

A Novel Energy Detection Scheme Based on Dynamic Threshold in Cognitive Radio Systems^{*}

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Abstract

Traditional energy detection algorithm is bad in anti-noise. In this paper, the relationship of energy detection performance and detection sensitivity with average noise power fluctuation in short time is investigated. Detection performance and detection sensitivity drops quickly with the increment of average noise power fluctuation and becomes worse in low signal-to-noise ratio. To the characteristic, a new energy detection algorithm based on dynamic threshold is presented. Theoretic results and simulations show that the proposed scheme removes the falling proportion of performance and detection sensitivity caused by the average noise power fluctuation with a choice threshold, and also improves the antagonism of the average noise power fluctuation in short time and obtains a good performance. Detection sensitivity and performance improves as the dynamic threshold factor increasing.

Keywords: Cognitive Radio; Energy Detection; Average Noise Power Fluctuation; Average Noise Power Invariability; Detection Sensitivity; Dynamic Threshold

1 Introduction

Cognitive radio spectrum sensing under cognitive problems is that the cognitive user can detect the signal in time and feedback the existence or else when an authorized user sends a signal. Clearly, the spectrum sensing affects largely the accuracy of the authorized user and cognitive user communication quality. Spectrum sensing algorithm is better robustness against noise, i.e., the disruption to authorized users is lower. So the sensing scheme in low SNR environment will be an important focus. Most programs now are based on the energy detection, and the energy detection scheme is sensitive to noise, small fluctuations in noise power may cause a sharp decline in energy detection performance. Most energy detection schemes are based on constant noise power [1-6], also same papers research on non-constant noise power [11-15]. In fact this is not possible because of the background noise by the thermal noise, quantization noise, and the non-ideal filter due to power leakage, interference between authorized users, cognitive interference

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between users and other components. Thus, the noise in the detection time can not be a constant; otherwise, noise average power is fluctuation.

2 Energy Detection Model Description

Assume the signal is independent of the noise. Random processes are also assumed to be stationary and ergodic unless otherwise specified. The problem of signal detection in additive Gaussian noise can be formulated as a binary hypothesis testing problem with the following hypotheses:

$$\begin{cases} \mathcal{H}_0 : Y(n) = W(n), & n = 1, 2, \dots, N; \\ \mathcal{H}_1 : Y(n) = X(n) + W(n), & n = 1, 2, \dots, N. \end{cases} \quad (1)$$

Where $Y(n)$, $X(n)$ and $W(n)$ are the received signals at CR nodes, transmitted signals at primary nodes and white noise samples, respectively; \mathcal{H}_1 and \mathcal{H}_0 stand for the decision that the licensed user is present or not, respectively. Noise samples $W(n)$ are from additive white Gaussian noise process with power spectral density σ_n^2 , i.e. $W(n) \sim \mathcal{N}(0, \sigma_n^2)$.

Assume absolutely there is no deterministic knowledge about the signal $X(n)$ besides the average power of the signal. In this case the optimal detector is an energy detector or a radiometer [8], the test statistic is given by:

$$D(Y) = \frac{1}{N} \sum_{n=0}^{N-1} Y^2(n) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma \quad (2)$$

Where $D(Y)$ is the decision variable and γ is the decision threshold, N is the number of samples. If the noise variance is known and no noise uncertainty, the central limit theorem gives the following approximations [7][8]:

$$\begin{cases} D(Y) | \mathcal{H}_0 \sim \mathcal{N}(\sigma_n^2, \frac{2}{N}\sigma_n^4); \\ D(Y) | \mathcal{H}_1 \sim \mathcal{N}(P + \sigma_n^2, \frac{2}{N}(P + \sigma_n^2)^2). \end{cases} \quad (3)$$

Where $P = \sum_{n=1}^N |X(n)|^2/N$ is the average signal power, σ_n^2 is the noise variance. With these approximations, one obtains the detection probability P_D and false alarm probability P_{FA} [7][8]:

$$P_D = \text{prob}(D(Y) > \gamma | \mathcal{H}_1) = Q\left(\frac{\gamma - (P + \sigma_n^2)}{\sqrt{\frac{2}{N}(P + \sigma_n^2)^2}}\right) \quad (4)$$

$$P_{FA} = \text{prob}(D(Y) > \gamma | \mathcal{H}_0) = Q\left(\frac{\gamma - \sigma_n^2}{\sqrt{\frac{2}{N}\sigma_n^4}}\right) \quad (5)$$

Where $Q(\cdot)$ is the standard Gaussian complementary cumulative distribution function (CDF). P_D , P_{FA} and P_{MD} represent detection probability, false alarm probability and missed detection probability, respectively.

3 Noise Average Power Fluctuation and Detection Performance

To simplify the problem, energy detection algorithm based on average noise power without uncertainty has been discussed. United (4) and (5), eliminate the variable of decision threshold γ , and we can get:

$$N = 2[Q^{-1}(P_{FA}) - Q^{-1}(P_D)(1 + \text{SNR})]^2 \text{SNR}^{-2} \tag{6}$$

Where Q^{-1} is the inverse standard Gaussian complementary cumulative distribution function (CDF), $\text{SNR} = P/\sigma_n^2$ is the signal-to-noise ratio.

Set the variables used in Fig. 1, i.e., Signal-to-noise ratio SNR; Detection probability P_D ; Probability of false alarm P_{FA} ; Number of samples N with following numerical values: $\text{SNR}=0.1$, denoted in dB is $\text{snr}=10 \lg(\text{SNR})=-10$ (dB), false alarm probability $P_{FA} \in (0, 0.5)$. Fig. 1 is the numerical results of (6). It shows that the performance has been improved gradually as N increases, and an accurate detection probability can be obtained even if the SNR is lower, as long as N is large enough without noise uncertainty. In other words, a weak signal can be detected.

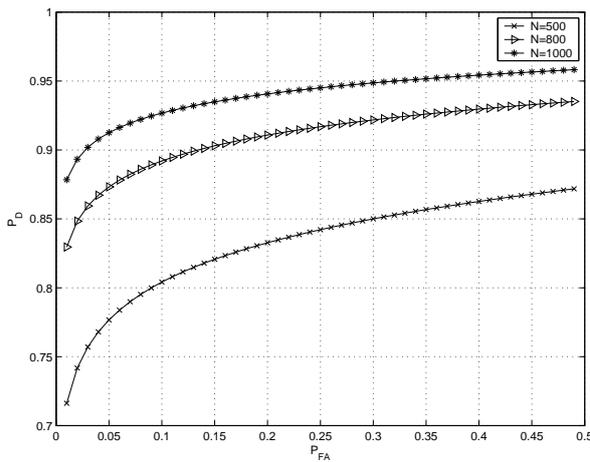


Fig. 1: P_D VS. P_{FA}

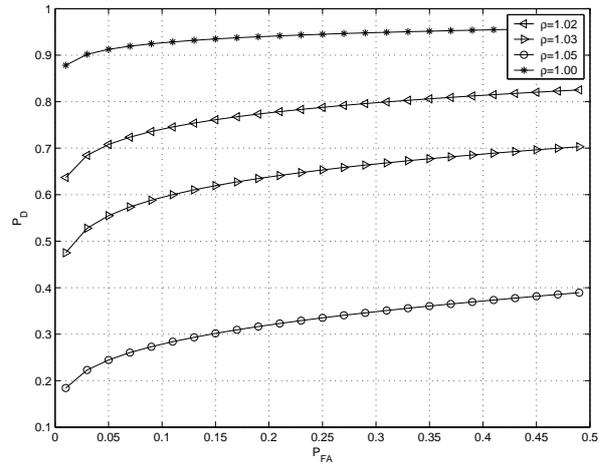


Fig. 2: P_D VS. P_{FA}

We have discussed and analyzed the case without noise uncertainty. Now, consider the case with uncertainty in the noise model. The variance of noise with uncertainty can be included in a single interval $\sigma^2 \in [\sigma_n^2/\rho, \rho\sigma_n^2]$, where ρ is the noise uncertainty factor and the value of ρ is closer to 1, that is $\rho > 1$ and $\rho \approx 1$. Thus (4) and (5) are modified to get:

$$P_D = \min_{\sigma^2 \in [\sigma_n^2/\rho, \rho\sigma_n^2]} Q \left(\frac{\gamma - (P + \sigma^2)}{\sqrt{2/N} (P + \sigma^2)} \right) = Q \left(\frac{\gamma - (P + \sigma_n^2/\rho)}{\sqrt{2/N} (P + \sigma_n^2/\rho)} \right) \tag{7}$$

$$P_{FA} = \max_{\sigma^2 \in [\sigma_n^2/\rho, \rho\sigma_n^2]} Q \left(\frac{\gamma - \sigma^2}{\sqrt{2/N} \sigma^2} \right) = Q \left(\frac{\gamma - \rho\sigma_n^2}{\sqrt{2/N} \rho\sigma_n^2} \right) \tag{8}$$

Eliminate γ and we can get the expression of P_D , P_{FA} , N , ρ and SNR:

$$N = 2[\rho Q^{-1}(P_{FA}) - (1/\rho + \text{SNR}) Q^{-1}(P_D)]^2 (\text{SNR} - (\rho - 1/\rho))^{-2} \tag{9}$$

Comparing (9) with (6), by the property of $Q^{-1}(\cdot)$ and $\rho \approx 1$, therefore, there is almost no contribution to the whole expression results if there is a tiny change of ρ ; however, the second half, i.e. SNR^{-2} and $(\text{SNR} - (\rho - 1/\rho))^{-2}$ should be mainly discussed and compared. When $\rho \approx 1$, then $\text{SNR} \approx (\text{SNR} - (\rho - 1/\rho))^{-2}$, the numerical value of (9) and (6) are almost the same; When ρ is larger and suppose $\rho = 1.05$, then $(\rho - 1/\rho) = 0.0976 \approx 0.1$, if $\text{SNR} = 0.1$, well then $(\text{SNR} - (\rho - 1/\rho))^{-2} \approx 0$, substituting into equation (9) to be $N \rightarrow \infty$. In other words, only an infinite detection duration can complete detection, which is impracticable. A tiny fluctuation of average noise power causes performance drop seriously, especially with a lower SNR.

Fig. 2 is the numerical results of 9), set parameters before simulation as this: $\text{SNR} = 0.1$, expressed in dB is $\text{snr} = 10 \lg(\text{SNR}) = -10$ (dB), $P_{FA} \in (0, 0.5)$ and $N = 1000$.

In Fig. 2, $\rho = 1.00$ represents the case without noise uncertainty, namely the average noise power keeps constant in short time. We can see that the performance gradually drops as the noise uncertainty factor increasing. When $\rho = 1.05$, the performance dropped seriously. For example, if $P_{FA} = 0.1$, then $P_D < 0.30$, even when $P_{FA} = 0.5$, the detection probability is still less than 40%. It means that rental users decide the spectrum is idle no matter whether there are primary users present. Consequently, rental users will be harmful to licensed users when primary users are present. This situation often occurs in cognitive radio systems, particularly in lower signal-to-noise ratio environments.

This illustrates that Energy detector is very sensitive to noise uncertainty. In order to guarantee a good performance, choosing a suitable threshold is very important. Traditional energy detection algorithms are based on a fixed threshold, and we have verified that the performance decreased under noise uncertainty environments. This show that the choice of a fixed threshold is no longer valid under noise uncertainty and threshold should be chosen flexible as necessary. In next section, energy detection algorithm based on dynamic threshold will be discussed in detail.

4 Noise Average Power Fluctuation and Detection Sensitivity

By comparing (6) with (9), since ρ is close to 1, so the two equations of the first half of the results has few effects on the entire expression. We focus on the relationship between the second half SNR^{-2} and $(\text{SNR}(\rho - 1/\rho))^{-2}$. When $\rho \approx 1$, $\text{SNR}^{-2} \approx (\text{SNR}(\rho - 1/\rho))^{-2}$, (6) and (9) is almost the same; when ρ is larger, such as $\rho = 1.05$, $(\rho - 1/\rho) \approx 0.1$. In the low SNR case and $\text{SNR} = 0.1$, $(\text{SNR}(\rho - 1/\rho))^{-2} \approx 0$, $N \rightarrow \infty$, this manifests that the detection duration is infinite. It is impossible to realize, especially in the low SNR environment. This shows in the cognitive radio systems, cognitive performance is influenced greatly by the noise average power fluctuation and the signal to noise ratio with noise average power fluctuations and detection duration have a close relation.

Next, the relationship of noise average power fluctuations factor, detection length, received signal to noise ratio and sensitivity will be studied. Define the detection sensitivity $\text{SNR}_s = \rho - 1/\rho$, in (9), if the signal to noise ratio cognitive radio received satisfies $\text{SNR} = (\rho - 1/\rho) \approx 0$, it can not complete the detection even if test duration long infinitely. Therefore, $\text{SNR} = \text{SNR}_s$. Let $S = 10 \lg(\rho - 1/\rho) = 10 \lg(\text{SNR}_s)$ with dB, so the detection sensitivity is the signal to noise ratio threshold SNR_s . If the signal to noise ratio cognitive user received SNR is lower than SNR_s , then one can not complete the spectrum sensing no matter how long sensing time takes.

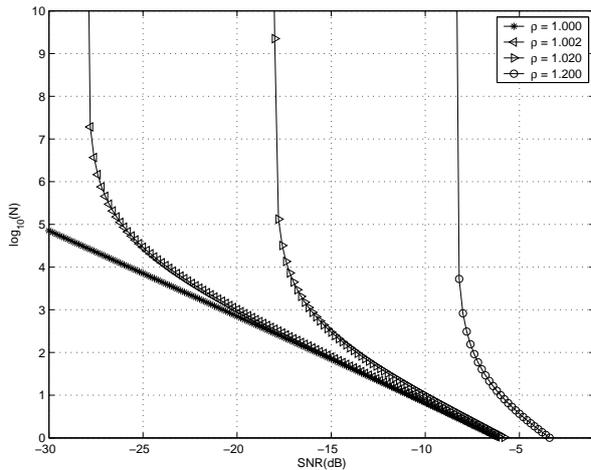


Fig. 3: $10 \lg N$ VS. SNR (dB)

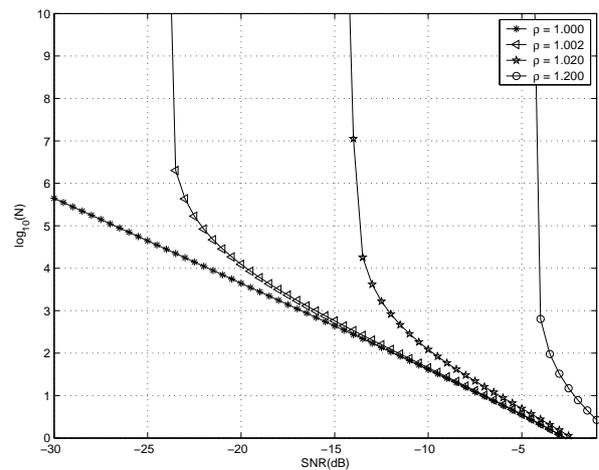


Fig. 4: $10 \lg N$ VS. SNR (dB)

Set the expected detection performance before discussion, detection probability $P_D = 0.9$ and false alarm probability $P_{FA} = 0.1$. When $\rho = 1.000$, $10 \lg(\rho) = 0$ (dB), $S \rightarrow -\infty$ (dB); if $\rho = 1.002$, $10 \lg(\rho) = 0.0087$ (dB), $S \approx -23.98$ (dB); if $\rho = 1.020$, $10 \lg(\rho) = 0.086$ (dB), $S \approx -14.02$ (dB); when $\rho = 1.200$, $10 \lg(\rho) = 0.7918$ (dB), $S \approx -4.36$ (dB). Fig. 3 shows the relation of detection sensitivity with detection duration and noise average power fluctuation factor. From Fig. 3, one can see that the cognitive user detection sensitivity of the spectrum will be very high, as long as the detection duration long enough when $\rho = 1.0$ and the cognitive user is able to detect any low power signal without introducing interference to the authorized user. If $\rho = 1.002$, the cognitive user detection sensitivity is -23.98 (dB), in other words, when the cognitive user receives the signal to noise ratio below -23.98 (dB), then cognitive user declares there is no authorized users in this band, if cognitive users occupied by the spectrum at this time will cause interference to the authorized users. When $\rho = 1.020$, the sensitivity of sensing is -14.02 (dB), that is, when the SNR the second user received is lower than -14.02 (dB), the cognitive user decides this band can dynamically access. If $\rho = 1.200$, the sensitivity of detection is -4.36 (dB), that is, when the SNR the second user received is below -4.36 (dB), the cognitive user considers this band are idle. Thus, the detection sensitivity of cognitive user declines with the noise average power fluctuation factor increased, especially in low SNR environment. If there is a big undulation, such as, $\rho = 1.200$, the spectral sensing sensitivity is higher than -4.36 (dB), which is fatal for cognitive radio. As in [7] and [10], cognitive radio detection sensitivity should be up to -22 (dB) for the authorization system ATSC (the Advanced Television Systems Committee).

5 Dynamic Threshold Algorithm

Noise average power fluctuation causes the decline of sensing sensitivity, which makes the detection accuracy drop quickly, and introduces cognitive user interference to the authorized users. Motivated by this, we present a dynamic threshold algorithm based on energy detection to repress the influence of noise fluctuation and improve the detection sensitivity.

Let ρ represent the noise average power fluctuation factor and the average power $\sigma^2 \in [\sigma_n^2/\rho, \rho\sigma_n^2]$,

then P_D and P_{FA} are:

$$P_D = \min_{\gamma' \in [\gamma/\rho', \rho'\gamma]} \min_{\sigma^2 \in [\sigma_n^2/\rho, \rho\sigma_n^2]} Q \left(\frac{\gamma' - (P + \sigma^2)}{\sqrt{\frac{2}{N}} (P + \sigma^2)} \right) = Q \left(\frac{\gamma/\rho' - (P + \sigma_n^2/\rho)}{\sqrt{\frac{2}{N}} (P + \sigma_n^2/\rho)} \right) \quad (10)$$

$$P_{FA} = \max_{\gamma' \in [\gamma/\rho', \rho'\gamma]} \max_{\sigma^2 \in [\sigma_n^2/\rho, \rho\sigma_n^2]} Q \left(\frac{\gamma' - \sigma^2}{\sqrt{\frac{2}{N}} \sigma^2} \right) = Q \left(\frac{\rho'\gamma - \rho\sigma_n^2}{\sqrt{\frac{2}{N}} \rho\sigma_n^2} \right) \quad (11)$$

Eliminating γ yields:

$$N = \frac{2 [(\rho/\rho')Q^{-1}(P_{FA}) - \rho'(1/\rho + \text{SNR})Q^{-1}(P_D)]^2}{\left(\rho'\text{SNR} + \frac{\rho'}{\rho} - \frac{\rho}{\rho'}\right)^2} \quad (12)$$

In (10), when $\rho' \approx \rho$, $\rho'(1/\rho + \text{SNR}) \approx (1 + \text{SNR})$ and $\left(\rho'\text{SNR} + \frac{\rho'}{\rho} - \frac{\rho}{\rho'}\right)^{-2} \approx (\text{SNR})^{-2}$. Substituting these approximate expressions into (12) and comparing it with (6), one can see that (12) and (6) are almost the same at this time. Therefore, the degradation of detection performance caused by noise average power fluctuation can be completely eliminated with a choice dynamic threshold factor. Comparing (12) with (9), one gets $\left(\rho'\text{SNR} + \frac{\rho'}{\rho} - \frac{\rho}{\rho'}\right)^{-2} \approx \text{SNR}$ and $\left(\rho'\text{SNR} + \frac{\rho'}{\rho} - \frac{\rho}{\rho'}\right)^{-2} \gg [\text{SNR} - (\rho - 1/\rho)]^{-2}$ in low SNR environment. Therefore, to achieve the same detection performance, the detection duration of dynamic threshold detection scheme is significantly shorter.

Set the expected detection performance $P_D = 0.9$ and $P_{FA} = 0.1$. As discussed earlier in Section III, the detection sensitivity S is about -14.02 (dB) when $\rho = 1.020$ and $\rho' = 1.000$. In this section, the dynamic threshold algorithm based on energy detection is introduced. Let S_d be detection sensitivity and $\rho = 1.020$ unchanged. The relationship between S_d and $\rho'/\rho - \rho/\rho'$ is discussed with different dynamic threshold factors. If $\rho' = 1.020$, then $S_d = 10 \lg(\rho'/\rho - \rho/\rho') \rightarrow -\infty$ (dB), the degradation of detection performance caused by noise average power fluctuation can be completely eliminated. If $\rho' = 1.015$ (dB), then $S_d \approx -20.08$ (dB), thus $S_d - S = -6.06$ (dB), the detection sensitivity increases about 6.06 (dB). If $\rho' = 1.010$, then $S_d \approx -17.05$ (dB) and $S_d - S = -3.03$ (dB), the sensing sensitivity increases about 3.03 (dB). If $\rho' = 1.000$, then $S_d \approx -14.02$ (dB), which is equivalent to the result without dynamic threshold, so the detection sensitivity unchanged, as shown in Fig. 4.

Fig. 5 is the numerical results of (6), (9) and (12). Set the parameters $N = 1000$ and $\text{SNR} = 0.1$ (-10 dB). The performance curve marked with ' \triangleleft ' corresponds to constant noise average power and without adopting the dynamic threshold algorithm, i.e., $\rho = \rho' = 1.000$. Curve Labeled with '*' corresponds to the noise average power fluctuation and not using dynamic threshold detection algorithm, here $\rho = 1.002$ and $\rho' = 1.000$. The curve marked with ' \triangleright ' represents the average noise power fluctuation factor $\rho = 1.020$ and the dynamic threshold factor $\rho' = 1.015$. The curve with ' \circ ' represents $\rho = 1.020$ and $\rho' = 1.010$. It is shown in Fig. 5 that the dynamic threshold detection algorithm based on energy detection improves the rivalry of noise average power fluctuation. It is very useful for energy detection algorithm to solve the vulnerability to noise-sensitive. This method significantly increases the robustness against noise average power fluctuation without increasing the detection duration, that is, the introduction of dynamic threshold improves the detection performance in the noise average power fluctuation environment.

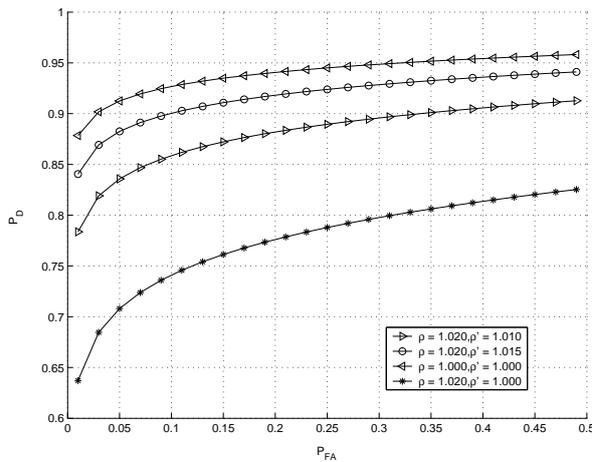


Fig. 5: P_D VS. P_{FA}

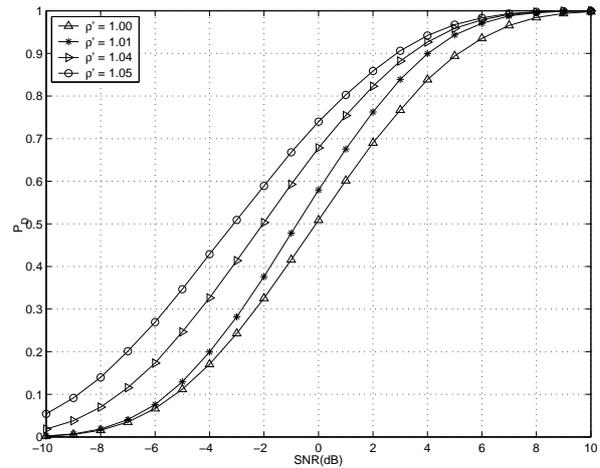


Fig. 6: Performance curves

6 Simulation Result

To further validate the above analysis, Fig. 6 is the computer Monte Carlo simulation result. In the processing, 5×10^5 signals were used and the authorize users to use the probability of channel is 50%. Noise is AWGN. Simulation parameter settings: computer environment is SNR $\in (-10, 10)$ (dB), false alarm probability is $P_{FA} = 0.01$, detection duration is $N = 500$, the average noise power fluctuation factor is $\rho = 1.02$.

In Fig. 6, curve labeled ‘ Δ ’ is the fixed threshold detection algorithm, that is dynamic threshold factor is $\rho' = 1.00$; and ‘*’ curve correspond to the dynamic threshold detection algorithm and the dynamic threshold factor is $\rho' = 1.01$; Marked ‘ \triangleright ’ is the curve correspond to the dynamic threshold factor is $\rho' = 1.04$; The last curve is $\rho' = 1.05$ and labeled with ‘o’.

From the figure we can see that when the noise is fluctuation, the dynamic threshold algorithm is superior to fixed threshold energy detection scheme. With the dynamic threshold factor value increases, detection performance improved significantly. The range of dynamic threshold factor increases is relatively small, while detection performance trends are obvious. When SNR close to 0 (dB), the value of dynamic threshold detection probability is better than fixed threshold detection probability is [0.08 0.17 0.23]. It is very helpful to improve cognitive radio system detection performance, especially work in low SNR environment.

Theoretical analysis and simulation results show that the dynamic threshold energy detection algorithm has a better robustness of anti-noise average power fluctuations.

7 Conclusion

In this paper, the relationship of energy detection performance with detection duration, detection sensitivity and noise average power fluctuation in short time is investigated. A fractional fluctuation of noise average power in short time will result in quick drop of spectrum detection performance and sensing sensitivity. In low SNR environment, the noise power average fluctuation increases to a certain extent, even if the detection duration is infinite long, it can not complete correct spectrum sensing. To the characteristic, a new energy detection algorithm based on dynamic

threshold is presented. Theoretic results and simulations show that the proposed scheme improves antagonism of noise average power fluctuation in short time with a good detection performance as long as we choose a suitable dynamic threshold, and improves the detection sensitivity visibly. In other words, the proposed scheme enhances the robustness of against noise and improves the capacity of spectrum sensing and detection sensitivity.

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