Computational Verb Cellular Networks: Part III–Solutions of One-Dimensional Computational Verb Cellular Networks

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Abstract— **The theoretical study of conditions, under which three kinds of pattern solutions are generated in 1D computational verb cellular networks(CVCN), is performed. Homogenous pattern solutions are found universal in 1D CVCNs while checkerboard and flip-flop pattern solutions are not rare in 1D CVCNs. Theorems, which provide constraints of parameters of 1D CVCN for generating these three kinds of pattern solutions, are constructed and proved. Numerical experimental results meet the predictions of the theorems.** *Copyright © 2009 Yang's Scientific Research Institute, LLC. All rights reserved.*

Index Terms— **Computational verb, 1D CVCN, cellular network, pattern formation, local rule, computational verb rule.**

I. Introduction

Computational verb cellular networks (CVCNs) (verb cellular networks, for short) are cellular networks, of which the local rules are computational verb rules. This paper is the third part of a paper series on CVCNs. In the first part[61] of this series 2D CVCN's were studied. In the second part[62] of this series 1D CVCN's were studied. In this paper, three kinds of pattern solutions, which are homogenous, checkerboard and flip-flop patterns, of 1D CVCNs will be studied theoretically. The basics of 1D CVCN can be found from [62].

The organization of this paper is as follows. In Section II, the brief history of computational verb theory will be given. In Section III, the checkerboard pattern solutions of 1D CVCN will be studied. In Section IV, the homogenous pattern solutions of 1D CVCN will be investigated. In Section V, the flip-flop pattern solutions of 1D CVCN will be explored. In Section VI, some concluding remarks will be included.

II. A Brief History of Computational Verb Theory

As the first paradigm shift for solving engineering problems by using verbs, the computational verb theory[31] and physical linguistics[34], [51], [25] have undergone a rapid growth since the birth of computational verb in the Department of Electrical Engineering and Computer Sciences, University of California at Berkeley in 1997[16], [17]. The paradigm of implementing verbs in machines was coined as *computational verb theory*[31]. The building blocks of computational theory are *computational verbs*[26], [20], [18], [27], [32]. The relation between verbs and adverbs was mathematically defined in [19]. The logic operations between verb statements were studied in [21]. The applications of verb logic to verb reasoning were addressed in [22] and further studied in [31]. A logic paradox was solved based on verb logic[28]. The mathematical concept of set was generalized into verb set in[24]. Similarly, for measurable attributes, the number systems can be generalized into verb numbers[29]. The applications of computational verbs to predictions were studied in [23]. In [33] fuzzy dynamic systems were used to model a special kind of computational verb that evolves in fuzzy spaces. The relation between computational verb theory and traditional linguistics was studied in [31], [34]. The theoretical basis of developing computational cognition from a unified theory of fuzzy and computational verb theory is the theory of the Unicogse that was studied in [34], [39]. The issues of simulating cognition using computational verbs were studied in [35]. In [64] the correlation between computational verbs was studied. A method of implementing feelings in machines was proposed based on grounded computational verbs and computational nouns in [41]. In [48] a theory of how to design stable computational verb controllers was given. In [42] the rule-wise linear computational verb systems and their applications to the design of stable computational verb controllers and chaos in computational verb systems were presented. In [46] the concept of computational verb entropy was used to construct computational verb decision tree for data-mining applications. In [45] the relation between computational verbs and fuzzy sets was studied by using computational verb collapses and computational verb extension principles. In [47] the distances and similarities of saturated computational verbs were defined as normalized measures of the distances and similarities between computational verbs. Based on saturated computational verbs, the verb distances and similarities are related to each other with a simple relation. The distances and similarities between verbs with different life spans can be defined based on saturated computational verbs as well. In [49] the methods of using computational verbs to cluster trajectories and curves were presented. To cluster a bank of trajectories into a few representative computational

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verbs is to discover knowledge from database of time series. We use cluster centers to represent complex waveforms at symbolic levels. In [14] computational verb controllers were used to control a chaotic circuit model known as Chua's circuit. Computational verb controllers were designed based on verb control rules for different dynamics of the region-wise linear model of the control plant. In [13] computational verb controllers were used to synchronize discrete-time chaotic systems known as Hénon maps. Different verb control rules are designed for synchronizing different kinds of dynamics. In [53], how can computational verb theory functions as the most essential building block of cognitive engineering and cognitive industries was addressed. Computational verb theory will play a critical important role in personalizing services in the next fifty years. In [50], [52] computational verb theory was used to design an accurate flame-detecting systems based on CCTV signal. In [56] the learning algorithms were presented for learning computational verb rules from training data. In [54] the structures and learning algorithms of computational verb neural networks were presented. In [63] the ambiguities of the states and dynamics of computational verbs were studied. In [55] the history and milestones in the first ten years of the studies of computational verb theory were given. In [4] a case study of modeling adverbs as modifiers of computational verbs was presented. In [15] computational verb rules were used to improve the training processes of neural networks. In [57] the classifications and interactions between computational verb rule bases were presented. In [58] the simplest verb rules and their verb reasoning were connected to many intuitive applications of verb rules before the invention of computational verbs. In [59] the interactions between computational verbs were used as a powerful tools to understand the merging and splitting effects of verbs. In [3] computational verb rules were trained by using prescribed training samples of functions. In [60] the trend-based computational verb similarity was given as a way to decrease the computational complexities of verb similarities. In [5] computational verb PID controller was used to control linear motors. In [12] computational verb controller was used to control an auto-focusing system.

The theory of computational verb has been taught in some university classrooms since 2005[1]. The latest active applications of computational verb theory are listed as follows.

1) Computational Verb Controllers. The applications of computational verbs to different kinds of control problems were studied on different occassions[30], [31]. For the advanced applications of computational verbs to control problems, a few papers reporting the latest advances had been published[37], [36], [48], [42], [65]. The design of computational verb controllers was also presented in a textbook in 2005[1].
2) Computational Verb Image Processing and Image Understanding. The recent results of image processing by using computational verbs can be found in[38]. The applications of computational verbs to image understanding can be found in [40]. The authors of [2] applied computational verb image processing to design the vision systems of RoboCup small-size robots.
3) Stock Market Modeling and Prediction based on computational verbs. The product of Cognitive Stock Charts[8] was based on the advanced modeling and computing reported in [43]. Computational verb theory was used to study the trends of stock markets known as Russell reconstruction patterns [44].

Computational verb theory has been successfully applied to many industrial and commercial products. Some of these products are listed as follows.

1) Visual Card Counters. The *YangSky-MAGIC* card counter[10], developed by Yang's Scientific Research Institute and Wuxi Xingcard Technology Co. Ltd., was the first visual card counter to use computational verb image processing technology to achieve high accuracy of card and paper board counting based on cheap webcams.
2) CCTV Automatic Driver Qualify Test System. The *DriveQfy* CCTV automatic driver qualify test system[11] was the first vehicle trajectory reconstruction and stop time measuring system using computational verb image processing technology.
3) Visual Flame Detecting System. The *FireEye* visual flame detecting system[6] was the first CCTV or webcam based flame detecting system, which works under color and black & white conditions, for surveillance and security monitoring system.
4) Smart Pornographic Image and Video Detection Systems. The *PornSeer*[9] pornographic image and video detection systems are the first cognitive feature based smart porno detection and removal software.
5) Webcam Barcode Scanner. The *BarSeer*[7] webcam barcode scanner took advantage of the computational verb image processing to make the scan of barcode by using cheap webcam possible.
6) Cognitive Stock Charts. By applying computational verbs to the modeling of trends and cognitive behaviors of stock trading activities, cognitive stock charts can provide the traders with the "feelings" of stock markets by using simple and intuitive indexes.
7) TrafGo ITS SDK. Computational verbs were applied to model vehicle trajectories and dynamics of optical field and many other aspects of dynamics in complex environments for applications in intelligent transportation systems (ITS).

III. CHECKERBOARD PATTERNS

Figure 1 shows two local evolving patterns, $\wp_1$ and $\wp_2$, for the purpose of forming a checkerboard pattern in a 1D CVCN. In $\wp_1$ and $\wp_2$, $a \in \mathbb{R}$ and $b \in \mathbb{R}$ are two constants. The first

[1]Some computational verb theory related college courses are

- Dr. G. R. Chen, EE 64152 - Introduction to Fuzzy Informatics and Intelligent Systems, Department of Electronic Engineering, City University of Hong Kong.
- Dr. D. H. Guo, Artificial Intelligence, Department of Electronic Engineering, Xiamen University.
- Prof. T. Yang, Computational Methodologies in Intelligent Systems, Department of Electronic Engineering, Xiamen University.
- Dr. Mahir Sabra, EELE 6306: Intelligent Control, Electrical and Computer Engineering Department, The Islamic University of Gaza.

and the second row show the values of three cells at iterations $k-1$ and k, respectively. The third row shows the value of the central cell at the $k+1$ evolving iteration.

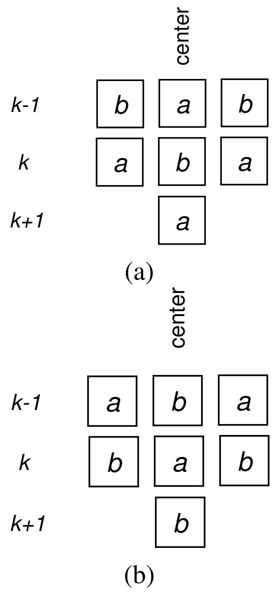


Fig. 1. Two local evolving patterns of generating checkerboard patterns in a 1D CVCN. (a) Local pattern $\wp_1$. (b) Local pattern $\wp_2$.

As long as in a 1D CVCN, local evolving patterns $\wp_1$ and $\wp_2$ can flip-flop to each other, a checkerboard pattern might form. To find the conditions of forming a checkerboard pattern in a 1D CVCN, we need to find the conditions under which $\wp_1$ and $\wp_2$ exist for a pair of constants a and b.

In this paper, the state of a cell in a 1D CVCN is calculated by using Eq. (10) of [62].

A. Local Evolving Pattern $\wp_1$

For $\wp_1$, the off-center cells have the following verb similarities.

$$\begin{aligned} S_{oI} &\triangleq S(\mathsf{increase}, x_{i\pm1}(k)) = \frac{1}{1+e^{-(a-b)/\Delta}}, \\ S_{oD} &\triangleq S(\mathsf{decrease}, x_{i\pm1}(k)) = \frac{1}{1+e^{(a-b)/\Delta}}. \end{aligned} \tag{1}$$

The central cell has the following verb similarities.

$$\begin{aligned} S_{cI} &\triangleq S(\mathsf{increase}, x_{i}(k)) = \frac{1}{1+e^{-(b-a)/\Delta}}, \\ S_{cD} &\triangleq S(\mathsf{decrease}, x_{i}(k)) = \frac{1}{1+e^{(b-a)/\Delta}}. \end{aligned} \tag{2}$$

It follows from Eqs. (1) and (2) that

$$S_{oI} = S_{cD}, S_{oD} = S_{cI}. \tag{3}$$

Let y_c and y_o denote the outputs of the central cell and the off-center cells, respectively, then we have

$$\begin{aligned} y_c &\triangleq f(x_i(k)) = \frac{1}{1+e^{-b}}, \\ y_o &\triangleq f(x_{i\pm1}(k)) = \frac{1}{1+e^{-a}}. \end{aligned} \tag{4}$$

It follows from Eq. (10) of [62] that the state of the central cell in the next evolving step, $x_i(k+1) = a$, is calculated as follow.

$$\begin{aligned} \Sigma_s &\triangleq S_{oD}S_{cD}S_{oD} + S_{oD}S_{cD}S_{oI} + S_{oD}S_{cI}S_{oD} \\ &\quad + S_{oD}S_{cI}S_{oI} + S_{oI}S_{cD}S_{oD} + S_{oI}S_{cD}S_{oI} \\ &\quad + S_{oI}S_{cI}S_{oD} + S_{oI}S_{cI}S_{oI} \\ &= (S_{oD}+S_{oI})^2(S_{cD}+S_{cI}) \\ &= (S_{cD}+S_{cI})^3, \\ \frac{\Sigma_{sf}}{y_o} &\triangleq g_1S_{oD}S_{cD}S_{oD} + g_2S_{oD}S_{cD}S_{oI} + g_3S_{oD}S_{cI}S_{oD} \\ &\quad + g_4S_{oD}S_{cI}S_{oI} + g_5S_{oI}S_{cD}S_{oD} + g_6S_{oI}S_{cD}S_{oI} \\ &\quad + g_7S_{oI}S_{cI}S_{oD} + g_8S_{oI}S_{cI}S_{oI}, \\ &= S_{oD}^2(g_1S_{cD}+g_3S_{cI}) + S_{oI}^2(g_6S_{cD}+g_8S_{cI}) \\ &\quad + S_{oD}S_{oI}[(g_2+g_5)S_{cD}+(g_4+g_7)S_{cI}] \\ &= S_{cI}^2(g_1S_{cD}+g_3S_{cI}) + S_{cD}^2(g_6S_{cD}+g_8S_{cI}) \\ &\quad + S_{cD}S_{cI}[(g_2+g_5)S_{cD}+(g_4+g_7)S_{cI}], \\ a &= \frac{\Sigma_{sf}}{\Sigma_s}. \end{aligned} \tag{5}$$

B. Local Evolving Pattern $\wp_2$

Based on the symmetrical correspondence between $\wp_1$ and $\wp_2$, it follows from Eq. (5) that

$$\begin{aligned} \widetilde{\Sigma}_s &\triangleq (\widetilde{S}_{cD}+\widetilde{S}_{cI})^3, \\ \frac{\widetilde{\Sigma}_{sf}}{\widetilde{y}_o} &\triangleq \widetilde{S}_{cI}^2(g_1\widetilde{S}_{cD}+g_3\widetilde{S}_{cI}) + \widetilde{S}_{cD}^2(g_6\widetilde{S}_{cD}+g_8\widetilde{S}_{cI}) \\ &\quad + \widetilde{S}_{cD}\widetilde{S}_{cI}[(g_2+g_5)\widetilde{S}_{cD}+(g_4+g_7)\widetilde{S}_{cI}], \\ b &= \frac{\widetilde{\Sigma}_{sf}}{\widetilde{\Sigma}_s} \end{aligned} \tag{6}$$

where

$$\begin{aligned} \widetilde{y}_o &= \frac{1}{1+e^{-b}}, \\ \widetilde{S}_{cD} &= \frac{1}{1+e^{(a-b)/\Delta}} = S_{cI}, \\ \widetilde{S}_{cI} &= \frac{1}{1+e^{-(a-b)/\Delta}} = S_{cD}. \end{aligned} \tag{7}$$

Thus it follows from Eqs. (6) and (7) that

$$\begin{aligned} \widetilde{\Sigma}_s &= (\widetilde{S}_{cD}+\widetilde{S}_{cI})^3 = \Sigma_s, \\ \frac{\widetilde{\Sigma}_{sf}}{\widetilde{y}_o} &= S_{cD}^2(g_3S_{cD}+g_1S_{cI}) + S_{cI}^2(g_8S_{cD}+g_6S_{cI}) \\ &\quad + S_{cD}S_{cI}[(g_4+g_7)S_{cD}+(g_2+g_5)S_{cI}], \\ b &= \frac{\widetilde{\Sigma}_{sf}}{\Sigma_s}. \end{aligned} \tag{8}$$

In order to find the conditions under which checkerboard patterns form, we need to solve Eqs. (5) and (8). Therefore we proved the following theorem.

Theorem 1 (checkerboard pattern): Given the parameters of a 1D CVCN Δ, g_I, g_D, a checkerboard pattern with two states a and b is a solution of the 1D CVCN if and only if the following conditions hold.

$$\begin{aligned}
S_{cI} &= \frac{1}{1+e^{-(b-a)/\Delta}}, S_{cD} = \frac{1}{1+e^{(b-a)/\Delta}}, \\
y_o &= \frac{1}{1+e^{-a}}, \widetilde{y}_o = \frac{1}{1+e^{-b}}, \\
\Sigma_s &= (S_{cD}+S_{cI})^3, \\
\frac{\Sigma_{sf}}{y_o} &= S_{cD}^2(g_6S_{cD}+g_8S_{cI}) + S_{cI}^2(g_1S_{cD}+g_3S_{cI}) \\
&\quad +S_{cD}S_{cI}[(g_2+g_5)S_{cD}+(g_4+g_7)S_{cI}], \\
\frac{\widetilde{\Sigma}_{sf}}{\widetilde{y}_o} &= S_{cD}^2(g_3S_{cD}+g_1S_{cI}) + S_{cI}^2(g_8S_{cD}+g_6S_{cI}) \\
&\quad +S_{cD}S_{cI}[(g_4+g_7)S_{cD}+(g_2+g_5)S_{cI}], \\
a &= \frac{\Sigma_{sf}}{\Sigma_s}, b = \frac{\widetilde{\Sigma}_{sf}}{\Sigma_s} \qquad (9)
\end{aligned}$$

where $g_i, i = 1, \ldots, 8$, are chosen from the set $\{g_I, g_D\}$ based on the local rules of the 1D CVCN.

Remark 1: It will be very difficult, if not impossible, to find an explicit solution to Eq. (9) in general. In this paper, I will only study the solutions of some special cases of Eq. (9).

Theorem 2: "Rule 0" and "Rule 255" 1D CVCNs can not form checkerboard patterns.

Proof: In either "Rule 0" or "Rule 255" 1D CVCNs, the parameters $g_i, i = 1, \ldots, 8$, have the same value; namely, $g_i = g, i = 1, \cdots, 8$. It follows from Eqs. (5) and (8) that

$$\begin{aligned}
\Sigma_s &= (S_{cD}+S_{cI})^3, \\
\Sigma_{sf} &= gy_o\Sigma_s, \\
a &= gy_o, \\
\widetilde{\Sigma}_{sf} &= g\widetilde{y}_o\Sigma_s, \\
b &= g\widetilde{y}_o. \qquad (10)
\end{aligned}$$

Therefore, we have the following conditions to satisfy

$$\begin{aligned}
a &= gy_o = \frac{g}{1+e^{-a}}, \\
b &= g\widetilde{y}_o = \frac{g}{1+e^{-b}} \qquad (11)
\end{aligned}$$

from which we have

$$g = a(1+e^{-a}), b(1+e^{-b}) = a(1+e^{-a}). \qquad (12)$$

The only solution is $a = b$, which leads to a homogenous pattern. ■

Remark 2: In Eq. (12), $a = b$ is the solution of equation

$$x + xe^{-x} - g = 0. \qquad (13)$$

It is difficult to find the explicit solution. Instead, we can find some numerical solutions. For example, when we choose $g = 1$, the solution is $a = b = 0.659046068407406660984386495592887$. It follows Theorem 2 that the solution $a = b$ is independent to parameter Δ. Thus, homogenous patterns are formed only relevant to the value of g.

Example 1 ("Rule 0 1D CVCN"): Figure 2(a) and (b) show the evolving patterns and the time series of the cell states of a "rule 0 1D CVCN", respectively. There are 11 cells in this 1D CVCN. The initial conditions are chosen as $x_i(-1) = 0$ and $x_i(0) \in U(0,1)$, $i = 1, \ldots, 11$. The parameter $\Delta = 0.5$. Observe that all cells converge to the homogamous solution, which is predicted by using Theorem 2. Simulation results verified that "rule 255 1D CVCN" behaved the same.

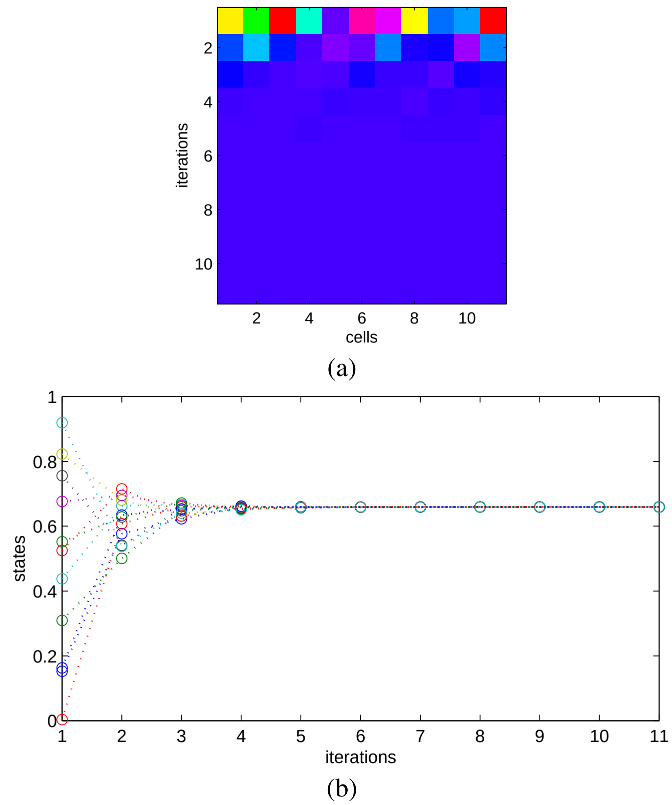


Fig. 2. Evolving process of "rule 0 1D CVCN" of 11 cells with $g = 1$. (a) Evolving pattern of the 1D CVCN array. (b) Time series of the states of cells in the 1D CVCN array.

Theorem 3: If $g_6 = g_3$, $g_8 = g_1$ and $g_2 + g_5 = g_4 + g_7$, then homogenous patterns are solutions to 1D CVCNs.

Proof: If $g_6 = g_3$, $g_8 = g_1$ and $g_2 + g_5 = g_4 + g_7$, then it follows from Eq. (9) that

$$\widetilde{\Sigma}_{sf} = \frac{\widetilde{y}_o}{y_o}\Sigma_{sf} \qquad (14)$$

which leads to

$$\frac{b}{a} = \frac{\widetilde{y}_o}{y_o} = \frac{1+e^{-a}}{1+e^{-b}} \qquad (15)$$

which leads to

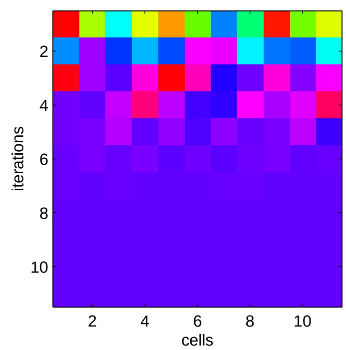
$$a(1+e^{-a}) = b(1+e^{-b}). \qquad (16)$$

The only solution is $a = b$. Therefore, the homogenous pattern is a solution to this 1D CVCN. ■

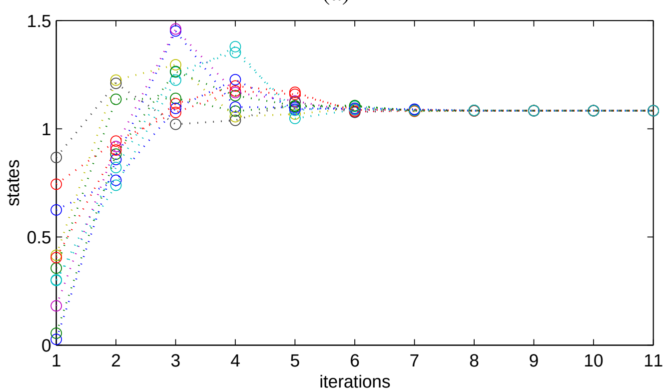
Remark 3: The cases in Theorem 2 are special cases of Theorem 3. The results presented in Theorem 2 is independent to the parameter Δ as well. Although these CVCNs have homogenous patterns as solutions, these solutions might not be asymptotically stable.

Example 2 ("Rule 102 1D CVCN"): As an example, the evolving processes for "rule 102 1D CVCN" of 11 cells with initial conditions $x_i(-1) = 0$ and $x_i(0) \in U(0,1)$,

$i = 1, \ldots, 11$, and parameters $\Delta = 0.5$, $g_I = 0.4$ and $g_D = 2.5$ are shown in Fig. 3. Observe from Fig. 3(b) that the states of all cells converge to the same value, which can be calculated based on the results in Section IV, from random initial conditions.



(a)



(b)

Fig. 3. Evolving process of "rule 102 1D CVCN" of 11 cells. (a) Evolving pattern of the 1D CVCN array. (b) Time series of the states of cells in the 1D CVCN array.

All 1D CVCNs that satisfy the conditions of Theorem 3 are listed in Table I. Simulation results with $g_I = g_D = 1$ show that all these 1D CVCNs converge to homogenous patterns. When $g_I = 0.4$ and $g_D = 2.5$, then rules 10, 46, 80, 116, 139, 209, and 245 don't approach homogenous patterns asymptotically. The reason that some of these 1D CVCNs asymptotically converge to homogenous while the others are not needs further study.

TABLE I

ALL 1D CVCNS THAT HAVE HOMOGENOUS PATTERNS AS STABLE SOLUTIONS.

b_8	b_7	b_6	b_5	b_4	b_3	b_2	b_1	rules	stable
0	0	0	0	0	0	0	0	0	yes
0	0	0	0	1	0	1	0	10	no
0	0	0	1	1	0	0	0	24	yes
0	0	1	0	0	1	0	0	36	yes
0	0	1	0	1	1	1	0	46	no
0	0	1	1	1	1	0	0	60	yes
0	1	0	0	0	0	1	0	66	yes
0	1	0	1	0	0	0	0	80	no
0	1	0	1	1	0	1	0	90	yes
0	1	1	0	0	1	1	0	102	yes
0	1	1	1	0	1	0	0	116	no
0	1	1	1	1	1	1	0	126	yes
1	0	0	0	0	0	0	1	129	yes
1	0	0	0	1	0	1	1	139	no
1	0	0	1	1	0	0	1	153	yes
1	0	1	0	0	1	0	1	165	yes
1	0	1	0	1	1	1	1	175	yes
1	0	1	1	1	1	0	1	189	yes
1	1	0	0	0	0	1	1	195	yes
1	1	0	1	0	0	0	1	209	no
1	1	0	1	1	0	1	1	219	yes
1	1	1	0	0	1	1	1	231	yes
1	1	1	1	0	1	0	1	245	no
1	1	1	1	1	1	1	1	255	yes

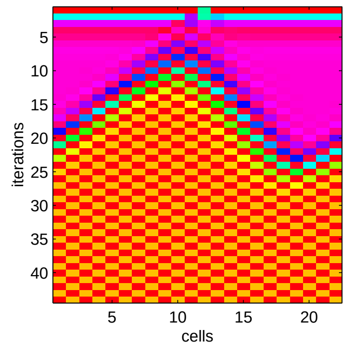
C. Checkerboard Patterns in "Rule 4 1D CVCN"

In a "rule 4 1D CVCN" we have $g_3 = g_I$ and all other g_i's have the same value of g_D, thus Eq. (9) can be rewritten as

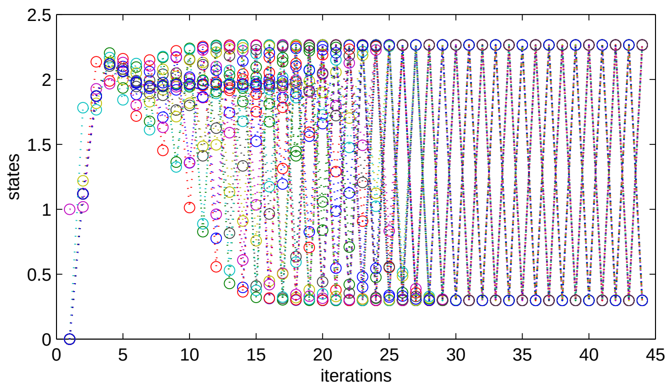
$$\begin{aligned}
S_{cI} &= \frac{1}{1+e^{-(b-a)/\Delta}}, S_{cD} = \frac{1}{1+e^{(b-a)/\Delta}}, \\
y_o &= \frac{1}{1+e^{-a}}, \widetilde{y}_o = \frac{1}{1+e^{-b}}, \\
\Sigma_s &= (S_{cD}+S_{cI})^3, \\
\frac{\Sigma_{sf}}{y_o} &= (S_{cD}^2 g_D + 2S_{cD}S_{cI}g_D)(S_{cD}+S_{cI}) \\
&\quad + S_{cI}^2(g_D S_{cD} + g_I S_{cI}), \\
\frac{\widetilde{\Sigma}_{sf}}{\widetilde{y}_o} &= S_{cD}^2(g_I S_{cD} + g_D S_{cI}) \\
&\quad + (S_{cI}^2 g_D + 2S_{cD}S_{cI}g_D)(S_{cD}+S_{cI}), \\
a &= \frac{\Sigma_{sf}}{\Sigma_s}, b = \frac{\widetilde{\Sigma}_{sf}}{\Sigma_s}
\end{aligned} \tag{17}$$

Solve this set of equations numerically, we get the following solution $a = 0.2975$, $b = 2.2648$, $\Delta = 0.5000$, $g_D = 2.5001$, and $g_I = 0.3999$. The simulation result of a 22-cell array is shown in Fig. 4. Toroidal boundary condition is used. The initial conditions are $x_i(-1) = 0$ and $x_i(0) = 0$, $i = 1, \ldots, 22$, except that the center cell has an initial state $x_{12}(0) = 1$. Observe from Fig. 4(b) that a checkerboard pattern formed with the values in coordinate with the theoretical prediction. At the end of the evolution, all cells oscillate between a and b values.

Since checkerboard patterns have a spatial period of two, if the number of cell in the 1D CVCN array is odd, then a defect propagates through the entire cell array. The simulation result of a 21-cell 1D CVCN is shown in Fig. 5. Toroidal boundary condition is used. The initial conditions are $x_i(-1) = 0$ and $x_i(0) = 0$, $i = 1, \ldots, 21$, except that the central cell has an

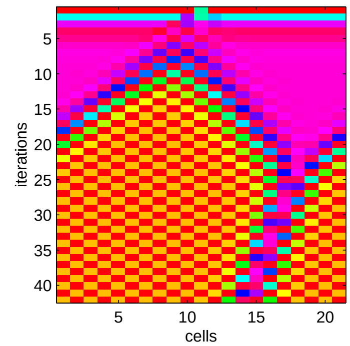


(a)

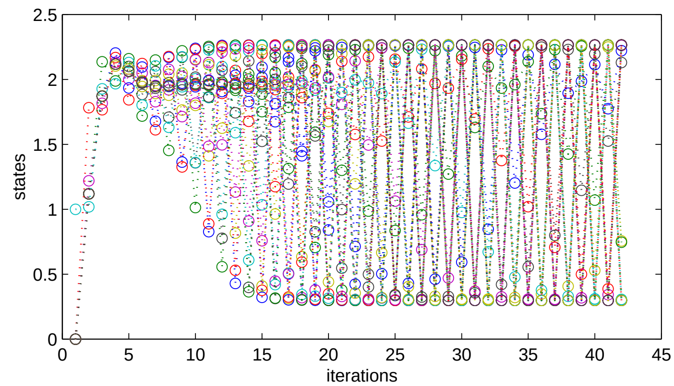


(b)

Fig. 4. Evolving process of "rule 4 1D CVCN" of 22 cells. (a) Evolving pattern of the 1D CVCN array. (b) Time series of the states of cells in the 1D CVCN array.



(a)



(b)

Fig. 5. Evolving process of "rule 4 1D CVCN" of 21 cells. (a) Evolving pattern of the 1D CVCN array. (b) Time series of the states of cells in the 1D CVCN array.

initial state $x_{11} = 1$. Observe from Fig. 5(a) that a defect travels from right to left in a checkerboard pattern. The defect is formed when the left wave front of the checkerboard pattern hits the left-hand side boundary before iteration 20. After iteration 20, the defect travels to the left at a speed of less than 1 cell every 5 iterations.

IV. Homogenous Patterns

The evolving pattern of a homogenous pattern in 1D a CVCN is shown in Fig. 6.

The off-center cells have the following verb similarities.

$$\begin{aligned} S_{oI} &\triangleq S(\text{increase}, x_{i\pm1}(k)) = \frac{1}{2}, \\ S_{oD} &\triangleq S(\text{decrease}, x_{i\pm1}(k)) = \frac{1}{2}. \end{aligned} \tag{18}$$

The central cell has the following verb similarities.

$$\begin{aligned} S_{cI} &\triangleq S(\text{increase}, x_{i}(k)) = \frac{1}{2}, \\ S_{cD} &\triangleq S(\text{decrease}, x_{i}(k)) = \frac{1}{2}. \end{aligned} \tag{19}$$

It follows from Eqs. (18) and (19) that

$$S_{oD} = S_{oI} = S_{cD} = S_{cI} = 1/2. \tag{20}$$

Let y_c and y_o denote the output of the central cell and the

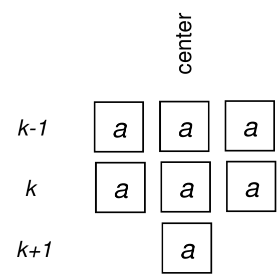


Fig. 6. The local evolving pattern of generating homogenous patterns in a 1D CVCN.

off-center cells, respectively, then we have

$$\begin{aligned} y_c &\triangleq f(x_i(k)) = \frac{1}{1+e^{-a}}, \\ y_o &\triangleq f(x_{i\pm1}(k)) = \frac{1}{1+e^{-a}}. \end{aligned} \tag{21}$$

It follows from Eq. (5) that the state of the central cell in the next evolving step, $x_i(k+1) = a$, is calculated as follow.

$$\begin{aligned} \Sigma_s &= (S_{cD} + S_{cI})^3 = 1, \\ \frac{8\Sigma_{sf}}{y_o} &= \sum_{i=1}^{8} g_i, \\ a &= \frac{\Sigma_{sf}}{\Sigma_s} = \Sigma_{sf}, \end{aligned} \tag{22}$$

from which we have

$$a = \Sigma_{sf} = \frac{y_o}{8}\sum_{i=1}^{8} g_i = \frac{\sum_{i=1}^{8} g_i}{8(1+e^{-a})}, \tag{23}$$

from which we have the following theorem.

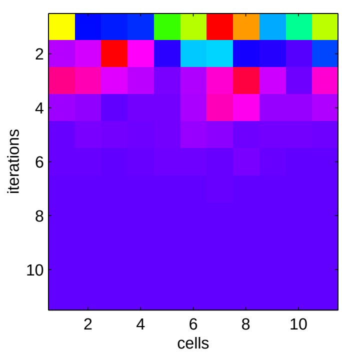
Theorem 4 (Homogenous patterns): Let a be a solution of equation

$$x(1+e^{-x}) - \frac{1}{8}\sum_{i=1}^{8} g_i = 0 \tag{24}$$

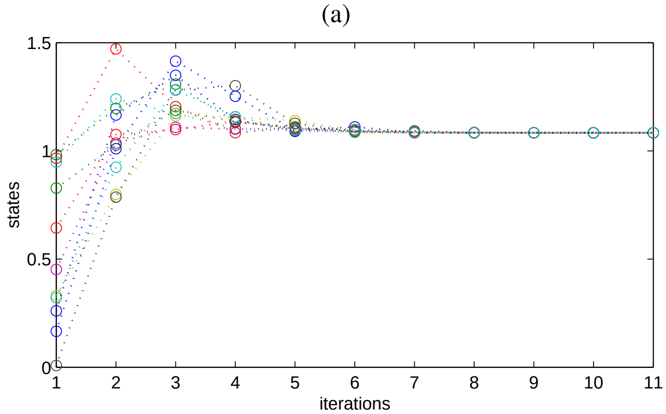
then a homogenous pattern with value of a is a solution to the 1D CVCN.

Remark 4: Note that the parameter Δ plays no role of determining the values of homogenous patterns. It follows from Theorem 4 that all 1D CVCNs have homogenous solutions. However, the stability of homogenous patterns is not guaranteed by Theorem 4.

Example 3: As an example, the evolving processes for "rule 60 1D CVCN" of 11 cells with initial conditions $x_i(-1) = 0$ and $x_i(0) \in U(0,1)$, $i = 1, \ldots, 11$, and parameters $\Delta = 0.5$, $g_I = 0.4$ and $g_D = 2.5$ are shown in Fig. 7. Toroidal boundary condition is used. In this case, the solution of Eq. (24) yields $a = 1.0833290005971588790923179511891$, which is observed from Fig. 7(b). Observe from Fig. 7(b) that all cells with different initial conditions converge asymptotically to the same value. In this case, since all cells converge to the same value, the toroidal boundary condition introduce no interference to the pattern formation.



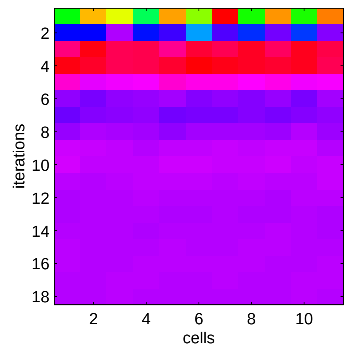
(a)



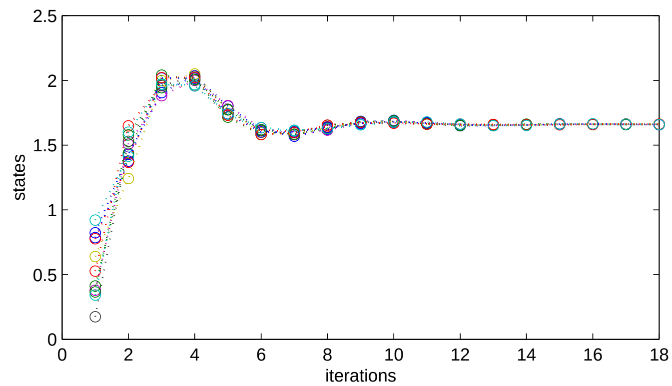
(b)

Fig. 7. Evolving process of "rule 60 1D CVCN" of 11 cells. (a) Evolving pattern of the 1D CVCN array. (b) Time series of the states of cells in the 1D CVCN array.

Example 4: As an example, the evolving processes for "rule 18 1D CVCN" of 11 cells with initial conditions $x_i(-1) = 0$ and $x_i(0) \in U(0,1)$, $i = 1, \ldots, 11$, and parameters $\Delta = 0.5$, $g_I = 0.4$ and $g_D = 2.5$ are shown in Fig. 8. In this case, the solution of Eq. (24) yields $a = 1.6592788059761405677037578778463$, which is observed from Fig. 8(b) where all cells with random initial states converge to the same value.



(a)



(b)

Fig. 8. Evolving process of "rule 18 1D CVCN" of 11 cells. (a) Evolving pattern of the 1D CVCN array. (b) Time series of the states of cells in the 1D CVCN array.

V. Flip-Flop Patterns

Two local evolving patterns, $\overline{\wp}_1$ and $\overline{\wp}_2$, for the purpose of generating flip-flop patterns in 1D CVCNs are shown in Fig. 9. The solutions defined by $\overline{\wp}_1$ and $\overline{\wp}_2$ are called *flip-flop patterns*.

A. Local Evolving Pattern $\overline{\wp}_1$

For $\overline{\wp}_1$, the off-center cells have the following verb similarities.

$$\begin{aligned} S_{oI} &\triangleq S(\mathsf{increase}, x_{i\pm1}(k)) = \frac{1}{1+e^{-(b-a)/\Delta}}, \\ S_{oD} &\triangleq S(\mathsf{decrease}, x_{i\pm1}(k)) = \frac{1}{1+e^{(b-a)/\Delta}}. \end{aligned} \tag{25}$$

The central cell has the following verb similarities.

$$\begin{aligned} S_{cI} &\triangleq S(\mathsf{increase}, x_{i}(k)) = \frac{1}{1+e^{-(b-a)/\Delta}}, \\ S_{cD} &\triangleq S(\mathsf{decrease}, x_{i}(k)) = \frac{1}{1+e^{(b-a)/\Delta}}. \end{aligned} \tag{26}$$

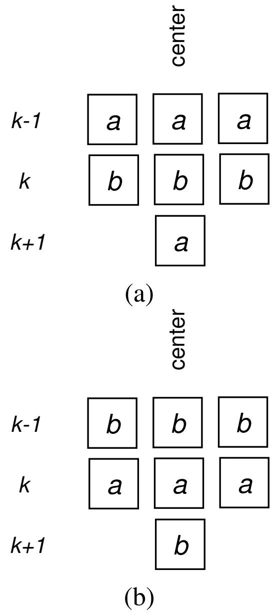


Fig. 9. Two local evolving patterns of generating flip-flop patterns in a 1D CVCN. (a) Local evolving pattern $\overline{\wp}_1$. (b) Local evolving pattern $\overline{\wp}_2$.

It follows from Eqs. (25) and (26) that

$$S_{oD} = S_{cD}, S_{oI} = S_{cI}. \tag{27}$$

Let y_c and y_o denote the output of the central cell and the off-center cells, then we have

$$\begin{aligned} y_c &\triangleq f(x_i(k)) = \frac{1}{1+e^{-b}}, \\ y_o &\triangleq f(x_{i\pm1}(k)) = \frac{1}{1+e^{-b}}. \end{aligned} \tag{28}$$

It follows from Eq. (5) that the state of the central cell in the next evolving step, $x_i(k+1) = a$, is calculated as follow.

$$\begin{aligned} \Sigma_s &= (S_{cD} + S_{cI})^3, \\ \frac{\Sigma_{sf}}{y_o} &= S_{cD}^2(g_1 S_{cD} + g_3 S_{cI}) + S_{cI}^2(g_6 S_{cD} + g_8 S_{cI}) \\ &\quad + S_{cD}S_{cI}[(g_2+g_5)S_{cD} + (g_4+g_7)S_{cI}], \\ a &= \frac{\Sigma_{sf}}{\Sigma_s}. \end{aligned} \tag{29}$$

B. Local Evolving Pattern $\overline{\wp}_2$

Based on the symmetrical correspondence between $\overline{\wp}_1$ and $\overline{\wp}_2$, it follows from Eq. (29) that

$$\begin{aligned} \widetilde{\Sigma}_s &= (\widetilde{S}_{cD} + \widetilde{S}_{cI})^3, \\ \frac{\widetilde{\Sigma}_{sf}}{\widetilde{y}_o} &= \widetilde{S}_{cD}^2(g_1 \widetilde{S}_{cD} + g_3 \widetilde{S}_{cI}) + \widetilde{S}_{cI}^2(g_6 \widetilde{S}_{cD} + g_8 \widetilde{S}_{cI}) \\ &\quad + \widetilde{S}_{cD}\widetilde{S}_{cI}[(g_2+g_5)\widetilde{S}_{cD} + (g_4+g_7)\widetilde{S}_{cI}], \\ b &= \frac{\widetilde{\Sigma}_{sf}}{\widetilde{\Sigma}_s} \end{aligned} \tag{30}$$

where

$$\begin{aligned} \widetilde{y}_o &= \frac{1}{1+e^{-a}}, \\ \widetilde{S}_{cD} &= \frac{1}{1+e^{(a-b)/\Delta}} = S_{cI}, \\ \widetilde{S}_{cI} &= \frac{1}{1+e^{-(a-b)/\Delta}} = S_{cD}. \end{aligned} \tag{31}$$

Thus it follows from Eqs. (30) and (31) that

$$\begin{aligned} \widetilde{\Sigma}_s &= (\widetilde{S}_{cD} + \widetilde{S}_{cI})^3 = \Sigma_s, \\ \frac{\widetilde{\Sigma}_{sf}}{\widetilde{y}_o} &= S_{cI}^2(g_3 S_{cD} + g_1 S_{cI}) + S_{cD}^2(g_8 S_{cD} + g_6 S_{cI}) \\ &\quad + S_{cD}S_{cI}[(g_4+g_7)S_{cD} + (g_2+g_5)S_{cI}], \\ b &= \frac{\widetilde{\Sigma}_{sf}}{\Sigma_s}. \end{aligned} \tag{32}$$

In order to find the conditions under which flip-flop patterns form, we need to solve Eqs. (29) and (32). Therefore, we proved the following theorem.

Theorem 5 (Flip-flop patterns): Given the parameters of a 1D CVCN Δ, g_I, g_D, a flip-flop pattern with two states a and b is a solution of the 1D CVCN if and only if the following conditions are satisfied.

$$\begin{aligned} S_{cI} &= \frac{1}{1+e^{-(b-a)/\Delta}}, S_{cD} = \frac{1}{1+e^{(b-a)/\Delta}}, \\ y_o &= \frac{1}{1+e^{-b}}, \widetilde{y}_o = \frac{1}{1+e^{-a}}, \\ \Sigma_s &= (S_{cD} + S_{cI})^3, \\ \frac{\Sigma_{sf}}{y_o} &= S_{cD}^2(g_1 S_{cD} + g_3 S_{cI}) + S_{cI}^2(g_6 S_{cD} + g_8 S_{cI}) \\ &\quad + S_{cD}S_{cI}[(g_2+g_5)S_{cD} + (g_4+g_7)S_{cI}], \\ \frac{\widetilde{\Sigma}_{sf}}{\widetilde{y}_o} &= S_{cD}^2(g_8 S_{cD} + g_6 S_{cI}) + S_{cI}^2(g_3 S_{cD} + g_1 S_{cI}) \\ &\quad + S_{cD}S_{cI}[(g_4+g_7)S_{cD} + (g_2+g_5)S_{cI}], \\ a &= \frac{\Sigma_{sf}}{\Sigma_s}, b = \frac{\widetilde{\Sigma}_{sf}}{\Sigma_s} \end{aligned} \tag{33}$$

where $g_i, i = 1, \ldots, 8$, are chosen from the set $\{g_I, g_D\}$ based on the local rules of the 1D CVCN.

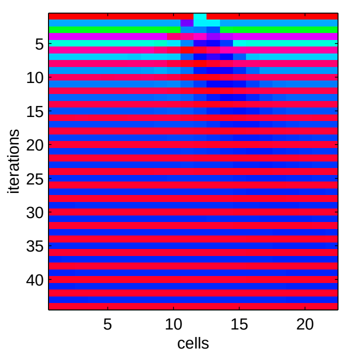
Remark 5: It will be very difficult, if not impossible, to find an explicit solution to Eq. (33) in general. In this paper, I will only study the solutions of a special case of Eq. (33).

C. Flip-Flop Patterns in "Rule 128 1D CVCN"

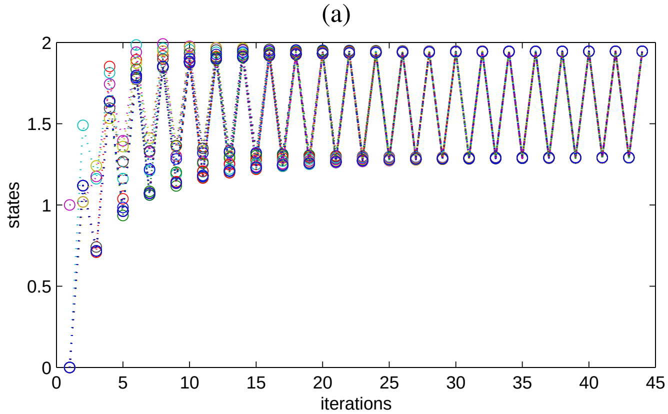
In a "rule 128 1D CVCN" we have $g_8 = g_I$ and all other g_i's have the same value of g_D, thus Eq. (33) can be rewritten as

$$
\begin{aligned}
S_{cI} &= \frac{1}{1+e^{-(b-a)/\Delta}}, S_{cD} = \frac{1}{1+e^{(b-a)/\Delta}},\\
y_o &= \frac{1}{1+e^{-b}}, \widetilde{y}_o = \frac{1}{1+e^{-a}},\\
\Sigma_s &= (S_{cD}+S_{cI})^3,\\
\frac{\Sigma_{sf}}{y_o} &= g_D(S_{cD}^2+2S_{cD}S_{cI})(S_{cD}+S_{cI})\\
&\quad +S_{cI}^2(g_D S_{cD}+g_I S_{cI}),\\
\frac{\widetilde{\Sigma}_{sf}}{\widetilde{y}_o} &= g_D(S_{cI}^2+2S_{cD}S_{cI})(S_{cD}+S_{cI})\\
&\quad +S_{cD}^2(g_I S_{cD}+g_D S_{cI}),\\
a &= \frac{\Sigma_{sf}}{\Sigma_s}, b = \frac{\widetilde{\Sigma}_{sf}}{\Sigma_s}. \qquad (34)
\end{aligned}
$$

Solve this set of equations numerically, we get the following solution $a = 1.2917$, $b = 1.9452$, $\Delta = 0.5000$, $g_D = 2.5000$, and $g_I = 0.4000$. The simulation result is shown in Fig. 10 for a 22-cell CVCN. The initial conditions are $x_i(-1) = 0$ and $x_i(0) = 0$, $i = 1, \ldots, 22$, except that the central cell has initial state $x_{12}(0) = 1$. Observe that a flip-flop pattern formed with the values in coordinate with the theoretical prediction. Figure 10(b) shows the process of cells with random initial states converges to a synchronized periodic oscillation for all cells in the 1D CVCN.



(a)



(b)

Fig. 10. Evolving process of "rule 128 1D CVCN" of 22 cells. (a) Evolving pattern of the 1D CVCN array. (b) Time series of the states of cells in the 1D CVCN array.

VI. Concluding Remarks

To analytically study the solutions of patterns generated by 1D CVCNs is extremely difficult. The author presented the conditions under which three kinds of patterns; namely, homogenous, checkerboard and flip-flop patterns, might be generated in 1D CVCNs. The author took advantage of the symmetrical structures in the local evolving patterns of 1D CVCNs such that a solution of the entire 1D CVCN can be studied locally by considering only two historical records of three cells. Simulation results show the good predicting abilities of the theorems presented. It shows that homogenous patterns are universal in the sense that all 1D CVCNs may have homogenous patterns as their solutions. Checkerboard and flip-flop patterns are not rare as well. The theory predicts that some CVCNs can have different types of checkerboard and flip-flop patterns with different values.

Numerical experiments show that the stability of the patterns predicted by the theory is different from case to case. The next research direction is to study the stability of these patterns.

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