

Achievable Rate of Two-Hop Channels under Statistical Delay Constraints

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Abstract

This paper analyzes the impact of statistical delay constraints on the achievable rate of a two-hop wireless communication link, in which the communication between a source and a destination is accomplished via an intermediate full-duplex relay node. It is assumed that there is no direct link between the source and the destination, and the relay forwards the information to the destination by employing the decode-and-forward scheme. Both the queues at the source and relay node are subject to statistical queueing constraints imposed on the limitations of buffer violation probability. Given statistical delay constraints specified via maximum delay and delay violation probability, the tradeoff between the statistical delay constraints imposed to any two concatenated queues is identified. With this characterization, the maximum constant arrival rates that can be supported by this two-hop link are obtained by determining the effective capacity of such links as a function of the statistical delay constraints and signal-to-noise ratios (SNR) at the source and relay, and the fading distributions of the links. It is shown that imposing unbalanced statistical delay constraints to the queues at the source and relay can improve the achievable rate. Overall, the impact of statistical delay constraints on the achievable throughput is provided.

Index Terms

Two-hop wireless links, statistical delay constraints, quality of service (QoS) constraints, fading channels, effective capacity, delay violation probability, full-duplex relaying.

I. INTRODUCTION

With the widespread of smart-phones and tablets, the volume of global mobile traffic has increased explosively in recent years. The portion of multimedia data has surged significantly in the wireless traffic,

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such as mobile video and voice over IP (VoIP)[1]. In such traffic, delay is an important consideration. Meanwhile, providing deterministic quality of service (QoS) guarantees is challenging for the wireless systems, since the instantaneous rate of the channel is vulnerable to numerous factors, such as mobility, changing environment and multipath fading [2]. Therefore, guaranteeing statistical QoS guarantees is more favorable.

Effective bandwidth theory has been developed to analyze high-speed systems operating under statistical queueing constraints[3][4]. The queueing constraints are imposed on buffer violation probabilities and are specified by the QoS exponent θ , which is defined as

$$\lim_{Q_{\max} \rightarrow \infty} \frac{\log \Pr\{Q > Q_{\max}\}}{Q_{\max}} = -\theta, \quad (1)$$

where Q is the queue length in steady state, Q_{\max} is the maximal queue length. With the above characterization, the statistical delay violation probabilities can be characterized through the effective bandwidths of the arrival and departure processes jointly [3][6][8]. Also, Chang and Zajic have characterized the effective bandwidths of the time varying departure processes in [5], which can be utilized to analyze the volatile wireless systems. Moreover, Wu and Negi in [6] defined the dual concept of effective capacity, which provides the maximum constant arrival rate that can be supported by a given departure process while satisfying statistical delay constraints. The analysis and application of effective capacity in various settings have attracted much interest recently (see e.g. [7]-[21] and references therein). For instance, in [7], the authors derived the optimal power control policies that maximize the effective capacity of a point-to-point link. In [9], the authors obtained the resource allocation and scheduling policies for video transmissions under the framework of effective capacity. In [11], the authors characterized the effective capacity in a time division downlink system and proposed the optimal scheduling schemes that can achieve points on the boundary of the effective capacity region.

In this paper, we study the effective capacity in relay channels under statistical end-to-end delay constraints. In particular, we assume that there are buffers at both the source and the relay nodes, and consider the queueing delay introduced by the buffers. Note that [13]-[21] have also recently investigated the effective

capacity of the relay channels. For instance, Tang and Zhang in [13] analyzed the power allocation policies of relay networks, where the relay node is assumed to have no queue, i.e., the packets arriving to the relay node are forwarded immediately. In [14], Liu *et. al.* considered the cooperation of two users for data transmission, where the interchanged data goes through only the queue of the other user. Parag and Chamberland in [15] provided a queueing analysis of a butterfly network with constant rate for each link, while assuming that there is no congestion at the intermediate nodes. The effective capacity of the two-hop link in the presence of the statistical queueing constraints at the source and relay node is given in [16], and the performance for multi-relay links is analyzed in [17].

As a stark difference from previous work, we consider the performance of two-hop wireless communication systems under the statistical delay constraints in the form of limitations on the end-to-end delay violation probabilities in this work. Note that statistical end-to-end delay analysis can also be found in [18]-[21]. In [18], Wu and Negi considered statistical end-to-end delay constraints for half-duplex relays, and gave an effective capacity formulation with time allocation to the different hops. In [19]-[21], the authors considered the statistical end-to-end delay constraints of multi-hop links, while assuming that the statistical delay violation probability of the queues are equal. However, it is possible that the relay can tolerate more stringent delay constraints while not affecting the system performance. Additionally, we note that for the analysis of link selection in half-duplex buffer-aided systems, the authors considered the case that only the relay node has queue, and analyzed the average queueing delay[22].

Our contributions and major findings in this paper can be summarized as follows. We consider the end-to-end delay for the information passing the queues at the source and relay node of the two-hop links, and analyze the impact of statistical end-to-end delay constraints, imposed as the limitations on the maximum delay violation probability. First, for the general case of two concatenated queues, we characterize the tradeoff between the statistical delay constraints imposed to the queues, which provides a framework for dynamically adjusting the delay constraints at the two interacting queues. With the obtained interplay, we derive the effective capacity of the two-hop links under target statistical end-to-end delay constraint. Unlike the results in [16][17] with given statistical queueing constraints, the effective capacity obtained is for target end-to-end delay constraints, and we optimize over the statistical queueing constraints at the queues of the

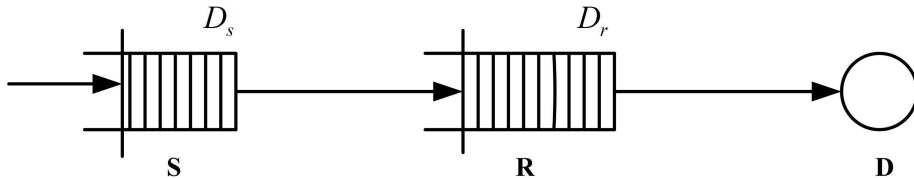


Fig. 1. The system model.

source and relay node to achieve this effective capacity. We also describe a method for analysing the effective capacity in such settings. Additionally, we show that balancing the delay constraints between the two queues is not always an optimal way. Instead, having bias towards one queue, i.e., lessening the delay constraint at one queue, can lead to larger achievable rate, which is verified by numerical results later. Moreover, it is demonstrated that the improvement is affected by the statistical delay constraints, the signal-to-noise ratio (SNR) levels and the channel conditions of the links.

The rest of this paper is organized as follows. In Section II, the system model and necessary preliminaries are described. In Section III, we present the tradeoff between the statistical delay constraints of any two concatenated queues. We describe our main results for block-fading channels in Section IV, with numerical results provided in Section V. Finally, in Section VI, we conclude the paper.

II. PRELIMINARIES

A. System Model

The two-hop communication link is depicted in Figure 1. In this model, source **S** is sending information to the destination **D** with the help of the intermediate relay node **R**. We assume that there is no direct link between **S** and **D** (which, for instance, holds, if these nodes are sufficiently far apart in distance). Both the source and the intermediate relay nodes are equipped with buffers. Hence, for the information flow of such links, the queueing delay experienced is given by

$$D = D_s + D_r, \quad (2)$$

where D_s and D_r denote the stationary delay experienced in the queue at the source and relay node, respectively.

We consider the full-duplex relay, where reception and transmission can be performed simultaneously at the relay node. In the i th symbol duration, the signal Y_r received at the relay from the source and the signal Y_d received at the destination from the relay can be expressed as

$$Y_r[i] = g_1[i]X_1[i] + n_1[i], \quad (3)$$

$$Y_d[i] = g_2[i]X_2[i] + n_2[i], \quad (4)$$

where X_j for $j = \{1, 2\}$ denote the inputs for the links $\mathbf{S} - \mathbf{R}$ and $\mathbf{R} - \mathbf{D}$, respectively. More specifically, X_1 is the signal sent from the source and X_2 is sent from the relay. The inputs are subject to individual average energy constraints $\mathbb{E}\{|X_j|^2\} \leq \bar{P}_j/B, j = \{1, 2\}$ where B is the bandwidth. Assuming that the symbol rate is B complex symbols per second, we can easily see that the symbol energy constraint of \bar{P}_j/B implies that the channel input has a power constraint of \bar{P}_j . We assume that the fading coefficients $g_j, j = \{1, 2\}$ are jointly stationary and ergodic discrete-time processes, and we denote the magnitude-square of the fading coefficients by $z_j[i] = |g_j[i]|^2$. Above, in the channel input-output relationships, the noise component $n_j[i]$ is a zero-mean, circularly symmetric, complex Gaussian random variable with variance $\mathbb{E}\{|n_j[i]|^2\} = N_j$ for $j = 1, 2$. The additive Gaussian noise samples $\{n_j[i]\}$ are assumed to form an independent and identically distributed (i.i.d.) sequence. We denote the signal-to-noise ratios as $\text{SNR}_j = \frac{\bar{P}_j}{N_j B}$.

B. Statistical Delay

We first state the following result from [5], which characterizes the statistical queueing constraint for given arrival and departure processes under certain conditions.

Theorem 1 ([5]): Suppose that the queue is stable and that both the arrival process $a[n], n = 1, 2, \dots$ and service process $c[n], n = 1, 2, \dots$ satisfy the Gärtner-Ellis limit, i.e., for all $\theta \geq 0$, there exists a differentiable

logarithmic moment generating function (LMGF) $\Lambda_A(\theta)$ such that¹

$$\lim_{n \rightarrow \infty} \frac{\log \mathbb{E}\{e^{\theta \sum_{i=1}^n a^{[n]}}\}}{n} = \Lambda_A(\theta), \quad (5)$$

and a differentiable LMGF $\Lambda_C(\theta)$ such that

$$\lim_{n \rightarrow \infty} \frac{\log \mathbb{E}\{e^{\theta \sum_{i=1}^n c^{[n]}}\}}{n} = \Lambda_C(\theta). \quad (6)$$

If there exists a unique $\theta^* > 0$ such that

$$\Lambda_A(\theta^*) + \Lambda_C(-\theta^*) = 0, \quad (7)$$

then

$$\lim_{Q_{\max} \rightarrow \infty} \frac{\log \Pr\{Q > Q_{\max}\}}{Q_{\max}} = -\theta^*. \quad (8)$$

where Q is the stationary queue length. ■

Consider a single stable first-come first-serve (FCFS) queue with statistical queueing constraint θ satisfying (8). The queueing delay D experienced by the information flow going through the queue can be expressed as [3][6]

$$\lim_{D_{\max} \rightarrow \infty} \frac{-\log \Pr\{D > D_{\max}\}}{D_{\max}} = \theta\delta, \quad (9)$$

where δ is decided by the arrival and departure processes jointly. Define[6]

$$J(\theta) = \theta\delta = -\Lambda_C(-\theta) \quad (10)$$

as the statistical delay exponent associated with the queue. Note that $J(\theta)$ is a function of the statistical queueing constraint θ , and larger $J(\theta)$ implies more stringent delay constraints. Above, $\Lambda_C(\theta)$ is the LMGF of the service process. Then assume that the queue is not empty, the delay violation probability can be

¹Throughout the text, logarithm expressed without a base, i.e., $\log(\cdot)$, refers to the natural logarithm $\log_e(\cdot)$.

written equivalently as [8][9]

$$\Pr\{D > D_{\max}\} \doteq e^{-J(\theta)D_{\max}}, \quad (11)$$

where we defined $f(x) \doteq e^{-cx}$ when $\lim_{x \rightarrow \infty} \frac{-\log f(x)}{x} = c$.

With the previous characterization, we can obtain the probability density function of random variable D as [20]

$$p_D(x) = \frac{\partial}{\partial x} (1 - \Pr\{D > x\}) \doteq J(\theta)e^{-J(\theta)x}. \quad (12)$$

Now consider two concatenated queues as depicted in Fig. 1. For the queueing constraints specified by θ_1 and θ_2 with (7) satisfied for each queue, we define

$$J_1(\theta_1) = -\Lambda_{C,1}(\theta_1), \text{ and } J_2(\theta_2) = -\Lambda_{C,2}(\theta_2), \quad (13)$$

where $\Lambda_{C,1}(\theta_1)$ and $\Lambda_{C,2}(\theta_1)$ are the LMGF functions of the service rate of queue 1, 2, respectively. In the two-hop system, we can express the end-to-end delay violation probability as

$$\Pr\{D_1 + D_2 > D_{\max}\} = 1 - \int_0^{D_{\max}} \int_0^{D_{\max}-D_1} p_D(D_1)p_D(D_2)dD_2dD_1 \quad (14)$$

$$\doteq \begin{cases} \frac{J_1(\theta_1)e^{-J_2(\theta_2)D_{\max}} - J_2(\theta_2)e^{-J_1(\theta_1)D_{\max}}}{J_1(\theta_1) - J_2(\theta_2)}, & J_1(\theta_1) \neq J_2(\theta_2), \\ (1 + J_1(\theta_1)D_{\max})e^{-J_1(\theta_1)D_{\max}}, & J_1(\theta_1) = J_2(\theta_2). \end{cases} \quad (15)$$

We need to guarantee that the statistical delay performance of the two-hop link is not worse than the statistical delay performance specified by (ϵ, D_{\max}) , where ϵ is the limitation on the statistical delay violation probability, and D_{\max} is the maximum tolerable delay. Then, we should have

$$\Pr\{D_1 + D_2 > D_{\max}\} \leq \epsilon. \quad (16)$$

C. Effective Capacity

Under the statistical queueing delay constraints, we can dynamically control the delay constraint $J_1(\theta_1)$ and $J_2(\theta_2)$ at the queue of the source and relay node as long as the statistical end-to-end delay performance

(16) can be guaranteed. At the same time, for each realization of (θ_1, θ_2) , assume that the constant arrival rate at the source is $R \geq 0$, and the channels operate at their capacities. To satisfy the queueing constraint at the source, we must have

$$\tilde{\theta} \geq \theta_1, \quad (17)$$

where $\tilde{\theta}$ is the solution to

$$R = -\frac{\Lambda_{sr}(-\tilde{\theta})}{\tilde{\theta}}, \quad (18)$$

and $\Lambda_{sr}(\theta)$ is the LMGF of the instantaneous capacity of the **S** – **R** link.

According to [5], the LMGF of the departure process from the source, or equivalently the arrival process to the relay node, is given by

$$\Lambda_r(\theta) = \begin{cases} R\theta, & 0 \leq \theta \leq \tilde{\theta}, \\ R\tilde{\theta} + \Lambda_{sr}(\theta - \tilde{\theta}), & \theta > \tilde{\theta}. \end{cases} \quad (19)$$

Therefore, in order to satisfy the queueing constraint of the intermediate relay node **R**, we must have

$$\hat{\theta} \geq \theta_2, \quad (20)$$

where $\hat{\theta}$ is the solution to

$$\Lambda_r(\hat{\theta}) + \Lambda_{rd}(-\hat{\theta}) = 0. \quad (21)$$

Above, $\Lambda_{rd}(\theta)$ is the LMGF of the instantaneous capacity of the **R** – **D** link.

Note that we can characterize the effective capacity $R_E(\theta_1, \theta_2)$ with (θ_1, θ_2) following the method provided in [16, Theorem 2]. Denote Ω as the set of pairs (θ_1, θ_2) such that (16) can be satisfied. After these characterizations, effective capacity of the two-hop communication model under statistical delay constraints (ϵ, D_{\max}) can be formulated as follows.

Definition 1: The effective capacity of the two-hop communication link with statistical delay constraints

specified by (ϵ, D_{\max}) is given by

$$R_\epsilon(\epsilon, D_{\max}) = \sup_{(\theta_1, \theta_2) \in \Omega} R_E(\theta_1, \theta_2) \quad (22)$$

where Ω is the set of all feasible (θ_1, θ_2) satisfying (16). Hence, effective capacity is now the maximum constant arrival rate that can be supported by the two-hop channels under the statistical delay constraints.

III. STATISTICAL DELAY TRADEOFF

For the following analysis, we first characterize the interrelationship between $J_1(\theta_1)$ and the associated minimum $J_2(\theta_2)$ satisfying the statistical delay constraint (16). We have the following results.

Lemma 1: Consider the following function

$$\vartheta(J_1(\theta_1), J_2(\theta_2)) = \frac{J_2(\theta_2)e^{-J_1(\theta_1)D_{\max}} - J_1(\theta_1)e^{-J_2(\theta_2)D_{\max}}}{J_2(\theta_2) - J_1(\theta_1)} = e^{-J_0D_{\max}} = \epsilon, \text{ for } 0 \leq \epsilon \leq 1, \quad (23)$$

where J_0 is defined as the statistical delay exponent associated with (ϵ, D_{\max}) . Denote $J_2(\theta_2) = \Phi(J_1(\theta_1))$ as a function of $J_1(\theta_1)$, we have

a) $\Phi(J_1(\theta_1))$ is continuous. For $J_1(\theta_1) = J_{th}(\epsilon)$, we have

$$\Phi(J_1(\theta_1)) = J_{th}(\epsilon) \quad (24)$$

where

$$J_{th}(\epsilon) = -\frac{1}{D_{\max}} \left(1 + \mathcal{W}_{-1} \left(-\frac{\epsilon}{e} \right) \right), \quad (25)$$

where $\mathcal{W}_{-1}(\cdot)$ is the Lambert W function, which is the inverse function of $y = xe^x$ in the range $(-\infty, -1]$.

b) Φ is strictly decreasing in $J_1(\theta_1)$.

c) Φ is convex in $J_1(\theta_1)$.

d) $J_1(\theta_1) \in [J_0, \infty)$, and $J_2(\theta_2) = \Phi(J_1(\theta_1)) \in [J_0, \infty)$.

Proof: See Appendix A.

Remark 1: The above properties can be understood intuitively. Larger $J_1(\theta_1)$ enforces more stringent

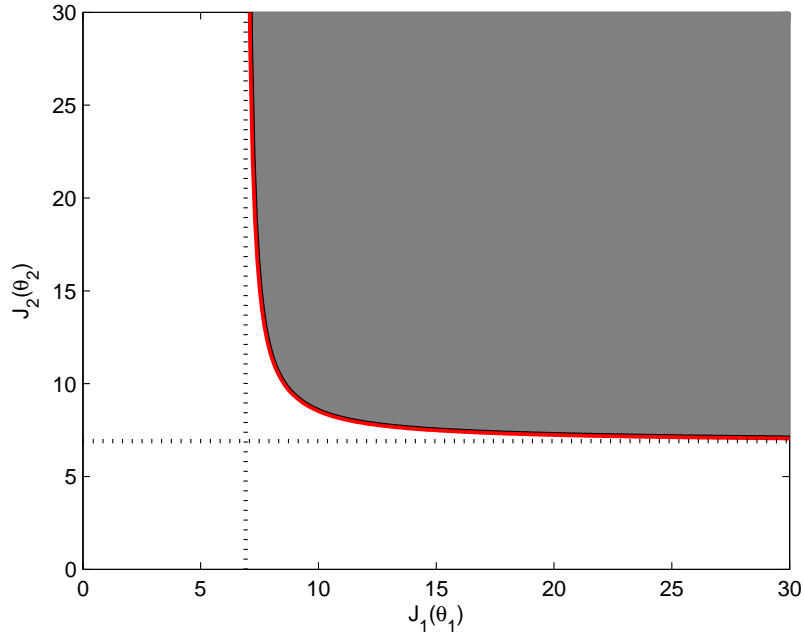


Fig. 2. J_2 v.s. J_1 . $D_{\max} = 1$. $\epsilon = 0.001$.

delay constraints for the queue 1, and we can have loosened delay constraints for the queue 2; vice versa. When either queue is subject to a deterministic constraint, i.e., $\theta = \infty$, the delay violation only occurs at the other queue. In Fig. 2, we plot J_2 as a function of J_1 for the case $\epsilon = 0.001$ and $D_{\max} = 1$ sec for illustration. Note that only (J_1, J_2) in the dark region can be acceptable to achieve the statistical delay performance. As can be seen from the figure, the curve given by the lower boundary matches the properties in the Lemma.

IV. EFFECTIVE CAPACITY IN BLOCK-FADING CHANNELS

In this section, we seek to identify the constant arrival rates R that can be supported by the two-hop system while satisfying the statistical delay constraints specified by (ϵ, D_{\max}) . We consider a block fading scenario in which the fading stays constant for a block of T seconds and change independently from one block to another.

The instantaneous capacities of the $\mathbf{S} - \mathbf{R}$ and $\mathbf{R} - \mathbf{D}$ links in each block are given, respectively, by

$$C_1 = TB \log_2(1 + \text{SNR}_1 z_1), \quad \text{and} \quad C_2 = TB \log_2(1 + \text{SNR}_2 z_2), \quad (26)$$

in the units of bits per block or equivalently bits per T seconds. These can be regarded as the service processes at the source and relay.

A. Buffer Stability and Log-Moment Generating Function of Block Fading Channels

To ensure the stability of the queues, we need to enforce the following condition[5]

$$\mathbb{E}_{z_1}\{C_1\} < \mathbb{E}_{z_2}\{C_2\}. \quad (27)$$

That is, the average arrival rate for the queue at the relay should be less than the average service rate.

Under the block fading assumption, the logarithmic moment generating functions for the service processes of queues at the source \mathbf{S} and the relay \mathbf{R} as functions of θ are given by [7]²

$$\Lambda_{sr}(\theta) = \log \mathbb{E}_{z_1}\{e^{\theta C_1}\}, \quad \text{and} \quad \Lambda_{rd}(\theta) = \log \mathbb{E}_{z_2}\{e^{\theta C_2}\}. \quad (28)$$

Therefore, the LMGF for the arrival process of the queue at the relay is

$$\Lambda_r(\theta) = \begin{cases} R\theta, & 0 \leq \theta \leq \tilde{\theta}, \\ R\theta + \log \mathbb{E}_{z_1}\{e^{(\theta-\tilde{\theta})C_1}\}, & \theta > \tilde{\theta}. \end{cases} \quad (29)$$

B. Effective Capacity under Statistical Delay Constraints

In the following, we first assume that there exists θ_1 and θ_2 such that (16) is satisfied. We identify the effective capacity associated with the given θ_1 and θ_2 values. Following the statistical delay tradeoff indicated

²Now, due to the assumption that the fading changes independently from one block to another, we can, for instance, simplify (5) as $\Lambda_A = \lim_{n \rightarrow \infty} \frac{\log \mathbb{E}\{e^{\theta \sum_{i=1}^n a^{[i]}}\}}{n} = \lim_{n \rightarrow \infty} \frac{\log \prod_{i=1}^n \mathbb{E}\{e^{\theta a^{[i]}}\}}{n} = \lim_{n \rightarrow \infty} \frac{\sum_{i=1}^n \log \mathbb{E}\{e^{\theta a^{[i]}}\}}{n} = \lim_{n \rightarrow \infty} \frac{n \log \mathbb{E}\{e^{\theta a^{[1]}}\}}{n} = \log \mathbb{E}\{e^{\theta a^{[1]}}\}$. If fading is correlated, such simplifications are in general not possible and analysis needs to be based on the limit forms of the asymptotic logarithmic moment generating functions. However, if the service rates can be regarded as Markov modulated processes, then it is shown in [23, Section 7.2] that $\lim_{n \rightarrow \infty} \frac{\log \mathbb{E}\{e^{\theta \sum_{i=1}^n a^{[i]}}\}}{\theta n} = \frac{1}{\theta} \log \text{sp}(\phi(\theta)r)$ where $\text{sp}(A)$ denotes the spectral radius or equivalently the maximum of the absolute values of the eigenvalues of the matrix A , and $\phi(\theta)r$ is a matrix which depends on the transition probabilities of the Markov process. In such cases, an analysis similar to the one given in this paper can be pursued to identify the effective capacity of the two-hop system under the statistical delay constraints.

in Lemma 1, we can obtain the effective capacity over all possible θ_1 and θ_2 , which is the effective capacity under the statistical delay constraint in Definition 1.

From (13) and (28), we have

$$J_1(\theta) = -\log \mathbb{E}_{z_1}\{e^{-\theta C_1}\}, \text{ and } J_2(\theta) = -\log \mathbb{E}_{z_2}\{e^{-\theta C_2}\}. \quad (30)$$

To proceed, we need the following properties of $J(\theta)$.

Lemma 2: Consider the function

$$J(\theta) = -\log \mathbb{E}_z\{e^{-\theta C}\} \quad \text{for } \theta \geq 0, \quad (31)$$

where $C = TB \log_2(1 + \text{SNR}z)$. This function has the following properties.

- a) $J(0) = 0$.
- b) $\dot{J}(0) = \mathbb{E}_z\{C\} > 0$, i.e., the first derivative of $J(\theta)$ with respect to θ at $\theta = 0$ is given by the average service rate.
- c) $J(\theta)$ is a concave function of θ .
- d) $\lim_{\theta \rightarrow \infty} J(\theta) = -\log \Pr\{C = 0\}$, i.e., the probability of the event that the service rate is 0.

Proof: See Appendix B.

Remark 2: From the properties above, we can see that $J(\theta)$ is equal to 0 at $\theta = 0$, and then it increases sublinearly, and achieves upperbound, if it exists, for $\theta \rightarrow \infty$. Therefore, $J(\theta)$ is a bijective function of θ , and for each value of J , we can find the associated θ .

Assumption 1: Throughout this article, we consider the fading distributions that satisfy the following conditions:

- 1) $\Pr\{z_1 = 0\} = 0$.
- 2) $\Pr\{z_2 = 0\} = 0$.

Remark 3: Under this assumption, we can see that $J_1(\theta)$ and $J_2(\theta)$ approaches to ∞ as θ increases. Note that for the continuous distributions of the fading states, such as Rayleigh and Rician fading, the above assumption is justified immediately. If the above assumption does not hold, we can see that the upperbounds

for $J_1(\theta_1)$ and $J_2(\theta_2)$ are finite values, and the following analysis still holds while only considering a sliced part of (J_1, J_2) of the $J_1 - J_2$ curve characterized in Lemma 1.

Note that we can also derive the following properties of effective capacity.

Lemma 3: ([16]) Consider the functions

$$\varphi(\theta) = -\frac{1}{\theta} \log \mathbb{E}_{z_1} \{e^{-\theta C_1}\} = \frac{J_1(\theta)}{\theta} \quad \text{for } \theta \geq 0, \quad (32)$$

$$\phi(\theta) = -\frac{1}{\theta} \log \mathbb{E}_{z_2} \{e^{-\theta C_2}\} = \frac{J_2(\theta)}{\theta} \quad \text{for } \theta \geq 0. \quad (33)$$

where C_1 and C_2 are given by (26). We have

- a) The functions are decreasing in θ .
- b) $\varphi(\theta)$ is increasing in SNR_1 , and $\phi(\theta)$ is increasing in SNR_2 .

Remark 4: According to Lemma 2 and the conditions specified in (17) and (20), we can see that the effective capacity obtained always satisfy the statistical delay constraints as long as θ_1 and θ_2 satisfy (16). Therefore, with the definitions of $J_1(\theta_1)$ and $J_2(\theta_2)$ in (30), we can find the associated θ_1 and θ_2 on the lower boundary curve indicated by Lemma 1. Iterating over this set of θ_1 and θ_2 , we can obtain the effective capacity under the statistical delay constraints. For other values of θ_1 and θ_2 , either the delay constraint cannot be satisfied, or one of the queues is subject to more stringent constraint than necessary, leading to worse performance due to Lemma 3.

For the following analysis, we define

$$\Omega_\epsilon = \{(\theta_1, \theta_2) : J_1(\theta_1) \text{ and } J_2(\theta_2) \text{ are solutions to (23)}\}. \quad (34)$$

Additionally, we need the following upperbound on the arrival rates supported by the two-hop system.

Proposition 1: ([16]) The constant arrival rates, which can be supported by the two-hop link in the presence of queueing constraints θ_1 and θ_2 at the source and relay, respectively, are upperbounded by

$$R \leq \min \left\{ -\frac{1}{\theta_1} \log \mathbb{E}_{z_1} \{e^{-\theta_1 C_1}\}, -\frac{1}{\theta_2} \log \mathbb{E}_{z_2} \{e^{-\theta_2 C_2}\} \right\} = \min \left\{ \frac{J_1(\theta_1)}{\theta_1}, \frac{J_2(\theta_2)}{\theta_2} \right\}. \quad (35)$$

Remark 5: In the rest of the text, we use the following definitions

$$R_1 = \frac{J_1(\theta_1)}{\theta_1}, \quad \text{and} \quad R_2 = \frac{J_2(\theta_2)}{\theta_2}. \quad (36)$$

They represent the two terms inside the minimization of (35).

The effective capacity of the two-hop system, i.e., maximum of the arrival rates that can be supported in the two-hop system in the presence of queueing constraints θ_1 and θ_2 , is given by the the following result.

Theorem 2: ([16]) The effective capacity of the two-hop system given $\theta_1 > 0$ and $\theta_2 > 0$ is given by the following:

Case I: If $\theta_1 \geq \theta_2$,

$$R_E(\theta_1, \theta_2) = \min \left\{ -\frac{1}{\theta_1} \log \mathbb{E}_{z_1} \{e^{-\theta_1 C_1}\}, -\frac{1}{\theta_2} \log \mathbb{E}_{z_2} \{e^{-\theta_2 C_2}\} \right\}. \quad (37)$$

Case II: If $\theta_1 < \theta_2$ and $\theta_2 \leq \bar{\theta}$,

$$R_E(\theta_1, \theta_2) = -\frac{1}{\theta_1} \log \mathbb{E}_{z_1} \{e^{-\theta_1 C_1}\} \quad (38)$$

where $\bar{\theta}$ is the unique value of θ for which we have the following equality satisfied:

$$-\frac{1}{\theta_1} \log \mathbb{E}_{z_1} \{e^{-\theta_1 C_1}\} = -\frac{1}{\theta_1} \left(\log \mathbb{E}_{z_2} \{e^{-\theta C_2}\} + \log \mathbb{E}_{z_1} \{e^{(\theta-\theta_1)C_1}\} \right). \quad (39)$$

Case III: Assume $\theta_1 < \theta_2$ and $\theta_2 > \bar{\theta}$.

III.a: If

$$-\frac{1}{\theta_2} \log \mathbb{E}_{z_2} \{e^{-\theta_2 C_2}\} \geq -\frac{1}{\theta_2} \log \mathbb{E}_{z_1} \{e^{-\theta_2 C_1}\}, \quad (40)$$

then

$$R_E(\theta_1, \theta_2) = -\frac{1}{\bar{\theta}^*} \log \mathbb{E}_{z_1} \{e^{-\bar{\theta}^* C_1}\} \quad (41)$$

where $\tilde{\theta}^*$ is the smallest solution to

$$-\frac{1}{\tilde{\theta}} \log \mathbb{E}_{z_1} \left\{ e^{-\tilde{\theta} C_1} \right\} = -\frac{1}{\tilde{\theta}} \left(\log \mathbb{E}_{z_2} \left\{ e^{-\theta_2 C_2} \right\} + \log \mathbb{E}_{z_1} \left\{ e^{(\theta_2 - \tilde{\theta}) C_1} \right\} \right). \quad (42)$$

III.b: Otherwise,

$$R_E(\theta_1, \theta_2) = -\frac{1}{\theta_2} \log \mathbb{E}_{z_2} \left\{ e^{-\theta_2 C_2} \right\}. \quad (43)$$

Recall that we are seeking to identify the effective capacity of the two-hop system under statistical delay constraints specified by (ϵ, D_{\max}) . Combining the behavior of $R_E(\theta_1, \theta_2)$ delineated in Theorem 2 and the tradeoff between $J_1(\theta_1)$ and $J_2(\theta_2)$ in Lemma 1, we have the following result.

Theorem 3: The effective capacity of the two-hop wireless communication systems subject to statistical delay constraints specified by (ϵ, D_{\max}) is given by the following:

Case I: If $\theta_{1,th} = \theta_{2,th}$,

$$R_\epsilon(\epsilon, D_{\max}) = \frac{J_{th}(\epsilon)}{\theta_{1,th}}, \quad (44)$$

where $(\theta_{1,th}, \theta_{2,th})$ is the unique solution pair to $J_1(\theta_1) = J_{th}(\epsilon)$, and $J_2(\theta_2) = J_{th}(\epsilon)$.

Case II: If $\theta_{1,th} > \theta_{2,th}$,

$$R_\epsilon(\epsilon, D_{\max}) = \begin{cases} \frac{J_0}{\theta_{1,0}}, & TB \log_2(1 + \text{SNR}_2 z_{2,\min}) \geq TB \log_2(1 + \text{SNR}_1 z_{1,\max}), \\ \frac{J_1(\theta_1)}{\theta_1}, & \text{otherwise.} \end{cases} \quad (45)$$

where $\theta_{1,0}$ is the solution to $J_1(\theta_1) = J_0$, θ_1 is given by $(\theta_1, \theta_2) \in \Omega_\epsilon$ with

$$\theta_1 = \theta_2, \quad (46)$$

and θ_1 is the smallest value of θ_1 with $(\theta_1, \theta_2) \in \Omega_\epsilon$ satisfying

$$-\frac{1}{\theta_1} \log \mathbb{E}_{z_1} \left\{ e^{-\theta_1 C_1} \right\} = -\frac{1}{\theta_1} \left(\log \mathbb{E}_{z_2} \left\{ e^{-\theta_2 C_2} \right\} + \log \mathbb{E}_{z_1} \left\{ e^{(\theta_2 - \theta_1) C_1} \right\} \right). \quad (47)$$

Moreover, if $\left. \frac{dJ_2(\theta)}{d\theta} \right|_{\theta=\theta_1} \leq \left. \frac{dJ_1(\theta)}{d\theta} \right|_{\theta=\theta_1}$, the solution to (47) with $(\theta_1, \theta_2) \in \Omega_\epsilon$ is unique.

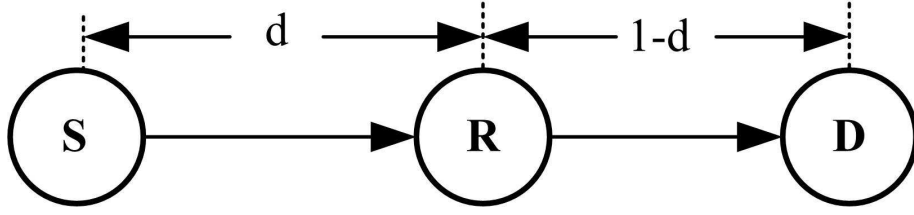


Fig. 3. The relay model.

Case III: If $\theta_{1,th} < \theta_{2,th}$,

$$R_\epsilon(\epsilon, D_{\max}) = \begin{cases} \frac{J_0}{\theta_{2,0}}, & TB \log_2(1 + \text{SNR}_1 z_{1,\min}) \geq \frac{J_0}{\theta_{2,0}} \\ \frac{J_2(\check{\theta}_2)}{\check{\theta}_2}, & \text{otherwise.} \end{cases} \quad (48)$$

where $\theta_{2,0}$ is the solution to $J_2(\theta_2) = J_0$, and $(\check{\theta}_1, \check{\theta}_2)$ is the unique solution to

$$\frac{J_1(\theta_1)}{\theta_1} = \frac{J_2(\theta_2)}{\theta_2} \quad (49)$$

with $(\theta_1, \theta_2) \in \Omega_\epsilon$.

Proof: See Appendix C.

Remark 6: Although implicitly, when $\theta_{1,th} = \theta_{2,th}$, we can also show that $\theta_{1,th}$ is the smallest θ_1 with $(\theta_1, \theta_2) \in \Omega_\epsilon$ satisfying (47) following the same argument in the proof. Note that $J_{th}(\epsilon)$ is a value decided by only ϵ and D_{\max} , while $\theta_{1,th}$ and $\theta_{2,th}$ also depend on SNR_1 , and SNR_2 , and the fading distributions.

Remark 7: The condition given in (45) or (48) indicate that either $J_2(\theta_2)$ or $J_1(\theta_1)$ can approach to infinity, and hence the only delay introduced is the queue at the source or the relay node, respectively.

Remark 8: Note that the effective capacity under statistical delay constraints is achieved when the queue at the relay is about to be the bottleneck of the two-hop system. Depending on the fading distributions and SNR levels, the operation point can be one such that the delay constraint at the queue of the source or the relay node can be lessened. This provides us insight for the design of wireless systems, and resource allocations.

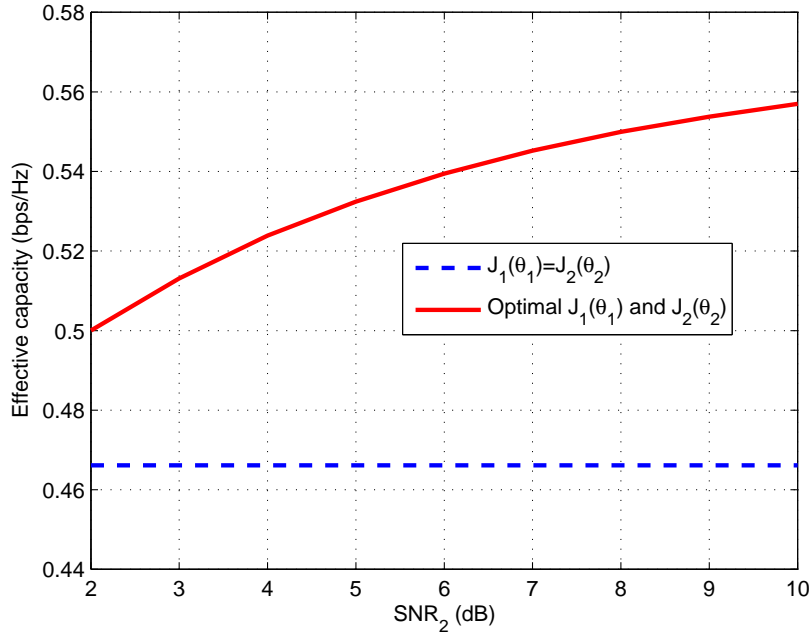


Fig. 4. Effective capacity in SNR_2 . $\text{SNR}_1 = 0$ dB. $\epsilon = 0.001$. $D_{\max} = 1$ sec.

V. NUMERICAL RESULTS

We consider the relay model depicted in Fig. 3. The source, relay, and destination nodes are located on a straight line. The distance between the source and the destination is normalized to 1. Let the distance between the source and the relay node be $d \in (0, 1)$. Then, the distance between the relay and the destination is $1 - d$. We assume the fading distributions for $\text{S} - \text{R}$ and $\text{R} - \text{D}$ links follow independent Rayleigh fading with means $\mathbb{E}\{z_1\} = 1/d^\alpha$ and $\mathbb{E}\{z_2\} = 1/(1-d)^\alpha$, respectively, where we assume that the path loss $\alpha = 4$. We assume that $\text{SNR}_1 = 0$ dB in the following numerical results.

In Fig. 4, we plot the effective capacity as a function of SNR of the relay node. We also plot the effective capacity with balanced delay constraints for the two queues, i.e., $J_1(\theta_1) = J_2(\theta_2) = J_{th}(\epsilon)$. We fix $d = 0.5$, in which case the $\text{S} - \text{R}$ and $\text{R} - \text{D}$ links have the same channel conditions. We assume that the statistical delay constraint is given by $\epsilon = 10^{-3}$ and $D_{\max} = 1$ sec. From the figure, we can see that the effective capacity of two-hop system increases with SNR_2 . And, in all cases, the achievable rate is greater than the one achieved with balanced delay constraints. In Fig. 5, we plot the associated $J_2(\theta_2)$ as a function of $J_1(\theta_1)$.

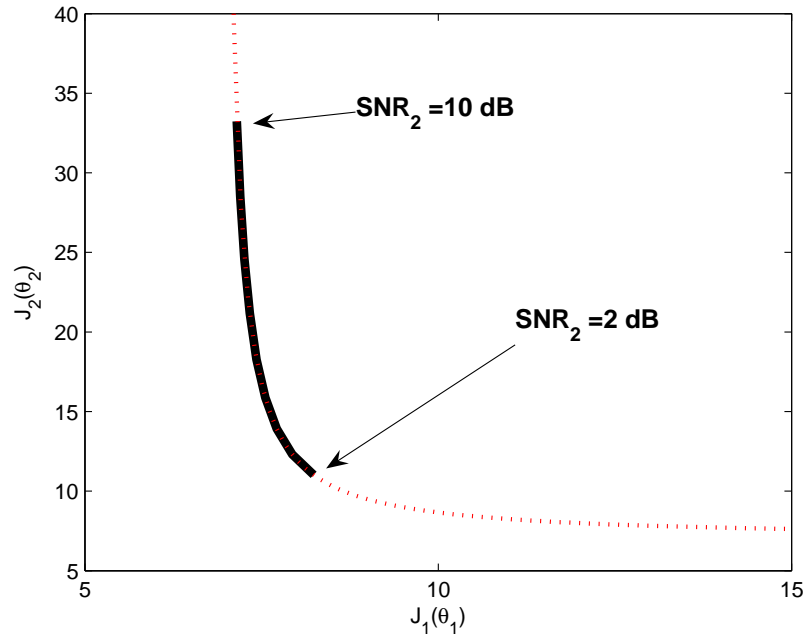


Fig. 5. $J_2(\theta_2)$ v.s. $J_1(\theta_1)$ as SNR_2 varies. $\text{SNR}_1 = 0$ dB. $\epsilon = 0.001$. $D_{\max} = 1$ sec.

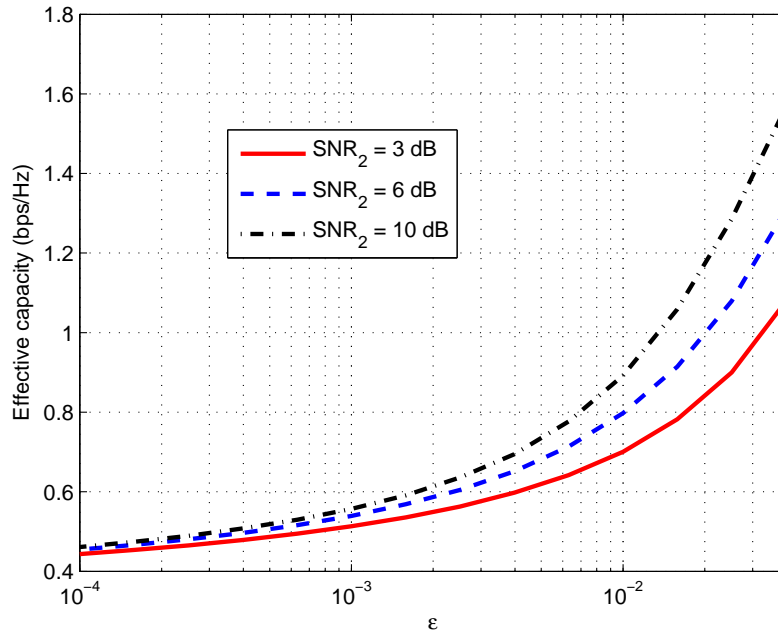


Fig. 6. Effective capacity as a function of ϵ . $\text{SNR}_1 = 0$ dB. $D_{\max} = 1$ sec.

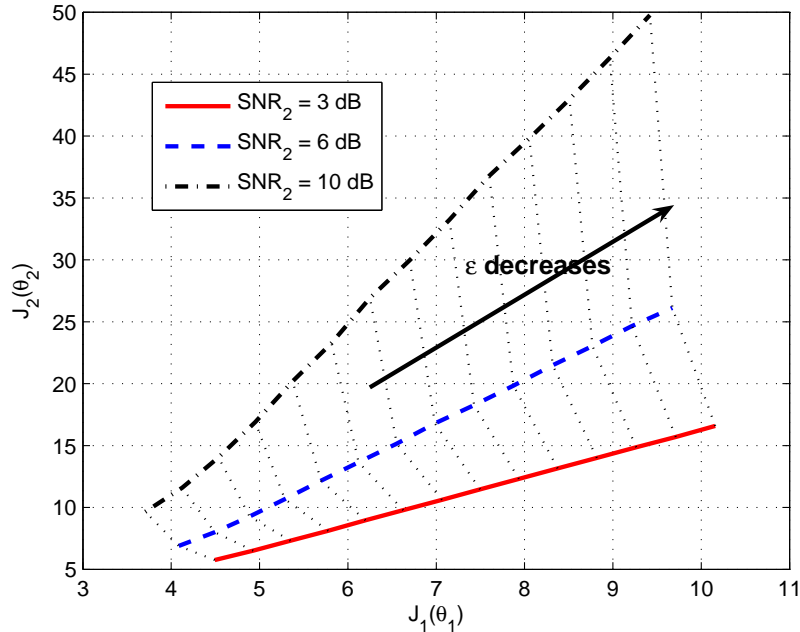


Fig. 7. $J_2(\theta_2)$ v.s. $J_1(\theta_1)$ as ϵ varies. $\text{SNR}_1 = 0$ dB. $D_{\max} = 1$ sec.

As can be seen from the figure, $J_2(\theta_2)$ increases as SNR_2 increases, i.e., we can put more stringent constraint to the queue at the relay, and hence the delay constraint at the source can be less. In this way, the effective capacity of the two-hop system can be improved.

We are interested in the impact of the delay violation probability ϵ on the achievable performance. In Fig. 6, we plot the effective capacity as ϵ varies for $\text{SNR}_2 = \{3, 6, 10\}$ dB. It is interesting that as ϵ decreases, the performance gap between different curves vanishes, i.e., the improvement caused by the increase of the signal-to-noise ratio at the relay can be negligible. To get more insights, we also plot the associated values of $J_1(\theta_1)$ and $J_2(\theta_2)$ as ϵ decreases in Fig. 7. It can be found that the increase in $J_2(\theta_2)$ is becoming larger while the decrease in $J_1(\theta_1)$ is smaller as ϵ decreases. Considering the convexity of $J_2(\theta_2)$ in $J_1(\theta_1)$ in Lemma 1, loosening the queueing constraint at one queue will require the other queue to operate in a much more conservative way, which provides little gain under more stringent delay constraints, i.e., smaller ϵ .

In Fig. 8, we plot the effective capacity as d varies. We assume $\text{SNR}_2 = \{3, 6, 10\}$ dB, $\epsilon = 0.001$. We can see from the figure that as d increases, i.e., the channel condition at the $\text{S} - \text{R}$ link is worse, the effective

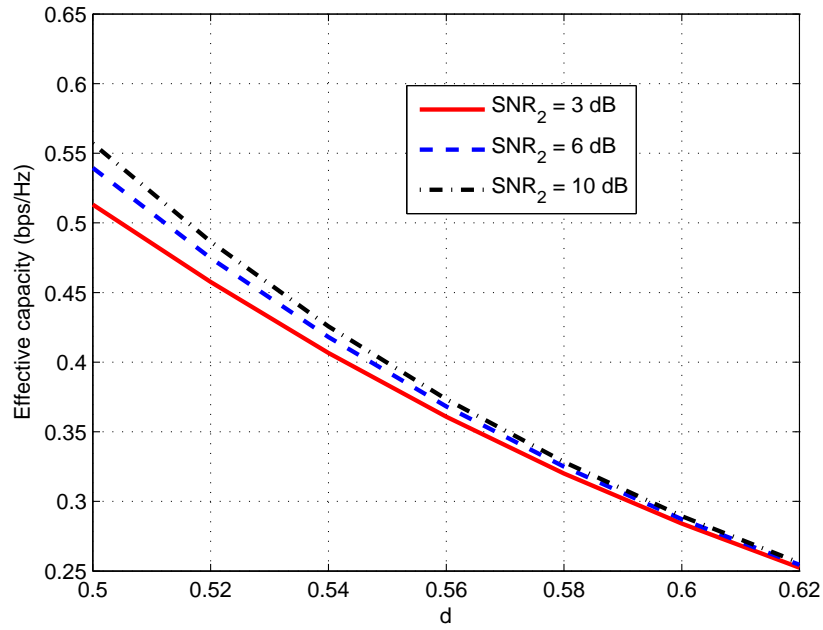


Fig. 8. Effective capacity as a function of d . SNR₁ = 0 dB. $\epsilon = 0.001$. $D_{\max} = 1$ sec.

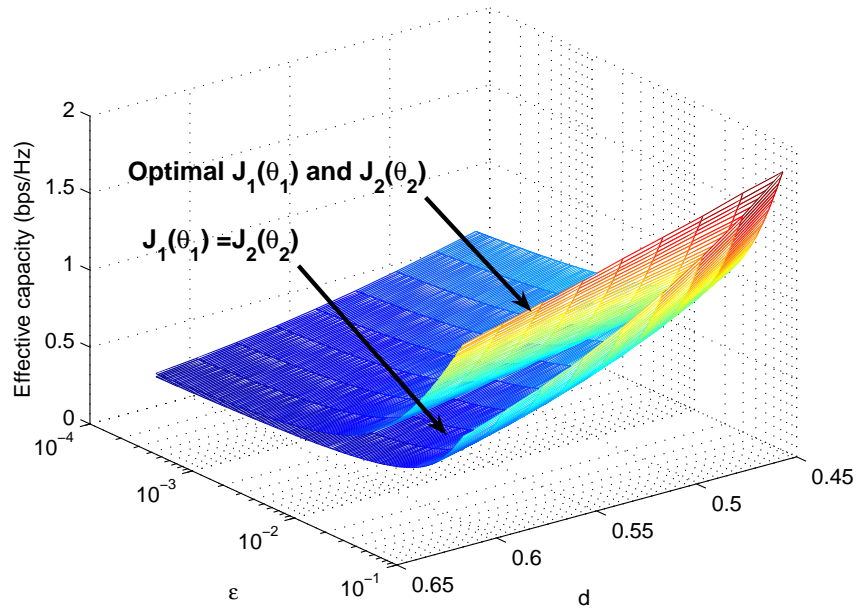


Fig. 9. Effective capacity as a function of d and ϵ . SNR₁ = 0 dB. SNR₂ = 3 dB. $D_{\max} = 1$ sec.

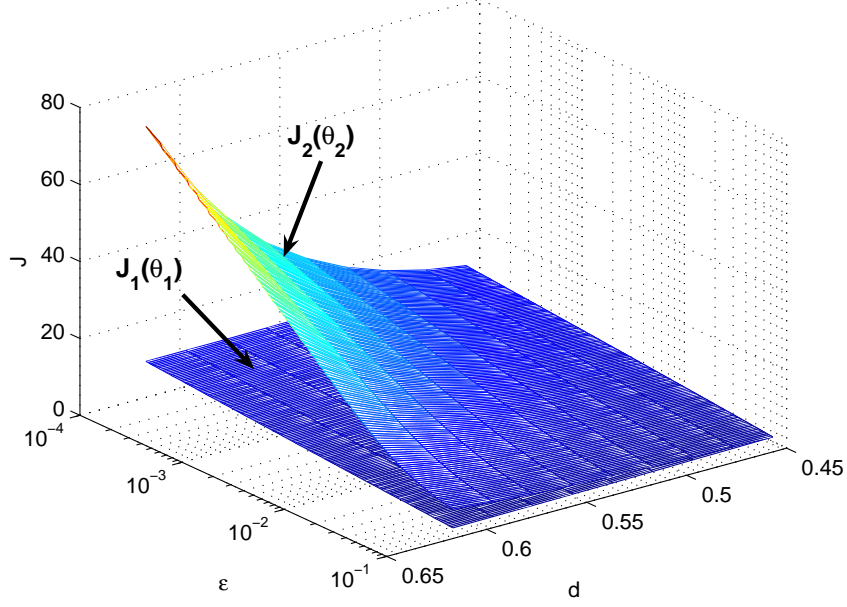


Fig. 10. $J_1(\theta_1)$ and $J_2(\theta_2)$ as functions of d and ϵ . $\text{SNR}_1 = 0$ dB. $\text{SNR}_2 = 3$ dB. $D_{\max} = 1$ sec.

capacity decreases, and the increase of SNR at the relay node helps little. This is mainly because of the severe channel conditions between the $\mathbf{S} - \mathbf{R}$ link, which is the bottleneck of the system. Finally, we plot the effective capacity as d and ϵ varies in Fig. 9, with the associated delay tradeoff $J_1(\theta_1)$ and $J_2(\theta_2)$ in Fig. 10. We assume $\text{SNR}_2 = 3$ dB. In order to ensure the stability of the queues, the minimum $d_{\min} = 0.4569$. Note that due to the definition of supremum for the effective capacity, the performance at d_{\min} can be achieved via some d arbitrarily close to d_{\min} . As can see from the figure, for all cases, effective capacity decreases as d increases or ϵ decreases, even with strong bias towards the queue at the source indicated by the larger $J_2(\theta_2)$. It is interesting that for large ϵ , the performance improvement by adjusting the delay constraints at the queues can be larger, albeit the improvement provided by increasing SNR at the relay vanishes with d . Motivated by this observation, we plot the effective capacity as d varies for $\text{SNR}_2 = 3$ dB and $\epsilon = 0.05$ in Fig. 11. It is obvious that the performance improvement by statistical delay tradeoff first increases with d , and after some point, it again decreases due to the poor channel conditions between the $\mathbf{S} - \mathbf{R}$ link. It is obvious that as d approach to 1, i.e., $TB \log_2(1 + \text{SNR}_2 z_{2,\min}) \geq TB \log_2(1 + \text{SNR}_1 z_{1,\max})$, the effective

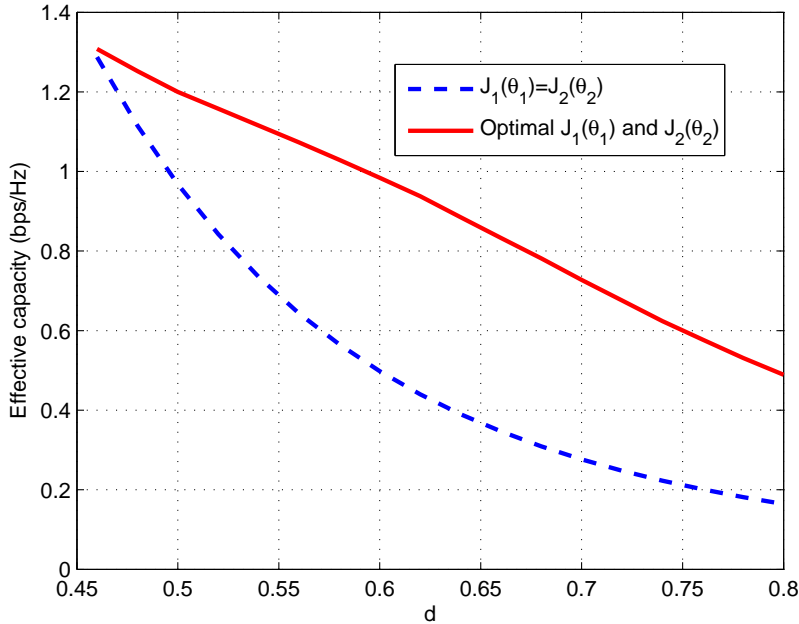


Fig. 11. Effective capacity as a function of d . $\text{SNR}_1 = 0$ dB. $\text{SNR}_2 = 3$ dB. $\epsilon = 0.05$. $D_{\max} = 1$ sec.

capacity is limited by the $S - R$ link, and the two curve will merge each other.

VI. CONCLUSION

In this paper, we have investigated the maximum constant arrival rates that can be supported by a two-hop communication link with full-duplex relay under statistical delay constraints. We have provided a unified framework for achieving statistical delay tradeoff imposed to the source and relay node while satisfying the statistical delay constraints. We have determined the effective capacity in the block-fading scenario as a function of the statistical delay constraints, the signal-to-noise ratio levels SNR_1 and SNR_2 , and the fading distributions. It is interesting that having bias towards the delay constraints at one queue can help improve the effective capacity of the two-hop system, especially when the delay violation probability can be large. Also, we have shown that increasing the SNR level at the relay node can further improve the achievable rate, while the improvement is negligible when either the delay constraint is too stringent or the channel conditions between the source and relay node are considerably poor. Moreover, even when the channel conditions between the source and the relay are becoming worse, we can still obtain non-negligible

performance improvement by the statistical delay tradeoff when the delay violation probability is large.

APPENDIX

A. Proof of Lemma 1

1) When $J_1(\theta_1) \neq J_2(\theta_2)$, the continuity is obvious since there is no pole to the equation (23). Consider $J_1(\theta_1) = J_2(\theta_2)$. We can see that

$$\lim_{J_2(\theta_2) \rightarrow J_1(\theta_1)-} \vartheta(J_1(\theta_2), J_2(\theta_2)) = \lim_{J_2(\theta_2) \rightarrow J_1(\theta_1)-} \frac{J_2(\theta_2)e^{-J_1(\theta_1)D_{\max}} - J_1(\theta_1)e^{-J_2(\theta_2)D_{\max}}}{J_2(\theta_2) - J_1(\theta_1)} \quad (50)$$

$$= \lim_{J_2(\theta_2) \rightarrow J_1(\theta_1)-} e^{-J_2(\theta_2)D_{\max}} \frac{J_2(\theta_2)e^{-(J_1(\theta_1)-J_2(\theta_2))D_{\max}} - J_1(\theta_1)}{J_2(\theta_2) - J_1(\theta_1)} \quad (51)$$

$$= \lim_{J_2(\theta_2) \rightarrow J_1(\theta_1)-} e^{-J_2(\theta_2)D_{\max}} \left(1 + J_2(\theta_2) \frac{1 - e^{-(J_1(\theta_1)-J_2(\theta_2))D_{\max}}}{J_1(\theta_1) - J_2(\theta_2)} \right) \quad (52)$$

$$= e^{-J_2(\theta_2)D_{\max}} (1 + J_2(\theta_2)D_{\max}) \quad (53)$$

Similarly, we can show that

$$\lim_{J_2(\theta_2) \rightarrow J_1(\theta_1)+} \vartheta(J_1(\theta_2), J_2(\theta_2)) = e^{-J_1(\theta_1)D_{\max}} (1 + J_1(\theta_1)D_{\max}). \quad (54)$$

From (15), we can see that at $J_1(\theta_1) = J_2(\theta_2)$, $\vartheta(J_1(\theta_2), J_2(\theta_2))$ is continuous, i.e., $J_2(\theta_2) = \Phi(J_1(\theta_1))$ is continuous, and from (16), we should have

$$(1 + J_1(\theta_1)D_{\max}) e^{-J_1(\theta_1)D_{\max}} \leq \epsilon \quad (55)$$

which gives us (25) immediately by solving the above equation with equality.

2) Taking the partial derivative of $\vartheta(J_1(\theta_1), J_2(\theta_2))$ in $J_1(\theta_1)$ and noting that the right-hand-side (RHS) of (23) is constant, we have

$$\begin{aligned} \frac{\partial \vartheta(J_1(\theta_1), J_2(\theta_2))}{\partial J_1(\theta_1)} &= \frac{1}{(J_2(\theta_2) - J_1(\theta_1))^2} \left(\left(\dot{J}_2(\theta_2)e^{-J_1(\theta_1)D_{\max}} - J_2(\theta_2)D_{\max}e^{-J_1(\theta_1)D_{\max}} - e^{-J_2(\theta_2)D_{\max}} \right. \right. \\ &\quad \left. \left. + J_1(\theta_1)\dot{J}_2(\theta_2)D_{\max}e^{-J_2(\theta_2)D_{\max}} \right) (J_2(\theta_2) - J_1(\theta_1)) - (\dot{J}_2(\theta_2) - 1) \right. \\ &\quad \left. \times (J_2(\theta_2)e^{-J_1(\theta_1)D_{\max}} - J_1(\theta_1)e^{-J_2(\theta_2)D_{\max}}) \right) = 0, \end{aligned} \quad (56)$$

which, after combining the coefficients of $\dot{J}_2(\theta_2)$ and rearrangement, gives us

$$\dot{\Phi}(J_1(\theta_1)) = \dot{J}_2(\theta_2) = \frac{J_2(\theta_2)}{J_1(\theta_1)} e^{(J_2(\theta_2) - J_1(\theta_1))D_{\max}} \frac{(J_2(\theta_2) - J_1(\theta_1))D_{\max} + e^{-(J_2(\theta_2) - J_1(\theta_1))D_{\max}} - 1}{(J_2(\theta_2) - J_1(\theta_1))D_{\max} + 1 - e^{-(J_2(\theta_2) - J_1(\theta_1))D_{\max}}} \quad (57)$$

In the following, we will show that $\dot{\Phi}(J_1(\theta_1)) < 0$. Denote $x = (J_2(\theta_2) - J_1(\theta_1))D_{\max}$, and define

$$\nu(x) = \frac{x + e^{-x} - 1}{x + 1 - e^x}. \quad (58)$$

Then, we can rewrite $\dot{\Phi}(J_1(\theta))$ as

$$\dot{\Phi}(J_1(\theta_1)) = \dot{J}_2(\theta_2) = \frac{J_2(\theta_2)}{J_1(\theta_1)} e^{(J_2(\theta_2) - J_1(\theta_1))D_{\max}} \nu(x). \quad (59)$$

Note that $\frac{J_2(\theta_2)}{J_1(\theta_1)} e^{(J_2(\theta_2) - J_1(\theta_1))D_{\max}}$ is positive. Taking the first derivative of $\nu(x)$, we obtain

$$\dot{\nu}(x) = \frac{4 - 2(e^x + e^{-x}) + x(e^x - e^{-x})}{(x + 1 - e^x)^2} \quad (60)$$

We can show that $\dot{\nu}(x) \geq 0$. Suppose $x > 0$. Considering the numerator of above equation, we have

$$4 - 2(e^x + e^{-x}) + x(e^x - e^{-x}) = -2(e^{\frac{x}{2}} - e^{-\frac{x}{2}})^2 + x(e^{\frac{x}{2}} - e^{-\frac{x}{2}})(e^{\frac{x}{2}} + e^{-\frac{x}{2}}) \quad (61)$$

$$= (e^{\frac{x}{2}} - e^{-\frac{x}{2}}) \left(-2(e^{\frac{x}{2}} - e^{-\frac{x}{2}}) + x(e^{\frac{x}{2}} + e^{-\frac{x}{2}}) \right) \quad (62)$$

$$= \frac{e^{-\frac{x}{2}}}{x + 2} (e^{\frac{x}{2}} - e^{-\frac{x}{2}}) \left(\frac{x - 2}{x + 2} e^x + 1 \right) \quad (63)$$

$$\geq 0 \quad (64)$$

where $\frac{x-2}{x+2}e^x \geq -1$ is incorporated since it is an increasing function of x , and its value at $x = 0$ is -1. Therefore, $\dot{\nu}(x) > 0$ for $x > 0$, i.e., $\nu(x)$ is increasing for $x > 0$. In a similar way, we can show that $\dot{\nu}(x) > 0$ for $x < 0$. Additionally, we can show $\lim_{x \rightarrow 0} \dot{\nu}(x) = 0$ by considering the Taylor expansions of e^x and e^{-x} at $x = 0$ and noting that the numerator goes to 0 in an order $o(x^4)$ while the denominator goes to 0 in the order of x^4 (detail is omitted since it is trivial). Therefore, ν is increasing

in x . Meanwhile,

$$\lim_{x \rightarrow \infty} \nu(x) = \lim_{x \rightarrow \infty} \frac{x + e^{-x} - 1}{x + 1 - e^x} = \lim_{x \rightarrow \infty} \frac{1 - e^{-x}}{1 - e^x} = 0. \quad (65)$$

Hence, $\nu(x) < 0$, which in turn, tells us that $\dot{\Phi}(J_1(\theta_1)) < 0$ in (59). Therefore, $J_2(\theta_2) = \Phi(J_1(\theta_1))$ is strictly decreasing in $J_1(\theta)$.

- 3) We will show the convexity of Φ by consider the branches for $J_2(\theta_2) > J_1(\theta_1)$ and $J_2(\theta_2) < J_1(\theta_1)$, respectively.

For $J_1(\theta_1) < J_{th}(\epsilon)$, we know that $J_2(\theta_2) > J_1(\theta_1)$. Consider

$$\dot{J}_2(\theta_2) = \frac{J_2(\theta_2)}{J_1(\theta_1)} e^{(J_2(\theta_2) - J_1(\theta_1))D_{\max}} \frac{(J_2(\theta_2) - J_1(\theta_1))D_{\max} + e^{-(J_2(\theta_2) - J_1(\theta_1))D_{\max}} - 1}{(J_2(\theta_2) - J_1(\theta_1))D_{\max} + 1 - e^{-(J_2(\theta_2) - J_1(\theta_1))D_{\max}}} \quad (66)$$

$$= \frac{J_2(\theta_2)}{J_1(\theta_1)} e^x \nu(x) \quad (67)$$

where again $x = (J_2(\theta_2) - J_1(\theta_1))D_{\max}$. Note that as x increases, $\frac{J_2(\theta_2)}{J_1(\theta_1)}$ should increase since $J_1(\theta_1)$ decreases and $J_2(\theta_2)$ increases. From the above discussion, we know $\nu(x) < 0$, for $x > 0$. Define $\eta(x) = e^x \nu(x)$, $\eta(x) < 0$ for $x > 0$. Then if we can show that $\eta(x)$ is decreasing as x increases, then $\dot{J}_2(\theta_2) = \dot{\Phi}(J_1(\theta_1))$ will decrease with x , since a decreasing minus value multiplying an increasing positive value will lead to smaller minus value. Taking the first derivative of $\eta(x)$, we have

$$\dot{\eta}(x) = e^x(\nu(x) + \dot{\nu}(x)) = e^x \frac{2 + x^2 - (e^x + e^{-x})}{(x + 1 - e^x)^2}. \quad (68)$$

Note that the numerator $2 + x^2 - (e^x + e^{-x})$ can be shown to be less than 0 for $x > 0$. More specifically, consider that its second derivative $2 - (e^x + e^{-x})$ is less than 0 for $x > 0$ and the first derivative $2x - (e^x - e^{-x})$ at $x = 0$ is 0, and hence its first derivative is always less than 0, which tells us that it is a decreasing function in x with the maximum value at $x = 0$ as 0. Therefore, $\dot{\eta} < 0$. Hence, $\dot{J}_2(\theta_2) < 0$ is decreasing as $J_1(\theta_1)$ decreases for $J_1(\theta_1) < J_{th}(\epsilon)$, i.e., $\ddot{\Phi}(J_1(\theta)) \geq 0$. Similarly, we can show that $\ddot{\Phi}(J_1(\theta_1)) \geq 0$ for $J_1(\theta_1) > J_{th}(\epsilon)$. Together, we know that $\ddot{\Phi}(J_1(\theta_1)) \geq 0$, and hence $J_2(\theta_2) = \Phi(J_1(\theta_1))$ is a convex function in $J_1(\theta_1)$.

4) Let $J_1(\theta)$ go to infinity, we can see that

$$\lim_{J_1(\theta) \rightarrow \infty} \vartheta(J_1(\theta), J_2(\theta)) = \lim_{J_1(\theta) \rightarrow \infty} e^{-J_2(\theta_2)D_{\max}} = e^{-J_0D_{\max}} \quad (69)$$

which indicates $\lim_{J_1(\theta) \rightarrow \infty} J_2(\theta_2) = J_0$. On the other hand, if we let $J_2(\theta)$ go to infinity, we can show that $\lim_{J_2(\theta) \rightarrow \infty} J_1(\theta_1) = J_0$. Together, we obtain the result in the lemma. \square

B. Proof of Lemma 2

a) This property can be readily seen by evaluating the function at $\theta = 0$.

b) The first derivative of J with respect to θ can be evaluated as

$$\dot{J}(\theta) = \frac{\mathbb{E}_z \{e^{-\theta C} C\}}{\mathbb{E}_z \{e^{-\theta C}\}} > 0. \quad (70)$$

Then, $\dot{J}(0)$ can be obtained by evaluating the above equation at $\theta = 0$.

c) The second derivative of J with respect to θ can be expressed as

$$\ddot{J}(\theta) = -\frac{1}{(\mathbb{E}_z \{e^{-\theta C}\})^2} \left(\mathbb{E}_z \{e^{-\theta C} C^2\} \mathbb{E}_z \{e^{-\theta C}\} - (\mathbb{E}_z \{e^{-\theta C} C\})^2 \right). \quad (71)$$

By Cauchy-Schwarz inequality, we know that $\mathbb{E}\{X^2\}\mathbb{E}\{Y^2\} \geq (\mathbb{E}\{XY\})^2$. Then, denoting

$X = \sqrt{e^{-\theta C} C^2}$ and $Y = \sqrt{e^{-\theta C}}$, we easily see that $\ddot{J}(\theta) \leq 0$ for all θ . Thus, $J(\theta)$ is a concave function.

d) Note that as long as $C \neq 0$, $\lim_{\theta \rightarrow \infty} e^{-\theta C} = 0$, and whenever $C = 0$, $e^{\theta C} = 1$. Therefore, we have

$$\lim_{\theta \rightarrow \infty} \mathbb{E}_{z \neq 0} \{e^{-\theta C}\} = 0. \quad (72)$$

Then $\lim_{\theta \rightarrow \infty} J(\theta) = \lim_{\theta \rightarrow \infty} -\log (\mathbb{E}_{z \neq 0} \{e^{-\theta C}\} + \mathbb{E}_{z=0} \{1\}) = -\log \Pr\{C = 0\}$. \square

C. Proof of Theorem 3

With the delay tradeoff specified in Lemma 1, we can see that there is potential improvement of effective capacity by adjusting the statistical delay constraint imposed to the queues at the source and the relay nodes.

As a start point, we consider $J_1(\theta_1) = J_2(\theta_2)$. According to the Lemma 2 and the subsequent discussions,

we can always find $\theta_{1,th}$ and $\theta_{2,th}$ for $J_{th}(\epsilon)$ defined in (25). Now, with the values of $\theta_{1,th}$ and $\theta_{2,th}$, we have different behaviors of the effective capacity depending on the conditions indicated in Theorem 2. We seek to find the optimal $J_1(\theta_1)$ and $J_2(\theta_2)$ with $(\theta_1, \theta_2) \in \Omega_\epsilon$ to maximize the effective capacity, where Ω_ϵ is defined in (34).

Case I: Assume $\theta_{1,th} = \theta_{2,th}$.

For this case, we should have

$$R_E(\theta_{1,th}, \theta_{2,th}) = R_1 = \frac{J_{th}(\epsilon)}{\theta_{1,th}} = \frac{J_{th}(\epsilon)}{\theta_{2,th}} = R_2. \quad (73)$$

We assert that this value is the effective capacity of the two-hop system, i.e.,

$$R_\epsilon(\epsilon, D_{\max}) = \sup_{(\theta_1, \theta_2) \in \Omega} R_E(\theta_1, \theta_2) = R_E(\theta_{1,th}, \theta_{2,th}). \quad (74)$$

We can show this by contradiction. From Lemma 3, we know that the effective capacity is a decreasing function in θ . Suppose that there exists some $R > R_E(\theta_{1,th}, \theta_{2,th})$ that can be supported by the two-hop system with θ_1 and θ_2 . With Lemma 3, we must have $\theta_1 < \theta_{1,th}$. Then $J_1(\theta_1) < J_1(\theta_{1,th})$. According to the statistical delay tradeoff shown in Lemma 1, we can see that $J_2(\theta_2) > J_2(\theta_{2,th})$, which tells us that $\theta_2 > \theta_{2,th}$ according to Lemma 2, since $J_2(\theta)$ is increasing in θ . Now from the Proposition 1, we obtain

$$R \leq \min \left\{ \frac{J_1(\theta_1)}{\theta_1}, \frac{J_2(\theta_2)}{\theta_2} \right\} = \frac{J_2(\theta_2)}{\theta_2} < \frac{J_2(\theta_{2,th})}{\theta_{2,th}} = R_E(\theta_{1,th}, \theta_{2,th}) \quad (75)$$

which leads to a contradiction.

Case II: Assume $\theta_{1,th} > \theta_{2,th}$.

In this case, we can see that

$$R_1 = \frac{J_1(\theta_{1,th})}{\theta_{1,th}} = \frac{J_{th}(\epsilon)}{\theta_{1,th}} < \frac{J_{th}(\epsilon)}{\theta_{2,th}} = \frac{J_2(\theta_{2,th})}{\theta_{2,th}} = R_2. \quad (76)$$

The effective capacity associated with $\theta_{1,th}, \theta_{2,th}$ specializes into **Case I** of Theorem 2. Therefore, $R_E(\theta_{1,th}, \theta_{2,th}) = \min\{R_1, R_2\} = R_1$. Obviously, the queueing constraint imposed at the source is more stringent. To achieve better performance, we should try to relieve the queueing constraints at the source, i.e., decrease θ_1 , or $J_1(\theta_1)$

equivalently. Correspondingly, from Lemma 1, $J_2(\theta_2)$ should increase, and we have $J_2(\theta_2) > J_{th}(\epsilon) > J_1(\theta_1)$. In the following, we will provide a characterization of θ_1 as we iterate over $(\theta_1, \theta_2) \in \Omega_\epsilon$ to find the optimal pair that maximizes the effective capacity.

First, noting that as $J_1(\theta_1)$ decreases from $J_{th}(\epsilon)$ to J_0 , we can see that θ_1 decreases from $\theta_{1,th}$ to some finite value $\theta_{1,0}$, which is the solution to $J_1(\theta) = J_0$. To the opposite, θ_2 increases from $\theta_{2,th} < \theta_{1,th}$ to ∞ . Clearly, from the continuity of $J_2(\theta_2) = \Phi(J_1(\theta_1))$, θ_2 and θ_1 should be continuous as well. Hence, there must be one point $(\underline{\theta}_1, \underline{\theta}_2) \in \Omega_\epsilon$ such that

$$\underline{\theta}_1 = \underline{\theta}_2, \quad (77)$$

and for all $\theta_1 < \underline{\theta}_1$, we will have $\theta_2 > \underline{\theta}_2 = \underline{\theta}_1 > \theta_1$. According to Lemma 2, we know $J_1(\theta)$ and $J_2(\theta)$ are increasing functions of θ . Therefore, at this point, we have

$$R_1 = \frac{J_1(\underline{\theta}_1)}{\underline{\theta}_1} < \frac{J_1(\theta_{1,th})}{\underline{\theta}_1} = \frac{J_{th}(\epsilon)}{\underline{\theta}_1} = \frac{J_2(\theta_{2,th})}{\underline{\theta}_1} < \frac{J_2(\underline{\theta}_2)}{\underline{\theta}_1} = \frac{J_2(\underline{\theta}_2)}{\underline{\theta}_2} = R_2. \quad (78)$$

That is, the queue at the source is still the bottleneck of the two-hop system. We can further relieve the queueing constraint at the source.

Now, as θ_1 further decreases, $\theta_1 < \theta_2$. Consequently, the effective capacity associated with (θ_1, θ_2) now specializes into **Case II** of Theorem 2. As can be seen from [16, Lemma 2], the queue at the relay will not affect the performance as long as θ_1 and θ_2 satisfy the following inequality given by

$$-\frac{1}{\theta_1} \log \mathbb{E}_{z_1} \{ e^{-\theta_1 C_1} \} \leq -\frac{1}{\theta_1} \left(\log \mathbb{E}_{z_2} \{ e^{-\theta_2 C_2} \} + \log \mathbb{E}_{z_1} \{ e^{(\theta_2 - \theta_1) C_1} \} \right). \quad (79)$$

Note that as θ_1 decreases from $\underline{\theta}_1$ to $\theta_{1,0}$, the LHS of the above inequality increases from $\frac{J_1(\underline{\theta}_1)}{\underline{\theta}_1}$ to $\frac{J_0}{\theta_{1,0}}$. On the other hand, at $\theta_1 = \underline{\theta}_1$, we have $\underline{\theta}_2 = \underline{\theta}_1$, and the value of the RHS of the above inequality at $(\underline{\theta}_1, \underline{\theta}_2)$ is given by

$$\text{RHS} = \frac{J_2(\underline{\theta}_2)}{\underline{\theta}_1} > \frac{J_1(\underline{\theta}_1)}{\underline{\theta}_1}. \quad (80)$$

As $\theta_1 \rightarrow \theta_{1,0}$, or $J_1(\theta_1) \rightarrow J_0$, we know that

$$\begin{aligned} \lim_{J_1(\theta_1) \rightarrow J_0} \text{RHS} &= \lim_{J_1(\theta_1) \rightarrow J_0} -\frac{1}{\theta_1} \left(\log \mathbb{E}_{z_2} \{e^{-\theta_2 C_2}\} + \log \mathbb{E}_{z_1} \{e^{(\theta_2 - \theta_1) C_1}\} \right) \\ &= \lim_{J_1(\theta_1) \rightarrow J_0} \frac{\theta_2}{\theta_1} \left(-\frac{1}{\theta_2} \log \mathbb{E}_{z_2} \{e^{-\theta_2 C_2}\} - \frac{1}{\theta_2} \log \mathbb{E}_{z_1} \{e^{(\theta_2 - \theta_1) C_2}\} \right). \end{aligned} \quad (81)$$

Note further that $J_2(\theta_2)$, and hence θ_2 , approaches to infinity as $J_1(\theta_1) \rightarrow J_0$. The first term inside the parenthesis goes to the minimum rate of **R** – **D** link, i.e., $TB \log_2(1 + \text{SNR}_2 z_{2,\min})$ and the second term goes to the largest rate of the link **S** – **R**, i.e., $TB \log_2(1 + \text{SNR}_1 z_{1,\max})$. So as long as the smallest rate of **R** – **D** is less than the largest rate of the link **S** – **R**, the limit in (81) goes to $-\infty$. It is important to note that if the highest rate of **S** – **R** can be supported by the link **R** – **D**, i.e.,

$$TB \log_2(1 + \text{SNR}_2 z_{2,\min}) \geq TB \log_2(1 + \text{SNR}_1 z_{1,\max}), \quad (82)$$

then there is no congestion at the relay node at all. In this case, θ_2 can take any value greater than 0, and the only delay caused is the queue at the source. Therefore, the arrival rates are limited by the **S** – **R** link, and to satisfy the statistical delay constraints, we have

$$R_\epsilon(\epsilon, D_{\max}) = \frac{J_0}{\theta_{1,0}}. \quad (83)$$

Now, we consider the case when (82) is not satisfied. In such cases, $\theta_2 \rightarrow \infty$ as $J_2(\theta_2) \rightarrow \infty$. From the continuity of the functions, we know that there must be some $(\theta_1, \theta_2) \in \Omega_\epsilon$ such that the above inequality in (79) is satisfied with equality. Denote the smallest θ_1 as $\overset{\circ}{\theta}_1$. Then, for all $\theta_1 < \overset{\circ}{\theta}_1$, (79) cannot be satisfied.

Moreover, consider Lemma 3, we know as θ_1 decreases, R_1 increases from $\frac{J_{th}(\epsilon)}{\theta_{1,th}}$ to $\frac{J_0}{\theta_{1,0}}$. At the same time, as θ_2 approaches to infinity, R_2 decreases from $\frac{J_{th}(\epsilon)}{\theta_{2,th}}$ to $TB \log_2(1 + \text{SNR}_2 z_{\min})$. Therefore, there must be some value such that

$$R_1 = \frac{J_1(\theta_1)}{\theta_1} = R = \frac{J_2(\theta_2)}{\theta_2} = R_2 \quad (84)$$

with the associated statistical queueing constraints denoted as $\check{\theta}_1$ and $\check{\theta}_2$, respectively. For $\theta_1 < \check{\theta}_1$, we have

$$R_1 = \frac{J_1(\theta_1)}{\theta_1} > \frac{J_2(\theta_2)}{\theta_2} = R_2. \quad (85)$$

In the following, we can establish the comparison between $\check{\theta}_1$ and $\overset{\circ}{\theta}_1$ as

$$\check{\theta}_1 \leq \overset{\circ}{\theta}_1. \quad (86)$$

Note here that if $\frac{J_0}{\theta_{1,0}} < TB \log_2(1 + \text{SNR}_2 z_{\min})$, there is no θ_1 for (84) to be satisfied, and hence we can set $\check{\theta}_1$ to be 0, which satisfies the above claim obviously. Suppose that $\check{\theta}_1 > \overset{\circ}{\theta}_1$. Since at $\overset{\circ}{\theta}_1$, the condition for **Case II** of Theorem 2 can be satisfied, we immediately see that

$$R_E(\overset{\circ}{\theta}_1, \overset{\circ}{\theta}_2) = \frac{J_1(\overset{\circ}{\theta}_1)}{\overset{\circ}{\theta}_1}. \quad (87)$$

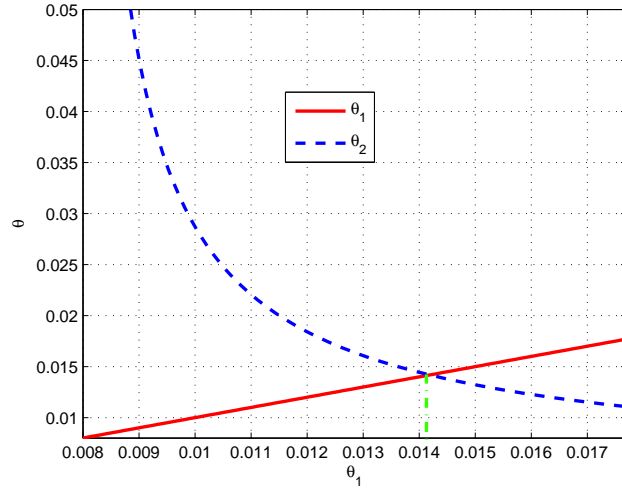
However, according to Proposition 1 and (85), we have

$$R_E(\overset{\circ}{\theta}_1, \overset{\circ}{\theta}_2) \leq \min \left\{ \frac{J_1(\overset{\circ}{\theta}_1)}{\overset{\circ}{\theta}_1}, \frac{J_2(\overset{\circ}{\theta}_2)}{\overset{\circ}{\theta}_2} \right\} = \frac{J_2(\overset{\circ}{\theta}_2)}{\overset{\circ}{\theta}_2} < \frac{J_1(\overset{\circ}{\theta}_1)}{\overset{\circ}{\theta}_1} \quad (88)$$

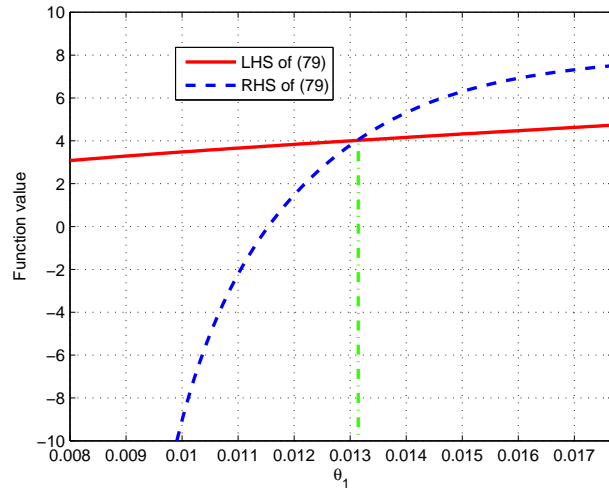
leading to contradiction. A numerical result provides a visualization of the aforementioned discussions on $\underline{\theta}_1$, $\overset{\circ}{\theta}_1$, and $\check{\theta}_1$. We consider the the delay constraint given by $(\epsilon, D_{\max}) = (0.05, 1)$ in Rayleigh fading channel. We assume that $\text{SNR}_1 = 0$ dB, $\text{SNR}_2 = 3$ dB, $T = 1$ ms, and $B = 180$ kHz. We obtain $\theta_{1,th} = 0.0178$, and $\theta_{2,th} = 0.011$. Now, as θ_1 decreases within Ω_ϵ , we plot the values of θ_1 and θ_2 in Fig. 12(a), the LHS and RHS of (79) in Fig. 12(b), and the values of R_1 and R_2 in Fig. 12(c). We can obtain $\underline{\theta}_1 = 0.0142$, $\overset{\circ}{\theta}_1 = 0.0131$, and $\check{\theta}_1 = 0.0109$. Obviously, we can see that $\check{\theta}_1 < \overset{\circ}{\theta}_1 < \underline{\theta}_1$. Note that we have $\Pr\{z_1 = 0\} = \Pr\{z_2 = 0\} = 0$ for Rayleigh fading channel, and hence $J_1(\theta_2) \rightarrow \infty$ as $\theta_2 \rightarrow \infty$. Note also that $z_{1,\max} = \infty$ and $z_{2,\min} = 0$ for Rayleigh fading channels.

Proposition 2: The effective capacity in this case is given by

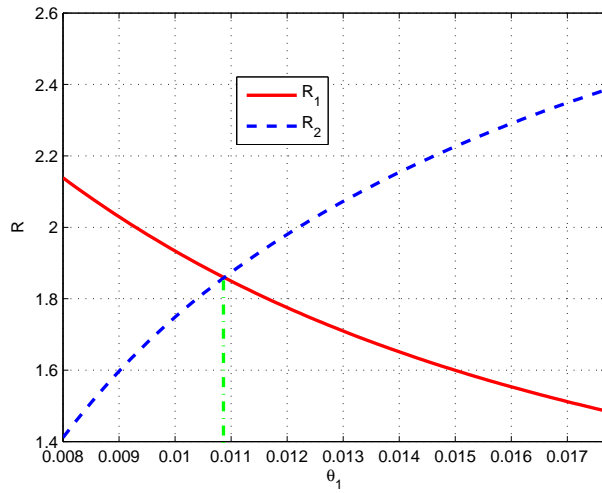
$$R_\epsilon(\epsilon, D_{\max}) = \sup_{(\theta_1, \theta_2) \in \Omega} R_E(\theta_1, \theta_2) = R_E(\overset{\circ}{\theta}_1, \overset{\circ}{\theta}_2) = \frac{J_1(\overset{\circ}{\theta}_1)}{\overset{\circ}{\theta}_1}. \quad (89)$$



(a) θ_1 and θ_2 v.s. θ_1 .



(b) LHS and RHS of (79) v.s. θ_1 .



(c) R_1 and R_2 v.s. θ_1 .

Fig. 12. The illustration of $\underline{\theta}_1$, $\overset{\circ}{\theta}_1$, and $\check{\theta}_1$. From a)-c), the cross points give us $\underline{\theta}_1$, $\overset{\circ}{\theta}_1$, and $\check{\theta}_1$. $\mathbb{E}_{z_1}\{z_1\} = \mathbb{E}_{z_2}\{z_2\} = 16$.

Proof: In order to prove the proposition, we have to show that there is no other arrival rate larger than the value specified above that can be supported by the two-hop link while satisfying the statistical delay constraint. We know that for all $\theta_1 > \overset{\circ}{\theta}_1$,

$$R_E(\theta_1, \theta_2) \leq \frac{J_1(\theta_1)}{\theta_1} < \frac{J_1(\overset{\circ}{\theta}_1)}{\overset{\circ}{\theta}_1} = R_\epsilon(\epsilon, D_{\max}), \quad (90)$$

due to Lemma 3. Suppose that there exists $R > R_E(\overset{\circ}{\theta}_1, \overset{\circ}{\theta}_2)$ can be supported by the two-hop system with θ_1 and θ_2 . Then, $\theta_1 < \overset{\circ}{\theta}_1$. As shown above, for $\theta_1 < \overset{\circ}{\theta}_1$, the inequality defined in (79) cannot be satisfied, and hence $R_E(\theta_1, \theta_2)$ falls into **Case III** of Theorem 2. In addition, with the previous characterization in (77), we know $\theta_2 > \underline{\theta}_2 = \underline{\theta}_1 > \overset{\circ}{\theta}_1$.

For **Case III.b** of Theorem 2, if (40) cannot be satisfied, i.e.,

$$\frac{J_2(\theta_2)}{\theta_2} < \frac{J_1(\theta_2)}{\theta_2}, \quad (91)$$

we know from Lemma 3 that the effective capacity is decreasing in θ , and as a result

$$R_E(\theta_1, \theta_2) = \frac{J_2(\theta_2)}{\theta_2} < \frac{J_1(\theta_2)}{\theta_2} < \frac{J_1(\underline{\theta}_2)}{\underline{\theta}_2} = \frac{J_1(\underline{\theta}_1)}{\underline{\theta}_1} \leq \frac{J_1(\overset{\circ}{\theta}_1)}{\overset{\circ}{\theta}_1} = R_E(\epsilon, D_{\max}) \quad (92)$$

where $\theta_2 > \underline{\theta}_2 = \underline{\theta}_1 > \overset{\circ}{\theta}_1$ is incorporated.

For **Case III.a** of Theorem 2, if (40) is satisfied, there exists $\tilde{\theta}_1^* \in (\theta_1, \theta_2)$ such that $\tilde{\theta}_1^*$ is the smallest solution to (42). With the assumption $R > R_E(\overset{\circ}{\theta}_1, \overset{\circ}{\theta}_2)$ and Lemma 3, we must have $\theta_1 < \tilde{\theta}_1^* < \overset{\circ}{\theta}_1$, and hence $J_1(\theta_1) < J_1(\tilde{\theta}_1^*) < J_1(\overset{\circ}{\theta}_1)$. Considering the statistical delay tradeoff characterized in Lemma 1, we must have the associated $J_2(\theta_2) > J_2(\tilde{\theta}_2^*) > J_2(\overset{\circ}{\theta}_2)$, and hence $\theta_2 > \tilde{\theta}_2^* > \overset{\circ}{\theta}_2$. Note that with [16, Lemma 2], we can obtain the following inequality

$$-\frac{1}{\tilde{\theta}_1^*} \log \mathbb{E}_{z_1} \left\{ e^{-\tilde{\theta}_1^* C_1} \right\} = -\frac{1}{\tilde{\theta}_1^*} \left(\log \mathbb{E}_{z_2} \left\{ e^{-\theta_2 C_2} \right\} + \log \mathbb{E}_{z_1} \left\{ e^{(\theta_2 - \tilde{\theta}_1^*) C_1} \right\} \right) \quad (93)$$

$$< -\frac{1}{\tilde{\theta}_1^*} \left(\log \mathbb{E}_{z_2} \left\{ e^{-\tilde{\theta}_2^* C_2} \right\} + \log \mathbb{E}_{z_1} \left\{ e^{(\tilde{\theta}_2^* - \tilde{\theta}_1^*) C_1} \right\} \right) \quad (94)$$

since the RHS of (93) is always greater than the LHS for all $\theta \in [0, \theta_2]$ with given $\tilde{\theta}_1^*$. That is, the condition

in (79) is satisfied at $\tilde{\theta}_1^*$. This violates the definition of $\overset{\circ}{\theta}_1$, which is the smallest solution to (79).

Combining the above discussions, we arrive at the conclusion that there is no other θ_1 that can achieve higher effective capacity than (89). Hence, it is indeed the largest achievable constant arrival rate in this case. \blacksquare

The aforementioned discussions show the existence of the solution to (47) under the statistical delay constraints. To show the uniqueness, we need the following Lemma.

Lemma 4: Consider the function

$$f(\theta_1) = J_2(\theta_2) - J_1(\theta_1) - \log \mathbb{E}_{z_1} \{ e^{(\theta_2 - \theta_1)C_1} \}, \text{ for } \theta_1 \leq \underline{\theta}_1 \quad (95)$$

where $(\theta_1, \theta_2) \in \Omega_\epsilon$, and $(\underline{\theta}_1, \underline{\theta}_2)$ is defined in (77). If the following condition

$$\left. \frac{dJ_2(\theta)}{d\theta} \right|_{\theta=\underline{\theta}_2} \leq \left. \frac{dJ_1(\theta)}{d\theta} \right|_{\theta=\underline{\theta}_1} \quad (96)$$

is satisfied, then $f(\theta_1)$ is increasing in θ_1 .

Proof: Following the proof in Appendix A, we view θ_2 as a function of θ_1 . Now taking the first derivative of f over θ_1 , we have

$$\frac{df(\theta_1)}{d\theta_1} = \frac{dJ_2(\theta_2)}{dJ_1(\theta_1)} \frac{dJ_1(\theta_1)}{d\theta_1} - \frac{dJ_1(\theta_1)}{d\theta_1} - \frac{\mathbb{E}_{z_1} \{ e^{(\theta_2 - \theta_1)C_1} C_1 \} \left(\frac{d\theta_2}{d\theta_1} - 1 \right)}{\mathbb{E}_{z_1} \{ e^{(\theta_2 - \theta_1)C_1} \}} \quad (97)$$

$$\begin{aligned} &= \frac{dJ_2(\theta_2)}{dJ_1(\theta_1)} \frac{\mathbb{E}_{z_1} \{ e^{-\theta_1 C_1} C_1 \}}{\mathbb{E}_{z_1} \{ e^{-\theta_1 C_1} \}} - \frac{d\theta_2}{d\theta_1} \frac{\mathbb{E}_{z_1} \{ e^{(\theta_2 - \theta_1)C_1} C_1 \}}{\mathbb{E}_{z_1} \{ e^{(\theta_2 - \theta_1)C_1} \}} \\ &+ \frac{\mathbb{E}_{z_1} \{ e^{(\theta_2 - \theta_1)C_1} C_1 \}}{\mathbb{E}_{z_1} \{ e^{(\theta_2 - \theta_1)C_1} \}} - \frac{\mathbb{E}_{z_1} \{ e^{-\theta_1 C_1} C_1 \}}{\mathbb{E}_{z_1} \{ e^{-\theta_1 C_1} \}}. \end{aligned} \quad (98)$$

where $\frac{dJ_1(\theta_1)}{d\theta_1} = \frac{\mathbb{E}_{z_1} \{ e^{-\theta_1 C_1} C_1 \}}{\mathbb{E}_{z_1} \{ e^{-\theta_1 C_1} \}}$ is substituted into (98).

First, similar to Lemma 2, we can show that the function $g(\theta_2) = \log \mathbb{E}_{z_1} \{ e^{(\theta_2 - \theta_1)C_1} \}$ is convex in θ_2 , i.e., $\frac{d^2 g(\theta_2)}{d\theta_2^2} \geq 0$. This tells us that the derivative of $g(\theta_2)$ is increasing in θ_2 , and

$$\left. \frac{dg(\theta_2)}{d\theta_2} \right|_{\theta_2=0} = \frac{\mathbb{E}_{z_1} \{ e^{-\theta_1 C_1} C_1 \}}{\mathbb{E}_{z_1} \{ e^{-\theta_1 C_1} \}}. \quad (99)$$

Therefore,

$$\frac{\mathbb{E}_{z_1} \{e^{(\theta_2-\theta_1)C_1} C_1\}}{\mathbb{E}_{z_1} \{e^{(\theta_2-\theta_1)C_1}\}} - \frac{\mathbb{E}_{z_1} \{e^{-\theta_1 C_1} C_1\}}{\mathbb{E}_{z_1} \{e^{-\theta_1 C_1}\}} \geq 0. \quad (100)$$

Considering the definition of $(\underline{\theta}_1, \underline{\theta}_2)$ in (77), we know that for all $\theta_1 \leq \underline{\theta}_1$, we have $\theta_2 \geq \underline{\theta}_2$. Note that $J_1(\theta_1)$ and $J_2(\theta_2)$ are concave functions according to Lemma 2, i.e., their first derivatives decreases with θ_1 and θ_2 , respectively. Therefore, we have

$$\left. \frac{dJ_1(\theta)}{d\theta} \right|_{\theta=\theta_1} \geq \left. \frac{dJ_1(\theta)}{d\theta} \right|_{\theta=\underline{\theta}_1}, \quad (101)$$

$$\left. \frac{dJ_2(\theta)}{d\theta} \right|_{\theta=\theta_2} \leq \left. \frac{dJ_2(\theta)}{d\theta} \right|_{\theta=\underline{\theta}_2}, \quad (102)$$

which, after combining with the assumption in (96), gives us

$$\frac{dJ_1(\theta_1)}{d\theta_1} \geq \frac{dJ_2(\theta_2)}{d\theta_2}. \quad (103)$$

Next, recalling the statistical delay tradeoff characterized in Lemma 1, we can see that $d\theta_2 < 0$ for $d\theta_1 > 0$, i.e., θ_2 decreases as we increase θ_1 . Then, we can get from (103) that

$$\frac{d\theta_2}{d\theta_1} \leq \frac{dJ_2(\theta_2)}{dJ_1(\theta_1)}. \quad (104)$$

Note that both $\frac{d\theta_2}{d\theta_1}$ and $\frac{dJ_2(\theta_2)}{dJ_1(\theta_1)}$ are negative values. Considering the expression in (98), we now have

$$\frac{df(\theta_1)}{d\theta_1} \geq \left(1 - \frac{dJ_2(\theta_2)}{dJ_1(\theta_1)}\right) \left(\frac{\mathbb{E}_{z_1} \{e^{(\theta_2-\theta_1)C_1} C_1\}}{\mathbb{E}_{z_1} \{e^{(\theta_2-\theta_1)C_1}\}} - \frac{\mathbb{E}_{z_1} \{e^{-\theta_1 C_1} C_1\}}{\mathbb{E}_{z_1} \{e^{-\theta_1 C_1}\}}\right) \geq 0. \quad (105)$$

That is, $f(\theta_1)$ is an increasing function in θ_1 . ■

Note that after eliminating the denominator of both sides of the equation (47), and moving the LHS of the obtained equation to the right side, we can obtain the function given in (95), which is increasing in θ_1 for $\theta_1 \leq \underline{\theta}_1$. Therefore, the solution to the equation (47) is unique.

Case III: Assume $\theta_{1,th} < \theta_{2,th}$.

For this case, at $\theta_{1,th}$, we know that

$$R_1 = \frac{J_1(\theta_{1,th})}{\theta_{1,th}} = \frac{J_{th}(\epsilon)}{\theta_{1,th}} > \frac{J_{th}(\epsilon)}{\theta_{2,th}} = \frac{J_2(\theta_{2,th})}{\theta_{2,th}} = R_2. \quad (106)$$

The queue at the relay becomes the bottleneck. We need to be careful about the effective capacity in this case. To improve the system performance, we may instead increase the queueing constraint θ_1 at the source, and correspondingly, the queueing constraint θ_2 at the relay can be less. Actually, decreasing the queueing constraint at the source node will not improve the performance, as will be justified later.

First, according to Lemma 2, we can see that as $J_1(\theta_1)$ increases from $J_{th}(\epsilon)$ to ∞ , θ_1 increases from $\theta_{1,th}$ to ∞ . To the opposite behavior, θ_2 decreases from $\theta_{2,th}$ to some finite value $\theta_{2,0}$, which is the solution to $J_2(\theta) = J_0$. Therefore, from the continuity of θ_1 and θ_2 , we again have one point $(\underline{\theta}_1, \underline{\theta}_2)$ such that

$$\underline{\theta}_1 = \underline{\theta}_2. \quad (107)$$

and for all $\theta_1 < \underline{\theta}_1$, we have $\theta_1 < \underline{\theta}_1 = \underline{\theta}_2 < \theta_2$. Also, we know that R_1 decreases from $\frac{J_{th}(\epsilon)}{\theta_{1,th}}$ to $TB \log_2(1 + \text{SNR}_1 z_{1,\min})$, while R_2 increases from $\frac{J_{th}(\epsilon)}{\theta_{1,th}}$ to some finite value $\frac{J_0}{\theta_{2,0}}$. Therefore, there must be a value such that

$$R_1 = \frac{J_1(\theta_1)}{\theta_1} = R = \frac{J_2(\theta_2)}{\theta_2} = R_2 \quad (108)$$

with the associated statistical queueing constraints denoted as $\check{\theta}_1$ and $\check{\theta}_2$, respectively. For all $\theta_1 < \check{\theta}_1$, we have

$$R_1 = \frac{J_1(\theta_1)}{\theta_1} > \frac{J_2(\theta_2)}{\theta_2} = R_2. \quad (109)$$

Note that the above result implicitly assume that $TB \log_2(1 + \text{SNR}_1 z_{1,\min}) < \frac{J_0}{\theta_{2,0}}$. If this condition does not hold, then θ_1 can take any value, and the only delay is introduced by the queue at the relay node. Hence, the effective capacity under the statistical delay constraint is given by

$$R_\epsilon(\epsilon, D_{\max}) = \frac{J_0}{\theta_{1,0}}. \quad (110)$$

Consider the queue stability condition (27), this is possible when the average rate of **R – D** link is larger but has more severe fading conditions.

Now, as a stark difference from the previous case, we should have

$$\check{\theta}_1 \geq \underline{\theta}_1. \quad (111)$$

Suppose that $\check{\theta}_1 < \underline{\theta}_1$, we can show the following contradiction. First, at $\check{\theta}_1$, from the definition of $\underline{\theta}_1$ in (107), we have

$$\check{\theta}_1 < \underline{\theta}_1 = \underline{\theta}_2 < \check{\theta}_2. \quad (112)$$

According to the definition of $\check{\theta}_1$ in (108), we can obtain

$$\frac{J_1(\check{\theta}_1)}{\check{\theta}_1} = \frac{J_2(\check{\theta}_2)}{\check{\theta}_2} \Rightarrow J_1(\check{\theta}_1) < J_2(\check{\theta}_2). \quad (113)$$

On the other hand, according to Lemma 1, we should have

$$J_1(\check{\theta}_1) > J_1(\theta_{1,th}) = J_{th}(\epsilon) = J_2(\theta_{2,th}) > J_2(\check{\theta}_2) \quad (114)$$

leading to contradiction.

Since $\check{\theta}_1 > \underline{\theta}_1$, with (107), we can see that

$$\check{\theta}_1 > \underline{\theta}_1 = \underline{\theta}_2 > \check{\theta}_2. \quad (115)$$

Now, the effective capacity $R_E(\check{\theta}_1, \check{\theta}_2)$ specializes into **Case I** of Theorem 2, we have

$$R_E(\check{\theta}_1, \check{\theta}_2) = \min \left\{ \frac{J_1(\check{\theta}_1)}{\check{\theta}_1}, \frac{J_2(\check{\theta}_2)}{\check{\theta}_2} \right\} = \frac{J_1(\check{\theta}_1)}{\check{\theta}_1} = \frac{J_2(\check{\theta}_2)}{\check{\theta}_2}. \quad (116)$$

Next, we can show the following result.

Proposition 3: The effective capacity in this case is given by

$$R_\epsilon(\epsilon, D_{\max}) = \sup_{(\theta_1, \theta_2) \in \Omega} R_E(\theta_1, \theta_2) = R_E(\check{\theta}_1, \check{\theta}_2) = \frac{J_2(\check{\theta}_2)}{\check{\theta}_2} = \frac{J_1(\check{\theta}_1)}{\check{\theta}_1}. \quad (117)$$

Proof: From Proposition 1, we know that

$$R \leq \min \left\{ \frac{J_1(\theta_1)}{\theta_1}, \frac{J_2(\theta_2)}{\theta_2} \right\}. \quad (118)$$

Now, for $\theta_1 > \check{\theta}_1$, we can see from Lemma 3 that

$$R_1 = \frac{J_1(\theta_1)}{\theta_1} < \frac{J_1(\check{\theta}_1)}{\check{\theta}_1} = R_\epsilon(\epsilon, D_{\max}) \quad (119)$$

and for $\theta_1 < \check{\theta}_1$, we have $\theta_2 > \check{\theta}_2$, and hence

$$R_2 = \frac{J_2(\theta_2)}{\theta_2} < \frac{J_2(\check{\theta}_2)}{\check{\theta}_2} = R_\epsilon(\epsilon, D_{\max}). \quad (120)$$

Therefore, $R_\epsilon(\epsilon, D_{\max})$ in (117) is the largest achievable constant rate in this case. ■

REFERENCES

- [1] Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 2013 - 2018.
- [2] A. Goldsmith, *Wireless Communications*, 1st ed. Cambridge University Press, 2005.
- [3] C.-S. Chang, "Stability, queue length, and delay of deterministic and stochastic queuing networks," *IEEE Trans. Auto. Control*, vol. 39, no. 5, pp. 913-931, May 1994.
- [4] F. Kelly, "Notes on effective bandwidth," in *Stochastic Networks: Theory and Applications* Royal Statistical Society Lecture Notes Series, 4. Oxford University Press, 141-168, 1996.
- [5] C.-S. Chang and T. Zajic, "Effective bandwidths of departure processes from queues with time varying capacities," In *Proceedings of IEEE Infocom*, pp. 1001-1009, 1995.
- [6] D. Wu and R. Negi, "Effective capacity: a wireless link model for support of quality of service," *IEEE Trans. Wireless Commun.*, vol.2,no. 4, pp.630-643. July 2003
- [7] J. Tang and X. Zhang, "Quality-of-service driven power and rate adaptation over wireless links," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 3058-3068, Aug. 2007.
- [8] J. Tang and X. Zhang, "Cross-layer-model based adaptive resource allocation for statistical QoS guarantees in mobile wireless networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 6, pp.2318-2328, June 2008.
- [9] A. A. Khalek, C. Caramanis, and R.W. Heath, "Delay-constrained video transmission: Quality-driven resource allocation and scheduling," to appear in *IEEE J. Sel. Topics in Sig. Process.*.

- [10] E. A. Jorswieck, R. Mochaourab, and M. Mittelbach, "Effective capacity maximization in multi-antenna channels with covariance matrix," *IEEE Trans. Wireless Commun.*, vol. 9, no. 10, pp. 2988-2993, Oct. 2010.
- [11] A. Balasubramanian and S.L. Miller, "The effective capacity of a time division downlink scheduling system," *IEEE Trans. Commun.*, vol. 58, no. 1, pp. 73-78, Jan. 2010.
- [12] D. Qiao, M. C. Gursoy, and S. Velipasalar, "Transmission strategies in multiaccess fading channels with statistical QoS constraints," *IEEE Trans. Inform. Theory*, vol. 58, no. 3, pp. 1578- 1593, Mar. 2012.
- [13] J. Tang and X. Zhang, "Cross-layer resource allocation over wireless relay networks for quality of service provisioning," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 4, pp. 645-656, May 2007.
- [14] L. Liu, P. Parag, and J.-F. Chamberland, "Quality of service analysis for wireless user-cooperation networks," *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 3833-3842, Oct. 2007.
- [15] P. Parag, and J.-F. Chamberland, "Queueing analysis of a butterfly network for comparing network coding to classical routing," *IEEE Trans. Inform. Theory*, vol. 56, no. 4, pp. 1890-1907, Apr. 2010.
- [16] D. Qiao, M. C. Gursoy, and S. Velipasalar, "Effective capacity of two-hop wireless communication systems," *IEEE Trans. Inform. Theory*, vol. 59, no. 2, pp. 873- 885, Feb. 2013.
- [17] K.T. Pan and L.-N. Tho, "Effective capacity of dual-hop networks with a concurrent buffer-aided relaying protocol", in Proc. IEEE International Conf. Commun. (ICC 2014), Sydney, NSW, June 2014.
- [18] D. Wu and R. Negi, "Effective capacity-based quality of service measures for wireless networks", *Mobile Networks and Applications*, vol. 11, no. 1, pp. 527 - 536, 2005.
- [19] A. A. Khalek and Z. Dawy, "Energy-efficient cooperative video distribution with statistical QoS provisions over wireless networks", *IEEE Trans. Mobile Comp.*, vol. 11, no. 7, pp. 1223 -1236, July 2012.
- [20] Q. Du, Y. Huang, P. Ren, and C. Zhang, "Statistical delay control and QoS-driven power allocation over two-hop wireless relay links", *IEEE Global Communications Conference (Globecom 2011)*, Houston, 2011.
- [21] X. Lei, X. Chen, R. Q. Hu, and G. Wu, "Mobile association for heterogeneous wireless relay networks with statistical QoS guarantees," in Proc. IEEE Global Commun. Conf. (Globecom 2013), Atlanta, GA, Dec. 2013.
- [22] N. Zlatanov and R. Schober, "Buffer-aided relaying with adaptive link selection-fixed and mixed rate transmission", *IEEE Trans. Inform. Theory*, vol. 59, no. 5, pp. 2816-2840, May 2013.
- [23] C.-S. Chang, *Performance Guarantees in Communication Networks*, New York: Springer, 1995.