GPS/TDOA Hybrid Location Algorithm Based on Federal Kalman Filter

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doi: 10.4156/jcit.vol5.issue7.6

Abstract

Generally, accuracy of GPS(Global Positioning System) is higher than that of cellular network. But in tall building urban area or indoor, relatively weak GPS signal makes the positioning unstable and inaccurate. TDOA(Time Difference of Arrival) is one of the most widely used positioning methods in cellular mobile communication systems, but its accuracy is not high enough to meet the growing demand. In urban where there are more base stations to support big communication capacity than in rural, the mobile terminal positioning accuracy and stability is relatively better. In order to make up for these inherent deficiency of GPS or TDOA separate positioning, this paper deduces a GPS/TDOA hybrid positioning algorithm based on federated kalman filter by giving error model, state equation and measurement equation of each local kalman filters. Simulation results show that the algorithm effectively improves the data fusion reliability and positioning accuracy.

Keywords: *Federal Kalman Filter, Hybrid Positioning, GPS, TDOA.*

1. Introduction

In recent years, communications technology, especially cellular mobile communications technology is expanding rapidly, and the number of users is also increasing rapidly. This makes mobile terminal positioning become a more urgent demand. It becomes a great business and market prospect to use the cellular network system to locate mobile terminal. The traditional positioning methods are GPS in the sky and cellular network positioning on the ground. TDOA is one of the most commonly used positioning methods based on the cellular network.

GPS method requires receiving at least four satellites signals to locate mobile terminal. In outdoor, the positioning can be very accurate for enough localization signals. But in indoor, roads surrounded dense high buildings, bridges tunnels or other areas where GPS signals are too weak to search or track, GPS positioning accuracy becomes instable or the positioning will fail.

TDOA positioning method is by means of measuring the difference of transmission times from the mobile terminal to base stations to calculate mobile terminal location. That required three or more cellular base stations whose system times must be synchronized for receiving mobile terminal signals. However, the communications systems and signal propagation environment exert a great influence on the positioning accuracy and speed.

So the upper traditional positioning technologies all have obvious deficiencies in terms of accuracy, sensitivity, speed or usability, which greatly limit the development and application of positioning service. To overcome those deficiencies, this paper constructs a hybrid positioning algorithm combining GPS and TDOA to improve positioning accuracy and availability.

2. Mathematical model of Federated Kalman Filter

The core of GPS/TDOA hybrid positioning algorithm is data fusion mode. How to fuse two kinds of positioning data is the key of high navigation and positioning accuracy. Federated kalman filter is currently the most widely used method of multi-sensor data fusion.

Federated kalman filter is actually a special kind of decentralized kalman filter. Its particularity lies in containing an information distribution process. In this process, dynamic information of master filter is allocated to each local filter. In local filter, the observed information collected through local sensor is combined with the feedback information outputted from master filter for a local estimation process. The local estimation result will be sent to master filter for further information fusion to make a global optimal estimation [1].

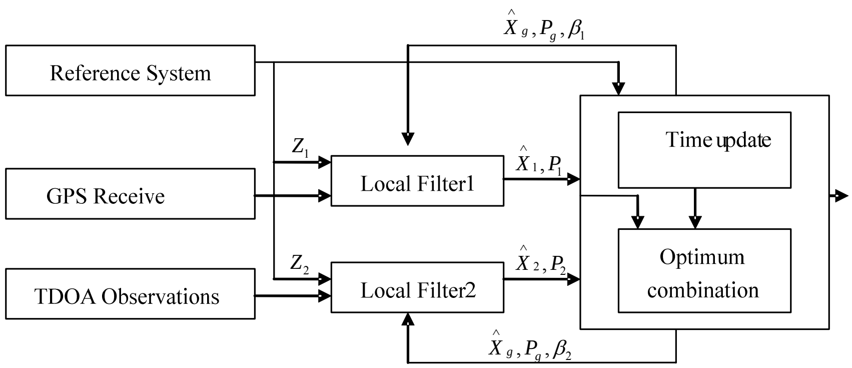

Figure 1. Federal Kalman Filter Model

Figure 1 shows the simplified structure of federal kalman filter. Here GPS receiver is used as local sensor 1, and TDOA observations receiver as local sensor 2.

In figure1, $\hat{X}_1, P_1$ and $\hat{X}_2, P_2$ are estimations of local filters. $\hat{X}_g, P_g$ are global optimal estimation outputted from master filter. The data fusion algorithm is described as the following:

$$\begin{aligned} \hat{X}_g(k) &= P_g(k)(P_1^{-1}(k)\hat{X}_1(k) + P_2^{-1}(k)\hat{X}_2(k)) \\ P_g(k) &= (P_1^{-1}(k) + P_2^{-1}(k))^{-1} \end{aligned} \tag{1}$$

β_1, β_2 are the coefficients of data allocation between the local filters. While the following conditions are met, $\hat{X}_g(k)$ is the best global optimal estimation result.

$$\begin{aligned} &X_i(k) = X_g(k), \quad \beta_1 + \beta_2 = 1 \\ &Q_i^{-1}(k) = \beta_i Q_g^{-1}(k), \quad P_i^{-1}(k) = \beta_i P_g^{-1}(k) \end{aligned} \tag{2}$$

While federal kalman filter is used in GPS/TDOA hybrid positioning, both GPS signal and TDOA observation will be used in location solution process simultaneously, so that the state of system can be amended continuously in the filtering process. The output of hybrid positioning could provide more accurate initial position and orientation information, thus when one positioning method is failure, another can maintain a high positioning accuracy in a long time [2].

3. Algorithms Design of Federal Kalman Filter of GPS/TDOA Hybrid Positioning System

3.1 Error Model, State Equation and Measurement Equation of Local Filter 1

GPS positioning errors include a variety of components, such as the distance error resulted from clock bias, speed error resulted from clock frequency drift. All of those errors show random noise characteristics clearly. It is generally considered that the GPS positioning errors in the east and north direction are respectively equivalent to a first-order Markov process. The error model is shown as below [3].

$$\tilde{\varepsilon}_e = -\tau_{\varepsilon e}\varepsilon_e + \omega_{\varepsilon e}, \quad \tilde{\varepsilon}_n = -\tau_{\varepsilon n}\varepsilon_n + \omega_{\varepsilon n} \tag{3}$$

In Eq. (3), ε_e and ε_n respectively indicate total GPS positioning error in the east and north direction respectively, $\tau_{\varepsilon e}$ and $\tau_{\varepsilon n}$ are corresponding to the reciprocal of time constant correlated with Markov processes, $\omega_{\varepsilon e}$ and $\omega_{\varepsilon n}$ indicate Gaussian white noise whose mean is zero and variance is $\sigma_{\varepsilon e}{}^2$ and $\sigma_{\varepsilon n}{}^2$.

It is generally believed that the movement model of mobile terminal is compliant with Singer model. Singer model is used in this case and the system state variables can be described as the following.

$$X = [l_e, v_e, a_e, l_n, v_n, a_n, \varepsilon_e, \varepsilon_n]^T \tag{4}$$

l_e, v_e, a_e and l_n, v_n, a_n respectively indicate mobile terminal's location, velocity, acceleration in east and north direction. ε_e and ε_n respectively indicate the total positioning errors caused by various error source along east and north direction. GPS positioning error model can be derived as below:

$$\begin{aligned} a_e &= a_e(t) + a_e(t), \quad v_e = -\tau_{ae} a_e(t) + \omega_{ae}(t) \\ a_e &= \bar{a}_n(t) + a_n(t), \quad v_e = -\tau_{an} a_n(t) + \omega_{an}(t) \end{aligned} \tag{5}$$

In Singer model, velocity of mobile terminal is also considered as a first-order Gauss-Markov process.

The system state equation can be written as:

$$\tilde{X}(t) = AX(t) + U + W(t) \tag{6}$$

The location values outputted from GPS receiver, respectively named as l_{e0} and l_{n0} in east and north, are considered as external observations. The relationship of external observations and state variables is:

$$\begin{aligned} l_{e0} &= l_e + \varepsilon_e + \tau_1 \\ l_{n0} &= l_n + \varepsilon_n + \tau_2 \end{aligned} \tag{7}$$

In Eq. (7), τ_1 and τ_2 are noise of position observation, l_{e0} and l_{n0}, outputted from GPS receiver, also are Gaussian white noise whose mean is zero and variance is σ_1^2 and σ_2^2. So the measurement equation can be written as:

$$Z(k) = \begin{bmatrix} l_{e0}(k) \\ l_{n0}(k) \end{bmatrix} = \begin{bmatrix} l_e(k) + \varepsilon_e(k) \\ l_n(k) + \varepsilon_n(k) \end{bmatrix} + \begin{bmatrix} \tau_1(k) \\ \tau_2(k) \end{bmatrix} \tag{8}$$

Eq. (8) can be abbreviated as:

$$Z(k) = H(k)X(k) + V(k) \tag{9}$$

And:

$$H(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

3.2 Error Model, State Equation and Measurement Equation of Local Filter 2

Propagation time from mobile terminal to base station i is denoted as the following

$$t_i = \overset{0}{t_i} + \overset{0}{\tau_i} + \tau_{ei} \tag{11}$$

In Eq. (11), $\overset{0}{t}_i$ indicates signal LOS propagation time, also is a linear function of distance between mobile terminal and base station. $\overset{0}{\tau}_i$ indicates system error and it is a zero-mean Gaussian random variable compliant with $\overset{0}{\tau} \sim N(0,\sigma_0^2)$, in which σ_0^2 depends on testing equipment accuracy and testing techniques. τ_{ei} indicates the additional delay error caused by NLOS, it is a random variable closely related to signal propagation environment and distance between mobile terminal and base station. By Eq. (11), we can deduce the error model of TDOA[4]:

$$\begin{aligned} t_{i,1} &= \overset{0}{t}_{i,1} + \overset{0}{\tau}_{i,1} + \tau_{ei,1} \\ u_{ei,1} &= E(\tau_{ei} - \tau_{el}) = u_{ei} - u_{el} \\ \sigma_{ei,l}^2 &= D(\tau_{ei} - \tau_{el}) = \sigma_{ei}^2 + \sigma_{el}^2 \end{aligned} \tag{12}$$

$\overset{0}{t}_{i,1}$ indicates the TDOA observed value in LOS environment. $\overset{0}{t}_{i,1}$ indicates system error and its characteristic like $\overset{0}{\tau}_i$. $\overset{0}{\tau}_{i,1}$ indicates the additional delay error caused by NLOS, its mean is $u_{ei,1}$ and variance is $\sigma_{ei,l}^2$.

TDOA errors contain not only the Gaussian distributed random variable, but also the exponential distributed random variables. In order to facilitate statistics analysis, normal distribution method is be used to fit τ_{ei} and τ_{ej}, channel environment errors which obeying exponential distribution.

The errors can be considered as part of the state variables. In order to eliminate the errors to improve the positioning accuracy, we use the second-order Markov process to fit the TDOA positioning error model.

In TDOA positioning, movement model of mobile terminal is still compliant with Singer model. System state variables can be described as the following.

$$X = [l_e, v_e, a_e, l_n, v_n, a_n, \delta_\theta, \delta_s]^T \tag{13}$$

According to the physical meaning of state variables, we can deduce the dynamic equation of terminal mobility and the state equation of kalman filter as following.

$$\begin{bmatrix} l_e(k+1) \\ v_e(k+1) \\ l_n(k+1) \\ v_n(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_e(k) \\ v_e(k) \\ l_n(k) \\ v_n(k) \end{bmatrix} + \begin{bmatrix} 0 \\ a_e \\ 0 \\ a_n \end{bmatrix}$$

$$\tilde{X}(t) = AX(t) + W(t) \tag{14}$$

Mobile terminal driving direction and distance information are considered as external observations. The relationship of external observations and state variables is:

$$\begin{aligned} \theta &= T\sqrt{v_{e(k)}^2 + v_{n(k)}^2} + \omega_1 \\ s &= \arctan\left(\frac{v_{n(k)}}{v_{e(k)}}\right) + \omega_2 \end{aligned} \tag{15}$$

ω_1 and ω_2 respectively denote total error of motion direction and distance of mobile terminal, and they are zero-mean Gaussian white noise and their variances are $\delta_\theta^2, \delta_s^2$.

Observation equation can be described as the following and it is nonlinear.

$$Z(k) = h\left[X(k)\right] + V(k) \tag{16}$$

We must dispose linearization on it when using optimal estimation method to estimate the system state variable.

3.3 Data Fusion Algorithm Based on Federated Kalman Filter

According to patterns of information distributing and feedback, federated kalman filter can be classified into four structural classes: no-reset, fusion-reset, zero-reset and re-adjust. Considering all the difficulties, such as system operation efficiency, fault tolerance and reliability and data fusion algorithm in master filter, No-reset structure is adopted in this design [5].

The master filter fuses the data outputted from local filters based on the information distribution coefficients, without information feedback to effectively prevent the error back propagation. Summary of the fusion algorithm is:

$$\hat{X}_k = \beta_{GPS} \hat{X}_{GPS(k)} + \beta_{TDOA} \hat{X}_{TDOA(k)} \tag{17}$$

In Eq. (17), β_{GPS} and β_{TDOA} respectively indicate information distribution coefficients of GPS and TDOA filter. The key of data fusion algorithm is how to choose β_{GPS} and β_{TDOA}. Here, they are chosen in accordance with the following formula:

$$\beta_{GPS} = P_{(k)} P^{-1}_{GPS(k)}\ ,\ \beta_{TDOA} = P_{(k)} P^{-1}_{TDOA(k)} \tag{18}$$

In Eq. (18),

$$\begin{gathered} P^{-1}_{(k)} = (P^{-1}_{GPS(k)} + P^{-1}_{TDOA(k)})^{-1} (P^{-1}_{GPS(k)} X_{GPS(k)} + P^{-1}_{TDOA(k)} X_{TDOA(k)}) \\ \beta_{GPS} + \beta_{TDOA} = 1 \end{gathered} \tag{19}$$

All of the above lead up to the following equation:

$$\hat{X}_g(k) = (P^{-1}_{GPS(k)} + P^{-1}_{TDOA(k)})^{-1} (P^{-1}_{GPS(k)} \hat{X}_{GPS}(k) + P^{-1}_{TDOA(k)} \hat{X}_{TDOA}(k)) \tag{20}$$

From the Eq. (20), we conclude the worse the estimation quality of state $\hat{X}_i$, the smaller the corresponding information matrix P_i^{-1}.

So in the data fusion process of GPS/TDOA hybrid positioning, if the accuracy of state estimation of one local filter is higher, its corresponding availability weight allocated by master filter is bigger. On the contrary, the availability weight should be smaller.

4. Algorithm Simulation

We choose a test area covered by at least four cellular base stations and a test mobile terminal carrying a GPS receiver. In the test process, supposing the initial velocity is 2m/s, mobile terminal is moving along a particular east-north path. The movement velocity may changes in this process, that is to say it have a disturbed acceleration which can be considered as a zero-mean random process and be described with a first-order Markov process [6].

The following diagrams respectively indicate the simulation result of positioning errors of TDOA, GPS and hybrid positioning based on federal kalman filter.

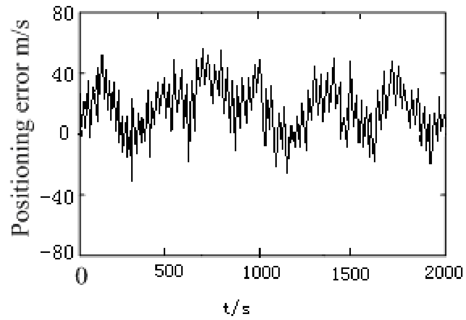

Figure 2. Total Positioning Error of TDOA in East and North

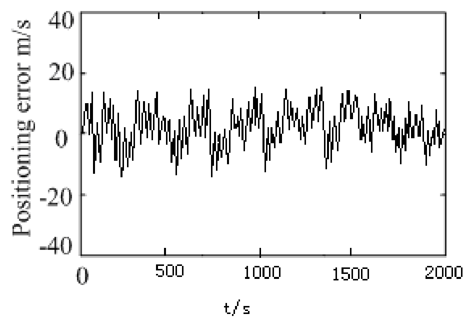

Figure 3. Total Positioning Error of GPS in East and North

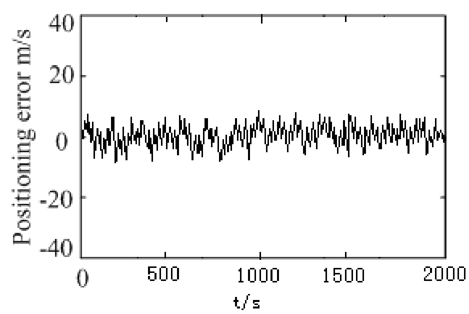

Figure 4. Total Positioning error of Hybrid Positioning in East and North

As can be seen from the figures, federal kalman filter can make the error lower than TDOA and GPS obviously.

The following table 1 shows the comparative results of error before and after filtering.

Table 1. Comparative results of error before and after filtering

		standard error before filtering	**standard error after filtering**
TDOA	in east	47.1572	22.5219
	in north	48.5148	23.1258
GPS	in east	20.5849	14.4586
	in north	21.4697	15.3741
Hybrid Positioning	in east	12.4521	11.9874
	in north	11.5412	10.1245

From the data of above table, we can see that TDOA or GPS separate positioning error is highly reduced based on kalman filter. Also the hybrid positioning error is reduced to 10-12 meters, this is a striking result. So, the simulation results show the hybrid positioning algorithms is effective.

5. Summary

In this paper, a data fusion algorithm based on federal kalman filter is designed for GPS/TDOA hybrid positioning. Theoretical analysis and experimental result show that the federal kalman filter is an effective method to deal with data fusion in mobile terminal positioning system. In order to meet the dynamic characteristics of mobile terminal movement, optimizing the filter structure and allocating coefficient to ensure performance of data fusion are still yet to be further studied [7].

6. References

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