *parameters* in rows.

3 If a **Solution** data set is selected, under **Selection**:

a Select **All** or **Manual**.

b If **Manual** is selected choose geometry directly from the **Graphics** window. Select **All** to add the applicable geometry or any other predefined grouping. Use the **Add**

to Selection ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required in the **Settings** window.

4 Under **Expression**, for all data sets, including **None**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c Select the **Description** check box to enter a description of the data set (or edit the default).

5 Under **Integration Settings**:

a Select a **Method**.

- **Auto**—the default and computes the integral for fields by numerical integration and for reaction forces by summation. Predefined quantities for reaction forces use summation instead of integration. Also, if you specify an expression for the integrand using the `reacf` operator, the automatic setting chooses the summation method.
- **Integration**—the standard numerical integration method (quadrature).
- **Summation**—a summation method useful for computing reaction forces. The summation method finds all nodes on the boundary, evaluates the expression in the nodes and sum up the values. Reaction force variables are predefined in the structural mechanics interfaces.

b COMSOL automatically determines an appropriate **Integration order** for the expression. To change this, click to clear the check box and enter a number. COMSOL performs the integration elementwise using numeric quadrature of the selected order.

Note: For 2D axisymmetric models, **Volume** and **Surface Integration** is available. COMSOL multiplies the expression (integrand) with `2*pi*r` prior to integration to compute the corresponding volume or surface integral. Complete the next step.

---

c For **Surface Integration**, select the **Compute volume integral (axial symmetry)** check box.

d For **Line Integration**, select the **Compute surface integral (axial symmetry)** check box.

6 Click the **Evaluate** button ( ).

7 The **Results** window opens to view a **Table** of the **Derived values**. By default four digits are displayed; click the **Full precision** button ( ) to display as many significant digits as possible. The **Table display precision** level can be changed from the **Options>Preferences>General** menu.

Defining a Point Evaluation Derived Value

1 In the **Model Builder**, under **Results**, right-click **Derived Values** and select **Point Evaluation** ( ) to evaluate expressions or variables defined in a point.

The **Point Evaluation** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Data** select a **Data set**.

If a **Cut Point** data set is selected, skip the next step.

3 If a **Solution** data set is selected, under **Selection**:

a Select **All** or **Manual**.

b If **Manual** is selected choose geometry directly from the **Graphics** window. Select **All** to add the applicable geometry or any other predefined grouping. Use the **Add to Selection** ( ), **Remove from Selection** ( ), and **Clear Selection** ( ) buttons as required in the **Settings** window.

4 Under **Expression**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

a Select a **Unit**.

b Select the **Description** check box to enter a description of the data set (or edit the default).

5 Click the **Evaluate** button ( ).

6 Open the **Results** window to view a **Table** of the **Derived values**. By default four digits are displayed; click the **Full precision** button ( ) to display as many significant digits as possible. The **Table display precision** level can be changed from the **Options>Preferences>General** menu.

Defining a Global Evaluation Derived Value

1 In the **Model Builder**, under **Results**, right-click **Derived Values** and select **Global Evaluation** ( ).

The **Global Evaluation** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Data** select a **Data set**. The result is computed by integrating over the data set selected.

3 Under **Expression**:

a Click the **Replace Expression** ( ) or **Insert Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression**.

b Select a **Unit**.

c Select the **Description** check box to enter a description of the data set (or edit the default).

4 Click the **Evaluate** button ( ).

5 Open the **Results** window to view a **Table** of the **Derived values**. By default four digits are displayed; click the **Full precision** button ( ) to display as many significant digits as possible. The **Table display precision** level can be changed from the **Options>Preferences>General** menu.

Adding and Evaluating Tables for a Derived Value

1 Add a **Derived Value** node to the model.

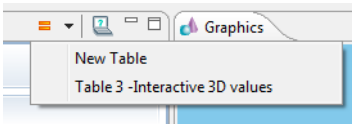
2 There are two ways to add a **Table**:

- In the right corner of the **Volume**, **Surface**, or **Line Integration** page, click the **New Table** button ( ). This button is also used as the **Evaluate** button.
- In the **Model Builder**, right-click the **Tables** node, and select **Table**.

  A **Table** node is added under **Tables**. Click the node to display a table with the selected integration feature's description and values in the **Results** window—by default it opens below the **Graphics** window.

3 There are two ways to evaluate a **Table**:

- In the right corner of the **Volume**, **Surface**, or **Line Integration** page, click the **Evaluate** button ( ). This button is also used as the **New Table** button.
- In the **Model Builder**, right-click the **Derived Values** node, and select **Evaluate**.
- After evaluating for the first time, you can click the **Evaluate** button ( ) a second time to evaluate the same table. Or you can click the down arrow next to the **Evaluate** button to select the target table on your own. This can be used, for example, to put the results from several evaluation features into the same table.



Editing and Organizing Results Tables

When a **Table** is generated and displays in the **Results** window, a variety of editing options are available as listed in Table 22-6 and in Figure 22-36.

TABLE 22-6: RESULTS TABLE EDITING BUTTONS

BUTTON AND FUNCTION	DESCRIPTION
Full precision ( )	Click to display as many significant digits as possible.
Clear Table ( )	Click to clear the data from the table, but keep the table itself. Click the Evaluate button ( ) to regenerate the table data.
Delete Table ( )	Click to delete the table. There is no undo. If required, click the Evaluate button ( ) to regenerate the table.
Plot ( )	Click to plot the table in the **Graphics** window.
Copy Table to Clipboard ( )	Click the button or right-click anywhere in the table and select this option from the context menu. Paste the data in a spreadsheet, for example.
Delete column	Right-click a column header or anywhere in the table, and select Delete column. There is no undo. If required, click the Evaluate button ( ) to regenerate the table.
Copy selection to clipboard	Select the rows to copy, then right-click anywhere in the table and select this option from the context menu or press Ctrl C.

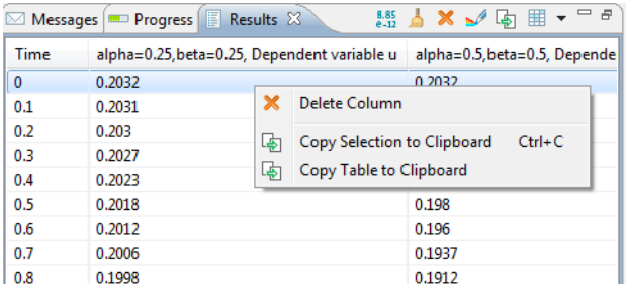


Figure 22-36: Context menu and button options for editing results tables.

Creating Reports and Exporting Data

The **Report** node is where you can export data from plot groups as images, animations, and numerical data. You can export the **Image**, **Plot**, or **Data** information directly for use in external reports or COMSOL generated reports.

ADDING REPORT TYPES

After a model is completed, you can add various components to the **Report** node and then generate report types (animation, data, images, player, plot, or export), or export the information to your computer for use in external reports.

- In the **Model Builder**, under the **Report** node, right-click and select an option—**Animation**, **Player**, **Image 1D**, **2D**, or **3D**, **Plot**, or **Data**.

TABLE 22-7: REPORT TYPES

REPORT	DESCRIPTION	GO TO
Animation	To define and export a movie or series of images based on a plot group. Play the animation in a web browser or use it in presentations or on a web site.	page 745
Data	Exports numerical data to file. Data export operates directly on data sets.	page 748
Image	To export plot images from 1D, 2D and 3D Plot Groups	page 750
Player	To create interactive animations.	page 752
Plot	To export a plot from a 1D, 2D, or 3D Plot Group.	page 751

Defining and Exporting Plot Animations

Use **Animation** to define and export a movie or series of images based on a plot group. Play the animation in a web browser or use it in presentations or on a web site.

1 In the **Model Builder**, right click **Report** and select **Animation** ().

The **Animation** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Scene**, select a **Subject**. The **Subject** list contains the **1D**, **2D**, or **3D Plot Groups** previously defined.

3 Under **Output**, select an **Output type**—**Image sequence** generates multiple image files; **Movie** generates a single file containing all the images. If **Movie** is selected, go to the next numbered step.

If **Image sequence** is selected:

- Enter a **Filename** including a path to save it to your computer.

  - OR -

- Click **Browse** and navigate to where you want to **Save** the output. For example, navigate to the computer desktop and enter a **File name** in the **Export Image Sequence** window then select an option from the **Save as type** list—**.png**, **.bmp**, or **.jpg**.

  The text entered in the **File name** field is used for all the images generated. For example, if you enter `image`, select **.png** as the file type, and there are 11 frames in the movie, 11 files are created: `image01.png`, `image02.png`, …, `image11.png`.

4 If **Movie** is selected as the **Output type**, select a **File type**—**GIF**, **Flash**, or **AVI**.

a For all **File types**, enter a path and include a **Filename**. Or click **Browse** and navigate to where you want to **Save** the output. For example, navigate to the computer desktop and enter a **File name** in the **Export Image Sequence** window.

b For all **File types**, enter a number of **Frames per second**.

Enter information into the following fields based on the **File type**:

c If **Flash** is selected, the **Interpolate between frames** check box is selected by default. Click to clear the check box if required.

d If **AVI** is selected, enter a **Quality**. The default is 0.75.

AVI is a file format that can contain video encoded in different ways.

e For **Flash** and **GIF**, select the **Open in browser** check box to launch the default web browser to view the output file.

5 Under **Frame Settings**:

a Select the **Lock aspect ratio** check box to keep the original animation width and height.

b In the **Width** and **Height** fields, enter the number of pixels (px) for the generated image size.

c If required, select the **Record in reverse order** check box.

6 Under **Parameter Sweep**, select a **Sweep Type**—**Result parameter**, **Dynamic data extension**, or **Stored solutions** (the default). Each time selected creates a frame in the movie or an individual image file.

If **Stored solutions** is selected:

a Select **Stored output times** from the **Select via** list, and enter any vector of times in the **Times** field. For example, enter `range(0,0.01,1)` or `0 10^(range(-3,3))`.

b For time-dependent problems, select **Interpolated times** from the **Select via** list. Enter **Times** or click the **Vector Input** button () to select and define specific times. See Entering Ranges and Vector-Valued Expressions for details.

If **Result parameter** is selected:

a Enter a **Parameter**, **Start**, and **Stop** interval.

b Enter the **Number of frames**. The number of frames divided by the number of frames per second gives the total playing time for a movie of a static or eigenvalue solution.

If **Dynamic data extension** is selected:

a When animating static and eigenvalue solutions, select a **Cycle type**:

- **Full harmonic**—a full sine wave (the solution phase grows linearly from 0 to 360°)
- **Half harmonic**—half a sine wave (the solution phase grows linearly from 0 to 180°)
- **Linear**—a linear ramp ($\mathrm{Re}(e^{i\alpha})$, where α is the phase) grows linearly from 0 to 1)

The cycle starts from the angle specified in the **Solution at angle (phase)** field when defining a **Solution Data Set** (see Defining a Solution Data Set).

b To visualize the time evolution of a time-harmonic model, enter the **Number of frames** to include in the animation.

The number of frames divided by the number of frames per second gives the total playing time for a movie of a static or eigenvalue solution.

7 Under **Parameter Sweep**, for **Parametric Sweep** studies, select an option from the **Solutions** list—**Inner solutions** or **Outer solutions**.

- If **Outer solutions** is selected (that is to say, the parameters from the parametric sweep), the **Parameter values** section lists all options. Select as required, then select an option from the **Inner type** list—**First**, **Last** or **All**.
- If **Inner solutions** is selected, select the **Parameter value** to animate, then the time steps. See Defining a Volume, Surface, or Line Integration Derived Valueand Creating a 1D Global Plotfor more information about the inner and outer solutions.

8 Under **Layout**:

   a By default, the **Title**, **Legend**, and **Axes** text is included. To edit the default, select the **Include** check box and click to clear one or all of the **Title**, **Legend**, and **Axes** (3D only) check boxes.

   b Enter a **Font size** (pt).

   c Select a **Background**—**Current** or **Color**. Click **Color** to select a **Custom** color background to replace the default, which is blue.

9 Under **Advanced**, enter a **Resolution**. The default is 96 DPI (dots per inch).

10 Select the **Antialiasing** check box to reduce stairstep-like lines and to smooth lines and edges.

11 Click the **Export** button ( ) in the **Settings** window or right-click the node and select **Export**.

   The file is exported to the location on your computer previously specified in Step 3. The **Messages** window confirms where the files are exported as specified in the **Output** section.

Note: Use Windows Media Player to play AVI files.

Exporting Data from Data Sets

The **Data** report exports numerical data to file. Data export operates directly on data sets.

1 In the **Model Builder**:

   Right click **Report** and select **Data** ( ).

   - OR -

   Right-click any **Data Set** node, for example, **Solution**, and select **Add to Report**. Click the **Data** node under **Report**.

   The **Data** page opens in the **Settings** window and a node is added to the **Model Builder**.

Note: The nodes are numbered sequentially. To make it easier to organize, you can right-click and **Rename** the node.

---

2 Under **Data** select a **Data set**.

The **Data set** list contains the 1D, 2D, or 3D **Data Sets** previously defined, for example, **Solution** or **Cut Line**.

a For **Parametric Sweep** studies select an option from the **Parameter values** list, which has the associated data set values.

b Select an option from the **Select via** list —**Stored output times** or **Interpolated times** (time dependent models only). If **Stored output times** is selected, the **Times** section is auto-filled with information from the **Data set** selected.

c If **Interpolated times** is selected, enter **Times** or click the **Vector Input** button ( ) to select and define specific times. See Entering Ranges and Vector-Valued Expressions for details.

3 Under **Expressions**:

a Specify the **Expression** to output. Click the **Replace Expression** ( ) or **Add Expression** ( ) buttons to select predefined expressions based on the physics of the model. Or enter an **Expression** in the table or field.

b Enter a **Description** in the table or field.

Use the **Move Up** ( ), **Move Down** ( ), and **Delete** ( ) buttons as required to move rows in the table.

4 Under **Output**, enter a **Filename** including a path to save it to your computer.

- OR -

Click **Browse** and navigate to where you want to **Save** the output. For example, navigate to the computer desktop and enter a **File name** in the **Export Data** window. All are saved as **.txt** files.

5 Under **Output**:

a Select a **Data format**—**Sectionwise** or **Spreadsheet**.

b Select a **Space dimension**—**Take from data set** or **Manual**. If **Manual** is selected, enter a **Dimension**.

c Select a **Geometry level**—**Take from data set**, **Volume**, **Surface**, **Line** or **Point**.

6 Under **Advanced**:

a The **Include header** check box is selected by default. Click to clear the check box if required.

b Select a **Resolution**—**Finer**, **Fine**, **Normal** (default), or **Custom**. If **Custom** is selected, enter a **Lagrange-element node-point order**.

7 Click the **Export** button ( ) in the **Settings** window or right-click the node and select **Export**.

The file is exported to the location on your computer previously specified. The **Messages** window confirms where the files are exported as specified in the **Output** section.

Exporting Plot Images (1D, 2D or 3D)

Use the **Image** report to export plot images from 1D, 2D and 3D Plot Groups.

1 In the **Model Builder**:

Right click **Report** and select **Image 1D**, **Image 2D**, or **Image 3D** ( ).

- OR -

Right-click any **Plot Group** node, for example, **3D Plot Group** or **1D Plot Group** and select **Add Image to Report**. Click the **Image** node under **Report**.

The **Image** page opens in the **Settings** window and a node is added to the **Model Builder**.

Note: The nodes are numbered sequentially. To make it easier to organize, you can right-click and **Rename** the node.

---

2 Under **Scene**:

a Select a **Subject** from the list.

The **Subject** list contains the **1D**, **2D**, or **3D Plot Groups** previously defined. **Image 1D** uses data from **1D Plot Groups**, **Image 2D** uses data from **2D Plot Groups** and **Image 3D** uses data from **3D Plot Groups**.

b *2D and 3D only*: Select a **View**—**From plot group** uses the plot group settings, or select a predefined **View** setting from the list.

On the toolbar, click the **Refresh** button ( ) to refresh the **Graphics** window with your settings. This is useful when you change the Plot group or View selection.

3 Under **Image**:

a Select a **Unit** of dimension—**Pixels (px)**, **Millimeters (mm)**, or **Inches (in)**.

b Select the **Lock aspect ratio** check box to keep the original image width and height.

c In the **Width** and **Height** fields, enter the number of pixels, millimeters, or inches for the final image size.

d Enter a **Resolution**. The default is 96 DPI (dots per inch).

e *2D and 3D only*: Select the **Antialiasing** check box to reduces stairstep-like lines and smooth lines and edges.

4 Under **File**, enter a **Filename** including a path to save it to your computer.

- OR -

Click **Browse** and navigate to where you want to **Save** the output. For example, navigate to the computer desktop and enter a **File name** in the **Export Image** window then select an option from the **Save as type** list—**.png**, **.bmp**, or **.jpg**.

- The text entered in the **File name** field is used for all the images generated. For example, if you enter `image`, select **.png** as the file type, and there are 11 frames in the movie, 11 files are created: `image01.png`, `image02.png`, …, `image11.png`.

5 Under **Layout**:

a By default, the **Title**, **Legend**, and **Axes** (3D only) text is included. To edit the default, select the **Include** check box and click to clear one or all of the **Title**, **Legend**, and **Axes** (3D only) check boxes.

b Enter a **Font size** (pt).

c Select a **Background**—**Current** or **Color**. Click **Color** to select a **Custom** color background to replace the default, which is blue.

6 Click the **Export** button ( ) in the **Settings** window or right-click the node and select **Export**.

The file is exported to the location on your computer previously specified. The **Messages** window confirms where the files are exported as specified in the **File** section.

Exporting Individual Plot Data

Use the **Plot** feature to export a plot from a 1D, 2D, or 3D Plot Group.

1 In the **Model Builder**:

Right click **Report** and select **Plot** ().

- OR -

Right-click any **Plot type** in any or all **Plot Groups**, for example, the **Surface** and **Streamline** plots in a 3D Plot Group, select **Add Plot Data to Report**. Click the **Plot** node under **Report**.

The **Plot** page opens in the **Settings** window and a node is added to the **Model Builder**.

Note: The nodes are numbered sequentially. To make it easier to organize, you can right-click and **Rename** the node.

---

2 Under **Plot**:

a Select a **Plot group**. The **Plot group** list contains the **1D**, **2D**, or **3D Plot Groups** previously defined.

b Select a **Plot** to export its data. **Plot Groups** can contain one or more individual plots.

3 Under **Output**, enter a **Filename** including a path to save it to your computer.

- OR -

Click **Browse** and navigate to where you want to **Save** the output. For example, navigate to the computer desktop and enter a **File name** in the **Export Data** window. All are saved as **.txt** files.

4 Under **Output**, from the **Data format** list, select **Sectionwise** or **Spreadsheet**.

5 Under **Advanced**, the **Include header** check box is selected by default. Click to clear the check box if required.

6 Click the **Export** button () in the **Settings** window or right-click the node and select **Export**.

The file is exported to the location on your computer previously specified. The **Messages** window confirms where the files are exported as specified in the **File** section.

Exporting Players

A **Player** report is used to create interactive animation.

1 In the **Model Builder**, right click **Report** and select **Player** ( ).

The **Player** page opens in the **Settings** window and a node is added to the **Model Builder**.

2 Under **Scene**, select a **Subject**. The **Subject** list contains the **1D**, **2D**, or **3D Plot Groups** previously defined.

3 Under **Parameter Sweep**, select a **Sweep Type**—**Model parameter**, **Dynamic data extension**, or **Stored solutions** (the default).

Note: Each time selected creates a frame in the movie or an individual image file.

If **Stored solutions** is selected and there is no data auto filled in the field:

a Select **Stored output times** from the **Select via** list, and enter any vector of times in the **Times** field. For example, enter `range(0,0.01,1)` or `0 10^(range(-3,3))`.

b For time-dependent problems, select **Interpolated times** from the **Select via** list. **Times** are automatically added from the Stored solutions.

If **Model parameter** is selected:

a Enter a **Parameter**, **Start**, and **Stop** interval.

b Enter the **Number of frames**. The number of frames divided by the number of frames per second gives the total playing time for a movie of a static or eigenvalue solution.

If **Dynamic data extension** is selected:

a When animating static and eigenvalue solutions, select a **Cycle type**:

- **Full harmonic**—a full sine wave (the solution phase grows linearly from 0 to 360°)
- **Half harmonic**—half a sine wave (the solution phase grows linearly from 0 to 180°)
- **Linear**—a linear ramp ($\mathrm{Re}(e^{i\,\mathrm{phase}})$ grows linearly from 0 to 1)

The cycle starts from the angle specified in the **Solution at angle (phase)** field when defining a **Solution Data Set**.

b To visualize the time evolution of a time-harmonic model, enter the **Number of frames** to include in the animation.

The number of frames divided by the number of frames per second gives the total playing time for a movie of a static or eigenvalue solution.

4 Under **Settings**:

   a Enter the **Maximum frames**. The default is 25.

   b Enter the **Frame** number or select it using the slider. Observe the geometry in the **Graphics** window to see the **Shown frame** number.

5 Click the **Export** button ( ) in the **Settings** window or right-click the node and select **Export**.

   The file is exported to the location on your computer previously specified. The **Messages** window confirms where the files are exported as specified in the **File** section.

23

Running the COMSOL Software

This section provides an overview of the different ways that you can run the COMSOL software in addition to the COMSOL Desktop graphical user interface, including:

- Running COMSOL
- COMSOL Client/Server Architecture
- Running COMSOL Client/Server
- Running COMSOL in Parallel
- Running COMSOL with MATLAB
- COMSOL API
- Running COMSOL as a Client/Server
- COMSOL Batch

Running COMSOL

The primary way to access COMSOL's functionality is through the COMSOL Desktop. This section describes alternative means of accessing COMSOL's functionality.

COMSOL Client/Server

COMSOL's client/server architecture lets you access the COMSOL server - COMSOL's computational engine - as a separate process. For example, the COMSOL Desktop can act as a COMSOL client when connected to a COMSOL server.

You must have a floating network license (FNL) to run the COMSOL server and the COMSOL client on separate computers. Any valid COMSOL license is sufficient to run the client and the server run on the same computer.

The client and server need not run on the same platform. For example, you can run the COMSOL Desktop on Windows connecting to a COMSOL server on a Linux or Mac server. In this way you can interactively access a more powerful remote computer.

To start the COMSOL server under windows, just click **COMSOL Multiphysics Server** in the **Client Server** folder under your COMSOL installation on the start menu. On Linux and Mac, type the command `comsol server` to start the COMSOL server. For more options for starting the COMSOL server, see the *COMSOL Installation and Operations Guide* for how to launch a COMSOL server on the different platforms. The section "The COMSOL Command" for your platform describes all options for launching the COMSOl server.

You can connect to a COMSOL server from the COMSOL Desktop by selecting **Connect to Server** on the **File** menu in the COMSOL desktop. Disconnect from the sever by selecting **Disconnect from Server**.

You can also export and import a model from the COMSOL desktop to a running COMSOL server and vice versa. This functionality is primarily intended for the LiveLink for MATLAB. Select **Export Model to Server** or Import Model from Server on the **File** menu to transfer a model between the COMSOL Desktop and the LiveLink for MATLAB.

Parallel COMSOL

COMSOL supports two mutual modes of parallel operation.

SHARED MEMORY COMSOL

The shared memory parallel mode is suitable for running COMSOL on modern multicore or multiprocessor computers.

This parallel mode of operation is available for all platforms and all license types.

By default, COMSOL uses the shared memory parallel mode and allocates all cores on the computer.

DISTRIBUTED MEMORY COMSOL

This parallel mode lets you run COMSOL on a Windows HPC cluster or a Linux cluster.

You must have a floating network license (FNL) to run COMSOL in distributed memory mode.

You can control the options for running COMSOL on a cluster from the **Study** node in the **Model Builder**. Activate **Show More Options** from the **Model Builder**'s **View** menu to see the Cluster Computing item when right clicking See the section Cluster Computing in the *COMSOL Multiphysics Reference Guide* to get more information on cluster computing (or see Where Do I Access the Documentation and Model Library?).

COMSOL API

The COMSOL API is a Java-based programming interface for COMSOL functionality. The COMSOL API can be used for developing standalone applications based on COMSOL functionality.

The most basic use of the COMSOL API is to run a Model Java-file from the COMSOL Desktop or with the COMSOL batch command.

To run a Model Java-file from the COMSOL Desktop, compile it using the COMSOL compile command. This gets you a model class file corresponding to the model Java-file. Launch the model class file by selecting **Open** on the **File** menu, and selecting a **Model Class File** under **File name**.

To create a standalone application using the COMSOL API you need to develop a command-line based or GUI-based interface to the functionality and compile it using

the COMSOL compile command. The application needs to be run together with a COMSOL server.

The COMSOL API is described in the *COMSOL Reference Guide* (or see Where Do I Access the Documentation and Model Library?).

See the *Installation Guide* for how to launch the COMSOL compile command on the different platforms. The sections the COMSOL Command for the respective platforms describe how the COMSOL server can be launched.

COMSOL Batch

COMSOL batch allows you to run COMSOL without a graphical user interface. The COMSOL batch mode of operations allows you both to run Model class-files and Model MPH-files.

You can control the options for running COMSOL in batch mode from the **Study** node in the **Model Builder**. Activate **Show More Options** from the **Model Builder**'s **View** menu to see the **Batch** option when right-clicking the **Study** node. See the section Batch of the *COMSOL Multiphysics Reference Guide* to get more information on cluster computing (or see Where Do I Access the Documentation and Model Library?).

You can also run COMSOL batch entirely from a command prompt. See the *COMSOL Installation and Operations Guide* for how to launch the COMSOL batch command on the different platforms.

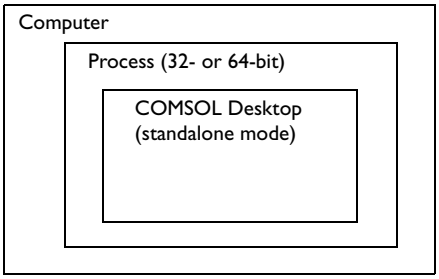
LiveLink for MATLAB

The LiveLink for MATLAB provides access to a COMSOL Model from MATLAB. On the MATLAB prompt you access the COMSOL model through a client/server connection to a running COMSOL server. You access the model through the COMSOL API and the MATLAB Java interface. In addition, there are M-file wrapper functions that help you perform tasks such as displaying graphics using MATLAB figure windows or fetching data from the model object.

COMSOL Client/Server Architecture

Standalone COMSOL

The most straightforward way of running COMSOL is as a standalone application.



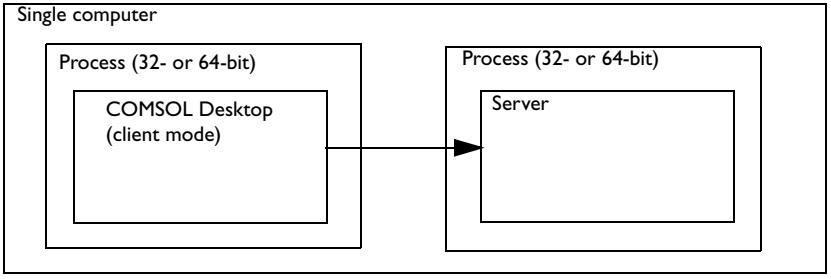
Running COMSOL as a Client/Server

The COMSOL client and server applications are available on all platforms.

Note: The license server is not the same as a COMSOL server. The license manager can run on a computer different from both the ones used by COMSOL Desktop and COMSOL server.

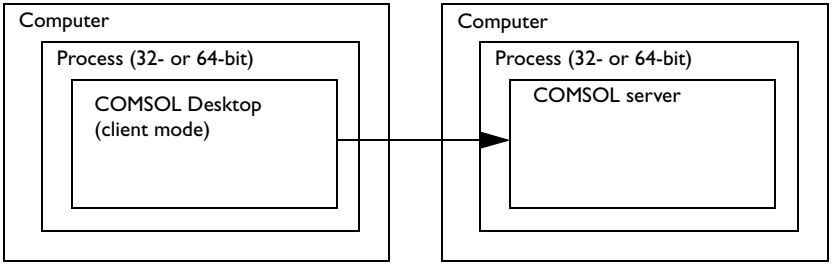
RUNNING COMSOL MULTIPHYSICS CLIENT/SERVER ON THE SAME COMPUTER

Both the COMSOL client and the COMSOL server can run on the same computer and with all available license types: named user license (NSL), CPU locked license (CPU), and floating network license (FNL).



RUNNING COMSOL CLIENT/SERVER ON DIFFERENT COMPUTERS

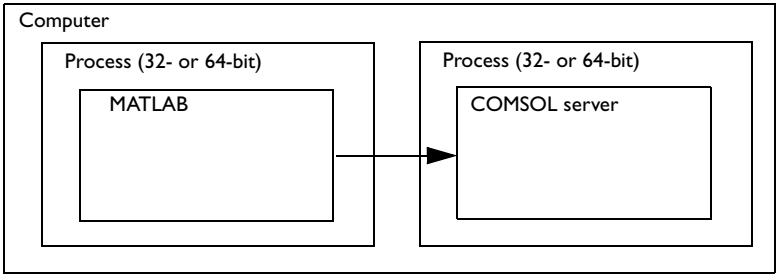
The COMSOL client and COMSOL Multiphysics server can also run on different computers, but this configuration requires a floating network license (FNL).



Running COMSOL with MATLAB

COMSOL can run together with MATLAB on the same machine using COMSOL's client/server architecture. The command `comsol server matlab` launches this

configuration. The command `comsol matlab` can be use to launch just the client and to connect to a remote server.



RUNNING A THIN CLIENT

When running inside MATLAB the COMSOL client runs in a so-called thin client mode. This means that few COMSOL shared libraries are loaded. Thus all operations that required shared libraries are performed on the COMSOL server.

Running COMSOL Client/Server

The COMSOL Desktop can run in a separate process as a client to a COMSOL server. The COMSOL Desktop client uses a TCP/IP connection to connect to the COMSOL server. The client and server need not run on the same platform. You must have a floating network license (FNL) to run the COMSOL server and the COMSOL client on separate computers.

You can also use the COMSOL client/server when running COMSOL with MATLAB. To do so, start a COMSOL client on a separate computer and connect to a COMSOL server started from within a MATLAB process.

Advantages of Using COMSOL Client/Server

The COMSOL client/server configuration frees your desktop computer of lengthy computations, dispatching your jobs to a dedicated computer. The computer that runs the COMSOL server could have more memory and a faster CPU than your desktop computer.

Note, too, that running the COMSOL server and the COMSOL client separately on the same computer increases the total memory available to solve problems. This is particularly interesting because the 32-bit limit on addressable memory can be the limiting factor for complex models. The COMSOL server components do not use the memory required for the graphical user interface, freeing that memory for the actual computations on the server.

Running COMSOL Client/Server

STARTING A COMSOL SERVER

When you have access to the Windows desktop, start the COMSOL server from the **Start** menu. Go to **Programs**, select **COMSOL 4.0a**, and then **Server**. If starting the COMSOL server from a terminal window in Windows, use the command `<COMSOL installation directory>\bin\win32\comsolserver.exe`.

On Linux, use the `comsol server` command to start a COMSOL server.

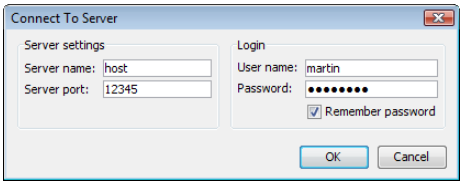
On Mac OS X, use the **COMSOL Multiphysics Server** application, or if you connect to Mac OS X from another computer, use the `comsol server` command in the terminal window.

INITIALIZING THE COMSOL SERVER

The first time you start a COMSOL server on a computer, you are asked for a user name and password. By default, your user name and a hashed password is stored on your computer's hard drive. You can avoid storing user name and password on disk by providing the option `-passwd nostore` target option to the COMSOL server command. As the COMSOL server is started, the server displays the port number. The server also displays a message each time you login from a client.

CONNECTING THE DESKTOP TO A SERVER

From the COMSOL Desktop select **Connect to Server** on the **File** menu. In the **Connect to Server** dialog box, specify the hostname of the server, the TCP/IP port number of the server (the port number of displayed by the server).



The user name and password are the ones you used when starting the server, and the port number is printed by the server when it is started.

DISCONNECTING THE DESKTOP FROM A SERVER

You can close the connection to the server or MATLAB using the menu item **Disconnect from Server**. Doing so transfers all data from the server to the client and makes it a standalone version of COMSOL Multiphysics. Unless the server was started with the option `-multi on` it will exit.

CONNECTING TO A SERVER FROM MATLAB

From MATLAB you can use the commands `Modelutil.connect` and `Modelutil.disconnect` to connect and disconnect from a COMSOL server. See the *COMSOL Reference Guide* for details (or see Where Do I Access the Documentation and Model Library?). The connection to the server is necessary to access and manipulate a model.

DISCONNECTING MATLAB FROM A SERVER

To connect to a COMSOL server, use the menu item **Connect to Server**. This opens a dialog box where you can enter the server and login information

EXPORTING TO A COMSOL SERVER

Primarily for use with the LiveLink for MATLAB, use **Export Model to Server** to transfer the model in the COMSOL Desktop to a live MATLAB session. This model transfer works while another client is connected to the server. You have this situation when running the LiveLink for MATLAB.

IMPORTING FROM A COMSOL SERVER

Primarily for use with the LiveLink for MATLAB, use **Import Model to Server** to transfer the model in the COMSOL Desktop to a live MATLAB session. This model transfer works while another client is connected to the server. You have this situation when running the LiveLink for MATLAB.

Running COMSOL in Parallel

COMSOL supports two mutual modes of parallel operation. One mode is based on the distributed memory model and runs on several nodes on a Linux or Windows cluster; see Distributed-Memory Parallel COMSOL. The other mode, described below, is based on the parallel shared memory model.

Shared-Memory Parallel COMSOL

Most multiprocessor machines and dual-core/multicore machines support the shared memory model, however, it is not supported by several nodes on a cluster. The solvers, assembly, and meshing in COMSOL Multiphysics benefit from shared memory parallelism. By default COMSOL uses all cores available on the machine for shared-memory parallelism.

BENEFITS OF RUNNING COMSOL SHARED-MEMORY PARALLEL

All iterative solvers and smoothers except Incomplete LU are parallelized. Some smoothers have blocked versions. The blocked versions are usually more parallel than the nonblocked versions. The finite element assembly also runs in parallel. Usually the speedup depends on the problem size; problems using a lot of memory usually have better speedup.

The PARDISO sparse direct linear solver runs in parallel. The SPOOLES sparse direct linear solver also runs in parallel. The direct solver MUMPS benefit from shared memory parallelism; however, to a slightly lesser extent than PARDISO and SPOOLES.

The free mesher in 3D runs in parallel over the faces and subdomains of the geometry object being meshed. For this reason, the speedup when running on several processors depends strongly on the domain partitioning of the corresponding geometry. Meshing a geometry with only one subdomain, such as an imported CAD part, gives almost no speedup at all. On the other hand, meshing a geometry with several subdomains, such as an imported CAD assembly with many parts, can give significant speedup, especially if the number of elements in the mesh is large.

The following postprocessing plots run in parallel in 3D: slice plots, isosurface plots, subdomain plots, boundary plots, edge plots, deformed-shape plots, and streamline plots. In 2D, surface plots, contour plots, and boundary plots, and deformed-shape plots run in parallel.

A significant part of the parallel speedup in computations comes from functions of the type BLAS (basic linear algebra subprogram; see the next section). If you want to run the software in parallel, it is important that the BLAS library you use supports parallelism. The BLAS libraries shipped with COMSOL do that.

Running in parallel usually requires extra memory. If you run out of memory, try to lower the number of used cores as explained in the Installation and Operations Guide. The speedup depends on the processor load. For instance, if your system has m processors and n of them are used by other active programs, do not set the number of processors to a number that is greater than $m - n$. The reason is that the programs compete for the same resources, which slows all of them considerably.

COMSOL and BLAS

BLAS is a set of functions for basic linear algebra operations. Vendors often supply BLAS libraries optimized for their hardware. A large portion of the computational engine in COMSOL relies on BLAS. Included with COMSOL are the BLAS libraries ACML (AMD Core Math Library) optimized for AMD processors with SSE2 support, MKL (Math Kernel Library) optimized for Intel processors. You can also supply your own BLAS library optimized for your hardware. By default COMSOL automatically tries to detect an appropriate BLAS library. The defaults in COMSOL are:

- MKL on Intel processors.
- ACML on AMD processors that support SSE2 instructions; otherwise MKL is used.

You can override the default as explained in the Installation and Operations Guide. If the library you want to use is unavailable or incorrectly installed, COMSOL switches back to the default library.

Distributed-Memory Parallel COMSOL

The Linux and Windows versions of COMSOL support a distributed memory mode. The distributed mode starts a number of computational nodes set by the user. Each computational node is a separate process running a COMSOL instance. A computational node is not the same as a physical node (computer), but they can

coincide. When running in distributed mode, COMSOL uses MPI for communicating between the processes in the distributed environment.

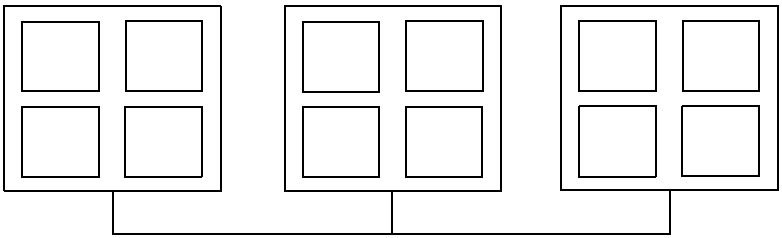


Figure 23-1: Schematic of a cluster with 3 physical nodes (computers) with 4 processors each.

The distributed-memory mode can be combined with COMSOL's ability to benefit from the shared-memory model. All modes that COMSOL can run in is able to use distributed memory mode.

Note: In client/server mode, the computer or cluster acting as server must be accessible from the client through a TCP/IP connection. If you are not able to connect to the server, you can use the COMSOL Batch to solve models on the cluster or do parametric sweeps. You can also use the Cluster Computing study to setup a COMSOL Batch job from within the GUI.

For the schematic in Figure 23-1, you can choose any number of computational nodes between 1 and 12. Each node, in turn, can use between 4 and 1 processors for shared memory. By default, COMSOL uses as many processors as are available on each physical node for shared-memory parallelism on Windows. This is suboptimal if the number of computational nodes is not the same as the number of physical nodes. Therefore we recommend that you explicitly set the number of processors. For the schematic example, if you run 6 computational nodes, the optimal value for number of processors is 2. The number of processors used is `6*2=12`.

For the same example, assuming you are the sole user of the system for the duration of the computation and that your problem requires a lot of memory, use 3 computational nodes with 4 shared memory cores each. If, on the other hand, your problem is small, use 12 computational nodes with 1 shared memory core each. This way you make the best use of shared-memory and distributed-memory parallelism for each problem.

You do not need a cluster to benefit from COMSOL's ability to utilize the distributed-memory model. On a multiprocessor computer you can use multiple computational nodes. This can be useful for small-sized parameter sweeps, for example. Make sure that the number of computational nodes times the number of processors does not exceed the number of available processors; otherwise performance deteriorates significantly. See the section Benefits of Running COMSOL Shared-Memory Parallel.

BENEFITS OF RUNNING COMSOL IN A DISTRIBUTED MODE

The solvers MUMPS and SPOOLES run distributed when running COMSOL in distributed mode. COMSOL can runs parameter sweeps using the distributed mode. The simplest way to start a distributed parameter sweep is to check the distribute parameters check box in the Cluster Computing study. The simplest way to modify an existing model is to add the Cluster Computing study and Compute. Make sure that Show more options is enabled.

24

Glossary

This glossary contains terms related to finite element modeling, mathematics, geometry, and CAD as they relate to the COMSOL Multiphysics software and documentation. For more application-specific terms, see the glossaries in the documentation for the following modules: *AC/DC Module*, *Acoustics Module*, *CAD Import Module*, *CFD Module*, *Chemical Reaction Engineering Module*, *Earth Science Module*, *Heat Transfer Module*, *MEMS Module*, *RF Module*, and *Structural Mechanics Module*. For references to further information about a term, see the index.

Glossary of Terms

adaptive mesh refinement A method of improving solution accuracy by adapting the mesh to the problem's physical behavior.

affine transformations Geometric transformations that are combinations of linear transformations and translations.

algebraic multigrid (AMG) An *algebraic multigrid* solver or preconditioner performs one or more cycles of a multigrid method using a coarsening of the discretization based on the coefficient matrix. Compare to *geometric multigrid (GMG)*.

anisotropy Variation of material properties with direction.

application programming interface (API) An *API* provides a set of documented functions and methods for interacting with a software product.

arbitrary Lagrangian-Eulerian formulation (ALE formulation) A formulation for a moving mesh where dependent variables represent the mesh displacement or mesh velocity. The COMSOL Multiphysics solvers have built-in support for the mesh movement.

arc A segment of the circumference of a circle or ellipse.

Argyris element A 2D, 6-node triangular finite element with a 5th-order *basis function* providing continuous derivatives between elements.

aspect ratio The ratio between the longest and shortest element or geometry dimension.

assemble Taking the local element stiffnesses, masses, loads, and constraints to form the *stiffness matrix*, *mass matrix*, load vector, constraint matrix, and constraint residual vector.

associative geometry An algorithm that maps data associated with a geometry to the new geometry entities when the geometry is modified.

backward differentiation formula (BDF) A multistep formula based on numerical differentiation for solutions to *ordinary differential equations*. A BDF method of

order *n* computes the solution using an *n*th-grade polynomial in terms of backward differences.

basis function A function φ_i in the *finite element space* such that the *i*th degree freedom is 1, while all other degrees of freedom are 0. For the Lagrange finite element space, φ_i is a linear or higher order polynomial on each mesh element with value 1 in node *i* and 0 in all other nodes.

Bernstein polynomial See *Bézier basis.*

Bézier basis A set of polynomial functions that occur in the definition of a *Bézier curve*. These polynomial functions are often called *Bernstein polynomials.*

Bézier curve A *rational Bézier curve* is a parameterized *curve* formed as the quotient of two polynomials expressed in the Bézier basis. It is a vector-valued function of one variable. The coefficients of a rational Bézier curve are geometrically interpreted as *control points* and *control weights.* A *nonrational Bézier curve* is a rational Bézier curve with all weights equal, thereby making the denominator polynomial equal to a constant. A nonrational Bézier curve is also called an *integer Bézier curve*.

Bézier patch, Bézier surface A *Bézier patch* or *Bézier surface* is a surface extension of the *Bézier curve*. The *Bézier patch* is a function of two variables with an array of control points.

bidirectional constraint See *constraint*.

Boolean operations Boolean operations are used to construct a *geometry object* from other solid geometry objects and rebuild it in a new form. At least two primary geometry objects are required to create a resultant new geometry object. That new object depends on the type of Boolean operation:

- Union (add): the resultant geometry object occupies all the space of the initial geometry objects
- Difference (subtract): the resultant geometry object occupies all the space of the first geometry object except for the space inside the second geometry object.
- Intersection: the resultant geometry object occupies only the space common to the initial geometry objects

boundary A geometric entity with a space dimension one less than the space dimension for the geometry (for example, a *face* in a 3D geometry). In a mathematical

context, the symbol $\partial\Omega$ represents the boundary of the domain Ω. Sometimes *boundary* is used in a narrower sense meaning an *exterior boundary*. See also *interior boundary*, *exterior boundary*.

border The interface between two parts in an *assembly*.

boundary modeling A geometry modeling method to create a geometry by defining its boundaries. Compare to *solid modeling* and *surface modeling*.

brick element See *hexahedral element*.

chamfer A CAD feature that trims off a corner with a plane or straight line.

Cholesky factorization A memory-saving version of *LU factorization* where U is the transpose of L. It requires that the coefficient matrix A ($A = LU$) be a symmetric positive definite matrix. See also *LU factorization* and *positive definiteness*.

coefficient form PDE A PDE in the coefficient form is a PDE formulation suited for linear PDEs

$$\begin{cases} e_a\dfrac{\partial^2 u}{\partial t^2} + d_a\dfrac{\partial u}{\partial t} + \nabla\cdot(-c\nabla u - \alpha u + \gamma) + \beta\cdot\nabla u + au = f & \text{in } \Omega \\ \mathbf{n}\cdot(c\nabla u + \alpha u - \gamma) + qu = g - h^T\mu & \text{on } \partial\Omega \\ hu = r & \text{on } \partial\Omega \end{cases}$$

coerce To convert a *geometry object* from one type to another, for example, from a *curve object* to s a *solid object*.

composite geometry object, composite solid object Geometric objects made up by combining *primitive geometry objects* and other composite objects. See also *constructive solid geometry*, *primitive geometry object*, and *Boolean operations*.

COMSOL Desktop The integrated simulation environment for the COMSOL Multiphysics products with a number of windows such as the Model Builder window, the Graphics window, and the Settings window.

COMSOL Multiphysics binary file A binary data file with the extension `.mphbin` that contains geometry objects or mesh objects. Earlier versions of COMSOL Multiphysics used the file extension `.flb`.

COMSOL Multiphysics text file A text data file with the extension `.mphtxt` that contains geometry objects or mesh objects.

condition number A measure of the possible error in a solution due to ill-conditioning of the equations. See also *ill-conditioning*.

constant A named model property that has a constant numeric value.

constraint Restriction imposed upon the dependent variables, typically as a *Dirichlet boundary condition*. *Neumann boundary conditions* are not regarded as constraints. When Dirichlet boundary conditions are introduced, the finite element algorithm makes a corresponding change to the Neumann boundary conditions so that the resulting model becomes solvable. For a *bidirectional constraint*, COMSOL Multiphysics accomplishes this change by adding the transpose of the constraint matrix h times a vector of *Lagrange multipliers* to the right-hand side of the Neumann boundary condition. For a *unidirectional constraint*, the extra term is often some matrix times the vector of Lagrange multipliers. In a mechanical model, the extra term is called a *constraint force*.

constructive solid geometry (CSG) A solid-modeling method that combines simple solid shapes, or *primitives*, to build more complex models using Boolean operations. See also *solid modeling* and *primitive*.

control point *Bézier* and *NURBS* curves and surfaces are defined by a set of points known as *control points*. The locations of the these points control the curve's shape.

control weight Scalar values assigned to *control points* to further control the shape of a curve or surface.

contour plot A plot that shows the variation of a solution component or other quantity. Points with equal values of the plotted quantity are connected with contour lines.

convergence The tendency for a finite element solution to approach the exact solution within well-defined and specified tolerances, for example, by reducing the mesh element size or the time step.

coupling operator An operator used to couple data within a model (geometry) or between different models (geometries). See also *extrusion coupling operator*, *projection coupling operator*, and *integration coupling operator*. Coupling variables provided similar functionality in earlier version of COMSOL Multiphysics, but coupling operators can be reused with different arguments (for example, for integrating different quantities over the same domain).

curl element See *vector element*.

curve The path of a point moving through space. See also *Bézier curve*, *NURBS*, and *manifold*.

curve object A geometry object consisting of only *edges* and *vertices*, for example a geometry object representing a *curve*.

curve segment An individual polynomial or rational polynomial curve. Compounded curves consist of several *curve segments*.

degree of freedom (DOF) One of the unknowns in a discretized finite element model. A degree of freedom is defined by a name and a *node point*. The degree of freedom names often coincide with the names of the dependent variables. The local degrees of freedom are all degrees of freedom whose node points are in one mesh element.

deformed geometry A geometry where the shape changes with a moving-mesh algorithm. It also the name of a *physics interface* for modeling deforming geometries. This is similar to the Parameterized Geometry physics interface in earlier versions of COMSOL Multiphysics.

dependent variable A varying quantity whose changes are arbitrary, but they are regarded as produced by changes in other variables. For example, temperature is a function of the space coordinates and time. In a narrower sense, the dependent variables, or *solution components*, are the unknowns in a mathematical PDE model. Compare to *independent variable*.

differential-algebraic equation (DAE) A set of equations that includes both differential and algebraic equations. A DAE is classified in terms of its *index*, a positive integer, which is related to the minimum number of differentiations needed to transform a DAE to an ODE form.

direct solver A solver for a system of linear equation that uses some variant of Gaussian elimination. Compare to *iterative solver*.

Dirichlet boundary condition A Dirichlet boundary condition specifies the value of the function (dependent variable) on a boundary. Dirichlet boundary conditions are sometimes called *essential boundary conditions* or *constraints*. For a coefficient form PDE the Dirichlet boundary condition is

$$hu = r.$$

See also *constraint*.

discretization The process of dividing a continuous system into a finite number of elements with finite size. The difference between the finite-element representation and the real system, the discretization error, drops as the size of the elements decrease. For a time-dependent analysis, a discretization of time into steps provides an idealized behavior of the variations in the solution during these steps.

divergence element A finite element often used for electromagnetic vector fields. The degrees of freedom on the boundary of a mesh element correspond to normal components of the field. Also *Nédélec's divergence element*.

domain A topological part of the modeling space in a geometry model. The geometric representation of a domain is a line segment (interval) in 1D, an area in 2D, and a volume in 3D. In a mathematical context, the symbol Ω represents the domain where the equations are defined.

drop tolerance A nonnegative scalar used in the incomplete LU preconditioner for the iterative solvers. See *incomplete LU factorization*.

dynamic model See *time-dependent model*.

edge, edge segment A geometric entity representing a bounded part of a *curve*. An *edge* or *edge segment* is a *boundary* in a 2D geometry. See also *domain*.

edge element See *vector element*.

eigenvalue PDE A PDE that describes an eigenvalue problem with unknown eigenmodes (eigenfunctions) u and eigenvalues λ. The *coefficient form* eigenvalue PDE is:

$$\lambda^2 e_a u - \lambda d_a u + \nabla \cdot (-c\nabla u - \alpha u) + \beta \cdot \nabla u + au = 0$$

elliptic PDE A linear stationary 2nd-order elliptic PDE has the form

$$\nabla \cdot (-c\nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + au = f$$

where c is positive or negative definite, for example, Poisson's equation.

embed To insert a 2D geometry into a 3D geometry model.

error Deviations from the correct solution, primarily due to: poor modeling; *discretization* (such as insufficiently fine mesh, poor elements, or insufficiently short time steps); and roundoff and truncation (depending on numerical representation, *ill-conditioning*, or the solution algorithms).

error estimate An estimation of the error in the numeric solution to a problem, either locally or globally, primarily for use by an adaptive mesh refinement. See also *adaptive mesh refinement*, *error*.

equivalent boundaries *Boundaries* that are rigid transformations of each other and have compatible meshes. See also *periodic boundary condition*.

essential boundary condition See *Dirichlet boundary condition*.

expression variable A user defined variable that is defined on any geometry domain in terms of *dependent variables*, *independent variables*, *constants*, *application scalar variables*, and other *expression variables*. Global expression variables are valid in all geometries; scalar expression variables are valid in the current geometry.

extended mesh A data structure that includes the full finite element mesh. See also *mesh*, *node point*.

extended multiphysics A model that includes nonlocal couplings and dependencies between variables, where the value at a point is the result of a computation elsewhere in the domain or in another geometry defined in the same model. *Coupling variables* provide the ability to project or extrude values from one geometry or domain to another. Compare to *multiphysics*.

exterior boundary An *exterior boundary* for a dependent variable u is a *boundary* such that u is defined only on one of the adjacent subdomains, that is, a boundary to the exterior of the computational domain. See also *boundary*.

extrude To create a 3D geometry object from a 2D geometry object in a *work plane* by translating (extruding) it along a path, often a straight line.

extruded mesh A 3D mesh created by extrusion of a 2D mesh. An extruded mesh can contain *hexahedral elements* and *prism elements*.

extrusion coupling operator An operator in the destination that takes values from the source by interpolation at points that depend on the position of the evaluation points in the destination.

face, face segment A domain describing a bounded part of a *surface* in a 3D geometry. A *face* or *face segment* is a *boundary* in a 3D geometry. See also *domain*.

face object A geometry object with no topological information on subdomains. Typically a trimmed surface is represented as a face object.

FEM See *finite element method*.

FEM structure The main data structure, containing all data for a model.

fillet A curved transition from one boundary to another, creating a rounded corner.

finite element In the mathematical sense, a *mesh element* together with a set of *shape functions* and corresponding *degrees of freedom*. The linear combinations of the shape functions form a space of functions called the *finite element space*. In the traditional FEA sense, the concept of a finite element also includes the discretized form of the PDEs that govern the physics. COMSOL Multiphysics generally uses *finite element* in the mathematical sense.

finite element analysis (FEA) A computer-based analysis method for field problems using the *finite element method*.

finite element method (FEM) A computational method that subdivides an object into very small but finite-size elements. The physics of one element is approximately described by a finite number of *degrees of freedom (DOFs)*. Each element is assigned a set of characteristic equations (describing physical properties, boundary conditions,

and imposed forces), which are then solved as a set of simultaneous equations to predict the object's behavior.

finite element space The linear space of functions where the finite element approximation to the solution of a PDE problem is sought. The functions in the finite element space are linear combinations of *basis functions* (*shape functions*).

flux vector The vector $\Gamma = -c\nabla u - \alpha u + \gamma$. See also *generalized Neumann boundary condition* and *normal flux*.

free mesh An *unstructured mesh* that can represent any geometry. Compare to *mapped mesh*.

free mesher The mesh generator creating *free meshes*. The mesh generator creating *triangular elements* is also referred to as the *free triangle mesher*, and the mesh generator creating *quadrilateral elements* is also referred to as the *free quad mesher*.

free quad mesher The mesh generator creating unstructured quadrilateral meshes.

free triangle mesher The mesh generator creating unstructured triangular meshes.

Gauss point Sometimes improperly used as a synonym for *integration point*. See also *integration point*.

general form PDE A PDE in the general form is a PDE formulation suited for nonlinear PDEs

$$\begin{cases} e_a\dfrac{\partial^2 u}{\partial t^2} + d_a\dfrac{\partial u}{\partial t} + \nabla\cdot\Gamma = F & \text{in } \Omega \\ -\mathbf{n}\cdot\Gamma = G + \left(\dfrac{\partial R}{\partial u}\right)^T \mu & \text{on } \partial\Omega \\ 0 = R & \text{on } \partial\Omega \end{cases}$$

generalized Neumann boundary condition A generalized Neumann boundary condition (also called a *mixed boundary condition* or a *Robin boundary condition*) specifies the value of a linear combination of the *normal flux* and the *dependent variables* on a boundary. For a coefficient form PDE, the generalized Neumann boundary condition is

$$\mathbf{n} \cdot (c\nabla u + \alpha u - \gamma) + qu = g - h^T \mu$$

The generalized Neumann condition is often called just *Neumann condition* in the documentation.

geometric multigrid (GMG) A *geometric multigrid* solver or preconditioner performs one or more cycles of a multigrid method, using a coarsening of the discretization based on a coarsening of the mesh or a reduction in the order of the shape functions. Compare to *algebraic multigrid (AMG)*.

geometry level The types of topological entities within a geometry model that describes bounded parts of the manifolds in the model, and also the relations between different manifolds in the geometry. The different geometry levels are the *vertex*, *edge*, *face*, and *domain* levels. An entity of dimension one less than the space dimension is referred to as a *boundary*. See also *manifold*.

geometry model A collection of topological and geometric entities that form a complete geometric description of the model.

geometry object The objects that represent a geometry model. See also *curve object*, *face object*, *primitive geometry object*, *solid object*.

grid A *grid* usually refers to sets of evenly-spaced parallel lines at particular angles to each other in a plane, or the intersections of such lines. Compare to *mesh*.

Hermite element A finite element similar to the *Lagrange element*. The difference is that there are degrees of freedom for the (1st-order) space derivatives at the mesh vertices. See also *Lagrange element*.

hexahedral element A 3D mesh element with eight corners and six faces, also referred to as *brick element*; sometimes also called *hex element* as a short form.

higher-order element A finite element with *basis functions* that consists of polynomials of degree 2 or higher.

hybrid geometry modeling Creating a geometry model using a combination of *boundary modeling/surface modeling* and *solid modeling*.

hyperbolic PDE A typical example of a linear 2nd-order hyperbolic PDEs is the *wave equation*

$$e_a \frac{\partial^2 u}{\partial t^2} + \nabla \cdot (-c\nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + au = f$$

where e_a and c are positive.

IGES file An *IGES file* contains 3D CAD data, including the 3D geometry, in an open format according to the *Initial Graphics Exchange Specification*. You can import an IGES file to COMSOL Multiphysics using the CAD Import Module.

ill-conditioning An ill-conditioned system is sensitive to small changes in the inputs and is susceptible to roundoff errors. See also *condition number*.

imprint An imprint of the smaller boundary on the larger boundary that makes the parts in a *pair* match. An imprint inserts points on the boundary in 2D and creates a curve on the boundary in 3D.

incomplete LU factorization An approximate *LU factorization* where small matrix elements are discarded to save memory and computation time. The *drop tolerance* is a relative measure of the smallness of the elements that should be discarded. See also *LU factorization*.

independent variable A variable that can cause variation in a second, *dependent variable*. The independent variables are most often space coordinates and time. Compare to *dependent variable*.

index, for DAE See *differential-algebraic equation*.

index vector See *domain group*.

initial condition The starting values for the dependent variables in a time-dependent analysis and for *nonlinear iterations* or other iterative solvers.

integration coupling operator An operator that evaluates integrals of expressions over the source and returns a single scalar value when used in the destination. Similar functionality is available to evaluate the average, minimum, and maximum values.

integration point See *numerical integration formula*.

interactive meshing Building a mesh in an incremental fashion where each meshing operation acts on a set of geometry domains.

interior boundary An *interior boundary* for a dependent variable u is a *boundary* such that u is defined on both adjacent subdomains or in no adjacent subdomain. See also *boundary*.

interval The domain between two vertices in a 1D geometry. Also called a *subdomain*.

isoparametric element A finite element that uses the same *shape function* for the element shape coordinates as for the *dependent variables*.

isosceles triangle A triangle with at least two equal sides (and two equal angles).

iteration See *iterative solver*.

iterative solver A solver for a system of linear equations that uses an iterative method, computing a sequence of more and more accurate approximations to the solution. Each step in this sequence is one *linear iteration*. This should not be confused with the Newtons iterations (*nonlinear iterations*) that occur in the solution of a nonlinear system of equations. Compare to *direct solver* and *nonlinear iteration*.

Jacobian matrix A matrix containing the first derivative of a vector-valued function of a vector variable. In particular, it is the derivative of the *residual vector* with respect to the *solution vector*. When used in this narrower sense, the term *stiffness matrix* is sometimes used.

Lagrange element A *finite element* with polynomial shape functions of a certain *order*. The value of the function is used as the *degree of freedom*, and the node points are evenly distributed within the mesh element.

Lagrange multiplier An extra dependent variable introduced in the Neumann boundary condition when a constraint is added. See also *constraint*.

linear iteration A step in a linear iterative solver. See *iterative solver*. Compare to *nonlinear iteration*.

linear PDE An equation where both sides are sums of a known function, the unknown functions, and their partial derivatives, multiplied by known coefficients that only depend on the *independent variables*. Other PDEs are called *nonlinear*.

LU factorization For a linear system of equations, a version of Gaussian elimination that produces a factorization $A = LU$ of the coefficient matrix, where L and U are the lower and upper triangular matrices, respectively. This makes it easy to quickly solve a number of systems with the same coefficient matrix. See also *direct solver*.

manifold A mathematical function describing a *surface*, *curve*, or *point* in a geometry model of any dimension.

mapped mesh A *structured mesh* with *quadrilateral elements* generated by mapping using transfinite interpolation.

mapped mesher The mesh generator creating *mapped meshes*.

mass matrix The matrix E that multiplies the second time derivative of the *solution vector* in the linearized discretized form of a PDE problem. If there are no second time derivatives (that is, if $E = 0$), then the term mass matrix is often used for the matrix D that multiplies the first derivative of the solution vector (the D matrix is otherwise called the *damping matrix*).

mesh A subdivision of the domains of a geometric model into, for example, triangles (2D) or tetrahedra (3D). These are examples of *mesh elements*. See also *grid*, *structured mesh*, and *unstructured mesh*.

mesh element The individual elements in the mesh that together form a partitioning of the geometry, for example, *triangular elements* and *tetrahedral elements*. See also *finite element*.

mesh vertex An endpoint or corner of a mesh element. See also *node point* and *vertex*.

method of lines A method for solving a time-dependent PDE through a space discretization, resulting in a set of ODEs.

mixed boundary condition See *generalized Neumann boundary condition*.

mode reduction A model-reduction technique for reducing systems with many degrees of freedom, for example large finite element models, to a form with fewer degrees of freedom for dynamic system simulations and analysis. See also *state-space model*.

model input *Model inputs* are fields such as temperature and velocities that act as inputs for material models and model equations. The model inputs can be fields computed by other physics interfaces or user defined values.

Model MPH-file A binary data file with the extension `.mph` that contains a COMSOL Multiphysics model. Often also just called model file. Earlier versions of COMSOL Multiphysics used the file extension `.fl`.

Model M-file An M-file containing commands that create a COMSOL Multiphysics model. Model M-files are text files that can be modified and used with MATLAB. If you have a MATLAB license and a license for the COMSOL LiveLink for MATLAB, the COMSOL Desktop can load a Model M-file. Compare with *Model MPH-file*.

MRI data *Magnet resonance imaging (MRI) data* is an image data format, primarily for medical use. MRI produces high-quality images of the inside of the human body. 3D MRI data is usually represented as a sequence of 2D images.

multidisciplinary models Multidisciplinary models combine PDE-based finite element modeling with other mathematical modeling techniques such as dynamic simulation in areas like automatic control and signal processing.

multigrid A solver or preconditioner for a linear system of equations that computes a sequence of increasingly accurate approximations of the solution by using a hierarchy of coarsened versions of the linear system (having fewer degrees of freedom). See also *algebraic multigrid*, *geometric multigrid*.

multiphysics Multiphysics models include more than one equation and variable from different types of physics. These variables can be defined in different subdomains. The equations can be coupled together through equation coefficients that depend on variables from other equations. Compare to *extended multiphysics*.

natural boundary condition See *Neumann boundary condition*.

Neumann boundary condition A Neumann boundary condition specifies the value of the *normal flux* across a boundary. Neumann boundary conditions are sometimes called *natural boundary conditions*. Compare to *generalized Neumann conditions*.

Newton's method An iterative solver method, also called the *Newton-Raphson method*, for solving nonlinear equations. See also *nonlinear iterations*.

Newton-Raphson method See *Newton's method*.

node point Any point in the mesh element where the degrees of freedom are defined. The node points often include the mesh vertices and possible interior or midpoint locations. See also *degree of freedom* (DOF) and *mesh vertex*.

nonlinear iteration A Newton step in the solution of a nonlinear PDE problem. Each nonlinear iteration involves the solution of a linear system of equations. Compare to *linear iteration*.

nonlinear PDE See *linear PDE*.

norm A scalar measure of the magnitude of a vector or a matrix. Several types of norms are used to measure the accuracy of numerical solutions.

numerical integration formula A numeric-integration method that approximates an integral by taking the weighted sum of the integrand evaluated at a finite number of points, the *integration points* (sometimes improperly called *Gauss points*). Also called *quadrature formula*.

normal flux The normal component of the *flux vector* at a boundary.

NURBS The *nonuniform rational B-spline (NURBS)* is a popular curve and surface representation scheme. A NURBS representation can be divided into a number of *rational Bézier curves* or surfaces.

order of a finite element The degree of the polynomials that define the *shape functions* (*basis functions*).

ordinary differential equation (ODE) An equation involving functions and their derivatives. The derivatives are with respect to one independent variable only. Compare to *partial differential equation (PDE)*.

parabolic PDE A typical example of a linear 2nd-order parabolic PDE is the *heat equation*

$$d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c\nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + a u = f$$

where d_a and c are positive.

parameter A constant that takes on different values for each model in a parametric analysis. See also *constant*.

partial differential equation (PDE) An equation involving functions and their partial derivatives; that is, an equation that includes derivatives with respect to more than one independent variable. Compare to *ordinary differential equation (ODE)*.

periodic boundary condition A boundary condition where the values of the solution appear in a periodic pattern, typically so that the value of the solution on one boundary is equal to the value on another boundary. See also *equivalent boundaries*.

phasor A complex number or a vector of complex numbers representing a sinusoidally varying current or voltage.

physics interfaces Sets of features for different types of physics in the COMSOL Desktop environment. The physics interfaces contain predefined equations and boundary conditions and a set of features for setting up models for a certain type of physics. The application modes in earlier versions of COMSOL Multiphysics provides similar functionality.

pivot Usually a value on the main diagonal of the *stiffness matrix*. *Pivoting* is the interchanging of rows and columns in order to place a particularly large element in the diagonal position. The value of the diagonal term when it is used to eliminate values below it is called the *pivot value*.

point A location in space.

point object A geometry object with only *vertices*.

positive definiteness A symmetric matrix is *positive definite* when all its eigenvalues are positive.

preconditioner The convergence rate of iterative methods depends on the spectral properties of the coefficient matrix. A *preconditioner* is a matrix that transforms the linear system into one that has the same solution but that has more favorable spectral properties. See also *algebraic multigrid*, *geometric multigrid*, *incomplete LU factorization*, *iterative solver*, and *SSOR*.

primitive, primitive geometry object A geometry object with a basic shape such as a cube or a sphere. You can add primitives to a model, using arbitrary sizes and positions,

and combine them to form complex shapes. See also *constructive solid geometry*, *composite geometry object*, and *Boolean operations*.

prism element A 3D mesh element with six corners and five faces, also referred to as *wedge element*.

projection coupling operator An operator that takes values from the source by evaluating line integrals over lines whose positions are dependent on the position of the evaluation points in the destination.

quadrature formula See *numerical integration formula*.

quadrilateral element A 2D mesh element with four corners and four edges; sometimes also called *quad element* as a short form.

rational Bézier curve See *Bézier curve*.

residual vector The vector L in the discretized form of a PDE problem. In the absence of *constraints*, the discrete form of a stationary equation is $0 = L(U)$ where U is the *solution vector*.

revolve To create a 3D geometry object from a 2D geometry object in a *work plane* by rotating it around an axis.

revolved mesh A 3D mesh created by revolving a 2D mesh. A revolved mesh can contain *hexahedral elements* and *prism elements*.

Robin boundary condition See *generalized Neumann boundary condition*.

shape function A *basis function* described in local element coordinates. See also *basis function*.

shift A value σ around which an eigensolver searches for eigenvalues.

simplex element *Triangle element* in 2D and *tetrahedral element* in 3D.

solid A description of a part of the modeling space. See also *domain*.

solid modeling A 3D geometry modeling method that describes both the boundary and interior of the geometry using solid objects. See also *constructive solid geometry (CSG)* and *solid*.

solid object A geometry object representing one or several *solids*.

solution component See *dependent variable*.

solution matrix A matrix that contains a sequence of solutions as columns. A steady-state problem results in a *solution vector*, but eigenvalue problems, time-dependent problems, and parametric analyses produce a *solution matrix*. See also *solution structure*.

solution structure A data structure that includes the *solution vector* or *solution matrix* and any associated data such as parameter values, output times, or eigenvalues.

solution vector A vector with all the *degrees of freedom* (values of the *dependent variables*) as its components. See also *solution matrix* and *solution structure*.

solver sequence A recorded sequence of named solver settings and commands that can be replayed by a single solver call.

sparse matrix Matrix for which the number of zero elements is large enough to justify special data types and algorithms that avoid operations on zero elements.

split To divide a geometry object into its minimal parts.

stability A solver for a time-dependent model is *unconditionally stable* if the initial conditions are not amplified artificially and the roundoff errors do not grow, regardless of the size of the time step. A solver is *conditionally stable* if there is a maximum value of the time step above which the numerical solution is unstable.

state-space model A linear time-invariant representation of a dynamic system as a set of 1st-order *ODEs* of the form

$$\dot{x} = Ax + Bu$$
$$\dot{y} = Cx + Du$$

where x is the state vector, u is the input, and y is the output. A, B, C, and D are the constant dynamics, input, output, and direct transmission matrices, respectively.

static model See *stationary model*.

stationary model A model where the dependent variables do not change over time. It typically represents a steady-state solution. Also called *static model* or *steady model*.

steady model See *stationary model*.

stiffness matrix See *Jacobian matrix*.

streakline The locus of particles that have earlier passed through a prescribed point in space. See also *streamline*.

streamline A curve that is everywhere tangent to the vector field (in particular a velocity field) at a given instant of time. Sometimes called a *flow line* or *flux line*. See also *streakline*.

streamline-diffusion stabilization A numerical technique for stabilization of the numeric solution to a PDE by artificially adding diffusion in the direction of the *streamlines*.

strong form A partial differential equation in the *strong form* is the standard formulation as an equality of functions. The strong form is divided into the *coefficient form* and the *general form*. Compare to *coefficient form*, *general form*, and *weak form*.

structured mesh A mesh for which all elements and nodes have the same topology. Compare to *unstructured mesh*.

surface A mathematical function (manifold) from 2D to 3D space.

surface normal A vector perpendicular to the surface.

surface modeling A 3D geometry modeling method to describe a geometry by defining its bounding surfaces. Compare *boundary modeling* and *solid modeling*.

swept mesh A 3D mesh generated by sweeping a face mesh along a subdomain.

symmetric matrix A matrix that equals its own transpose.

symmetric successive overrelaxation (SSOR) A *symmetric successive overrelaxation (SSOR)* preconditioner uses classic SSOR iterations.

symmetry The invariance of an object attribute or of the object itself under a well-defined operation or transformation such as inversion, rotation, or reflection. A *symmetry* allows for a reduction of the model geometry so that appropriate boundary

conditions account for the redundant portions of the geometry. Axisymmetry is a common type of *symmetry*.

symmetry boundaries See *equivalent boundaries*.

test function See *weak form*.

tetrahedral element A 3D mesh element with four corners, six edges, and four triangular faces.

time-dependent model See *transient model*.

transient model A model where at least one of the dependent variables changes over time, for example, the heat equation or the wave equation. Also called *dynamic model*, *time-dependent model*, or *unsteady model*.

triangular element A 2D mesh element with three corners and three edges.

trimmed surface If the parameter space of a surface is divided into "valid" and "invalid" regions, the image of the valid regions is called the *trimmed surface*. This corresponds to the part of the surface limited by a closed loop of edges lying on the surface.

unidirectional constraint See *constraint*.

unstructured mesh A mesh without a specific pattern where the elements can have different shapes and the nodes can have different connectivities. Compare to *structured mesh*.

unsteady model See *time-dependent model*.

vector element A finite element often used for electromagnetic vector fields. Each mesh element has degrees of freedom corresponding only to tangential components of the field. Also called *curl element*, *Nédélec's edge element*, or just *edge element*.

vertex A point in a geometry model, often an endpoint of a geometry segment or an intersection of geometry entities of higher degree such as *edges* or *faces*. A vertex is referred to as a *point* for the specification of point sources and other PDE modeling. See also *domain*.

weak constraint A reformulation of a *Dirichlet boundary condition* as a *weak form* equation. When using a weak constraint, the corresponding *Lagrange multiplier* becomes a *solution component* (*dependent variable*).

weak form A partial differential equation in the *weak form* is a more general formulation than the strong form. It is produced by multiplying the *strong form* PDE with an arbitrary function called the *test function* and integrating over the computational domain. Most physics interfaces in COMSOL Multiphysics are implemented using a weak form. Compare to *strong form*.

wedge element See *prism element*.

work plane An embedded 2D work space that can be positioned relative to the coordinate planes or an already existing geometry. Using *work planes* makes it possible to define a geometry in terms of previously created geometry objects such as *points*, *edges*, and *faces*.

INDEX

materials