The Capacity of Wireless Ad Hoc Networks With Multi-Packet Reception

Hamid R. Sadjadpour, *Senior Member, IEEE,* Zheng Wang, *Student Member, IEEE,*
and J.J. Garcia-Luna-Aceves, *Fellow, IEEE, ACM*

***Abstract*—We compute the throughput capacity of random dense wireless ad hoc networks for multi-pair unicast traffic in which nodes are endowed with multi-packet reception (MPR) capabilities. We show that $\Theta\left(\frac{(R(n))^{(1-\frac{2}{\alpha})}}{n^{1/\alpha}}\right)$ and $\Theta\left(R(n)\right)$ bits per second constitute tight bounds for the throughput capacity under the physical and protocol model assumptions, respectively, where n is the total number of nodes in the network, $\alpha > 2$ is the path-loss parameter in the physical model, and $R(n)$ is the MPR communication range. In so doing, we close the gap between the lower and upper bounds of throughput capacity in the physical model. Compared to the capacity of point-to-point communication reported by Gupta and Kumar [1], MPR increases the order capacity of random wireless ad hoc networks under both protocol and physical models by at least $\Theta(\log n)$ and $\Theta\left((\log n)^{\frac{\alpha-2}{2\alpha}}\right)$, respectively. We address the cost incurred in increasing the throughput capacity of wireless ad hoc networks over what can be attained when sources and destinations communicate over multi-hop paths under the physical model assumption. We define the *power efficiency* $\eta(n)$ as the bits of information transferred per unit time (second) in the network for each unit power, and compute such power efficiency for different techniques. We show that a lower power efficiency is attained in order to achieve higher throughput capacity.**

I. Introduction

The work by Gupta and Kumar [1] demonstrated that wireless ad hoc networks do not scale well for the case of multi-pair unicasts when nodes are able to encode and decode at most one packet at a time. This has motivated the study of different approaches to "embrace interference" in order to increase the capacity of wireless ad hoc networks. Embracing interference consists of increasing the concurrency with which the channel is accessed.

One approach to embracing interference consists of allowing a receiver node to decode correctly multiple packets transmitted concurrently from different nodes, which we call multi-packet reception (MPR) [2]. In practice, MPR can be achieved with a variety of techniques, including multiuser detection (MUD) [3], directional antennas [4], [5] or multiple input multiple output (MIMO) techniques. A complementary approach to embracing interference consists of increasing the amount of information sent per channel usage. Network coding (NC) [6] was introduced and shown to achieve the optimal capacity for single-source multicast in directed graphs corresponding to wired networks in which nodes are connected by point-to-point links. Since then, many attempts have been made to apply NC to wireless ad hoc networks, and Liu et al. [7] have shown that NC cannot increase the order capacity of wireless ad hoc networks for multi-pair unicast traffic. However, recent work [8]–[12] has shown promising results on the advantage of NC in wireless ad hoc networks subject to multicast traffic. An interesting aspect of these works is that nodes are also assumed to have multi-packet transmission (MPT) and MPR capabilities in addition to using NC for multicasting. Recently, Katti et al. [8] and Zhang et al. [9] proposed analog network coding (ANC) and physical-layer network coding (PNC) respectively, as ways to embrace interference. Interestingly, a careful review of ANC and PNC reveals that they consist of the integration of NC with a form of MPR, in that receivers must be allowed to decode successfully concurrent transmissions from multiple senders by taking advantage of the modulation scheme used at the physical layer (e.g., MSK modulation in ANC [8]). Furthermore, the prior work, which we summarize in Section II, has not addressed the contribution that MPR can make on the scaling laws of wireless ad hoc networks.

This paper focuses on multi-pair unicast traffic in wireless ad hoc networks when nodes are endowed with MPR. Section III describes the network model we use to obtain upper and lower bounds on the throughput capacity of wireless networks with MPR. Section IV presents the derivation of these bounds, which constitutes the first contribution of this paper. We show that $\Theta\left(R(n)\right)$ and $\Theta\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$ bits per second constitute tight bounds for the throughput capacity per node in random wireless ad hoc networks for protocol and physical models respectively, where $R(n)$ is the MPR communication range and α is the channel path loss parameter. MPR achieves higher throughput capacity under physical model than techniques proposed in [1], [13]. When $R(n) = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$,

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H. Sadjadpour and Z. Wang are with the Department of Electrical Engineering, University of California, Santa Cruz, 1156 High Street, Santa Cruz, CA 95064, USA (e-mail:{hamid, wzgold}@soe.ucsc.edu)

J.J. Garcia-Luna-Aceves is with the Department of Computer Engineering, University of California, Santa Cruz, 1156 High Street, Santa Cruz, CA 95064, USA and the Palo Alto Research Center (PARC), 3333 Coyote Hill Road, Palo Alto, CA 94304, USA (e-mail:jj@soe.ucsc.edu)

the throughput capacity is tight bounded by $\Theta\left(\sqrt{\frac{\log n}{n}}\right)$ and $\Theta\left(\frac{(\log n)^{\frac{1}{2}-\frac{1}{\alpha}}}{\sqrt{n}}\right)$ for protocol and physical models respectively. This is a gain of $\Theta(\log n)$ and $\Theta\left((\log n)^{\frac{\alpha-2}{2\alpha}}\right)$ compared to the bound in [1]. The assumptions we use to obtain these results are similar to those made by Gupta and Kumar [1], except that each node is equipped with MPR.

Several schemes have been proposed in the recent past [1], [13] that achieve different capacities for multi-pair unicast under the physical model. Intuitively, there must be a price paid in any scheme aimed at increasing the capacity of wireless networks, including MPR of course. This price is the energy required to transport information across the network. Section V presents our second contribution, which is to compare all these existing techniques in terms of power efficiency. We introduce a new parameter to quantify how many bits/sec of information are transferred across the network per each unit of power. We call this metric *power efficiency*, computed by normalizing the throughput capacity by the total transmitted power. We compute the power efficiency of many existing techniques [1], [13] and compare them to the power efficiency of MPR. We show that MPR provides a tradeoff between throughput capacity, node decoding complexity, and power efficiency in random wireless ad hoc networks. We also show that achieving higher throughput capacity leads to a lower power efficiency in all techniques, including MPR. Note that we define the throughput capacity for random wireless ad hoc networks in bits per second as defined in [1]. The focus of this paper is only on random wireless ad hoc networks. Section VI discusses several possible implications of this study. The paper conclusion and future work are discussed in Section VII.

II. Related Work

Since the landmark work by Gupta and Kumar [1] on the scalability of wireless networks, considerable attention has been devoted to improving or analyzing their results, and we only mention a very small fraction of these works due to space limitations. Grossglauser and Tse [14] demonstrated that a non-vanishing capacity can be attained at the price of long delivery latencies by taking advantage of long-term storage in mobile nodes. The throughput capacity can also be increased by using multiple channels [15] or sender-receiver cooperation[1] [16]. Recently, Ozgur et al. [17] demonstrated that the capacity of random wireless ad hoc network scales linearly with n by allowing nodes to cooperate intelligently using distributed MIMO communications.

Under the physical model assumption, Gupta and Kumar [1] showed that the throughput capacity of random wireless ad hoc networks has lower and upper bounds of $\Theta(\sqrt{1/n \log n})$ and $\Theta(\sqrt{1/n})$, respectively. Franceschetti et al. [13] closed the gap between these two bounds and obtained a tight bound of $\Theta(\sqrt{1/n})$ using percolation theory. In this approach, all communications are simple point-to-point without any cooperation between senders and receivers.

[1]Note that these two approaches [15], [16] can be considered as two different forms of implementing MPR technique.

We note that previous work [18] has suggested the concept of bits per joule capacity to evaluate how much information can be transmitted with each unit energy. Our definition of power efficiency is an extension of this prior work for wireless ad hoc networks.

III. Network Model

We consider a dense wireless ad hoc network with n nodes distributed uniformly in a square of unit area. There are two types of networks, namely, dense and extended networks. The area of a dense network is constant independent of the number of nodes while the area of extended network increases with n. The network is assumed as static which means that the nodes are not mobile. We follow this assumption throughout the paper. Our capacity analysis is based on the extension of protocol and physical models for dense networks which is introduced by Gupta and Kumar [1]. Throughout this paper, the distribution of nodes in random networks is uniform, and non-uniform distribution is the topic of future work.

According to the Gupta-Kumar protocol model for point-to-point communications, a common transmission range $r(n)$ for all nodes is defined. Node i at location X_i can successfully transmit to node j at location X_j if for any node k at location $X_k, k \neq i$, that transmits at the same time as i, then $|X_i - X_j| \leq r(n)$ and $|X_k - X_j| \geq (1+\Delta)r(n)$, where X_i, X_j and X_k are the cartesian position in the unit square network for these nodes. The parameter Δ is a function of characteristics of the channel model. We next define the protocol model for MPR.

Definition 3.1: Protocol Model with Multi-packet Reception: In wireless ad hoc networks with MPR, the protocol model assumption allows MPR capability at nodes as long as they are within a radius of $R(n)$ from the receiver and all other transmitting nodes are at a distance larger than $(1+\Delta)R(n)$. The difference is that we allow the receiver node to receive multiple packets from different nodes within its disk of radius $R(n)$ simultaneously in MPR scheme.

$R(n)$ denotes the communication range for MPR model which is a function of decoding complexity of nodes and node density. $r(n)$ denotes the communication range for point-to-point communication, and it is a function of nodes density in the network. Because the distribution of nodes is uniform, these parameters are not a function of node distribution. However when the node distribution in the network is not uniform, these parameters will be a function of node distribution.

We assume that nodes cannot transmit and receive at the same time, which is equivalent to half duplex communication [1]. The data rate for each link pair is a constant value of W bits/second and does not depend on n. Given that W does not change the order of throughput capacity of the network, we normalize its value to one. The MPR protocol model is shown in Fig. 1.

It has been proven [19] that the minimum communication range $R(n)$ in a random geometric graph to assure connectivity in the network, is given in the following lemma.

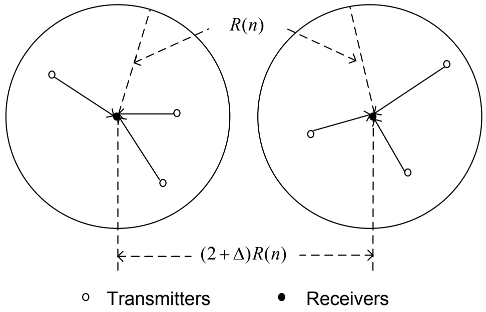

Fig. 1. MPR protocol model

Lemma 3.2: For any $\epsilon > 0$ and $n \to \infty$, we have

$$\begin{aligned} \text{Prob(existence of an isolated node)} &= 1 \\ \text{when } R(n) &= (1-\epsilon)\sqrt{\frac{\log n}{n\pi}} \\ \text{Prob(existence of an isolated node)} &= 0 \\ \text{when } R(n) &= (1+\epsilon)\sqrt{\frac{\log n}{n\pi}} \end{aligned}$$

Our definition of throughput capacity is based on the assumption that all the nodes in the network can achieve this capacity. Therefore, in order to guarantee connectivity in the network, the communication range should satisfy the following criterion[2].

$$R(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right) \tag{1}$$

Note that this result is independent of the physical layer model we use for the network and it is a characteristic of random geometric graphs [19]. Similar to the results in [1], we have adopted same minimum communication range $R(n)$ to assure connectivity in the network for the protocol model. Note that the successful communication in the physical model is based on signal to interference and noise ratio and not the distance between nodes, therefore we can no longer use the condition of (1) for successful communication in the physical model.

Based on physical model for dense random wireless ad hoc networks in [1], a successful communication occurs if signal to interference and noise ratio (SINR) of the pair of transmitter i and receiver j satisfies

$$\text{SINR}_{i\to j} = \frac{Pg_{ij}}{BN_0 + \sum_{k\neq i,k=1}^{n} Pg_{kj}} \geq \beta, \tag{2}$$

where P is the transmit power of a node, g_{ij} is the channel attenuation factor between nodes i and j, and BN_0 is the total noise power. The channel attenuation factors g_{ij} and g_{kj} are only functions of the distance under the simple path loss propagation model, i.e., $g_{ij} = |X_i - X_j|^{-\alpha}$.

However, based on the physical model definition for MPR, each receiving node has a communication range such that all the nodes transmitting within this range will be decoded by the receiver. Consequently, the definition of physical model should incorporate this fact in order to better represent this new many-to-one communication scheme. The following statement describes the decoding procedure for MPR. Note that, with MPR, we can either decode the received signal for multiple transmitters jointly using maximum likelihood (ML) decoding or decode transmitters sequentially utilizing successive interference cancelation (SIC). ML decoding is computationally more complex than SIC but it provides optimal performance. Our SIC decoding requires all nodes inside transmission range to be grouped into several smaller sets with each set satisfying the SINR condition in (2). Because the channel model is based on path loss propagation model, the SIC decoding starts from a set of nodes that has the closest distance to the receiver node. Each set may consist of either a single node or multiple nodes. If a set consists more than one node, then the decoding of these nodes are performed jointly. Definition 3.3 below describes the successful transmission for MPR under physical model.

Definition 3.3: Physical Model with Multi-packet Reception: In the physical model of dense random wireless ad hoc networks [1], the active transmissions from all of the transmitters centered around the corresponding receiver j with a distance smaller or equal to $R(n)$ occur successfully if the SINR of the transmitter $Z(R(n))$ near to the edge of the circle of the receiver satisfies

$$\mathit{SINR}_{Z(R(n))\to j} = \frac{Pg_{Z(R(n))j}}{BN_0 + \sum_{k,\forall X_k \notin A_{Z(R(n))}} Pg_{kj}} \geq \beta, \tag{3}$$

where $g_{Z(R(n))j}$ is the channel attenuation factor between nodes $Z(R(n))$ and j and BN_0 is the total noise power. $A_{Z(R(n))} = \pi R^2(n)$ is the area of the circle centered around the receiver j, whose radius is $R(n)$.

Any transmission outside the communication range is considered interference while all the transmissions inside communication range will be decoded jointly or separately depending on the location of nodes inside the transmission circle. The decoding is carried by dividing all the transmitters inside the communication range (circle) into many subsets. The first set of nodes have the closest distance to the receiver. The total number of nodes in each set is selected such that if they are decoded jointly by the receiver, they will satisfy the SINR condition while the remaining nodes inside the transmission circle are considered as interference. Once this set of nodes are decoded jointly, they are subtracted from the received signal and then the next set of nodes are decoded. The selection of nodes for each set depends on the relative locations of nodes with respect to the receiver node. Note that this approach is suboptimal as compared to joint decoding of the entire transmitting nodes inside the communication range which is equivalent to maximum likelihood (ML) decoding. For this reason, we denote the interference inside area $A_{Z(R(n))}$ as constructive interference, because it consists of transmissions that will be eventually decoded, while all the transmissions from nodes outside of area A are called destructive interference and are not decoded. Note that in the physical model for the MPR scheme, the communication range $R(n)$ defines the area where the receiver is capable of decoding, which contrasts with point-to-point communication [1], for which the transmission range $r(n)$ defines the possible area where the receiver can decode,

[2]Given two functions $f(n)$ and $g(n)$. We say that $f = O(g(n))$ if $\sup_n (f(n)/g(n)) < \infty$. We say that $f(n) = \Omega(g(n))$ if $g(n) = O(f(n))$. If both $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$, then we say $f(n) = \Theta(g(n))$.

given that only one transmission is successful at a receiver.

Definition 3.4: Feasible throughput capacity of unicast: A throughput of $\lambda(n)$ bits per second for each node is feasible if we can define a scheduling transmission scheme that allows each node in the network to transmit $\lambda(n)$ bits per second on average to its destination.

The per-node throughput capacity of the network is defined as the number of bits per second in Definition 3.4 that every node can transmit to its destination.

Definition 3.5: Order of throughput capacity: $\lambda(n)$ is said to be of order $\Theta(f(n))$ bits per second if there exist deterministic positive constants c and c' such that

$$\begin{cases} \lim_{n\to\infty} \text{ Prob } (\lambda(n) = cf(n) \text{ is feasible}) = 1 \\ \liminf_{n\to\infty} \text{ Prob } (\lambda(n) = c'f(n) \text{ is feasible}) < 1. \end{cases} \quad (4)$$

The distribution of nodes in random networks is uniform. Therefore, if there are n nodes in a unit square, then the density of nodes equals n. Hence, if $|S|$ denotes the area of space region S, the expected number of the nodes, $E(N_S)$, in this area is given by $E(N_S) = n|S|$. Let N_j be a random variable defining the number of nodes in S_j . Then, for the family of variables N_j, we have the following standard results known as the Chernoff bound [20]:

Lemma 3.6: Chernoff bound

- For any $\delta > 0$, $P[N_j > (1+\delta)n|S_j|] < \left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}}\right)^{n|S_j|}$
- For any $0 < \delta < 1$, $P[N_j < (1-\delta)n|S_j|] < e^{-\frac{1}{2}n|S_j|\delta^2}$

Combining these two inequalities we have, for any $0 < \delta < 1$:

$$P[|N_j - n|S_j|| > \delta n|S_j|] < e^{-\theta n|S_j|}, \quad (5)$$

where $\theta = (1+\delta)\ln(1+\delta) - \delta$ in the case of the first bound, and $\theta = \frac{1}{2}\delta^2$ in the case of the second bound.

Therefore, for any $\theta > 0$, there exist constants such that deviations from the mean by more than these constants occur with probability approaching zero as $n \to \infty$. It follows that we can get a very sharp concentration on the number of nodes in an area, so we can find the achievable lower bound, provided that the mean for the upper bound is given.

IV. Capacity with MPR

We compute the capacity of wireless ad hoc networks for both protocol and physical models. To accomplish this, we first present some definitions and preliminary results from our earlier work [21].

A cut Γ is a partition of the vertices (i.e. nodes in the wireless networks) of a graph into two sets. The cut capacity is defined to be the sum of the capacity of all the active edges crossing the cut that transmit simultaneously and successfully. In this paper, we use random geometric graph (RGG). An edge is active (communication link) in RGG if the protocol or physical model is satisfied for successful communications between the two nodes which is directly a function of distance between nodes. However, an edge in a general graph is not necessarily an active edge for an RGG. Min-cut is a cut whose capacity is the minimum value among the capacity of all cuts. For the wireless networks, we use the concept of *sparsity cut*, which is defined by Liu et al. [7], instead of min-cut, to take into account the differences between wired and wireless links. l_Γ is defined as the length of the cut. For the square region illustrated in Fig. 2, the middle line induces a sparsity cut Γ. Because nodes are uniformly deployed in a random network, such a sparsity cut captures the traffic bottleneck of these random networks on average [7]. The sparsity-cut capacity is upper bounded by the maximum number of simultaneous transmissions across the cut.

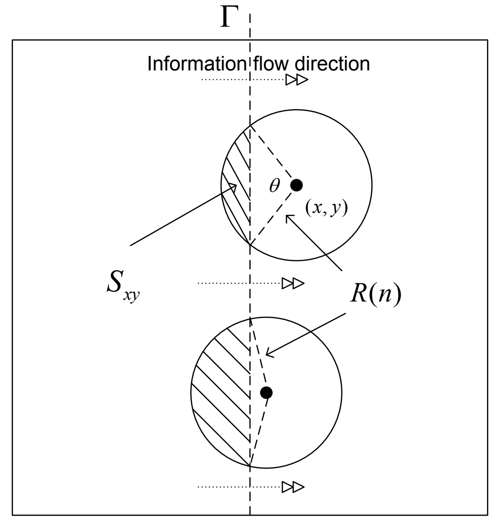

Fig. 2. For a receiver at location (x, y), all the nodes in the shaded region S_{xy} can send a message successfully and simultaneously.

Definition 4.1 (Sparsity Cut:): A sparsity cut for a random network is defined as a cut induced by the line segment with the minimum length that separates the region into two equal area subregions. Note that the definition of sparsity cut does not depend on a specific realization of a random network, it rather focuses on the asymptotic order of some spatial-statistical property of the collection of random networks as a whole. The cut capacity is defined as the transmission bandwidth W multiplied by the maximum possible number of simultaneous transmissions across the cut. This cut capacity constrains the information rate that the nodes from one side of the cut as a whole can deliver to the nodes at the other side. The cut length l_Γ is defined as the length of the cut line segment in 2-D space. Similarly, in 3-D volume, the sparsity cut is a plane, and the cut plane has an area. In another word, sparsity cut can be seen for random geometric graph (RGG) similar to min-cut concept in graph theory.

Let $R(n)$ be the radius of the receiver area A, i.e., $A = \pi R^2(n)$. Given that we assume omnidirectional antennas for all nodes, the information from any node inside this area is decode-able while the information from all transmitting nodes outside of this region are considered as interference.

We assume that each disk with radius $R(n)$ centered at any receiver is disjoint from the other disks centered at the other receivers. It will be shown later that this assumption is necessary in order to guarantee that the physical model condition, SINR $\geq \beta$, is satisfied.

A. Upper Bound for Protocol Model

We first derive the sparsity cut for a random wireless ad hoc network under the protocol model.

Lemma 4.2: The asymptotic throughput capacity of a sparsity cut Γ for a unit square region has an upper bound of $c_1 l_\Gamma n R(n)$, where, $c_1 = \pi/2(2+\Delta)$.

Proof: The cut capacity is the maximum number of simultaneous transmissions across the cut.

We define S_{xy} as the area in the left side of the cut Γ that contains nodes sending packets to the receiver node located at (x,y) (see Fig. 2)

These nodes lie in the left side of the cut Γ within an area called S_{xy}. The assumption is that all these nodes are sending packets to the right side of the cut Γ.

From the definition of the MPR, for a node at location (x,y), any node in the disk of radius $R(n)$ can transmit information to this receiver simultaneously and the node can successfully decode those packets. In order to obtain an upper bound, we only need to consider edges that cross the cut. Let us first consider all possible nodes in the S_{xy} region that can transmit to the receiver node. Because nodes are uniformly distributed, the average number of transmitters located in S_{xy} is $n \times S_{xy}$. The number of nodes that are able to transmit at the same time from left to right is upper bounded as a function of S_{xy}.

The area of S_{xy} is

$$S_{xy} = \frac{1}{2}R^2(n)(\theta - \sin\theta). \tag{6}$$

This area is maximized when $\theta = \pi$,

$$\max_{0\le\theta\le\pi}[S_{xy}] = \frac{1}{2}\pi R^2(n). \tag{7}$$

The total number of nodes that can send packets across the cut is upper bounded as

$$\frac{l_\Gamma}{(2+\Delta)R(n)}\frac{1}{2}\pi R^2(n)n = c_1 l_\Gamma n R(n), \tag{8}$$

where $c_1 = \pi/2(2+\Delta)$. ∎

Corollary 4.3: For any arbitrary shape unit area random network, if the minimum cut length l_Γ is not a function of n, then the sparsity cut capacity has an upper bound of $\Theta(nR(n))$.

Proof: Regardless of the shape of the unit area region, it is clear that the length of l_Γ is $\Theta(1)$. because the network area is unity. If l_Γ is not a function of n, then the capacity is always upper bounded as $\Theta(nR(n))$. ∎

Theorem 4.4: The per-node throughput of *MPR* scheme in a 2-D random network is upper bounded by $\Theta(R(n))$.

Proof: For a sparsity cut Γ in the middle of the unit plain, on average, there are $\Theta(n)$ pairs of source-destination nodes that need to cross Γ in one direction, i.e., $n_{\Gamma_{l,r}} = n_{\Gamma_{r,l}} = \Theta(n)$. Combining this result with Corollary 4.3, we can easily prove this theorem. Note that $n_{\Gamma_{l,r}}$ and $n_{\Gamma_{r,l}}$ are the transmissions from left to right and from right to left respectively. ∎

B. Lower Bound for Protocol Model

We now prove that, when n nodes are distributed uniformly over a unit square area, we have simultaneously at least $\frac{l_\Gamma}{(2+\Delta)R(n)}$ circular regions in Fig. 2, each one contains $\Theta(nR^2(n))$ nodes. The objective is to find the achievable lower bound using Chernoff bound such that the distribution of the number of edges across the cut is sharply concentrated around its mean, and hence in a randomly chosen network, the actual number of edges crossing the sparsity cut is indeed $\Theta(nR(n))$.

Theorem 4.5: Each area A_j with circular shape contains $\Theta(nR^2(n))$ nodes uniformly for all values of $j, 1 \le j \le \lceil \frac{l_\Gamma}{(2+\Delta)R(n)} \rceil$, with high probability (w.h.p.). [3]

This theorem can be expressed as

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{\lceil l_\Gamma/(2+\Delta)R(n)\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] = 1, \tag{9}$$

where δ is a positive small value arbitrarily close to zero.

Proof: Since l_Γ is not a function of n, using Chernoff bound (Lemma 3.6) and Eq. (5), for any given $0 < \delta < 1$, we can find $\theta > 0$ such that

$$P\left[|N_j - E(N_j)| > \delta E(N_j)\right] < e^{-\theta E(N_j)} = e^{-\theta n |A_j|}. \tag{10}$$

Thus, we can conclude that the probability that the value of the random variable N_j deviates by an arbitrarily small constant value from the mean tends to zero as $n \to \infty$. This is a key step in showing that when all the events $\bigcap_{j=1}^{\lceil l_\Gamma/(2+\Delta)R(n)\rceil} |N_j - E(N_j)| < \delta E(N_j)$ occur simultaneously, then all N_j's converge uniformly to their expected values. Utilizing the union bound, we arrive at

$$\begin{aligned}
& P\left[\bigcap_{j=1}^{\lceil l_\Gamma/(2+\Delta)R(n)\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] \\
= \;& 1 - P\left[\bigcup_{j=1}^{\lceil l_\Gamma/(2+\Delta)R(n)\rceil} |N_j - E(N_j)| \ge \delta E(N_j)\right] \\
\ge \;& 1 - \sum_{j=1}^{\lceil l_\Gamma/(2+\Delta)R(n)\rceil} P\left[|N_j - E(N_j)| \ge \delta E(N_j)\right] \\
> \;& 1 - \lceil \frac{l_\Gamma}{(2+\Delta)R(n)} \rceil e^{-\theta E(N_j)} \\
= \;& 1 - \lceil \frac{l_\Gamma}{(2+\Delta)R(n)} \rceil e^{-\frac{\theta\pi n R^2(n)}{2}}.
\end{aligned} \tag{11}$$

The last term is derived from the fact that $E(N_j) = \frac{\pi}{2} n R^2(n)$. In order to guarantee connectivity, we need $R(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right)$ [1]. Thus we have the following equations.

$$\frac{e^{-\frac{\theta\pi n R^2(n)}{2}}}{R(n)} = O\left(\frac{1}{n^{\frac{\theta\pi}{2}-\frac{1}{2}}\log n}\right) = O\left(\frac{1}{n}\right). \tag{12}$$

[3] An event happens with high probability if the probability of this event is greater than $1 - \frac{1}{n}$ when n goes to infinity.

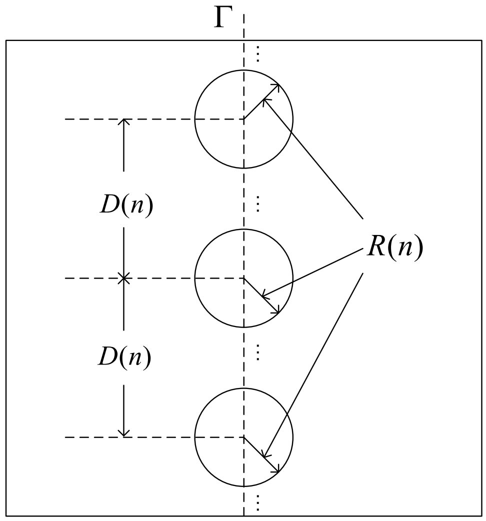

Fig. 3. Upper bound design of the network

provided that $\theta > 3/\pi$. Then

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{\lceil l_\Gamma/(2+\Delta)R(n)\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] > 1-\frac{1}{n}, \tag{13}$$

which proves this theorem.

■

The next theorem demonstrates that this capacity is an achievable lower bound.

Corollary 4.6: The per-node throughput of *MPR* scheme for a 2-D random network has a lower bound of $\Theta(R(n))$.

Proof: It is proved in Theorem 4.5, there are $\lceil \frac{l_\Gamma}{(2+\Delta)R(n)}\rceil$ different circles of radius $R(n)$ each of them having $\Theta(nR^2(n))$ nodes. Therefore, per-node is the multiplications of these two values which is divided by the total number of nodes.

$$\left\lceil \frac{l_\Gamma}{(2+\Delta)R(n)}\right\rceil \times \frac{nR^2(n)}{n} = R(n) \tag{14}$$

■

C. Upper Bound for Physical Model

We define division range $D(n)$ as the minimum distance required between receiving nodes such that each node can decode all transmitters within the communication range $R(n)$ successfully. Equivalently, $D(n)$ is the minimum distance that separates simultaneous active receivers far from each other such that receiver nodes can have successful communications. Based on the above, $D(n)$ is a function that depends on n which we want to minimize between two concurrent receivers as shown in Fig. 3 such that the physical model constraint is satisfied. We will prove that $D(n)$ is a function of $R(n)$.

Lemma 4.7: The asymptotic throughput capacity of a sparsity cut Γ for a unit square region has an upper bound of $\pi l_\Gamma n\frac{R^2(n)}{D(n)}$, where, $R(n)$ and $D(n)$ are communication range and division range of MPR respectively as illustrated in Fig. 3.

Proof: The cut capacity is upper bounded by the maximum number of simultaneous transmissions across the cut. Based on the results from section IV-A and the total number of nodes in each area S_{xy}, we can compute the total information capacity (i.e. the total capacity) C_j for one receiver j at the right side of the cut as

$$C_j = \frac{1}{2}\pi n R^2(n). \tag{15}$$

The constraint to guarantee that Eq. (15) is true for all of the nodes inside the circle of radius $R(n)$, is to satisfy $\text{SINR}_{i\in S_{xy}} \geq \beta$. For this reason, the circles in which nodes are transmitting concurrently must be away from each other far enough to satisfy SINR criterion.

Therefore, the total throughput capacity $C(n)$ across the sparsity cut is

$$C(n) \leq \left(\left\lfloor\frac{l_\Gamma}{D(n)}\right\rfloor + 1\right) C_j < \frac{\pi n R^2(n)(l_\Gamma + D(n))}{2D(n)}. \tag{16}$$

Since $l_\Gamma \geq D(n)$, then $l_\Gamma + D(n) \leq 2l_\Gamma$ and the proof follows. ■

Lemma 4.8: The per-node throughput of MPR scheme in a 2-D random network is upper bounded by $O\left(\frac{R^2(n)}{D(n)}\right)$.

Proof: From lemma 4.7, there are $\lceil l_\Gamma/D(n)\rceil$ different circles of radius $R(n)$ each of them having $\Theta(nR^2(n))$ nodes on average. Therefore, the average per node throughput capacity can be derived as

$$\lambda(n) = \frac{C(n)}{n} = O\left(\frac{R^2(n)}{D(n)}\right). \tag{17}$$

■

To derive an upper bound for the throughput capacity, we need to obtain a minimum $D(n)$, such that it guarantees $\text{SINR}_{Z(R(n))} \geq \beta$. The decoding is conducted from the nearest nodes to the farthest nodes by decoding the strongest signals first and then subtract them from the received signal. So if the SINR of the outmost node can be decoded, then all of the nodes inside that circle can be decoded successfully because the nodes closer to the receiver provide higher SINR if they are decoded either jointly or separately depending on the location of nodes in the network. Based on this assumption, we only need to compute the SINR of the farthest nodes $Z(R(n))$ (i.e., at the conjunction edge of the communication circle) to make sure that $\text{SINR}_{Z(R(n))} \geq \beta$. Hence, to obtain the upper bound of the capacity is equivalent to maximize the following function.

$$\max_{\text{SINR}_{Z(R(n))}\geq\beta} \lambda(n) = \max_{\text{SINR}_{Z(R(n))}\geq\beta} O\left(\frac{R^2(n)}{D(n)}\right) \tag{18}$$

Note that the throughput capacity is maximized by minimizing $D(n)$ since $R(n)$ is a network parameter that is determined in advance. If the value of $D(n)$ is too small, then Eq. (3) will not be satisfied. Our aim is to find the optimum value for $D(n)$ such that both conditions are satisfied. The following theorem and its applications establish the optimum value that will satisfy Eq. (3).

Theorem 4.9: The per-node throughput of MPR scheme in a 2-D random network is upper bounded by $O\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$.

The proof can be found in the Appendix.

The above upper bound is derived based on the assumption that the SINR for the nodes that are located on the circumference of communication circle A of radius $R(n)$ satisfy the physical model, i.e., $\text{SINR}_{Z(R(n))} \geq \beta$. We will show that this upper bound in Theorem 4.9 is also an achievable capacity.

D. Lower Bound for Physical Model

Given the upper bound derived in the previous section, the Chernoff Bound is used to prove the achievable lower bound. We prove that, when n nodes are distributed uniformly over a square area, we have simultaneously $\lceil \frac{l_\Gamma}{D(n)} \rceil$ circular regions (see fig. 2), each one containing $\Theta(nR^2(n))$ nodes. The objective is to find the achievable lower bound using the Chernoff bound, such that the distribution of the number of edges across the cut is sharply concentrated around its mean, and hence in a randomly chosen network, the actual number of edges crossing the sparsity cut is indeed $\Theta\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{(1/\alpha)}}\right)$.

Theorem 4.10: Each area A_j with circular shape of radius $R(n)$ contains $\Theta(nR^2(n))$ nodes uniformly and w.h.p. for all values of $j, 1 \leq j \leq \lceil \frac{l_\Gamma}{D(n)} \rceil$ under the condition that $R(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right)$. Equivalently, this can be expressed as

$$\lim_{n \to \infty} P\left[\bigcap_{j=1}^{\lceil l_\Gamma / D(n) \rceil} |N_j - E(N_j)| < \delta E(N_j)\right] = 1, \quad (19)$$

where δ is a positive arbitrarily small value close to zero.

The proof can be found in the Appendix.

Eq. (42) is equivalent to the connectivity condition in the protocol model [1], [21]. It is interesting to note that we did not really use connectivity criterion in the physical model, however, it turns out that the minimum distance for the communication range in MPR model is equivalent to the connectivity constraint in protocol model for random networks.

The above theorem demonstrates that there are indeed $\Theta(nR^2(n))$ nodes in each communication region with the constraint in (42). The achievable capacity is only feasible when the communication range of each node in MPR scheme is at least equal to the connectivity criterion of transmission range in point-to-point communication [1]. Combining the result of Eq. (38) in Theorem 4.9 and (42) in Theorem 4.10, we can state the following theorem for the lower bound of throughput capacity. It implies that the lower bound order capacity achieves the upper bound in physical model.

Theorem 4.11: The per-node throughput capacity of MPR scheme in a 2-D wireless ad hoc network is bounded by $\Omega\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$, provided that $R(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right)$. The achievable lower bound is $\Omega\left(\frac{(\log n)^{\frac{1}{2}-\frac{1}{\alpha}}}{\sqrt{n}}\right)$ for $\alpha > 2$.

The proof can be found in the Appendix.

The above theorem demonstrates that a gain of at least $\Theta\left((\log n)^{\frac{\alpha-2}{2\alpha}}\right)$ can be achieved compared with the results by Gupta and Kumar [1] and Franceschetti et al. [13]. Combining Theorems 4.9 and 4.11, we arrive at our next major contribution of this paper.

Theorem 4.12: The per-node throughput capacity of MPR scheme in a 2-D wireless ad hoc network is tight bounded as $\Theta\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$. The communication range is lower bounded as $R(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right)$, which implies a bound of $\Theta\left(\frac{(\log n)^{\frac{1}{2}-\frac{1}{\alpha}}}{\sqrt{n}}\right)$.

Note that this result shows that we can close the gap in the physical model similar to the results derived by Franceschetti et al. [13] but achieving higher throughput capacity with MPR.

V. POWER EFFICIENCY

Many wireless sensor and ad hoc networks are energy and power limited systems and it is natural to ask what the price of achieving higher capacities in wireless ad hoc networks is.

The capacity was originally defined in [1] based on bits per second for random networks. The definition of bits-per Joule was defined in [18]. To incorporate the effect of energy consumption for communication in wireless networks, we define *bits per second per Watts for random networks as "power efficiency" in the followings.* This new metric is a measure for evaluating the power efficiency of the capacity in wireless sensor and ad hoc networks. The formal definition is as follows.

Definition 5.1 (power efficiency:): In wireless ad hoc networks with limited energy, the power efficiency is defined as

$$\eta(n) = \frac{\lambda(n)}{P(n)}, \quad (20)$$

where $\lambda(n)$ is the capacity of the network and $P(n)$ is the total minimum average power required to achieve $\lambda(n)$ for each source-destination pair in the network. The metric is "bits per Joule" or "bits per second per Watts" in random wireless ad hoc networks."

With this definition of efficiency, we compute the relationship between the capacity and the power efficiency for the various approaches defined to increase the throughput capacity of wireless ad hoc networks, including our own.

A. Power efficiency in approach by Gupta and Kumar [1]

It is easy to show [22] that the minimum transmit power P for each hop to guarantee SINR $\geq \beta$ is

$$\min(P) = \Theta(s_n^\alpha) = \Theta\left(\left(\frac{\log n}{n}\right)^{\frac{\alpha}{2}}\right), \quad (21)$$

Where, $s_n = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$. The total average power to transmit this information is

$$P(n) = \min(P) \times \text{total number of hops} = \Theta\left(\left(\frac{\log n}{n}\right)^{\frac{\alpha}{2}-\frac{1}{2}}\right). \quad (22)$$

The power efficiency for this scheme can be computed by dividing the throughput capacity by the total average power required to achieve this capacity. This renders

$$\eta(n) = \Theta\left(\frac{n^{\frac{\alpha}{2}-1}}{(\log n)^{\frac{\alpha}{2}}}\right). \tag{23}$$

B. Power efficiency in approach by Franceschetti et al. [13]

The communication in the approach by Franceschetti et al. [13] is based on dividing the transfer of packets into four phases. In the first phase, the source transmits a packet to a relay inside a path that is called "highway path." The distance between the source and highway path is considered a long range communication and is proportional to $\Theta(\frac{\log n}{\sqrt{n}})$. Inside the highway path in phases two and three, multiple hop communication occurs horizontally and vertically respectively. The communication range is of short range and proportional to $\Theta(\frac{1}{\sqrt{n}})$. Communication in phase four is similar to phase one and it is between relay and destination.

Assume that $P_h(n)$ is the transmit power at the highway path in phases two and three. Following the definition in [13], the interference from the other cells can be expressed as

$$I(d,n) \leq P_h(n)\,(s_n(d+1))^{-\alpha}\, c_6. \tag{24}$$

where c_6 is a constant value. The signal power at the receiver is lower bounded as

$$S(d,n) \geq P_h(n)\left(s_n\sqrt{2}(d+1)\right)^{-\alpha}. \tag{25}$$

Using the above results, the SINR is derived as

$$\text{SINR} = \frac{S(d,n)}{BN_0 + I(d,n)} \geq \frac{P_h(n)\left(\sqrt{2}\right)^{-\alpha}}{BN_0(s_n(d+1))^{\alpha} + P_h(n)c_6}. \tag{26}$$

In the limit, the minimum required power to guarantee that the SINR satisfies the physical model when $n \to \infty$ is $\min(P_h(n)) = \Theta((s_n(d+1))^{\alpha}) = \Theta\left((n)^{-\alpha/2}\right)$.

For the long-range communications in the first and fourth phase, there is no interference. Therefore, the SINR can be expressed as

$$\text{SINR} = \frac{P_u(n)\left(\frac{\log n}{\sqrt{n}}\right)^{-\alpha}}{BN_0}. \tag{27}$$

The minimum required power for this case to guarantee the physical model condition is given by

$$\min(P_u(n)) = \Theta\left(\left(\frac{(\log n)^2}{n}\right)^{\frac{\alpha}{2}}\right). \tag{28}$$

Using the definition of power efficiency, we can compute its value for this case as

$$\begin{aligned}\eta(n) &= \lambda(n)/P(n) \\ &= \frac{\lambda(n)}{2\min(P_u(n)) + \sqrt{n}\min(P_h(n))} \\ &= \Theta\left(n^{\frac{\alpha}{2}-1}\right).\end{aligned} \tag{29}$$

C. Power efficiency with MPR

In this paper, we demonstrated that MPR closes the gap between the upper and lower bounds of the capacity of wireless ad hoc networks by achieving higher throughput capacity. However, it is important to find out the power efficiency of this approach. From the derivation of throughput capacity for MPR in Eq. (43), the SINR is given by

$$\text{SINR} \geq \frac{P(R(n))^{-\alpha}}{BN_0 + \frac{\pi}{2}nR^2(n)\sum_{i=1}^{\lceil l_\Gamma/2D(n)\rceil} 2P(iD(n)-R(n))^{-\alpha}} \tag{30}$$

The physical model constraint is guaranteed for SINR asymptotically when the minimum transmit power $P_{\text{MPR}}(n)$ is

$$\min(P_{\text{MPR}}(n)) = \Theta\left(R^{\alpha}(n)\right) = \left(\frac{\log n}{n}\right)^{\frac{\alpha}{2}}. \tag{31}$$

Eq. (31) is derived using Eqs. (42) and (44) when $n \to \infty$.

The relationship between $\lambda(n)$ and $P_{\text{MPR}}(n)$ can be computed from Theorem 4.12 as

$$\lambda(n) = n^{-1/\alpha}\left(P_{\text{MPR}}(n)\right)^{\frac{\alpha-2}{\alpha^2}}. \tag{32}$$

Because the communication range in MPR is equal to $R(n)$, the total minimum transmit power from source to destination is equal to $\frac{P_{\text{MPR}}(n)}{R(n)}$.

The power efficiency of MPR scheme is given by

$$\begin{aligned}\eta(n) &= \frac{\lambda(n)R(n)}{P_{\text{MPR}}(n)} \\ &= \lambda(n)(R(n))^{1-\alpha} \\ &= n^{-\frac{\alpha-1}{\alpha-2}}\lambda(n)^{\frac{-(\alpha-1)^2-1}{\alpha-2}}.\end{aligned} \tag{33}$$

VI. Discussion

The reason for the significant increase in capacity with MPR is that, unlike point-to-point communication in which nodes compete to access the channel, MPR embraces (strong) interference by utilizing higher decoding complexity for all nodes. As we have pointed out, recent work on network coding [8], [9] implicitly assumes some form of MPR. These results clearly demonstrate that embracing interference is crucial to improve the performance of wireless ad hoc networks, and that MPR constitutes an important component of that.

Another interesting observation is the fact that increasing the communication range $R(n)$ increases the throughput capacity. This is in sharp contrast with point-to-point communication in which increasing the communication range actually decreases the throughput capacity and it is again due to the fact that MPR embraces the interference.

Figure 4 shows the tradeoff between the total minimum transmit power and the throughput capacity. From this figure, it is clear that the total transmit power for the network must be increased in order to increase the per-node throughput capacity in random wireless ad hoc networks.

Fig. 5 shows that, by increasing the throughput capacity in wireless ad hoc networks, the power efficiency of all the schemes we analyzed decreases. Many wireless ad hoc networks are limited in total available energy or power for each node. Therefore, increasing the throughput capacity may not be feasible if the required power to do so is not available. This result also shows that the throughput capacity should not be the only metric used in evaluating and comparing the merits of different schemes. The power efficiency of these schemes

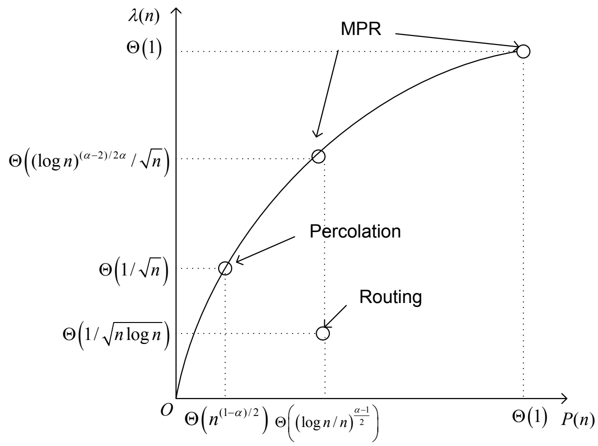

Fig. 4. Power and capacity relationship

is also very important. Based on different values for $R(n)$, different throughput capacities can be attained. In general, MPR allows to have tradeoff between receiver complexity and throughput capacity.

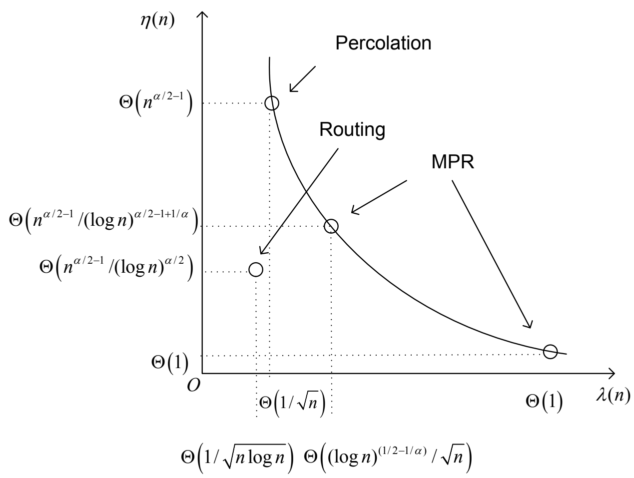

Fig. 5. Capacity and power efficiency tradeoff

There are certain issues that we did not discuss in this paper. Our analysis does not include the energy required for increased decoding complexity, which is necessary for MPR. Our analysis also does not include the additional required overhead related to cooperation among nodes. Such topics are the subject of future studies.

VII. Conclusion and Future Work

This paper shows that the use of MPR can close the gap for the throughput capacity in random wireless ad hoc networks under the physical model, while achieving much higher capacity gain than that of [13]. The tight bounds are $\Theta(R(n))$ and $\Theta\left(\frac{(R(n))^{1-2/\alpha}}{(n^{(1/\alpha)})}\right)$ where $R(n)$ is the communication range in MPR model for protocol and physical models respectively.

We introduced a new definition related to power efficiency. Our results show that increasing the throughput capacity by means of MPR or any of the other techniques proposed to date [1], [13] results in a reduction of power efficiency in the network. Accordingly, there is a tradeoff to be made between increasing capacity and decreasing power efficiency. Determining what is the optimum tradeoff between capacity and power efficiency is an open problem.

This paper discusses homogeneous networks where the distribution of nodes is uniform and all nodes have the same communication range. However in many practical applications, the distribution of nodes is not uniform and nodes may have different communication range. The impact of non-uniform node distribution and asymmetric transmission ranges on the throughput capacity and power efficiency is the subject of future study. Our channel is modeled based on path loss parameter and the effect of fading was not considered in this paper. It is important to investigate the impact of more complicated channel models in the future work.

VIII. Acknowledgments

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IX. Appendix

A. Proof of Theorem 4.9

Proof: In order to compute the upper bound, we derive the SINR for the node that is in a circle close to the edge of the network. For this receiver node, the Euclidean distances of interfering nodes are at $(iD(n) + R(n))$ if we assume all interfering nodes are at the farthest distance from the receiver node. Then the SINR of the transmitter node that is located at the circumference of the communication circle is given by

$$\begin{aligned} \text{SINR}_{Z(R(n))} &\leq \frac{P/R^{\alpha}(n)}{\frac{\pi}{2}nR^2(n)\sum_{i=1}^{\lceil l_\Gamma/D(n)\rceil}\frac{P}{(iD(n)+R(n))^{\alpha}}} \\ &\leq \left(\frac{D(n)}{R(n)}\right)^{\alpha}\frac{1}{\frac{\pi}{2}nR^2(n)\sum_{i=1}^{\lceil l_\Gamma/D(n)\rceil}\frac{1}{(i+\frac{1}{2})^{\alpha}}} \end{aligned} \tag{34}$$

The second inequality above stems from the fact that $\frac{R(n)}{D(n)} \leq \frac{1}{2}$. Note that $\lceil l_\Gamma/D(n)\rceil$ approaches infinity when $n \to \infty$; therefore, the summation $\sum_{i=1}^{\lceil l_\Gamma/D(n)\rceil}\frac{1}{(i+\frac{1}{2})^{\alpha}}$ converges to a bounded value when $\alpha > 2$. This means that there are constant values c_3 and c_4 such that

$$c_3 \leq \sum_{i=1}^{\lceil l_\Gamma/D(n)\rceil}\frac{1}{(i+\frac{1}{2})^{\alpha}} \leq \sum_{i=1}^{\lceil l_\Gamma/D(n)\rceil}\frac{1}{(i)^{\alpha}} \leq c_4. \tag{35}$$

Combining (34) and (35), the SINR constraint can be revised as

$$\beta \leq \text{SINR}_{Z(R(n))} \leq \left(\frac{D(n)}{R(n)}\right)^{\alpha}\frac{2}{\pi c_3 nR^2(n)}. \tag{36}$$

Then the relationship between $R(n)$ and $D(n)$ can be expressed as

$$D(n) \geq \left(\frac{c_3\beta\pi}{2}\right)^{\frac{1}{\alpha}} n^{\frac{1}{\alpha}}(R(n))^{(1+2/\alpha)}. \tag{37}$$

From Eqs. (17) and (37), the upper bound of the throughput capacity is computed as

$$\lambda(n) = O\left(\frac{R^2(n)}{D(n)}\right) = O\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right). \quad (38)$$

■

B. Proof of Theorem 4.10

Proof: From Equation (5), for any given $0 < \delta < 1$, there exists a $\theta > 0$ such that

$$P\left[|N_j - E(N_j)| > \delta E(N_j)\right] < e^{-\theta E(N_j)} = e^{-\theta n |A_j|}. \quad (39)$$

Thus, we can conclude that the probability that the value of the random variable N_j deviates by an arbitrarily small constant value from the mean tends to zero as $n \to \infty$. This is a key step in showing that when all the events $\bigcap_{j=1}^{\lceil l_\Gamma / D(n) \rceil} |N_j - E(N_j)| < \delta E(N_j)$ occur simultaneously, then all N_js converge uniformly to their expected values. Utilizing the same technique as in IV-B, we obtain

$$\begin{aligned} & P\left[\bigcap_{j=1}^{\lceil l_\Gamma / D(n) \rceil} |N_j - E(N_j)| < \delta E(N_j)\right] \\ \geq \quad & 1 - \sum_{j=1}^{\lceil l_\Gamma / D(n) \rceil} P\left[|N_j - E(N_j)| \geq \delta E(N_j)\right] \\ > \quad & 1 - \lceil \frac{l_\Gamma}{D(n)} \rceil e^{-\theta E(N_j)}. \end{aligned} \quad (40)$$

Because $E(N_j) = \frac{\pi}{2} n R^2(n)$, the final result is

$$\begin{aligned} & \lim_{n\to\infty} P\left[\bigcap_{j=1}^{\lceil l_\Gamma / D(n) \rceil} |N_j - E(N_j)| > \delta E(N_j)\right] \\ \geq \quad & 1 - \lceil \frac{l_\Gamma}{D(n)} \rceil e^{-\frac{\theta \pi n R^2(n)}{2}} \\ \geq \quad & 1 - \lceil \frac{l_\Gamma}{2R(n)} \rceil e^{-\frac{\theta \pi n R^2(n)}{2}}. \end{aligned} \quad (41)$$

If $R(n) \geq \sqrt{\frac{c_5 \log n}{n}}$ and as $n \to \infty$, then $\frac{e^{-\frac{\theta \pi n R^2(n)}{2}}}{R(n)} \to 0$, when $\theta > 1/\pi c_5$. Here, the key constraint of $R(n)$ is given as

$$R(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right). \quad (42)$$

■

C. Proof of Theorem 4.11

Proof: We first prove that Eq. (38) is an achievable bound and then by applying the minimum communication range constraint in Eq. (42), we derive the lower bound for this theorem.

To derive the achievable lower bound, we design a scheme for separating decode-able transmitter nodes inside the communication circle and interference, such that $\text{SINR}_{Z(R(n))} \geq \beta_1$. Similar to the derivations in Eq. (34) and using Fig. 3, it is clear that the SINR is minimized when the largest value for interference is considered. This value is achieved when we compute the interference for a receiver node in the middle of the network and use the closest possible distance to the receiver node[4]. This lower bound can be written as

$$\text{SINR}_{Z(R(n))} \geq \frac{\frac{P}{R^\alpha(n)}}{BN_0 + \frac{\pi}{2} n R^2(n) \sum_{i=1}^{\lceil l_\Gamma / 2D(n) \rceil} \frac{2P}{(iD(n) - R(n))^\alpha}}. \quad (43)$$

Assume that $D(n)$ satisfies the condition in Eq. (37). If we use the constraint for $R(n)$ in (42), we arrive at

$$\frac{D(n)}{R(n)} \geq \left(\frac{c_3 \beta \pi}{2}\right)^{\frac{1}{\alpha}} n^{\frac{1}{\alpha}} (R(n))^{2/\alpha} \geq \Theta\left((\log n)^{\frac{1}{\alpha}}\right), \quad (44)$$

which illustrates that $R(n)$ can be ignored compared with $D(n)$ for large values of n, i.e., $n \to \infty$. We now evaluate the asymptotic behavior of (43) when $n \to \infty$. Combining Eqs. (44) and (43), $\text{SINR}_{Z(R(n))}$ can be lower bounded by

$$\begin{aligned} \lim_{n\to\infty} \text{SINR}_{Z(R(n))} \quad & \geq \quad \left(\frac{D(n)}{R(n)}\right)^\alpha \frac{1}{\pi n R^2(n) \sum_{i=1}^{\lceil l_\Gamma / D(n) \rceil} \frac{1}{i^\alpha}} \\ & \geq \quad \left(\frac{D(n)}{R(n)}\right)^\alpha \frac{1}{\pi c_4 n R^2(n)} \\ & \geq \quad \frac{c_3}{2c_4} \beta = \beta_1. \end{aligned}$$

This inequality is derived using Eqs. (37) and (35), together with the fact that the second term in the denominator of SINR goes to infinity when $n \to \infty$ and, therefore, we can drop the first term related to the noise. Using the same arguments introduced for the computation of the upper bound, we can show that a non-zero value for $\text{SINR}_{Z(R(n))}$ can be achieved which implies that the throughput capacity can be achieved asymptotically. ■

REFERENCES

[1] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, 2000.

[2] S. Ghez, S. Verdu, and S. C. Schwartz, "Stability properties of slotted aloha with multipacket reception capability," *IEEE Transactions on Automatic Control*, vol. 33, no. 7, pp. 640–649, 1988.

[3] S. Verdu, *Multiuser Detection*. Cambridge University Press, 1998.

[4] C. Peraki and S. Servetto, "On the maximum stable throughput problem in random networks with directional antennas," in *Proc. of ACM MobiHoc 2003*, Annapolis, Maryland, USA., June 1-3 2003.

[5] S. Yi, Y. Pei, and S. Kalyanaraman, "On the capacity improvement of ad hoc wireless networks using directional antennas," in *Proc. of ACM MobiHoc 2003*, Annapolis, Maryland, USA., June 1-3 2003.

[6] R. Ahlswede, C. Ning, S. Y. R. Li, and R. W. Yeung, "Network information flow," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1204–1216, 2000.

[7] J. Liu, D. Goeckel, and D. Towsley, "Bounds on the gain of network coding and broadcasting in wireless networks," in *Proc. of IEEE INFOCOM 2007*, Anchorage, Alaska, USA., May 6-12 2007.

[8] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," in *Proc. of ACM SIGCOMM 2007*, Kyoto, Japan, August 27-31 2007.

[9] S. Zhang, S. C. Liew, and P. P. Lam, "Hot topic: Physical-layer network coding," in *Proc. of ACM MobiCom 2006*, Los Angeles, California, USA., September 23-29 2006.

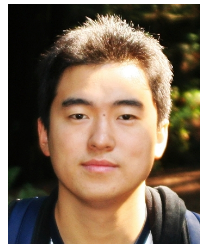
[10] A. Ramamoorthy, J. Shi, and R. Wesel, "On the capacity of network coding for random networks," *IEEE Transactions on Information Theory*, vol. 51, no. 8, pp. 2878–2885, 2005.

[4]Note that the difference between maximum and minimum value of interference is a constant value.

[11] S. A. Aly, V. Kapoor, J. Meng, and A. Klappenecker, "Bounds on the network coding capacity for wireless random networks," in *Proc. of Third Workshop on Network Coding, Theory, and Applications (NetCod 2007*, San Diego, California, USA, January 29 2007.
[12] Z. Kong, S. A. Aly, E. Soljanin, E. M. Yeh, and A. Klappenecker, "Network coding capacity of random wireless networks under a signal-to-interference-and noise model," *Submitted to IEEE Transactions on Information Theory*, 2007.
[13] M. Franceschetti, O. Dousse, D. N. C. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory," *IEEE Transactions on Information Theory*, vol. 53, no. 3, pp. 1009–1018, 2007.
[14] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Transactions on Networking*, vol. 10, no. 4, pp. 477–486, 2002.
[15] P. Kyasanur and N. H. Vaidya, "Capacity of multi-channel wireless networks: Impact of number of channels and interfaces," in *Proc. of ACM MobiCom 2005*, Cologne, Germany, August 28-September 2 2005.
[16] R. M. de Moraes, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "Many-to-many communication: A new approach for collaboration in manets," in *Proc. of IEEE INFOCOM 2007*, Anchorage , Alaska , USA., May 6-12 2007.
[17] A. Ozgur, O. Leveque, and D. Tse, "Hierarchical cooperation achieves linear capacity scaling in ad hoc networks," in *Proc. of IEEE INFOCOM 2007*, Anchorage, Alaska, USA., May 6-12 2007.
[18] S. Verdu, "On channel capacity per unit cost," *IEEE Transactions on Information Theory*, vol. 36, no. 5, pp. 1019–1030, 1990.
[19] M. D. Penrose, "The longest edge of the random minimal spanning tree," *The Annals of Applied Probability*, vol. 7, no. 2, pp. 340–361, 1997.
[20] R. Motwani and P. Raghavan, *Randomized Algorithms*. Cambridge University Press, 1995.
[21] J. J. Garcia-Luna-Aceves, H. R. Sadjadpour, and Z. Wang, "Challenges: Towards truly scalable ad hoc networks," in *Proc. of ACM MobiCom 2007*, Montreal, Quebec, Canada, September 9-14 2007.
[22] Z. Wang, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "The capacity and energy efficiency of wireless ad hoc networks with multi-packet reception," in *Proc. of ACM MobiHoc 2008*, Hong Kong SAR, China, May 26-30 2008.

Hamid R. Sadjadpour (S'94-M'95-SM'00) received his B.S. and M.S. degrees from Sharif University of Technology with high honor and Ph.D. degree from University of Southern California in 1986, 1988 and 1996, respectively. After graduation, he joined AT&T as a member of technical staff, later senior technical staff member, and finally Principal member of technical staff at AT&T Lab. in Florham Park, NJ until 2001. In fall 2001, he joined University of California, Santa Cruz (UCSC) where he is now an Associate Professor.

Dr. Sadjadpour has served as technical program committee member in numerous conferences and as chair of communication theory symposium at WirelessCom 2005, and chair of communication and information theory symposium at IWCMC 2006, 2007 and 2008 conferences. He has been also Guest editor of EURASIP on special issue on Multicarrier Communications and Signal Processing in 2003 and special issue on Mobile Ad Hoc Networks in 2006, and is currently Associate editor for Journal of Communications and Networks (JCN). He has published more than 110 publications. His research interests include space-time signal processing, scaling laws for wireless ad hoc networks, performance analysis of ad hoc and sensor networks, and MAC layer protocols for MANETs. He is the co-recipient of International Symposium on Performance Evaluation of Computer and Telecommunication Systems (SPECTS) 2007 best paper award and the IEEE Fred W. Ellersick Award for Best Unclassified Paper at the 2008 Military Communications (MILCOM) conference. He holds more than 13 patents, one of them accepted in spectrum management of T1.E1.4 standard.



Zheng Wang (S'05) received his B.S. and M.S. degree from Department of Electrical Engineering, Peking University, China in 2003 and 2006 respectively. Since 2006, he has been pursuing Ph.D. degree with the Department of Electrical Engineering at University of California, Santa Cruz, USA.

His research interests include wireless networks, wireless communications and network information theory. He is the recipient of IEEE Fred W. Ellersick Award for Best Unclassified Paper at MILCOM 2008.

J.J. Garcia-Luna-Aceves (S'75-M'77-SM'02-F'06) received the B.S. degree in Electrical Engineering from the Universidad Iberoamericana, Mexico City, Mexico in 1977; and the M.S. and Ph.D. degrees in Electrical Engineering from the University of Hawaii at Manoa, Honolulu, HI in 1980 and 1983, respectively. He holds the Jack Baskin Endowed Chair of Computer Engineering at the University of California, Santa Cruz (UCSC), and is a Principal Scientist at the Palo Alto Research Center (PARC). Prior to joining UCSC in 1993, he was a Center Director at SRI International (SRI) in Menlo Park, California. He has been a Visiting Professor at Sun Laboratories and a Principal of Protocol Design at Nokia.

Dr. Garcia-Luna-Aceves holds 26 U.S. patents, and has published a book and more than 290 papers. He has directed 26 Ph.D. theses and 22 M.S. theses since he joined UCSC in 1993. He has been the General Chair of the ACM MobiCom 2008 Conference; the General Chair of the IEEE SECON 2005 Conference; Program Co-Chair of ACM MobiHoc 2002 and ACM MobiCom 2000; Chair of the ACM SIG Multimedia; General Chair of ACM Multimedia '93 and ACM SIGCOMM '88; and Program Chair of IEEE MULTIMEDIA '92, ACM SIGCOMM '87, and ACM SIGCOMM '86. He has served in the IEEE Internet Technology Award Committee, the IEEE Richard W. Hamming Medal Committee, and the National Research Council Panel on Digitization and Communications Science of the Army Research Laboratory Technical Assessment Board. He has been on the editorial boards of the IEEE/ACM Transactions on Networking, the Multimedia Systems Journal, and the Journal of High Speed Networks.

He is an IEEE Fellow and an ACM Fellow, and is listed in Marquis Who's Who in America and Who's Who in The World. He is the co-recipient of the IEEE Fred W. Ellersick 2008 MILCOM Award for best unclassified paper. He is also co-recipient of Best Paper Awards at the IEEE MASS 2008, SPECTS 2007, IFIP Networking 2007, and IEEE MASS 2005 conferences, and of the Best Student Paper Award of the 1998 IEEE International Conference on Systems, Man, and Cybernetics. He received the SRI International Exceptional-Achievement Award in 1985 and 1989.