Network Coding for Wireless Applications: A Brief Tutorial

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***Abstract*— The advent of network coding promises to change many aspects of networking. Network coding moves away from the classical approach of networking, which treats networks as akin to physical transportation systems. We overview some of the main features of network coding that are most relevant to wireless networks. In particular, we discuss the fact that random distributed network coding is asymptotically optimal for wireless networks with and without packet erasures. These results are extremely general and allow packet loss correlation, such as may occur in fading wireless channels. The coded network lends itself, for multicast connections, to a cost optimization which not only outperforms traditional routing tree-based approaches, but also lends itself to a distributed implementation and to a dynamic implementation when changing conditions, such as mobility, arise. We illustrate the performance of such optimization methods for energy efficiency in wireless networks and propose some new directions for research in the area.**

I. Introduction

The notion of coding at the packet level—commonly called network coding—has attracted significant interest since the publication of [1], which showed the utility of network coding for multicast in wireline packet networks. But the utility of network coding reaches much further. And, in particular, it reaches to include various wireless applications. In fact, wireless packet networks are a most natural setting for network coding because the very characteristics of wireless links that complicate routing, namely, their unreliability and broadcast nature, are the very characteristics for which coding is a natural solution. Couple this with the fact that we are not nearly as constrained in our protocol design choices in the wireless case as we are in the wireline one, and applying network coding to wireless packet networks seems an ideal way of achieving appreciable efficiency gains.

The attractiveness of marrying network coding and wireless packet networks has not escaped the notice of researchers, and numerous papers (e.g., [2]–[8]) have appeared on the subject. In this paper, we overview some of the main features of network coding that are most relevant to wireless networks. In particular, we discuss, first, the asymptotic optimality of random distributed network coding for wireless networks with and without packet erasures; and, second, the superior performance and relative ease of cost optimization in coded wireless networks as opposed to traditional routed wireless networks. The overview we give will only be very brief, and we refer readers interested in more details to the research papers [4], [5], [6] and the more detailed review paper [9].

II. Network model

We model the network with a directed hypergraph $\mathcal{H} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{A}$ is the set of hyperarcs. A hypergraph is a generalization of a graph, where, rather than arcs, we have hyperarcs. A hyperarc is a pair (i, J), where i, the start node, is an element of $\mathcal{N}$ and J, the set of end nodes, is a non-empty subset of $\mathcal{N}$.

Each hyperarc (i, J) represents a broadcast link from node i to nodes in the non-empty set J. This link may be lossless or lossy, i.e. it may or may not experience packet erasures. Let A_{iJK} be the counting process describing the arrival of packets that are injected on hyperarc (i, J) and received by the set of nodes $K \subset J$, i.e. for $\tau \geq 0$, $A_{iJK}(\tau)$ is the total number of packets received between time 0 and time τ by all nodes in K due to (i, J). We assume that A_{iJK} has an average rate z_{iJK}; more precisely, we assume that

$$\lim_{\tau \to \infty} \frac{A_{iJK}(\tau)}{\tau} = z_{iJK} \tag{1}$$

almost surely. When links are lossless, we have $z_{iJK} = 0$ for all $K \subsetneq J$, and, for convenience, we write z_{iJ} for z_{iJJ}. The rate vector z is called the coding subgraph and can be varied within a constraint set Z dictated to us by lower layers. We reasonably assume that Z is a convex subset of the positive orthant containing the origin.

We associate with the network a cost function f that maps feasible coding subgraphs to real numbers and that we seek to minimize. For wireless networks, it is common for the cost function to reflect energy consumption, but it could also represent, for example, average latency, monetary cost, or a combination of these considerations.

III. Random distributed network coding

In the section, we discuss how random distributed network coding can achieve the capacity of single multicast connections in a given coding subgraph z. As a consequence, in setting up optimal single multicast connections in a network, there is no loss of optimality in separating the problems of subgraph selection and coding, i.e. separating the optimization for a minimum-cost subgraph, which we discuss in the following section, and the construction of a code for a given subgraph, which we now discuss. We consider multicast connections as they are the most general type of single connection, including unicast and broadcast as special cases.

Given a coding subgraph z, it is shown in [4], [5] that a multicast of rate arbitrarily close to R is achievable with coding from source node s to sink nodes in the set T if and only if there exists, for all $t \in T$, a flow vector $x^{(t)}$ satisfying

$$\sum_{\{J|(i,J)\in\mathcal{A}\}} \sum_{j\in J} x_{iJj}^{(t)} - \sum_{\{j|(j,I)\in\mathcal{A}, i\in I\}} x_{jIi}^{(t)} = \sigma_i^{(t)} \tag{2}$$

for all $i \in \mathcal{N}$, and

$$\sum_{j\in K} x_{iJj}^{(t)} \leq \sum_{\{L\subset J|L\cap K\neq\emptyset\}} z_{iJL} \tag{3}$$

for all $(i, J) \in \mathcal{A}$ and $K \subset J$, where

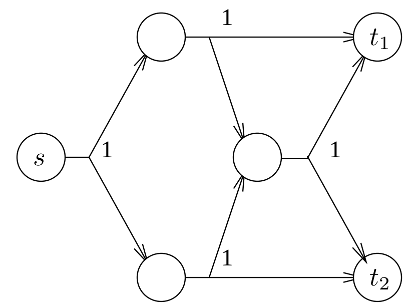
$$\sigma_i^{(t)} = \begin{cases} R & \text{if } i = s, \\ -R & \text{if } i = t, \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, a coding scheme that achieves a connection of rate arbitrarily close to R from such a coding subgraph is capacity achieving. In the general lossy case, the only practical capacity-achieving scheme we currently have is the one described in [4], [5]. This scheme is a random distributed coding scheme, just as the schemes in [10], [11], but it deals specifically with lossy networks.

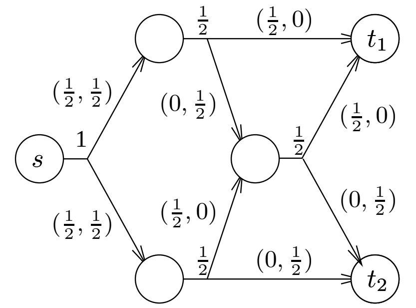
We give a brief description of the scheme. We suppose that, at the source node s, we have K message packets $w_1, w_2, \ldots, w_K$, which are vectors of length ρ over the finite field $\mathbb{F}_q$. (If the packet length is b bits, then we take $\rho = \lceil b/\log_2 q \rceil$.) The message packets are initially present in the memory of node s.

The coding operation performed by each node is simple to describe and is the same for every node: Received packets are stored into the node's memory, and packets are formed for injection with random linear combinations of its memory contents whenever a packet injection occurs on an outgoing hyperarc. The coefficients of the combination are drawn uniformly from $\mathbb{F}_q$. Since all coding is linear, we can write any packet x in the network as a linear combination of $w_1, w_2, \ldots, w_K$, namely, $x = \sum_{k=1}^{K} \gamma_k w_k$. We call γ the *global encoding vector* of x, and we assume that it is sent along with x, in its header. The overhead this incurs (namely, $K \log_2 q$ bits) is negligible if packets are sufficiently large.

A sink node collects packets and, if it has K packets with linearly-independent global encoding vectors, it is able to recover the message packets. Decoding can be done by Gaussian elimination. In addition, the scheme can be operated ratelessly, i.e. it can be run indefinitely until all sink nodes in T can decode (at which stage that fact is signaled to all nodes, requiring only a small amount of feedback).



(a) Each hyperarc is marked with its cost per unit rate.



(b) Each hyperarc is marked with z_{iJ} at the start and with the pair $(x_{iJj}^{(1)}, x_{iJj}^{(2)})$ at the ends.

Fig. 1. A wireless network with multicast from s to $T = \{t_1, t_2\}$.

The scheme is remarkably robust: If run for a sufficiently long period of time, it achieves the maximum feasible rate of a given subgraph, with only assumption (1) on the arrival of received packets on a link. Assumption (1) makes no claims on loss correlation or lack thereof—all we require is that a long-run average rate exists. This fact is particularly important in wireless packet networks, where slow fading and collisions often cause packets not be received in a steady stream.

IV. Cost optimization

We now turn to the subgraph selection problem, which we see is the problem of finding a coding subgraph z of minimum cost satisfying (2) and (3):

$$\begin{aligned} &\text{minimize } f(z) \\ &\text{subject to } z \in Z, \\ &\sum_{j\in K} x_{iJj}^{(t)} \leq \sum_{\{L\subset J|L\cap K\neq\emptyset\}} z_{iJL}, \\ &\qquad\qquad \forall\ (i, J) \in \mathcal{A},\ K \subset J,\ t \in T, \\ &\sum_{\{J|(i,J)\in\mathcal{A}\}} \sum_{j\in J} x_{iJj}^{(t)} - \sum_{\{j|(j,I)\in\mathcal{A}, i\in I\}} x_{jIi}^{(t)} = \sigma_i^{(t)}, \\ &\qquad\qquad \forall\ i \in \mathcal{N},\ t \in T, \\ &x_{iJj}^{(t)} \geq 0, \qquad \forall\ (i, J) \in \mathcal{A},\ j \in J,\ t \in T. \end{aligned} \tag{4}$$

As an example, consider the wireless network depicted in Figure 1(a). Suppose the network is lossless. We wish to achieve multicast of unit rate to two sinks, t_1 and t_2. We have $Z = [0, 1]^{|\mathcal{A}|}$ and $f(z) = \sum_{(i,J)\in\mathcal{A}} a_{iJ} z_{iJ}$, where a_{iJ} is

Network size	Approach	Average multicast energy			
		2 sinks	4 sinks	8 sinks	16 sinks
20 nodes	MIP algorithm	30.6	33.8	41.6	47.4
	Network coding	15.5	23.3	29.9	38.1
30 nodes	MIP algorithm	26.8	31.9	37.7	43.3
	Network coding	15.4	21.7	28.3	37.8
40 nodes	MIP algorithm	24.4	29.3	35.1	42.3
	Network coding	14.5	20.6	25.6	30.5
50 nodes	MIP algorithm	22.6	27.3	32.8	37.3
	Network coding	12.8	17.7	25.3	30.3

TABLE I

AVERAGE ENERGY OF RANDOM MULTICAST CONNECTIONS OF UNIT RATE FOR VARIOUS APPROACHES IN RANDOM WIRELESS NETWORKS OF VARYING SIZE. NODES WERE PLACED RANDOMLY WITHIN A 10×10 SQUARE WITH A RADIUS OF CONNECTIVITY OF 3. THE ENERGY REQUIRED TO TRANSMIT AT UNIT RATE TO A DISTANCE d WAS TAKEN TO BE d^2. SOURCE AND SINK NODES WERE SELECTED ACCORDING TO AN UNIFORM DISTRIBUTION OVER ALL POSSIBLE SELECTIONS.

the cost per unit rate shown beside each hyperarc. An optimal solution to problem (4) is shown in Figure 1(b). We have flows $x^{(1)}$ and $x^{(2)}$ of unit size from s to t_1 and t_2, respectively, and, for each hyperarc (i, J), $z_{iJ} = \max(\sum_{j \in J} x^{(1)}_{iJj}, \sum_{j \in J} x^{(2)}_{iJj})$, as we expect from the optimization.

One specific problem of interest is that of minimum-energy multicast (see, for example, [12], [13]). In this problem, we wish to achieve minimum-energy multicast in a lossless wireless network without explicit regard for throughput or bandwidth, so the constraint set Z can be dropped altogether. Moreover, the cost function is separable and linear, i.e. $f(z) = \sum_{(i,J)\in\mathcal{A}} a_{iJ} z_{iJ}$, where a_{iJ} represents the energy required to transmit to nodes in J from node i for some fixed time interval. Hence problem (4) becomes a linear optimization problem with a polynomial number of constraints, which can therefore be solved in polynomial time. By contrast, the same problem using traditional routing-based approaches is NP-complete—in fact, the special case of broadcast in itself is NP-complete [13]. The problem must therefore be addressed using polynomial-time heuristics such as the Multicast Incremental Power (MIP) algorithm in [12]. Even if an optimal routing solution is found, it is in general worse than an optimal coding solution because coding subsumes routing. Thus coding promises to significantly outperform routing for practical multicast. In simulations, we observed reductions ranging from 13% to 49% in the average total energy of random multicast connections in random wireless networks of varying size as a result of coding as opposed to routing with the MIP algorithm (see Table I).

Not only can subgraphs for minimum-energy multicast be computed in polynomial time in coded wireless networks, however, they can be computed by a distributed algorithm. The algorithm, which is described in [6], requires each node only to know the costs of its incoming and outgoing hyperarcs and to communicate the results of its computations with its neighbors. When coupled with distributed random network coding, the algorithm yields a complete distributed solution.

We have thus far discussed static networks. But, in many wireless networks, conditions change. We should then, when given some predictive information regarding movement or traffic trends, adjust our optimization to account in balanced fashion for both current and future network states and demands. Some results [6] indicate that using network coding in dynamic environments may lead to dynamic programming approaches that naturally extend the static optimization techniques we have discussed. Exploring this extension is just one of many avenues for future work; others are exploring the extension to correlated sources and studying specific instances where problem (4)—posed in a very general form—simplifies.

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