On the Optimization of Hybrid Raman/Erbium-Doped Fiber Amplifiers

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***Abstract*—A comprehensive theoretical study on the optimal configuration of hybrid Raman/erbium-doped fiber amplifiers has been carried out yielding a closed form analysis. In order to compare different system configurations, a weight for the impact of fiber nonlinearities has been introduced. The maximum reachable distance has been evaluated as a function of the span length and nonlinear weight, given a target optical signal-to-noise ratio.**

***Index Terms*—Nonlinearities, optical fiber amplifiers, optical noise, Raman scattering.**

I. INTRODUCTION

HYBRID Raman/erbium-doped fiber amplifiers (HFAs) are an enabling and promising technology for future dense wavelength-division-multiplexing (DWDM) multiterabit systems, as it has been shown in recent experimental results [1], [3]. Hybrid Raman/erbium-doped fiber amplifiers are designed in order to maximize the span length and/or to minimize the impairments of fiber nonlinearities, and to enhance the bandwidth of erbium-doped fiber amplifiers (EDFAs). In this letter, we do not look at the bandwidth issue and concentrate on the other two.

The design of an optimal HFA is a complex problem with several degrees of freedom. In [4], a preliminary theoretical analysis was presented. This letter extends this analysis to a general system setup in order to draw some general rules. An expression for the optical signal-to-noise ratio (OSNR) at the receiver is analytically derived in Section II, taking into account Rayleigh backscattering effects on the amplified spontaneous emission (ASE) noise introduced by a Raman amplifier (RA) [5]. Then, an optimization technique is described to evaluate the optimum gain balance between RAs and EDFAs that allows to achieve the maximum reachable distance.

In Section III, the developed analysis is applied to two different systems based on SMF and nonzero dispersion-shifted fiber (NZDSF), respectively. The maximum reachable distance is plotted as a function of span length and nonlinear weight given a target OSNR.

II. ANALYSIS

The analyzed system setup is shown in Fig. 1. We considered a long-haul system composed of N_{span} spans.

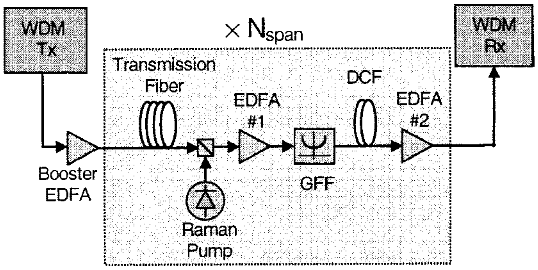

Fig. 1. The analyzed system setup. We also considered the situation where the DCF is replaced by a fiber grating.

Each span is made up of a stretch of transmission fiber backward Raman pumped, an EDFA #1 with gain G_{E1}, a gain flattening filter (GFF), a dispersion compensating fiber (DCF), and an EDFA #2 with gain G_{E2}. Fiber length and dispersion parameters are L_{span} and D for the transmission fiber, and L_{DCF} and D_{DCF} for the DCF fiber.

The total link length is $L_{\mathrm{tot}} = N_{\mathrm{span}} L_{\mathrm{span}}$. The transmission fiber is backward pumped in order to get a Raman on–off gain G_{RA}[6]. L_{DCF} is set by the degree k_{comp} of dispersion compensation: $L_{\mathrm{DCF}} = -k_{\mathrm{comp}} D / D_{\mathrm{DCF}}$. Note that $k_{\mathrm{comp}} = 1$ means that dispersion is completely compensated every span. We also assume that gains are set so as to perfectly compensate for the total loss of the link yielding

$$\begin{aligned} G_{\mathrm{tot}} &= G_{RA} G_{E1} G_{E2} \\ &= \frac{1}{\exp(-\alpha_S L_{\mathrm{span}}) T_F \exp(-\alpha_{\mathrm{DCF}} L_{\mathrm{DCF}})} \end{aligned} \tag{1}$$

where α_S and α_{DCF} are the fiber loss coefficients for the transmission and DCF fiber, respectively, and T_F is the loss introduced by the GFF. Equation (1) sets the value of G_{tot}, but leaves individual gains undetermined.

If we consider the propagation of signal and noise over the system represented in Fig. 1 under the transparency condition of (1), the OSNR at the receiver can be expressed as shown in (2), shown at the bottom of the next page, where P_{TX} is the average optical power-per-channel at the input of each span, h is Planck's constant, f is the optical carrier frequency, B_O is the bandwidth over which optical noise is integrated in order to calculate the OSNR. $n_{\mathrm{eq,RA}}$ is the *equivalent input noise factor* [7] for the inline RAs that can be derived from the noise figure definition introduced in [4], it includes the effects of Rayleigh backscattering on noise. $n_{sp,E1}$ and $n_{sp,E2}$ are the *spontaneous emission factors* [7] for the two EDFAs. The factor $n_{\mathrm{eq,RA}}$ depends on pump power, fiber length, the Raman efficiency of the fiber, and on the Rayleigh backscattering capture factor.

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In order to compare system configurations with different Raman G_{RA}'s, and therefore, with different power profiles along z, we need an estimate of the impact of the Kerr nonlinearity over the whole link. We use the parameter $k_{\rm NL}$ [4] called *nonlinear weight*, which is, mathematically, the overall nonlinear phase-shift experienced by a single channel over the whole transmission link. It is defined as

$$\begin{aligned} k_{\rm NL} &= \int_{\rm link} \gamma(z) P(z)\, dz \\ &= P_{TX} N_{\rm span} [\gamma L_{\rm eff} + \gamma_{\rm DCF} \exp(-\alpha_S L_{\rm span}) \\ &\qquad \cdot G_{RA} G_{E1} T_F L_{\rm eff,\,DCF}] \end{aligned} \tag{3}$$

where γ and $\gamma_{\rm DCF}$ are the nonlinear coefficients, $L_{\rm eff}$ and $L_{\rm eff,DCF}$ are effective lengths for the transmission and DCF fibers, respectively. $k_{\rm NL}$ is [rad]. Note that the effective length of the transmission fiber depends on the Raman gain, because it is defined as the integral of the power profile along the fiber. This already shows that if one wants to keep the impact of nonlinearity within set limits, the launched power must be reduced when Raman amplification is used.

We then define P_1 as the value of P_{TX}, which makes $k_{\rm NL} = 1$ after only one span. P_1 is found forcing (3) to 1 with $N_{\rm span} = 1$. We also define N_1 as the amount of noise entering the system every span, i.e., the denominator of (2) with $N_{\rm span} = 1$. Then, it turns out that the transmitted power can be written as $P_{TX} = k_{\rm NL} P_1 / N_{\rm span}$, and the total amount of noise at the end of the link is $N_{\rm tot} = N_{\rm span} N_1$. Therefore, the OSNR at the receiver can be expressed as follows

$$\mathrm{OSNR} = \frac{k_{\rm NL} P_1}{N_{\rm span}^2 N_1} = \frac{k_{\rm NL}}{N_{\rm span}^2} \mathrm{OSNR}_1 \tag{4}$$

where OSNR_1 is the ratio of P_1 to N_1 defined above. OSNR_1 depends on the span length $L_{\rm span}$, and given the span length, it assumes different values for each individual gain of the amplifiers even if their product is fixed by the transparency condition of (1). The optimization process that we propose is the search of the individual gain of each amplifier that maximizes OSNR_1. The parameters that we vary are $k_1 = G_{RA,\rm dB}/(G_{RA,\rm dB} + G_{E1,\rm dB})$, indicating how much gain is provided by the RA out of the overall gain before the GFF, and $k_2 = (G_{RA,\rm dB} + G_{E1,\rm dB})/G_{\rm tot,dB}$, showing how much of $G_{\rm tot,dB}$ is provided before the GFF. Note that the expressions of k_1 and k_2 refer to gains ($G_{RA,\rm dB}$, $G_{E1,\rm dB}$, and $G_{\rm tot,dB}$) expressed in decibels. The goal of the optimization process is to find a lookup table giving, for each chosen value of $L_{\rm span}$, the maximum $\mathrm{OSNR}_1^{\rm opt}$ and the optimal (k_1, k_2), called $(k_1, k_2)_{\rm opt}$. Note that $(k_1, k_2)_{\rm opt}$ sets the gain of each amplifier, and therefore, also determines the required Raman pump power. Given this lookup table, further optimizations can be performed such as the maximization of $L_{\rm span}$, the minimization of the impact of nonlinearities $k_{\rm NL}$, and the evaluation of the maximum reachable distance. We focus our attention on the maximum reachable distance evaluation as a function of $L_{\rm span}$, given a minimum $\mathrm{OSNR}=\mathrm{OSNR}_{\min}$, and a maximum tolerable $k_{\rm NL} = k_{\rm NL}^{\max}$. From (4), remembering that $N_{\rm span}$ is equal to the ratio of the total length to the span length, the expression of the maximum reachable distance can be derived and assumes the following form

$$L_{\max} = \mathrm{int}\left\{ \sqrt{\frac{k_{\rm NL}^{\max}}{\mathrm{OSNR}_{\min}} \mathrm{OSNR}_1^{\rm opt}} \right\} L_{\rm span}. \tag{5}$$

TABLE I
PARAMETERS OF THE CONSIDERED FIBERS

Fiber	Loss [dB/km] Signal	Loss [dB/km] Pump	D [ps/nm/km]	A_{eff} [µm²]	γ [1/W/km]	Raman eff [dB/W]
SMF	0.2	0.3	16	80	1.27	26
NZDSF	0.2	0.3	5	55	1.85	38
DCF	0.5	-	-100	25	4.10	-

Notice that a different optimal HFA configuration corresponds to each $L_{\rm span}$, as well as a different gain balance between RA and EDFAs, a different Raman pump, and a different level of launched power-per-channel. From (5), one can also see that $L_{\max}$ grows only as $\sqrt{k_{\rm NL}^{\max}}$.

III. RESULTS AND DISCUSSION

We applied the above analysis to systems using SMF or NZDSF. The attributes of these fibers are reported in Table I. The values are typical and are not necessarily related to specific commercial fibers. Moreover, we assumed to completely compensate dispersion ($k_{\rm comp} = 1$) at each span. We assumed that compensation was done by either inserting a DCF span of proper length, as shown in Fig. 1, or using a fiber grating (FG) in place of the DCF. FGs cause extra loss, but do not add any nonlinearity. We assumed that the GFF brings 4 dB of loss (T_F) referred to the most attenuated channel. Erbium-doped fiber amplifier noise figure has been set to 4.5 dB.

Fig. 2 depicts contour levels of the OSNR_1 surface in the plan of (k_1, k_2) for the analyzed situation based on $\mathrm{SMF} + \mathrm{DCF}$ with $L_{\rm span} = 50$ km and $L_{\rm DCF} = 8$ km. Every scenario is characterized by a different surface, one for each of the tested span lengths. Searching for the optimal (k_1, k_2) means detecting the maximum of the surface. For the plotted case, the maximum $\mathrm{OSNR}_1 = 61.78$ dB corresponds to $(k_1, k_2) = (0.8, 0.85)$, that implies $G_{RA,\rm dB} = 12.2$ dB, $G_{E1,\rm dB} = 2.9$ dB, and $G_{E2,\rm dB} = 2.7$ dB. It can be observed $OSNR_1$ is less sensitive to a suboptimal choice of k_2 than of k_1. It means that the product $G_{RA}\, G_{E1}$ must be set more carefully than their ratio.

Fig. 3(a) and (b) show results of the maximum reachable distance $L_{\max}$ as a function of $L_{\rm span}$ given a target OSNR of

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$$\mathrm{OSNR} = \frac{P_{TX}}{hfB_O N_{\rm span}\left[\left(n_{\rm eq,\,RA} + n_{sp,\,E1}\dfrac{G_{E1} - 1}{G_{E1} G_{RA}}\right)\exp(+\alpha_S L_{\rm span}) + n_{sp,\,E2}(G_{E2} - 1)\right]} \tag{2}$$

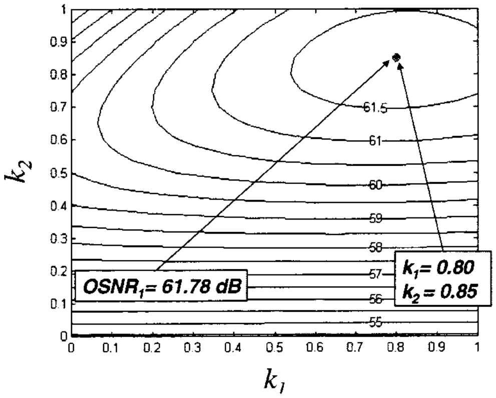

Fig. 2. Contour plot of OSNR_1 in the (k_1, k_2) plan for a system based on SMF + DCF with span-length $L_{\mathrm{span}} = 50$ km.

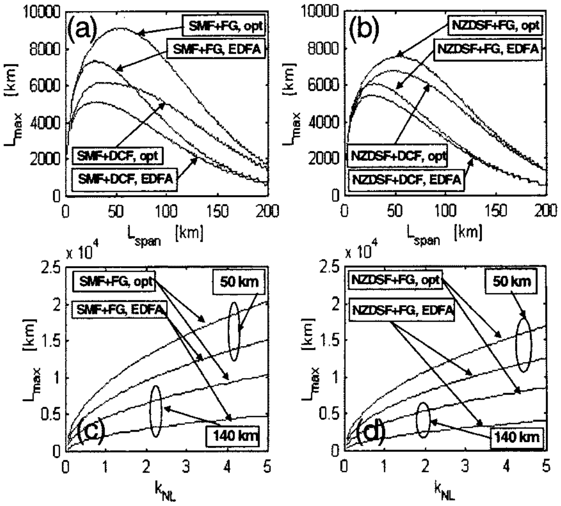

Fig. 3. Maximum reachable distance versus span length with OSNR $= 20$ dB over 0.1 nm and $k_{\mathrm{NL}} = 1$. (a) SMF-based system. (b) NZDSF-based system. For both systems, compensation based on either DCF or FG is considered. Results for amplification employing only EDFAs are plotted for comparison. For the same system setup, the maximum reachable distance is plotted against k_{NL}. (c) SMF. (d) NZDSF fiber. Span lengths of 50 and 140 km are considered for these plots.

20 dB over 0.1 nm, and a nonlinear weight $k_{\mathrm{NL}} = 1$. Results of systems based on only EDFAs reported for comparison. Fig. 3(c) and (d) show $L_{\max}$ plotted as a function of k_{NL} with OSNR$= 20$ dB over 0.1 nm, for two different span lengths of 50 and 140 km. All of the mentioned plots refer to the optimal configuration, which means that each point of the curves represents a different system configuration. In Fig. 3(a) and (b), an optimal L_{span} can always be observed. Its value is around 30 km when only EDFAs are used, while it is increased up to 50–60 km in case HFAs are used. For both the SMF and NZDSF systems, the best results are obtained using FGs instead of the DCF. The reason is that we assume that FGs do not induce nonlinear effects, but just loss. In addition, we assume the same loss for any amount of dispersion to be recovered. SMF allows a maximum reachable distance greater than the one obtained with NZDSF. The explanation is the different nonlinearity of the two fibers. In fact, the ratio of the maximum reachable distances (~9000 km versus 7500 km) is roughly equal to the square root of the ratio of the effective areas (80/55). When the DCF is used, the system based on SMF presents a stronger degradation, because of the longer (~3 times) stretch of highly nonlinear DCF needed to fully compensate for the dispersion. In this case, the NZDSF performs better. Regardless of fiber, the combined use of RA and EDFA always allows a substantial increase in L_{span}. For instance, to reach 4000 km using NZDSF with OSNR$= 20$ dB over 0.1 nm in a system tolerating $k_{\mathrm{NL}} = 1$, a maximum $L_{\mathrm{span}} \approx 80$ km is allowed when only EDFAs are used. If hybrid amplification is used, L_{span} can be as large as 140 km [see Fig. 3(b)].

Fig. 3(c) and (d) tell us how much nonlinearity the system must tolerate, expressed as k_{NL}, to reach a certain distance $L_{\max}$, given L_{span}. The plotted results refer to setups, where compensation is achieved by means of FG, both for the optimal HFA and for the EDFA-only case. It can be observed that the use of a proper amount of RA is especially advantageous for systems that must operate with low nonlinearity (small k_{NL}). For instance [see Fig. 3(d)], a 5000-km-long system based on NZDSF + FG, with $L_{\mathrm{span}} = 50$ km, must tolerate $k_{\mathrm{NL}} = 1$ if only EDFAs are used. The nonlinear impact is reduced to $k_{\mathrm{NL}} = 0.5$ when choosing the optimal HFA. Enlarging the fiber span to 140 km, the optimal HFA configuration yields $k_{\mathrm{NL}} = 3$, whereas if only EDFAs are used, the system should strive with much bigger nonlinearity $(k_{\mathrm{NL}} \gg 5)$.

Once the system has been designed using the presented analysis, a full phenomena split-step simulation is needed in order to verify the penalty induced by nonlinear propagation effects. If the penalty is too large, the design process must be redone reducing k_{NL}.

IV. CONCLUSION

We have derived an analytical expression for the receiver OSNR in transmission systems based on HFAs. The analysis could be easily extended to other system setups than the one shown in Fig. 1.

The results show that Raman amplification, combined with EDFAs, allow the increase of the maximum reachable distance and/or the span length. Raman amplification can also be used to substantially reduce the impact of fiber nonlinearity.

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