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Measurements and Mathematical Characterization of Uncertain Information

NING Xu[1, a], JING Fang Guo[1,b] and JIN Fang Han[1,c]

[1]College of Science, Hebei University of Science and Technology,

Shijiazhuang Hebei 050018, P.R. China

[a]ym731@sina.com, [b]fsqsjz@163.com, [c]jfhanemail@126.com

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Abstract. This paper, the problems of mathematical characteristics and measurements for three kind of uncertainty information are descussed by means of the "logical analysis method". Firstly, By virtue of the analysis for the importance of uncertain information research in scientific development, a research chain: "uncertainty information $\leftrightarrow$ information theory $\leftrightarrow$ complexity" is presented. Secondly, the mathematical characterization and measurements for three kind of uncertainty information are obtained in terms of the characteristic analysis for uncertain information .

Introduction

The world is an information-oriented society. Information plays a significant role with the increasing applications of information science. Some information from the information source is certain, while more of them is uncertain. The certain information or uncertain information is in itself neither good nor bad. Currently, there are mainly five different uncertain information known to all.They are random information, fuzzy information, gray information, unascertained information, and pan-gray information[5] . For these uncertain information, some of them has been deeply investigated in recent years, and some nice important results have been proposed, such as "Shannon entropy of random information"[15] . And the research for the most of other uncertain information is still not enough to go deep into , a lot of problems will solve urgently, for example: the measurement of information. For the results of uncertain information the interested reader is referred to Refs.[5,6,7,8,9,10,11,12,etc.] Under the enlightenment of paper [5-18], the author tries to consider the mathematical characterization and measurements of uncertain information. Combining existing results and by means of the logical analysis method, we want to present some viewpoints about "the research chain in now scientific development" and "mathematical characterization and measurements of uncertainty information". so that to be discussed with the people of the same trade and experts.

Importance on Studying Uncertain Information

It is well known, the concept of Hall for Workshop of Meta-synthetic Engineering has been proposed by the scientist of our country professor Xue-sen Qian firstly presented in 1990-1992. It is coping with opened complex giant systems. where "complex giant systems"（1990）, "Hall for Workshop of Meta-synthetic Engineering"(1992). and he thinks that complexity problem is actually the dynamics property problem of opened complex giant system. which with the complexity scientific / complex systematic was proposed by abroad is expressing the same idea in different words. Originally, Xue-sen Qian point out: complexity science is a scientific new field. afterwards, famous academician Ru-wei Dai with it to summarize the science of the 21 century. This has explained the importance of complexity science. On the other hand, Information idea is that the people carry information as a kind of the basic existence for difference from material, as well as the generality understanding, stipulation and knowledge that made for its essence, existent way, meaning and value. And according to corresponding understanding, stipulation and knowledge, since the structure of extant thing organizes

and concerns the evolution program and generation course pattern and type of movable mould, holds and describes method and the way of property, characteristic and the essence of thing , is program course, relation and the structure of extant thing the carrier of information or is consistent yard, and from this decoding happen in which implication method and the way about the indirect existent contents such as thing historical state, realistic relation and future trend, as well as will realistic object thing or information again pedestrian is symbolism, and method and the way of composing the acting type relation that it specifies have formed the information way or information thought way of knowing , namely information thought.

Reasonable theory considers that the information theory, system theory, cybernetics，dissipative structure, synergetics and hypercycle theory etc. are all complex system theory. However, for the feelings boundary of initial origin of above-mentioned theories, they can not be at last complexity research theory of standard. Though the information theory research can not be simple directly regard as complexity research , but compared with the systems scientific research and from organization research , information theory research is not only more fundament and also more synthesized , but also it can be embody the related basic feature of complexity research. Information theory research is more basic, which because of the development of information science has first revealed a brand-new existent field in the sense of world this body existent theory, and in the layer with normally abstract philosophy surface on, have demonstrated the brand-new world view of a kind of material world and information world double existence and double evolution , and then have again founded information intermediary theory of a kind of philosophy theory of knowledge.

Information thought is a kind of brand-new complexity thought way. Obviously, by from the above-mentioned is related explain, we can see distinctly, that the related theory, viewpoint, way and method was provided by the information theory research which embodies information thought, it able better embody general feature of holographic comprehensive for complexity research, and then it offered one brand-new explain dimension degree for the complexity. On this new dimension degree, not only systematic complex behavior can get explanation from the angle of information campaign and information campaign is complex. Is just based on this, we just reasonable from say , the information theory in develop- ment of present science, which with very unique position and role, it's brand-new research viewing angle is that the other already some research all cannot be replaced. That is just information theory research may offer certain kind the most fundamental theoretical normal-form for the complex system research, and it is that has nucleus theoretical meaning.

From the course of producing of uncertainty information, we can find out , it is the result in human’s process of cognition, it is also the inevitable result of mass sport. If we say that information is determined ,then we can only say it is source information, (but from the spread course of information has known: What we can get can only be old information, that source information is direct got is impossible) or say thing fairly simple, we can man-made regard it as determinate. However because of the complexity in information course, that can be regarded as definite old information is very few. Our conclusion is: In the old information that obtained by mankind, uncertainty information is absolute, deterministic information is relative. So we study information theory and must study uncertainty information.

Sum up the above, we don’t difficult to get that a research chain: “uncertainty information↔ information theory ↔ complexity”. It obviously reflects the important significance on uncertain information research. furthermore, because the produce source of uncertainty information is not one-side, so besides classical random information, we still study other several uncertainty information, such as fuzzy information, grey information, and unascertained information itc..

Measurements and Mathematical Characterization of Uncertain Information

The world is an information-oriented society. Information plays a significant role with the increasing applications of information technology. Some information from the information source is certain, while more of them is uncertain.To identify a system is to identify its information characteristic,

information characteristic is mainly the unity of system's essential factor, structure, function. A system can't be described and expressed without these information characteristic. But, in respective aspects from the generation of information to the description of information characteristic, the information grasped is uncertain owing to various reasons. For example, in the process of information generation, transmission and receiving, owing to the disturbance of outside disturbance source, some distortion phenomenon usually happen when source information arrive at clinic information through channel. When recognizing and describing system's information characteristic, owing to the objective thing's complexity, people subjective knowledge's limitation, natural language's intrinsic uncertainty, the quality's appearance, the quantity's provision, relationship's expression and law's find, people can't truthfully reflect objectively existing things. Distortion and no easily exact reflection make information uncertain. For uncertainty is universal in objective reality, people have to study, build up and apply uncertain information system. Via analyzing several kinds of uncertain information, different uncertain systems are expounded

3.1. Random Information

A lot of actual problems involve the analysis and design of system disturbed by environment uncertainty. The uncertain disturbance may be caused by different source. Among them, a kind of disturbance source is a certain causality that does not happen between condition and event because the condition is insufficient. Hence, it is uncertain whether the event happen or not (namely the event maybe happen or not). The kind of uncertainty is called randomness. For random information have been relatively matured, we give directly its measure and mathematical characterization as follows.

Deffinition3.1.1: let x be an element of which want to known, U be a Cantor set, $x \in A \subset U$, α_i denote possibility of $x \in A$, $\alpha_i \in [0,1]$, and $\sum_i \alpha_i = 1$, then the information that offers by x, it is called as random information.

Deffinition3.1.2: let e be an element of random information., $\{p_i\}_{i=1}^n$ is it's posterior probability distribution about n classification attached, then the $H(e) = -\sum_{i=1}^{n} p_i \log p_i$ is called as measure of random information. That is "Shannon entropy".

If e be an element of continuous-type random information, $p(x)$ is corresponding density function, then it's measure can be rewrite as $H(e) = -\int_{-\infty}^{+\infty} p(x) \log p(x) dx$ .it is a generalization of Shannon entropy. By the Lebesgue integral ,both can unified as $H(e) = -\int_E p(x) \log p(x) dm, a.e.$

3.2. Fuzzy Information

In present, the fuzziness of things considered is mainly uncertainty caused by the distinct definition and judgment standard that can't be given for something. Or say the difference between things display either this or that in middle transition. As inconsistent law expound "when the complexity of a system add, people's capacity to make it exact decrease, when arrive at a certain threshold, complexity and exactness repel each other, fuzziness accompany with complexity" [10]. In fuzzy system, or when people handle fuzziness problem in reality, the tool adopted is fuzzy mathematics. For fuzzy mathematics, its characteristic is depending on experience, Fuzzy mathematics faces uncertainty caused by inadequate knowledge, namely, solves the uncertain problem of "knowledge uncertainty". Depending on experience talked here is membership function is given via experience. In practice, constructing a continuous membership functions usually needs infinite experience data (there are exceptions in specific). Hence, we may say, the information criterion of fuzzy theory is experience information. The involved is experience's infinite information space (there are exceptions). For the characteristic of fuzzy set is denotation is uncertain, fuzzy mathematics' thinking way is making uncertain denotation expressed by membership degree.that is, denotation quantification. Finally, we give its measure and mathematical characterization as follows

Deffinition3.2.1: let x be an element of which want to known, U be a Cantor set, $x \in A \subset U$, α_i denote membership stage of $x \in A$, $\alpha_i \in [0,1]$, then the information that offers by x, it is called as fuzzy information

Deffinition3.2.2: let $U = \{x_1, x_2, \cdots x_n\}$ be an universe of discourse, $F(U)$ denote all information on the U, $e \in F(U)$, $\mu_e(x)$ denote membership function of e, then the measure of e defined as $H(e) = \sum_{i=1}^{n} \mu_e(x_i)(1-\mu_e(x_i)), \forall e \in F(U)$, it is called fuzzy entropy. Another fuzzy entropy is $H(e) = -K\sum_{i=1}^{n}[\mu_e(x_i)\log\mu_e(x_i) + (1-\mu_e(x_i))\log(1-\mu_e(x_i))], \forall e \in F(U)$. where K is a normalization constant

3.3. Grey Information

Problems in complex big system often to involve a lot of information (data and knowledge). Because the noise in channel (objective) disturb, or the capacity of receiving system (subjective) is limited, We only infer the general range that information reflect, but didn't know all the confirmed content, in addition, when some problem is handled, all information concerned is unnecessarily or impossibly obtained (only a part is known), which is information incompleteness. Grey system is the system which partial information is distinct and partial information is indistinct. It can be said information incomplete system.

Grey system theory's intension is "small sample uncertainty". Grey system faces "the uncertainty" caused by a few data, namely, solves the problem of "a few data uncertainty". It demands data is arbitrary distribution. Its goal is "reality law". Because a few data studied is the data distributing in reality time zone. The data embody the reality law. The basis of grey system theory is "grey hazy set". Grey hazy set is the set, under a certain proposition, whose element is from distinct to indistinct, information is from a little to a lot, and can be replenished unceasingly; the set from grey to white, from abstract to concrete; the set can evolve, have "life", have "effectiveness for a given period of time"; the set is compatible to 0,1 property, cantor set and [0,1] property; the set have four kinds of forms (embryo, growth, maturity, actual evidence).[3], The grey system theory is on the basis of information covering, that is, via information covering to describe, analyze, handle grey target whose information is incomplete, uncertain. The intension of information covering is to contain, cover information of given proposition with a group of information. The basis means of grey system theory is grey generation. grey generation or generation is the data processing (concluding accumulation,transform, reject, interpolation…). Its goal is to provide comparable data for analysis; to provide reasonable data base for building model; to provide extremity uniform sample for making decision. Its characteristic is a few data; its thinking way is many angle of view; its information criterion is a little information. On the basis of "uncertainty of a few data", only a little information can be obtained, hence, grey system theory involves finite information space. As final of this section, we give its measure and mathematical characterization as follows

Deffinition3.3.1: let x be an element of which want to known, U be a Cantor set, $x \in A \subset U$, $A = [x_1, x_2]$, α denote possibility of $x \in A$, then the α is called as grey information.

Deffinition3.3.2: the information with can make object having been known completely clear is called as complete information, it is denoted by Ω [7].

Deffinition3.3.3: Let α be a grey information, $\mu(\alpha)$ denote the potency or the amount of information of α (enumerable infinite or finite), $\mu(\Omega)$ denote the potency or the amount of information of Ω (enumerable infinite or finite), then the measure of α defined as $H(\alpha) = \mu(\Omega) - \mu(\alpha)$, it is also called grey information entropy.

Deffinition3.3.4: Let α be a grey information about $x \in A \subset U$, $F(U)$ denote information field on the x , then the measure of α defined as $H(\alpha) = \log(\mu(F) - \mu(\alpha))$,it is also called grey information entropy.

Note: The above can discuss according to continuaus and discrete, respectively. It is omitted here.

3.4. Unascertained Information

There exists another kind of uncertain information, which our country's famous scholar Guangyuan Wang academician finds and first provided, that is, unascertained information, called weak uncertain information too. "Unascertained information" is the information that is must be used but unknown when making a strategy owing to restriction of objective condition. or say, owing to inadequate evidence (information) grasped by decision maker is not enough to ascertain thing's real state and quantity relation, which lead to the unascertained information of pure objective knowledge. This uncertainty information on knowledge and pure subjective is called as "unascertained information ". This kind of information produce can sum up is to act blindly experiment, it can seemingly repeat under same condition ,but the experimental results be not completely known. For instance, depreciation capital produces in two wears: Mechanical wear and spiritual wear. Produce factor and natural factor in which mechanical wear to arouse, it is known to be able to regard as. Spiritual wear is to use reduction because of the manufacturing cost of similar machine, or since having produced new more effective machine equipment the devaluation of the original fixed assets that arouses. The result of this respect is nearly impossible to be accurate to quantify , before it is artificial to guess and estimate is unknown, the result of guessing and estimate can only be x % of true value.

The system with unascertained information is unascertained system. For unascertained system, its mathematics tool is "unascertained mathematics", its basis is unascertained set. Reader can consult reference [12-14] As final of this section, we give its measure and mathematical characterization as follows

Deffinition3.4.1: let x be an element of which want to known, U be a Cantor set, $x \in A \subset U$, α_i denote possibility of $x \in A$, $\alpha_i \in [0,1]$, then the information that offers by x ,it is called as Unascertained information.

Deffinition3.4.2: Let e be an element of unascertained information, $U = \{x_1, x_2, \cdots x_n\}$ be a discrete "universe of discourse", it's Faith distribution is $F = (f_1, f_2, \cdots f_n)$, where $f_i = f(x_i), i = 1,2, \cdots n$, $F(U)$ denote all information on the U , $e \in F(U)$, then the $H(e) = -\sum_{i=1}^{n} f_i \log f_i$ is called as measure of unascertained information. If $f_i, i = 1,2, \cdots n$. denote subjective membership degree of e ,then measure of unascertained information e can be defined as $H(e) = \sum_{i=1}^{n} f_e(x_i)(1 - f_e(x_i)), \forall e \in F(U)$.

We no harm name both are unascertained information entropy. If U be a continuous "universe of discourse" , e be an element of continuous-type unascertained information, $f(x)$ is corresponding density of Faith distribution, then it's measure can be rewrite as $H(e) = -\int_{-\infty}^{+\infty} f(x) \log(f(x)) dx$.

Similarly, if $f(x)$ is corresponding distribution of subjective membership degree, then measure of e is $H(e) = \int_{-\infty}^{+\infty} f(x)(1 - f(x)) d(x), \forall e \in F(U)$

Note: The demonstration and the detailed analysis of this problem will be given in another writing .

Conclusions

In this paper, the important position of information theory in scientific development are described, the importance on studying uncertain information is emphasized, and a kind of research chain : "uncertainty information $\leftrightarrow$ information theory $\leftrightarrow$ complexity" is presented. Secondly, several kinds

of uncertain information are discussed by analyzing uncertainty of information. That the measurements and mathematical characterization are also given to these uncertainty information.

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