

# Computational Verb Neural Networks

Tao Yang

**Abstract**—When any attribute value in a conventional neural network is verified, the result is a computational verb neural network(VNN). We can verbify inputs, outputs, biases, weights and many other attributes of conventional neural networks. In this paper, we reported two types of VNNs. The first one consists of computational verb inputs and numerical output. The second one consists of computational verb inputs and outputs. The learning algorithms for both types of VNNs were provided. The existence of solutions and continuities of single-layer feedforward type-I VNN were studied. Copyright © 2007 Yang's Scientific Research Institute, LLC. All rights reserved.

**Index Terms**—Computational verb neural network, VNN, computational verb, single-layer feedforward type-I VNN.

## I. INTRODUCTION

ONE of the main drawbacks of neural networks is its lack of high-level representing power of the knowledge learnt and represented in their weights. One way to overcome this problem is to implement the representing power of natural languages into the structures of neural networks. The physical linguistics is the very framework to make natural language measurable and therefore, the best tool to implement the representing power of natural languages into the structures of neural networks. The author will present a neural network structure where computational verbs will be workhorses to bring the high-level knowledge representing power into the domain of measurements.

In [38], the author presented the learning algorithms for sets of computational verb rules. These learning algorithms can be readily transformed to fit into the configurations of computational verb neural networks. A computational verb neural network is the result of verbifying a conventional neural network. There are many ways to verbify the attributes of a conventional neural network. For example, one can verbify the inputs, outputs, weights, and biases of a conventional neural network. The author will study the following two types of computational verb neural networks.

- 1) Type-I computational verb neural network consists of verbified inputs and real outputs. All other attributes of are numerical.
- 2) Type-II computational verb neural network consists of verbified inputs and outputs. All other attributes of are numerical.

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The organization of this paper is as follows. In Section II, the brief history of computational verb theory will be given. In Section III, the structure of neuron of type-I computational verb neural networks and its learning algorithm will be presented. In Section IV, the structure of neuron of type-II computational verb neural networks and its learning algorithm will be given. In Section V, the existence of solutions and the continuities of single-layer feedforward type-I VNN will be studied. In Section VI, some concluding remarks will be presented.

## II. A BRIEF HISTORY OF COMPUTATIONAL VERB THEORY

As the first paradigm shift for solving engineering problems by using verbs, the computational verb theory and physical linguistics have undergone a rapid growth since the birth of computational verb in the Department of Electrical Engineering and Computer Sciences, University of California at Berkeley in 1997[8], [9]. The paradigm of implementing verbs in machines were coined as *computational verb theory*[22]. The building blocks of computational theory are *computational verbs*[17], [12], [10], [18], [23]. The relation between verbs and adverbs was mathematically defined in [11]. The logic operations between verb statements were studied in [13]. The applications of verb logic to verb reasoning were addressed in [14] and further studied in [22]. A logic paradox was solved based on verb logic[19]. The mathematical concept of set was generalized into verb set in[16]. Similarly, for measurable attributes, the number systems can be generalized into verb numbers[20]. The applications of computational verbs to predictions were studied in [15]. In [24] fuzzy dynamic systems were used to model a special kind of computational verb that evolves in a fuzzy space. The relation between computational verb theory and traditional linguistics was studied in [22], [25]. The theoretical basis of developing computational cognition from a unified theory of fuzzy and computational verb theories is the theory of the Unicogse that was studied in [25], [30]. The issues of simulating cognition using computational verbs were studied in [26]. A way to implementing feelings in machines was proposed based on grounded computational verbs and computational nouns in [32]. In [39] a new definition of the similarity between computational verbs was studied. The theory of computational verb has been taught in some university classrooms since 2005<sup>1</sup>.

The latest active applications of computational verb theory are listed as follows.

- 1) Computational Verb Controllers. The applications of computational verbs to different kinds of control prob-

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lems were studied on different occasions[21], [22]. For the advanced applications of computational verbs to control problems, two papers reporting the latest advances had been published[28], [27]. The design of computational verb controller was also presented in a textbook in 2006[1].

- 2) Computational Verb Image Processing and Image Understanding. The recent results of image processing by using computational verbs can be found in[29]. The applications of computational verbs to image understanding can be found in [31].
- 3) Stock Market Modeling and Prediction based on computational verbs. The product of Cognitive Stock Charts[4] was based on the advanced modeling and computing reported in [33]. Applications of computational verbs was used to study the trends of stock markets known as Russell reconstruction patterns [34].

Computational verb theory has been successfully applied to many industrial and commercial products. Some of these products are listed as follows.

- 1) Visual Card Counters. The *YangSky-MAGIC* card counter[6], developed by Yang's Scientific Research Institute and Wuxi Xingcard Technology Co. Ltd., was the first visual card counter to use computational verb image processing technology to achieve high accuracy of card and paper board counting based on cheap webcams.
- 2) CCTV Automatic Driver Qualify Test System. The *DriveQfy* CCTV automatic driver qualify test system[7] was the first vehicle trajectory reconstruction and stop time measuring system using computational verb image processing technology.
- 3) Visual Flame Detecting System. The *FireEye* visual flame detecting system[2] was the first CCTV or webcam based flame detecting system, that works under color and black & white conditions, for surveillance and security monitoring system[36], [37].
- 4) Smart Pornographic Image and Video Detection Systems. The *PornSeer*[5] pornographic image and video detection systems are the first cognitive feature based smart porno detection and removal software.
- 5) Webcam Barcode Scanner. The *BarSeer*[3] webcam barcode scanner took advantage of the computational verb image processing to make the scan of barcode by using cheap webcam possible.
- 6) Cognitive Stock Charts. By applying computational verbs to the modeling of trends and cognitive behaviors of stock trading activities, cognitive stock charts can provide the traders with the "feelings" of stock markets by using simple and intuitive indexes.

### III. TYPE-I COMPUTATIONAL VERB NEURAL NETWORKS

A type-I computational verb neural network(VNN)<sup>2</sup> takes implementations of computational verbs as inputs, compares the input verbs with some template verbs and output real numbers. Therefore, a type-I VNN has verbs as inputs and

<sup>2</sup>Instead of using CVNN, we use VNN to stand for computational verb neural network to avoid clutter.

real numbers as outputs. In a type-I computational verb neural network, for input waveforms  $x_i(t) \in \mathbb{R}, t \in [0, T], i = 1, \dots, n$ , there are template computational verbs  $V_i$  such that the computational verb similarity between  $x_i(t)$  and the evolving function of  $V_i, \mathcal{E}_i(t)$ , will be weighted and results in the output  $y \in \mathbb{R}$  of the neuron as follow.

$$y = f \left( \sum_{i=1}^n w_i S(x_i(t), \mathcal{E}_i(t)) - \theta \right), t \in [0, T] \quad (1)$$

where  $S(\cdot, \cdot)$  is a computational verb similarity[35],  $\theta \in \mathbb{R}$  is the bias of the neuron and the function  $f(\cdot)$  is the transfer function for the neuron. Given  $m$  canonical computational verbs,  $\{\tilde{\mathcal{E}}_j\}_{j=1}^m$ , we can represent a computational verb as

$$\mathcal{E}_i(t) = \sum_{j=1}^m \alpha_{ij} \tilde{\mathcal{E}}_j(t) \quad (2)$$

where  $\alpha_{ij} \in \mathbb{R}$  are constants and can be viewed as adverbs.

Assume that we have  $K$  set of training examples  $\{u_{k1}(t), \dots, u_{kn}(t), d_k\}_{k=1}^K$ , where  $u_{ki}(t), i = 1, \dots, n$ , are  $n$  input waveforms and  $d_k$  is the corresponding output. Our goal is to learn all computational verbs  $\mathcal{E}_i(t)$  and bias  $\theta$  from the training examples. For the  $k$ th training sample, we construct an output  $y_k$  as,

$$y_k = f \left( \sum_{i=1}^n w_i S(u_{ki}(t), \mathcal{E}_i(t)) - \theta \right), \quad (3)$$

based on which we construct the following error function.

$$E = \sum_{k=1}^K (y_k - d_k)^2. \quad (4)$$

The learning rules are given by

$$\begin{aligned} w_i(l+1) &= w_i(l) + \gamma_i^w(l) \Delta w_i(l), \\ \theta(l+1) &= \theta(l) + \gamma^\theta(l) \Delta \theta(l), \\ \alpha_{ij}(l+1) &= \alpha_{ij}(l) + \gamma_{ij}^\alpha(l) \Delta \alpha_{ij}(l), \quad l = 0, \dots, \end{aligned} \quad (5)$$

where  $\gamma_i^w(l) \in \mathbb{R}^+, \gamma^\theta(l) \in \mathbb{R}^+$  and  $\gamma_{ij}^\alpha(l) \in \mathbb{R}^+, i = 1, \dots, n; j = 1, \dots, m$ , are learning rates for training iteration  $l$ , and

$$\begin{aligned} \Delta w_i &= -\frac{\partial E}{\partial w_i} \\ &= -\frac{\partial \sum_{k=1}^K (y_k - d_k)^2}{\partial w_i} \\ &= -2 \sum_{k=1}^K (y_k - d_k) \frac{\partial y_k}{\partial w_i} \\ &= -2 \sum_{k=1}^K (y_k - d_k) \dot{f} \left( \sum_{h=1}^n w_h S(u_{kh}(t), \mathcal{E}_h(t)) - \theta \right) \\ &\quad \times S(u_{ki}(t), \mathcal{E}_i(t)), \end{aligned} \quad (6)$$

$$\begin{aligned}
 \Delta\theta &= -\frac{\partial E}{\partial\theta} \\
 &= -\frac{\partial \sum_{k=1}^K (y_k - d_k)^2}{\partial\theta} \\
 &= -2 \sum_{k=1}^K (y_k - d_k) \frac{\partial y_k}{\partial\theta} \\
 &= 2 \sum_{k=1}^K (y_k - d_k) f \left( \sum_{h=1}^n w_h S(u_{kh}(t), \mathcal{E}_h(t)) - \theta \right), \tag{7}
 \end{aligned}$$

$$\begin{aligned}
 \Delta\alpha_{ij} &= -\frac{\partial E}{\partial\alpha_{ij}} \\
 &= -\frac{\partial \sum_{k=1}^K (y_k - d_k)^2}{\partial\alpha_{ij}} \\
 &= -2 \sum_{k=1}^K (y_k - d_k) \frac{\partial y_k}{\partial\alpha_{ij}} \\
 &= 2 \sum_{k=1}^K (y_k - d_k) f \left( \sum_{h=1}^n w_h S(u_{kh}(t), \mathcal{E}_h(t)) - \theta \right) \\
 &\quad \times w_i \frac{\partial S(u_{ki}(t), \mathcal{E}_i(t))}{\partial\alpha_{ij}} \tag{8}
 \end{aligned}$$

where  $\frac{\partial S(u_{ki}(t), \mathcal{E}_i(t))}{\partial\alpha_{ij}}$  is given by Eq. (10) where  $s_t(\cdot)$  is a *saturate function*[35] given by

$$s_t(x) = \frac{1}{1 + e^{-x}}, \text{ and } \dot{s}_t(x) = \frac{e^{-x}}{(1 + e^{-x})^2}. \tag{9}$$

#### IV. TYPE-II COMPUTATIONAL VERB NEURAL NETWORKS

A type-II VNN has verbs as inputs and verbs as outputs. A neuron in a type-II computational verb neural network is represented as

$$\mathcal{E}_y(t) = \sum_{i=1}^n w_i S(x_i(t), \mathcal{E}_i(t)) \mathcal{E}_i(t) - \mathcal{E}_\theta(t), \quad t \in [0, T] \tag{11}$$

where  $\mathcal{E}_y(t)$  is the evolving function of the output computational verb,  $\mathcal{E}_\theta(t) \in \mathbb{R}$  is the verb bias of the neuron. Given  $m$  canonical computational verbs,  $\{\tilde{\mathcal{E}}_j\}_{j=1}^m$ , computational verbs  $\mathcal{E}_i(t), i = 1, \dots, n$ , are constructed as in Eq. (2) and  $\mathcal{E}_\theta(t)$  is constructed as

$$\mathcal{E}_\theta(t) = \sum_{j=1}^m \alpha_j^\theta \tilde{\mathcal{E}}_j(t) \tag{12}$$

where  $\alpha_j^\theta \in \mathbb{R}$  are constants and can be viewed as adverbs.

Assume that we have  $K$  set of training examples  $\{u_{k1}(t), \dots, u_{kn}(t), d_k(t)\}_{k=1}^K$ , where  $u_{ki}(t), i = 1, \dots, n$ , are  $n$  input waveforms and  $d_k(t)$  is the corresponding output verb. Our goal is to learn all computational verbs  $\mathcal{E}_i(t)$  and

bias verb  $\mathcal{E}_\theta(t)$  from the training examples. For the  $k$ th training sample, we construct an output  $y_k(t)$  as,

$$y_k(t) = \sum_{i=1}^n w_i S(u_{ki}(t), \mathcal{E}_i(t)) \mathcal{E}_i(t) - \mathcal{E}_\theta(t), \tag{13}$$

based on which we construct the following error function.

$$E = \sum_{k=1}^K \int_0^T [y_k(t) - d_k(t)]^2 dt. \tag{14}$$

The learning rules are given by

$$\begin{aligned}
 w_i(l+1) &= w_i(l) + \gamma_i^w(l) \Delta w_i(l), \\
 \alpha_j^\theta(l+1) &= \alpha_j^\theta(l) + \gamma_j^\theta(l) \Delta \alpha_j^\theta(l), \\
 \alpha_{ij}(l+1) &= \alpha_{ij}(l) + \gamma_{ij}^\alpha(l) \Delta \alpha_{ij}(l), \quad l = 0, \dots, \tag{15}
 \end{aligned}$$

where  $\gamma_i^w(l) \in \mathbb{R}^+$ ,  $\gamma_j^\theta(l) \in \mathbb{R}^+$  and  $\gamma_{ij}^\alpha(l) \in \mathbb{R}^+, i = 1, \dots, n; j = 1, \dots, m$ , are learning rates for training iteration  $l$ , and

$$\begin{aligned}
 \Delta w_i &= -\frac{\partial E}{\partial w_i} \\
 &= -\frac{\partial \sum_{k=1}^K \int_0^T [y_k(t) - d_k(t)]^2 dt}{\partial w_i} \\
 &= -2 \sum_{k=1}^K \int_0^T [y_k(t) - d_k(t)] \frac{\partial y_k(t)}{\partial w_i} dt \\
 &= -2 \sum_{k=1}^K \int_0^T [y_k(t) - d_k(t)] S(u_{ki}(t), \mathcal{E}_i(t)) \mathcal{E}_i(t), \tag{16}
 \end{aligned}$$

$$\begin{aligned}
 \Delta \alpha_j^\theta &= -\frac{\partial E}{\partial \alpha_j^\theta} \\
 &= -\frac{\partial \sum_{k=1}^K \int_0^T [y_k(t) - d_k(t)]^2 dt}{\partial \alpha_j^\theta} \\
 &= -2 \sum_{k=1}^K \int_0^T [y_k(t) - d_k(t)] \frac{\partial y_k(t)}{\partial \alpha_j^\theta} dt \\
 &= 2 \sum_{k=1}^K \int_0^T [y_k(t) - d_k(t)] \tilde{\mathcal{E}}_j(t), \tag{17}
 \end{aligned}$$

$$\begin{aligned}
 \Delta \alpha_{ij} &= -\frac{\partial E}{\partial \alpha_{ij}} \\
 &= -\frac{\partial \sum_{k=1}^K \int_0^T [y_k(t) - d_k(t)]^2 dt}{\partial \alpha_{ij}} \\
 &= -2 \sum_{k=1}^K \int_0^T [y_k(t) - d_k(t)] \frac{\partial y_k(t)}{\partial \alpha_{ij}} dt \tag{18}
 \end{aligned}$$

$$\begin{aligned}
\frac{\partial S(u_{ki}(t), \mathcal{E}_i(t))}{\partial \alpha_{ij}} &= \frac{\partial \left\{ 1 - \sqrt{\frac{1}{T} \int_0^T \left[ s_t(u_{ki}(t)) - s_t \left( \sum_{j=1}^m \alpha_{ij} \tilde{\mathcal{E}}_j(t) \right) \right]^2 dt} \right\}}{\partial \alpha_{ij}} \\
&= \frac{\int_0^T \left[ s_t(u_{ki}(t)) - s_t \left( \sum_{j=1}^m \alpha_{ij} \tilde{\mathcal{E}}_j(t) \right) \right] \dot{s}_t \left( \sum_{j=1}^m \alpha_{ij} \tilde{\mathcal{E}}_j(t) \right) \tilde{\mathcal{E}}_j(t) dt}{T \sqrt{\frac{1}{T} \int_0^T \left[ s_t(u_{ki}(t)) - s_t \left( \sum_{j=1}^m \alpha_{ij} \tilde{\mathcal{E}}_j(t) \right) \right]^2 dt}}. \tag{10}
\end{aligned}$$

where  $\frac{\partial y_k(t)}{\partial \alpha_{ij}}$  is given by

$$\begin{aligned}
\frac{\partial y_k(t)}{\partial \alpha_{ij}} &= w_i \frac{\partial S(u_{ki}(t), \mathcal{E}_i(t))}{\partial \alpha_{ij}} \mathcal{E}_i(t) \\
&\quad + w_i S(u_{ki}(t), \mathcal{E}_i(t)) \frac{\partial \mathcal{E}_i(t)}{\partial \alpha_{ij}} \\
&= w_i \mathcal{E}_i(t) \frac{\partial S(u_{ki}(t), \mathcal{E}_i(t))}{\partial \alpha_{ij}} \\
&\quad + w_i S(u_{ki}(t), \mathcal{E}_i(t)) \tilde{\mathcal{E}}_j(t) \tag{19}
\end{aligned}$$

where  $\frac{\partial S(u_{ki}(t), \mathcal{E}_i(t))}{\partial \alpha_{ij}}$  is given by Eq. (10).

#### V. SINGLE-LAYER FEEDFORWARD TYPE-I VNN

The structure of single-layer feedforward type-I VNN is shown in Fig. 1. We assume that there are  $n$  input nodes,  $m$  hidden nodes and 1 output node. The weight  $w_{ij}$ ,  $i = 1, \dots, n; j = 1, \dots, m$ , connects between the  $i$ th input node and the  $j$ th hidden node. The weight  $v_j$  connect the  $j$ th hidden node to the only output node.

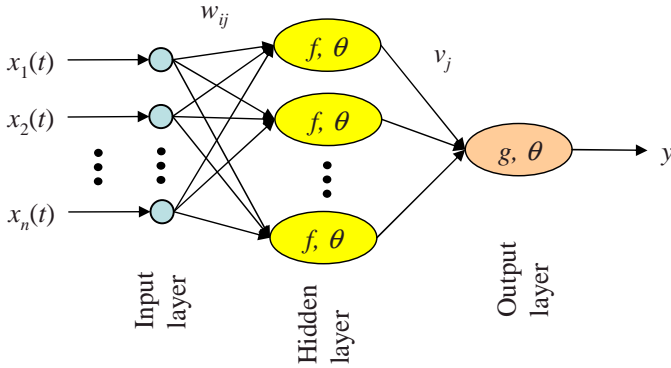


Fig. 1. The structure of single-layer feedforward type-I VNN.

The input-output relation of this VNN is given by

$$y = g \left( \sum_{j=1}^m v_j f \left( \sum_{i=1}^n w_{ij} S(x_i(t), \mathcal{E}_i(t)) - \theta_j \right) - \theta \right) \tag{20}$$

where each computational verb is constructed based on  $p$  canonical computational verbs

$$\mathcal{E}_i(t) = \sum_{h=1}^p \alpha_{ih} \tilde{\mathcal{E}}_h(t). \tag{21}$$

**Remark.** If in Eq. (20) we choose a computational verb similarity and a set of computational verbs such that  $S(x_i(t), \mathcal{E}_i(t)) = x_i(t)$  and only consider one sample point of the input waveforms, then VCNN (20) degenerates into the following conventional feedforward neural network with one hidden layer.

$$y = g \left( \sum_{j=1}^m v_j f \left( \sum_{i=1}^n w_{ij} x_i - \theta_j \right) - \theta \right). \tag{22}$$

The general conclusion is that *all* conventional neural networks can be verified into computational verb neural networks.

**Theorem 1.** Assume that  $\{\mathbf{u}_k(t), d_k\}, k = 1, \dots, K$ , are  $K$  training samples that are independent and have the same distribution.  $\mathbf{u}_k(t) \in \underbrace{C[0, T] \times C[0, T] \dots \times C[0, T]}_n$  and

$d_k \in \mathbb{R}$ .  $g(\cdot)$  is monotonic increasing. Then for any a given approximation error  $\varepsilon \in \mathbb{R}^+$ , the VNN (20) can approximate the input-output relation defined by the training set if the number of hidden nodes,  $m$ , is big enough.

*Proof.* The  $k$ th training sample is

$$(\mathbf{u}_k(t), d_k) = (u_1^{(k)}, \dots, u_n^{(k)}, d_k). \tag{23}$$

It follows from Eq. (20) that the  $k$ th training sample satisfies Eq. (24). Then for the training samples  $\{\mathbf{u}_k(t), d_k\}, k = 1, \dots, K$  and a given error  $\varepsilon$  Eq. (25) is satisfied. In Eq. (25)  $w_{ij}, v_j, \alpha_{ih}$  and  $\theta$  are adjustable parameters to be determined. Therefore, Eq. (25) defines a region in the parameter space to include the solution to the input-output relation defined by the training samples. The existence of such a solution is guaranteed by principles of nonlinear programming for each fixed  $\theta$  when we choose the total number of parameters no less than the number of training samples; namely,  $nm + m + np \geq K$ . ■

**Remark.** It follows from the conclusion in [38] that the set of nonlinear equations can be solved numerically. The details can be found from Sec. IV of [38].

**Theorem 2.** For VNN (20), assume that  $\{\mathbf{x}_k(t), y_k\}, k = 1, 2$ , are 2 input-output pairs where  $\mathbf{x}_k(t) \in \underbrace{C[0, T] \times C[0, T] \dots \times C[0, T]}_n$  and  $y_k \in \mathbb{R}$ , and assume

that  $f(\cdot)$  and  $g(\cdot)$  are continuous, then for any  $\varepsilon > 0$ , there

$$\begin{aligned}
 d_k &= g \left( \sum_{j=1}^m v_j f \left( \sum_{i=1}^n w_{ij} S \left( u_i^{(k)}(t), \mathcal{E}_i(t) \right) - \theta_j \right) - \theta \right) \\
 &= g \left( \sum_{j=1}^m v_j f \left( \sum_{i=1}^n w_{ij} S \left( u_i^{(k)}(t), \sum_{h=1}^p \alpha_{ih} \tilde{\mathcal{E}}_h(t) \right) - \theta_j \right) - \theta \right). \tag{24}
 \end{aligned}$$

$$\left\{ \begin{array}{l}
 \sum_{j=1}^m v_j f \left( \sum_{i=1}^n w_{ij} S \left( u_i^{(1)}(t), \sum_{h=1}^p \alpha_{ih} \tilde{\mathcal{E}}_h(t) \right) - \theta_j \right) = g^{-1}(d_1 + \theta), \\
 \sum_{j=1}^m v_j f \left( \sum_{i=1}^n w_{ij} S \left( u_i^{(2)}(t), \sum_{h=1}^p \alpha_{ih} \tilde{\mathcal{E}}_h(t) \right) - \theta_j \right) = g^{-1}(d_2 + \theta), \\
 \vdots \\
 \sum_{j=1}^m v_j f \left( \sum_{i=1}^n w_{ij} S \left( u_i^{(K)}(t), \sum_{h=1}^p \alpha_{ih} \tilde{\mathcal{E}}_h(t) \right) - \theta_j \right) = g^{-1}(d_K + \theta).
 \end{array} \right. \tag{25}$$

exists such a  $\delta > 0$  that when  $\|\mathbf{x}_1(t) - \mathbf{x}_2(t)\| < \delta$ , we have  $|y_1 - y_2| < \varepsilon$ .

*Proof.* To avoid clutter, let us use the following notation.

$$v_j^{(k)} \triangleq \sum_{i=1}^n w_{ij} S \left( x_i^{(k)}(t), \mathcal{E}_i(t) \right) - \theta_j, \quad k = 1, 2. \tag{26}$$

Since  $g(\cdot)$  is continuous, for any  $\varepsilon > 0$ , there exists such a  $\delta_1 > 0$  that when

$$\begin{aligned}
 &\left| \left( \sum_{j=1}^m v_j f \left( v_j^{(1)} \right) - \theta \right) - \left( \sum_{j=1}^m v_j f \left( v_j^{(2)} \right) - \theta \right) \right| \\
 &= \left| \sum_{j=1}^m v_j \left[ f \left( v_j^{(1)} \right) - f \left( v_j^{(2)} \right) \right] \right| < \delta_1, \tag{27}
 \end{aligned}$$

we have  $|y_1 - y_2| < \varepsilon$ . The next step is to prove that for  $\delta_1 > 0$ , there exists such a  $\delta > 0$  that when  $\|\mathbf{x}_1(t) - \mathbf{x}_2(t)\| < \delta$ , Eq. (27) satisfies. Since  $f(\cdot)$  is continuous, for  $\delta_1 > 0$ , there exist such a  $\delta_2 > 0$  that when  $|v_j^{(1)} - v_j^{(2)}| < \delta_2$ , we have

$$|f(v_j^{(1)}) - f(v_j^{(2)})| < \frac{\delta_1}{m \max_{j=0}^m |v_j|}. \tag{28}$$

Therefore, if we choose a  $\delta > 0$  satisfying

$$\begin{aligned}
 &\left| \sum_{j=1}^m v_j \left[ f \left( v_j^{(1)} \right) - f \left( v_j^{(2)} \right) \right] \right| \\
 &\leq \sum_{j=1}^m |v_j| \left| f \left( v_j^{(1)} \right) - f \left( v_j^{(2)} \right) \right| \\
 &\leq m \max_{j=0}^m |v_j| \frac{\delta_1}{m \max_{j=0}^m |v_j|} = \delta_1. \tag{29}
 \end{aligned}$$

■

## VI. CONCLUDING REMARKS

The results presented in this paper revealed the brand new opening to a new world of learning from dynamical data or time series. The potentials of applying VNN to financial world are tremendous if we view the records of intra-day stock prices as different computational verbs and transform the data-mining problem of intra-day stock prices into a learning problem of VNN. Needless to say, many similar cases exist in applications where dynamical observations are of regular time windows and the knowledge discovery from these data becomes impossible if we only view each observations isolated, without considering the embedded natural time-line in them. It is the author's vision that within ten years, computational neural network alone will become a comprehensive discipline in the family of computational verb theory.

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