Reactive Relay Selection in Underlay Cognitive Networks with Fixed Gain Relays

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***Abstract*—Best relay selection is a bandwidth efficient technique for multiple relay environments without compromising the system performance. The problem of relay selection is more challenging in underlay cognitive networks due to strict interference constraints to the primary users. Generally, relay selection is done on the basis of maximum end-to-end signal to noise ratio (SNR). However, it requires large amounts of channel state information (CSI) at different network nodes. In this paper, we present and analyze a reactive relay selection scheme in underlay cognitive networks where the relays are operating with fixed gains near a primary user. The system model minimizes the amount of CSI required at different nodes and the destination selects the best relay on the basis of maximum relay to destination SNR. We derive close form expressions for the received SNR statistics, outage probability, bit error probability and average channel capacity of the system. Simulation results are also presented to confirm the validity of the derived expressions.**

I. Introduction

The fact that the licensed spectrum is used at a fraction of its full capacity helped maturing the idea of cognitive radio in which non-licensed or secondary users can opportunistically use the spectrum dedicated for the licensed or primary users [1], [2]. Several approaches have been suggested in the literature to access and share the primary spectrum with the secondary users. Most commonly, secondary users sense and detect the unused primary spectrum or a part of it, known as spectrum hole, and use it until the primary user becomes active [1]. This is known as interweave approach and does not allow simultaneous in-band existence of both the users. In overlay approach, secondary users can access the spectrum being used by the primary user; however, the secondary users should implement some interference avoidance techniques to guarantee that the primary transmission is not affected. Similarly, in underlay approach, both the primary and secondary users can exist in the same band simultaneously but the secondary users must strictly meet the interference threshold at the primary user. This threshold limits the transmission power of the secondary users and eventually their area of coverage. A well known technique to reach distant users in case of limited or weak coverage is cooperation through relays. Therefore, it is highly expected that the nodes in underlay cognitive networks would make use of cooperative relaying.

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The main purpose of cooperative relaying is to improve the diversity order of the received signal at the destination. Higher diversity orders can be achieved by seeking help from multiple relays [3]. The use of multiple relays is spectrally inefficient as, in most cases, they need to transmit on orthogonal channels. However, this could be overcome by selecting the best relay from the available ones based on a certain criterion [4]. Selective relaying is a well investigated topic in non-cognitive networks [4], [5]. The most common criterion for selecting the best relay in these networks is the end-to-end signal to noise ratio (SNR) offered by a relay. However, the selective relaying defines an entirely different problem in underlay cognitive settings due to the stringent interference thresholds.

Recently, few papers studied selective relaying in cognitive networks. A relay selection and channel allocation method is discussed in [6]; however, it considers interweaved approach and can not be adopted in underlay mode. A similar relay selection and power allocation scheme with limited interference to the primary users is proposed in [7] which is more suited to underlay settings. In a multi-hop network, the same problem can be defined as hop selection and a related scheme is proposed in [8] which involves power control as well to co-exist with the primary users. A modified relay selection criterion is proposed in [9] which takes into account the interference constraint and the relays in the network are assumed to be operating in decode-and-forward (DF) mode. Another relay selection criterion is proposed in [10] which selects the best relay under the constraint of satisfying a required outage probability of the primary network. The outage probability of the secondary network is derived where the relays are operating in DF mode. A common denominator in all these papers is that the secondary nodes are assumed to adapt their transmission power in order to always satisfy the interference constraint in underlay settings. However, this may not be the case in every network and the secondary nodes may have fixed transmission power. In this situation, depending upon their locations and channel conditions, secondary nodes may violate the interference constraint. A relay selection scheme in an underlay cognitive network with fixed transmission power nodes operating near a primary user is analyzed in [11]. This scheme first excludes the relays from the selection process which do not satisfy the interference constraint and then selects the best relay based on maximum end-to-end SNR. The relays need instantaneous channel state information (CSI) of the source to relay links to adjust their amplification factors in order to generate fixed output power. Also, the decision about the best relay is taken at the destination requiring the global CSI knowledge which may not be practical in some cases.

In order to ease the CSI burden, we consider a system where the relays operate in amplify-and-forward (AF) mode with fixed gains. Hence, they do not need to know the source to relay channel, but the output power of each relay is different depending upon the strength of the received signal. The destination performs reactive selection, i.e., selects the best relay based on the maximum relay to destination SNR which requires limited CSI at the destination.

II. System Model

Our system model is comprised of a secondary source S which is transmitting its signal to a secondary destination D with the help of L secondary relays represented by $R_i, i = 1, 2, \cdots, L$, in Fig. 1. This whole network is operating in underlay mode near a primary user P. A traditional two time slot communication procedure is followed in AF mode with fixed gain relays. The source S with transmission power E_s broadcasts its signal in the first time slot. This signal is received by the destination, all the relays and the primary user with channel gains h_0, h_{1i} and h_{SP}, respectively. We assume that each relay is aware of the interference channel h_{iP} from itself to the primary user. The relays can gather this information either when the primary user is transmitting or when it is acknowledging any received signal. We also assume that each hop in the system, either communication or interference link, is subjected to additive white Gaussian noise (AWGN) with zero mean and variance N_0.

In AF mode, relays can amplify the received signal either by using CSI based gain or fixed gain. Using CSI, each relay sets its amplification factor to $g_i = \sqrt{\frac{E_r}{E_s|h_{1i}|^2+N_0}}$ to reciprocate the first hop's channel fading and transmit at a fixed output power E_r. As this needs instantaneous knowledge of CSI, we assume that each relay amplifies the received signal with a fixed gain g without knowing the CSI. Due to the fixed gains at the relays, each of them transmits at a different power depending upon the first hop's channel gain. From the i^{th} relay to the destination, the channel gain is h_{2i} which is known to the destination.

Underlay cognitive networks are required to operate under stringent interference limits which guarantees that the primary network is not affected by the secondary communication. Let λ be the interference threshold; however, for some relays the interference channel may be strong enough that they would not satisfy this threshold. We assume that such relays send a single bit "yes or no" decision about satisfying the interference threshold to the destination on a dedicated feedback channel. Therefore, the destination excludes such relays from the group it is going to pick the best relays, no matter what SNR they could provide over the secondary relay link.

Let us assume that ℓ relays out of L satisfy the interference threshold. So, we define a set $\mathcal{U}$ which contains the indexes of all the relays, another set $\mathcal{A} \subseteq \mathcal{U}$ which contains the indexes of the relays satisfying interference threshold whereas $\mathcal{B} = \mathcal{U} - \mathcal{A}$ contains the remaining indexes.

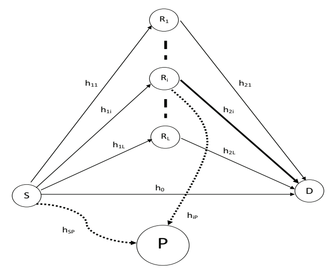

Fig. 1. System Model: A multi-relay cognitive network near a primary user.

Relay selection takes place in the second time slot and the destination chooses the best one among ℓ relays based on maximum relay to destination SNR. The chosen best relay then forwards the source's message to the destination in AF mode with fixed gain. This transmission is shown by a bold arrow in Fig. 1. We assume that all the channels are Rayleigh distributed and therefore their squared amplitudes have exponential distributions.

The end-to-end SNR of the i^{th} relayed link (secondary SNR) can be given as

$$\gamma_i = \frac{\gamma_{1i}\gamma_{2i}}{C + \gamma_{2i}}, \tag{1}$$

where $\gamma_{1i} = \frac{E_s|h_{1i}|^2}{N_0}$ is the SNR of the first hop, $\gamma_{2i} = \frac{E_s|h_{2i}|^2}{N_0}$ is the SNR of the second hop and $C = \frac{E_s}{g^2 N_0}$.

The interference from the source and each relay to the primary user over the two time slots can be given, respectively, as

$$\mathcal{I}_{SP} = E_s|h_{SP}|^2 \quad \text{and} \quad \mathcal{I}_{iP} = E_s g^2 |h_{1i}|^2 |h_{iP}|^2. \tag{2}$$

III. Reactive Relay Selection Scheme

Based on the above discussion, the reactive relay selection scheme can be described mathematically as

$$\hat{i} = \max_{i \in \mathcal{A}} (\gamma_{2i}) \qquad \text{such that} \qquad \mathcal{I}_{SP}, \mathcal{I}_{iP} < \lambda, \tag{3}$$

where $\hat{i}$ is the index of the selected best relay.

It is important to note that if $\mathcal{I}_{SP} > \lambda$, the source would refrain from starting the transmission and the proposed scheme could not be analyzed. Since $\mathcal{I}_{SP}$ is exponentially distributed, the probability of this event is $e^{-\frac{\lambda}{\sigma_{SP}}}$, where σ_{SP} is the average strength of the source to primary interference channel. Hence, from the analysis point of view, we assume a situation when $\mathcal{I}_{SP} < \lambda$. To simplify the analysis and reach some more insightful results, we can assume that the average SNRs of each $S \to R_i$ and $R_i \to D$ links are α and β, respectively. Similarly, the average strengths of $R_i \to P$ links are σ.

Now, the probability density function (PDF) of the selected link ($R_{\hat{i}} \to D$) SNR $\gamma_s = \max_{i \in \mathcal{A}} \gamma_{2i}$ among the ℓ relays satisfying the interference constraint is given by

$$p_{\gamma_s}(\gamma|\ell) = \frac{\ell}{\beta} e^{-\frac{\gamma}{\beta}} \left(1 - e^{-\frac{\gamma}{\beta}}\right)^{\ell-1}. \tag{4}$$

It is worthy to note that the value of ℓ may vary from 0 to L. There exists a non-zero probability that none of the relays could satisfy the interference constraint resulting in $\ell = 0$ and the system would operate on the direct link only. A choice in relay selection becomes available when $\ell \geq 2$ whereas maximal ratio combining (MRC) is possible at the destination for $\ell \geq 1$. As given in (2), $\mathcal{I}_{iP}$ is a product of two independent and non-identically distributed exponential random variables whose cumulative distribution function (CDF) and PDF can be given, respectively, as

$$P_{\mathcal{I}_{iP}}(y) = 1 - 2\sqrt{\frac{y}{\alpha\sigma}} K_1\Big(2\sqrt{\frac{y}{\alpha\sigma}}\Big) \text{ and } p_{\mathcal{I}_{iP}}(y) = \frac{2}{\alpha\sigma} K_0\Big(2\sqrt{\frac{y}{\alpha\sigma}}\Big), \tag{5}$$

where where $K_1(\cdot)$ and $K_0(\cdot)$ are the first and zero-order modified Bessel function of the second kind, respectively.

Using the above, we can find out the probability P_λ of satisfying the interference constraint by each relay. Hence, a relay can become a candidate for the best one with probability P_λ and could be excluded from the selection process with a probability $\bar{P}_\lambda = 1 - P_\lambda$. With these probabilities, finding ℓ relays in the selection pool out of L suggests a binomial distribution as follows

$$p_\ell(\ell; L, P_\lambda) = \binom{L}{\ell} P_\lambda^\ell \bar{P}_\lambda^{L-\ell}, \tag{6}$$

where $\binom{L}{\ell} = \frac{L!}{\ell!(L-\ell)!}$.

Now, the unconditional but truncated PDF of γ_s can be found by averaging (4) over (6), as given below

$$p_{\gamma_s}(\gamma) = \sum_{\ell=1}^{L} \binom{L}{\ell} \frac{\ell P_\lambda^\ell \bar{P}_\lambda^{L-\ell}}{\beta}\, e^{-\frac{\gamma}{\beta}} \big(1 - e^{-\frac{\gamma}{\beta}}\big)^{\ell-1}. \tag{7}$$

The above is a truncated PDF because $\ell = 0$ is not considered; hence, excluding the probability that the system is operating on the direct link only. This probability will be included in the later derivation. To find out the PDF of the SNR at the destination using the proposed reactive selection scheme, we begin with the outage probability of the relay links. It is defined as the probability of having the received SNR below a certain threshold γ_{th}.

$$P_{out}^{\hat{i}} = Pr\big[\gamma_{\hat{i}} < \gamma_{th}\big] = Pr\Big[\frac{\gamma_{1i}\gamma_s}{C + \gamma_s} < \gamma_{th}\Big], \tag{8}$$

where $\gamma_{\hat{i}}$ is the secondary SNR at the destination through the selected best relay and $P_{out}^{\hat{i}}$ is the outage due to the relay links only without considering the direct link.

For a given γ_s, (8) can be evaluated over the PDF of γ_s in (7) as in [12]

$$P_{out}^{\hat{i}} = \int_0^\infty Pr\Big[\frac{\gamma_{1i}\gamma_s}{C + \gamma_s} < \gamma_{th}|\gamma_s\Big] p_{\gamma_s}(\gamma) d\gamma = \sum_{\ell=1}^{L} \binom{L}{\ell} \frac{\ell P_\lambda^\ell \bar{P}_\lambda^{L-\ell}}{\beta} \times \int_0^\infty \big[1 - e^{-\frac{\gamma_{th}}{\alpha}(1+\frac{C}{\gamma})}\big] e^{-\frac{\gamma}{\beta}} \big(1 - e^{-\frac{\gamma}{\beta}}\big)^{\ell-1} d\gamma. \tag{9}$$

Now, using [13, Eqs. (3.312.1), (8.384.1) and (3.324.1)] and binomial expansion in the above, we get

$$P_{out}^{\hat{i}} = (1 - \bar{P}_\lambda^L) - 2\sum_{\ell=1}^{L} f(\ell, \lambda)\, e^{-\frac{\gamma_{th}}{\alpha}} \sum_{n=0}^{\ell-1} \binom{\ell-1}{n} \frac{(-1)^n}{n+1} \times \sqrt{\frac{\gamma_{th} C(n+1)}{\alpha\beta}}\, K_1\left(2\sqrt{\frac{\gamma_{th} C(n+1)}{\alpha\beta}}\right), \tag{10}$$

where $f(\ell, \lambda) = \binom{L}{\ell} \ell P_\lambda^\ell \bar{P}_\lambda^{L-\ell}$.

The PDF of $\gamma_{\hat{i}}$ can be obtained by differentiating (10) with respect to γ_{th} and some straightforward manipulations using [13, Eq. (8.486.12)] as follows

$$p_{\gamma_{\hat{i}}}(\gamma) = \frac{2}{\alpha}\, e^{-\frac{\gamma}{\alpha}} \sum_{\ell=1}^{L} f(\ell, \lambda) \sum_{n=0}^{\ell-1} \binom{\ell-1}{n} \frac{(-1)^n}{n+1} \left[\sqrt{\frac{\gamma C(n+1)}{\alpha\beta}} \times K_1\left(2\sqrt{\frac{\gamma C(n+1)}{\alpha\beta}}\right) + \frac{C(n+1)}{\beta} K_0\left(2\sqrt{\frac{\gamma C(n+1)}{\alpha\beta}}\right)\right]. \tag{11}$$

The destination combines the directly received signal and the signal through the selected relay using MRC. Therefore, the total SNR at the destination becomes $\gamma_T = \gamma_0 + \gamma_{\hat{i}}$, where $\gamma_0 = \frac{E_s|h_0|^2}{N_0}$ is the direct link SNR which is also exponentially distributed with parameter $\overline{\gamma}_0$ representing the average SNR of the direct link.

$$p_{\gamma_0}(\gamma) = \frac{1}{\overline{\gamma}_0} e^{-\frac{\gamma}{\overline{\gamma}_0}}. \tag{12}$$

Since γ_0 and $\gamma_{\hat{i}}$ are mutually independent, the PDF of γ_T is simply the convolution between (11) and (12).

$$p_{\gamma_T}(\gamma) = \int_0^\infty p_{\gamma_0}(\gamma - x) p_{\gamma_{\hat{i}}}(x) dx. \tag{13}$$

Solving (13) using [13, Eq. (6.643.3)] and simplifying through [14, Eqs. (13.1.33), (13.6.28), (13.6.30) and (6.5.19)], we get

$$p_{\gamma_T}(\gamma) = e^{-\frac{\gamma}{\overline{\gamma}_0}} \sum_{\ell=1}^{L} f(\ell, \lambda) \sum_{n=0}^{\ell-1} \binom{\ell-1}{n} \frac{(-1)^n}{n+1} \left[\frac{1}{\overline{\gamma}_0 - \alpha} - \frac{\alpha C(n+1)}{\beta(\overline{\gamma}_0 - \alpha)^2}\, e^{\frac{\overline{\gamma}_0 C(n+1)}{\beta(\overline{\gamma}_0 - \alpha)}} E_1\Big(\frac{\overline{\gamma}_0 C(n+1)}{\beta(\overline{\gamma}_0 - \alpha)}\Big)\right], \tag{14}$$

where $E_1(\cdot)$ is the exponential integral function defined in [14, Eq. (5.1.1)].

The CDF of the received SNR at the destination can be obtained by integrating (14) as follows

$$P_{\gamma_T}(\gamma) = \overline{\gamma}_0(1 - e^{-\frac{\gamma}{\overline{\gamma}_0}}) \sum_{\ell=1}^{L} f(\ell, \lambda) \sum_{n=0}^{\ell-1} \binom{\ell-1}{n} \frac{(-1)^n}{n+1} \times \left[\frac{1}{\overline{\gamma}_0 - \alpha} - \frac{\alpha C(n+1)}{\beta(\overline{\gamma}_0 - \alpha)^2}\, e^{\frac{\overline{\gamma}_0 C(n+1)}{\beta(\overline{\gamma}_0 - \alpha)}} E_1\left\{\frac{\overline{\gamma}_0 C(n+1)}{\beta(\overline{\gamma}_0 - \alpha)}\right\}\right]. \tag{15}$$

IV. Performance Analysis

A. Outage Probability

According to the conventional definition of outage probability, it represents the probability of having the received SNR below a certain threshold. Hence, the outage probability could be directly derived through the CDF of the total SNR by replacing $\gamma = \gamma_{th}$, where γ_{th} is the outage threshold SNR. The CDF of the total SNR in (15) is for the situation when at least one relay satisfies the interference constraint. However, as mentioned earlier, it is possible that none of the relays satisfies the constraint and the destination receives the direct signal only. The probability of this event is $Pr[\ell = 0] =$

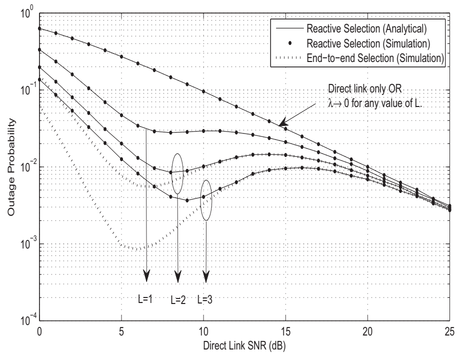

Fig. 2. Outage probability with $\lambda = 10$ and $\gamma_{th} = 1$ for $L = 0, 1, 2, 3$.

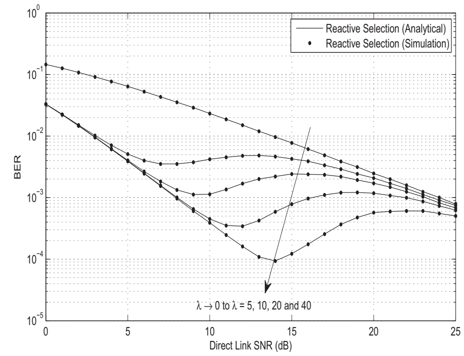

Fig. 3. BER with $L = 3$ and different values of λ.

$(1 - P_\lambda)^L = \bar{P}_\lambda^L$. Furthermore, the probability of having the SNR less than γ_{th} with the direct signal only is $(1 - e^{\frac{\gamma_{th}}{\overline{\gamma}_0}})$. Hence, the outage probability of the system becomes

$$P_{out} = P_{\gamma_T}(\gamma_{th}) + \bar{P}_\lambda^L (1 - e^{\frac{\gamma_{th}}{\overline{\gamma}_0}}). \tag{16}$$

B. Average Bit Error Probability

In order to find the average bit error probability, we first express the error probability conditioned over a given SNR in AWGN. This could be written terms of standard Q function which could then be averaged over the derived total SNR PDF. We assume that the modulation scheme used in the network is linear in nature.

$$P_e = \underbrace{Pr[\ell = 0]\int_0^\infty P_e(\varepsilon|\gamma_0) p_{\gamma_0}(\gamma) d\gamma}_{\text{Direct link only}} + \underbrace{\int_0^\infty P_e(\varepsilon|\gamma_T) p_{\gamma_T}(\gamma) d\gamma}_{\text{Direct and best relay links}}, \tag{17}$$

where $P_e(\varepsilon|\gamma) = Q(\sqrt{\eta\gamma})$ and η is a constant depending upon the modulation scheme.

Instead of using PDF, we apply the technique given in [15] to evaluate P_e using the derived CDF.

$$\int_0^\infty P_e(\varepsilon|\gamma) p_\gamma(\gamma) d\gamma = \frac{1}{\sqrt{2\pi}} \int_0^\infty P_\gamma\Big(\frac{t^2}{\eta}\Big) e^{-\frac{t^2}{2}} dt. \tag{18}$$

Replacing (15) and the CDF of the direct link SNR in (18) and solving using [13, Eq. (3.321.3)], we obtain the average probability of bit error for the proposed scheme as

$$P_e = \frac{1}{2}\left(1 - \sqrt{\frac{\eta\overline{\gamma}_0}{2+\eta\overline{\gamma}_0}}\right)\left[\bar{P}_\lambda^L + \overline{\gamma}_0 \sum_{\ell=1}^{L} f(\ell,\lambda) \sum_{n=0}^{\ell-1} \binom{\ell-1}{n} \frac{(-1)^n}{n+1} \times \left[\frac{1}{\overline{\gamma}_0 - \alpha} - \frac{\alpha C(n+1)}{\beta(\overline{\gamma}_0 - \alpha)^2} e^{\frac{\overline{\gamma}_0 C(n+1)}{\beta(\overline{\gamma}_0-\alpha)}} E_1\left\{\frac{\overline{\gamma}_0 C(n+1)}{\beta(\overline{\gamma}_0 - \alpha)}\right\}\right]\right]. \tag{19}$$

C. Average Channel Capacity

The channel capacity of the system model considered here is slightly different from the classical Shannon's definition, $\mathfrak{C} = \mathfrak{B} \log_2(1+SNR)$, where $\mathfrak{B}$ is the signal bandwidth. The reason for this difference is due to the fact that the information is conveyed to the destination in two time slots. Therefore, the channel capacity of this system is actually half of the Shannon's capacity. Therefore, the average channel capacity can be given as

$$\mathfrak{C} = \frac{\mathfrak{B}}{2}\Big[\int_0^\infty \log_2(1+\gamma_T) p_{\gamma_T}(\gamma) d\gamma + \bar{P}_\lambda^L \int_0^\infty \log_2(1+\gamma_0) p_{\gamma_0}(\gamma) d\gamma\Big]. \tag{20}$$

Replacing (12) and (14) in (20) and solving using [13, Eqs. (4.337.1 or 2)] and [14, Eq. (5.1.7)], we get

$$\mathfrak{C} = \frac{\mathfrak{B}}{2\ln 2} e^{\frac{1}{\overline{\gamma}_0}} E_1(1/\overline{\gamma}_0)\Bigg[\bar{P}_\lambda^L + \overline{\gamma}_0 \sum_{\ell=1}^{L} f(\ell,\lambda) \sum_{n=0}^{\ell-1} \binom{\ell-1}{n} \frac{(-1)^n}{n+1} \times \left[\frac{1}{\overline{\gamma}_0 - \alpha} - \frac{\alpha C(n+1)}{\beta(\overline{\gamma}_0 - \alpha)^2} e^{\frac{\overline{\gamma}_0 C(n+1)}{\beta(\overline{\gamma}_0-\alpha)}} E_1\Big(\frac{\overline{\gamma}_0 C(n+1)}{\beta(\overline{\gamma}_0 - \alpha)}\Big)\right]\Bigg]. \tag{21}$$

V. Simulation Results

Simulation results are obtained by varying the average SNR of the direct link $\overline{\gamma}_0$ whereas the average SNRs of the first and the second hops are set at $\alpha = 1.8\overline{\gamma}_0$ and $\beta = 1.3\overline{\gamma}_0$, respectively. The interfering channels are generated with parameter $\sigma = 0.8\overline{\gamma}_0$. The noise in each hop is considered to be unit variance AWGN with zero mean. The transmission power at the source is also assumed to be $E_s = 1$ while the amplification gain at each relay is $g = 1$. Binary phase shift keying (BPSK) with $\eta = 2$ is used as the modulation technique. System configurations with different number of relays are compared with equal power conditions.

The outage probability of the system is shown in Fig. 2 with $\gamma_{th} = 1$ and $\lambda = 10$. The top most curve is plotted for $\lambda \to 0$ and no relay could satisfy such interference constraint. Hence, the system operates on the direct link only. The remaining curves are for $L = 1, 2$, and 3, respectively. The performance of selective relaying on the basis of end-to-end SNR is also shown for comparison only. The outage probability curves are different from the traditional ones. At low SNR, all the relays in the system could satisfy the interference constraint ($\ell = L$) and the system works with a diversity order of $L + 1$. As the SNR increases, with fixed gains and no transmission power control at the relays, interference to the primary user also increases. In this situation, some relays could not satisfy λ and hence excluded from the selection process ($\ell < L$), resulting in reduced diversity order of the system. Eventually, at high

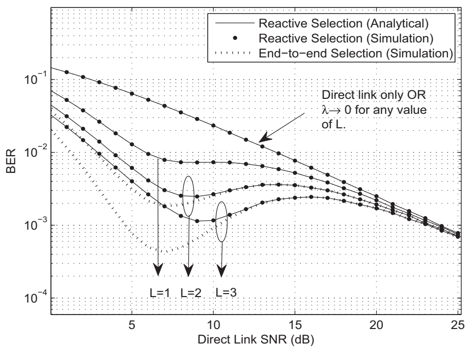

Fig. 4. BER with $\lambda = 10$ for different number of relays.

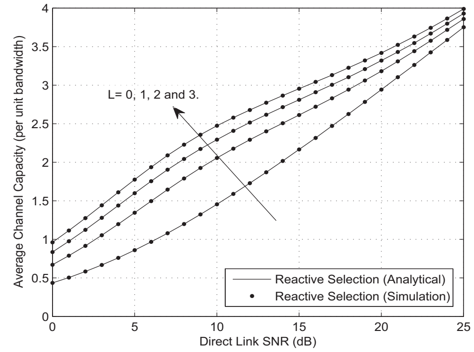

Fig. 5. Channel capacity with $\lambda = 10$ for different number of relays.

SNR, none of the relays could meet the interference constraint and the system operates on the direct link only. Hence, all the curves merge into the direct link curve at high SNR. Simulation and analytical results are in perfect agreement.

Fig. 3 depicts the bit error probability (BER) of the system with the same configurations. Similar trends could be observed due to the same reasons as mentioned above. It is evident that there exists an optimum SNR after which the performance start to degrade. Hence, these curves can help in choosing the number of relays required to maintain a specific BER at a certain value of λ. The system can operate normally in extended SNR regions with higher number of relays. Analytical results are again closely verified through the simulation results.

The system performance is also affected by the interference tolerance i.e. λ at the primary user. Fig. 4 explains the variations in the BER performance with three relays due to different values of λ. As the interference constraint is relaxed or increased, relays with relatively stronger interference channels could be accommodated in the selection process and the diversity order of the system remains $L + 1$ at comparatively higher SNRs. Hence, the optimum operating SNR moves gradually forward.

Average channel capacity of the system is presented in Fig. 5. The bottom curve shows the minimum capacity with the direct link only. The channel capacity gradually increases with the number of relays. However, a slight decrease can be seen after a certain SNR in each case, which again results from decreasing diversity order of the system due to interference constraint violation.

VI. Conclusion

We proposed and analyzed a reactive relay selection scheme for an underlay cognitive network operating near a primary user. The system settings were defined in a way to minimize the amount of CSI required at each node. The destination which made the selection decision only needed the CSI of second hops in the relay links. The relays were equipped with fixed gains and do not need first hop CSI. We derived the necessary statistics of the received SNR at the destination in closed form and used it to evaluate important system performance parameters. Analysis revealed that with fixed gain relays in underlay cognitive networks, the relay selection is feasible at only low SNR. The derived analytical results were verified through simulations.

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