# Asymptotically Fair Scheduling on Fading Channels


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***Abstract*— The problem of scheduling data for a DS-CDMA downlink, over a fading channel is considered. We show that high speed downlink data is most efficiently supported by time division of the channel, by letting only one single user in each cell access the channel at a time. For efficient resource utilization, some form of scheduling is required to determine which user should transmit at any given instant of time. Scheduling algorithms of different adaptation rate are suggested and compared. To avoid unfair performance, an algorithm that schedules a user to transmit when its channel is the "relatively best" is analyzed. We show that this algorithm asymptotically provides the same fairness as a round robin scheduler, but the throughput is significantly improved. For a Rayleigh fading channel, we show that the scheduling gain is in fact equal to the gain of a selection diversity scheme.**


## I. Introduction

Forthcoming wireless systems are supposed to support a variety of services, requiring different Quality of Service (QoS). Unlike voice services, which exhibit a stable stream of data flow, having tight delay requirements, packet data is more delay tolerant and suitable for nonreal time services. Since these services may require significantly larger amount of data to be delivered, along with the low reliability and time varying capacity of the wireless channel, efficient bandwidth allocation is a priority.

To satisfy increasing demands for high-speed packet data, emerging standards for next-generation DS-CDMA systems are currently extended to cope with higher data rates. Both the outlined HDR mode [1] and HSDPA [2] consider a time divided downlink, in some cases offering peak rates around 10 Mbps. Transmitting the users' data in a one-by-one fashion can be regarded as providing an average data rate, as opposed to the continuous rate of voice. It has been shown that this multiple access concept has merits in terms of substantially increased capacity [3]. As the larger delay tolerance provides more freedom to adaptively plan the transmissions, it is anticipated that the channel fading can be easier to cope with, or even be in favor. The concept of "water filling" in time is well known and gives a possibility for unequal service and higher capacity for users with favorable channel states. To reap the benefits of the asynchronous time variations, some form of channel estimation is needed as input to the scheduler. Rapid estimation and feedback of channel states have been suggested for HDR and HSDPA [1, 2], for supporting fast scheduling, ARQ and link adaptation. Given the estimated channel quality, a problem for the scheduler is to determine which user gives the "best" utilization.

Although always scheduling the user with the highest link quality may maximize capacity, it can result in too unfair a performance among the users. Such fairness issues have been studied for many type of systems, not only wireless. In [4], Kelly extended the classical max-min fairness, where user throughputs are made as equal as possible subject to channel capacities, to an alternative proportional fairness criterion, aiming to maximize a sum of utilities. Other performance criteria can be found in [5–8]. In [9–11], asymptotical analysis of scheduling was studied. In [9, 10], the purpose was to schedule the users to get access to the channel the same asymptotical fraction of time but taking advantage of instantaneous channel variations. The algorithm was based on the criterion in [4]. It was shown that the performance gain increased with the variance of the channel fading. In [11], the objective was to maximize the minimum long-run expected average throughput, subject to required throughput targets. In this work we will, similarly to [9, 10], consider an asymptotical analysis, where fairness accounts for providing certain channel access time fractions among the users. That is, equal expected throughput is not necessarily guaranteed, rather the access to the channel.

We consider two classes of schedulers; slow (nonadaptive) and fast (adaptive). The slow scheduler base its decisions on slow channel variations, like attenuation and shadowing, or does not adapt to the channel at all. Clearly, this form is robust and requires small overhead, to the price of performance. A simple type of channel nonadaptive scheduling, known to allocate the access times fairly, is the Round Robin (RR) scheduler. However, although the time allocation of RR is fair, the performance is mostly not very encouraging in fading environments. We suggest a channel adaptive scheduler, taking into account the different channel conditions, where the user with the "Relatively Best" (RB) channel state is scheduled. The underlying principle is that a user which has a good quality relative to its


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expected quality, transmits. The gain from this is that the same asymptotical fairness as RR is maintained but all users will experience higher throughput. In particular for Rayleigh fading, the scheduling gain of RB over RR is shown to be the same as the gain of a selection diversity scheme.

In Section II, the assumptions and system model are given. Two classes of schedulers are analyzed in Section III and numerically verified in Section IV. The conclusions end the paper in Section V.

## II. System Model

Consider a single cell in a DS-CDMA system with a spreading bandwidth $W$. The required $E_b/I_0$ is set to $\Gamma_i$ for any user $1 \leq i \leq N$. The data buffers are considered to be full all the time and the downlink transmission power is fixed at $P$. With $I_i + \nu_i$ denoting the intercell plus background interference level and $g_i(t)$ the time variant link gain, we define the instantaneous channel state as $\xi_i(t) = g_i(t)P/(I_i + \nu_i)$. The instantaneous transmission rate of user $i$ is assumed to follow

$$r_i(t) = \frac{W}{\Gamma_i} \frac{\xi_i(t)\Delta_i}{\theta_i(1-\Delta_i)\xi_i(t)+1} \tag{1}$$

where $\Delta_i \in [0,1]$ denotes the fraction of the transmission power allocated to user $i$ and $\theta_i \in (0,1]$ denotes the orthogonality factor, which is assumed to be fixed.

If one-by-one transmission is employed, all the transmission power in the cell can be given to the user when it transmits. Consequently, $\Delta_i = 1$ and the instantaneous transmission rate of a transmitting user becomes

$$r_i(t) = \frac{W}{\Gamma_i}\xi_i(t). \tag{2}$$

We assume that $\xi_i(t)$ can be accurately estimated and and that it is a wide sense stationary and ergodic stochastic process for all users $i$. The channel state of each user has a mean $\mathbb{E}_{\xi_i}[\xi_i(t)] = \bar{\xi}_i$ for all $t$. From the these assumptions, we have the asymptotical data rate

$$R_i \stackrel{\text{def}}{=} \lim_{T\to\infty} \frac{1}{T}\int_0^T r_i(t)dt = \frac{W}{\Gamma_i}\bar{\xi}_i. \tag{3}$$

where the limit denotes convergence almost surely. For the numerical evaluation, the channel fading is assumed to follow the noncentral $\chi^2$ distribution with two degrees of freedom,

$$f_{\xi_i}(x) = \frac{K+1}{\bar{\xi}_i} e^{-K-\frac{(K+1)x}{\bar{\xi}_i}} I_0\left(\sqrt{\frac{4K(K+1)x}{\bar{\xi}_i}}\right) \tag{4}$$

where $K$ is the Rice factor [12]. The Rice factor is the ratio of the power in the specular and scattered components. When $K = 0$, the channel exhibits Rayleigh fading and when $K \to \infty$ the channel is not fading at all.

## III. One-by-one scheduling

In [3], it was shown by using snapshot analysis that one-by-one scheduling outperforms simultaneous transmission in terms of throughput for nonfading channels. The next proposition shows that this property holds even if we let the link gains be time varying and stochastic. This justifies our choice to focus on scheduling one-by-one transmissions. Let $\phi_i$ be the fraction of time user $i$ transmits, where $0 \leq \phi_i \leq 1$ and $\sum_{i=1}^N \phi_i = 1$.

**Proposition 1.** *The total throughput $\sum_{i=1}^N R_i$ of a one-by-one transmission scheme is greater than that of simultaneous transmission.*

**Proof.** Consider the asymptotical fraction of time $\phi_i$ that is required to obtain the same average data rate as using simultaneous transmission:

$$R_i = \phi_i \frac{W}{\Gamma_i}\bar{\xi}_i = \frac{W}{\Gamma_i}\mathbb{E}_{\xi_i}\left[\frac{\xi_i(t)\Delta_i}{\theta_i(1-\Delta_i)\xi_i(t)+1}\right] \tag{5}$$

Since the channel state $\xi_i(t)$ is a stationary ergodic process, it follows that the channel at certain time instant $t$ is determined by a density function $f_{\xi_i}(x)$, which is independent of $t$. Furthermore, it is reasonable to assume that $\xi_i(t) \geq 0$ and the probability that $\xi_i = 0$ is strictly less than one. That is, $f_{\xi_i}(x) = 0$ for all $\xi_i < 0$ and $f_{\xi_i}(x) \neq \delta(\xi_i)$ where $\delta(x)$ denotes the Dirac delta function. It follows for any $i$ with $\Delta_i < 1$ that

$$\mathbb{E}_{\xi_i}\left[\frac{\xi_i(t)\Delta_i}{\theta_i(1-\Delta_i)\xi_i(t)+1}\right] = \int_0^\infty \frac{\xi_i\Delta_i}{\theta_i(1-\Delta_i)\xi_i+1} f_{\xi_i}(\xi_i)d\xi_i$$
$$< \int_0^\infty \xi_i\Delta_i f_{\xi_i}(\xi_i)d\xi_i = \Delta_i\bar{\xi}_i.$$

Hence, we get

$$R_i = \phi_i\frac{W}{\Gamma_i}\bar{\xi}_i < \frac{W}{\Gamma_i}\Delta_i\bar{\xi}_i.$$

Solving for the sum of $\phi_i$ yields

$$\sum_{i=1}^N \phi_i < \sum_{i=1}^N \Delta_i = 1.$$

The excess time fraction $1 - \sum_{i=1}^N \phi_i > 0$ could be utilized to transmit more data. Consequently, the throughput of a one-by-one scheduling scheme is greater than that of simultaneous transmission. □

Since (1) is concave in $\xi_i$, Jensen's inequality provides a simple upper bound of the throughput, which alternatively proves Proposition 1,

$$R_i \leq \frac{W}{\Gamma_i}\frac{\bar{\xi}_i\Delta_i}{\theta_i(1-\Delta_i)\bar{\xi}_i+1}. \tag{6}$$

The bound becomes an equality for $\Delta_i = 1$. Now, consider two different classes of schedulers.

### A. Slow Scheduling

A simple and robust type of scheduling is to ignore the instantaneous channel variations and determine a transmission schedule in advance. Thus, the input parameters to the scheduler are measured on long term basis and do not adapt to the fast variations. As the time average of $\xi_i(t)$ equals its ensemble mean $\bar{\xi}_i$, the expected throughput of a user will asymptotically be

$$R_i = \phi_i \frac{W}{\Gamma_i} \bar{\xi}_i. \tag{7}$$

A resource fair algorithm is round robin (RR), where

$$\phi_i = \frac{1}{N} \tag{8}$$

and the channel access is evenly allocated over all users. A fully performance fair allocation would give the same throughput rather than time fractions to all users. This can be achieved by an equal throughput (ET) scheduler

$$\phi_i = \frac{\Gamma_i/\bar{\xi}_i}{\sum_{j=1}^{N} \frac{\Gamma_j}{\bar{\xi}_j}} \tag{9}$$

which gives each user a throughput of $R_0 = W/\sum_{i=1}^{N} \frac{\Gamma_i}{\bar{\xi}_i}$. The next result shows that the resource fair strategy results in better system throughput.

**Proposition 2.** *The total throughput $\sum_{i=1}^{N} R_i$ of RR is greater than that of ET.*

**Proof.** By observing that $f(x) = x + x^{-1} \geq 2$ for nonnegative $x$, we can write

$$\sum_{i=1}^{N} x_i \sum_{i=1}^{N} \frac{1}{x_i} \geq N + 2\sum_{i=1}^{N}(N-i) = N^2$$

which means that

$$N \leq \sum_{i=1}^{N} \frac{\Gamma_i}{\bar{\xi}_i} \frac{1}{W} \sum_{i=1}^{N} \frac{1}{N} \frac{W}{\Gamma_i} \bar{\xi}_i$$

or equivalently

$$NR_0 = \frac{NW}{\sum_{i=1}^{N} \frac{\Gamma_i}{\bar{\xi}_i}} \leq \sum_{i=1}^{N} \frac{1}{N} \frac{W}{\Gamma_i} \bar{\xi}_i.$$

□

Another strategy for allocating the $\phi_i$ could be by normalized expected channel states. Such a fractionally fair (FF) assignment yields

$$\phi_i = \frac{\bar{\xi}_i/\Gamma_i}{\sum_{j=1}^{N} \frac{\bar{\xi}_j}{\Gamma_j}}. \tag{10}$$

This causes users which have $\frac{\bar{\xi}_i}{\Gamma_i} < \frac{1}{N}\sum_{j=1}^{N} \frac{\bar{\xi}_j}{\Gamma_j}$ to obtain lower throughput than for RR, however the next result shows that the total throughput in the cell will increase.

**Proposition 3.** *The total throughput $\sum_{i=1}^{N} R_i$ of FF is greater than that of RR.*

**Proof.** By using the Chebyshev sum inequality, we find that

$$\sum_{i=1}^{N} \frac{1}{N} \frac{W}{\Gamma_i} \bar{\xi}_i \sum_{i=1}^{N} \frac{\bar{\xi}_i}{\Gamma_i} \leq \sum_{i=1}^{N} \bar{\xi}_i/\Gamma_i \frac{W}{\Gamma_i} \bar{\xi}_i$$

which can be rewritten as

$$\sum_{i=1}^{N} \frac{1}{N} \frac{W}{\Gamma_i} \bar{\xi}_i \leq \sum_{i=1}^{N} \frac{\bar{\xi}_i/\Gamma_i}{\sum_{j=1}^{N} \bar{\xi}_j/\Gamma_j} \frac{W}{\Gamma_i} \bar{\xi}_i.$$

□

In Proposition 2 and 3, equality is achieved iff, for all $i$, $\bar{\xi}_i/\Gamma_i = C$, where $C > 0$ is an arbitrary constant. Hence, we have shown that the resulting throughputs are related as $ET \leq RR \leq FF$.

### B. Fast Scheduling

Now, consider a situation where for any time instant the sample $\xi_i$ is perfectly available. The suggested relatively best (RB) algorithm decides from a normalized channel state and gives an instantaneous throughput of

$$r_i(t) = \begin{cases} \frac{W}{\Gamma_i}\xi_i(t), & \text{if } \frac{\xi_i(t)-\bar{\xi}_i}{c_i} > \max_{j\neq i}\left[\frac{\xi_j(t)-\bar{\xi}_j}{c_j}\right] \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

Ties are broken with equal probability. The $c_i$ are positive control parameters, resulting in different time fractions $\phi_i$. Thus, the scheduling decision is made from a normalized state, referred to as the "relatively best". The consequence is that each user only transmits on good instants, but with a certain asymptotical channel access, controlled by the $c_i$. The expected throughput is,

$$\begin{aligned} R_i &= \frac{W}{\Gamma_i}\mathbb{E}_{\xi_i}\left[\xi_i \Pr\left\{\frac{\xi_i-\bar{\xi}_i}{c_i} > \max_{j\neq i}\left[\frac{\xi_j-\bar{\xi}_j}{c_j}\right]\right\}\right] \\ &= \frac{W}{\Gamma_i}\mathbb{E}_{\xi_i}\left[\xi_i \prod_{j\neq i}\Pr\left\{\frac{\xi_i-\bar{\xi}_i}{c_i} > \frac{\xi_j-\bar{\xi}_j}{c_j}\right\}\right] \end{aligned} \tag{12}$$

where the last equality follows from that the channels are assumed to be mutually independent. If we consider a Rayleigh fading channel, $K = 0$, the conditional probability that user $i$ has a relatively better channel than user $j$ is given by:

$$\Pr\left\{\xi_j \leq \frac{c_j}{c_i}(\xi_i - \bar{\xi}_i) + \bar{\xi}_j | \xi_i\right\} =$$

$$\begin{cases} 0, & \frac{c_j}{c_i}(\xi_i - \bar{\xi}_i) + \bar{\xi}_j < 0 \\ 1 - e^{-\frac{\frac{c_j}{c_i}(\xi_i - \bar{\xi}_i) + \bar{\xi}_j}{\bar{\xi}_j}}, & \frac{c_j}{c_i}(\xi_i - \bar{\xi}_i) + \bar{\xi}_j \geq 0 \end{cases}$$

If we let $\xi_0 = \max_{j \neq i}[0, \bar{\xi}_i - \frac{c_i}{c_j}\bar{\xi}_j]$, the resulting asymptotical time fraction assignment can be written as

$$\phi_i = \int_{\xi_0}^{\infty} \prod_{j \neq i} (1 - e^{-\frac{\frac{c_j}{c_i}(\xi_i - \bar{\xi}_i) + \bar{\xi}_j}{\bar{\xi}_j}}) \frac{1}{\bar{\xi}_i} e^{-\frac{\xi_i}{\bar{\xi}_i}} \, d\xi_i. \tag{13}$$

The feasible time fractions cannot take any values. Consider an example with $N = 2$, $c_1 = a\bar{\xi}_1$ and $c_2 = \bar{\xi}_2$. Evaluating the integral (13) we get

$$\phi_1 = \begin{cases} (1 - \frac{a}{1+a})e^{a-1}, & 0 < a < 1 \\ 1 - \frac{a}{1+a} e^{-(1-1/a)}, & a \geq 1. \end{cases} \tag{14}$$

Thus, the feasible time fraction is here limited to $e^{-1} \leq \phi_i \leq 1 - e^{-1}$. For further generalization, consider two classes of users where there are $N_A$ users with $c_i = a\bar{\xi}_i$ and $N - N_A$ users with $c_j = \bar{\xi}_j$. For $0 < a < 1$, the resulting time fractions become for any of the $N_A$ users

$$\phi_i = \sum_{k=0}^{N_A - 1} \sum_{l=0}^{N - N_A} \binom{N_A - 1}{k} \binom{N - N_A}{l} (-1)^{k+l} \times \frac{e^{-(1-1/a)l - (k+1+l/a)(1-a)}}{k + 1 + l/a}$$

and for $a \geq 1$

$$\phi_i = \sum_{k=0}^{N_A - 1} \sum_{l=0}^{N - N_A} \binom{N_A - 1}{k} \binom{N - N_A}{l} (-1)^{k+l} \times \frac{e^{-(1-1/a)l}}{k + 1 + l/a}$$

where a binomial expansion is performed in (13). In Fig. 1, this is plotted for $N = 10$ and $N_A = 1, 2, \ldots, N$. The arrows denote increasing values of $N_A$. As expected, $\phi_i = 0.1$ at $a = 1$. It should be pointed out that the characteristic of the time fraction curves is very much dependent on the parameters $N$ and $N_A$, compare, e.g., with the two dimensional case (14) above. For fair comparison between scheduling algorithms, their asymptotical time fractions should be the same. Importantly, it can easily be shown that when $c_i = \bar{\xi}_i$, the integral (13) reduces to $\phi_i = 1/N$. Therefore, the RB algorithm can provide the same asymptotical fairness as RR. This also holds for $K > 0$.

## IV. Scheduling Gain Function

The expected throughput of RB can for the Rayleigh fading channel, be evaluated by

$$R_i = \int_{\xi_0}^{\infty} \frac{W}{\Gamma_i} \xi_i \prod_{\substack{j=1 \\ j \neq i}}^{N} (1 - e^{-\frac{\frac{c_j}{c_i}(\xi_i - \bar{\xi}_i) + \bar{\xi}_j}{\bar{\xi}_j}}) \frac{1}{\bar{\xi}_i} e^{-\frac{\xi_i}{\bar{\xi}_i}} \, d\xi_i. \tag{15}$$

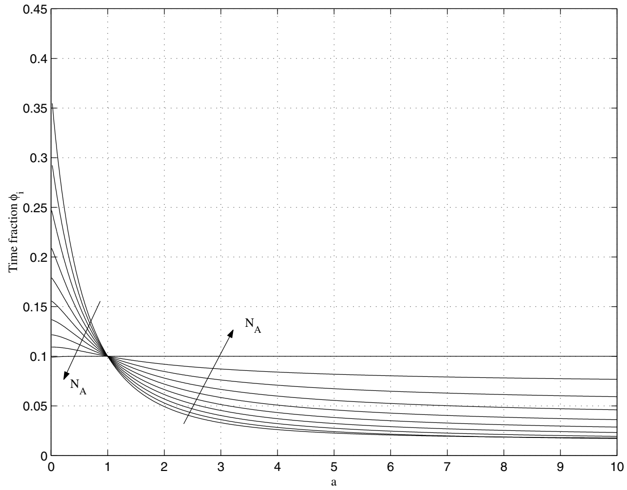


Fig. 1. Feasible time fractions for different $N_A$ when $N = 10$.

To compare with RR, we let $c_i = \bar{\xi}_i$, which, after some algebra, results in that (15) equals

$$R_i = \frac{W}{\Gamma_i} \frac{\bar{\xi}_i}{N} \sum_{k=1}^{N} \frac{1}{k}. \tag{16}$$

This should be compared with $R_i = \frac{W}{\Gamma_i}\frac{\bar{\xi}_i}{N}$, the expected throughput for RR. Thus, we can define a scheduling gain

$$G = \sum_{k=1}^{N} \frac{1}{k} \approx \ln N + \gamma + \frac{1}{2N} \tag{17}$$

where $\gamma \approx 0.577$ is the Euler constant. The approximation becomes tight for large $N$ [13]. Therefore the scheduling gain from RB compared to RR increases with the load $N$. It can be noted that (17) describes the total throughput gain in the cell as well. Furthermore, this scheduling gain is exactly as that for selection diversity on a Rayleigh fading channel [12] with $\bar{\xi}_i/N$ as the SNR in each branch. Although $G$ is increasing in $N$, the absolute throughput of an individual user decreases, and asymptotically $R_i = O(\ln N/N)$ which is captured in Fig. 2. There, the normalized throughputs of RB and RR, $\frac{1}{N}\sum_{k=1}^{N}\frac{1}{k}$ and $1/N$, are plotted, describing the relative loss by increasing the number of users. For RR, the normalized throughput decreases by half when adding one more user to $N = 1$. For the same degradation, the RB can support at least 3 more users.

The closed form expression (16) accounts for Rayleigh fading. If we consider general $K$ factors, it can be shown that [14]

$$\Pr[\xi_i \leq x] = 1 - Q_1\left(\sqrt{2K}, \sqrt{\frac{2(K+1)}{\bar{\xi}_i}x}\right) \tag{18}$$

where $Q_1(\cdot,\cdot)$ is the Marcum $Q$-function. Generalizing (15)

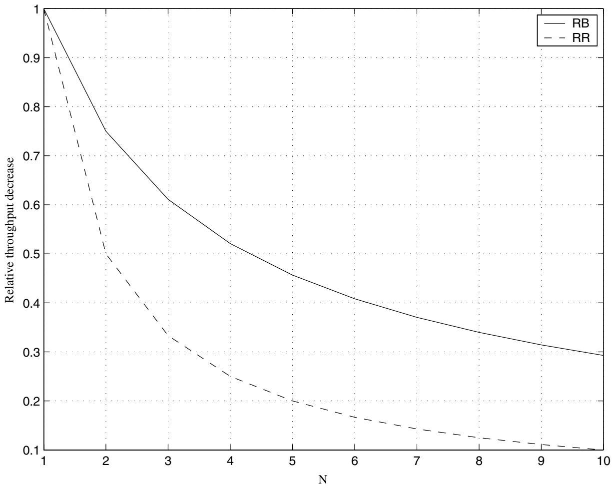


Fig. 2. The relative throughput loss as function of the number of users. The asymptotical time fractions are set to $\phi_i = 1/N$ for both RB and RR.

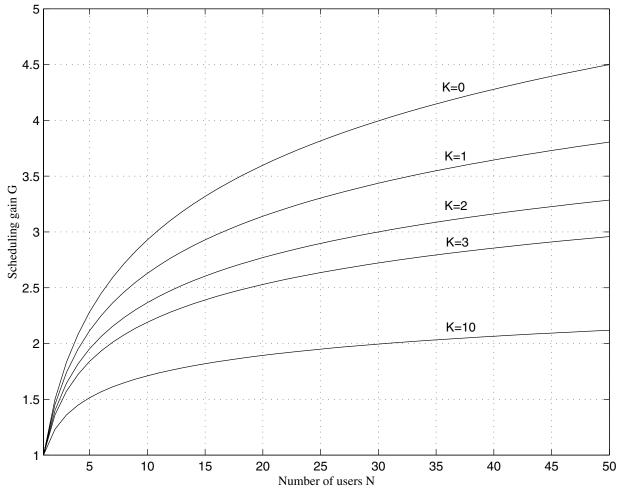


Fig. 3. The scheduling gain for RB over RR for different Ricean factors $K$. The asymptotical time fractions are set to $\phi_i = 1/N$ for both RB and RR.

and using (4) and (18),

$$R_i = \int_{\xi_0}^{\infty} \frac{W}{\Gamma_i} \xi_i \prod_{j \neq i} \Pr[\xi_j \leq \frac{c_j}{c_i}(\xi_i - \bar{\xi}_i) + \bar{\xi}_j] f_{\xi_i}(\xi_i)\, d\xi_i, \quad (19)$$

the throughput can be found by numerical integration. In Fig. 3, we plot the scheduling gain for different values of $N$and $K$. The result of (17) corresponds to the curve of $K = 0$. The results show that, the larger Rice factor, the less gain from the proposed scheduler but the gain is still significant.

## V. Concluding Remarks

From Proposition 1, one-by-one transmission was shown to be superior to simultaneous transmission for downlink scheduling in DS-CDMA over fading channels. This assumed a linear relation between rate and signal-to-interference ratio. That assumption may be less valid when adaptive modulation and coding are used. However, even if Proposition 1 would not hold under a nonlinear relation, higher throughput may still be obtained than simultaneous transmission, using the RB scheduler. This is due to that the proof is based on averaging and does not consider multiuser diversity aspects, captured by the additional scheduling gain. The fairness criteria was here to give the users access to the channel the same fraction of time. Alternatively, users could be given different service in terms of individual asymptotical time fractions, controlled by the $c_i$. Our numerical results confirm those of [9, 10], where the throughput increased with the channel variability. It should be noted that for the RB scheduler, the expected throughputs will not necessarily be equal, but such requirements can be met by adjusting the transmission power $P$.

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