Precision Doppler measurements with steep dispersion

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Controlling the group velocity of light is a valuable resource for quantum and classical optical processing and high performance sensor technologies. In this context, slow-light (SL) and the associated steep dispersion have been proposed to increase the sensitivity of certain types of interferometers. Here, we show that the interaction of two intensity-balanced light beams in a SL medium can be used to detect Doppler shifts with extremely high sensitivity. By using this effect in a liquid crystal light-valve, we have been able to measure Doppler shifts as low as 1 μHz with an integration time of only 1 s. The shot noise limited sensitivity inversely depends on the steepness of the beam-coupling dispersive response. This method allows for remote sensing of very slowly moving objects with a linear response over 5 orders of magnitude.

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The ability to slow the group velocity of light has attracted significant attention [1]. Applications include interferometry [2,3], quantum information [4,5], and optical sensing. Several slow-light (SL) schemes have been proposed and realized, based on different effects and different materials, such as electromagnetically induced transparency (EIT) in atomic media [6–8], in solids [9,10] and in optomechanical systems [11], coherent population oscillations (CPO) in solids [12,13], stimulated Brillouin and Raman scattering in optical fibers [14–20], beam coupling in photorefractive media [21–23] and wave-mixing in nematic liquid crystals [24,25]. Controlling the group velocity is of both fundamental and practical interest (e.g., temporal cloaking [26,27]). Here, we propose a novel application of the enhanced steep dispersion provided by a SL medium, showing that it allows the detection of Doppler shifts with very high sensitivity. The principle of the measurement is schematically illustrated in Fig. 1. Two coherent beams with the same intensity (same mean number of photons $\langle N \rangle$) and of slightly different frequencies, ν and $\nu + \Delta\nu$, respectively, interact in a SL medium. Because of the steep dispersion provided by the SL process, at the output of the medium the amplitude of each beam strongly depends on its frequency. Therefore, a small Doppler shift has the effect to unbalance the two output intensities, that is, one beam is slightly amplified while the other one is symmetrically depleted. The difference ΔN in the number of photons of the two output beams is measured through a balanced detection, with the intensity imbalance directly proportional to the frequency shift. Note that slightly nondegenerate two-wave mixing experiments have been previously performed in photorefractive crystals, however, in these media the beam-coupling is asymmetric (because of the crystallographic axis orientation and the anisotropy of the electro-optic coefficients), therefore, Doppler shifts always enhance the transfer from the same beam to the other [28–31], inhibiting the use of a balanced detection. Our method is based on a symmetric beam-coupling, hence, leads to a photon imbalance directly proportional to frequency shifts between the two interacting beams. As a result, the system realizes a noninterferometric detector that is able to reveal the Doppler shift by direct measurement of the intensity, without the need of preserving the relative phase between the two interfering beams as required in classical interferometers. In the experimental procedure, the Doppler shift is derived from the measured intensity imbalance ΔN. Theoretically, let us assume that there are N photons in each of the input beams. After the interaction, the number of photons will be increased on one of the beams and symmetrically decreased in the other one of a quantity $dN/d\nu$. The total output signal is proportional to the linear part of the gain curve $\Delta N = 2(dN/d\nu)\Delta\nu = \chi N \Delta\nu$, were χ represents the maximum slope of the gain curve as a function of the frequency detuning. χ will be characterized below for the case considered here. The minimum detectable signal is determined by the intensity of the photon shot noise. Since we are using coherent states, the uncertainty of the signal, for small Doppler shift $\Delta\nu$ is $\sqrt{N}$. The signal to noise ratio is, therefore, $\mathrm{SNR} \sim |\chi|\sqrt{N}\Delta\nu$. By taking into account the quantum efficiency η of the detector we obtain the expression for the signal to noise ratio $\mathrm{SNR} = |\chi|\sqrt{\eta N}\Delta\nu$.

The minimum detectable Doppler shift can be calculated by taking the signal to noise ratio equal to the unity.

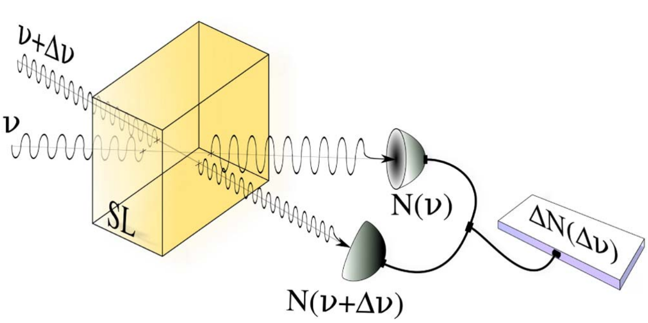

Fig. 1. Two-beam interaction in a slow-light (SL) medium. Two nondegenerate light beams of the same intensity and with a slightly different frequencies, ν and $\nu + \Delta\nu$, respectively, interact in the SL medium. The difference ΔN between the output intensities, which is measured through a balanced detection, strongly depends on the Doppler shift $\Delta\nu$.

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The order of magnitude for the minimum detectable Doppler shift is

$$\Delta\nu_{\min} \sim \frac{1}{|\chi|}\sqrt{\frac{h\nu}{\eta T}}P^{-\frac{1}{2}}, \tag{1}$$

where P is the optical power impinging on the detector, T is the measurement integration time, and h is the Planck's constant. We note that the minimum Doppler shift is inversely proportional to the slope of the gain/absorption curve associated to the SL process. Therefore, the higher the group index provided by the SL medium, the greater the sensitivity of the Doppler shift detection.

In our experiment SL is achieved by performing two-beam coupling in a liquid crystal light-valve (LCLV). The setup is shown in Fig. 2. A laser beam of wavelength $\lambda = 490$ nm is spatially filtered and collimated, a beam-splitter is used to divide the incoming beam in two beams of the same intensity. The two beams are sent to the LCLV and then detected through a balanced photodetector. The beams are linearly polarized and their intensities are finely tuned in order to have a balanced signal in the absence of Doppler shift. As shown in Fig. 2, one of the mirrors is mounted on a piezo-electric crystal (PZT) and, by applying an appropriate voltage across the PZT, is displaced at constant velocity v_p. Therefore, one of the beams acquires a Doppler shift $\Delta\nu = 2(v_p/c)\nu \cos\theta$, where θ is the light incidence angle on the moving mirror and ν the optical frequency. At the exit of the LCLV, the output beams are sent to a balanced detector composed by two photodiodes, PD_1 and PD_2, and their intensity difference is measured as an output voltage $V(t)$, $V = 0$ for exactly balanced intensities. Due to the dispersion properties associated with the two-beam coupling and its gain features, two-wave interaction in the LCLV leads to SL effects with large group delays [24,25]. In our case, the two interacting beams have the same intensity; therefore, at the exit of the medium there is a balanced exchange of photons between the two output beams. By using the expression for the output order beams in the Raman–Nath regime of diffraction, where the LCLV usually operates [24,25], we can calculate the intensity difference ΔN between the two output beams, which reads as

$$\Delta N = 8\pi J_0(\rho)J_1(\rho)\frac{\Delta\nu\tau}{\sqrt{1+(2\pi\Delta\nu\tau)^2}}N, \tag{2}$$

where $\rho = 2k_0 dn_2 I/\sqrt{1+(2\pi\Delta\nu\tau)^2}$ is the amplitude of the refractive index grating induced in the LCLV, I is the input intensity of each beam, $\Delta\nu$ the frequency detuning between the two interacting beams, τ the medium response time, and J_0, J_1 are the Bessel functions of the first kind and of order zero and one, respectively. The model parameters considered here are: the optical wavenumber, $k_0 = 2\pi/\lambda$; the nonlinear coefficient characterizing the LCLV response, $n_2 = -6\ \mathrm{cm^2/W}$; the liquid crystal relaxation time, $\tau = 110$ ms; the intensity of each beam, $I = 0.3\ \mathrm{mW/cm^2}$; and the thickness of the liquid crystal layer, $d = 25$ μm. The angle between the two interacting beams is ≃5 mrad. The imbalance dispersion curve $\Delta N/N$ is plotted in Fig. 3 as a function of $\Delta\nu$. Close to zero Doppler shift the detection is linear and the balanced signal is symmetrically negative or positive depending on the sign of the Doppler shift. This feature allow us to determine the sign of the Doppler shift. Note that the response is purely linear in a frequency bandwidth that is dictated by the SL effect in the medium. In our case, the bandwidth is limited to a few Hz, therefore, the method can be used to measure displacements occurring with very low speeds. The linearity of the response is expressed as $\Delta N = \chi N \Delta\nu$, with the parameter χ reading as

$$\chi = 8\pi\tau J_0(2k_0 dn_2 I)J_1(2k_0 dn_2 I), \tag{3}$$

and, by using our experimental parameters, we obtain $\chi = -0.9324\ \mathrm{Hz^{-1}}$. The minimum measurable Doppler

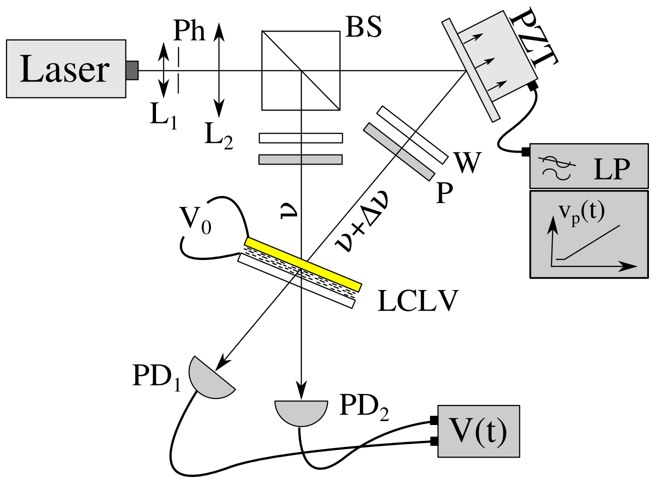

Fig. 2. Experimental setup of the two-beam coupling in a liquid crystal light-valve (LCLV). The laser beam is spatially filtered with a pinhole (Ph) and enlarged through the telescope composed of the two lenses L_1 and L_2. The beam-splitter (BS) is used to separate the input laser beam into two beams whose intensity and polarization are finely controlled through a polarizer (P) and a wave-plate (W). V_0 is the voltage applied to the LCLV. The output beams are sent to a balanced detector composed by two photodiodes, PD_1 and PD_2, where their intensity difference $V(t)$ is measured. An ultra-low noise voltage source (LP) is used to drive a piezo-electric mirror (PZT); the PZT moves at a constant speed v_p.

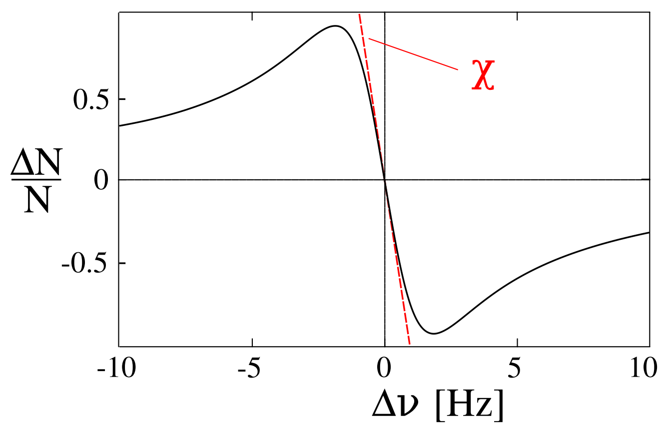

Fig. 3. Balanced signal as a function of the Doppler frequency shift. Close to zero the response is linear with a coefficient equal to $\chi = -0.9324\ \mathrm{Hz^{-1}}$.

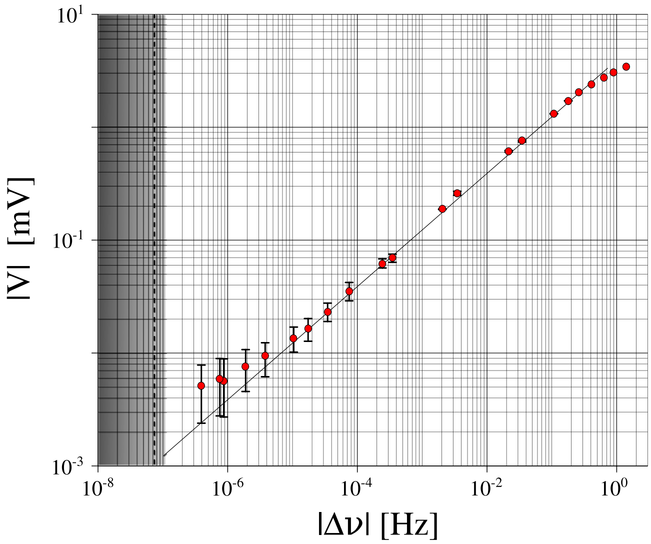

Fig. 4. Measured voltage $|V|$ as a function of the Doppler shift $\Delta\nu$. The dotted line and the gray region represent the photon shot noise. The continuous line is the fit of the linear part of the response.

shift in the case of two-beam coupling in the LCLV can then be calculated by substituting the above expression into Eq. (1). By taking a detector with a quantum efficiency $\eta = 0.35$, we obtain a sensitivity of $\Delta\nu_{\min} \sim 70\ \mathrm{nHz}/\sqrt{\mathrm{Hz}}$. Correspondingly, the equivalent minimum measurable linear speed of the mirror for one second of integration time is $v_p \sim 20$ fm/s. The combination of the sensitivity and simplicity of this setup, may make it valuable in infrasound detection, gyroscope systems, and remote infrastructure monitoring.

The linear response itself is used to calibrate the detector. Indeed, by measuring the output voltage $V(t)$ for a couple of known frequency detunings, determined with a standard interferometric technique, it is possible to calculate the slope of the curve V versus $\Delta\nu$. In Fig. 4, it is reported the voltage $|V|$ measured for different frequency detunings. The response is linear, as predicted. The continuous line corresponds to the slope measured during the calibration stage. In this case, we have determined the slope by measuring two frequency detunings, $\Delta\nu = \pm 100$ mHz, with a standard Mach–Zehnder interferometer and by measuring the consequent output voltage change in the SL detector. A fine calibration of the system is achieved with this method. Moreover, by switching on and off the voltage applied to the LCLV, a direct comparison can be made in order to check that the intensity imbalance ΔN results from a Doppler shift and it is not due to spurious effects, such as a differential attenuation or polarization variation along the beam paths. Indeed, when the voltage is turned off, the beam coupling in the liquid crystal device is disabled and any dependence of ΔN on the frequency detuning must disappear. The minimum measured Doppler shift is around 900 nHz. This experimental limit is attributed to the noise added by the electronics used to drive the piezo-electric crystal. Note that the system not only has a high sensitivity but also provides a robust and stable measurement of the Doppler shift. Indeed, the measurement is completed within only one second of integration time, hence, largely avoiding $1/f$ noise. We can also observe that, in principle, the detection limit could be further reduced by using a nonclassical input state of light.

In conclusion, we have shown that two-beam coupling in a SL system can be efficiently used to realize ultraprecise and stable Doppler measurements. The method proposed is robust and based on balanced detection. The physical mechanism is general and can be applied in different systems provided they are characterized by a steep dispersion, as other SL media or, for instance, acoustic systems. An estimation of the shot noise limited sensitivity is reported and demonstrated to inversely depend on the steepness of the gain versus frequency curve. Unlike interferometric methods, this system does not require phase stability, making it a viable remote sensing technique.

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