Comparative Study of Localization Techniques for Mobile Robots based on Indirect Kalman Filter

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Abstract: This paper reviews some popular localization techniques used in mobile robotics. Furthermore, an adaptation of the classical formulation of the Indirect Kalman Filter (IKF) has been implemented by means of fusing different kinds of sensors (such as odometry and radar-compass system). Two different formulations of the IKF have been compared with popular localization techniques, through real tests using a tracked mobile robot available at the University of Almería.

Keywords: Mobile Robots, Robot navigation, Kalman filters, Estimation, Kinematics.

1. INTRODUCTION

It is well-known that one of the main issues in mobile robotics is the robot localization, that is the process in which a mobile robot determines its current position and orientation. Two of the most popular solutions are odometry and dead-reckoning (Barshan and Durrant-Whyte [1995]), (Borenstein and Feng [1996]). These techniques can be considered as relative or local localization. Other authors use absolute or global techniques as GPS (Durrant-Whyte and Leonard [1991]) or a combination of both. On the other hand, some efforts are being developed at the probabilistic estimation techniques field. The more representative approach is the Kalman Filter (Goel et al. [1999]), (Kim et al. [1996]), (Nebot et al. [1997]), (Roumeliotis et al. [1998]).

Currently, a tracked mobile robot is available at the University of Almeria (Spain), see Figure 7. Some works have been developed studying the navigation control of this vehicle (González et al. [2007]), (González et al. [2008]). The goal of this vehicle is to operate inside greenhouses. For that purpose, this paper presents a comparative study of some of the most popular localization techniques in the mobile robotics community. The objective of this study is to check what is the most appropriate localization technique for the integration in the navigation control architecture.

One of the techniques analyzed has been odometry, where the calibration of the wheel radius is the main issue, so that a comparison has been performed between an ideal situation and that with wheel radius uncertainty. Another selected technique has been a combination of radar and magnetic compass measurements, this solution has been adopted to remove the typical drawback of Inertial Navigation Systems (growth of errors due to integration of measurements). Finally, a Kalman Filter is used to estimate the localization of the mobile robot. This solution has been adopted because it is expected that the position estimate fusing both previous localization techniques (odometry and radar-compass) will be more accurate (Maybeck [1979]).

The paper has been organized as follows: in section 2 a review of the localization strategies used in mobile robotics is performed. In section 3 a brief introduction to the Kalman Filter is addressed and the proposed solution based in the adaptation to the Indirect Kalman Filter is detailed. Section 4 shows real tests carried out using a real mobile robot available at the University of Almería. Finally, some conclusions and future research are presented in section 5.

2. LOCALIZATION IN MOBILE ROBOTS

This section reviews the basic ideas about localization in mobile robotics. The main groups of localization strategies are: relative or local localization and absolute or global localization. An alternative to previous strategies are the techniques which use probability to fuse different sources of information to estimate the position of the vehicle. The main advantages and drawbacks of each technique are also summarized.

2.1 Relative or local localization

Relative localization techniques are based on determining incrementally the position and orientation of a robot from an initial point. To provide this information it uses various onboard sensors, such as encoders, gyroscopes, accelerometers, etc. (Siegwart and Nourbakhsh [2004]).

The most used relative localization techniques are odometry and dead-reckoning (odometry and inertial navigation systems). Odometry employs simple geometric equations (kinematics of mobile robot) with wheel encoders that provide angular velocities of the wheels (or tracks). By integrating previous equations the position and orientation of the vehicle is calculated.

The main drawbacks of odometry (and dead-reckoning) are (Borenstein and Feng [1996]), (González et al. [2008]): (i) Due to encoder measurements are integrated, the noise is also integrated, thus it causes an unbounded growth of the error along time and distance. (ii) Odometry is based on the assumption that wheel revolutions can be translated into linear displacement relative to the floor. This assumption is limited on slip surfaces. These are called *non-systematic errors*. (iii) Other type of errors are called *systematic errors* which relates to unequal wheel diameters, uncertainty about the effective distance between wheels centers, limited encoder resolution, etc.

2.2 Absolute or global localization

The absolute localization techniques determine the position of the robot with respect to a global reference frame (Durrant-Whyte and Leonard [1991]), for example, using beacons or landmarks. The most popular technique is GPS which is based on satellite signals to determine the absolute position (longitude, latitude and altitude) of an object on the Earth.

In the case of absolute localization the error growth is mitigated when measurements are available. The robot position does not depend on time and initial position.

The main problems of the techniques based on landmarks are: (i) It requires a costly installation of the markers on the area where the robot operates. (ii) The mobile robot can only navigate over the area in which landmarks are located. (iii) Between landmarks the robot cannot determine its localization (Wang [1988]).

In relation to the GPS the main drawbacks are: (i) Small accuracy of data. This problem is solved using Differential GPS or GPS with RTK corrections (Lenain et al. [2004]). (ii) The sampling time is relatively large (greater than 1 second). For some kind of mobile robot applications this sampling time is quite slow. (iii) The GPS signal can be lost in closed spaces. It is only useful in outdoor free spaces. (iv) High cost of Differential GPS systems.

2.3 Probabilistic localization

Probabilistic techniques are based on estimating the localization of the mobile robot combining measurement data and prior knowledge about the system and measuring devices, in such a way that the error is statistically minimized. The most extended techniques are based on Kalman Filter (Welch and Bishop [2001]), (Maybeck [1979]). One of these techniques is SLAM (Simultaneous Localization and Mapping) (Durrant-Whyte and Bailey [2006]), (Tardos et al. [2002]). In SLAM, localization proceeds as Kalman Filter does, but updating the map. The new features or observations added to the map are represented by a statistical distribution because the unknown exact value of robot's position. One of the drawbacks of SLAM is the associated computational effort because large data structures (map, covariance matrices, etc.) needs to be created and updated to each feature obtained by the robot (Siegwart and Nourbakhsh [2004]).

Another research works are based on the Extended Kalman Filter (EKF) which is used in nonlinear systems. The basic idea of EKF is to start with a nonlinear system, and then find a linear system whose states represent the deviations from a nominal trajectory of the nonlinear system (Simon [2006]). Particle filtering also constitutes a localization method for mobile robots (Thrun et al. [2005]). It uses multiple copies (particles) of the state, each one associated with a weight. An estimate of the state is obtained by the weighted sum of all the particles. The main drawback is that the quality of the solution increases with the number of samples, that is, it may require exponential time (Thrun et al. [2005]).

3. KALMAN FILTER

A Kalman Filter is simply an optimal recursive data processing algorithm (Maybeck [1979]). It combines all available measurement data, plus prior knowledge about the system and measuring devices, to produce an estimate of the desired variables in such a manner that the error is minimized statistically (Maybeck [1979]). Furthermore, it supposes the system can be described through a linear model, and system and measurement noises are white and Gaussian ones (Thrun et al. [2005]).

There are two formulations of the Kalman Filter: total state space (direct formulation) and error state space (indirect formulation). In the *direct formulation*, the filter estimates the states of the system, which are used by the control loop, that is, the filter actuates as an observer (figure 1). The advantage of this formulation is that the available information is weighted optimally rather than operated upon by fixed gains and integrator. Some drawbacks are (Maybeck [1979]): (i) Due to the fact that the filter is needed in the control loop (observer), if the filter fails, the entire navigation system fails. The mobile robot cannot operate without the filter. (ii) The filter requires a sampling period similar to the controller, which could cause that the filter cannot operate correctly.

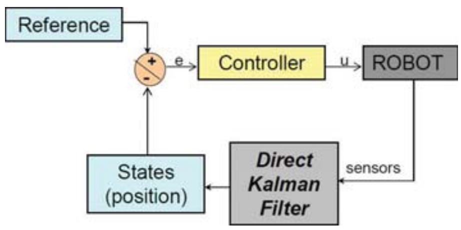

Fig. 1. Direct Kalman Filter

In the *indirect formulation*, the state is obtained combining a primary set of sensors (i.e. odometry) and the difference between these sensors with auxiliary measurements (i.e. radar-compass), such as shown in Figure 2. The advantages of this configuration are: (i) The system can operate without the filter, because at least the main set of sensors can calculate the position of the robot. (ii) The filter can run at relatively slow rate. The drawback is that the real states of the system are not estimated, only the errors between different sources.

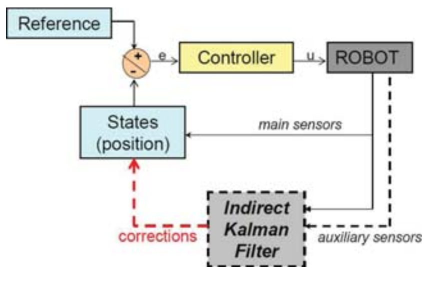

Fig. 2. Indirect Kalman Filter

Depending on the treatment of the error estimates, there are two types of linear Indirect Kalman Filter (IKF) implementations: feedforward and feedback (Maybeck [1979]).

These formulations use odometry with calibrated wheel radius and effective distance between wheels centers to avoid systematic errors and the radar-compass system to avoid non-systematic errors due to slip compensation (González et al.

[2008]). The strategy followed is summarized in figure 3. Firstly, the error between the reference position and odometry, and the error covariance matrix are determined. Then, the Kalman gain is computed using the radar-compass noise covariance matrix. Finally the difference between odometry and radar-compass positions are used to estimate the error, and the error covariance matrix is updated.

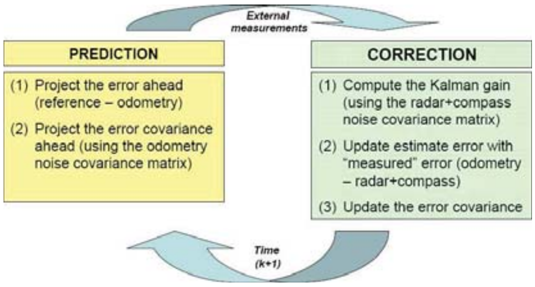

Fig. 3. Proposed strategy using IKF with odometry and radar-compass

3.1 Kinematic Model

As well-known the motion of a mobile robot is expressed as the relative movement of the attached frame (R) to the mid-point of the vehicle (O') with respect to a global or inertial base frame (G), such as Figure 4 shows. The approach presented in this paper to implement the two formulations of the IKF are based in the kinematic model of a differential-drive mechanism (Siegwart and Nourbakhsh [2004]) to determine the position, because the mobile robot used for our tests has this configuration (González et al. [2007]).

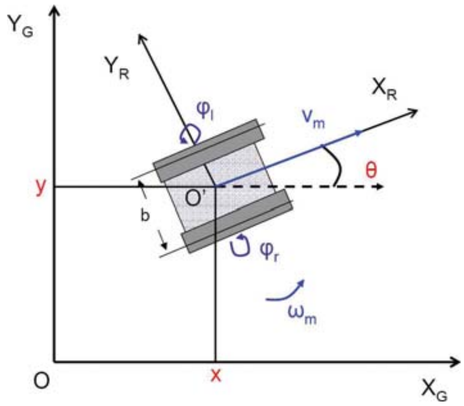

Fig. 4. Kinematic model of a differential-drive mechanism

In the case of odometry the kinematic model is

$$\begin{aligned} x_{k+1}^{odo} &= x_k^{odo} + T_s \frac{w_r}{2}[\phi_r + \phi_l] \cos\theta_k^{odo} + \epsilon^x, \\ y_{k+1}^{odo} &= y_k^{odo} + T_s \frac{w_r}{2}[\phi_r + \phi_l] \sin\theta_k^{odo} + \epsilon^y, \\ \theta_{k+1}^{odo} &= \theta_k^{odo} + T_s \frac{w_r}{b}[\phi_r - \phi_l] + \epsilon^\theta, \end{aligned} \quad (1)$$

where $p^{odo} = [x^{odo} \quad y^{odo} \quad \theta^{odo}]^T$ is the position and orientation of the robot using odometry, k is the discrete sampling time, T_s is the sampling time, ϕ_i is the angular velocity measured by the encoders of wheel j, $j = \{r, l\} = \{right, left\}$, b is the distance between wheels centers of the vehicle, w_r is the wheel radius, and ϵ is a Gaussian white noise.

Using the radar-compass system, the previous equation can be rearranged as

$$\begin{aligned} x_{k+1}^{rc} &= x_k^{rc} + T_s v_m \cos\theta_k^{rc} + \delta^x, \\ y_{k+1}^{rc} &= y_k^{rc} + T_s v_m \sin\theta_k^{rc} + \delta^y, \\ \theta_{k+1}^{rc} &= \theta_k^{rc} + T_s \omega_m + \delta^\theta, \end{aligned} \quad (2)$$

where v_m is the linear velocity of the vehicle measured with the radar, ω_m is the angular velocity of the vehicle measured with the magnetic compass, and δ is a Gaussian white noise.

3.2 Feedforward Indirect Kalman Filter

The *Feedforward formulation* estimates the error to compensate the position determined by odometry. In this formulation, the reference position, and those estimated by odometry and by radar-compass are used to determine the error estimation. The disadvantage of this approach is that error due to odometry can grow unbounded because the filter has not any feedback.

As commented above, firstly the error between reference and odometry position is determined,

$$\hat{e}_{k|k} = \begin{bmatrix} \hat{e}_{k|k}^x \\ \hat{e}_{k|k}^y \\ \hat{e}_{k|k}^\theta \end{bmatrix} = \begin{bmatrix} x_{k|k}^{ref} - x_{k|k}^{odo} \\ y_{k|k}^{ref} - y_{k|k}^{odo} \\ \theta_{k|k}^{ref} - \theta_{k|k}^{odo} \end{bmatrix}, \quad (3)$$

where ref is related to the reference.

The predicted error is the same that the error between reference and odometry,

$$\hat{e}_{k+1|k} = A\hat{e}_{k|k}, \quad (4)$$

generally in the IKF formulation state matrix A is equal to the identity matrix of the same size that the state space. In this case $A = I_3$. Then, error covariance matrix P is calculated. Later this matrix is used to determine the Kalman gain,

$$P_{k+1|k} = AP_{k|k} + Q_{odo}, \quad (5)$$

where Q_{odo} matrix is the noise covariance matrix of odometry. This matrix will be experimentally determined using the variance of each component of the position,

$$Q_{odo} = \begin{bmatrix} \sigma^2(e_{odo}^x) & 0 & 0 \\ 0 & \sigma^2(e_{odo}^y) & 0 \\ 0 & 0 & \sigma^2(e_{odo}^\theta) \end{bmatrix}. \quad (6)$$

In the next step, the Kalman gain K is calculated. This gain weights the difference between the external sources of information (odometry and radar-compass positions) and estimated error,

$$K = P_{k+1|k}[P_{k+1|k} + R_{rc}]^{-1}, \quad (7)$$

where R_{rc} matrix is the noise covariance matrix of radar-compass. This matrix is obtained following the same procedure that Q_{odo} matrix,

$$R_{rc} = \begin{bmatrix} \sigma^2(e_{rc}^x) & 0 & 0 \\ 0 & \sigma^2(e_{rc}^y) & 0 \\ 0 & 0 & \sigma^2(e_{rc}^\theta) \end{bmatrix}. \quad (8)$$

Then, the error state is estimated using the Kalman gain (K) and the error between odometry and radar-compass as

$$\hat{e}_{k+1|k+1} = \hat{e}_{k+1|k} + K(z_k - \hat{e}_{k+1|k}), \tag{9}$$

where

$$z_k = \begin{bmatrix} x^{odo}_{k|k} - x^{rc}_{k|k} \\ y^{odo}_{k|k} - y^{rc}_{k|k} \\ \theta^{odo}_{k|k} - \theta^{rc}_{k|k} \end{bmatrix}. \tag{10}$$

The difference $z - \hat{e}_{k+1|k}$ is called residual or innovation.

Now, the error covariance matrix is updated,

$$P_{k+1|k+1} = (I_3 - K)P_{k+1|k}. \tag{11}$$

Finally, the optimal estimates of error committed by the odometry are subtracted from the odometry data, to yield optimally estimated localization,

$$\hat{p}_{k+1|k+1} = p^{odo}_{k+1|k+1} - \hat{e}_{k+1|k+1}. \tag{12}$$

This process should be repeated at each sampling time.

These steps are summarized in figure 5.

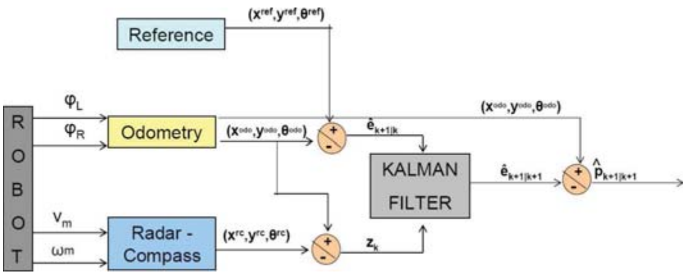

Fig. 5. Feedforward Indirect Kalman Filter

3.3 *Feedback Indirect Kalman Filter*

The process to implement the *Feedback formulation* is similar to the previous one. The key difference is that the errors are fedback into the odometry to correct it. The most straightforward means of generating the feedback implementation is to write the system and filter equations in terms of corrected states. Using the next equation

$$\hat{p}_{k|k} = p^{odo}_{k|k} - \hat{e}_{k|k}, \tag{13}$$

knowing that p^{odo} is defined as (1), and substituting $\hat{e}$ by (9), it holds,

$$p^{odo}_{k+1|k+1} - \hat{e}_{k+1|k+1} = (p^{odo}_{k|k} - \hat{e}_{k|k}) - \\ -K(p^{odo}_{k|k} - p^{rc}_{k|k} - \hat{e}_{k|k}) + T_s B_k \upsilon_k, \tag{14}$$

where

$$B_k = \begin{bmatrix} \cos\theta^{odo}_k & 0 & 0 \\ 0 & \sin\theta^{odo}_k & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \upsilon_k = \begin{bmatrix} v_m \\ \omega_m \end{bmatrix}. \tag{15}$$

Grouping terms in (14), it produces,

$$\hat{p}_{k+1|k+1} = \hat{p}_{k|k} - K(\hat{p}_{k|k} - p^{rc}_{k|k}) + T_s B_k \upsilon_k. \tag{16}$$

As in the previous formulation this process should be repeated at each sampling time. Previous equation represents the feedback formulation of IKF where K is calculated as in (7).

The feedback formulation is summarized in figure 6.

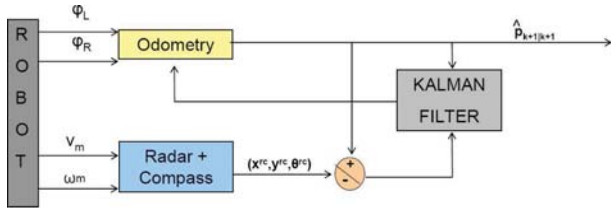

Fig. 6. Feedback Indirect Kalman Filter: strategy implemented

3.4 *Fault-tolerance in radar*

A fault-tolerant procedure to account for erroneous data from radar has been implemented. This is because radar sensitivity can become erroneous, with changes in reflection levels under not appropriate soil conditions. This issue have been solved modifying the Kalman gain. When radar readings are erroneous (during a long time are equal to zero and the encoders and compass readings are different from zero, and when radar reading are quite noisy), the Kalman gains is imposed to zero and the radar-compass position does not affect the error state estimation.

4. RESULTS

This section discusses a comparison of different localization techniques implemented in the real mobile robot. The mobile robot called *Fitorobot* is available at the University of Almería (figure 7). It is a tracked vehicle designed to work in greenhouses, and equipped with some sensors and a computer to operate autonomously (González et al. [2007]). The purpose of this tracked mobile robot is to operate in greenhouse where slip inevitably occurs. For that reason, the approach in which radar-compass is fused with odometry will obtain the best behavior in this type of terrains. As explained in (González et al. [2008]) radar and encoders are used to calculate the slip which is taken into account by the main controller.

Currently the mobile robot is equipped with few sensors for localization purposes, for that reason we have only compared: Odometry simulating a calibrated track's radius and a non-calibrated track's radius [1]; radar-compass with fault-tolerant procedure and without fault-tolerant procedure; feedforward IKF, and feedback IKF. In future works, we will compare absolute localization techniques as GPS to the current results.

The real track's radius of the testbed is 0.15 [m] but the calibrated track's radius is 0.10 [m]. The distance between tracks centers is 0.5 [m]. The noise covariance matrices Q_{odo} and R_{rc} experimentally determined are

$$Q_{odo} = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \quad R_{rc} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \tag{17}$$

[1] Because a track is different to a wheel some experimental tests have been carried out to find out the relationship between the linear velocity of the vehicle and the angular velocities of the tracks. That is the reason, by which appears two different radius.

Fig. 7. Mobile robot *Fitorobot*

In order to test the performance of the localization techniques the mobile robot was teleoperated over an U-shaped reference trajectory over a terrain with two different areas (one in which the radar performance is fine and another in which the radar does not work well). The size of the trajectory was approximately 14 meters long and 10 meters width. This type of trajectories are common in greenhouses. Because the vehicle was teleoperated the real trajectory followed by the mobile robot differs slightly from the reference trajectory. This is the reason of why errors in localization techniques are higher that expected.

As can be seen in Figure 8, the trajectories using Kalman Filters fit correctly the reference trajectory (taking into account the errors due to teleoperation described above). Techniques based on odometry show an unacceptable behavior in the turns. At the end of the test, there is a slight difference between the reference and the trajectories obtained using the localization methods, this fact is due to the commented difference between the reference trajectory and the real teleoperated trajectory.

The errors can be better observed from Figure 9. To calculate these errors, the Euclidean norm (between the error in the forward and lateral directions) has been used. As expected, the error using odometry grows considerably, because it has a bad behavior in long trajectories and in turns. Furthermore, the error for all the techniques grows unchecked, because the vehicle moves in an open-loop experiment. Seeing these results, it is demonstrated that odometry is not appropriate to estimate the position of a mobile robot, above all, when the trajectory has several turns. The lesser error is achieved using the feedback IKF. Relative errors between the other techniques with respect to the feedback IKF using the mean value (of the errors) are: feedforward IKF 2.70%, radar-compass (fault tolerant procedure) 5.26% and radar-compass (no fault tolerant procedure) 73.33%.

Finally, Figure 10 shows the measurements of encoders (angular velocity of wheels) and radar (linear velocity of the vehicle). At time instants (80-100) seconds is possible to see some erroneous peaks due to radar readings (terrain with different reflectivity properties). Figure 11 shows the orientation mea-

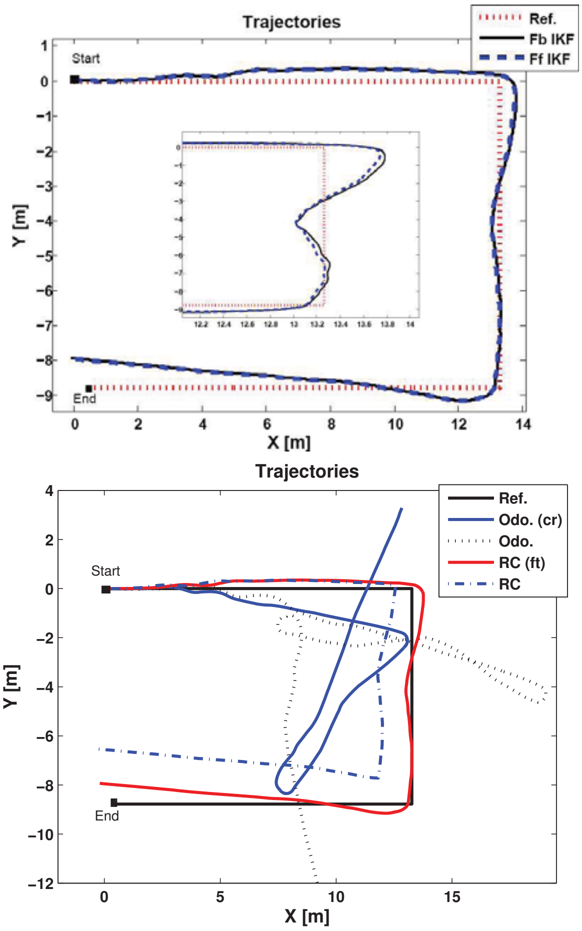

Fig. 8. Trajectories of the test

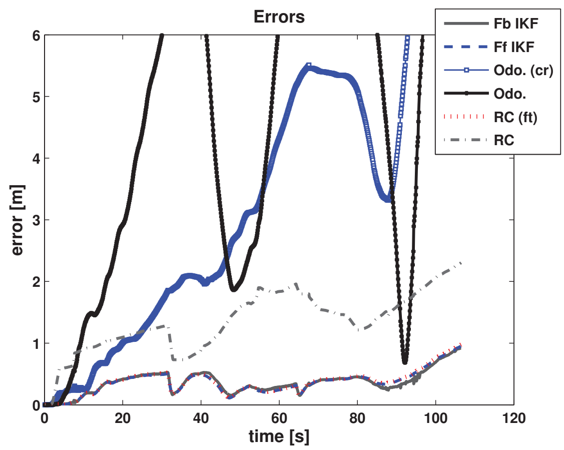

Fig. 9. Errors of the different localization techniques

sured by the magnetic compass. It is possible to check the two consecutive 90-degrees turns.

5. CONCLUSIONS AND FUTURE RESEARCH

This paper reviews some popular localization techniques used in mobile robotics. Special attention has been shown for the Indirect Kalman Filter. Real tests have demonstrated that both IKF formulations implemented have a minimum error. The feedback formulation has the best performance, because as dis-

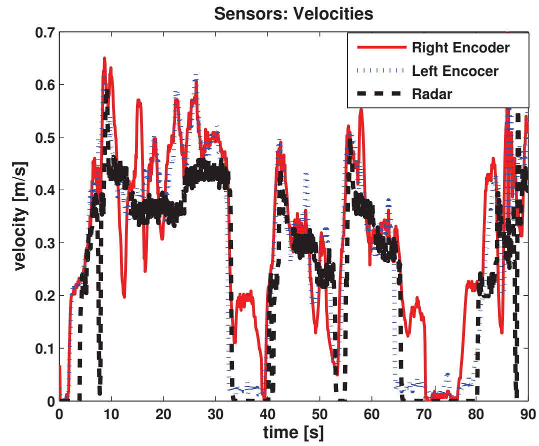

Fig. 10. Velocities during the test with encoders and radar

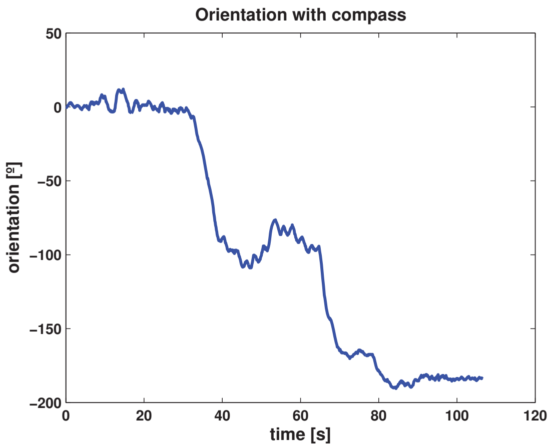

Fig. 11. Orientation during the test with magnetic compass

cussed, the filter is fedback with the current errors in odometry. As expected odometry shows a bad behavior, above all in turns.

In future, Kalman Filter localization techniques will be integrated with navigation controllers in a real mobile robot, and absolute localization techniques will be tested.

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