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A CFD Application to Optimize T-Shaped Fins: Comparisons to the Constructal Theory's Results

The problem of heat removal in energetic processes represents a big challenge especially because of the enhanced requirements of the modern industry. The thermal exchange systems therefore have to guarantee better performances in correspondence to ever more severe dimensional constraints. This paper shows a numerical approach, based on computational fluid dynamics (CFD) software, for the evaluation of the heat exchange performances of finned (straight fins) surfaces made of highly heat conductive material. The same geometric constraints assumed in a reference work were adopted. This research attempts to develop an easy-to-use method to face what was previously solved by the powerful approach of Bejan's Constructal theory. The results obtained show a good agreement between CFD and the Constructal theory's results, validating, therefore, the simplified approach proposed and encouraging its application to a broader variety of geometries. [DOI: 10.1115/1.2756852]

1 Introduction

In the last few decades, industry has been requiring devices that are better performing and ever smaller. This is true in many fields, and the lasting and quality of many industrial components relies on effective heat removal. Heat dissipation is, in fact, observed in each process involving energy exchange. Therefore, designing heat exchangers is one of the most important steps in the definition of a new industrial production, and the contribution of engineering in the shape optimisation is emphasized more every day. Scientific literature offers many experimental examples representing the thermal performances of finned surfaces in the presence of different kinds of boundary conditions. The work in [1] follows this approach. It presents the experimental results of research related to air-cooled finned heat exchangers. Similarly, the research in [2] focuses on the performance in the air side of heat exchangers. It is obvious that these studies limit their application to the few parameters imposed during the physic realization of the samples to be tested. This means that they do not determine a general view on the real phenomenon, characterized by a wide range of variables. The practical need to realize a new material model each time a physical parameter of interest must change makes the experimental approach very expensive, both from a time-evaluation point of view and from an economic one. Other studies in literature are based on a numerical-theoretical approach and give a better overview of the detailed trends regarding the effects in analysis: the mathematical nature of the models is, in fact, created in relation to the punctual characteristics of the examined systems. Papers [3,4] are valid examples of the latter kind of research because they analyze the performance of finned heat exchanger systems for different geometries. The paper from which this research begins [5] is, however, a theoretical analysis of the thermal behavior of a simple T-shaped structure, composed of single prismatic entities. The reference research gains a relation between some dimensionless parameters of interest and the dimensionless conductance, used as a performance evaluation tool. In the geometrical optimization of the T-shaped fins, Bejan and Almogbel [5] apply the Constructal theory [6] to study the trends of this reference parameter, defining the quality of heat exchange. Based on the evaluation of some ratios between the dimensions of the system, a final configuration is obtained that optimizes the target variable, following a maximization criterion. In particular, the approach presented in [5] is important for its numerical precision and for its absolute novelty, leading to a very powerful but not elementary implementation of the effective model presented. This paper, beginning with the approach in [5], is aimed at showing the effectiveness and the satisfactory quality of the results of a novel approach based on the use of a finite element CFD code. The advantage of this choice is that of making numerically complicated models more accessible for industrial use.

2 Model Definition

In accordance to the numerical domain defined in [5], a T-shaped straight fin composed of three parallelepiped subdomains is considered here. One is vertical with a thickness equal to t_1 and a length equal to L_1 two are horizontal with a thickness equal to t_0 and a length equal to L_0 (Fig. 1).

The third spatial dimension W is chosen to be long enough, as compared to L_1 and L_0 , to avoid checkerboard effects. The convection coefficient h, considered as a boundary condition, is uniform throughout the domain. The root temperature T_1 and the surrounding fluid temperature T_{∞} are also known. The objective of the following analysis is to determine the optimal geometric ratios L_1/L_0 and t_1/t_0 , which allow the highest heat flux q_1 through the root and that guarantee the biggest value of the dimensionless conductance q_1^*

$$q_1^* = \frac{q_1}{kW(T_1 - T_\infty)}$$
(1)

where q_1 is the thermal power through the fin root, k is the thermal conductivity; T_1 and T_{∞} are, as mentioned above, the root and fluid temperatures, respectively.

The geometric model supplied in input to the simulation code was realized in CAD environment and transferred with the IGES format. In their analytical approach [5], Bejan and Almogbel do not use numerical values for defining the geometrical dimensions of the heat exchanger, such as the thickness *t* and the length *L* of the fins: their investigation relates, on the contrary, to the dimensionless ratios $k_1=t_1/t_0$ and $k_2=L_1/L_0$. For the present approach, it is, instead, initially necessary to define some numerical values to then define the dimensions of the fin within the interface of the

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Fig. 1 Geometric definition of the model

code. This starts by assigning some geometrical constraints, such as the total volume occupied by the system and the volume materially occupied by the fin. The same constraints were adopted in [5] and will be better described later on. Obviously, having the third dimension W be constant and the thermal effect in this direction negligible, in the following treating the surfaces can substitute the volumes,

$$A = 2L_0L_1$$

$$A_f = 2L_0 \cdot t_0 + L_1t_1$$
(2)

Through these expressions, it is possible to define the volume ratio φ between the surface of the fin and the surface actually occupied by the system

$$\varphi = \frac{A_f}{A} \tag{3}$$

Using this expression, it is possible to obtain the analytical expression of L_0 , the only unknown datum, since the other dimensions are reckoned directly from the characteristic values of the *i*th test, once assigned k_1 and k_2

$$L_{0} = \frac{1}{2\varphi} t_{0} \left(\frac{2}{k_{2}} + k_{1} \right)$$
 (4)

The value of φ is initially imposed as equal to 0.1 and is kept constant for each test, so that the value of t_0 is considered equal to 1 throughout the computations. The boundary conditions are defined starting from the convection coefficient; *h* is uniform in each domain but varies from case to case, depending on the dimensions of the fin, as its definition proves,

$$h = \frac{a^2 k}{2\sqrt{A}} \tag{5}$$

It can be observed that h is directly proportional to the thermal conductivity k and to the value of the dimensionless parameter a, which analyzes the thermal effect due to convection in its different



Fig. 2 Example of used mesh

forms [5]. The effect of *a* on the global process is going to be studied later in this paper. The root temperature T_1 and that of the fluid T_{∞} , are set as 100°C and 20°C, respectively. This choice, completely arbitrary, does not affect the general applicability of this treating, as demonstrated by the definition of q_1^* , which is the parameter to be optimized. According to the same dimensionless approach and in agreement with [5], it is adopted a thermal conductivity *k* equal to 200 W/(m°C), which practically corresponds to aluminium. Finally, the opportune value for *W* was chosen to make the checkerboard effects negligible over the global performance of the model.

3 Method and Tests

The tool used in the numerical computations is COMSOL 3.2, a modern CFD software that provides a graphical interface that, by itself, allows for the generation of simple geometries. However, in the cases presented here, a CAD modeling software has been utilized, using the IGES format to export the files created to the solver. Once the model has been imported, an opportune mesh was generated. Its number of elements varied from case to case, but 121,700 was the mean value. Figure 2 displays an example of mesh.

The mesh has been generated to guarantee a better refinement in each critical zone. Among them, the intersections between horizontal and vertical surfaces have shown considerable thermal gradients. This study began by considering those values of k_1 and k_2 that, in [5], maximized the dimensionless conductance. Their definition can be expressed in the following relationships:

$$k_1 = \frac{t_1}{t_0} \quad k_2 = \frac{L_1}{L_0} \tag{6}$$

Based on the dimensionless ratios just quoted, the tests performed were those (in gray) in Table 1.

4 Results and Comments

The numerical implementations of the CAD models imported in COMSOL resulted, obviously, in different temperature distributions

Table 1 Tests performed

			$k_2 = L_1 / L_{\diamond}$										
			0,0600	0,0625	0,0650	0,0675	0,0700	0,0725	0,0750	0,0775	0,0800	0,0825	0,0850
	k ₁ =t ₁ /t ₀	2											
		з											
		4											
		5											
		6											
		7											

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Fig. 3 Graphic output of the thermal field

from case to case. Anyway, all of them had a qualitative trend similar to that in Fig. 3.

As the output shows, the temperature field has its highest value in correspondence to the root of the system and decreases gradually toward the lateral sides of the fin.

The parameters investigated in this analysis are the same as in [5]: the purpose of this research is, in fact, to obtain results with a more elementary method (because it is based on a commercial CFD code) that match to that of Bejan and Almogbel [5]. The first objective is therefore to optimize the dimensionless thermal conductance in dependence of k_1 and k_2 , previously defined. As it is shown in Fig. 4, the highest values of q_1^* independently from the parameter k_1 are obtained for values of k_2 in the vicinity of 0.07, which is in very good agreement with the results in [5], also represented in Fig. 4. The best performing configuration obtained

in this research, however, is that corresponding to a value of $k_1 = 5$, whereas in [5] $k_1=4$ was the "optimal" value (Fig. 4). The two different trends that were compared prove, anyway, to be very close one another, and this makes the reciprocal distance negligible. Again in analogy with the study in [5], starting from the optimal value for k_2 just found, the trend of the dimensionless thermal conductance q_1^* with the ratio k_1 was analyzed. A convex trend can be observed (represented in Fig. 5) which shows the maximum value in correspondence to a ratio k_1 of 5; $k_1=4$ and $k_1=6$ also determine a curve close to the "best" one. Also in this second part of the work, the numerical values that were reached show a good correspondence with [5], even for what relates to the shape of the curve.

The last part of the optimization process intended to study the



Fig. 4 Dimensionless conductance as a function of k_1 and k_2

326 / Vol. 129, SEPTEMBER 2007

Transactions of the ASME



Fig. 5 Variation of q_1^* in function of k_1

variation of the dimensionless thermal conductance in function of the parameters a and φ , previously defined, holding constant and equal to the optimum value the fundamental ratios k_1 and k_2 . The result of this analysis is displayed in Fig. 6. It can be observed that the curves are characterized by a strong parallelism, as proved by Bejan and Almogbel [5]. Their results show a tight coincidence with those obtained here.

5 Conclusions

The problem of heat removal from heat-generating systems has significant relevance and characterizes the whole industrial production in many forms. These applications require ever higher heat exchange performances associated with a clear trend of dimensional reduction of the systems involved. Such a dynamic behavior in the field of industrial heat exchange implies that projecting and testing new generation systems in a virtual environment could address both the requirements of high-quality design-



Fig. 6 Variation of q_1^{\star} in function of *a*

ing and of low expenses in research and development. With these purposes in mind, this research used a commercial CFD code to analyze the case of heat exchange in the presence of T-shaped fins, following an approach suggested by Bejan's Constructal theory. The comparative results we have gained showed a significant agreement with previous research taken as the main reference, and this result allows for the application of the approach here presented to a wider range of systems, as a result of its features of practicality and simplicity.

Nomenclature

- a = dimensionless parameter, Eq. (5)
- $A = \operatorname{area} (\mathrm{m}^2)$
- h = heat transfer coefficient (W m⁻² K⁻¹)
- $k = \text{fin thermal conductivity } (W \text{ m}^{-1} \text{ K}^{-1})$

 k_1, k_2 = dimensionless ratios

- L = length (m)
- q = heat flux (W)
- $t = \text{thickness}(\mathbf{m})$
- T = temperature (K)
- $V = \text{volume } (\text{m}^3)$
- W =width (m)

Greek Letters

 φ = volume ratio of the fin, Eq. (3)

Subscripts

0 = horizontal arm

- 1 = root and vertical arm
- f = related to the fin

Superscript

(*) = dimensionless variables, Eq. (1)

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