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Experimental study and finite element analysis for piezoelectric impact energy harvesting using a bent metal beam

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Abstract. Linear uniform beams have been extensively investigated as a typical piezoelectric vibration energy harvesting mechanism in both conventional cantilever-beam-based harvester and a novel internal impact type energy harvester, the latter is of broad bandwidth and frequency up-converting capability and therefore quite suitable to low frequency ambient vibrations. To sufficiently utilize the space of the existing internal impact type energy harvester, a bent metal beam with distributed piezoelectric patches is proposed and investigated as the vibration energy harvesting module. A prototype harvester has been built and performances including the voltage and output power have been tested by experiments and analyzed by the finite element analysis. Both experiment and finite element analysis show that the proposed structure can effectively harvest vibration energy. And the optimal position for energy harvesting in this structure is also found. This work indicates that the frame of the existing internal impact type energy harvester may be used to harvest vibration energy, if it is bonded with distributed piezoelectric patches, which gives a way to increase its power density.

Keywords: Piezoelectric energy harvesting, impact, bent beam, finite element analysis

1. Introduction

With rapid development of wireless and microelectromechanical system technology, portable electronics are becoming more popular and powerful. However, their extensive and further applications are limited by the comparatively slow development of conventional power sources such as batteries. Recharging or replacing batteries are always inconvenient and in some cases even impossible. For example, battery replacement or replenishment for wireless sensor networks with tens of thousands of sensor nodes and for implanted sensing system within wild animals [1] can be very costly, and that of health monitoring systems embedded into concrete structures, such as skyscrapers and bridges, is nearly impossible. With the development of low power electronics, energy harvesting, which converts the ambient energy in the working environment of electronic products into usable electric energy, will be a renewable power source sustaining intermittent or even continuous operation for its target electronics during their whole lifespan. Of all ambient energy sources, the most commons are light, wind, thermal gradient and vibration [2,3]. Vibration energy, which is clean, ubiquitous and renewable, has attracted

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worldwide researchers in disciplines ranging from mechanical, civil, material and electrical to aeronautical and has been, since its first emergence, one of the foci for energy harvesting, namely, vibration energy harvesting [3–6].

Among all of the vibration energy harvesting methods, piezoelectric method is an effective and mature way for its convenience of availability and miniaturization, and ease of integration with electronics. Lots of work has been done in this field on materials [7], harvester structures [8–13], modeling [14–20] and power conditioning circuits [21,22]. Of all piezoelectric vibration energy harvester structures, linear piezoelectric unimorph/bimorph cantilever beams with a proof mass attached at the tip have been most commonly used. Vibration energy harvester utilizing this kind of topology works mainly in the resonance condition and power output decreases dramatically when the working frequency shifts away from the resonance frequency. Hence, it has very narrow operation bandwidth which makes its application restricted to a very limited scope since the ambient vibrations in real environment have many frequency components.

To widen the operation bandwidth, nonlinear mechanisms were proposed [8,23]. An internal impact type energy harvester operating in integrated vibration modes has also been proposed by the authors [13]. The impact mechanism can transfer low frequency base excitations into high frequency vibrations of the harvester structure, in which the fundamental vibration mode will be excited under impact. This impact-based frequency up-converting method is very useful for vibration energy harvesting, particularly in low frequency ambient vibration environment since the power output is proportional to the third power of vibration frequency such that the harvested energy will be drastically augmented with the increase of vibration frequency of piezoelectric components. However, the power density of this type of energy harvester is low due to its poor spatial utilization. Most of the piezoelectric patches in this energy harvester are bonded onto the cantilever beam, and its frame and other parts are not utilized yet.

In this paper, to sufficiently utilize the space of the existing internal impact type energy harvester, the energy harvesting characteristics of a bent beam under impact is investigated experimentally and theoretically for low frequency ambient vibrations. Five distributed piezoelectric patches are bonded on different location of the beam. Output voltage and power of each piezoelectric patch are measured. A piezoelectric-circuit coupled finite element model is built, and the voltage and power output of each piezoelectric patch is analyzed and compared to the experimental results. The optimum location on the bent beam for maximum output power is found for bonding piezoelectric patches.

The arrangement of this paper is as follows. Section 2 introduces the concept and design of the internal impact type piezoelectric energy harvester, followed by Section 3 which details the experimental method and results. In Section 4, a piezoelectric-circuit coupled finite element model and simulation results are presented and discussed. Finally comes Section 5.

2. Piezoelectric impact energy harvester

2.1. Introduction to the internal impact type piezoelectric vibration energy harvester

As shown in Fig. 1(a), a generic internal impact type piezoelectric energy harvester consists of an impact module which includes a cantilever beam and an impact mass, and a piezoelectric energy harvesting module which contains a piezoelectric bimorph and cantilever generating beam bonded with the piezoelectric bimorph. As the base vibrates, the impact module will be excited and move toward the energy harvesting module. At reaching the energy harvesting module, the impact mass hits the generating beam within very short duration, during which they remain contact and vibration energy of the impact module,

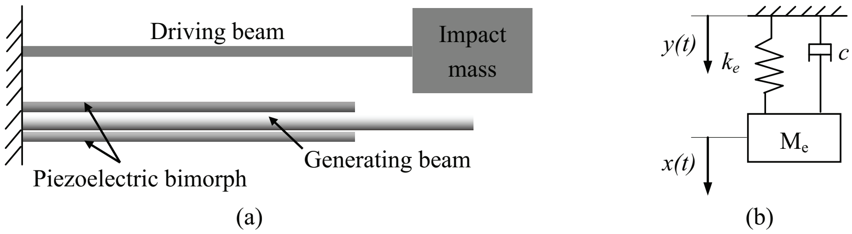


Fig. 1. Conventional structure of piezoelectric impact energy harvester. (a) Schematic; (b) an equivalent single degree of freedom (SDOF) system.

which originates from the ambient vibration, is transferred to the energy harvesting module by exciting its fundamental vibration mode (higher modes are neglected). After impact, the energy harvesting module will vibrate with damping till the next collision.

In such a case, the piezoelectric energy harvesting module can be modeled as a single degree of freedom (SDOF) system, as shown in Fig. 1(b). The differential equation of motion can be described by

$$M_e\ddot{x} + k_e\left[x(t) - y(t)\right] + \mathrm{c}[\dot{x}(t) - \dot{y}(t)] = 0 \tag{1}$$

Where M_e and k_e are the equivalent mass and stiffness coefficient of the energy harvesting module, respectively, c is the total damping, and $c = c_m + c_e$, in which, c_m is the mechanical damping corresponding to energy dissipation, and c_e the electrical damping corresponding to the energy harvesting, and $x\left(t\right)$ and $y\left(t\right)$ the displacement of the tip and base, respectively.

Assume that the relative displacement of M_e to the base is $z\left(t\right)$, then $z\left(t\right) = x\left(t\right) - y\left(t\right)$. Substituting it into Eq. (1), it is obtained

$$M_e\ddot{z}(t) + c\dot{z}(t) + k_e z(t) = -M_e\ddot{y}(t) \tag{2}$$

The steady state solution of Eq. (2) for a sinusoidal base excitation $y\left(t\right) = Y_0\sin\left(\omega t\right)$ is

$$z\left(t\right) = \frac{\omega^2}{\sqrt{\left(\frac{k_e}{M_e} - \omega^2\right)^2 + \left(\frac{c\omega}{M_e}\right)^2}}Y_0\sin\left(\omega t - \varphi\right) \tag{3}$$

The instantaneous power harvested by the piezoelectric energy harvesting module is

$$p\left(t\right) = c_e\dot{z}^2(t) \tag{4}$$

When the frequency of the base excitation equals to the resonance frequency of the SDOF system, energy output of the whole system reaches the maximum and the output power of energy harvesting is [17]

$$p_e = \frac{M_e\zeta_e\omega_n^3Y_0^2}{4\zeta_T^2} = \frac{M_e\zeta_e\omega_n^3Y_0^2}{4\left(\zeta_e + \zeta_m\right)^2} \tag{5}$$

where ζ_m is the equivalent mechanical damping ratio corresponding to energy dissipation, and ζ_e the equivalent electrical damping ratio corresponding to energy harvesting, ω_n the base excitation frequency that is equal to the resonance frequency of the energy harvesting module.

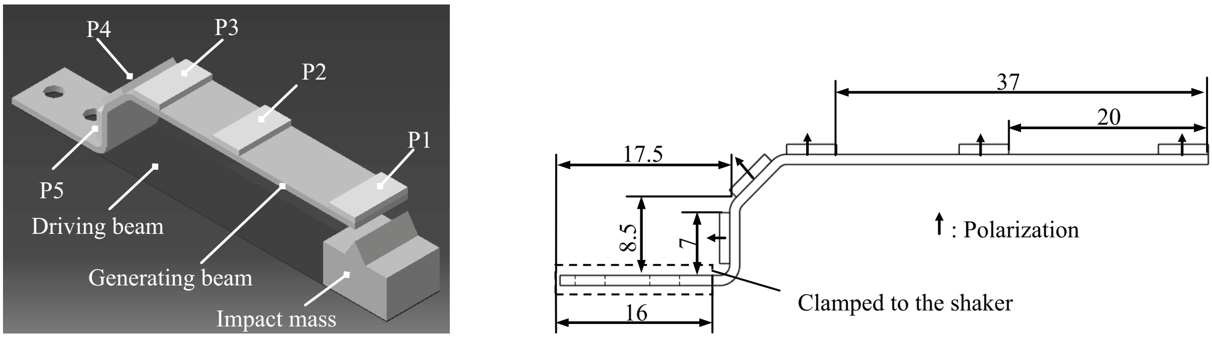
Setting the derivative of Eq. (5) to zero, it is found that the output power reaches the maximum when $\zeta_m = \zeta_e$ and the maximum output power is

$$p_e = \frac{M_e\omega_n^3Y_0^2}{16\zeta_m} \tag{6}$$

Obviously, with a large unwanted mechanical damping ratio ζ_m, the maximum output power and thus efficiency of energy harvesting will be significantly reduced.

Table 1
Material properties

Substrate material: Steel	
Density (kg/m^3)	7850
Young's modulus (GPa)	210
Poisson's ratio	0.3
Piezoelectric material: P-51	
Density (kg/m^3)	7450
Poisson's ratio	0.3
Elasticity coefficient (10^{-12} m^2/N)	
S_{11}	16.5
S_{12}	-4.78
S_{13}	-8.45
S_{22}	16.5
S_{33}	20.7
S_{44}	43.5
S_{55}	43.5
S_{66}	42.6
Piezoelectric constants (10^{-12} C/N)	
d_{31}	-185
d_{33}	400
d_{15}	650
Relative permittivity	
ε_{11}	2400
ε_{33}	2100

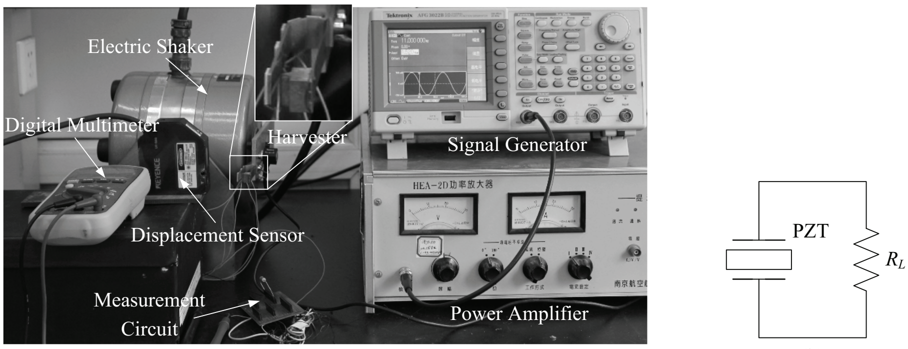


(a) (b)

Fig. 2. Schematic diagrams of (a) the bent piezoelectric beam and (b) impact energy harvester utilizing such a beam. The size is in mm.

2.2. *Concept and design of bent beam based piezoelectric impact energy harvester*

A piezoelectric energy harvesting module with a bent beam is proposed. As shown in Fig. 2(a), the beam with a bent section (65 mm $\times$ 12.9 mm $\times$ 10 mm) is bonded with five differently placed piezoelectric patches (10 mm $\times$ 5 mm $\times$ 1 mm). The position of each piezoelectric patch can be seen in Fig. 2(b). Of all piezoelectric patches, P1, P2 and P3 are bonded on the horizontal uniform section, and P4 and P5 the sloped and vertical sections, respectively. The impact module is similar to the conventional one, with an impact mass and linear uniform cantilever beam. The material for the substrate is steel and the piezoelectric material used here is Lead Zirconate Titanate (PZT) piezoelectric ceramic (P-51, HAIYING). The material properties are shown in Table 1.



(a) (b)

Fig. 3. Schematic diagrams of (a) experimental setup and (b) electrical connection.

3. Experimental study

3.1. Experimental setup and method

As shown in Fig. 3(a), in our experiments, the designed prototype was fixed to an electric shaker (HEV-200, NANJING FONENG) by bolts. A signal generator (AFG3022B, TEKTRONIX) was used to supply sinusoidal excitation signal to the power amplifier (HEA-2D, NANJING FONENG) through which the signal was power amplified and transferred to the electric shaker to generate harmonic base vibration for the harvester prototype under test. Each of the PZT patches was connected to a load resistor. The root-mean-square (rms) voltage across the resistors was measured by a digital multimeter (F17B, FLUKE). A laser displacement sensor (LK D30, KEYENCE) was used to measure the displacement of the free tip. The electrical connection is shown in Fig. 3(b).

3.2. Experimental results

The average output power P_{av} transferred to a load resistor can be calculated by

$$P_{av} = \frac{V_{rms}^2}{R_L} \tag{7}$$

where V_{rms} is the effective voltage across the load resistor R_L. P_{av} reaches the maximum when R_L equals to its optimal value [15]

$$R_{LO} = \frac{1}{\omega C_P} \tag{8}$$

where ω is the vibration angular frequency of the energy harvesting module and C_P is the clamped capacitance of the PZT patch. In the design, each PZT patch has the same theoretical optimal load resistor as they have identical vibration frequency and clamped capacitance.

Figure 4 shows the effective voltage across and average power consumed by the load resistor R_L for each PZT patch under the base vibration frequency of 11 Hz and acceleration of 0.5 g (rms). It can be seen in Fig. 4(a) that, with the increase of R_L, the effective voltage increases fast at first and begins to slow down after a critical point and finally almost comes to a constant. The critical point, as shown in Fig. 4(b), is the optimal value R_{LO} of load resistor. It can be found that PZT patches, P1, P2, P3,

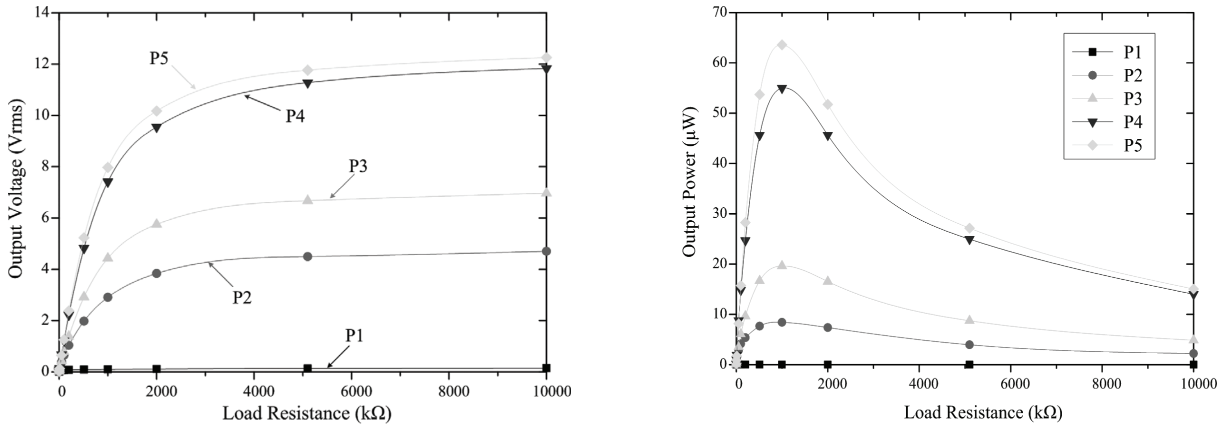


Fig. 4. (a) The output voltage across the load resistor and (b) the output power vs. load resistance at the base vibration frequency of 11 Hz and acceleration of 0.5 g (rms).

P4 and P5, have the same optimal load resistance and $R_{LO} \approx 1$ MΩ, which is in accordance with the aforementioned theoretical analysis. The maximum output powers of P1, P2, P3, P4 and P5 at the optimal load resistance are 0.001 μW, 8.428 μW, 19.625 μW, 54.988 μW and 63.596 μW, respectively. The output power of P4 and P5, which are bonded on bent and vertical sections, are much greater than that of P1, P2 and P3, which are bonded on linear uniform sections. P5 outputs more power than other patches, which means that it has the optimum location among the five piezoelectric patches.

In order to examine the effect of the impact module upon power output performance of the proposed harvester structure, a comparison was made for the maximum output power of piezoelectric patches between the two cases with and without the impact module at the base vibration frequency of 11 Hz and acceleration of 0.5 g (rms), as demonstrated in Fig. 5. Clearly, for the case without-impact-module, the power outputs of piezoelectric patches are at the level of approximately 0.001 μW, which is 3 to 4 magnitudes lower than that with-impact-module. Therefore, the impact module can effectively promote the energy harvesting capability of the proposed piezoelectric structure.

To further understand the energy harvesting characteristics of the proposed structure, the output power vs. excitation frequency was tested for piezoelectric patch P5 at a base excitation acceleration of 0.5 g (rms). As shown in Fig. 6, as the excitation frequency increases from 7 Hz to 12 Hz, the output power increases quickly and reaches the maximum at 12 Hz, after which, the output power begins to decrease and as the frequency goes from 12.1 Hz to 12.2 Hz, the output power drops steeply to almost zero. The resonance frequency of the cantilever beam with the impact mass without impact is measured to be 11 Hz. With the energy harvesting module and impact, the resonance frequency of the cantilever beam may increase a bit, which explains the phenomena shown in Fig. 6.

4. Finite element analysis

4.1. Finite element model

The finite element method (FEM) is an efficient way to analyze piezoelectric energy harvesting [18–20]. To understand the energy harvesting mechanism of the proposed piezoelectric impact energy harvester, a piezoelectric-circuit coupled finite element model has been built for the energy harvester, using

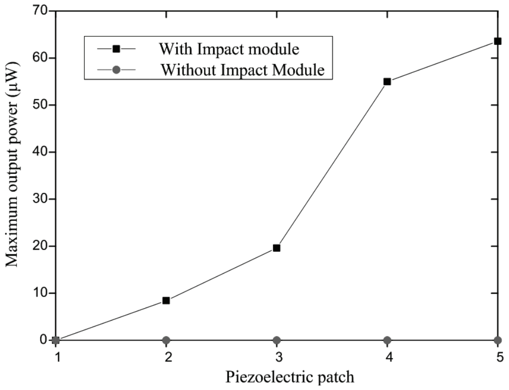


Fig. 5. A comparison of the maximum output power of piezoelectric patches between the cases with and without the impact module at the base vibration frequency of 11 Hz and acceleration of 0.5 g (rms).

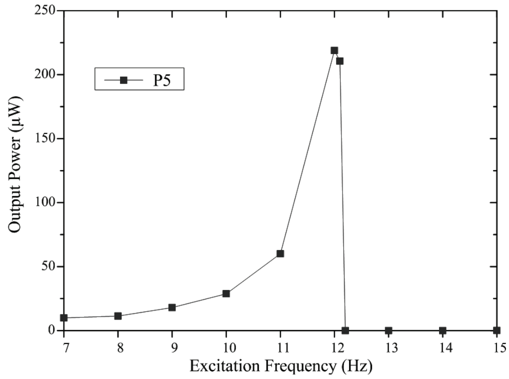


Fig. 6. The output power vs. base excitation frequency at the base excitation acceleration of 0.5 g (rms).

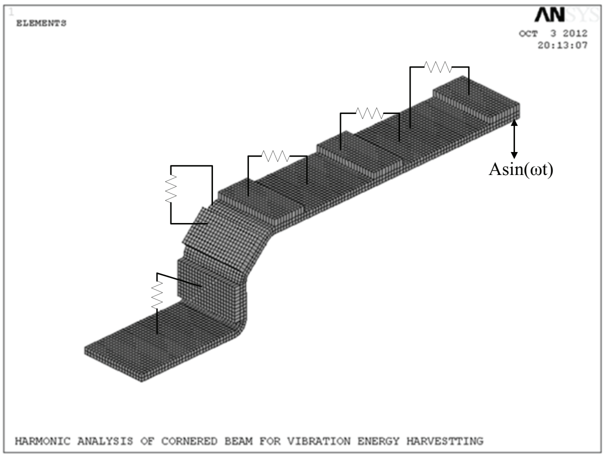


Fig. 7. The piezoelectric-circuit coupled finite element model.

commercial finite element analysis code ANSYS 12.0, as shown in Fig. 7. The substrate structure is meshed with the 3-D SOLID45 elements and the PZT patches with the 3-D SOLID5 elements, which have three and four degrees of freedom per node, respectively, i.e., translation in the x-, y- and z-directions and the electric potential V (for piezoelectric elements). The whole structure is discretized with 7140 elements, which are distributed in the two element layers in the bent beam in the thickness direction and the one in each piezoelectric patch. Moreover, CIRCU94 elements are used to model the load resistors, which are directly connected to the top and bottom electrodes of the piezoelectric patches. As the boundary conditions for the FEM model, the root of the bent beam is clamped; the tip is free; all the interfaces between the piezoelectric patches and bent beam are electrically grounded.

In this paper, the finite element analysis consists of two steps. The first step is a modal analysis to determine the fundamental natural frequency, and the second step is a harmonic analysis. Impact is a highly nonlinear process and needs essentially nonlinear transient dynamic analysis. Here for simplicity of analysis, a sinusoidal displacement is applied to the tip with a frequency equal to the fundamental natural frequency of the bent beam, as shown in Fig. 7, in which A is the vibration amplitude (0-peak).

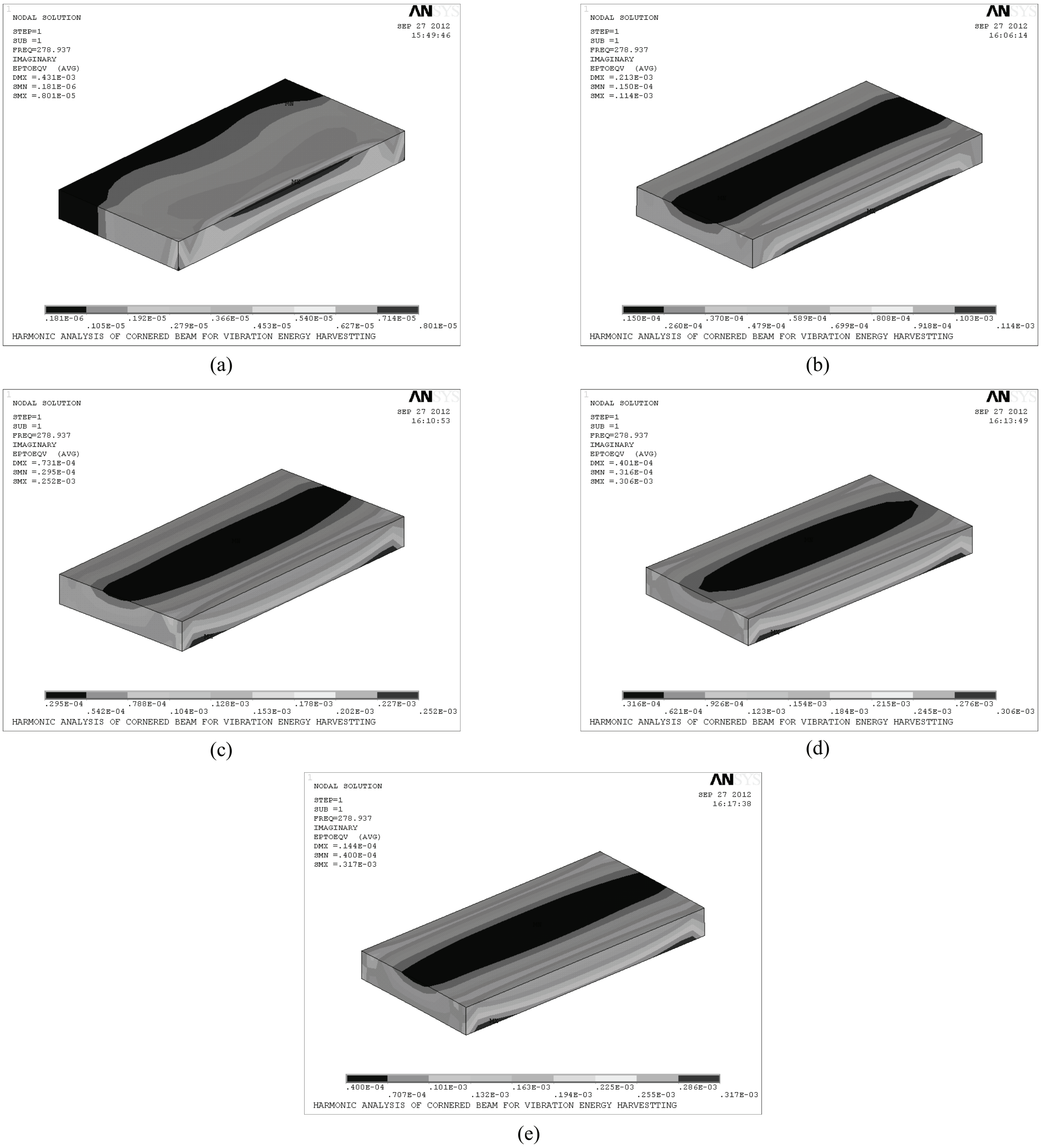


Fig. 8. Strain distributions of the PZT patches. (a) P1, (b) P2, (c) P3, (d) P4, and (e) P5.

4.2. Calculation results

From the modal analysis, it is known that the natural frequency f_n of the fundamental flexural vibration mode is 278.9 Hz when the piezoelectric patches are in the short-circuit condition. The harmonic

Table 2
ANSYS simulation results of voltage and output power

	P1	P2	P3	P4	P5
Output voltage (V_{rms})	0.2568	7.818	24.56	33.94	38.09
Output power (μW)	0.04083	37.14	366.9	701.5	883.5

Table 3
The rms volumetric strain of PZT patches

	P1	P2	P3	P4	P5
rms volumetric strain ($\mu\varepsilon$)	2.6	48.6	108.8	129.5	142.5

analysis is made by applying a harmonic displacement load at the tip of the bent beam and connecting five load resistors of 1 MΩ to each PZT patch. The angular frequency of this displacement load is determined by $\omega = \omega_n = 2\pi f_n$ and the tip's vibration amplitude is 405 μm, which is measured when the base excitation has a frequency of 11 Hz and acceleration of 0.5 g (rms). The rms output voltage and average output power of each piezoelectric patch are shown in Table 2.

To compare the strain levels of P1, P2, P3, P4 and P5, the rms volumetric element strains are calculated under the condition that the beam tip reaches its maximum displacement. Figure 8 shows the contour plots of strain distribution of piezoelectric patches P1, P2, P3, P4 and P5, from which the rms volumetric strains of P1, P2, P3, P4 and P5 can be derived, as shown in Table 3. Comparing the results in Tables 2 and 3, it is known that the greater the volumetric strain, the larger the output power. The calculation results can well explain the experimental phenomenon that the output power of P4 and P5, are much greater than that of P1, P2 and P3, and P5 outputs more power than other patches. Here the calculated results are larger than the measured ones because the bonding layer is not considered in the FEM model. The accurate values of output power can be calculated by proper increasing the loss factors of the piezoelectric and metal materials to equivalently include the loss factor of the bonding layer.

Based on the experimental and simulation results, it is known that the bending structure can effectively harvest vibration energy. This implies that the vertically bent frame in the existing internal impact type piezoelectric energy harvester [13] can also be used to harvest vibration energy, if piezoelectric patches are properly bonded onto the surfaces of the frame, which gives a way to increase the power density of the energy harvester.

5. Conclusion

A bent metal beam with distributed piezoelectric patches is proposed to be used as the energy harvesting module of an internal impact type vibration energy harvester. A prototype has been built, and the performance including output voltage and power has been experimentally tested and theoretically analyzed. Both experiment and finite element analysis shows that the bent metal beam with distributed piezoelectric patches can effectively harvest vibration energy. It is found that the optimal location to bond piezoelectric patches for energy harvesting is the vertical and tilt sections of the bent beam. This work indicates that the frame of the existing internal impact type energy harvester may be used to harvest vibration energy, if distributed piezoelectric patches are bonded to the vertical and tilt sections, which gives a useful guideline for increasing the power density.

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