On the Performance Analysis of Multirelay Cooperative Diversity Systems With Channel Estimation Errors

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***Abstract*—In this paper, we investigate the performance of an amplify-and-forward (AF) cooperative diversity system with multiple relays in the presence of channel estimation errors. We consider both conventional relaying (in which all relay nodes participate in the relaying phase) and opportunistic relaying (in which only a single relay is allowed to participate). We derive closed-form expressions for error probability, outage probability, and ergodic channel capacity. The derivations are confirmed through Monte Carlo simulations. We further deploy the derived expressions to obtain optimal power allocation rules for performance improvements.**

***Index Terms*—Amplify-and-forward (AF) relaying, channel estimation, cooperative diversity, fading channels, performance analysis.**

I. Introduction

COOPERATIVE diversity has emerged as a powerful technique to realize spatial diversity advantages in a distributed manner [1], [2]. Taking advantage of the broadcast nature of the radio-frequency transmission, cooperative diversity creates a virtual antenna among the nodes that are willing to share their resources. In the literature, various cooperation protocols, coupled with different relaying modes, have been proposed [3], [4]. A commonly employed form of cooperation protocol is the so-called receive diversity (RD) protocol, which is also known as orthogonal relaying [1]. In the first phase of this protocol, the source node broadcasts to the destination and the relay nodes. In the second phase, the source stops transmission, and the relay nodes forward their received signals within the first phase to the destination node. Either repetition coding in orthogonal time slots or orthogonal space-time block coding can be used among relay nodes in the second phase. The RD protocol realizes a maximum degree of broadcasting and exhibits no receive collision. Non-orthogonal cooperation protocols such as the transmit diversity (TD) protocol and the simplified TD protocol have been further proposed [5], [6] and allow receive collision. In an effort to address the spectral inefficiency of the conventional cooperation protocols, which assume the participation of all relays, relay selection has been proposed to improve the throughput [7]. Based on a predetermined criteria, e.g., the signal-to-noise ratio (SNR), the "best" relay is selected, and only a single relay is allowed to participate in the second phase.

A common assumption in the earlier literature on cooperative diversity is the availability of perfect channel state information (CSI) at the receiver. In coherent detection, the fading channel coefficients need to first be accurately estimated during the training period, and then, these imperfect estimates are used in the detection process at the destination node. Relay nodes operating in decode-and-forward (DF) mode need the CSI of the source-to-relay channel for their own decoding process. In amplify-and-forward (AF) relaying, relay nodes might need CSI for appropriately scaling the received signal to satisfy relay power constraints. The effect of channel estimation on the overall performance of cooperative systems is therefore critical.

Several researchers have investigated the performance of cooperative systems with imperfect channel estimation [8]–[10]; however, these works are mainly limited to simulation studies. A few exceptions are [11]–[14], which aim to analytically study the impact of channel estimation for single-relay cooperative systems. In [11], Mheidat and Uysal derived a pairwise error probability (PEP) expression for the TD protocol with AF relaying. In [12], Patel and Stüber obtained an approximate error rate performance expression for the RD protocol, assuming binary phase-shift keying (PSK). In [13], Wu and Patzold derived symbol error rate expressions for the RD protocol, assuming M-ary PSK and M-ary quadrature amplitude modulation (QAM). In [14], Gedik and Uysal considered both RD and TD protocols and presented PEP analysis for mismatched-coherent and partially coherent receivers under different degrees of CSI. While the aforementioned works assume single-relay scenarios, Han *et al.* [15] considered multirelay AF cooperative systems and derived the average bit error rate (BER) for the RD protocol, assuming all relays' participation in the relaying phase.

Manuscript received June 19, 2010; revised January 5, 2011; accepted February 3, 2011. Date of publication March 3, 2011; date of current version June 20, 2011. The work of O. Amin was supported by the Egyptian Higher Education Ministry. This paper was presented in part at the IEEE International Conference on Communication, Capetown, South Africa, May 23–27, 2010. The review of this paper was coordinated by Prof. Y. Ma.

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Digital Object Identifier 10.1109/TVT.2011.2121926

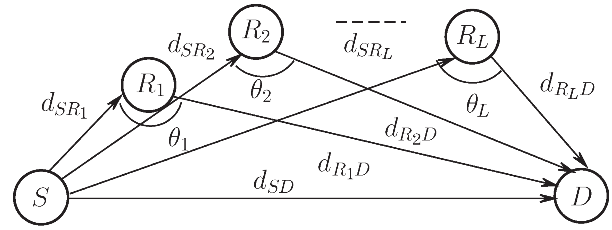

Fig. 1. Cooperative system model.

In this paper, we present a framework for the performance analysis of multirelay AF cooperative systems in the presence of channel estimation errors. We consider both *conventional relaying* (in which all relay nodes participate in the relaying phase) and *opportunistic relaying* (in which only a single relay is allowed to participate). We derive expressions for BER, outage probability, and channel capacity, demonstrating the effect of channel estimation on the performance.

The rest of this paper is organized as follows: In Section II, we present the system and channel models. In Section III, we derive the instantaneous effective SNR, which will be used for the derivations of performance measures under consideration in Section IV. The derived closed-form expressions can be used for system optimization. As an example, we use the derived BER expression to optimize power allocation in Section V. Section VI concludes this paper.

Notation: $|.|$ denotes the absolute value. $\mathbb{E}(.)$ is the expectation operator.

II. System and Channel Models

As shown in Fig. 1, we consider a multirelay network in which a source node (S) and a destination node (D) communicate through a number of relay nodes denoted by R_i, $i = 1, 2, \cdots, L$. The nodes are assumed to be located in a 2-D plane where d_{SD}, d_{SR_i}, and d_{R_iD} denote the distances of source-to-destination ($\mathrm{S} \rightarrow \mathrm{D}$), source-to- relay ($\mathrm{S} \rightarrow \mathrm{R}_i$), and relay-to-destination ($\mathrm{R}_i \rightarrow \mathrm{D}$) links, respectively. The angle between lines representing $\mathrm{S} \rightarrow \mathrm{R}_i$ and $\mathrm{R}_i \rightarrow \mathrm{D}$ links is θ_i. The complex fading coefficients between the source and the ith relay and between the ith relay and the destination are h_i and g_i, $i = 1, \ldots, L$, respectively. They are modeled as complex Gaussian with zero mean and variances of $\sigma^2_{h_i}$, $\sigma^2_{g_i}$, leading to Rayleigh fading. The fading coefficient for the direct link is denoted as g_0, and its amplitude distribution also follows Rayleigh distribution. Finally, in our analysis, we assume frequency-flat and time-flat (nonselective fading) channel models.

To take into account the relays' location, we also consider the long-term path loss. The path loss is inversely proportional to d^α, where d is the distance between nodes and α is the path loss exponent. By normalizing the path loss terms with respect to that of direct $\mathrm{S} \rightarrow \mathrm{D}$ link, the so-called geometrical gains [5] can be defined as $G_{SR_i} = (d_{SD}/d_{SR_i})^\alpha$ and $G_{R_iD} = (d_{SD}/d_{R_iD})^\alpha$, $i = 1, 2 \ldots L$.

We assume the RD cooperation protocol. In the first time slot (i.e., broadcasting phase), the source broadcasts its signal. The destination and all L relays receive faded noisy versions of the source signal. In the relaying phase, the source is silent. If *conventional relaying* is employed, all relay nodes participate in the relaying phase and forward the scaled versions of their received signals to the destination node in orthogonal time slots. The destination node combines all signals received through indirect/direct links using maximal ratio combining. On the other hand, if *opportunistic relaying* is employed, the destination combines only the "best" indirect link (which yields the highest SNR at the destination) and the direct link.

Mathematically speaking, the received signals from the source at the destination and the ith relay can, respectively, be written as

$$y_{SD} = \sqrt{E_S} g_0 x + n_{SD} \tag{1}$$

$$y_{SR_i} = \sqrt{G_{SR_i} E_S} h_i x + n_{SR_i} \qquad i = 1, 2, \ldots, L \tag{2}$$

where E_S is the source signal energy, and x is either an M-PSK or an M-QAM modulated signal with unit energy. In the second time slot, the relay normalizes the received signal (to comply with power constraints) and transmits the resulting signal x_{R_i} to the destination. The received signal at the destination from the ith relay is given by

$$y_{R_iD} = \sqrt{G_{R_iD} E_i} g_i x_{R_i} + n_{R_iD}, \qquad i = 1, 2, \ldots, L \tag{3}$$

where E_i is the relay signal energy, and x_{R_i} is given by

$$x_{R_i} = \sqrt{\frac{1}{E_S G_{SR_i} |\hat{h}_i|^2 + N_0}} y_{SR_i}. \tag{4}$$

In (1)–(4), n_{SD}, n_{SR_i}, and n_{R_iD} are zero-mean complex Gaussian noise terms with variance $N_0/2$ per dimension. We assume that total power consumption in the network is given by E_T. In the case of conventional relaying, the power of source and each of L relays is given by $E_T/(L+1)$, i.e., $E_S = E_i = E_T/(L+1)$. On the other hand, in the case of *opportunistic relaying*, the source and the selected relay are each assigned $E_T/2$.

The destination is assumed to have access only to imperfect channel estimates, which will be used at the combiner. Let the channel estimate of the $\mathrm{S} \rightarrow \mathrm{R}_i$ link be $\hat{h}_i$ for the ith relay link. We assume that h_i and $\hat{h}_i$ are jointly ergodic and stationary Gaussian processes. We can write

$$h_i = \hat{h}_i + e_{h_i} \tag{5}$$

where e_{h_i} denotes the channel estimation error, which is modeled as complex Gaussian with zero mean and variance $\sigma^2_{e_{h_i}}$. Assuming a linear-minimum-mean-square-error estimator, the variance of channel estimation error is $\sigma^2_{e_{h_i}} = \mathbb{E}(|h_i|^2) - \mathbb{E}(|\hat{h}_i|^2) = 1/(N_p \bar{\gamma}_{SR_i,t} + 1)$ [16]. Here, N_p is the number of pilot symbols, $\bar{\gamma}_{SR_i,t} = \mathbb{E}(\gamma_{SR_i,t}) = G_{SR_i} E_{S,t}/N_0$ is the average SNR of pilot symbols for $\mathrm{S} \rightarrow \mathrm{R}_i$ link, and $E_{S,t}$ is the source power for training period. Similarly, variances for channel estimates of $\mathrm{S} \rightarrow \mathrm{D}$ and $\mathrm{R}_i \rightarrow \mathrm{D}$ links are given by $\sigma^2_{e_{g_i}} = 1/(N_p \bar{\gamma}_{R_iD,t} + 1)$, where $\bar{\gamma}_{SD,t} = \mathbb{E}(\gamma_{SD,t}) = E_{S,t}/N_0$, and

$\bar{\gamma}_{R_iD,t} = \mathbb{E}(\gamma_{R_iD,t}) = G_{R_iD}E_{i,t}/N_0$ are the average SNRs of pilot symbols for S $\rightarrow$ D and $\mathrm{R}_i \rightarrow$ D links, with $E_{i,t}$ denoting the ith relay power for training period. The signal at the combiner's output can be written as [17]

$$\lambda = \begin{cases} W_{SD}y_{SD} + \sum_{i=1}^{L} W_i y_{R_iD}, & \text{Conventional} \\ W_{SD}y_{SD} + W_{i_{\text{sel}}} y_{R_{i_{\text{sel}}}D}, & \text{Opportunistic} \end{cases} \tag{6}$$

where the combiner coefficients are given by $W_{SD} = \sqrt{E_S}\hat{g}_0^*/N_0$ and $W_i = \sqrt{G_{R_iD}E_i}\hat{h}_i^*\hat{g}_i^*/(|\hat{h}_i|N_{\text{tot}})$, with $N_{\text{tot}} = N_0 + E_iG_{R_iD}|\hat{g}_i|^2N_0/(E_SG_{SR_i}|\hat{h}_i|^2 + N_0)$. In (6), i_{sel} denotes the index for the selected relay, which yields the highest effective SNR in S $\rightarrow \mathrm{R}_i \rightarrow$ D.

III. Statistical Characterization of Instantaneous Effective Signal-to-Noise Ratio

In this section, we will determine the probability density function (pdf) of the *effective* SNR (i.e., incorporating the effects of channel estimation) at the destination. Such a statistical characterization is essential for the derivation of performance measures under consideration.

A. Instantaneous Effective SNR

From (6), it can be noticed that the combiners output consist of two terms coming from either direct or indirect links. $\lambda_{SD} \stackrel{def}{=} W_{SD}y_{SD}$ is the term contributed by the S $\rightarrow$ D link. It can be expanded as

$$\lambda_{SD} = \underbrace{\frac{E_S|\hat{g}_0|^2}{N_0}x}_{\text{signal}} + \underbrace{\frac{\sqrt{E_s}\hat{g}_0^*}{N_0}\left(\sqrt{E_S}e_{g_0}x + n_{S,D}\right)}_{\text{effective noise}}. \tag{7}$$

The instantaneous effective SNR for this term can be therefore written as

$$\gamma_{SD} = \frac{\hat{\gamma}_{S,D}}{E_S\sigma^2_{e_{g_0}}/N_0 + 1} \tag{8}$$

where $\hat{\gamma}_{SD} = E_S|\hat{g}_0|^2/N_0$ is the estimated instantaneous SNR of the S $\rightarrow$ D link. The average effective SNR can be further written as

$$\bar{\gamma}_{SD} = \mathbb{E}(\gamma_{SD}) = \frac{E_S\left(\mathbb{E}\left(|g_0|^2\right) - \sigma^2_{e_{g_0}}\right)}{\left(E_S\sigma^2_{e_{g_0}} + N_0\right)}. \tag{9}$$

Similarly, it can be noticed from (5) that $\lambda_{SR_iD} \stackrel{def}{=} W_iy_{R_iD}$ is the term contributed by the indirect link via the ith relay node. It can be expanded as in (10), shown at the bottom of the page, where $G_i = E_SG_{SR_i}|h_i|^2 + N_0$. This lets us to write the corresponding instantaneous effective SNR as in (11), shown at the bottom of the page.

Further defining $\hat{\gamma}_{SR_i} = G_{SR_i}E_S|\hat{h}_i|^2/N_0$ and $\hat{\gamma}_{R_iD} = G_{R_iD}E_i|\hat{g}_i|^2/N_0$ as the estimated instantaneous SNR of the S $\rightarrow \mathrm{R}_i$ and $\mathrm{R}_i \rightarrow$ D links, (11) can be rewritten as in (12), shown at the bottom of the page.

Equation (12) can be simplified as

$$\gamma_{SR_iD} = \frac{\gamma_{SR_i}\gamma_{R_iD}}{\gamma_{SR_i} + \gamma_{R_iD} + \Omega_i} \tag{13}$$

where $\gamma_{SR_i} = \hat{\gamma}_{SR_i}/(G_{SR_i}E_S\sigma^2_{e_{h_i}}/N_0 + 1)$ and $\gamma_{R_iD} = \hat{\gamma}_{R_iD}/(G_{R_iD}E_i\sigma^2_{e_{g_i}}/N_0 + 1)$ are the instantaneous effective SNRs of S $\rightarrow \mathrm{R}_i$ and $\mathrm{R}_i \rightarrow$ D links, respectively, and Ω_i is given by

$$\Omega_i = \frac{G_{SR_i}G_{R_iD}E_SE_i\sigma^2_{e_{h_i}}\sigma^2_{e_{g_i}} + G_{R_iD}E_i\sigma^2_{e_{g_i}}N_0 + N_0^2}{\left(G_{SR_i}E_S\sigma^2_{e_{h_i}} + N_0\right)\left(G_{R_iD}E_i\sigma^2_{e_{g_i}} + N_0\right)}. \tag{14}$$

$$\lambda_{SR_iD} = \underbrace{\frac{\sqrt{G_{R_iD}E_i}\hat{h}_i^*\hat{g}_e^*}{|\hat{h}_i|N_{\text{tot}}}\sqrt{\frac{G_{SR_i}E_S}{G_i}}h_ig_i}_{\text{signal}} + \underbrace{\sqrt{\frac{G_{R_iD}E_i}{G_i}}\frac{\hat{h}_i^*\hat{g}_i^*}{|\hat{h}_i|N_{\text{tot}}}\left(\sqrt{G_{SR_i}E_S}(\hat{h}_ie_{g_i} + \hat{g}_ie_{h_i})x + (\hat{g}_i + e_{g_i})n_{S,R_i} + \sqrt{G_i}n_{R_iD}\right)}_{\text{effective noise}} \tag{10}$$

$$\gamma_{SR_iD} = \frac{G_{SR_i}G_{R_iD}E_SE_i|\hat{h}_i|^2|\hat{g}_i|^2}{G_{SR_i}E_S|\hat{h}_i|^2\left(G_{R_iD}E_i\sigma^2_{e_{g_i}} + N_0\right) + G_{R_iD}E_i|\hat{g}_i|^2\left(G_{SR_i}E_S\sigma^2_{e_{h_i}} + N_0\right) + G_{SR_i}G_{R_iD}E_SE_i\sigma^2_{e_{h_i}}\sigma^2_{e_{g_i}} + G_{R_iD}E_i\sigma^2_{e_{g_i}}N_0 + N_0^2} \tag{11}$$

$$\gamma_{SR_iD} = \frac{\hat{\gamma}_{SR_i}\hat{\gamma}_{R_iD}}{\hat{\gamma}_{SR_i}\left(\frac{G_{R_iD}E_i\sigma^2_{e_{g_i}}}{N_0} + 1\right) + \hat{\gamma}_{R_iD}\left(\frac{G_{SR_i}E_S\sigma^2_{e_{h_i}}}{N_0} + 1\right) + \frac{G_{SR_i}G_{R_iD}E_SE_i\sigma^2_{e_{h_i}}\sigma^2_{e_{g_i}}}{N_0^2} + \frac{G_{R_iD}E_i\sigma^2_{e_{g_i}}}{N_0} + 1} \tag{12}$$

Finally, using (8) and (13), the effective total output SNR can be obtained as

$$\gamma_{\text{tot}} = \begin{cases} \gamma_{SD} + \sum_{i=1}^{L} \gamma_{SR_iD}, & \text{Conventional} \\ \gamma_{SD} + \max_{i\in L} \gamma_{SR_iD}, & \text{Opportunistic.} \end{cases} \quad (15)$$

To simplify the ensuing performance analysis, we employ a tight upper bound on γ_{SR_iD}, which is given as $\gamma_{SR_iD} \le \gamma_i = \min(\gamma_{SR_i}, \gamma_{R_iD})$ [17]. This yields bounds on (15) as

$$\gamma_{ub}^{\text{Con}} = \gamma_{SD} + \sum_{i=1}^{L} \min(\gamma_{SR_i}, \gamma_{R_iD}) \quad (16)$$

for conventional relaying. Similarly, we have

$$\gamma_{ub}^{\text{Opp}} = \gamma_{SD} + \max_{i\in L} \min\left(\gamma_{SR_i}, \gamma_{R_iD}\right) \quad (17)$$

for opportunistic relaying.

B. PDFs of the Instantaneous Effective SNR

For conventional relaying, the pdf of (16) can be derived, following the steps in [18], which yields the following:

$$f_{\gamma_{ub}^{\text{Con}}}(\gamma) = \frac{\beta_{S,D}}{\bar{\gamma}_{SD}} \exp\left(\frac{-\gamma}{\bar{\gamma}_{SD}}\right) + \sum_{i=1}^{L} \frac{\beta_i}{\bar{\gamma}_i} \exp\left(\frac{-\gamma}{\bar{\gamma}_i}\right) \quad (18)$$

where

$$\bar{\gamma}_i = \frac{\bar{\gamma}_{SR_i}\bar{\gamma}_{R_iD}}{\bar{\gamma}_{SR_i} + \bar{\gamma}_{R_iD}} \quad (19)$$

$$\bar{\gamma}_{SR_i} = \mathbb{E}\left(\gamma_{SR_i}\right) = \frac{G_{SR_i}E_S\left(\mathbb{E}\left(|h_i|^2\right) - \sigma^2_{e_{h_i}}\right)}{\left(G_{SR_i}E_S\sigma^2_{e_{h_i}} + N_0\right)} \quad (20)$$

$$\bar{\gamma}_{R_iD} = \mathbb{E}\left(\gamma_{R_iD}\right) = \frac{G_{R_iD}E_S\left(\mathbb{E}\left(|g_i|^2\right) - \sigma^2_{e_{g_i}}\right)}{\left(G_{R_iD}E_S\sigma^2_{e_{g_i}} + N_0\right)} \quad (21)$$

$$\beta_{SD} = \prod_{i=1}^{L}\left(1 - \frac{\bar{\gamma}_i}{\bar{\gamma}_{SD}}\right)^{-1} \quad (22)$$

$$\beta_i = \left(1 - \frac{\bar{\gamma}_{SD}}{\bar{\gamma}_i}\right)^{-1} \prod_{k=1,k\neq i}^{L}\left(1 - \frac{\bar{\gamma}_k}{\bar{\gamma}_i}\right)^{-1}. \quad (23)$$

For opportunistic relaying, the pdf of (17) is obtained as

$$f_{\gamma_{ub}^{\text{Opp}}}(\gamma) = \sum_{i=1}^{L}(-1)^{i+1} \sum_{K_1=1}^{L-i+1} \sum_{K_2=K_1+1}^{L-i+2} \cdots \sum_{K_i=K_{i-1}+1}^{L} \frac{1}{1/\xi_i - \bar{\gamma}_{SD}}\left(\exp(-\gamma\xi) - \exp\left(-\frac{\gamma}{\bar{\gamma}_{SD}}\right)\right) \quad (24)$$

where $\xi_i = \sum_{j=1}^{i}(1/\bar{\gamma}_{K_j})$. Considering independent identical distribution (i.i.d) channels, i.e., $\bar{\gamma}_{SD} = \bar{\gamma}_{SR_i} = \bar{\gamma}_{R_iD} = \bar{\gamma}$, (24) reduces to

$$f_{\gamma_{ub,iid}}^{\text{Opp}}(\gamma) = \sum_{q=1}^{M}\binom{M}{q}\frac{(-1)^{q-1}}{\bar{\gamma}\left(1 - 1/(2q)\right)} \times \left[\exp\left(-\frac{\gamma}{\bar{\gamma}}\right) - \exp\left(-\frac{2q\gamma}{\bar{\gamma}}\right)\right]. \quad (25)$$

IV. Performance Analysis

A. Error Probability

The average error probability over frequency-flat fading channels can be found by averaging the conditional error probability in additive white Gaussian noise $P_b(e/\gamma_{ub})$. Mathematically, $P_b(e)$ is given by [19]

$$P_b(e) = \int_0^{\infty} P_b(e/\gamma_{ub}) f_{\gamma_{ub}}(\gamma_{ub}) d\gamma_{ub} \quad (26)$$

where $f_{\gamma_{ub}}(\gamma_{ub})$ is given by either (18) and (24) for the schemes under consideration. Note that, for several Gray bit-mapped constellations, $P_b(e/\gamma_{ub})$ is in the form of $a\text{erfc}(\sqrt{b\gamma_{ub}})$, with $\text{erfc}(x)$ being the complementing error function [19], and a, b are constants, depending on the type of modulation (e.g., BPSK: $a = 0.5$, and $b = 1$, QPSK: $a = 0.5$, and $b = 0.5$). By substituting (18) into (26) and solving the integration, a lower bound on the error probability for conventional relaying can be obtained as

$$P_b^{\text{Con}}(e) = a\beta_{SD}\left(1 - \sqrt{\frac{b\bar{\gamma}_{SD}}{1 + b\bar{\gamma}_{SD}}}\right) + a\sum_{i=1}^{L}\beta_i\left(1 - \sqrt{\frac{b\bar{\gamma}_i}{1 + b\bar{\gamma}_i}}\right). \quad (27)$$

Similarly, by substituting (24) into (26) and performing the integration operation, we find a lower bound on the error probability for opportunistic relaying as

$$P_b^{\text{Opp}}(e) = a\sum_{i=1}^{L}(-1)^{i+1} \sum_{K_1=1}^{L-i+1} \sum_{K_2=K_1+1}^{L-i+2} \cdots \sum_{K_i=K_{i-1}+1}^{L} \times 1 - \frac{1}{1 - \bar{\gamma}_{SD}\xi_i}\sqrt{\frac{b}{\xi_i + b}} + \frac{\bar{\gamma}_{SD}\xi_i}{1 - \bar{\gamma}_{SD}\xi_i}\sqrt{\frac{b\bar{\gamma}_{SD}}{1 + b\bar{\gamma}_{SD}}}. \quad (28)$$

As a sanity check, it can be noted that, in the perfect CSI case (i.e., $\sigma^2_{e_{h_i}} = \sigma^2_{e_{g_i}} = 0$), (28) reduces to the expression given by [20, eq. (14)].

To provide some further insight into the performance, we now consider the asymptotically high-SNR case. Following [21], we can obtain an approximation to (27) as follows:

$$P_b^{\text{Con}}(e) \approx \frac{aC(L)}{b^{L+1}} \frac{1}{\bar{\gamma}_{SD}} \prod_{i=1}^{L}\left(\frac{1}{\bar{\gamma}_{SR_i}} + \frac{1}{\bar{\gamma}_{R_iD}}\right) \quad (29)$$

where $C(L) = \prod_{i=1}^{L+1}(2i-1)/(2(L+1)!)$ is constant, which depends on the number of relay nodes L. It is observed from (29) that a full diversity order of $L+1$ is achieved as in the case of the perfect channel estimation. For $\sigma^2_{e_{h_i}} = \sigma^2_{e_{g_i}} = 0$,

(29) reduces to [22, eq. (21)], which is reported for perfect CSI case.

An asymptotical expression can be obtained for opportunistic relaying as (see the Appendix for the proof)

$$P_b^{\mathrm{Opp}}(e) \approx \frac{aU(L)}{b^{L+1}} \frac{1}{\bar{\gamma}_{SD}} \prod_{i=1}^{L} \left(\frac{1}{\bar{\gamma}_{SR_i}} + \frac{1}{\bar{\gamma}_{R_i D}} \right) \tag{30}$$

where $U(L) = L\prod_{i=1}^{L+1}(2i-1)/(2(L+1)!)$. A comparison of (29) and (30) reveals that both expressions have identical forms where the only difference comes from the coefficient term given by $U(L) = LC(L)$. This will eventually result in a horizontal shift between the performance of two schemes, both of which are able to achieve the full diversity.

B. Outage Probability

The mutual information between the source and the destination for conventional and opportunistic relaying can be written as

$$I_{\mathrm{Con}} = \frac{1}{L+1} \log_2\left(1 + \gamma_{ub}^{\mathrm{Con}}\right) \tag{31}$$

$$I_{\mathrm{Opp}} = \frac{1}{2} \log_2\left(1 + \gamma_{ub}^{\mathrm{Opp}}\right). \tag{32}$$

The reason for different factors in (31) and (32) is that we need $L+1$ time slots (or orthogonal channels) to transmit the data in conventional relaying and only two times slots in the case of opportunistic relaying.

The outage probability is defined as the probability that the mutual information falls below the required rate r. For conventional relaying, it is given by

$$P_{\mathrm{out}}^{\mathrm{Con}} = \Pr(I_{\mathrm{Reg}} \le r) = \Pr\left(\gamma_{ub}^{\mathrm{Con}} \le \gamma_t^{\mathrm{Con}} = 2^{(L+1)r} - 1\right). \tag{33}$$

It can be noted that outage probability is actually the cumulative CDF of $\gamma_{\mathrm{out}}^{\mathrm{Conv}}$ evaluated at $2^{(L+1)r} - 1$. Using (18), we can obtain a lower bound on the outage probability as

$$P_{\mathrm{out}}^{\mathrm{Con}} = \beta_{SD}\left(1 - \exp\left(-\frac{\gamma_t^{\mathrm{Con}}}{\bar{\gamma}_{SD}}\right)\right) + \sum_{i=1}^{L} \beta_i \left(1 - \exp\left(-\frac{\gamma_t^{\mathrm{Con}}}{\bar{\gamma}_i}\right)\right) \tag{34}$$

where β_{SD} and β_i are earlier defined in (22) and (23). Asymptotically, it can be approximated as

$$P_{\mathrm{out}}^{\mathrm{Con}}(e) \approx \frac{\left[2^{(L+1)r} - 1\right]^{L+1}}{\bar{\gamma}_{SD}(L+1)!} \prod_{i=1}^{L} \left(\frac{1}{\bar{\gamma}_{SR_i}} + \frac{1}{\bar{\gamma}_{R_i D}} \right). \tag{35}$$

Similarly, for opportunistic relaying, we obtain

$$\begin{aligned} P_{\mathrm{out}}^{\mathrm{Opp}} &= \Pr(I_{\mathrm{Opp}} \le r) = \Pr\left(\gamma_{ub}^{\mathrm{Opp}} \le \gamma_t^{\mathrm{Opp}} = 2^{2r} - 1\right) \\ &= \sum_{i=1}^{L} (-1)^{i+1} \sum_{K_1=1}^{L-i+1} \sum_{K_2=K_1+1}^{L-i+2} \cdots \sum_{K_i=K_{i-1}+1}^{L} 1 \\ &\quad - \frac{1}{1 - \bar{\gamma}_{SD}\xi_i} \exp\left(-\gamma_t^{\mathrm{Opp}} \xi_i\right) \\ &\quad + \frac{\bar{\gamma}_{S,D}\xi_i}{1 - \bar{\gamma}_{SD}\xi_i} \exp\left(-\frac{\gamma_t^{\mathrm{Opp}}}{\bar{\gamma}_{SD}}\right). \end{aligned} \tag{36}$$

Asymptotically, this yields

$$P_{\mathrm{out}}^{\mathrm{Opp}}(e) \approx \frac{[2^{2r} - 1]^{L+1}}{(L+1)} \frac{1}{\bar{\gamma}_{SD}} \prod_{i=1}^{L} \left(\frac{1}{\bar{\gamma}_{SR_i}} + \frac{1}{\bar{\gamma}_{R_i D}} \right). \tag{37}$$

C. Ergodic Channel Capacity

For conventional relaying, the average channel capacity is given by

$$\bar{C}^{\mathrm{Con}} = \frac{W}{L+1} \int_0^\infty \log_2(1+\gamma) f_{\gamma_{ub}^{\mathrm{Con}}}(\gamma) d\gamma. \tag{38}$$

By substituting (18) into (38), an upper bound on channel capacity can be obtained in a closed form as

$$\bar{C}^{\mathrm{Con}} = \frac{W}{(L+1)\ln(2)} \left[\beta_{SD} \exp\left(\bar{\gamma}_{SD}^{-1}\right) \mathrm{E}_1\left(\bar{\gamma}_{SD}^{-1}\right) + \sum_{i=1}^{L} \beta_i \exp\left(\bar{\gamma}_i^{-1}\right) \mathrm{E}_1\left(\bar{\gamma}_i^{-1}\right) \right]. \tag{39}$$

where $\mathrm{E}_1(x)$ is the exponential integral defined as $\mathrm{E}_1(x) = \int_x^\infty \exp(-t)/t dt$ [23]. It can be readily checked that, for $\sigma_{e_{h_i}}^2 = \sigma_{e_{g_i}}^2 = 0$, (39) reduces to [24, eq. (32)] reported for perfect CSI scenario.

For opportunistic relaying, we have

$$\bar{C}^{\mathrm{Opp}} = \frac{W}{2} \int_0^\infty \log_2(1+\gamma) f_{\gamma_{ub}^{\mathrm{Opp}}}(\gamma) d\gamma. \tag{40}$$

For this case, we obtain an upper bound as

$$\bar{C}^{\mathrm{Opp}} = \frac{W}{2\ln(2)} \sum_{i=1}^{L} (-1)^{i+1} \sum_{K_1=1}^{L-i+1} \cdots \sum_{K_i=K_{i-1}+1}^{L} \times \frac{\exp(\xi_i) E_1(\xi_i) - \bar{\gamma}_{SD}\xi_i \exp(1/\bar{\gamma}_{SD}) E_1(1/\bar{\gamma}_{SD})}{1 - \bar{\gamma}_{SD}\xi_i}. \tag{41}$$

For $\sigma_{e_{h_i}}^2 = \sigma_{e_{g_i}}^2 = 0$, (41) reduces to [25, eq. (24)] reported for perfect CSI scenario.

V. Adaptive Power Allocation (APA)

The derived closed-form expressions can be used for system optimization. In this section, as an example, we consider the derived error probability expression and obtain the optimal power distribution rules between the source and the relay nodes to minimize it.

A. Conventional Relaying

For conventional relaying, the optimization problem can be formulated as

$$\min_{\text{s.t. } E_S+\sum_{i=1}^{L} E_i=E_T} P^{\text{Con}}(e). \tag{42}$$

The power to be allocated to source and relay nodes can be written in terms of total power as $E_S = \rho_0 E_T$, $E_i = \rho_i E_T$, $i = 1, 2, \ldots, L$. Using (29), (42) can be rewritten as

$$\min_{\text{s.t. } E_s+\sum_{i=1}^{L} E_i=E_T} \left(\epsilon_0 + \frac{\alpha_0}{\rho_0 E_T}\right) \prod_{i=1}^{L} \left(\epsilon_i + \frac{\alpha_i}{\rho_0 E_T} + \frac{\beta_i}{\rho_i E_T}\right) \tag{43}$$

where ϵ_0, ϵ_i, α_0, and α_i are given by

$$\epsilon_0 = \frac{\sigma_{e_{g_0}} a C(L)}{b^{L+1}\left(\mathbb{E}|g_0|^2 - \sigma_{e_{g_0}}^2\right)} \tag{44}$$

$$\epsilon_i = \frac{\sigma_{e_{h_i}}^2}{\mathbb{E}\left(|h_i|^2\right) - \sigma_{e_{h_i}}^2} + \frac{\sigma_{e_{g_i}}^2}{\mathbb{E}\left(|g_i|^2\right) - \sigma_{e_{g_i}}^2} \tag{45}$$

$$\alpha_0 = \frac{a N_0 C(L)}{b^{L+1}\left(\mathbb{E}\left(|g_0|^2\right) - \sigma_{e_{g_0}}^2\right)} \tag{46}$$

$$\alpha_i = \frac{N_0}{G_{SR_i}\left(\mathbb{E}\left(|h_i|^2\right) - \sigma_{e_{h_i}}^2\right)} \tag{47}$$

$$\beta_i = \frac{N_0}{G_{R_iD}\left(\mathbb{E}\left(|g_i|^2\right) - \sigma_{e_{g_i}}^2\right)}. \tag{48}$$

Unfortunately, (43) cannot be solved in a closed form; however, ρ_i, $i = 1, 2, \ldots, L$ can be calculated through numerical optimization techniques. For perfect channel estimation (i.e., $\sigma_{e_{h_i}}^2 = \sigma_{e_{g_i}}^2 = 0, i = 0, \ldots, L$), (43), however, simplifies to

$$\min_{\text{s.t. } E_s+\sum_{i=1}^{L} E_i=E_T} \frac{aC(L)N_0^{L+1}}{b^{L+1}\rho_0 E_T^{L+1}} \prod_{i=1}^{L} \left(\frac{1}{G_{SR_i}\rho_0} + \frac{1}{G_{R_iD}\rho_i}\right). \tag{49}$$

Using Lagrange multiplier method, we can obtain optimum values for ρ_i, $i = 1, 2, \ldots, L$ as

$$\rho_i = \frac{-G_{SR_i}\rho_0 + \sqrt{G_{SR_i}^2\rho_0^2 + 4G_{R_iD}G_{SR_i}\rho_0/(L+1)}}{2G_{R_iD}} \tag{50}$$

where ρ_0 can be computed by solving

$$\sum_{i=1}^{L} \sqrt{\left(\frac{G_{SR_i}}{G_{R_iD}}\right)^2 \rho_0^2 + \frac{4}{(L+1)}\left(\frac{G_{SR_i}}{G_{R_iD}}\right)\rho_0} - 2(1-\rho_0) - \rho_0 \sum_{i=1}^{L} \frac{G_{SR_i}}{G_{R_iD}} = 0 \tag{51}$$

using any minimization bracketing method [26].

In Table I, we present the optimal values of ρ_i, $i = 0, 1, \ldots, L$ for both perfect and imperfect channel estimation cases. We assume 4-PSK modulation, $\alpha = 2$, and $N_p = 1$ and consider the following scenarios based on the number of relays and relay geometry.

1) $L = 1, \theta_1 = \pi$, and $G_{SR_1}/G_{R_1D} = -30$ dB.
2) $L = 2, \theta_1 = \pi, \theta_2 = \pi/3, G_{SR_1}/G_{R_1D} = -30$ dB, and $G_{SR_2}/G_{R_2D} = 0$ dB.
3) $L = 2, \theta_1 = \pi, \theta_2 = \pi/3, \theta_3 = \pi/6, G_{SR_1}/G_{R_1D} = -30$ dB, $G_{SR_2}/G_{R_2D} = 0$ dB, and $G_{SR_3}/G_{R_3D} = 30$ dB.
4) i.i.d case: $G_{SD} = G_{SR_i} = G_{R_iD} = 1$, and $\theta_i = \pi/3$, $i = 1, 2, 3$.

B. Opportunistic Relaying

For opportunistic relaying, the optimization problem can be formulated as

$$\min_{\text{s.t. } E_s+E_{R_{\text{sel}}}=E_T} P^{\text{Opp}}(e). \tag{52}$$

The power to be allocated to source and selected relay can be written in terms of the total power as $E_S = \rho E_T$ and $E_i = (1-\rho)E_T$. Using (30), (52) can be rewritten as

$$\min_{s.t.\ E_s+E_{R_{\text{sel}}}=E_T} L\left(\epsilon_0 + \frac{\alpha_0}{\rho E_T}\right) \prod_{i=1}^{L} \left(\epsilon_i + \frac{\alpha_i}{\rho E_T} + \frac{\beta_i}{(1-\rho)E_T}\right). \tag{53}$$

The optimal ρ parameter value can be calculated from equating $\partial P^{\text{Opp}}(e)/\partial\rho$ to zero, which yields

$$\left(\epsilon_0 + \frac{\alpha_0}{\rho E_T}\right) \sum_{k=1}^{L} \frac{\frac{\beta_k}{(1-\rho)^2 E_T} - \frac{\alpha_k}{\rho^2 E_T}}{\epsilon_k + \frac{\alpha_k}{\rho E_T} + \frac{\beta_k}{(1-\rho)E_T}} - \frac{\alpha_0}{\rho^2 E_T} = 0. \tag{54}$$

In the case of perfect channel estimation, (54) simplifies to

$$(L+1)\rho - 1 - \sum_{k=1}^{L} \frac{(1-\rho)G_{R_iD}\mathbb{E}\left(|g_i|^2\right)}{\rho G_{SR_i}\mathbb{E}\left(|h_i|^2\right) + (1-\rho)G_{R_iD}\mathbb{E}\left(|g_i|^2\right)} = 0. \tag{55}$$

Further imposing the assumption of identical channels (i.e., $\mathbb{E}(|g_i|^2) = \mathbb{E}(|h_i|^2)$, and $G_{SD} = G_{SR_i} = G_{R_iD} = 1$), optimal ρ is found as

$$\rho_{\text{iden}} = \frac{L+1}{2L+1}. \tag{56}$$

Optimal values of ρ for aforementioned scenarios with perfect and imperfect channel estimation can be found in Table II. From Tables I and II, we observe that power allocation values slightly vary with SNR for the case of imperfect channel estimation (as a result of the dependency of channel estimation error on SNR). On the other hand, they are independent of SNR in the case of perfect channel estimation.

VI. Simulation Results and Discussion

In this section, we provide numerical results to confirm the derived analytical expressions. We assume the same system

TABLE I
OPTIMAL POWER ALLOCATION VALUES FOR CONVENTIONAL RELAYING. (a) IMPERFECT CHANNEL ESTIMATION. (b) IMPERFECT CHANNEL ESTIMATION AND I.I.D CASE. (c) PERFECT CHANNEL ESTIMATION

(a)

SNR (dB)	L=1 $\rho_0,\ \rho_1$	L=2 $\rho_0,\ \rho_1, \rho_2$	L=3 $\rho_0,\ \rho_1, \rho_2, \rho_3$
0	0.9871–0.0129	0.7209–0.0076–0.2715	0.5564–0.0625–0.0625–0.3186
4	0.9835–0.0165	0.7107–0.0100–0.2793	0.5576–0.0625–0.0625–0.3174
8	0.9809–0.0191	0.7040–0.0119–0.2841	0.6250–0.1250–0.1250–0.1250
12	0.9794–0.0206	0.7006–0.0131–0.2863	0.5002–0.0091–0.2031–0.2876
16	0.9787–0.0213	0.6991–0.0137–0.2872	0.4983–0.0097–0.2038–0.2882
20	0.9784–0.0216	0.6984–0.0140–0.2876	0.4975–0.0099–0.2041–0.2884
24	0.9783–0.0217	0.6982–0.0141–0.2877	0.4972–0.0100–0.2042–0.2885
28	0.9782–0.0218	0.6981–0.0141–0.2878	0.4971–0.0101–0.2043–0.2886

(b)

SNR (dB)	L=1 (i.i.d.) $\rho_0,\ \rho_1$	L=2 (i.i.d.) $\rho_0,\ \rho_1, \rho_2$	L=3 (i.i.d.) $\rho_0,\ \rho_1, \rho_2, \rho_3$
0	0.6558–0.3442	0.5245–0.2378–0.2378	0.4511–0.1830–0.1830–0.1830
4	0.6518–0.3482	0.5196–0.2402–0.2402	0.4635–0.1793–0.1793–0.1780
8	0.6493–0.3507	0.5162–0.2419–0.2419	0.6250–0.1250–0.1250–0.1250
12	0.6481–0.3519	0.5143–0.2428–0.2428	0.4402–0.1866–0.1866–0.1866
16	0.6475–0.3525	0.5135–0.2432–0.2432	0.4392–0.1869–0.1869–0.1869
20	0.6473–0.3527	0.5132–0.2434–0.2434	0.4388–0.1871–0.1871–0.1871
24	0.6472–0.3528	0.5130–0.2435–0.2435	0.4386–0.1871–0.1871–0.1871

(c)

L=1	L=1 (i.i.d.)	L=2	L=2 (i.i.d.)	L=3	L=3 (i.i.d.)
0.9784	0.6667	0.7357	0.5352	0.5519	0.4606
0.0216	0.3333	0.0153	0.2324	0.0115	0.1798
		0.2490	0.2324	0.1868	0.1798
				0.2499	0.1798

TABLE II
OPTIMAL ρ VALUES FOR OPPORTUNISTIC RELAYING ($L = 1, 2, 3$). (a) IMPERFECT CHANNEL ESTIMATION. (b) PERFECT CHANNEL ESTIMATION

(a)

SNR (dB)	L=1 $\rho_0,\ \rho_1$	L=2 $\rho_0,\ \rho_1, \rho_2$	L=3 $\rho_0,\ \rho_1, \rho_2, \rho_3$
0	0.9871–0.0129	0.7209–0.0076–0.2715	0.5564–0.0625–0.0625–0.3186
4	0.9835–0.0165	0.7107–0.0100–0.2793	0.5576–0.0625–0.0625–0.3174
8	0.9809–0.0191	0.7040–0.0119–0.2841	0.6250–0.1250–0.1250–0.1250
12	0.9794–0.0206	0.7006–0.0131–0.2863	0.5002–0.0091–0.2031–0.2876
16	0.9787–0.0213	0.6991–0.0137–0.2872	0.4983–0.0097–0.2038–0.2882
20	0.9784–0.0216	0.6984–0.0140–0.2876	0.4975–0.0099–0.2041–0.2884
24	0.9783–0.0217	0.6982–0.0141–0.2877	0.4972–0.0100–0.2042–0.2885
28	0.9782–0.0218	0.6981–0.0141–0.2878	0.4971–0.0101–0.2043–0.2886

(b)

L=1	L=1 (i.i.d.)	L=2	L=2 (i.i.d.)	L=3	L=3 (i.i.d.)
0.9784	0.6667	0.7493	0.6000	0.5998	0.5714

parameters for non-identical channels considered in Tables I and II.

In Fig. 2, we present the error probability performance of conventional relaying for $L = 1, 2$ and 3. In this figure, the exact error probability (obtained through simulations) is plotted along with the lower bound given by (27) and the asymptotical expression by (29). The derived lower bound lies within 0.3 dB of the exact performance. The asymptotical expression further provides an excellent match in moderate-high SNR region ($>$ 15 dB). The performance with perfect CSI is also included as a benchmark. It is observed that, due to imperfect channel estimation, the performance is degraded by $\sim$3 dB. However, the slope of the performance curves and, therefore, the diversity order remain the same for both perfect and imperfect CSI.

In Fig. 3, we present the error probability performance of opportunistic relaying for $L = 1, 2$ and 3. In this figure, the lower bound given by (28) and the asymptotical expression given by (30) are provided along with the exact (simulated) expression. Similar to Fig. 2, the lower bound is within 0.3 dB of the exact performance, and the asymptotical expression provides tight results for SNR higher than 15 dB. We further observe that the performance is degraded by 3, 4, and 4.7 dB as a result of the imperfect channel estimation.

From the comparison of Figs. 2 and 3, we observe that opportunistic relaying outperforms conventional relaying, confirming the earlier reported results for the case of perfect channel estimation (see, e.g., [27]–[30]). It is also important to emphasize that imperfect channel estimates affect the opportunistic relaying more than the conventional relaying. This is due to the

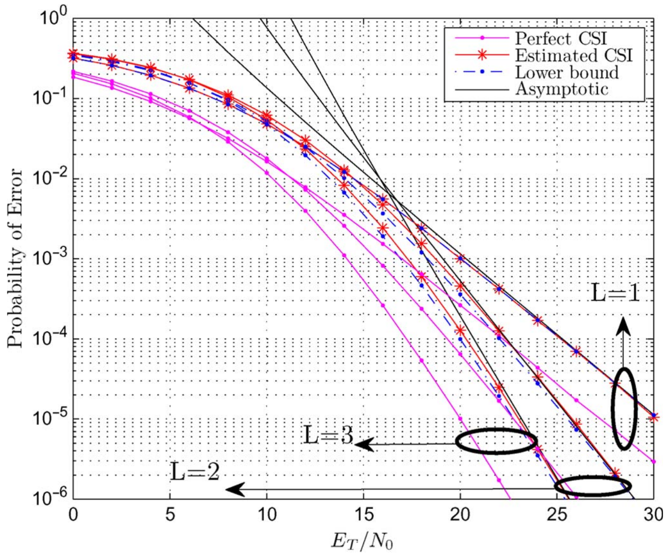

Fig. 2. Probability of error for conventional relaying.

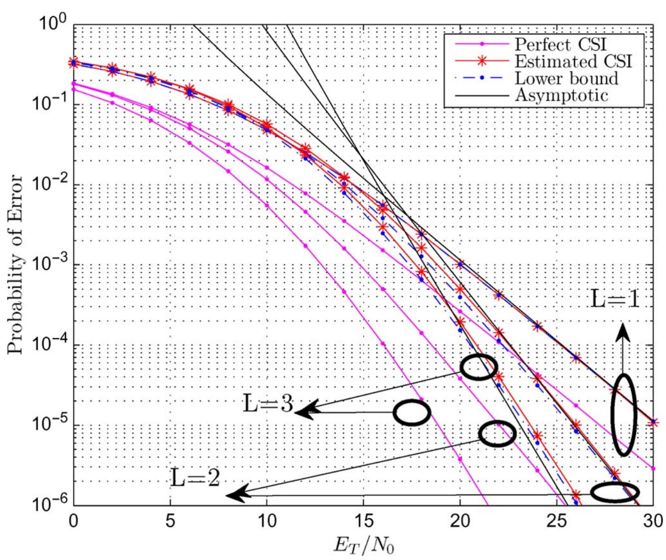

Fig. 3. Probability of error for opportunistic relaying.

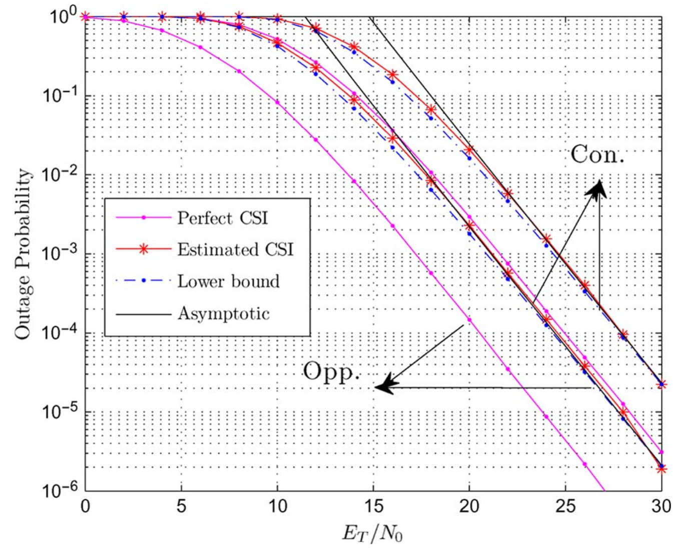

Fig. 4. Outage probability for conventional and opportunistic relaying with $L = 2$.

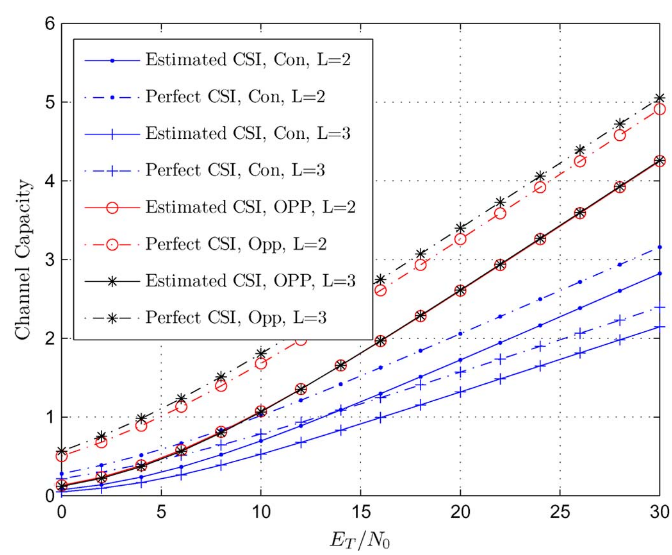

Fig. 5. Channel capacity for conventional and opportunistic relaying.

effect of imperfect channel estimation on both relay selection and the MRC combining processes in opportunistic relaying. On the other hand, it has impact only on the MRC combiner in the case of conventional relaying.

In Fig. 4, we return our attention to outage probability. Lower bounds on the outage probability for conventional and opportunistic relaying, respectively, given by (34) and (36), along with asymptotical expressions given by (35) and (37), are illustrated, assuming two relays. As expected from the similarity of corresponding expressions, tightness is similar to that observed in error probability.

In Fig. 5, we illustrate the upper bounds on ergodic channel capacity for conventional and opportunistic relaying, assuming $L = 2$ and 3. The degradation in the capacity for conventional relaying due to imperfect channel estimation is 3 dB for both $L = 2$ and 3. The degradation in opportunistic relaying climbs to 4 and 4.9 dB for $L = 2$ and 3, respectively. Despite the additional degradation, opportunistic relaying outperforms the conventional counterpart for the same number of relays. It should be noted that, in the case of opportunistic relaying, as L increases, the end-to-end SNR improves, and the average channel capacity of the opportunistic relaying increases. On the other hand, in conventional relaying, increasing L has an opposing factor to SNR improvement, because it uses $L + 1$ time slots and consequently reduces the channel capacity by a factor of $L + 1$. The second factor has the dominant effect, which degrades the channel capacity performance with increasing number of relays, as observed in Fig. 5.

Finally, in Figs. 6 and 7, we investigate the effect of power allocation on the error rate performance. For conventional relaying, it is observed from Fig. 6 that optimized power allocation provides a performance improvement of 1.15 dB for $L = 1$ at error probability of 10^{-4}. The improvement decreases to 1 and 0.77 dB for $L = 2$ and $L = 3$, respectively. On the other hand, for opportunistic relaying, it is observed from Fig. 7 that the performance improvement is 1.15 dB for $L = 1$. The

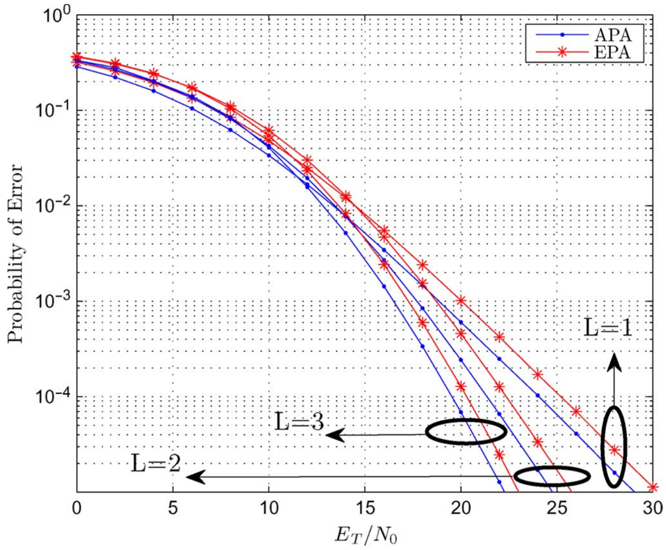

Fig. 6. Effect of power allocation on the performance of conventional relaying.

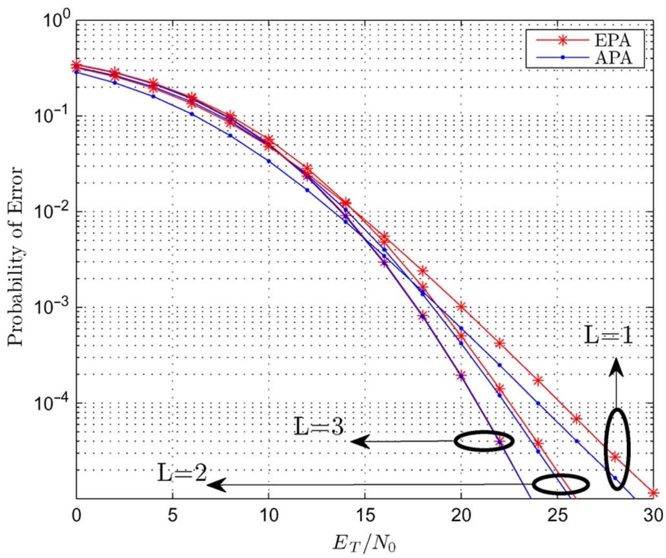

Fig. 7. Effect of power allocation on the performance of opportunistic relaying.

improvement gets smaller for increasing number of relays. It should be further emphasized that imperfect channel estimation reduces the improvement obtained through APA over equal power allocation (EPA) when compared with the perfect channel estimation case. For example, assume conventional relaying and $L = 2$. For the perfect channel estimation case, APA brings a gain of 2.5 dB with respect to EPA. When imperfect channel estimation is considered, this gain reduces to 1 dB. This is due to the fact that the imperfect estimation reduces the average received energy at both the relay and the destination nodes, i.e., $\hat{\gamma}_{SR_i}$ and $\hat{\gamma}_{R_i,D}$.

VII. Conclusion

In this paper, we have derived error probability, outage probability, and ergodic channel capacity expressions for a multirelay AF scheme in the presence of imperfect channel estimation. Utilizing a tight bound on the effective SNR, we have derived closed-form expressions for both conventional and opportunistic relaying. As demonstrated throughout this paper, our results generalize the existing results in the literature as to illustrate the effect of channel estimation errors. High-SNR analysis has also been made to provide insight into the achievable diversity orders. Derived results have been validated through extensive Monte Carlo simulation results. We have further used the derived error probability expressions to optimize the power allocation, which yields performance improvements of about 1 dB.

Appendix

In this Appendix, we present the proof of (30). First note that the nth differentiation of γ_i's pdf at zero, $(\partial^n f_{\gamma_i}/\partial\gamma^n)(0)$, has a limited nonzero value. Using the chain rule and the fact that $\Pr[\gamma_i \leq 0] = 0$, it can be shown that the pdf of the random variable $X = \max_{i\in L}(\gamma_i)$ can be written as

$$\frac{\partial^{L-1} f_X}{dx^{L-1}}(0) = L\prod_{i=1}^{L} f_{\gamma_i}(0) \tag{57}$$

where $f_X(x)$ is the pdf of the random variable X. Further, note that all the derivatives of the pdf of $\gamma_{\text{tot}}^{\text{Opp}}$, i.e., $f_{\gamma_{\text{tot}}^{\text{Opp}}}(\gamma)$, evaluated at zero up to order $L-1$ are zero, and the Lth-order derivative is given by

$$\frac{\partial^L f_{\gamma_{\text{tot}}^{\text{Opp}}}}{\partial\gamma^L}(0) = f_{\gamma_{SD}}(0)\frac{\partial^{L-1} f_X}{\partial\gamma^{L-1}}(0). \tag{58}$$

Since we are integrating at the value around zero, the initial value theorem of Laplace transforms can be used to find (58). Noting that $\gamma_{\text{tot}}^{\text{Opp}}$ is the sum of two independent random variables, we have $M_{\gamma_{\text{tot}}^{\text{Opp}}}(s) = M_{\gamma_{SD}}(S)M_X(s)$, where $M_{\gamma_{SD}}(s)$ and $M_X(s)$ are the Laplace transforms of $f_{\gamma_{SD}}(\gamma)$ and $f_X(\gamma)$, respectively. Using the initial value theorem, we obtain

$$\frac{\partial^L f_{\gamma_{\text{tot}}^{\text{Opp}}}}{\partial\gamma^L}(0) = \lim_{s\to\infty} s^{L+1} M_{\gamma_{\text{tot}}^{\text{Opp}}}(s). \tag{59}$$

On the other hand, we have $\lim_{s\to\infty} sM_{\gamma_{SD}}(s) = f_{\gamma_{SD}}(0)$, which yields

$$\lim_{s\to\infty} s^L M_X(s) = L\prod_{i=1}^{L} f_{\gamma_i}(0). \tag{60}$$

To find out the asymptotic behavior of the average error probability, we use the approximate expression given in [21]. Since the derivatives of $f_{\gamma_{\text{tot}}^{\text{Opp}}}(\gamma)$ up to the kth order are null at $\gamma = 0$, the approximate average error probability using the McLaurin series can be expressed as

$$f_{\gamma_{\text{tot}}^{\text{Opp}}}(\gamma) \approx \frac{\prod_{i=1}^{k+1}(2i-1)}{2(k+1)b^{k+1}}\frac{1}{k1}\frac{\partial^k f_{\gamma_{\text{tot}}^{\text{Opp}}}}{\partial\gamma^k}(0) \tag{61}$$

where $\partial^k f_\gamma(0)/\partial\gamma^k$ is the kth-order derivative of the pdf, and the derivatives of $f_\gamma(\gamma)$ up to order $k+1$ are zero. Applying (57) and (61), the approximate error probability can be written as in (30).

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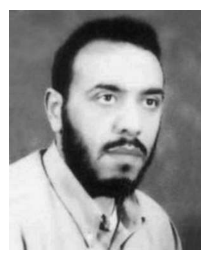

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