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INVESTMENT INCENTIVES IN PROPRIETARY AND OPEN-SOURCE TWO-SIDED PLATFORMS*

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Abstract. We study incentives to invest in platform quality in proprietary and open-source platforms. A comparison of monopoly platforms reveals that for a given level of user and developer adoption, investment incentives are stronger in proprietary platforms. However, open platforms may receive larger investment because they may benefit from wider adoption, which raises the returns to quality investment. We also study a mixed duopoly model of competition and examine how the price structure and investment incentives of the proprietary platform are affected by quality investments in the open platform. We find that access prices may increase or decrease as a result of investment in the open platform, and the sign of the change may be different for user and developer access prices. We also find that the proprietary platform may benefit from higher investment in the open platform when developers multi-home. This result helps explain why a proprietary platform such as Microsoft has chosen to contribute to the development of Linux.

Keywords: Two-Sided Markets, Platform Investment, Network Effects, Open Source, User Innovation, Complementarity (JEL: O31, L17, D43).

1. Introduction

While proprietary and open-source software have coexisted since the early days of the computing industry, competition between these two modes of development has intensified dramatically following the surge of the Internet in the mid-1990s. Prominent examples include Windows vs. Linux, Microsoft Office vs. Open Office, Safari vs. Firefox, MS Internet Server vs. Apache, and more recently, Apple's iOS vs. Google's Android.

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The open-source development model is characterized by two distinctive features: open access (the freedom to use the software free of charge) and open investment (the freedom to modify the source code).[1] Proprietary development, on the other hand, has closed access and closed investment: the platform sets access prices and invests centrally to improve its quality. The coexistence of these two diametrically opposed modes of platform governance has sparked a thriving literature on open source examining as to why individuals and profit-maximizing firms might choose to contribute to open-source development (see Lerner and Tirole, 2005; von Krogh and von Hippel, 2006; Fershtman and Gandal, 2011, for recent surveys).

While insightful and enlightening, theoretical developments on the economics of open source have fallen short of fully embracing the modeling breakthroughs offered by the literature on two-sided platforms of the past decade (e.g., Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Hagiu, 2006a; Spulber, 2006; Weyl, 2010). Likewise, while the literature on two-sided platforms has studied some aspects of open platforms, the most distinctive feature of open source (i.e., open investment) has not been considered.[2]

In this paper, we bring together these two streams of work to address the following questions: how are the incentives to invest in platform quality affected by the degree of platform openness? which of these two modes of governance leads to investments closer to the social optimum? and how are incentives to invest in platform quality moderated by competition between proprietary and open two-sided platforms?

We set up a model of a platform that brings together users (buyers) and developers (sellers) of applications. Users are heterogeneous in their willingness to pay for access to the platform. Developers are also heterogeneous in that they bear different costs of developing applications. A proprietary platform chooses how much to invest in platform quality and sets access prices for each side of the market. An open platform may be accessed for free and developers may invest in improving its quality.[3] Regardless of whether the platform is proprietary or open, after users and

[1]Open access and open investment are complementary but do not always go hand in hand. For example, while MS Explorer is an open-access program, it does not allow for open investment as the source code is not made available to users.

[2]To the literature on two-sided platforms, an open platform is one that offers open access, and a proprietary platform is one that has closed access (the platform sets access prices, positive for at least one side). Thus this literature is silent about the "investment side" of openness: an open source platform not only offers open access but it also allows for open investment in that users and developers are allowed invest in platform quality by modifying the source code.

[3]For concreteness, in this paper we focus on developer investment in open platforms and assume user investment is absent. Recent empirical evidence suggests that a large share of investments

developers have accessed the platform, developers compete to sell applications to users. As in Dixit and Stiglitz (1977), we assume that users prefer product variety but consider applications as interchangeable.

Our model has three distinctive features. First, there is endogenous investment in platform quality (by the platform owner if the platform is proprietary and by application developers if the platform is open). Second, along with the case of *substitute* applications whose value decreases with the number of applications available, we study the mirror case of *complement* applications. Finally, we let users have bargaining power to negotiate application prices. User bargaining power may emanate from structural features or from price sensitiveness due to the presence of substitute product categories, such as pirated versions of the software.

We divide the analysis into two parts. We first examine models of proprietary and open monopoly platforms; that is, we consider incentives to invest by proprietary and open platforms in isolation from each other and compare equilibrium outcomes. In the second part of the paper, we analyze a duopoly model with direct competition between both types of platforms.

Our model of monopoly platforms has two forces at play. First, under both proprietary and open platforms entry is below the first-best, but comparing between the two, open platforms often trigger more entry than proprietary platforms. Intuitively, with a proprietary platform we have the standard monopoly solution of producing less than the social optimum. With an open platform, each side does not internalize the benefit entry provides to the other side so again entry is suboptimal. We find that the latter effect can be weaker than the former, in which case there is more entry under the open platform.

The second force is that a proprietary platform sets the first-best level of investment for a given entry level. This is because a close platform internalizes the gains from investment as it charges fixed access fees from the two sides. The open platform has zero access prices and thus such internalization does not take place. Now, combining the two forces, an open platform may generate substantially more entry than a proprietary platform so that it will also generate more investment in quality (the first force is stronger than the second one).

in open source are made by firms rather than users. For example, a recent report by the Linux Foundation (Corbet, Kroah-Hartman, and McPherson, 2012) states that seventy-five percent of all Linux kernel development is done by developers who are being paid for their work, and that the top ten organizations sponsoring Linux development are Red Hat, Intel, Novell, IBM, Texas Instruments, Broadcom, Nokia, Samsung, Oracle, and Google. Extending the model to include user innovation would be trivial and only strengthen our results.

From a welfare point of view, proprietary and open platforms are both inefficient, but the reasons for the inefficiencies differ. In general terms, proprietary platforms are welfare superior if investments in platform quality are more important than the effect of product variety on user utility.

Our analysis of competition between a proprietary and an open platform reveals that equilibrium access prices depend on the equilibrium relation between entry and investments. Access prices may increase or decrease when taking into account investment incentives compared to a situation where there are no investments in platform quality. The effect of investment incentives on user and developer access prices may have the same or opposite signs. Specifically, the effect of investments in quality in the open platform on the structure of access prices depends on (1) how changes in the number of users and developers in the proprietary platform affect investments in the open platform, and (2) how investments in the open platform affect the revenues of the proprietary platform.

Finally, we show that when developers multi-home, the proprietary platform may benefit from higher quality investment in the open platform. This result explains why proprietary firms may choose to contribute to the development of competing open-source platforms. For example, in a recent report, Corbet, Kroah-Hartman, and McPherson (2012) show that Microsoft ranks 17 in the list of top contributors to Linux. Indeed, while in 2001 Microsoft's CEO Steve Ballmer famously claimed that "Linux is a cancer that attaches itself in an intellectual property sense to everything it touches," in 2010 Jean Paoli (general manager of Microsoft's interoperability strategy team) declared: "We love open source."

1.1. **Related literature.** Our paper contributes to the literatures on multisided markets and the economics of open source. A large share of the extant literature on two-sided platforms studies pricing in the presence of network effects (e.g., Spulber, 1996; Caillaud and Jullien, 2003; Rochet and Tirole, 2003; Parker and Van Alstyne, 2005; Rochet and Tirole, 2006; Armstrong, 2006; Hagiu, 2006a; Nocke, Peitz, and Stahl, 2007; Casadesus-Masanell and Ruiz-Aliseda, 2008; Weyl, 2010). In general terms, the structure of equilibrium prices depends on the relative size of demand elasticities and cross-group externalities, the costs of serving each side of the market, market structure, and whether end-users single-home or multi-home. Although we focus on the incentives to invest in platform quality, we also derive the access prices charged by proprietary platforms in equilibrium and obtain results congruous with the literature. Closer to our setting, Hagiu (2006b) and Economides and

Katsamakas (2006b) compare proprietary and open platforms. These papers model open platforms as open access platforms. While we also assume zero access prices to open platforms, we allow for developer innovation to improve platform quality.

Incentives to invest in platform quality in proprietary and open-source two-sided platforms have not been analyzed before. Hagiu (2007), Belleflamme and Peitz (2010), Zhao (2010), and Lin, Li, and Whinston (2011) study sellers' incentives to invest in the quality of the products they sell, rather than on the quality of the platform. Our work is closer to Economides and Katsamakas (2006a) who examine incentives to invest in a one-sided platform with one application developer. These authors compare proprietary and open-source operating systems. In a proprietary operating system, quality-enhancing investments are made by the platform owner; in an open operating system, investments are made by the application developer and advanced users.[4] They find that the incentives to invest in the application are generally larger when the platform is open, and that investment in the open source operating system is larger if there are strong reputation effects from participation in open-source development, and/or a significant part of the open source users are developers.

Rather than one-sided operating systems, we consider two-sided platforms. In our setting, the proprietary platform chooses access prices for two sides and may subsidize one to better exploit indirect network effects. Moreover, we allow for endogenous platform adoption by users and developers and, contrary to Economides and Katsamakas (2006a), in our model there is always a large number of users and developers. We do not consider the role of reputation from participation in open-source development on developers' incentives to invest. Our analysis thus shows that such reputational concerns are not necessary for an open platform to obtain higher investment than a proprietary one.

The early literature on open source was concerned with explaining why individual developers contributed to open-source projects allegedly for free (Lerner and Tirole, 2005; von Krogh and von Hippel, 2006; Fershtman and Gandal, 2011, present excellent surveys). The most common explanations were: altruism, personal gratification, peer recognition, and career concerns. Bagozzi and Dholakia (2006), for example, demonstrate that participation in open-source development is partly explained by social and psychological factors, and Roberts, Hann, and Slaughter (2006) find that status and career concerns significantly influence developers' levels

[4]Advanced users are users who invest in platform quality to maximize reputation.

of participation. We do not consider social preferences or career concerns. Rather, we focus on self-interested agents and examine the value of investments in the platform to the very developers who make those investments.

While the contributions of individual users have played a crucial role for the success of many open-source projects, the same is true of contributions by developers and commercial firms. In a carefully executed empirical piece, Fosfuri, Giarratana, and Luzzi (2008) find that firms with a larger stock of hardware patents and trademarks are more likely to participate in open source. Shah (2006) investigates the effects of sponsorship of open-source projects by commercial firms and finds that voluntary developers tend to contribute less, have different motivations for contributing, and take on fewer code maintenance tasks than in the absence of such sponsorship.

Our paper also contributes to an emerging literature in strategy that explores competitive interactions between organizations with different business models. While there are several formal models of asymmetric competition that exist in strategy (differences in costs, resource endowments, or information, mainly), the asymmetries that this literature wrestles with are of a different nature: firms with fundamentally different objective functions, opposed approaches to competing, or different governance structures. Within this literature, papers examining competition between open-source and proprietary software have considered duopoly models of a profit-maximizing, proprietary firm and a community of not-for-profit/non-strategic open-source user/developers selling at zero price (Mustonen, 2003; Bitzer, 2004; Gaudeul, 2005; Casadesus-Masanell and Ghemawat, 2006; Economides and Katsamakas, 2006b; Lee and Mendelson, 2008; Casadesus-Masanell and Llanes, 2011). These papers, however, assume that investment incentives are exogenously given (generally, investment in open source is a function of the number of users). The exception is Llanes and de Elejalde (2009), who assume investment is performed by sellers of complementary goods. In addition, for the most part, the literature on mixed duopoly presents models of one-sided firms. We contribute work in this area by endogenizing developer investment incentives and considering interactions between two-sided platforms.

The rest of the paper is organized as follows. In Section 2 we present the model. In Sections 3 and 4 we study the cases of proprietary and open platforms, and in Section 5 we compare outcomes across the two governance modes. Section 6 studies equilibrium investment in a mixed duopoly where a proprietary and an open platform compete for users. Section 7 concludes.

2. The model

We study a two-sided monopoly platform that brings together application developers and users.[5] The platform may be software (e.g., an operating system), hardware (e.g., a DVD player), or a combination of the two (e.g., a video game console). We focus on the incentives to invest in *platform quality*, that is, on the incentives to develop the software and/or hardware which constitute the platform. Although the number of applications is endogenous in our model, we do not study incentives to invest in application quality, which have been studied elsewhere (Hagiu, 2007; Belleflamme and Peitz, 2010; Zhao, 2010; Lin, Li, and Whinston, 2011).

There is a continuum of potential users and developers. Users demand applications and run them on the platform. The indirect utility of user i is

$$u(i) = v(n, x) - \int_0^n \rho(j)\, dj - h(i) - p^u, \tag{1}$$

where n is the measure of available applications, x is the investment in platform quality, $h(i)$ is a user-specific adoption cost, p^u is the platform access price for users, and $\rho(j)$ is the price of application j.[6]

Function $v(n, x)$ is the gross utility of consuming n applications when the platform has received quality investment x. We follow the usual convention of representing derivatives through subscripts (e.g., $v_{nx} = \frac{\partial^2 v(n,x)}{\partial n\, \partial x}$). Users prefer higher quality platforms and application variety ($v_x > 0$ and $v_n > 0$). The investment in platform quality and the measure of applications are complements ($v_{nx} \geq 0$). When $v_{nn} = 0$ applications are independent in that consuming more of any one application does not affect the marginal utility of consuming any other application. The cases $v_{nn} < 0$ and $v_{nn} > 0$ correspond to applications being substitutes and complements. When $v_{nn} < 0$, we have $v(n_1, x) + v(n_2, x) > v(n_1 + n_2, x)$ and applications detract from each other. The reverse is true for complements. Without loss of generality, let $h(0) = 0$. Consumers are ordered according to cost so that $h_i > 0$. Therefore, $h(i) > 0$.

[5] More generally, our model applies to any technology platform allowing the interaction between sellers and buyers.
[6] Function h may also be interpreted as a taste differentiation parameter or transportation cost.

Each developer may produce one application. Developer j's profits are

$$\pi(j) = \rho(j)\, m - c(j) - p^d - \sigma\, x(j), \tag{2}$$

where m is the measure of users, $c(j)$ is a developer-specific development cost, σ is the marginal cost of investing in platform quality, $x(j)$ is developer j's investment in platform quality, and p^d is the platform access price for developers. Developers are ordered according to cost so that $c_j > 0$. Assume $0 \leq c(0) \leq v_n(0, x)$, which means that having a positive number of applications is always desirable from a social point of view.

There are two types of platforms. In the proprietary platform case, the platform is provided by a profit-maximizing firm, which sets access prices p^u and p^d and invests in platform quality. Therefore, in this case, developers' investment $x(j)$ is null. In the open source platform case (hereinafter referred to as "open platform"), access to the platform is free, $p^u = 0$ and $p^d = 0$, and developers invest in platform quality. Therefore, $x(j)$ may be positive. As noted in the introduction, the existing literature on open platforms in multisided markets has only considered the zero-price dimension of open source (open access), and has not studied the implications of open source on the incentives for innovation (open investment). We include this important aspect of open platforms to our model and analysis.

Prices are determined through Nash bargaining. In particular, suppose the surplus (increase in gross utility) from buying application j is $\theta(j)$. Then, equilibrium price is $\rho(j) = \alpha\, \theta(j)$, where $\alpha \in [0, 1]$ represents the bargaining power of developers. If $\alpha = 1$, developers are price setters. If $\alpha = 0$, users have all the bargaining power and application prices are equal to zero. In a market in which software piracy is pervasive, for example, α will be close to zero, and prices will tend to be low.

In what follows, we compare market equilibria with the socially optimal allocation. The social planner chooses m, n, and x to maximize the sum of indirect utility and profits:

$$\begin{aligned} W &= \int_0^m u(i)\, di + \int_0^n \pi(j)\, dj, \\ &= m\, v(n, x) - \int_0^m h(i)\, di - \int_0^n c(j)\, dj - \sigma\, x. \end{aligned}$$

The equations characterizing the first best (obtained straightforwardly by differentiating W with respect to m, n, and x) are

$$v = h, \qquad m\, v_n = c, \qquad \text{and} \qquad m\, v_x = \sigma.$$

3. Proprietary platform

The timing of the game is the following: (i) the platform provider chooses x, p^u, and p^d; (ii) users and developers decide whether to join the platform; and (iii) developers and users bargain over $\rho(j)$, and users choose how many applications to buy. The equilibrium concept is subgame perfect equilibrium, and we solve the model backward.

Since developers cannot invest in platform quality, equations (1) and (2) become

$$\begin{aligned} u(i) &= v(n,x) - \int_0^n \rho(j)\,dj - h(i) - p^u, \text{ and} \\ \pi(j) &= \rho(j)\,m - c(j) - p^d. \end{aligned}$$

In the third stage, developers and users bargain over the price of applications. Let $\rho^*(j)$ be the third-stage equilibrium price of application j. Price is determined differently when applications are substitutes and complements. When applications are substitutes, the maximum price an application developer may charge is v_n (if the price of any application was greater than the marginal value of the last application, users would be better off not consuming that application). The actual price paid by users is determined through bargaining, and $\rho^*(j) = \alpha\, v_n$ for all j.

When applications are complements, the maximum price is no longer v_n. To see this, note that if price was v_n, the total cost of a bundle of n applications would be larger than its gross utility to users ($n\, v_n > v(n,x) - v(0,x)$), and thus users would be better off not buying any application. Therefore, in equilibrium we must have $\int_0^n \rho(j)dj \le v(n,x) - v(0,x)$. In a symmetric equilibrium, the maximum application price is $(v(n,x) - v(0,x))/n$. Let

$$w(n,x) = (v(n,x) - v(0,x))/n,$$

which is increasing in n ($w_n = (v_n - w)/n > 0$ when applications are complements). As in the substitutes case, the actual price paid by users is determined through bargaining, and $\rho^*(j) = \alpha\, w$ for all j.

In the second stage, users and developers choose whether to access the platform. The marginal entrants, m and n, satisfy $v(n,x) - n\,\rho^* = h(m) + p^u$, and $m\,\rho^* = c(n) + p^d$. From here, we obtain the inverse demand functions:

$$p^u = v(n,x) - n\,\rho^* - h(m), \tag{3}$$

$$p^d = m\,\rho^* - c(n). \tag{4}$$

Since ρ^* does not depend on m, $\partial p^u/\partial m = -h_m < 0$ with substitutes and complements. With substitutes, $\partial p^d/\partial n = \alpha\, m\, v_{nn} - c_n$, which is always negative. With complements, $\partial p^d/\partial n = \alpha\, m\, w_n - c_n$, which is negative if and only if $n\, c_n > \alpha\, m\,(v_n - w)$. We will assume that this condition holds whenever n solves (4), which in turn means there is only one (m, n) pair verifying (3) and (4) for a given set of prices p^u and p^d.

The following lemma shows that application prices do not affect the equilibrium allocation m, n, x arising from platform choices. Nonetheless, application prices do affect the level of access prices.

Lemma 1 (Neutrality of application prices)**.** *In a proprietary platform, the equilibrium allocation m, n, and x is the same, regardless of the size of payments between users and developers (application prices).*

Proof. The platform chooses p^u, p^d, and x to maximize total profits, $m\, p^u + n\, p^d - \sigma\, x$. There is a unique pair of prices for each pair m, n, so finding the optimal m and n is equivalent to finding the optimal p^u and p^d. Replacing prices by inverse demand functions in the profit function we obtain $m\,(v - n\,\rho^* - h(m)) + n\,(m\,\rho^* - c(n)) - \sigma\, x$. Rearranging terms, profits can be rewritten as $m\, v - m\, h(m) - n\, c(n) - \sigma\, x$, which clearly does not depend on ρ^*. ■

The platform provider internalizes the effect of payments on m, n, and x, and chooses p^u and p^d to neutralize their effect. For Lemma 1 to hold, two conditions must be satisfied. First, the platform provider must be able to price both sides of the market. If the platform provider cannot price one side of the market (for example, if platform access cannot be verified for one side), then it will not be able to transfer utility from one side to the other. Second, the market must exhibit pure membership externalities.[7] As we show in Section 4, Lemma 1 does not hold for open platforms. Thus, while the nature of payments between users and developers does not matter for proprietary platforms, it does play a role for open platforms.

In the first stage, the platform provider chooses x, p^u, and p^d to maximize profits $m\, p^u + n\, p^d - \sigma\, x$. The following proposition characterizes the equilibrium.

Proposition 1 (Proprietary platform)**.** *An equilibrium exists and is unique. The measure of users and developers $(m,\ n)$, and the investment in platform quality (x), satisfy $v = h + m\, h_m$, $m\, v_n = c + n\, c_n$, and $m\, v_x = \sigma$. When applications*

[7]See Armstrong (2006) and Rochet and Tirole (2006) for details.

are substitutes, $\rho^* = \alpha\, v_n$*,* $p^u = m\, h_m - \alpha\, n\, v_n$*, and* $p^d = n\, c_n - (1-\alpha)\, m\, v_n$*. When applications are complements,* $\rho^* = \alpha\, w$*,* $p^u = m\, h_m - \alpha\, n\, w$*, and* $p^d = n\, c_n - m\,(v_n - \alpha\, w)$*.*

Proof. According to Lemma 1, the platform provider's problem is

$$\max_{m,n,x} \quad m\, v(n,x) - m\, h(m) - n\, c(n) - \sigma\, x.$$

The first order conditions with respect to m, n, and x are $v = h + m\, h_m$, $m\, v_n = c + n\, c_n$ and $m\, v_x = \sigma$. Assuming h_{mm} and c_{nn} are positive, or negative but not too large in absolute value, the second order conditions will hold, and there will be at least one local maximum. If there is more than one local maximum, the firm will choose the one with the largest profit (i.e. the global maximum).

Substituting the first two expressions in the inverse demand functions, we obtain the optimal access prices. There is a unique pair (p^u, p^d) for a given triple (m, n, x).

Finally, even though in the second stage users and developers may coordinate in different second-stage equilibria for a given pair of access prices (i.e. there may be more than one pair m, n solving $v = h + m\, h_m$ and $m\, v_n = c + n\, c_n$), only one combination m, n will be part of the Nash equilibrium of the complete game (the one corresponding to the optimal prices p^u, p^d), which is a condition for subgame perfect equilibrium. Thus, the equilibrium is unique. ■

The marginal user and developer obtain zero utility and profit in equilibrium. Therefore, the net utility of user $i < m$ in equilibrium is $u(i) = h(m) - h(i)$, and the profit of developer $j < n$ is $\pi(j) = c(n) - c(j)$.

The condition determining x in the proprietary platform is the same as that of the first best. Therefore, if m and n were set at their socially optimal levels, investment would be optimal. A proprietary platform sets access prices in order to capture the full increase in user surplus due to an increase in x, and thus has strong incentives to invest in product quality.

However, the conditions determining m and n are different from those of the first best, which means that x will be set at an inefficient level. Efficiency requires that the value of the platform is equal to the entry cost of the marginal user ($v = h$), and that the marginal benefit of the marginal application is equal to the entry cost of the marginal developer, ($m\, v_n = c$). The platform provider does not fully internalize the marginal benefits of increases in m and n, and thus sets prices that lead to insufficient entry.

Turning to the analysis of prices, the platform adjusts p^u and p^d so that users obtain gross utility v and developers obtain a net revenue of $m\,v_n$. Developers may be subsidized in equilibrium only when α is low, and thus the prices they obtain from applications are also low. When $\alpha = 1$, developers are not subsidized. Likewise, users may be subsidized when $\alpha > 0$ but are not subsidized when $\alpha = 0$. When $\alpha = 0$, gross utility is already equal to v and no further adjustments through p^u are needed.

Equilibrium access prices can be written using the formulas for price elasticity of demand. In particular, $\varepsilon^m_{p^u} = \frac{1}{m\,h_m}\,p^u$ is the price elasticity of user demand and $\varepsilon^n_{p^d} = \frac{1}{n\,c_n - m\,n\,\rho^*_n}\,p^d$ is the price elasticity of developer demand. As shown by the previous literature (e.g., Armstrong, 2006; Rochet and Tirole, 2003, 2006), equilibrium prices can be expressed as a combination of elasticities and indirect network effects:

$$\frac{p^u + n\,\rho^*}{p^u} = -\frac{1}{\varepsilon^m_{p^u}}\,, \quad \text{and} \quad \frac{p^d + m\,(v_n - \rho^* - n\,\rho^*_n)}{p^d} = -\frac{1}{\varepsilon^n_{p^d}}.$$

Here, we show that the technical characteristics of applications (whether applications are complements or substitutes) and the nature of payments between users and developers (application prices) affect indirect network effects (i.e. the structure of access prices depends on the ability of developers to charge users for the use of their applications).

4. Open platform

We now turn to the case of an open platform. In open platforms, investment in quality is decentralized; each developer chooses independently how much to invest in platform quality. By their very nature, open platforms have unstructured entry and investment. Therefore, m, n, and $x(j)$ are determined simultaneously in the first stage. Application prices, $\rho(j)$, are set in a second stage. Since access to the platform is free, equations (1) and (2) become:

$$\begin{aligned} u(i) &= v(n,x) - \int_0^n \rho(j)\,dj - h(i), \text{ and} \\ \pi(j) &= \rho(j)\,m - c(j) - \sigma\,x(j), \end{aligned}$$

where $x = \int_0^n x(j)\,dj$.

In open platforms, payments between users and developers matter because they affect their incentives to join the platform and invest in platform quality. Because application prices are determined differently when they are substitutes and complements, we study both cases separately. Proposition 2 summarizes the equilibrium choices of users and developers when applications are substitutes.

Proposition 2 (Open platform with substitute applications). *An equilibrium exists. In equilibrium, the measure of users and developers (m, n) and the investment in platform quality (x), satisfy $h = v - \alpha\, n\, v_n$, $c = \alpha\, m\, v_n$, and $\alpha\, m\, v_{nx} = \sigma$. Application prices are $\rho^* = \alpha\, v_n$.*

Proof. By the arguments brought forward in Section 3, application price is $\alpha\, v_n$. In the first stage, users and developers choose whether to enter the platform and developers choose how much to invest in platform quality. In choosing how much to invest, developers solve

$$\max_{x(j)} \alpha\, m\, v_n(n, x) - c(j) - \sigma\, x(j).$$

The first order conditions yield $\alpha\, m\, v_{nx} = \sigma$. The marginal user and developer obtain zero utility and profit. The marginal agents do not invest in platform innovation. Therefore, in equilibrium we must have $v - \alpha\, n\, v_n - h(m) = 0$, and $\alpha\, m\, v_n - c(n) = 0$. ■

In equilibrium, users obtain $u(i) = h(m) - h(i)$, and developers earn $\pi(j) = c(n) - c(j) - \sigma\, x(j)$. Because $h_i > 0$ and $c_j > 0$, larger equilibrium entry by users and/or developers implies more user utility and developer profit.

Open source does not force developers to invest in platform quality. If $\alpha\, m\, v_n > c(j)$, developer j will find it optimal to enter the platform. There will be entry until $\alpha\, m\, v_n = c$ and marginal entrants do not invest in platform quality.

There is an efficiency trade-off in the conditions determining m and n. As α increases, the condition determining n gets closer to the first best condition, but the condition determining m moves away from the first best. The condition determining m is the same as that of the first best only when $\alpha = 0$. Note however, that even in this case, m will be suboptimal because n is determined inefficiently. Likewise, the condition determining n is the same as that of the first best only when $\alpha = 1$, but n will be suboptimal because m is determined inefficiently.

In Lemma 2 we show that an increase in α may lead to an increase (decrease) in both m and n, due to the existence of indirect network effects. For example, an increase in α means that applications become more expensive, which lowers user utility. However, the positive effect of the increase in n on user utility may more than offset the negative effect of the increase in application prices, thereby leading to a higher m.

Lemma 2 (Barganing power and entry in open platforms)**.** *Suppose x is fixed. Then,*

$$\frac{dm}{d\alpha} = -\frac{n\,c_n - (1-\alpha)\,m\,v_n}{D} \qquad \textit{and} \qquad \frac{dn}{d\alpha} = \frac{m\,h_m - \alpha\,n\,v_n}{D},$$

where

$$D = \frac{c_n\,h_m}{v_n} - \alpha\left(\frac{m\,h_m\,v_{nn}}{v_n} + (1-\alpha)\,v_n - \alpha\,n\,v_{nn}\right).$$

Proof. The equilibrium conditions are $v - \alpha\,n\,v_n - h(m) = 0$ and $\alpha\,m\,v_n - c(n) = 0$. Holding x constant, the total differential with respect to α is

$$\begin{aligned} h_m\,\frac{dm}{d\alpha} + (v_n - \alpha\,v_n + \alpha\,n\,v_{nn})\frac{dn}{d\alpha} + n\,v_n &= 0, \\ \alpha\,v_n\,\frac{dm}{d\alpha} + (\alpha\,m\,v_{nn} - c_{nn})\,\frac{dn}{d\alpha} + m\,v_n &= 0, \end{aligned}$$

which yields a system of two equations and two unknowns. Solving this system, we obtain $dm/d\alpha$ and $dn/d\alpha$. ■

A sufficient condition for $D > 0$ is $v_{nnn} < 0$. To see this, note that the denominator can be rewritten as

$$D = \frac{(1-\alpha)\,c_n\,h_m}{v_n} + \alpha\left(\frac{h_m\,(c_n - m\,v_{nn}) - v_n^2}{v_n} + \alpha\,(v_n + n\,v_{nn})\right).$$

The first term is always positive. The second-order condition of the social planner problem implies that $h_m\,(c_n - m\,v_{nn}) - v_n^2 > 0$. We are left with $v_n + n\,v_{nn}$, which is positive when $v_{nnn} < 0$. Note that the denominator may be positive even if $v_{nnn} > 0$, as long as v_{nnn} is not too large in absolute value.

If the denominator is positive, then $dm/d\alpha > 0$ if and only if $n\,c_n - (1-\alpha)\,m\,v_n < 0$, and $dn/d\alpha > 0$ if and only if $m\,h_m - \alpha\,n\,v_n > 0$. Therefore, if a *proprietary* platform would choose to subsidize *developers*, the number of *users* would increase with α in an *open* platform. Otherwise, $dm/d\alpha < 0$. Likewise, if a proprietary platform would choose to subsidize *users*, the number of *developers* would decrease with α in an open platform. Otherwise, $dn/d\alpha > 0$. In addition, note that the effects depend on the level of α. Holding everything else equal, $dm/d\alpha > 0$ is more likely for low α and $dn/d\alpha < 0$ is more likely for high α.

Intuitively, when α is very low, developers' revenue from the sale of applications is low and there is little entry on the developer side, which hurts consumer utility and leads to low entry on the user side. An increase in α leads to more developer entry, which in turn benefits users and improves user entry. This is exactly the same case when a proprietary platform would choose to subsidize developers. A similar intuition applies to $dn/d\alpha < 0$ when $p^u < 0$.

Equilibrium aggregate investment, x, may result from a large number of possible distributions of developer investments, $x(j)$. As long as $\alpha\, m\, v_{nx} > \sigma$, any developer will find it optimal to increase its investment in platform quality. In equilibrium $\alpha\, m\, v_{nx} = \sigma$, regardless of *who* is investing. Developer incentives to invest are not socially optimal because developers do not fully internalize the effect of an increase in x in user utility.

We turn now to the case of complement applications. Proposition 3 summarizes the equilibrium choices of users and developers in this case.

Proposition 3 (Open platform with complement applications)**.** *An equilibrium exists. In equilibrium, the measure of users and developers* (m, n) *and the investment in platform quality* (x)*, satisfy* $h = v - \alpha\, n\, w$*,* $c = \alpha\, m\, w$*, and* $\alpha\, m\, w_x = \sigma$*. Application prices are* $\rho^* = \alpha\, w$*.*

Proof. The proof follows similar steps as the proof of Proposition 2, taking into account that the price of applications is now $\alpha\, w$. ■

Most of the intuitions developed above for substitutes also apply to complements. An important difference is that in the case of complements, the condition for n is inefficient even if $\alpha = 1$. The reason is that when applications are complements, revenues of developers depend on their applications' *average* contribution to consumer gross utility instead of their *marginal* contribution. Therefore, in the case of complements, even if m and x were set at their optimal levels, developer entry would be inefficient.

5. Comparison

In this section, we compare the equilibrium conditions determining m, n, and x for proprietary and open platforms. Table 1 presents a summary of our results. We compare entry and investment incentives analyzing one condition at a time, holding everything else constant. Obviously, all variables are jointly determined, thus there are interactions that we are not considering in a one-on-one comparison.

<table>
<tr><th></th><th>Substitutes</th><th>Complements</th></tr>
<tr><td>Welfare optimum</td><td colspan="2">$\begin{aligned} v &= h \\ m\, v_n &= c \\ m\, v_x &= \sigma \end{aligned}$</td></tr>
<tr><td>Proprietary platform</td><td colspan="2">$\begin{aligned} v &= h + m\, h_m \\ m\, v_n &= c + n\, c_n \\ m\, v_x &= \sigma \end{aligned}$</td></tr>
<tr><td>Open platform</td><td>$\begin{aligned} v - \alpha\, n\, v_n &= h \\ \alpha\, m\, v_n &= c \\ \alpha\, m\, v_{nx} &= \sigma \end{aligned}$</td><td>$\begin{aligned} v - \alpha\, n\, w &= h \\ \alpha\, m\, w &= c \\ \alpha\, m\, w_x &= \sigma \end{aligned}$</td></tr>
</table>

TABLE 1. Comparison of results

The first difference is that α does not play a role in proprietary platforms, but matters for open platforms. As noted, a proprietary platform provider internalizes the effect of α on user and developer entry, and chooses access prices so that the desired level of entry occurs regardless of the distribution of bargaining power between both sides of the market.

Turning to entry, we see that whether proprietary platforms provide higher or lower incentives than open platforms for user entry depends on the comparison between $m\, h_m$ and $\alpha\, n\, v_n$ in the substitutes case, and between $m\, h_m$ and $\alpha\, n\, w$ in the complements case. Recall that the equilibrium access price for users is $p^u = m\, h_m - \alpha\, n\, v_n$ (substitutes) and $p^u = m\, h_m - \alpha\, n\, w$ (complements). Thus, incentives for user entry are stronger in proprietary platforms compared to open platforms when users are subsidized in equilibrium ($p^u < 0$). If α is sufficiently close to zero, open platforms provide stronger incentives for user entry. However, even when $\alpha = 1$, we cannot guarantee that proprietary platforms provide stronger incentives for user entry.

A similar comparison can be made for the relative strength of incentives for developer entry. Proprietary platforms provide stronger incentives when $n\, c_n$ is less than $(1 - \alpha)\, m\, v_n$ (substitutes) or $(1 - \alpha)\, m\, w$ (complements), which is the same condition determining whether developers are subsidized by the proprietary platform provider.

Finally, we compare equilibrium investment. The condition that determines quality investment in open platforms is $\alpha\, m\, v_{nx} = \sigma$ (substitutes) or $\alpha\, m\, w_x = \sigma$ (complements). For proprietary platforms, the condition is $m\, v_x = \sigma$. Holding everything else constant (i.e., taking m and n as given), equilibrium investment in an open platform is lower than in a proprietary platform, even when $\alpha = 1$. In the case of complements, this follows from $w_x < v_x$ which always holds. In the case of substitutes, even though v_{nx} could be larger than v_x from a mathematical point of view, it is only reasonable to assume that $v_{nx} < v_x$. To understand why, note that if the model had a discrete number of developers, v_{nx} would be defined as $v_x(n, x) - v_x(n - 1, x)$, which is always smaller than $v_x(n, x)$.

In any case, investment may be larger in an open platform compared to a proprietary one. The reason is that m, n, and x are determined jointly. In particular, n may be larger when the platform is open—which could lead to stronger incentives to invest in the open platform—as long as v_{nnx} is positive and sufficiently large. Similarly, an open platform may lead to a larger m, which also improves investment incentives. Indeed, in Section 5.1 we provide an example in which an open platform has larger quality investment than a proprietary platform.

5.1. **Example.** The following example illustrates that investment in platform quality may be larger when a platform is open. The example also demonstrates that a proprietary platform may wind up encouraging more user and developer entry than an open platform.

Let $v(x, n) = x^a\, n^b$, where $0 < a < 1$ and $0 < b < 1$. The assumption $b < 1$ implies that applications are substitutes. We also assume that $2\,a + b < 1$, which guarantees that the second order conditions for profit maximization are satisfied. Investment in platform quality and the measure of applications are complements, $v_{nx} > 0$. Finally, let $h(i) = i$, $c(j) = j$, and $\sigma = 1$.

Using the equations in Table 1, we derive equilibrium adoption and investment. The social planner's solution is

$$m^s = \left(a^a\, b^{-\frac{b}{2}}\right)^{\frac{1}{1-2\,a-b}}, \qquad n^s = \left(a^a\, b^{\frac{2\,a-1}{2}}\right)^{\frac{1}{1-2\,a-b}}, \qquad x^s = \left(a^{1-b}\, b^b\right)^{\frac{1}{1-2\,a-b}}.$$

For the proprietary platform, the equations are

$$m^p = \frac{m^s}{2^{\frac{1-a}{1-2\,a-b}}}, \qquad n^p = \frac{n^s}{2^{\frac{1-a}{1-2\,a-b}}}, \qquad x^p = \frac{x^s}{2^{\frac{1-a}{1-2\,a-b}}}.$$

Finally, for the open platform, we have

$$m^o = \left(a^a\,(\alpha\,b)^{\frac{a+b}{2}}\,(1-\alpha\,b)^{\frac{a+b-2}{2}}\right)^{\frac{1}{1-a-b}}, \quad n^o = \left(a^a\,(\alpha\,b\,(1-\alpha\,b))^{\frac{1}{2}}\right)^{\frac{1}{1-a-b}},$$

$$x^o = \left(a^{1-b}\,(\alpha\,b\,(1-\alpha\,b))^{\frac{1}{2}}\right)^{\frac{1}{1-a-b}}.$$

Due to the non-linearity of the equilibrium equations, it is not possible to find an explicit solution for the parameter values that lead to $x^o > x^p$. Figure 1 shows the region of parameters for which $x^o > x^p$ in our example. For a given α, combinations of a and b to the northeast of the depicted frontiers have $x^o > x^p$. As α increases, the area of parameters for which $x^o > x^p$ becomes larger.

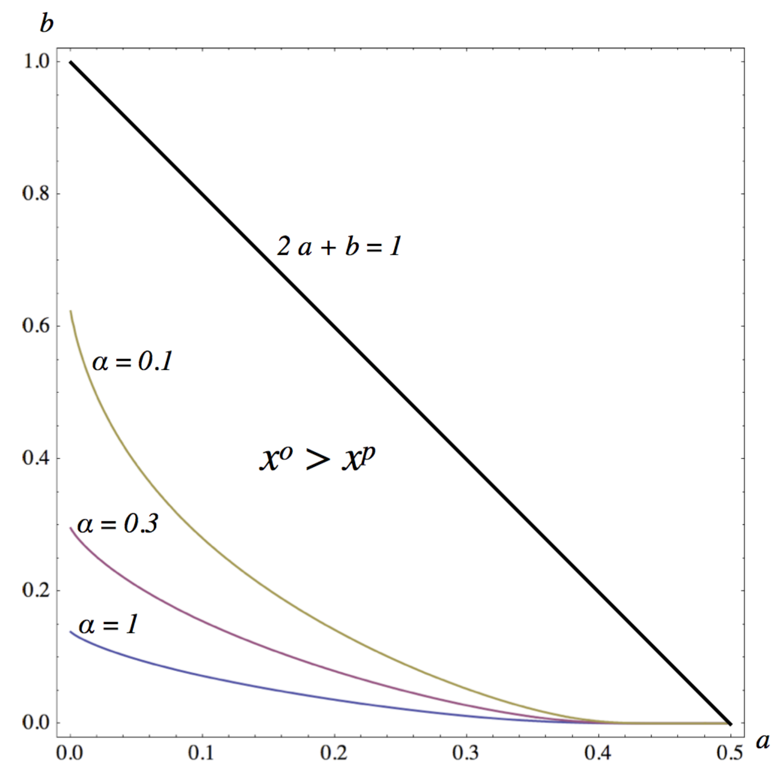

Figure 1. Parameter values for which $x^o > x^p$

6. Duopoly

In this section, we extend the model to analyze competition between a proprietary and an open platform. For concreteness, we will focus on the case of substitute applications, but similar results hold for the case of complements.

We show that access prices may increase or decrease when taking into account investment incentives compared to a situation where there are no investments in platform quality. The effect of investment incentives on user and developer access prices may have the same or opposite signs. Specifically, the effect of investments in quality in the open platform on the structure of access prices depends on (1) how changes in the number of users and developers in the proprietary platform affect investments in the open platform, and (2) how investments in the open platform affect the revenues of the proprietary platform. We also show that when developers multi-home, the proprietary platform may benefit from higher quality investment in the open platform.

We model the mixed duopoly as follows. There is one unit mass of single-homing users, indexed by $i \in [0,1]$. User i's utility of consuming n applications in a proprietary and an open platform are

$$
\begin{aligned}
u^p(i) &= v(n^p, x^p) - \int_0^{n^p} \rho^p(j)\,dj - p^u - h^p(i), \text{ and} \\
u^o(i) &= v(n^o, x^o) - \int_0^{n^o} \rho^o(j)\,dj - h^o(i),
\end{aligned}
$$

where superscripts p and o indicate whether the variable or function refers to the proprietary or to the open platform.

Access to the open platform is free. To guarantee that the market is covered, we assume that $\min_i h^p(i)$ and $\min_i h^o(i)$ are sufficiently low. The optimal choice of platform by users depends on $h(i) = h^p(i) - h^o(i)$, which measures the difference in the cost of learning how to use the proprietary vs. the open platform. Assume $h_i > 0$, with $\lim_{i\to 0} h(i) = -\infty$ and $\lim_{i\to 1} h(i) = \infty$. Let m indicate the indifferent user. Then, m is the measure of users choosing the proprietary platform, and $1-m$ is the measure of users choosing the open platform.

Developers multi-home. Thus we assume that it is inexpensive to adapt applications to run on both platforms. Even though the measure of applications is the same for both platforms, equilibrium application prices may differ across platforms because they depend on platform quality investments.

The timing is as follows: (i) the proprietary platform chooses p^u, p^d, and x^p; (ii) users choose which platform to join, and developers decide whether to develop an application and choose $x^o(j)$; and (iii) users and developers bargain over application prices $\rho^p(j)$ and $\rho^o(j)$. The timing reflects the fact that proprietary platforms are developed before they become accessible to users and developers, but that adoption and development are contemporaneous in open platforms. The equilibrium concept is subgame perfection, and we solve the model backward.

6.1. **Equilibrium entry by users and developers.** In the third stage, users and developers bargain over application prices. The price of applications running on the proprietary platform is $\alpha\, v_n^p$, and the price of applications running on the open platform is $\alpha\, v_n^o$.

In the second stage, the marginal user and developer satisfy $h(m) = v^p - v^o - n\,\alpha\,(v_n^p - v_n^o) - p^u$ and $c(n) = \alpha\,(m\, v_n^p + (1-m)\, v_n^o) - p^d$. The inverse demands are

$$p^u = v^p - v^o - n\,\alpha\,(v_n^p - v_n^o) - h, \tag{5}$$

$$p^d = \alpha\,(m\, v_n^p + (1-m)\, v_n^o) - c, \tag{6}$$

and the optimal investment in the open platform by developers is

$$\alpha\,(1-m)\, v_{nx}^o = \sigma. \tag{7}$$

In the first stage, the platform provider chooses p^u, p^d, and x^p to maximize profits, taking into account that the second-stage equilibrium levels of m, n, and x^o are functions of p^u, p^d, and x^p. We now turn to examining these choices.

6.2. **Pricing and investment.** To better understand the equilibrium choices in the first stage, it is helpful to study a generalization of the model which we later specialize to the functional forms in equations (5) to (7). In particular, let

$$\begin{aligned} m &= M\,(p^u, n, x^o, x^p), \\ n &= N\,(p^d, m, x^o, x^p), \quad \text{and} \\ x^o &= X\,(m, n). \end{aligned}$$

These equations show that changes in prices and investment have direct and indirect effects. For example, a change in p^u affects m directly, but it also affects n and x^o indirectly through m, and so on. In the first stage, the proprietary platform solves

$$\max_{p^u, p^d, x^p} p^u\, m(p^u, p^d, x^p) + p^d\, n(p^u, p^d, x^p) - \sigma\, x^p,$$

where $m(p^u, p^d, x^p)$ and $n(p^u, p^d, x^p)$ are the measure of users and developers arising from the equilibrium of the second stage. Proposition 4 characterizes the equilibrium.

Proposition 4 (Duopoly pricing and investment)**.** *Equilibrium prices are*

$$\begin{aligned} p^u &= -(1-M_{x^o}X_m)\,\frac{m}{M_{p^u}}+(N_m+N_{x^o}X_m)\,\frac{n}{N_{p^d}}, \\ p^d &= (M_n+M_{x^o}X_n)\,\frac{m}{M_{p^u}}-(1-N_{x^o}X_n)\,\frac{n}{N_{p^d}}, \end{aligned}$$

and investment solves

$$-M_{x^p}\,\frac{m}{M_{p^u}}-N_{x^p}\,\frac{n}{N_{p^d}}=\sigma.$$

Proof. The first order conditions are $m+p^u\,\frac{dm}{dp^u}+p^d\,\frac{dn}{dp^u}=0$, $p^u\,\frac{dm}{dp^d}+n+p^d\,\frac{dn}{dp^d}=0$ and $p^u\,\frac{dm}{dx^p}+p^d\,\frac{dn}{dx^p}-\sigma=0$. The optimal choices depend on the derivatives of $m(p^u,p^d,x^p)$ and $n(p^u,p^d,x^p)$ with respect to p^u, p^d, and x^p. We will show how to obtain dm/dp^u (the other derivatives are obtained similarly). The total differentials of equations M, N, and X with respect to p^u are

$$\begin{aligned} \frac{dm}{dp^u} &= M_{p^u}+\frac{dn}{dp^u}\,M_n+\frac{dx^o}{dp^u}\,M_{x^o}, \\ \frac{dn}{dp^u} &= \frac{dm}{dp^u}\,N_m+\frac{dx^o}{dp^u}\,N_{x^o}, \quad \text{and} \\ \frac{dx^o}{dp^u} &= \frac{dm}{dp^u}\,X_m+\frac{dn}{dp^u}\,X_n, \end{aligned}$$

which constitutes a system of three equations with three unknowns. Solving for dm/dp^u, we obtain

$$\frac{dm}{dp^u}=\frac{1-N_{x^o}X_n}{1-X_n\,N_{x^o}-(X_m+N_mX_n)\,M_{x^o}-(N_m+N_{x^o}\,X_m)\,M_n}\,M_{p^u}.$$

Introducing the derivatives in the first order conditions for the proprietary platform and solving for p^u, p^d, and x^p, we obtain the result stated in the proposition. ■

Proposition 4 shows that access prices are affected by investment incentives in the open platform. The proprietary platform provider takes into account that her decisions affect the incentives to invest in the open platform, which in turn affect platform membership decisions, and adjusts access prices to reflect this effect.

For example, consider p^u. If x^o was fixed (so that $X_m=0$ and $X_n=0$), we would have

$$p^u=-\frac{m}{M_{p^u}}+N_m\,\frac{n}{N_{p^d}}.$$

Allowing for changes in x^o, we have

$$p^u=-\frac{m}{M_{p^u}}+N_m\,\frac{n}{N_{p^d}}-\left(-\frac{M_{x^o}}{M_{p^u}}\,m-\frac{N_{x^o}}{N_{p^d}}\,n\right)X_m. \tag{8}$$

The new term in (8) measures the indirect effect of a change in m on profits as it operates through x^o. The expression inside the parenthesis in the second term of the right hand side of (8) measures the change in revenues on the user and developer side caused by a change in x^o.[8] X_m measures the change in x^o caused by a change in m.

In words, a higher access price for users leads to fewer users in the proprietary platform (i.e., it leads to more users joining the open platform). More users in the open platform lead to more open investment, which in turn leads to a higher quality open platform, affecting platform membership decisions (m and n) and proprietary platform's revenues. The proprietary platform provider takes these effects into account and adjusts prices accordingly.

The changes in p^u and p^d when taking into account investment incentives in the open platform, may go in the same or opposite direction. For example, suppose that $X_m < 0$ (an increase in m leads to a decrease in x^o), $X_n > 0$ (an increase in n leads to an increase in x^o), and $-\frac{M_{x^o}}{M_{p^u}}\, m - \frac{N_{x^o}}{N_{p^d}}\, n < 0$ (an increase in x^o lowers the proprietary platform's revenue). Then, the proprietary platform provider will set a higher p^u and a lower p^d, in comparison with the case of no investment. If $X_n < 0$, on the other hand, the proprietary platform provider will set a higher p^u and a lower p^d, in comparison with the case of no investment.

The equations in Proposition 4 can be written as modified Lerner equations:

$$\frac{p^u + a_{mn}}{p^u} = -(1 - b_m)\, \varepsilon^m_{p_u} \qquad \text{and} \qquad \frac{p^d + a_{nm}}{p^d} = -(1 - b_n)\, \varepsilon^n_{p_d},$$

where a_{mn} is the externality that users impose on developers, a_{nm} is the externality that developers impose on users, and b_m and b_n are the indirect effects of changes in m and n operating through x^o.

Note finally that the simple expression determining x^p coincides with the formula that we would have if there was no interaction with the open platforms.

[8]For instance, $-\frac{M_{x^o}}{M_{p^u}}$ measures the change in p^u caused by a change in x^o, holding m and n constant. Multiplying this ratio by m, we obtain the change in revenues from the user side caused by a change in x^o.

We now specialize Proposition 4 to the functional forms introduced above. Equilibrium prices are

$$\begin{aligned}
p^u &= m\,h_m - \alpha\,n\,(v_n^p - v_n^o) + (m\,v_x^o - \alpha\,n\;v_{nx}^o)\,\frac{v_{nx}^o}{(1-m)\,v_{nxx}^o}, \quad \text{and} \\
p^d &= n\,(c_n - \alpha\,v_{nn}^o) - (1-\alpha)\,m\,(v_n^p - v_n^o) - (m\,v_x^o - \alpha\,n\;v_{nx}^o)\,\frac{v_{nnx}^o}{v_{nxx}^o}.
\end{aligned}$$

Substituting in (5) and (6), we obtain equations determining the optimal measure of users and developers:

$$\begin{aligned}
h + m\,h_m &= (v^p - v^o) - (m\,v_x^o - \alpha\,n\;v_{nx}^o)\,\frac{v_{nx}^o}{(1-m)\,v_{nxx}^o}, \quad \text{and} \\
c + n\,c_n &= m\,(v_n^p - v_n^o) + \alpha\,(v_n^o + n\,v_{nn}^o) + (m\,v_x^o - \alpha\,n\;v_{nx}^o)\,\frac{v_{nnx}^o}{v_{nxx}^o}.
\end{aligned}$$

To understand these equations, suppose first that investment in the open platform was fixed ($X_m = 0, X_n = 0$). Then, we would have:

$$\begin{aligned}
h + m\,h_m &= (v^p - v^o), \quad \text{and} \\
c + n\,c_n &= m\,(v_n^p - v_n^o) + \alpha\,(v_n^o + n\,v_{nn}^o).
\end{aligned}$$

In contrast to the case of a monopolist proprietary platform, α matters in determining the equilibrium number of developers, even in the absence of investment in the open platform. Access on the developer side depends partly on the revenues that developers obtain on the open platform, which in turn depend on α.

Next, we interpret the terms related to investment in the open platform (x^o). In short, these capture the effects on profits due to the interactions between m and x^o and between n and x^o. It is straightforward to show that

$$\begin{aligned}
X_m &= \frac{v_{nx}^o}{(1-m)\,v_{nxx}^o}, \\
X_n &= -\frac{v_{nnx}^o}{v_{nxx}^o}, \quad \text{and} \\
\frac{M_{x^o}}{M_{p^u}}\,m + \frac{N_{x^o}}{N_{p^d}}\,n &= m\,v_x^o - \alpha\,n\,v_{nx}^o.
\end{aligned}$$

From these expressions, we can see that X_m is always negative: an increase in m decreases the market share of the open platform, and therefore lowers the incentives to invest in it. On the other hand, X_n may be positive or negative depending on the sign of v_{nnx}^o which is positive (negative) when n and x act as complements (substitutes) on application price $\rho^o = v_n^o$. If $v_{nnx}^o > 0$, then X_n is positive: an increase in n leads to a higher v^o_{nx}, and thus leads to more incentives to invest. The opposite is true when $v^o_{nnx} < 0$.

Finally, as we showed in our discussion of the general case, $-\frac{M_{x^o}}{M_{p^u}}\, m - \frac{N_{x^o}}{N_{p^d}}\, n$ measures the change in profits due to a change in x^o. This expression may also be positive or negative. First, note that $\frac{M_{x^o}}{M_{p^u}} = v^o_x - \alpha\, n\, v^o_{nx}$ may be positive or negative: on the one hand, an increase in x^o raises the quality of the open platform which lowers user demand for the proprietary platform, but it also leads to higher application prices for the open platform, which has the opposite effect. If $\frac{M_{x^o}}{M_{p^u}} > 0$, an increase in the quality of the open platform leads to a higher demand for the proprietary platform, i.e. to higher profits. Second, consider $\frac{N_{x^o}}{N_{p^d}} = -\alpha\,(1-m)\, v^o_{nx}$. This expression is always negative: an increase in the quality of the open platform leads to higher entry by developers, and therefore to higher profits for the proprietary platform. This seemingly counter-intuitive result is due to multi-homing. The proprietary platform provider gains more on the developer side when there is more entry, and entry is partly determined by developers' revenue in the open platform. If developers single-home, the sign of this expression will be negative.

Finally, the equation determining x^p is $m\, v^p_x = \sigma$, from which we obtain the following ratio in equilibrium:

$$\alpha\, \frac{1-m}{m} = \frac{v^p_n}{v^o_{nx}}.$$

Therefore, equilibrium investment in the proprietary platform increases relative to the investment in the open platform as the equilibrium market share of the proprietary platform increases, and as the bargaining power of developers decreases.

Summarizing, we find that the effect of investment in the open platform on the structure of access prices (p^u and p^d) depends on (1) the sign of the effect of changes in x^o on the revenues of the proprietary platform, and (2) the sign of the effect of changes in m and n on x^o. Under multi-homing, the proprietary platform provider may benefit from higher investment in the open platform. Access prices may increase or decrease when taking into account investment incentives. Finally, the effect of investment incentives on p^u and p^d may have the same or opposite signs.

7. Conclusion

We have examined a model of a proprietary and an open-source two-sided platform to study equilibrium investment in quality. The analysis has provided answers to three important questions that had not been tackled before in the literature: (i) how are the incentives to invest in platform quality affected by the degree of platform openness? (ii) which of these two modes of governance leads to investment closer to the social optimum? and (iii) how are incentives to invest in platform quality moderated by competition between proprietary and open two-sided platforms?

To the first question, our answer is twofold. First, while the nature of cross payments between users and developers plays a role in determining user and developer investments in open platforms (in the sense that the larger the bargaining power of agents on one side of the platform, the more that side is willing to invest), these play no role in monopoly proprietary platforms. Through access prices, a proprietary platform ensures that a particular level of investment takes place, regardless of how much users pay for applications. Second and relatedly, the effect of investments in quality on application prices plays an important role in determining the equilibrium efforts to innovate in open platforms. The stronger the effect, the less users (and the more developers) will want to invest.

We also present simple rules to determine the effects of a change in the bargaining power of developers *vis-à-vis* users on the number of users and developers in open platforms. In particular, an increase in the bargaining power of developers will lead to higher user adoption (and higher consumer surplus) in an open platform if developers would be subsidized if the platform was proprietary. Likewise, a decrease in the bargaining power of developers will lead to higher developer adoption (and higher developer profits) if users would be subsidized if the platform was proprietary.

To the question of social optimality, we find that a proprietary platform would invest efficiently if adoption by users and developers was efficient. Lower than efficient entry, however, implies that investment is always lower than what a social planner would choose. Free riding implies that investment is always socially suboptimal in open platforms. Nonetheless, investment may be larger than in the case of proprietary platforms due to larger entry. Therefore, open platforms may lead to investments in platform quality closer to social efficiency.

Finally, to the question of incentives to invest and competition, we find that a proprietary platform cannot fully internalize the effects of payments on entry and investment when in competition for users against an open platform. As a consequence, the extent to which users have bargaining power affects equilibrium entry. This is because when developers multi-home, entry depends partially on the revenues they obtain from selling applications in the open platform, which depend on their bargaining power. Likewise, equilibrium investment in the proprietary platform increases relative to investment in the open platform as developers' bargaining power in the application market decreases.

We hope to have provided a solid first step to better understand incentives to invest in proprietary and open platforms. An obvious next step is to extend the model to study mixed modes of governance (e.g., a platform open to one side only or an open platform with coordinated investment by developers). Having presented a thorough analysis of two extreme modes of governance, we leave these extensions for future work.

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