Network Coding With Wireless Applications

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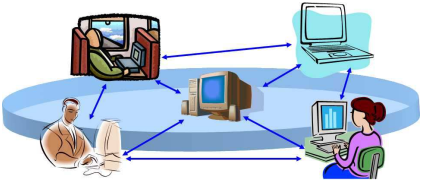
- **Goal of Talk:**

  Understand the main results in network coding to date.

Outline of Talk

- What is Network Coding?
- Theory of Network Coding
- Wireless Applications
- Conclusion

What is Network Coding?



- **Network coding is a strategy for sending data across a communication network.**
- Instead of forwarding the data, we **transform** it along the way.
- This allows us to **communicate more efficiently**!

Preliminaries: The XOR Operator

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- In network coding, we often like to transform data by using the **"XOR"** operator, denoted by $\oplus$.
- XOR is a binary operator that takes two bits as input, and returns one bit as output, as defined by this truth table:

a	b	$a \oplus b$
0	0	0
0	1	1
1	0	1
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- Summary: (Different inputs) $\Rightarrow$ 1. (Same inputs) $\Rightarrow$ 0.

- XOR naturally extends in a pairwise fashion to vectors of bits:

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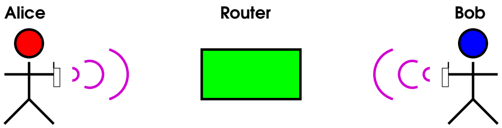
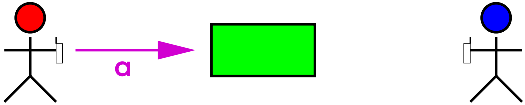
- **Main Point:**

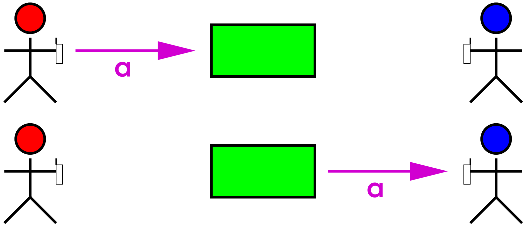
If I know **a**, and someone gives me $\mathbf{a} \oplus \mathbf{b}$, *I can decode* **b**.

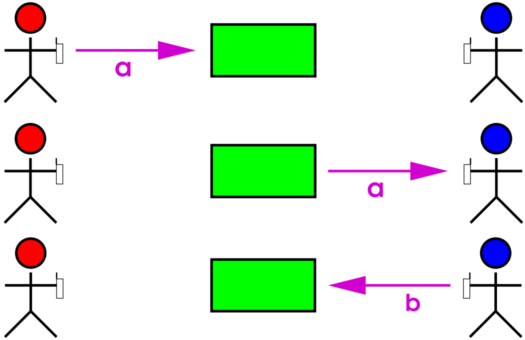
Wireless Exchange

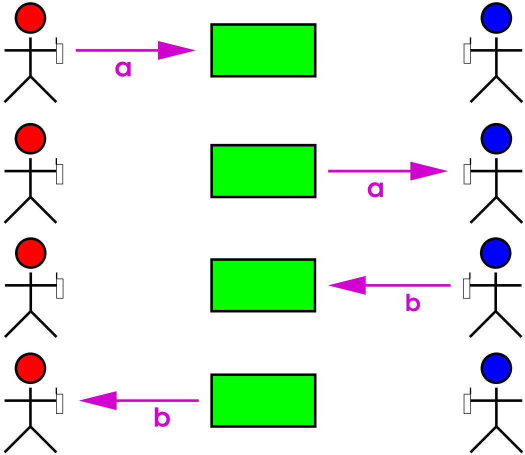


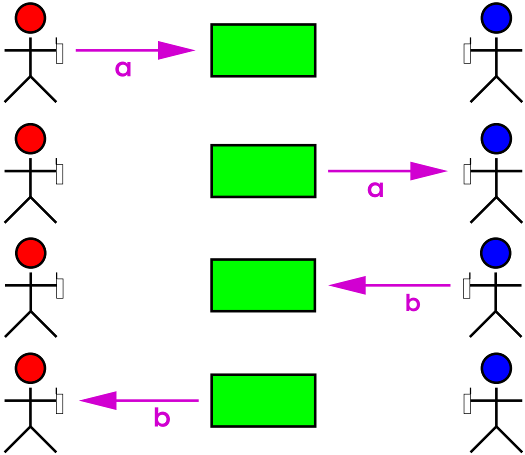
- Alice and Bob are wireless users.
- Alice wants to send message **a** to Bob.
- Bob wants to send message **b** to Alice.
- Because Alice and Bob are too far away from each other, they must send their messages to a router.


a

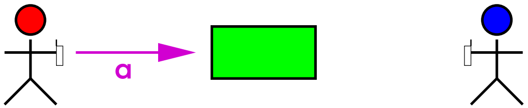

a
a


a
a
b


a
a
b
b



Number of Transmissions = 4. Can we do better?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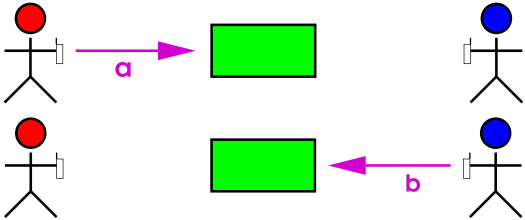
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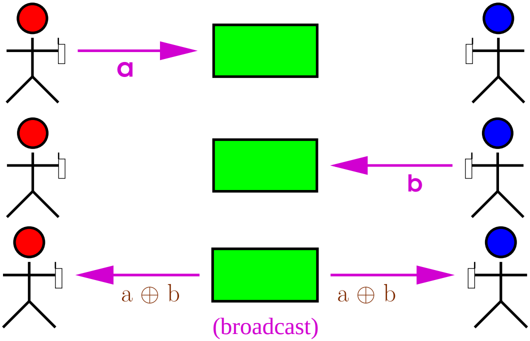
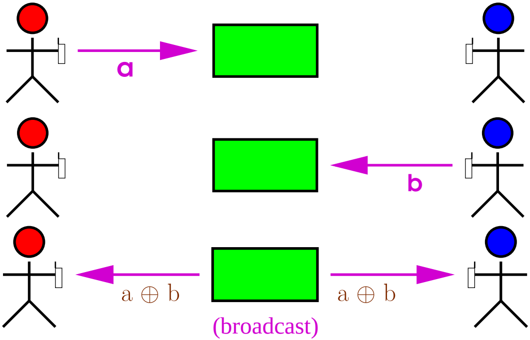
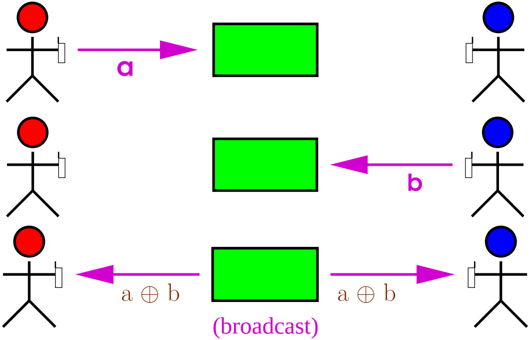


- Network coding at the router: broadcast **a** $\oplus$ **b**.



- Network coding at the router: broadcast $\mathbf{a} \oplus \mathbf{b}$.
- Alice knows **a** already (she sent it!).
- So Alice can decode $\mathbf{b} = \mathbf{a} \oplus (\mathbf{a} \oplus \mathbf{b})$. Similarly for Bob.



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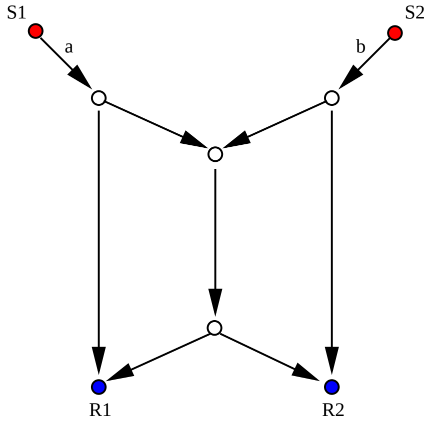
- Number of Transmissions = 3.

Multicast: Butterfly



- Senders: $S1, S2$
- Receivers: $R1, R2$
- Multicasting:
  - $S1$ wants to send **a** to *both* receivers.
  - $S2$ wants to send **b** to *both* receivers.
- Every edge in the communication network has the same capacity.

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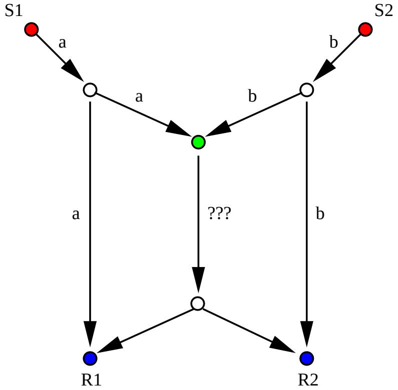

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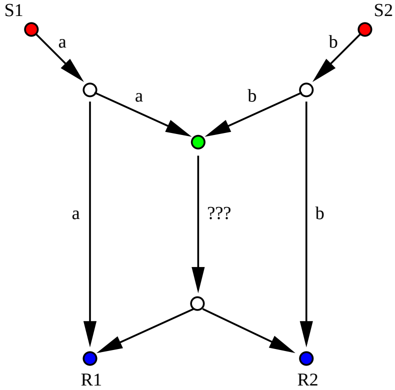
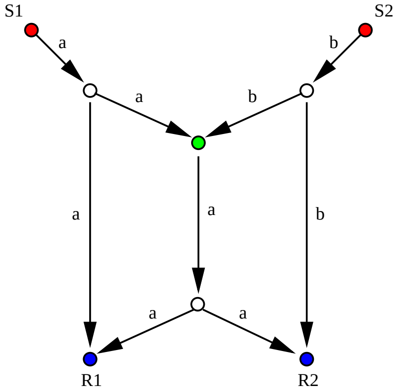


But what should the green node do? (ask audience)

Suppose he forwards one of the two messages; let's say **a** . . .



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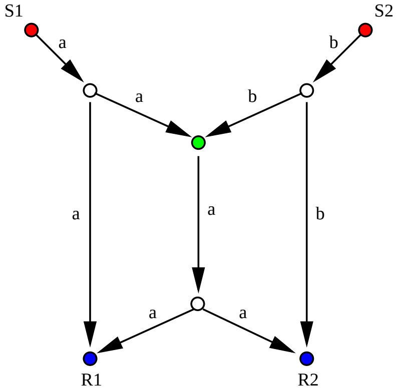
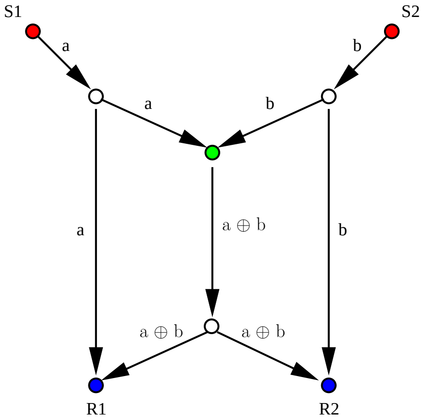


Then $R2$ receives both **a** and **b**, but $R1$ only receives **a**.

- Better idea: Use XOR to *mix* the information.



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- $R1$ receives **a** and $\mathbf{a} \oplus \mathbf{b}$. Decode $\mathbf{b} = \mathbf{a} \oplus (a \oplus b)$.
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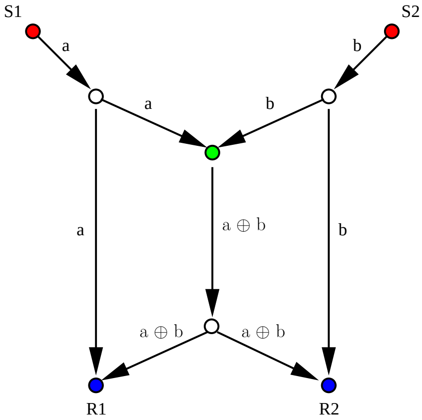

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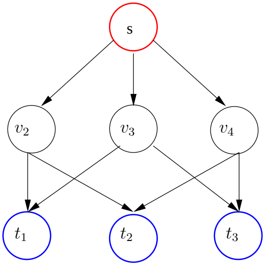
- $R2$ receives **b** and $\mathbf{a} \oplus \mathbf{b}$. Decode $\mathbf{a} = (\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{b}$.
- Both get two messages! Network coding increases **capacity**.

The Key Idea

Key Idea of Network Coding

- Information is not a physical commodity!
- We don't have to keep it in its original packaging. (*routing*)
- Sometimes we should open the package and change it! (*network coding*)

Multicast: 3-ary Graph



- Node s wants to send the *same set of messages* to three different receivers t_1, t_2, and t_3. (This is called "multicast".) Every edge has the same capacity.

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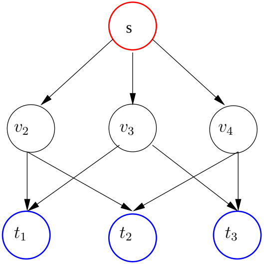

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- How many different messages can s send simultaneously? (ask audience)

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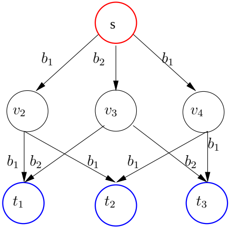

routing

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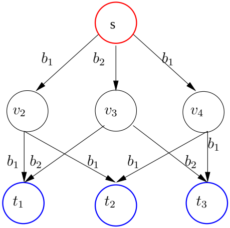

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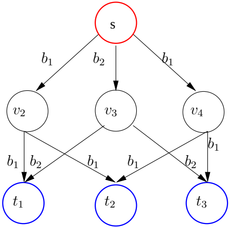

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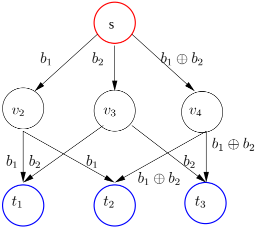

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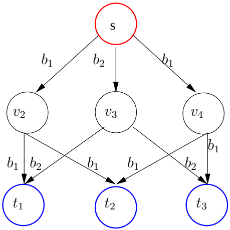

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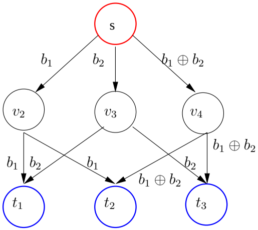

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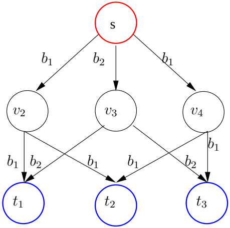

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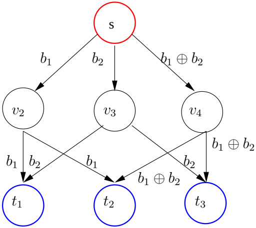

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- This was harder than the previous problems.
- How do we know that we cannot send three messages?

Natural Questions

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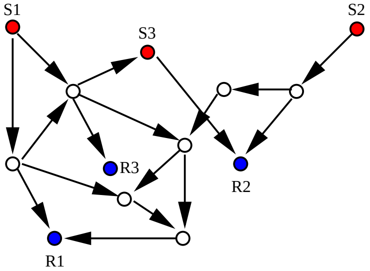
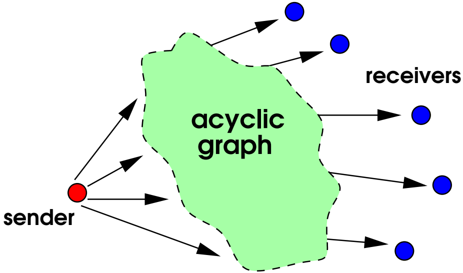
- How can we do network coding on a complex network?



(XORs aren't good enough ...)

- Satisfying answers to these questions are available for **one sender multicasting on an acyclic graph.**



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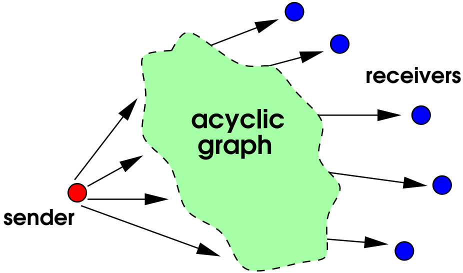

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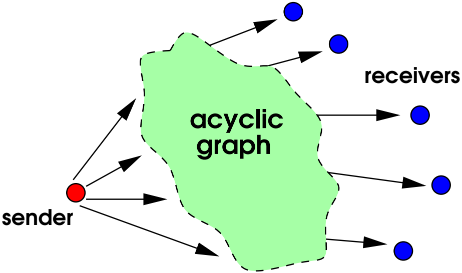

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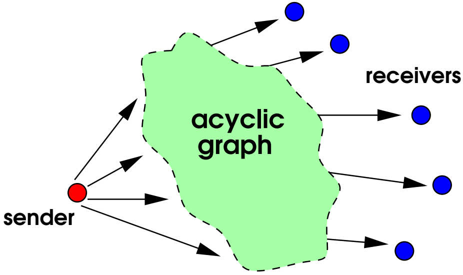
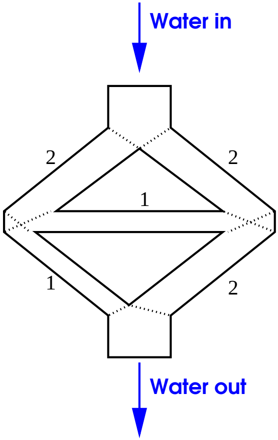


- Other scenarios are open problems.

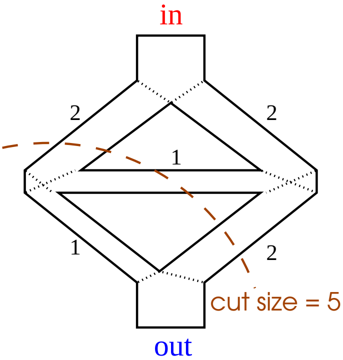
- To understand the existing answers, consider flowing water . . .

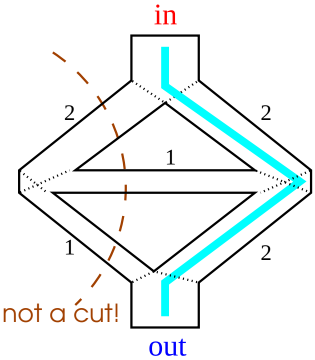
Preliminaries: Max-Flow Min-Cut Theorem

- Consider a network of water pipes. There is a single input pipe, and a single output pipe.
- Every pipe has a certain flow capacity that it can support (e.g., 2 gal/sec).
- **Question:** *What is the maximum water flow between input and output?*



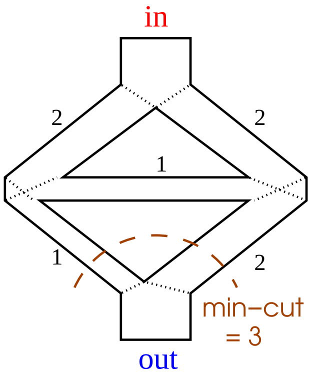
- **Definition:** A **cut** is a set of pipes that together completely separate the input and output.
- **Definition:** The **size** of a cut is the sum of the capacities of all the pipes in the cut.





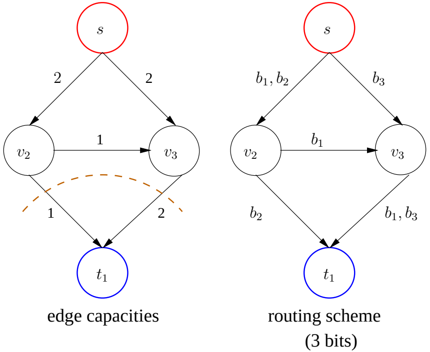
- **Ford-Fulkerson Max-Flow Min-Cut Theorem:**

The maximum flow is equal to the size of the smallest cut.



- The smallest cut is called the "min-cut".

- This result extends to information transfer!



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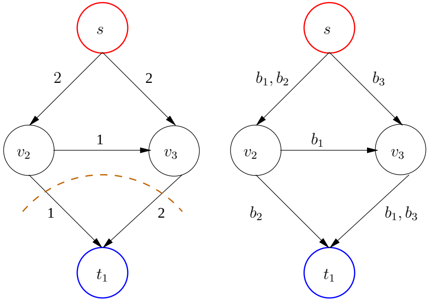

edge capacities

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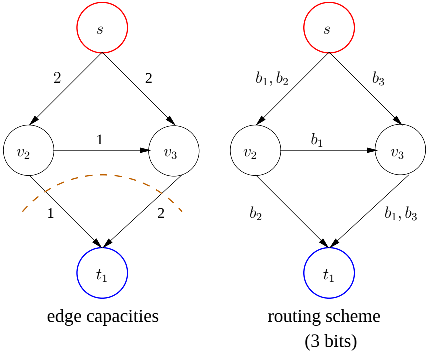

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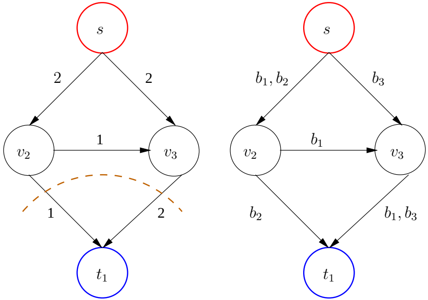

edge capacities
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Full solution to one-sender one-receiver problem.
Ford-Fulkerson routing achieves optimal throughput.

Multicasting Problem Statement

Now let's look at one sender and multiple receivers.

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  - V is the set of nodes,
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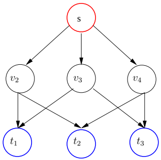
- **Problem 2 (Code):** Find the routing/coding scheme which achieves the maximum rate.

Example: 3-ary Multicast, Again



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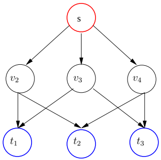
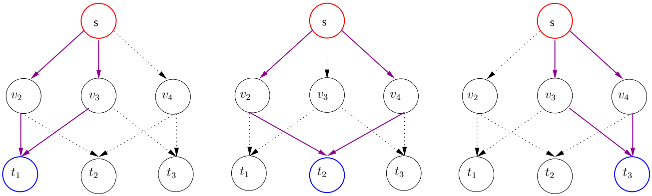
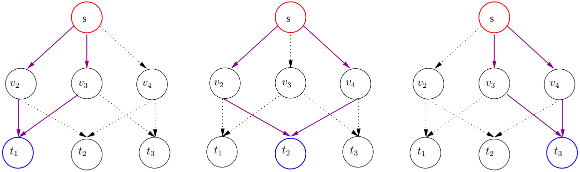


- One sender s, and three receivers $T \triangleq \{t_1, t_2, t_3\}$.

- For each $t \in T$, define the "subgraph" G_t to be the graph consisting of all paths which run from s to t.





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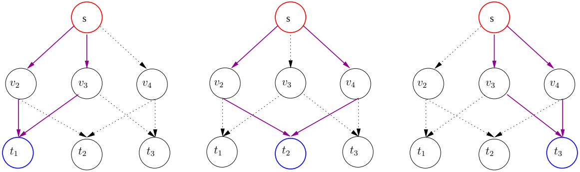

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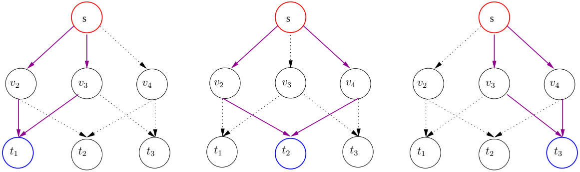

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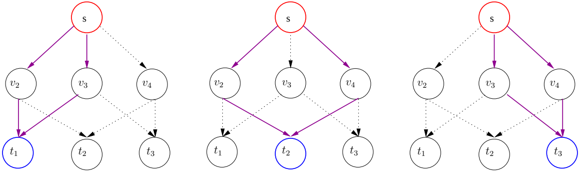

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- **The Theorem**: $MAXRATE = \min_t MINCUT(G_t)$.

Main Theorem of Network Coding

Main Theorem of (Multicast) Network Coding

Let G_t be the subgraph between s and $t \in T$. Then $MINCUT(s \to t)$ is the min-cut between s and t in G_t. Then, the *maximum reliable multicast rate* is:

$$MAXRATE = \min_{t \in T} MINCUT(s \to t)$$

This rate can be achieved with *linear codes* which can be found in polynomial time $O(|E| \cdot |T| \cdot (h^2 + |T|^2))$.

How To Find The Code?

1. **Key Idea:** With every edge $e_{ij} \in E$, we will associate a vector $\mathbf{b}(e_{ij})$ representing the information on that edge.

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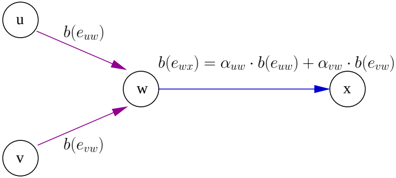
2. Find the maximum symbol rate $h \triangleq \min_{t \in T} MINCUT(s \rightarrow t)$.
3. Represent each of the h symbols generated at s by unit vectors:

$$\mathbf{b}(e_i) = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \mathbf{b}(e_2) = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \ldots, \mathbf{b}(e_h) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

4. Linear Coding

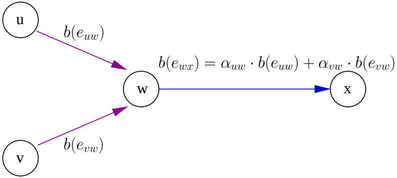


- $\mathbf{b}(e_{ij})$ is a *random linear combination* of information received from incoming edges $\mathbf{b}(e_{ki})$:

$$\mathbf{b}(e_{ij}) = \sum_{e_{ki} \in E} \alpha_e(e_{ki})\mathbf{b}(e_{ki})$$

where $\alpha_e(e_{ki})$ are drawn randomly from a field (set) $\mathcal{F}$.

4. **Linear Coding**



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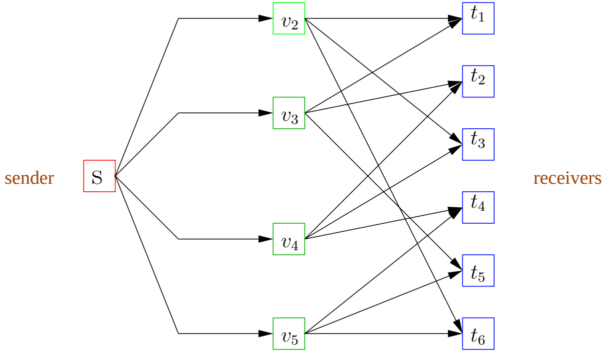
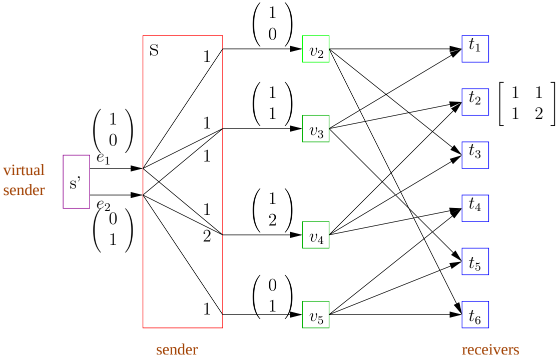
5. If $|\mathcal{F}| >> |T|$, we will successfully multicast at rate h with high probability.

Example



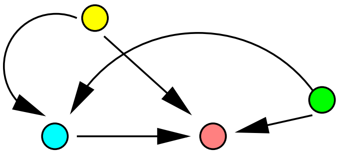
- The min-cut of each sender-to-receiver subgraph is 2.
- So $h = 2$.



- Introduce a virtual sender s' which supplies the symbols.
- Our code can multicast if and only if for every receiver t, the determinant of the matrix of vectors entering t is nonzero.

Toward Reality

- We have been looking at networks which are
  - noiseless
  - have clearly defined communication links.



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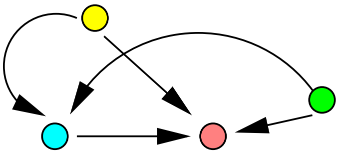
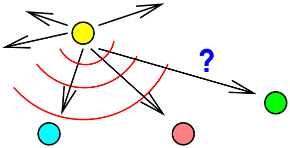


- Yet, real wireless networks
  - have noisy links
  - are broadcast in nature (unintended listeners).

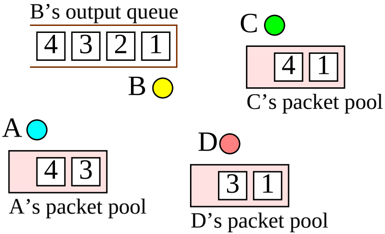


The Key Idea

Key Idea of Wireless Network Coding

- In wireless networks,
  - information is always broadcast to many users, and
  - information can be lost.
- Therefore,
  - Sometimes Alice will hear something that Bob didn't.
  - Sometimes Bob will hear something that Alice didn't.
- Network coding can exploit this *diversity*!
- The wireless channel is naturally suited for network coding.

"Coding Opportunistically" (COPE)



packet in B's queue		next hop
1	$\longrightarrow$	A
2	$\longrightarrow$	C
3	$\longrightarrow$	C
4	$\longrightarrow$	D

coding option	who can decode

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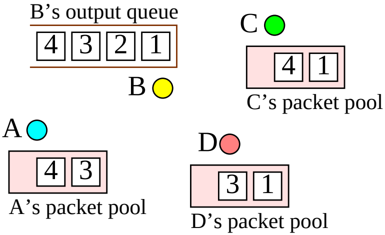

packet in B's queue		next hop
1	→	A
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coding option	who can decode
1 $\oplus$ 2	C

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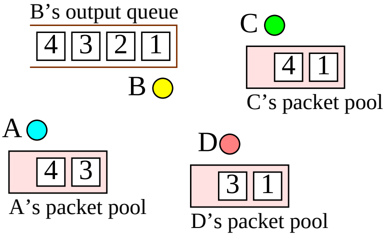

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1 ⊕ 2	C
1 ⊕ 3	A,C

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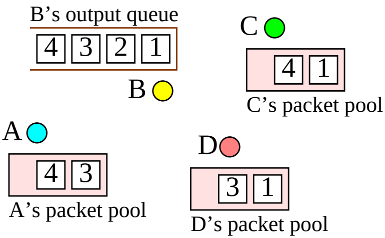

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1	→	A
2	→	C
3	→	C
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1 ⊕ 2	C
1 ⊕ 3	A,C
1 ⊕ 3 ⊕ 4	A,C,D

Framework of COPE

- Opportunistic Listening
  - Set all nodes to *promiscuous* mode.
  - Everyone *records* what they have heard for a while.
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- Opportunistic Coding
  - When sending, XOR together as many packets we can in order to maximize the number of intended receivers who can decode.
  - Never delay packets.

Experimental Results

Fully-implemented 20-node wireless testbed at MIT

- <u>Wireless Ad-Hoc Network</u>

Protocol	Throughput Gain
TCP	2-3% 20-30% when nodes are closely packed
UDP	300-400%

(TCP backs off excessively due to collision-based losses.)

- <u>Wireless Mesh Access</u>

Protocol	Throughput Gain
UDP	5-15% when UL/DL ratio $< 1/2$
UDP	70% when UL/DL ratio $> 1/2$

(higher uplink traffic = more diversity at output queues)

Reliable Broadcast

Sender s broadcasts to receivers $R1$ and $R2$. Packets are lost.

Received by R1	x	2	3	x	5	6	x	8	x
Received by R2	1	2	x	4	x	6	x	8	9

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From negative acknowledgements (opposite of ACK), s knows who did not receive what. Use XOR to retransmit efficiently.

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Received by R2	1	2	x	4	x	6	x	8	9

In practice, use a combination of FEC and network coding.

Analog Network Coding (ANC)

Idea: Increase throughput by letting analog signals collide.

traditional scheme	digital network coding	analog network coding

How can we get away with this?

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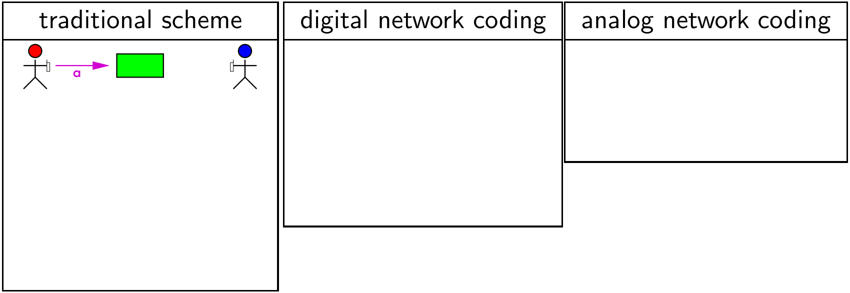

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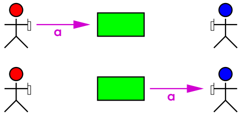		

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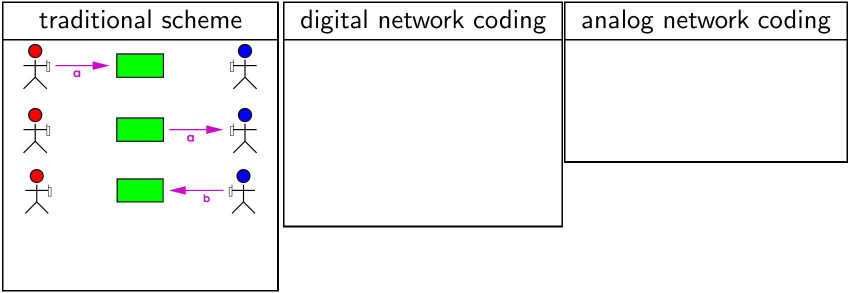

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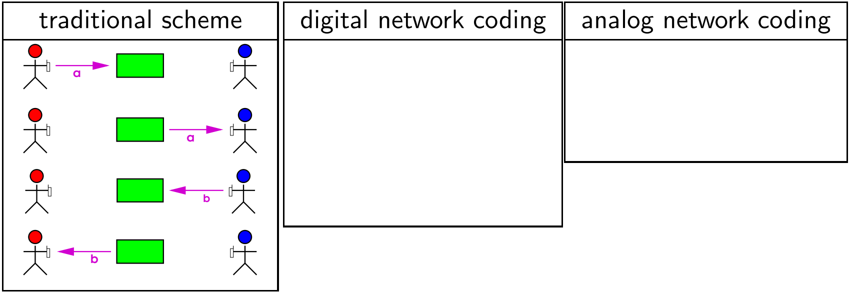

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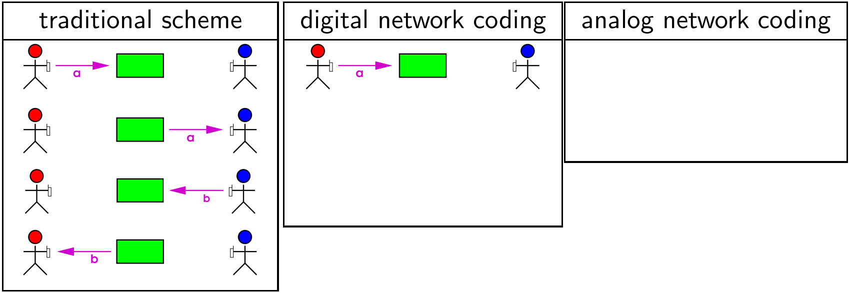

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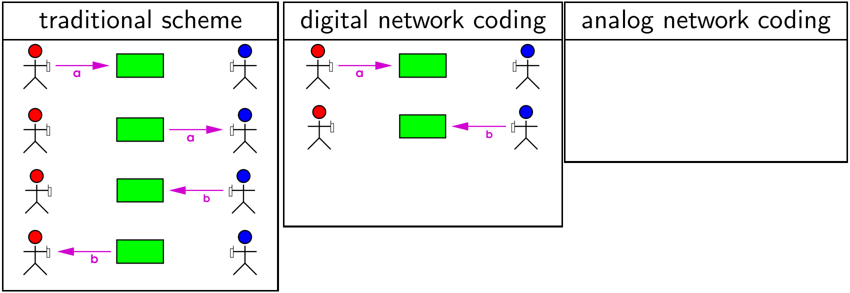

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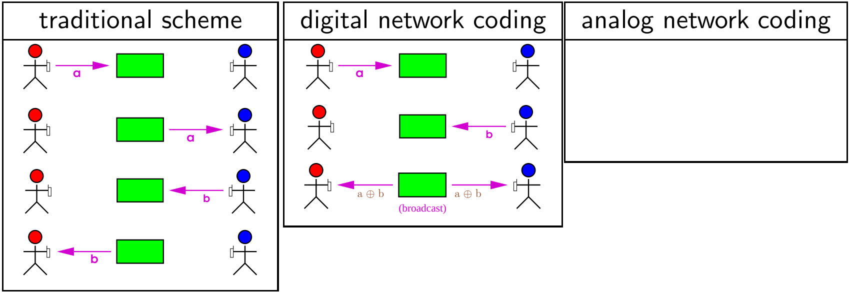

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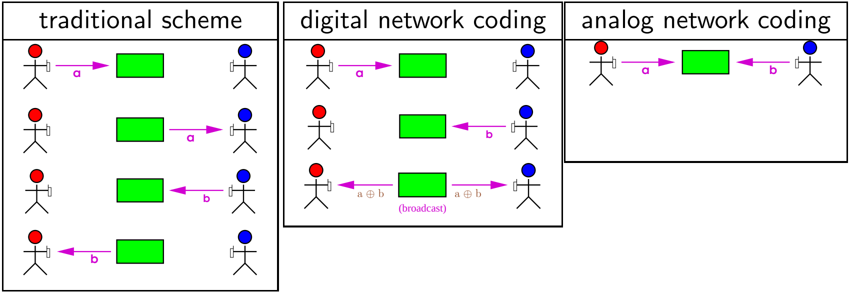

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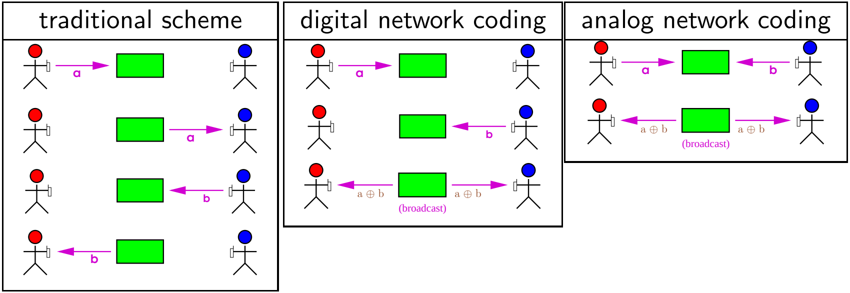
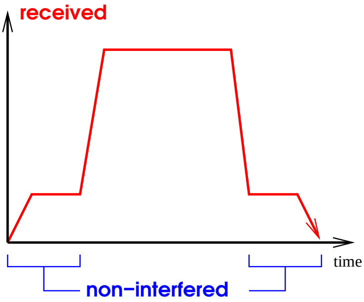


How can we get away with this?

Key Trick:

- The two simultaneously sent signals will not be exactly synchronized.
- By using *MSK modulation*, we can deduce the original signals by analyzing the non-interfered parts of the combined signal.



Result (software radios): Two senders $\Longrightarrow \sim 70\%$ gain

Summary

Summary of Key Ideas

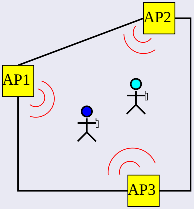
- Information is not a physical commodity. We can transform it at intermediate nodes.
- For multicasting between s and a set of receivers T,

$$MAXRATE = \min_{t \in T} MINCUT(s \rightarrow t).$$

  Achievable with linear codes found in polynomial time.
- The wireless channel is naturally suited for network coding, since there is diversity in the received information.

Further Investigation

- How to use network coding ideas effectively in an indoor Wireless LAN?
  - Wired APs in building
  - Wireless users



- How can we improve on COPE (Coding Opportunistically)?
- New ideas in applying network coding to ad-hoc networks?
- How to best use network coding ideas in unicast scenarios?
- Thanks for listening!
- Comments and collaboration: **willywutang@gmail.com**

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