Circular solitons do not exist in photorefractive media

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The shape of two-dimensional solitary beams propagating in photorefractive media with an externally applied field is studied. The analytical results indicate that, for both focusing and defocusing nonlinearities, radially symmetric self-channeled beams do not exist. Some recent experiments are interpreted in light of the present results. © 1998 Optical Society of America

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The current interest in the properties of photorefractive spatial solitons was initiated by the suggestion of Segev *et al.* in 1992 (Ref. 1) that self-channeled, spatially localized beams could be observed in photorefractive media in the presence of a strong, externally applied electric field. As was first pointed out in Ref. 2 and subsequently demonstrated by several groups,[3,4] in the case of a single transverse coordinate {(1 + 1-dimensional [(1 + 1)D] propagation} the photorefractive response is formally equivalent to a saturable Kerr nonlinearity[5]: $\delta n \sim |B|^2/(1+|B|^2)$, where δn is the nonlinear increment to the refractive index and B is the slowly varying envelope of the optical field. The (1 + 1)D spatial solitons have been observed for both focusing and defocusing nonlinearities.[2,6]

By contrast, the physics of three-dimensional {(2 + 1)-dimensional [(2 + 1)D]} beam propagation in photorefractive media are remarkably different from those observed in Kerr or saturable Kerr media that are describable by an isotropic and local nonlinearity. Analytical, numerical, and experimental studies[4,7–13] have revealed phenomena that are due to the strong anisotropy of the photorefractive nonlinearity in the plane transverse to the direction of propagation. In particular, an analytical solution describing a highly elliptical self-channeled beam with characteristic diameter along the y coordinate (perpendicular to the applied field) approximately 1.5 times larger than along the x coordinate (parallel to the applied field) was obtained in the limit of weak saturation of the nonlinearity.[10] Numerical calculations showed that (2 + 1)D solitary solutions are strongly elliptical over a wide range of intensities, including the limit of strong saturation, and convergence of a circular input beam to an elliptical profile was demonstrated experimentally.[10] This elliptical soliton is the analog of the bright circular soliton that exists in (2 + 1)D saturable Kerr-type media. Conversely, for a defocusing nonlinearity, numerical and experimental studies have indicated that localized vortex solitons do not exist in photorefractive media.[12] Concurrently, there have been a number of experimental reports of radially symmetric bright[14] and dark[15,16] solitons in photorefractive media. Furthermore, a recent analysis[17] of the same three-dimensional model as that used below [see Eqs. (1)] appeared to demonstrate circular solitary solutions.

The question of the spatial profile of solitary solutions is of central importance in the study of three-dimensional propagation effects, and the possible existence of structurally simple, circular (2 + 1)D solitary solutions in photorefractive media is considered here. It was shown in Ref. 4 that time-independent three-dimensional propagation in photorefractive media is described by the equations

$$\left(\frac{\partial}{\partial z} - \frac{i}{2}\nabla^2\right)B(\mathbf{r}) = is\,\frac{\partial\phi}{\partial x}\,B(\mathbf{r})\,, \tag{1a}$$

$$\nabla^2\phi + \nabla\phi\cdot\nabla\ln(+|B|^2) = \frac{\partial}{\partial x}\ln(1+|B|^2)\,. \tag{1b}$$

Equation (1a) is the parabolic wave equation in the presence of a nonlinear increment to the refractive index proportional to $\partial\phi/\partial x$. Equation (1b) determines the form of the potential ϕ induced by the optical beam in the presence of an external voltage applied along the x axis or of a photogalvanic current. This form follows immediately from Eq. (3) of Ref. 4 in the diffusionless approximation,[18] where $k_{\mathrm{Debye}} \to \infty$. Depending on the polarity of the external voltage, the factor s is equal to $+1$ or -1, corresponding to a focusing or a defocusing nonlinearity, respectively. The transverse gradient operator is $\nabla = \hat{x}(\partial/\partial x) + \hat{y}(\partial/\partial y)$, and ϕ satisfies the boundary conditions $\nabla\phi(r_\perp \to \infty) \to 0$. We show below that the anisotropy of Eqs. (1) is unavoidable in the sense that circular solitary solutions do not exist for any parameter values.

We seek a general soliton solution in the form

$$B(x,y,z) = b(r)\exp[i\Gamma z + i\psi(x,y)]\,, \tag{2}$$

where $r = \sqrt{x^2+y^2}$. Note that the above soliton ansatz has a radially symmetric intensity profile, but no assumptions are made about its phase distribution.

Separating real and imaginary parts of Eq. (1a) results in the set of equations

$$b\nabla^2\psi + 2(\nabla\psi)\cdot(\nabla b) = 0\,, \tag{3a}$$

$$\Gamma b + \frac{1}{2}[-\nabla^2 b + (\nabla\psi)^2 b] = s\,\frac{\partial\phi}{\partial x}\,b\,. \tag{3b}$$

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For $s = +1$ we seek localized bright soliton solutions satisfying $\lim_{r\to\infty} b(r) = 0$. It then follows from Eq. (3b) that the propagation constant $\Gamma > 0$ and that the soliton amplitude b has exponentially decaying asymptotics at infinity, $b(r \to \infty) \propto \exp(-\sqrt{2\Gamma}\, r)$. For $s = -1$ we seek dark soliton solutions with $\lim_{r\to\infty} b(r) = b_{\max}$, which leads to $\Gamma = 0$.

Owing to radial symmetry of the soliton amplitude $b(r)$, Eq. (1b) in cylindrical coordinates (r, θ) takes the form

$$\nabla^2 \phi + \frac{\partial \phi}{\partial r} \frac{\mathrm{d}}{\mathrm{d}r} \ln(1 + b^2) = \cos(\theta) \frac{\mathrm{d}}{\mathrm{d}r} \ln(1 + b^2). \quad (4)$$

The solution of Eq. (4) has the form $\phi = \cos(\theta) h(r)$, where h satisfies the equation

$$\frac{\mathrm{d}^2 h}{\mathrm{d}r^2} + \frac{1}{r} \frac{\mathrm{d}h}{dr} - \frac{h}{r^2} + \left(\frac{\mathrm{d}h}{\mathrm{d}r} - 1\right) \frac{\mathrm{d}}{\mathrm{d}r} \ln(1 + b^2) = 0. \quad (5)$$

The nonlinear refractive index $\partial\phi/\partial x$ has the form

$$\frac{\partial \phi}{\partial x} = \frac{1}{2}\left(\frac{\mathrm{d}h}{\mathrm{d}r} + \frac{h}{r}\right) + \frac{1}{2}\left(\frac{\mathrm{d}h}{\mathrm{d}r} - \frac{h}{r}\right)\cos(2\theta). \quad (6)$$

The function h in Eq. (5) has asymptotic behavior $h(r \to \infty) \propto 1/r$, so the nonlinear refractive index $\partial\phi/\partial x$ has asymptotic form $\partial\phi/\partial x \propto -\cos(2\theta)/r^2$.

From Eq. (3b) we have

$$(\nabla\psi)^2 = a(r) + s(\partial\phi/\partial x), \quad (7)$$

so the angular dependence of ψ must cancel the term proportional to $\cos(2\theta)$ in Eq. (6) to yield a solution.

The phase ψ is determined modulo(2π), so the general solution of Eq. (3a) can be written as

$$\psi = m\theta + \sum_{l>0} F_l(r) \cos(l\theta + \theta_l), \quad (8)$$

where m is an integer and the function F_l satisfies the equation

$$\frac{\mathrm{d}^2 F_l}{\mathrm{d}r^2} + \frac{1}{r} \frac{\mathrm{d}F_l}{\mathrm{d}r} - \frac{l^2}{r^2} F_l + 2 \frac{\mathrm{d}F_l}{\mathrm{d}r} \frac{\mathrm{d}}{\mathrm{d}r} \ln b = 0. \quad (9)$$

The term with $l = 0$ in Eq. (9) has been discarded, since

$$F_0(r) = \int^r \frac{\mathrm{d}\tilde{r}}{\tilde{r}} b^{-2} + \mathrm{const}. \quad (10)$$

is divergent for both bright and vortex solitons. In the first case $\lim_{r\to\infty} b(r) = 0$, and in the second case $b(r = 0) = 0$ and $\lim_{r\to\infty} b(r) = b_{\max}$.

Now consider the case of vortex solitons with topological charge $m \neq 0$ [see Eq. (8)]. Asymptotics of two linearly independent solutions for F_l are $r^{\pm l}$ for large arguments. Finiteness of the function F_l implies the r^{-l} asymptotics at infinity. At large distances the leading contribution to the phase gradient $\nabla\psi$ comes from the $m\theta$ term in Eq. (8) and has the form $\nabla\psi = \mathbf{e}_\phi m/r$, where $\mathbf{e}_\phi$ is the azimuthal unit vector. All the other contributions have at least $1/r^2$ dependence on the radial coordinate. This means that $(\nabla\psi)^2 \propto m^2/r^2$. This contribution cannot cancel the angle-dependent term $\partial\phi/\partial x$ on the right-hand side of Eq. (3b), and this concludes the proof.

For bright solitons asymptotics of two linearly independent solutions for F_l for small arguments are $r^{\pm l}$ and for large arguments are $\exp(2\sqrt{2\Gamma}\, r)$ and $(1 + l^2/2\sqrt{2\Gamma}\, r)$. Exponentially growing solutions do not satisfy Eq. (3b), so we have to require the $(1 + l^2/2\sqrt{2\Gamma}\, r)$ asymptotics for $r \to \infty$. For such asymptotics the main contribution to the phase gradient $\nabla\psi$ comes from the azimuthal derivative; the radial derivative is $1/r$ times smaller. Consequently, at large distances $(\nabla\psi)^2 = r^{-2}(\partial\psi/\partial\theta)^2$. Equation (7) in this limit yields $(\partial\psi/\partial\theta)^2 \propto \beta(r) - \cos(2\theta)$. Since $(\nabla\psi)^2 \geq 0$, we should require that $\beta \geq 1$. If $\beta > 1$, then $\psi(2\pi) \neq \psi(0)$, which contradicts the periodicity of the phase with respect to the azimuthal angle θ in the absence of topological charge. Hence $\beta = 1$, which results immediately in $\psi \propto \cos(\theta)$. This shows that only the $l = 1$ term in Eq. (8) can be nonzero. The angle-dependent part of Eq. (7) then yields

$$\left(\frac{\mathrm{d}F_1}{\mathrm{d}r}\right)^2 - \left(\frac{F_1}{r}\right)^2 = \frac{\mathrm{d}h}{\mathrm{d}r} - \frac{h}{r} \quad (11)$$

for all r. It is easy to show that Eq. (11) cannot be satisfied for all r. Indeed, for large r the left-hand side is proportional to $1/r^2 + O(1/r^3)$ (see asymptotics for F_1), whereas the right-hand side is proportional to $1/r^2$ plus exponentially small corrections [see Eq. (5)]. This observation concludes the proof.

The above proof is valid for any relative intensity of the light beam. Thus, in particular, in the weak saturation limit ($b^2 \ll 1$) circular solitons are not possible, contrary to the claims of Ref. 17. Equations (1) also exclude the possibility of the existence of almost circular solitons. Direct substitution of any circularly symmetric envelope into Eq. (1) shows that the anisotropic part of the nonlinear response is of the same order of magnitude as the isotropic part. Since there is no smallness parameter, a soliton beam (if it exists) must be considerably different from a circular one.

These results disagree with the experimental reports given in Ref. 14 of circular solitons for $s = 1$ and in Refs. 15 and 16 for $s = -1$. We have previously explained the apparent observations of bright circular solitons in terms of oscillatory self-focusing in the regime of high saturation ($b^2 \gg 1$).[10,11] For $s = -1$ previous numerical analysis of unit-charged vortex beams shows that the vortex core rotates and stretches on propagation, leading to complete delocalization of the core region.[12] Together with the stretching, which occurs along a direction at a small angle to the x coordinate, self-focusing and compression of the core region can be observed along the y coordinate. These effects are illustrated in Fig. 1, where we show the evolution of the full width at half-maximum of the core region calculated numerically from Eqs. (1) for an input field of the form $B = \exp(i\theta)[1 - \exp(-r/w_{\mathrm{core}})]\exp[-(r^2/w_{\mathrm{outer}}^2)^p]$, with $p = 7$. w_{core} was chosen to give an input core diameter of 26 μm (FWHM), while w_{outer} was chosen to be sufficiently large that the numerical

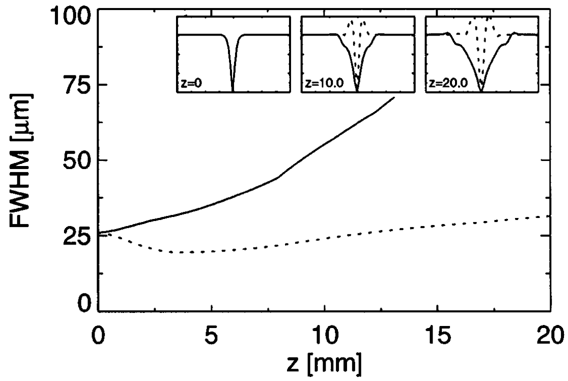

Fig. 1. Evolution of the core of a unit-charged vortex. The diameters were measured along the x (solid curve) and the y (dotted curve) coordinates. The insets show the intensity profiles along x and y in the central portion of the beam (-0.3 mm $< x, y <$ 0.3 mm).

results were unaffected by spreading of the outer core ($\mathrm{FWHM}_{\mathrm{outer}} \sim 2$ mm). The numerical parameters were chosen to correspond to a strontium barium niobate crystal with index of refraction $n = 2.3$, electro-optic coefficient $r_{33} = 340$ pm/V, an applied field of 850 V/cm, and normalized intensity $|B|^2_{\max} = 0.95$. These values are comparable with those reported in Ref. 16. Note that, while the core diameter measured along y is roughly constant, the diameter along x diverges. Additional numerical studies with both larger and smaller input beams show that the diameter along x always diverges.

Our analysis and numerical results concerning the propagation of vortex beams are at odds with experimental results given in Refs. 15 and 16 of circular photorefractive vortex solitons. To understand the apparent discrepancy one should recall that the numerical results given in Fig. 1 were obtained with a beam having an idealized, smooth background intensity. By contrast, a striking feature of the intensity profiles shown in Ref. 16 is that the background beam is strongly modulated with a peak-to-peak amplitude that exceeds 50% of the mean background level. Self-focusing of the vortex core along both x and y coordinates over a finite propagation distance can be observed with nonideal, modulated background profiles. The asymptotic behavior is, however, always characterized by spreading of the core. Strong background modulation also results in the fact that the input and the output profiles given in Fig. 1 of Ref. 16 coincide in the center of the core but differ by 50% or more at radial distances comparable with the core diameter. In our opinion, it is not possible to infer the existence of solitary solutions under such conditions.

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References

1. M. Segev, B. Crosignani, A. Yariv, and B. Fischer, Phys. Rev. Lett. **68,** 923 (1992).
2. M. D. Iturbe-Castillo, P. A. Márquez Aguilar, J. J. Sánchez-Mondragón, S. I. Stepanov, and V. Vysloukh, Appl. Phys. Lett. **64,** 408 (1994).
3. M. Segev, G. C. Valley, B. Crosignani, P. Di Porto, and A. Yariv, Phys. Rev. Lett. **73,** 3211 (1994); A. A. Zozulya and D. Z. Anderson, Opt. Lett. **20,** 837 (1995); P. A. Márquez Aguilar, J. J. Sánchez-Mondragón, S. Stepanov, and G. Bloch, Opt. Commun. **118,** 165 (1995); S. R. Singh and D. N. Christodoulides, Opt. Commun. **118,** 569 (1995); G. Montemezzani and P. Günter, Opt. Lett. **22,** 451 (1997).
4. A. A. Zozulya and D. Z. Anderson, Phys. Rev. A **51,** 1520 (1995).
5. S. Gatz and J. Herrmann, J. Opt. Soc. Am. B **8,** 2296 (1991).
6. G. Duree, G. Salamo, M. Segev, A. Yariv, B. Crosignani, P. Di Porto, and E. Sharp, Opt. Lett. **19,** 1195 (1994); G. Duree, M. Morin, G. Salamo, M. Segev, B. Crosignani, P. Di Porto, E. Sharp, and A. Yariv, Phys. Rev. Lett. **74,** 1978 (1995); M. Taya, M. C. Bashaw, M. M. Fejer, M. Segev, and G. C. Valley, Phys. Rev. A **52,** 3095 (1995); M. D. Iturbe-Castillo, J. J. Sánchez-Mondragón, S. I. Stepanov, M. B. Klein, and B. A. Weschler, Opt. Commun. **118,** 515 (1995); K. Kos, H. Meng, G. Salamo, M.-F. Shih, M. Segev, and G. C. Valley, Phys. Rev. E **53,** R4330 (1996).
7. N. Korneev, P. A. Márquez Aguilar, J. J. Sánchez-Mondragón, S. Stepanov, M. Klein, and B. Wechsler, J. Mod. Opt. **43,** 311 (1996); G. S. García Quirino, M. D. Iturbe-Castillo, J. J. Sánchez-Mondragón, S. Stepanov, and V. Vysloukh, Opt. Commun. **123,** 597 (1996); C. M. Goméz Sarabia, P. A. Márquez Aguilar, J. J. Sánchez-Mondragón, S. Stepanov, and V. Vysloukh, J. Opt. Soc. Am. B **13,** 2767 (1996).
8. A. V. Ilyenkov, A. I. Khiznyak, L. V. Kreminskaya, M. S. Soskin, and M. V. Vasnetsov, Appl. Phys. B **62,** 465 (1996).
9. A. V. Mamaev, M. Saffman, D. Z. Anderson, and A. A. Zozulya, Phys. Rev. A **54,** 870 (1996).
10. A. A. Zozulya, D. Z. Anderson, A. V. Mamaev, and M. Saffman, Europhys. Lett. **36,** 419 (1996).
11. A. A. Zozulya, D. Z. Anderson, A. V. Mamaev, and M. Saffman, Phys. Rev. A **57,** 522 (1998).
12. A. V. Mamaev, M. Saffman, and A. A. Zozulya, Phys. Rev. Lett. **77,** 4544 (1996); Phys. Rev. A **56,** R1713 (1997).
13. W. Królikowski, M. Saffman, B. Luther-Davies, and C. Denz, Phys. Rev. Lett. **80,** 3240 (1998).
14. M.-F. Shih, M. Segev, G. C. Valley, G. Salamo, B. Crosignani, and P. Di Porto, Electron. Lett. **31,** 826 (1995); M.-F. Shih, P. Leach, M. Segev, M. H. Garrett, G. Salamo, and G. C. Valley, Opt. Lett. **21,** 324 (1996).
15. Z. Chen, M. Segev, D. W. Wilson, R. E. Muller, and P. D. Maker, Phys. Rev. Lett. **78,** 2948 (1997).
16. Z. Chen, M.-F. Shih, M. Segev, D. W. Wilson, R. E. Muller, and P. D. Maker, Opt. Lett. **22,** 1751 (1997).
17. B. Crosignani, P. Di Porto, A. Degasperis, M. Segev, and S. Trillo, J. Opt. Soc. Am. B **14,** 3078 (1997).
18. Corrections due to finite Debye wave-number effects lead to additional terms in Eq. (1b) that cause bending of solitary solutions along the x axis, as do corrections to the denominators of the terms on the left-hand side of Eq. (1b) (see Refs. 4 and 11). For the parameters corresponding to experimental studies of solitary waves in photorefractive media, these additional terms are small and are neglected here.