The Multiattribute Linear Ballistic Accumulator Model of Context Effects in Multialternative Choice

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Context effects occur when a choice between 2 options is altered by adding a 3rd alternative. Three major context effects—similarity, compromise, and attraction—have wide-ranging implications across applied and theoretical domains, and have driven the development of new dynamic models of multiattribute and multialternative choice. We propose the multiattribute linear ballistic accumulator (MLBA), a new dynamic model that provides a quantitative account of all 3 context effects. Our account applies not only to traditional paradigms involving choices among hedonic stimuli, but also to recent demonstrations of context effects with nonhedonic stimuli. Because of its computational tractability, the MLBA model is more easily applied than previous dynamic models. We show that the model also accounts for a range of other phenomena in multiattribute, multialternative choice, including time pressure effects, and that it makes a new prediction about the relationship between deliberation time and the magnitude of the similarity effect, which we confirm experimentally.

Keywords: decision making, multialternative choice, preference reversal, context effects, dynamic models

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We often have to choose among multiple options, each with several attributes, such as when purchasing a car, where one option might have better gas mileage but another might require less maintenance. These types of decisions can be subject to "context effects," whereby a choice between two options is affected by the introduction of a third option. For example, an initial preference for a low-maintenance car with inferior gas mileage over a higher maintenance car with good gas mileage could be reversed when a third car with moderate gas mileage and high maintenance is also considered.

Three major context phenomena are the similarity (Tversky, 1972), attraction (Huber, Payne, & Puto, 1982), and compromise (Simonson, 1989) effects. These three effects are demonstrations of how preferences for two existing alternatives can be influenced by the inclusion of a third new option. The attraction effect occurs when two original choices are augmented with a dominated option (i.e., a new option that is slightly worse than one of the original

options) and the probability of selecting the better, dominant alternative increases. The similarity effect arises from the introduction of an option that is similar to, and competitive with, one of the original alternatives, and causes a reduction in the probability of choosing the similar alternative. The compromise effect occurs when there is an enhancement in the probability of choosing an alternative that becomes the intermediate option when a third extreme option is added.

Beyond their practical implications for areas such as consumer choice, the three effects are important for theories of preference because they violate the property of simple scalability (Krantz, 1964; Tversky, 1972). Simple scalability is a property of most utility models used to study choice behavior, including Luce's (1959) ratio of strengths model. Hence, developing a single theoretical framework able to accommodate all three effects is a significant challenge.

Proposals addressing this challenge have involved dynamic processes, that is, models that account for the time to make choices (deliberation time) as well as the choices that are made. Over the last decade, several dynamic models have been proposed, in particular, multialternative decision field theory (MDFT; Roe, Busemeyer, & Townsend, 2001) and the leaky competing accumulator (LCA) model (Usher & McClelland, 2004). Although these models have provided great insight into multialternative choice behavior, they are not without flaws. First, they are difficult to fit to data because they require computationally intensive simulations. As a result, evaluation of these models has rested almost entirely on qualitative analyses, such as demonstrating the presence of all three effects with the same set of parameters. Thus, it is unclear whether the models can actually provide a quantitative account of human behavior, although decision field theory has been fit to

human data involving two alternatives (Busemeyer & Townsend, 1993).

Besides computational issues, the LCA model is also limited in scope by its use of loss aversion to explain the attraction and compromise effects. The loss aversion hypothesis applies to high-level decision-making tasks where the options have hedonic attributes, such as consumer products with attributes of price and quality. The LCA model assumes that decision makers are particularly averse to losses of such quantities. However, recent studies have demonstrated the three context effects using nonhedonic stimuli, such as with perceptual choices about size (Trueblood, Brown, Heathcote, & Busemeyer, 2013). It is difficult to imagine how loss aversion could apply to such stimuli, where the concept of "loss" is not easily imagined. This suggests that context effects might be a general feature of choice behavior, not due to loss aversion.

We introduce a new dynamic account, the multiattribute linear ballistic accumulator (MLBA) model, as an alternative framework for modeling multialternative choice. Unlike MDFT and the LCA model, the MLBA model has an analytic likelihood function, making it easier to fit to experimental data. It differs from the other dynamic models by not including moment-to-moment random variations in preference, and it is also based on a new psychological theory about context effects that does not rely on loss aversion, so it naturally applies to both hedonic and nonhedonic choices. The MLBA model explains context effects through a front-end component that transforms choice stimuli into selection tendencies and a back-end process that transforms these tendencies into overt choices. The computational tractability of the MLBA means that it can easily be fit to data from individual subjects. This tractability arises mainly from the simplifying assumption that moment-by-moment randomness does not play an important role in decision processes—an assumption borne out by recent model comparisons in the response time literature (Donkin, Brown, Heathcote, & Wagenmakers, 2011).

In the first two sections of this article we describe the three context effects and discuss recent experimental results demonstrating these effects in inference (Trueblood, 2012) and perceptual (Trueblood et al., 2013) tasks. We then review MDFT and the LCA model and present the new MLBA model, outlining the similarities and differences between the three models. In the subsequent section we present qualitative analyses of the MLBA model and show that it makes a new prediction about the relationship between the magnitude of the similarity effect and deliberation time, which we support with experimental evidence. The remaining sections are devoted to quantitative analyses using the inference and perceptual context-effect data.

Three Context Effects

The three context effects are laboratory demonstrations of intuitively plausible ways in which our preferences for existing alternatives can be altered by the introduction of new alternatives. Suppose there are two options in some choice and that these are almost equally attractive. If a third option is introduced that is very similar to one of the existing two, the existing similar option becomes less attractive—intuitively, because it has to "share" space with the new, similar alternative. This is the similarity effect. If, instead, the new third alternative is set up to make one of the

two existing alternatives appear as a compromise between the other two options, then that compromise alternative becomes more attractive than previously—this is the compromise effect. Finally, if a third alternative is introduced that is quite like an existing alternative, but just a bit poorer, it makes that better alternative seem even better than it used to—the attraction effect.

The usual task used to examine these three standard context effects—attraction, similarity, and compromise—has choices among three alternatives that have two attributes each. In a typical consumer goods decision task, for example, subjects might be asked to choose among cars that vary on two attributes: price and quality. Figure 1 graphically represents the options by plotting price against quality.

The three context effects are sometimes studied using choices between two options at a time (binary choices), and other times using choices between three options (ternary choices). We discuss the effects in terms of the latter method as it is used in the studies we address. The all-ternary method also tends to produce larger effect sizes (Wedell, 1991).

The Attraction Effect

The attraction effect enhances the probability of choosing an option by introducing a similar, but inferior ("decoy") option (Choplin & Hummel, 2005; Huber et al., 1982; Maylor & Roberts, 2007). Consider the choice set $\{X, Y\}$ and the decoy R_X, which is
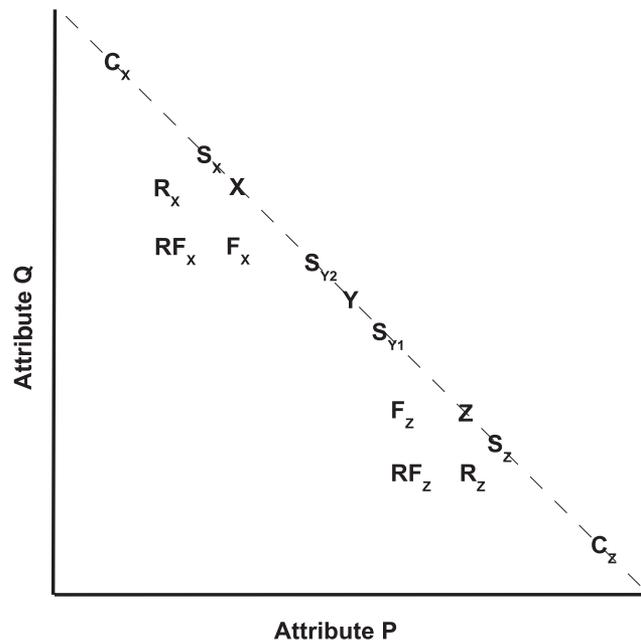
Figure 1. The various options producing context effects plotted in a two-dimensional space defined by two attribute values such as price and quality. The basic choice set includes three options: X, Y, and Z. Options labeled R, F, and RF refer to attraction decoys, where R represents range decoys, F represents frequency decoys, and RF represents range–frequency decoys. Options labeled S refer to similarity decoys, and options labeled C refer to compromise decoys. Although the figure shows all the different kinds of stimuli used in different conditions, only three of the stimuli are ever used in any single choice set.

similar but inferior to X. The attraction effect occurs when people show a stronger preference for option X when it is presented along with its inferior comparison (R_X) than otherwise. The attraction effect can similarly be demonstrated for option Z, by introducing the inferior decoy R_Z. Formally, the attraction effect occurs when the probability of choosing X is greater when the decoy favors X as compared to when it favors Z, and vice versa: $Pr[X \mid \{X, Z, R_X\}] > Pr[X \mid \{X, Z, R_Z\}]$ and $Pr[Z \mid \{X, Z, R_X\}] < Pr[Z \mid \{X, Z, R_Z\}]$.

In the attraction effect experiments by Trueblood (2012) and Trueblood et al. (2013), three types of decoys were tested: range, frequency, and range–frequency (Huber et al., 1982). The placement of these decoys in the attribute space is illustrated in Figure 1. The range decoy (R) refers to an option that is a little weaker than the focal alternative on the focal alternative's weakest attribute. (Throughout, we use the term *focal* to refer to the option that is enhanced by the presence of the decoy.) The frequency decoy (F) refers to an option that is a little weaker than the focal alternative on the focal alternative's strongest attribute. The range–frequency decoy (RF) combines both a range and frequency manipulation. The three types of decoys are important because previous research has demonstrated that they result in attraction effects with different magnitudes (Huber et al., 1982).

The Similarity Effect

The similarity effect occurs when a competitive option (i.e., an option that is worse on one attribute but better on another) that is similar to one of the existing alternatives is added to the choice set and the probability of selecting the dissimilar option increases (Maylor & Roberts, 2007; Tsetsos, Usher, & McClelland, 2011; Tversky, 1972). Consider the choice set $\{X, Y\}$ and two decoys, S_X and S_{Y1}, where S_X is similar to X and S_{Y1} is similar to Y, as illustrated in Figure 1. The similarity effect occurs when the probability of choosing X is greater when the decoy is similar to Y as compared to when it is similar to X, and vice versa: $Pr[X \mid \{X, Y, S_X\}] < Pr[X \mid \{X, Y, S_{Y1}\}]$ and $Pr[Y \mid \{X, Y, S_X\}] > Pr[Y \mid \{X, Y, S_{Y1}\}]$.

The Compromise Effect

The compromise effect occurs if an option is made more attractive when presented as a compromise between the other alternatives in the choice set compared to when it appears to be an extreme alternative (Pettibone & Wedell, 2000; Simonson, 1989). Consider a choice set $\{Y, Z\}$ along with options X and C_Z, where X is an extreme option that makes Y assume the middle ground and C_Z is an extreme option that makes Z assume the middle ground, as illustrated in Figure 1. Between these two choice sets, the option Y changes from a compromise to an extreme position. The compromise effect occurs when the probability of choosing Y is greater when Y appears as a compromise rather than an extreme alternative, and vice versa: $Pr[Y \mid \{X, Y, Z\}] > Pr[Y \mid \{Y, Z, C_Z\}]$ and $Pr[Z \mid \{X, Y, Z\}] < Pr[Z \mid \{Y, Z, C_Z\}]$.

Violations of Simple Scalability

All three effects violate the simple scalability property (Krantz, 1964; Tversky, 1972), which underlies most of the utility models used to study choice behavior including Luce's (1959) ratio of strengths model. In the ratio of strengths model, the probability of selecting a particular option is simply the strength s of the option divided by the sum of the strengths of all of the options:

$$Pr[X \mid \{X, Y, Z\}] = \frac{s(X)}{s(X) + s(Y) + s(Z)}.$$

To demonstrate a violation, consider the attraction effect. According to the simple scalability property, the inequality $Pr[X \mid \{X, Z, R_X\}] > Pr[X \mid \{X, Z, R_Z\}]$ implies that the strength of R_X is less than the strength of R_Z. However, the inequality $Pr[Z \mid \{X, Z, R_X\}] < Pr[Z \mid \{X, Z, R_Z\}]$ implies that the strength of R_Z is less than the strength of R_X. Because these two statements cannot both be true, the property is violated. Similar arguments apply to the similarity and compromise effects.

Previous Experimental Findings

The study of multialternative context effects began with Tversky's (1972) work on the similarity effect. Tversky was interested in demonstrating the inadequacy of the simple scalability property in accounting for multialternative choice behavior. He developed a series of three experiments examining the similarity effect in different domains. The first experiment examined the effect in perception, where participants were shown squares containing random dot patterns and asked to choose the square with the most dots. In the second experiment, participants were asked to select the most promising college applicant based on attributes of intelligence and motivation. In the third experiment, participants were asked to choose among two-outcome gambles where the attribute values were the probability of winning and the amount that could be won.

Tversky's (1972) perceptual experiment failed to produce a reliable similarity effect, whereas the remaining two experiments produced reliable effects. Tversky concluded that the psychophysical stimuli were perceived holistically, and that perceptual choices followed the simple scalability property. Tversky explained the similarity effect in the second and third experiments via his "elimination by aspects" theory. However, this theory (and other contemporary theories: Hausman & Wise, 1978; McFadden, 1980) predicted that choices should always obey the regularity principle: the principle that adding extra options to a choice set must always decrease the choice probability for existing options.[1] Huber et al. (1982) were interested in demonstrating that choices among consumer products could produce a violation of this principle. This work was the first to show that adding an option to a choice set can increase the probability of choosing an alternative from the original set. This result later became known as the attraction effect. Following the work of Huber et al., Simonson (1989) developed new experiments demonstrating the attraction and compromise effects in consumer products.

Since the experimental work by Simonson (1989), there have been numerous studies demonstrating context effects in high-level decision tasks were alternatives have hedonic attributes. These studies include choices among consumer products both in the

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[1] Formally, the regularity principle states that for an option $X \in W \subset U$, the probability of selecting X out of W cannot be less than the probability of selecting X out of U.

laboratory (Pettibone & Wedell, 2000) and with real in-store purchases (Doyle, O'Connor, Reynolds, & Bottomley, 1999), choices among candidates in elections (Pan, O'Curry, & Pitts, 1995), choices among gambles (Wedell, 1991), likelihood judgment problems (Windschitl & Chambers, 2004), selection of mates (Sedikides, Ariely, & Olsen, 1999), and hiring decisions (Highhouse, 1996). Although there has been extensive research on context effects in high-level tasks with hedonic attributes, there has been much less work in examining the existence of these effects in high-level tasks where the attributes are nonhedonic. One study that has explored this issue was an episodic memory experiment conducted by Maylor and Roberts (2007), which demonstrated both attraction and similarity effects. There is also little experimental work examining context effects in low-level tasks. Choplin and Hummel (2005) found the attraction effect with ovals and line segments in a similarity judgment paradigm. Also, Tsetsos et al. (2011) and Tsetsos, Chater, and Usher (2012) obtained the attraction and similarity effects using psychophysical stimuli, providing the first evidence of the similarity effect with perceptual stimuli since Tversky's (1972) unsuccessful attempt. These two studies involved the presentation of sequential information and reducing the possibility of holistic processing.

Although these studies have added to our understanding of context effects, the evidence for context effects in high-level tasks with nonhedonic attributes and in low-level tasks is scanty and distributed across different experimental paradigms and different subject populations (see Table 1 for a summary of the experimental results). Only recently has there been experimental evidence that all three context effects can arise in the same nonhedonic paradigm. These recent experiments show that all three effects can be obtained in both an inference task (Trueblood, 2012) and a simple perceptual decision-making task (Trueblood et al., 2013). These results suggest that context effects are a general property of human choice behavior, and so are important outside the field of consumer decision making and may be important for theories of memory and categorization.

Inference Context Effects Experiments

Trueblood (2012) developed an inference paradigm involving decisions about criminal suspects to examine attraction, similarity, and compromise effects. In this paradigm, participants were asked to infer which of three suspects was most likely to have committed a crime based on two eyewitnesses. The evidence from each eyewitness was given a strength rating ranging from 0 (*very weak evidence for guilt*) to 100 (*very strong evidence for guilt*). The three suspects were the choice options, and the two eyewitness strengths were the attributes. For example, a single choice set might be strength ratings of 64 (Eyewitness 1) and 32 (Eyewitness 2) for Suspect 1, strength ratings of 33 (Eyewitness 1) and 63 (Eyewitness 2) for Suspect 2, and strength ratings of 62 (Eyewitness 1) and 30 (Eyewitness 2) for Suspect 3.

Three experiments were conducted where each effect was tested separately. In each experiment, subjects completed 240 trials, half of which were filler trials used to monitor participants' accuracy. In each experiment, the trials used to test for a context effect were subdivided so that the decoy was similar to one alternative for some trials and similar to the other alternative for other trials. With the labels from Figure 1, the attraction effect was analyzed by comparing the range choice sets $\{X, Z, R_X\}$ and $\{X, Z, R_Z\}$, the frequency choice sets $\{X, Z, F_X\}$ and $\{X, Z, F_Z\}$, and the range–frequency choice sets $\{X, Z, RF_X\}$ and $\{X, Z, RF_Z\}$. The similarity effect was tested using four ternary choice sets: $\{X, Y, S_X\}$, $\{X, Y, S_{Y1}\}$, $\{Y, Z, S_Z\}$, and $\{Y, Z, S_{Y2}\}$. The first two choice sets in the previous list provided one evaluation of the similarity effect, and the second two choice sets provided a second evaluation of the effect. This allowed the effect to be tested in different ranges of the attribute space. The compromise effect was examined using the sets $\{C_X, X, Y\}$, $\{X, Y, Z\}$, $\{Y, Z, C_Z\}$.

All three experiments produced the intended effects. In the attraction experiment, the mean choice probability for the focal alternative averaged across range, frequency, and range–frequency decoys was approximately .53, as compared to .40 for the nonfocal

Table 1

Summary of Previous Research on Context Effects

Study	Stimuli	Attraction	Similarity	Compromise
Tversky (1972)	Candidates		X	
	Gambles		X	
Huber et al. (1982)	Consumer goods	X		
Simonson (1989)	Consumer goods	X		X
Wedell (1991)	Gambles	X		
	Consumer goods	X		
Pan et al. (1995)	Candidates	X		
Highhouse (1996)	Job candidates	X		
Doyle et al. (1999)	Consumer goods	X		
Sedikides et al. (1999)	Mates	X		
Pettibone & Wedell (2000)	Consumer goods			X
Choplin & Hummel (2005)	Perceptual objects	X		
Maylor & Roberts (2007)	Past events	X	X	
Tsetsos et al. (2011)	Perceptual objects		X	
Trueblood (2012)	Inference	X	X	X
Tsetsos et al. (2012)	Perceptual objects	X	X	
Trueblood et al. (2013)	Perceptual objects	X	X	X

Note. A successful demonstration of an effect is denoted by an X. Empty cells represent studies that did not examine a particular effect.

alternative. In the similarity experiment, the mean choice probability for the focal alternative was approximately .51, as compared to .31 for the nonfocal alternative. In the compromise experiment, the mean choice probability for the compromise alternative was approximately .48, as compared to .38 for the extreme alternative. All findings were statistically significant.

Perceptual Context Effects Experiments

In Trueblood et al. (2013) the three context effects were examined using a simple perceptual decision-making task: judging which of three rectangles had the largest area, with the rectangles' heights and widths acting as the two attributes. As in the inference experiments just discussed, three experiments were conducted testing the effects separately. In each experiment, participants completed 720 trials including filler trials used to assess accuracy.

As in the inference experiments, the trials used for testing a particular context effect were subdivided so that the decoy was similar to one alternative for some trials and similar to the other alternative for other trials. As before, three versions of the attraction effect corresponding to the three decoys (range, frequency, and range–frequency) were tested. The similarity effect used four ternary choice sets, where two choice sets tested the effect when width was greater than height and two choice sets tested the effect when height was greater than width. The compromise effect was tested using two choice sets, $\{X, Y, Z\}$ and $\{Y, Z, C_Z\}$, where all of the rectangles had equal area but height $X <$ height $Y <$ height $Z <$ height C_Z.

All three perceptual experiments produced the expected effects. In the attraction experiment, the range decoy produced the strongest effect, with a mean choice probability for the focal alternative of approximately .51, as compared to .46 for the nonfocal alternative. In the similarity experiment, the mean choice probability for the focal alternative was approximately .37, as compared to .32 for the nonfocal alternative. In the compromise experiment, the mean choice probability for the compromise alternative was approximately .42, as compared to .40 for the extreme alternative. Although the magnitudes of the effects were small, all were statistically reliable. Further, the effects were reliable at an individual-subject level, with between 60% and 70% of participants demonstrating the effects at an individual-subject level.

Theoretical Implications

The experimental work by Trueblood (2012) and Trueblood et al. (2013) shows that the three standard context effects from the consumer choice literature also occur in inference and perceptual choice tasks, and demonstrate the need for a unified account of context effects across domains. The experiments suggest that these context effects are a general feature of human choice behavior because they are a fundamental part of decision-making processes. Since the effects violate the property of simple scalability, they appear to illustrate a fundamental way in which cognition differs from rational judgment, providing an ongoing problem of interest for cognitive science.

There are two prominent models that explain the effects by postulating dynamic processing: multialternative decision field theory (MDFT; Roe et al. 2001) and the leaky competing accumulators (LCA) model (Usher & McClelland, 2004). Both models

assume that there is a single set of cognitive principles that underlie the effects. The inference and perceptual experiments discussed above confirm this assumption by demonstrating that the three effects can arise in the same experimental paradigm. Although MDFT and the LCA model share many features, a key difference between them is that only the LCA model accounts for the attraction and compromise effects using loss aversion. Loss aversion is the assumption that losses impact the choice process more than equivalent gains (Tversky & Kahneman, 1991; Tversky & Simonson, 1993).

The existence of all three context effects in inference and perception calls into question the loss aversion explanation. In these experiments, the attributes are nonhedonic, and there is no notion of gains or losses on which loss aversion could operate. It might be possible to reinvent the LCA's assumption of loss aversion as an asymmetric weighting of advantages and disadvantages (rather than losses and gains): a "disadvantage aversion" assumption. Although the assumption of disadvantage aversion might allow the LCA to predict context effects for perceptual choices, it is premature to accept this assumption as a replacement for loss aversion, because it has not been established how this change influences the LCA's account for the well-established context effects using traditional consumer choices.

Although the LCA model uses loss aversion to account for the attraction and compromise effects, MDFT accounts for these effects by comparing options along dominance and indifference dimensions (Hotaling, Busemeyer, & Li, 2010). This explanation is able to generalize to multiple domains including inference and perception because differences in attribute values are not arbitrarily weighted. Instead, an option's relevance is determined by whether it is viewed as dominated or indifferent to other options in the choice set. Even though MDFT provides a parsimonious and general explanation of context effects, it still requires computationally intensive simulations and is difficult to fit to data from multialternative choice experiments. In the following sections, we provide an overview of both MDFT and the LCA model and discuss their strengths and weaknesses.

Previous Dynamic Models

Both MDFT and the LCA model conceive choice behavior as the gradual and random accumulation of evidence over time, with a decision made when the accumulated evidence reaches a threshold amount. This same idea forms the basis for a number of different cognitive models used to study a wide range of phenomena including recognition memory (Ratcliff, 1978), perceptual discrimination (Link, 1992), sensory detection (Smith, 1995), conceptual categorization (Ashby, 2000; Nosofsky & Palmeri, 1997), cognitive architectures (Townsend & Wenger, 2004), and confidence (Pleskac & Busemeyer, 2010).

Both MDFT and LCA also assume that evidence accumulation is "leaky," corresponding to memory loss for previous preferences. Technically, the models assume an Ornstein–Uhlenbeck diffusion process. Both models also build on Tversky's (1972) elimination by aspects heuristic, by incorporating a sequential scanning of attributes. Details for both models can be found in the appendices.

Multialternative Decision Field Theory

MDFT extends decision field theory (Busemeyer & Townsend, 1992, 1993) to accommodate multialternative choice situations. Decision field theory can explain numerous phenomena in decision making including violations of stochastic dominance, violations of independence between alternatives, regret, relations between choice probability and decision time, effects of time pressure on decision accuracy, approach–avoidance, and violations of strong stochastic transitivity.

MDFT assumes that the preferences for each alternative evolve across time through a series of comparisons and evaluations of the alternatives. The preferences continue to evolve until the preference state for one of the options reaches a threshold level (a "criterion") and the corresponding option is selected. Preference states are determined by valences for each option and competition (lateral inhibition) among the options. The valence values represent the advantages or disadvantages of the alternatives at each moment in time and are constructed from three components: subjective values, stochastic attention weights, and a comparison mechanism.

The strength of the lateral inhibition is determined by the distance between two options in an "indifference/dominance" space (Hotaling et al., 2010). The indifference/dominance space is a way to represent how options are perceived in relationship to each other and the goal of the choice. Options that fall along lines of indifference are selected with equal probability. Options that are dominated by other options are selected less often. In Hotaling et al. (2010), it is assumed that the line of indifference corresponds to the direction of the unit-length vector $\frac{1}{\sqrt{2}} \cdot [-1, 1]'$ and the line of dominance corresponds to the direction of the unit-length vector $\frac{1}{\sqrt{2}} \cdot [1, 1]'$. In Figure 1, the line of indifference corresponds to the dotted line, and the line of dominance would be perpendicular to this line. Hotaling et al. assumed that lateral inhibition increases slowly along the line of indifference and rapidly along the line of dominance because dominated options are quickly discarded.

In total MDFT uses four free parameters for fitting choice probabilities. One parameter, $p(w_P)$, is the attention weight reflecting how important each attribute is to the decision maker. Another parameter, the dominance dimension weight, β, is used in the indifference/dominance distance function, which reflects how much more weight is given to improvements in both attributes versus trade-offs between the attributes. The final two parameters, ϕ_1 and ϕ_2, are associated with a Gaussian mapping used to convert distances to lateral inhibition strengths. The model also has a variance parameter σ^2 that determines the noise in the accumulation process and a threshold parameter associated with the decision criterion. These two additional parameters are often fixed for simulations. A formal description of the model is provided in Appendix A.

The Leaky Competing Accumulator Model

In the LCA model, each option in a choice set is associated with an activation value that changes across time. Similar to preferences in MDFT, the amount of activation is determined by a series of comparisons and evaluations of the alternatives. Activation changes across time until one of the options reaches a decision criterion and is selected. In contrast to the linear MDFT model, the LCA model assumes two types of nonlinearity. First, activation values are not allowed to become negative. Second, the model incorporates an asymmetric value function.

Unlike MDFT, the LCA model does not compute valences to compare options. Instead, the advantages or disadvantages of an alternative are calculated using an asymmetric value function that weights losses greater than gains following Tversky and Kahneman (1991) and Tversky and Simonson (1993). Activation states for different alternatives are determined by input from the asymmetric value function and lateral inhibition between options. The strength of lateral inhibition is constant across alternatives and does not depend on the distance between alternatives (as in MDFT). A complete description of the model can be found in Appendix B.

Discussion of MDFT and the LCA Model

MDFT and the LCA model have contributed greatly to our understanding of multialternative, multiattribute choice behavior. Both models can simulate choice tasks involving internally controlled stopping times where the decision maker is free to deliberate as long as he or she desires before making a choice (Ratcliff, 1978; Vickers, Smith, & Brown, 1985). This procedure is commonly used in multialternative decision tasks including all of the experiments presented here. However, because the simulations are very computationally demanding, it is difficult to fit the models to empirical data from a multialternative choice task using internally controlled stopping times.

Externally controlled stopping time experiments—where decisions are made at a fixed or designated time point—provide an alternative to the internally controlled paradigm (Ratcliff, 1978, 2006; Vickers et al., 1985). MDFT has an analytic solution in the externally controlled paradigm, and so can be easily fit to experimental data. In contrast the LCA model does not have an analytical solution, due to its nonlinearities, and so computer-intensive simulations are still required.

Another difficulty with the LCA model is its reliance on loss aversion to produce the attraction and compromise effects. As discussed previously, the loss aversion principle is difficult to extend equally to perceptual experiments as well as the standard consumer choice experiments. In perceptual experiments, there are no trade-offs between losses and gains of attribute values. Tsetsos, Usher, and Chater (2010) claimed that the LCA model follows Tversky and Kahneman (1991) in the assumption that the asymmetric relationship between losses and gains is a primitive and is hardwired in the neural system. MDFT does not build in loss aversion; instead loss aversion emerges from the dynamics of the MDFT.

A difficulty with MDFT is the possibility of unbounded preference states in situations with long external stopping times. For some parameter settings, the system becomes unstable and preference states grow without bound (Tsetsos et al., 2010). This problem can be avoided by allowing different parameters settings (particularly for the noise variance) between the three context effects. Hotaling et al. (2010) argued that allowing the variance to differ among effects captures differences in experimental stimuli. Although this explanation might have been reasonable for previous

experimental designs (e.g., Huber et al., 1982; Simonson, 1989; Tversky, 1972), the new experiments reported by Trueblood (2012) and Trueblood et al. (2013) found all three effects with the same stimuli. In these cases, it is unclear why there should be different noise parameters for the different effects.

Besides instability problems, MDFT is also restricted in explaining experimental results involving the compromise effect. In MDFT, the compromise effect is explained by the correlation of extreme options. Because the extreme options are far apart in the indifference/dominance space, there are weak inhibitory connections between these options. However, there are strong inhibitory connections between the compromise option and the two extremes. Due to these strong and weak interconnections, the momentary preference states for the extreme options become anticorrelated with the compromise option and correlated with each other. The correlated extremes split their winnings and are selected less often than the compromise option.

Usher, Elhalal, and McClelland (2008) experimentally tested temporal correlation in the compromise effect. In their experiment, subjects were given a ternary choice set and asked to choose one of the options. Then, after the selection was made, subjects were told that their preferred option was no longer available and were asked to choose again. If extreme options are correlated, as in MDFT, when one extreme is initially selected, the other also has a high level of activation, and so is more likely to be selected as the second choice. However, experimental results showed that the compromise option was selected more often following an initial choice of an extreme. Tsetsos et al. (2010) demonstrated through simulations that the LCA model makes the correct prediction, unlike MDFT. Hotaling et al. (2010) argued that MDFT can account for the results of Usher et al. (2008) if asymmetric attention weighting is allowed. They argued that an extreme option is only initially selected if an individual is attending to one dimension more than the other.

This discussion illustrates that there are several limitations to both MDFT and the LCA model. We propose the MLBA model in attempt to overcome these limitations. First, the model has simple analytic solutions for both internally and externally controlled decision tasks and can easily be fit to data using either procedure. The model does not assume loss aversion, so can explain the presence of context effects across a number of domains. Further, it does not suffer from the same instabilities as MDFT with regard to the compromise effect.

The Multiattribute Linear Ballistic Accumulator Model

The MLBA model is an extension of the linear ballistic accumulator (LBA) model developed by Brown and Heathcote (2008). It also incorporates the LCA model's use of pairwise differences in calculating the inputs to each accumulator. We begin with a discussion of the LBA model and then provide a detailed description of the MLBA model.

The Linear Ballistic Accumulator Model

The LBA models choice and response times using independent accumulators that race toward a threshold. The accumulators are linear and accumulate information deterministically during a trial. Deterministic accumulation leads to greater mathematically tractability than stochastic models with moment-to-moment fluctuations in information, but the LBA can still accommodate benchmark empirical phenomena including predicting fast and slow errors and the shape of speed–accuracy trade-off curves (Brown & Heathcote, 2008). The model has a closed-form analytic likelihood function for choices between any number of alternatives, making it easy to apply to data (derivations for the LBA model's likelihood functions can be found in the appendix of Brown & Heathcote, 2008). The analytic solutions are a consequence of the simplifying assumptions that evidence accumulation is both linear and deterministic. Although these assumptions probably do not reflect the true state of neurophysiological processes (at least at a single-cell level), the model's success in accounting for both behavioral and neural data suggests that the LBA's trade-off between veracity and simplicity is reasonable (Forstmann et al., 2008, 2010).

In the LBA model, a choice between three alternatives is represented by a race between three evidence accumulators. For example, Figure 2 illustrates a choice among options {X, Y, Z}. At the beginning of a trial, each accumulator starts at a randomly determined amount of evidence drawn independently for each accumulator from a uniform distribution on the interval [0, A]. The accumulators' activations increase at speeds defined by a drift rate associated with each response choice, until one of the accumula-
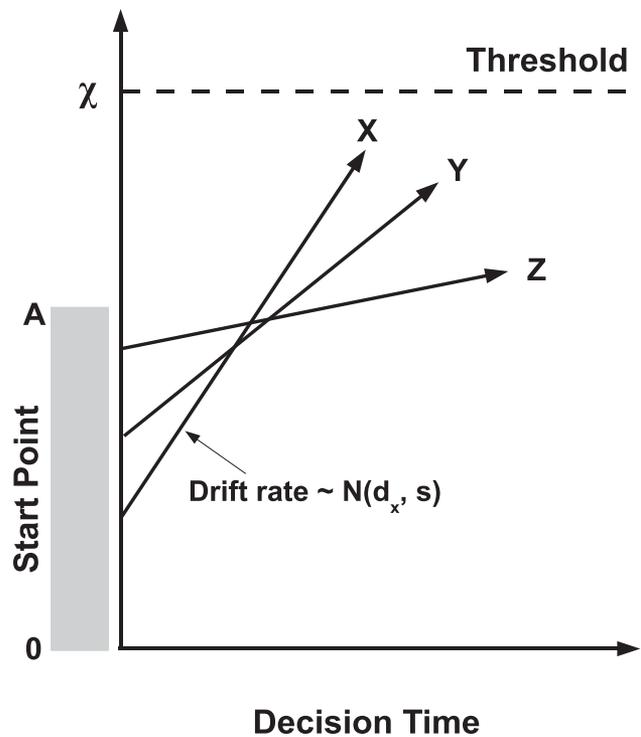

Figure 2. The linear ballistic accumulator model with three accumulators for options X, Y, and Z. Accumulators begin at randomly determined starting points and increase at speeds determined by the drift rates. The first accumulator to reach the threshold χ is selected.

tors reaches its threshold χ.[2] The alternative associated with the accumulator that reaches the threshold first is selected. On each trial, the drift rates are drawn from normal distributions. In many past applications, the normal distributions have freely estimated mean values, $d_1, d_2, d_3, \ldots$, and a common standard deviation s, which is often fixed.

The MLBA model adds to the LBA model by explicitly specifying how drift rates arise from the evaluation of choice stimuli. We call the LBA process the "back end" of the MLBA, because it describes how a final choice is selected after the "front-end" processes transform stimulus inputs into drift rates.

Description of the MLBA Model

Consider three options that vary along two attributes, P and Q. Let P_i and Q_i denote the actual value of option i on the two dimensions. For example, in the inference experiment, P_i would represent the testimony strength of eyewitness P for suspect i. Likewise, Q_i would represent the testimony strength of eyewitness Q for suspect i. Alternatively, in a consumer choice experiment, P and Q might indicate two attributes such as price and quality, with P_i indicating the price of option i. Formally, the mean drift rate d_i for each alternative i is defined by comparing that option against the other two:

$$d_1 = V_{12} + V_{13} + I_0, \tag{1a}$$

$$d_2 = V_{21} + V_{23} + I_0, \tag{1b}$$

$$d_3 = V_{31} + V_{32} + I_0. \tag{1c}$$

The term V_{ij} represents a comparison between options i and j, and I_0 is a positive constant. The mean drift rates resemble the inputs in the LCA model (see Equation B1), but are not determined by a loss aversion function and do not fluctuate during the trial.

To determine V_{ij}, we begin by calculating the subjective values u for each alternative. These are psychological representations of the raw stimulus values. In the LCA model, the subjective values are determined by a loss aversion function, whereas MDFT does not explicitly specify the form of subjective values. In the MLBA model, we calculate subjective values by mapping objective quantities (such as price, quality, eyewitness testimony strengths, or rectangle height) to psychological magnitudes.

Consider a pair of options (P_1, Q_1) and (P_2, Q_2) designed to be indifferent. In the inference experiments (Trueblood, 2012), for example, options were defined as indifferent by the additive rule: $P_1 + Q_1 = P_2 + Q_2$. In other words, two criminal suspects were assumed to be indifferent if the sum of their eyewitness testimony strengths were equal. In the perceptual experiments (Trueblood et al., 2013), a pair of options were defined as indifferent if they had equal area, that is, if $\log(P_1) + \log(Q_1) = \log(P_2) + \log(Q_2)$. However, these simple and rational definitions of indifference might not correspond to human perceptions of indifference. In other words, an individual's subjective valuation of the options might differ from the way the options were defined experimentally. For example, suppose (P_1, Q_1) represents a criminal suspect with extreme eyewitness testimony strengths such as (20, 80) and (P_2, Q_2) is another suspect with intermediate eyewitness testimony strengths such as (55, 45). Objectively, these two criminal suspects should be viewed as equally likely to have committed the crime

because $P_1 + Q_1 = P_2 + Q_2 = 100$. However, Chernev (2004) has argued that when options are described using the same metric, as in the inference and perceptual experiments, the option with less dispersion of its attribute values is preferred. With respect to our example, the suspect with eyewitness strengths of (55, 45) would be preferred to the one with eyewitness strengths of (20, 80).

To capture possible differences between extreme and intermediate alternatives, we introduce curvature to the attribute space as illustrated in Figure 3. The curves are defined using an exponent m and obey

$$\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 1. \tag{2}$$

The relationship between extreme and intermediate alternatives is governed by the value of the m parameter. If the curve is concave (i.e., $m > 1$), then intermediate options (i.e., those with less attribute dispersion) are preferred to extreme options. The opposite is true when the curve is convex ($0 < m < 1$). When $m = 1$, the curve reduces to a straight line, implying subjective and objective values are equal. The curves also preserve equal preference for symmetric options. That is, an option with attributes (P_1, Q_1) and another option with attributes (Q_1, P_1) are indifferent. A detailed description of the curvature mapping can be found in Appendix C.

Let (u_{Pi}, u_{Qi}) and (u_{Pj}, u_{Qj}) be the subjective values for options i and j determined by the mapping from objective to subjective values. Similar to Tversky and Simonson (1993) and Usher and McClelland (2004), we assume that options are evaluated in relation to each other. That is, each option is used as a reference point in the evaluation of the other options. In the valuation function V_{ij} in Equation 1, option i is the target and option j is evaluated relative to it. We assume this function is defined by the difference in the subjective values of the options:

$$V_{ij} = w_{Pij} \cdot (u_{Pi} - u_{Pj}) + w_{Qij} \cdot (u_{Qi} - u_{Qj}). \tag{3}$$

Note that because differences in subjective values can be both positive and negative, V is not necessarily symmetric (i.e., $V_{ij} \neq V_{ji}$).

The weights w_{Pij} and w_{Qij} reflect the amount of attention given to a particular comparison. Attention, quantified by visual fixation, has been shown to influence the valuation process in multialternative choice—options that are fixated on more are more likely to be chosen (Krajbich & Rangel, 2011). It is also well known that fixation duration increases with decreasing discriminability of the target (Gould, 1967, 1973; Hooge & Erkelens, 1998; Jacobs, 1986; Jacobs & O'Regan, 1987). Thus, we hypothesize the attention weights should be larger when attribute values are similar and smaller when they are easy to discriminate. We define the weights using Shepard's (1987) famous law of generalization in which similarity is an exponentially decaying function of distance:

$$\begin{aligned} w_{Pij} &= \exp(-\lambda \,|\, u_{Pi} - u_{Pj}\,|) \\ w_{Qij} &= \exp(-\lambda \,|\, u_{Qi} - u_{Qj}\,|). \end{aligned} \tag{4}$$

Tversky (1977) showed that similarity judgments often violate symmetry (the similarity of A to B can be different from that of B

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[2] In other writing, the threshold parameter of the LBA model is often denoted by b.
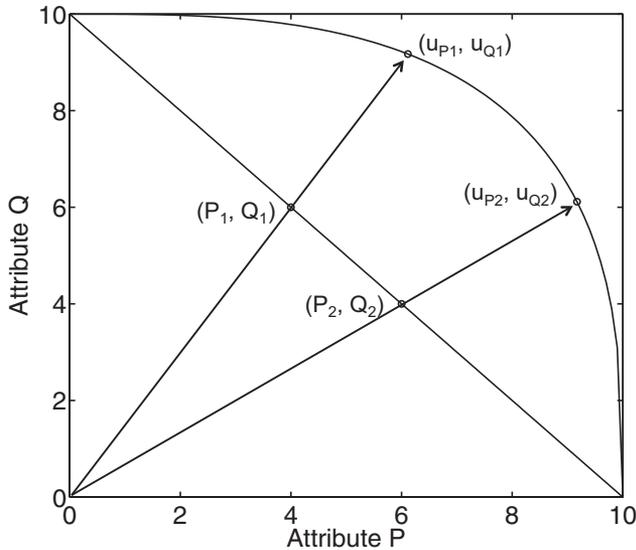
Figure 3. Function used to map experimentally defined options to subjective values. The curve is defined by one free parameter and can be concave or convex depending on the value of the parameter.

to A). To allow for such violations, we follow Nosofsky (1991) in using different parameterizations for w_{Pij} and w_{Pji} and likewise for w_{Qij} and w_{Qji}. If the difference in attribute values is positive (e.g., $u_{Pi} - u_{Pj} \geq 0$), then we set $\lambda = \lambda_1$. If the difference is negative, we set $\lambda = \lambda_2$.

Note that the attention weights do not add up to 1. The weights are not meant to quantify the exact distribution of attention during a trial. Rather, the weights capture the overall trend that similar, difficult-to-discriminate options receive more attention than those that are easy to discriminate. The weights are dependent on the values of the attributes similar to those in multiattribute utility theory (Keeney & Raiffa, 1976). However, unlike in multiattribute utility theory, the weights do not represent the importance of the attributes.

The constant I_0 in Equation 1 ensures that at least one of the mean drift rates is positive. This is necessary in order to avoid nontermination in the LBA model.[3] This value can be seen as a baseline rate of evidence accumulation for the available options, similar to I_0 in the LCA model given in Equation B1.

In total, the MLBA model uses four free parameters to define the mean drift rates: the curvature parameter m, the two decay parameters λ_1 and λ_2 used to define the attention weights, and the input constant I_0. The model also has three additional parameters, a starting point parameter A, a threshold parameter χ, and a drift rate noise parameter s. These additional parameters are fixed when only modeling choice probabilities, but must be estimated if fitting response times as well. Table 2 lists the model parameters and their allowable ranges.

The MLBA model accounts for the attraction effect through the attention weights w_{Pij} and w_{Qij}. Consider the choice set $\{X, Y, R_X\}$ where R_X is an inferior decoy near X. Because the distance between X and R_X is smaller than the distance between Y and R_X, the attentions weights are larger for the comparison of X and R_X than Y and R_X. Thus, the differences in subjective values for X and R_X

receive more weight than the differences in subjective values for Y and R_X. Psychologically, the model predicts the attraction effect occurs because X and R_X are more difficult to discriminate than Y and R_X, which leads to increased attention to X and R_X during the evaluation process.

The model predicts that the similarity effect can occur when people weight supportive information more than disconfirmatory evidence. Consider the choice set $\{X, Y, S_X\}$ where S_X is a competitive option near X. In the valuation function V_{XS_X}, option X acts as a reference point and option S_X is evaluated relative to it. If the decay constants obey $\lambda_1 < \lambda_2$, negative differences "decay" faster than positive differences. In other words, evidence supporting the reference option X is weighted more than evidence against X. Overall, this can lead to a small positive valuation V_{XS_X} because the differences in subjective values are small (i.e., the distance between X and S_X is small) and positive differences receive more weight than negative differences. Similarly, the comparison of option Y to option S_X can result in a large positive valuation V_{YS_X} because the differences in subjective values are large (i.e., the distance between Y and S_X is large) and positive differences receive more weight. Thus, the mean drift rate for Y can be larger than the mean drift rate for X, leading to the similarity effect. Asymmetry in attention weights where positive differences receive more weight than negative differences could reflect a confirmation bias (Nickerson, 1998). Research has suggested that people tend to seek information that supports a selected point of view rather than disconfirmatory evidence (Koriat, Lichtenstein, & Fischhoff, 1980). Although asymmetry of attention weights is a necessary condition for producing the similarity effect, it is not necessary for the attraction and compromise effects. These effects can arise for any relationship between λ_1 and λ_2.

The compromise effect arises through the subjective value function given in Equation 2. This function allows for curvature that can result in advantages for compromise options. For example, if the m parameter is greater than 1, midrange options (i.e., those with less attribute dispersion) are preferred to extremes, thus producing a compromise effect. The compromise effect is further enhanced through the attention weights w_{Pij} and w_{Qij}. Consider the choice set $\{X, Y, C_X\}$ where Y and C_X are extreme options and X is the compromise. Because the distance between the compromise and extremes is smaller than the distance between the two extremes, the attention weights are larger for the comparison of X to Y and X to C_X than comparisons of the extremes. This implies that the differences in subjective values involving X receive more weight than those involving extremes.

Model Predictions for Deliberation Time Effects

The MLBA model predicts that preferences are determined "in expectation" during the front-end, preprocessing stage, but these preferences are subject to random variation during the back-end

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[3] The MLBA model can be further constrained to ensure that all of the drift rates are positive. This could be done in several different ways. For example, I_0 could be defined as the absolute value of the minimum of the sums $\{V_{12} + V_{13}, V_{21} + V_{23}, V_{31} + V_{32}\}$ plus a constant. Another alternative is to define the drift rates as an exponential function of the binary comparisons, for example $d_1 = c \cdot \exp(V_{12} + V_{13})$. Positivity is not necessary for termination in the LBA model. As long as one of the drift rates is positive, the model can produce a decision.

Table 2
Parameters and Allowable Ranges for the Multiattribute Linear Ballistic Accumulator Model 2

Parameter	Description	Allowed ranges
m	Exponent transforming objective to subjective values	$m > 0$
λ_1	Decay constant for attention weights with positive differences	$\lambda_1 \geq 0$
λ_2	Decay constant for attention weights with negative differences	$\lambda_2 \geq 0$
I_0	Baseline input	$I_0 \geq 0$
A	Uniform distribution range for accumulator starting points	Fixed $A = 1$ for choices only
χ	Threshold amount of evidence required to trigger a choice	Fixed $\chi = 2$ for choices only
s	Drift rate variability across repeated decisions	Fixed $s = 1$ for choices only

selection process. In particular, the back-end process allows for both variability in the starting point and rate of evidence accumulation. The combination of both the front-end and back-end processes are crucial for understanding how preferences are influenced by deliberation time. The MLBA model predicts that preferences strengthen with deliberation time, which also means that context effects are predicted to increase as deliberation time increases. Below we discuss previous results demonstrating that the attraction and compromise effects increase with longer deliberation time. We then describe two new experiments showing that the similarity effect also increases with deliberation time.

Attraction and Compromise Time Pressure Effects

Recently, Pettibone (2012) tested the effects of time pressure on both the attraction and compromise effects, using choices about consumer products. In this experiment, subjects were assigned to either an attraction or compromise condition and also to one of four deliberation time conditions: 2, 4, 6, or 8 s. At the end of the deliberation time, the choice sets were removed from the computer screen and subjects were instructed to make a decision. Results showed that as deliberation time increased, the size of the attraction and compromise effects increased. There is also some past experimental evidence suggesting that the compromise effect increases with deliberation time. Simonson (1989) found that subjects who selected compromise options provided longer self-reports of their decision protocols. Also, Dhar, Nowlis, and Sherman (2000) found a decrease in the size of the compromise effect when deliberation time was limited, as compared to subjects with unlimited deliberation time.

Both MDFT and the LCA model predict that the attraction and compromise effects increase as deliberation time increases. In these models, the attraction and compromise effects are produced by comparisons of the options across time. As the number of comparisons increase, the effects grow in magnitude. In MDFT, this comparison process is based on distance-dependent lateral inhibition in indifference and dominance coordinates. In the LCA model, the comparison process is based on an asymmetric loss aversion function.

To examine the MLBA model's predictions for time pressure effects, we calculated the choice probabilities for the model as a function of deliberation time using the artificial stimuli given in Table 3. In this demonstration we fixed the parameters to the following values: the curvature parameter $m = 5$, the two decay parameters $\lambda_1 = .2$ and $\lambda_2 = .4$, and the initial input $I_0 = 5$. This set of parameters is one of many possible combinations that can produce all three effects simultaneously. For this demonstration,

we also set $A = 1$ and $s = 1$. We used an externally controlled stopping time procedure, so there was no response boundary. A more detailed discussion of the model's ability to simultaneously produce all three effects is provided in a later section. Figure 4 illustrates the relationship between effect size and deliberation time for all three context effects. As with MDFT and the LCA model, the MLBA model predicts that the attraction and compromise effects grow as deliberation time increases. The MATLAB programs that produced the predictions and Figure 4 are in the supplemental materials.

Similarity Time Pressure Effects

For the compromise and attraction effects, all three models discussed above make the same predictions—that the effects will grow with deliberation time. However, the similarity effect is different and might distinguish between the models. The MLBA model predicts the similarity effect will grow with deliberation time. In MDFT, the relationship between deliberation time and the magnitude of the similarity effect is governed by the attention switching mechanism. When equal weight is placed on the two attributes (i.e., $p(w_P) = p(w_Q) = .5$ in Appendix A), MDFT predicts the similarity effect diminishes with increased deliberation time (Busemeyer & Johnson, 2004). However, when there is unequal attribute weighting, so that more weight is placed on the most favorable attribute of the dissimilar alternative, MDFT predicts the similarity effect increases with increased deliberation time. Currently, it is unknown whether the LCA model predicts an increase or decrease in the similarity effect with deliberation time. However, the LCA model uses the same attention switching mechanism as MDFT, and thus it is likely that the relationship between

Table 3
Artificial Stimuli Used in the Qualitative Analyses of the Multiattribute Linear Ballistic Accumulator Model

| Effect	X	Y	Decoy
Attraction (range decoy) Set 1	(4, 6)	(6, 4)	(3.4, 6)
Attraction (range decoy) Set 2	(4, 6)	(6, 4)	(6, 3.4)
Attraction (frequency decoy) Set 1	(4, 6)	(6, 4)	(4, 5.4)
Attraction (frequency decoy) Set 2	(4, 6)	(6, 4)	(5.4, 4)
Attraction (range–frequency decoy) Set 1	(4, 6)	(6, 4)	(3.7, 5.7)
Attraction (range–frequency decoy) Set 2	(4, 6)	(6, 4)	(5.7, 3.7)
Similarity Set 1	(4, 6)	(6, 4)	(3.8, 6.2)
Similarity Set 2	(4, 6)	(6, 4)	(6.2, 3.8)
Compromise Set 1	(4, 6)	(6, 4)	(8, 2)
Compromise Set 2	(4, 6)	(6, 4)	(2, 8)
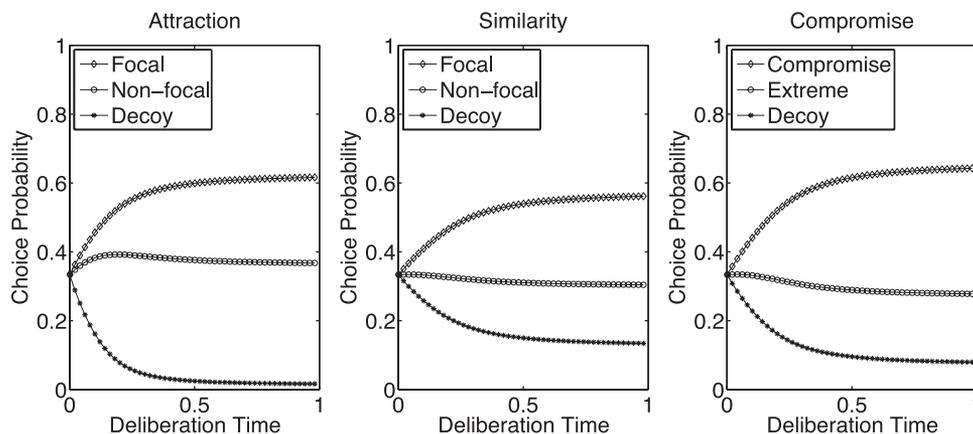
Figure 4. Multiattribute linear ballistic accumulator model predictions for the relationship between deliberation time and the size of context effects. The model predicts that all three effects grow over time, shown by the increasing probability of choosing the focal or compromise alternative.

the similarity effect and deliberation time is contingent on the attention weights as in MDFT.

We are not aware of any experimental work examining the relationship between time pressure and the similarity effect, which makes these model predictions truly a priori. Thus, we developed two new experiments to investigate this relationship. In the first experiment, participants made decisions about criminal suspects based on eyewitness testimony strengths, as in Trueblood (2012). In the second experiment, participants made decisions about the size of rectangles, as in Trueblood et al. (2013). In both experiments, deliberation time was manipulated similarly to Pettibone (2012).

Inference Time Pressure Experiment

Forty-seven undergraduate students from the University of California, Irvine, participated for course credit, completing the computer-based experiment online at a time of their choosing. Participants were told that on each trial they would see eyewitness testimony strengths for three criminal suspects from two eyewitnesses, and they should select the suspect most likely to have committed the crime. Participants were told that the eyewitness testimony strengths ranged from 0 to 100, with 0 implying very weak evidence of guilt and 100 implying very strong evidence of guilt. Eyewitness strengths were determined in the same manner as in Trueblood (2012). Participants were also told that the eyewitness strengths would only be shown for a brief period and would be hidden after this period. They were asked to make their decisions as quickly as possible once the testimony strengths had been removed. This method for creating time pressure is similar to the one used by Pettibone (2012).

Three deliberation times were used: 1, 2, and 5 s. Each participant completed a practice block of 15 trials with the shortest deliberation time of 1 s. After completing the practice trials, each participant completed three blocks of 48 randomized trials that were divided into 28 similarity trials and 20 filler trials. The 28 similarity trials were further divided so that the decoy was placed near one alternative for half of the trials and near the other alternative for half of the trials. The filler trials were used to assess

accuracy as in Trueblood (2012). The three blocks corresponded to the three deliberation times, and their presentation order was randomized across subjects. This provides a powerful within-subjects test of the relationship between time pressure and the similarity effect. Note that the same amount of information was presented on all trials. The only difference between blocks was the duration the information was on the screen.

Results for the Inference Time Pressure Experiment

One subject was excluded from the analyses because his or her accuracy on the filler trials was 2 standard deviations lower than the average. For all three deliberation times, the mean accuracy on the filler trials was above the probability of guessing of .33 ($M =$ .40 for the 1-s block, $M = .50$ for the 2-s block, and $M = .61$ for the 5-s block). Figure 5 shows the mean choice probability of the focal alternative (i.e., the dissimilar alternative) compared to the mean choice probability of the nonfocal alternative collapsed across the different positions of the decoy for the three deliberation times. To analyze the data, we conducted a 2 (context) $\times$ 3 (deliberation time) repeated-measures analysis of variance. The dependent variable was the percentage of times that a subject chose an option (focal or nonfocal). A main effect of context, $F(1, 45) = 95.76$, $p < .001$, indicated that participants preferred the focal option to the nonfocal option, demonstrating a strong similarity effect. There was no significant main effect of deliberation time, $F(2, 90) = 1.25$, $p = .29$. However, the interaction of context by deliberation was significant, $F(2, 90) = 7.51$, $p < .001$, indicating that the similarity effect increases with longer deliberation times as predicted by the MLBA model.

One explanation for the data is that participants were unable to read all pieces of information during the trials in the 1-s block. Thus, the results could be influenced by regression effects due to increased noise under time pressure. This is not problematic from a theoretical perspective, as the mechanism by which time pressure causes reduced similarity effects in the MLBA is consistent with regression toward chance responding: With greater time pressure comes greater influence of start point variability, just as in the
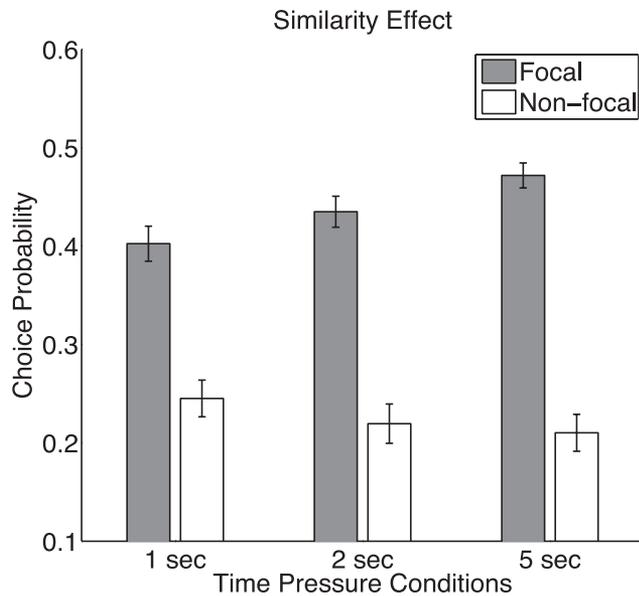
Similarity Effect

Figure 5. Mean choice probabilities for the inference time pressure experiment with error bars showing the standard error of the mean. The similarity effect increases with increasing deliberation time (1, 2, and 5 s), as predicted by the multiattribute linear ballistic accumulator model.

speed–accuracy trade-off described in the standard LBA model (Brown & Heathcote, 2008).

Perceptual Time Pressure Experiment

We wanted to demonstrate that the relationship between the size of the similarity effect and deliberation time was not specific to the inference paradigm discussed above. As such, we developed a new experiment to test this relationship in perception. For this experiment, 47 undergraduate students from the University of California, Irvine, participated for course credit, completing the computer-based experiment online at a time of their choosing. Participants were told that on each trial they would see three rectangles and to select the rectangle that had the largest area. As in the inference time pressure experiment discussed above, participants were told that the rectangles would only be shown for a brief period and would be hidden after this time. They were asked to make their decisions as quickly as possible once the rectangles had been removed.

Three deliberation times were used: 250, 400, and 800 ms. Each participant completed three practice trials with the shortest deliberation time of 250 ms. After completing the practice trials, each participant completed three blocks of 80 randomized trials, which were divided into 60 similarity trials and 20 filler trials. The 60 similarity trials were further divided so that the decoy was placed near one alternative for half of the trials and near the other alternative for half of the trials. The filler trials were used to assess accuracy as in Trueblood et al. (2013). The three blocks corresponded to the three deliberation times, and their presentation order was randomized across subjects.

Results for the Perceptual Time Pressure Experiment

For all three deliberation times, the mean accuracy on the filler trials was above the probability of guessing of .33 (M = .57 for the 250-ms block, M = .56 for the 400-ms block, and M = .73 for the 800-ms block). This shows that even at the shortest deliberation time, participants were not simply guessing. Figure 6 shows the mean choice probability of the focal alternative (i.e., the dissimilar alternative) compared to the mean choice probability of the non-focal alternative collapsed across the different positions of the decoy for the three deliberation times. To analyze the data, we calculated a 2 (context) $\times$ 3 (deliberation time) repeated-measures analysis of variance on choice proportions. A main effect of context, $F(1, 46) = 38.49$, $p < .001$, indicated that participants preferred the focal option to the nonfocal option, demonstrating a strong similarity effect. There was no significant main effect of deliberation time, $F(2, 92) = 1.31$, $p = .275$. However, the interaction of context by deliberation time was significant, $F(2, 92) = 3.98$, $p = .022$, indicating that the similarity effect increases with deliberation time as in the inference experiment and as predicted by the MLBA model.

Discussion of Experimental Results

Both the inference and perceptual experiments demonstrate that the similarity effect increases with deliberation time. The results of the experiments support the prediction of the MLBA model that context effects strengthen with longer deliberation. The MLBA model produces an increase in all three context effects with time due to variability in the starting points of the evidence accumulators. This initial randomness in the accumulated evidence results in the options being equally favored at very short deliberation times.
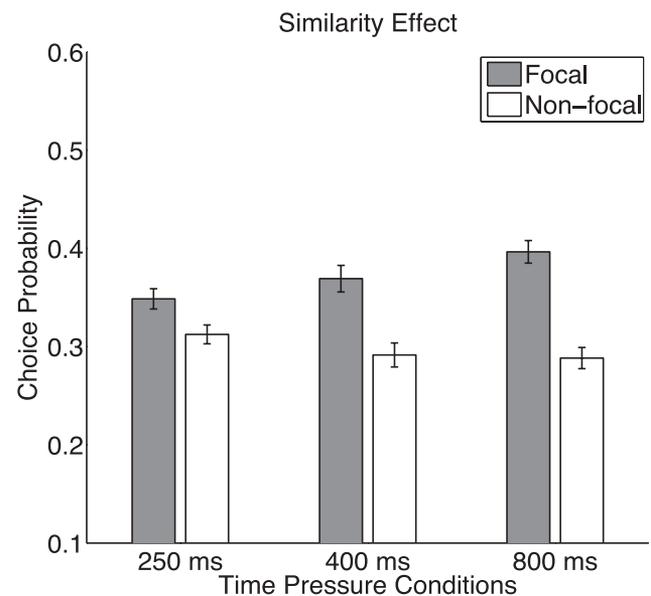
Similarity Effect

Figure 6. Mean choice probabilities for the perceptual time pressure experiment with error bars showing the standard error of the mean. The similarity effect increases with increasing deliberation time (250, 400, and 800 ms), as predicted by the multiattribute linear ballistic accumulator model.

As deliberation time increases, the initial randomness is overcome by the integration of evidence for each option (quantified by the drift rate) so that one option is favored over the others. This is the same mechanism that gives rise to the well-known speed–accuracy trade-off in the LBA model (Brown & Heathcote, 2008). This mechanism is consistent with, and is a mechanistic explanation for, the notion that context effects reduce with time pressure due to regression toward chance responding.

Qualitative Modeling Results

In this section, we discuss several qualitative results for the MLBA model. First, we illustrate that the model can account for all three context effects using the same set of parameters. We then demonstrate the model can account for the findings of Usher et al. (2008) involving unavailable options in the compromise effect. We conclude by showing that the model can also produce violations of the regularity principle.

Predictions for Three Context Effects

Both MDFT and the LCA model can account for all three context effects using a single set of parameter values. In previous literature, this has typically been demonstrated by selecting one set of parameters and running simulations showing the three effects with these parameters. Rather than find just one set of MLBA parameters that can produce the effects, we were interested in exploring the entire parameter space. Because the MLBA model has analytical solutions, this is computationally feasible. Our goal was to find multiple parameter combinations that can simultaneously produce five effects: the similarity effect, the compromise effect, and the three kinds of attraction effects (for range, frequency, and range–frequency decoys).

We used artificial stimuli in a two-dimensional attribute space, with a similar stimulus structure to Figure 1. These stimuli are given in Table 3. For each effect, two choice sets were defined with the decoy placed near one alternative in one set and near the other alternative in the other set, mirroring standard experimental procedures. The effects were assessed using the method of comparing ternary choice sets described previously in relation to the experimental work.

In our exploration of the parameter space, we examined the values of the four parameters used to define the mean drift rates: the curvature parameter m, the two decay parameters λ_1 and λ_2, and the initial input I_0. Because we were only analyzing choice probabilities, we fixed the starting point parameter to $A = 1$, the threshold parameter to $\chi = 2$, and the drift rate noise parameter to $s = 1$. These parameters are important for fitting response times, but not response proportions, unless they differ between accumulators and/or experimental conditions. In this case, the three parameters can also affect choice proportions.

We defined a grid over the parameter space and examined 3,840 parameter combinations across the ranges $.5 \le m \le 5$, $.1 \le \lambda_1 \le .8$, $.1 \le \lambda_2 \le .8$, and $5 \le I_0 \le 10$. Out of these combinations, 348 produced the five effects simultaneously. Although the parameter values producing the effects are specific to the stimuli used, other sets of stimuli will also give rise to the co-occurrence of the effects; the MLBA model's ability to account for the effects simultaneously does not depend on the specific stimuli given in Table 3.
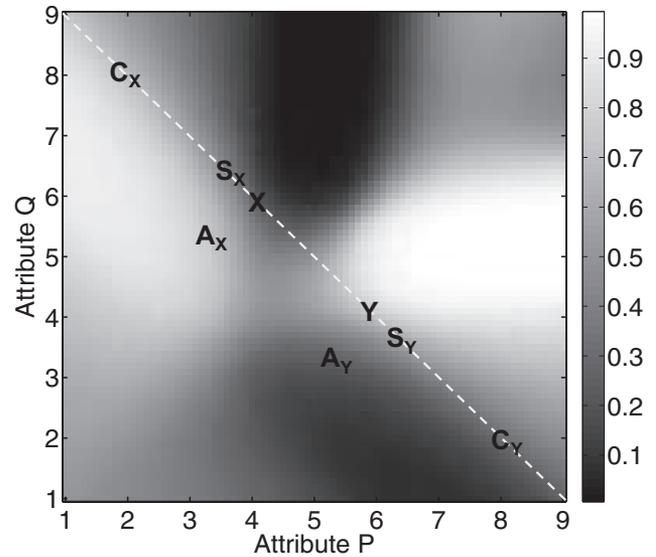

Figure 7. Relative choice shares for option $X = (4, 6)$ as compared to option $Y = (6, 4)$ for various positions of the decoy. Points labeled A, S, and C indicate positions of the decoy associated with the attraction, similarity, and compromise effects, respectively. The shade at any point reflects the relative choice share for X when the decoy at that location is included in the choice set. Lighter shades indicate increased preference for X over Y. Darker shades indicate the opposite.

Figure 7 plots the effects of different decoy options on choices between options $X = (4, 6)$ and $Y = (6, 4)$ for parameter values $m = 5$, $\lambda_1 = .2$, $\lambda_2 = .4$, and $I_0 = 5$. (As before, $A = 1$, $\chi = 2$, and $s = 1$.) The different decoys are defined as points in the attribute space with values ranging from 1 to 9, shown on the x- and y-axes of Figure 7. The shades capture the relative choice share for option X as compared to option Y, defined as $\frac{p(X)}{p(X) + p(Y)}$, where $p(X)$ is the choice probability from the MLBA model for option X in the presence of the decoy and likewise for option Y. Values near 1 (light shades in the plot) represent greater preference for X as compared to Y when the decoy is included in the choice set. Similarly, values near 0 (dark shades in the plot) represent greater preference for Y as compared to X. Values equal to .5 indicate equal preference between X and Y.

In the Figure, all three context effects are demonstrated.[4] For example, when the decoy is located at (3.8, 6.2), the relative choice share for X is .41, indicating a similarity effect. However, as the decoy is moved further away from X, the similarity effect is replaced by the compromise effect. For example, when the decoy

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[4] The phantom decoy effect cannot be observed in the figure (Pettibone & Wedell, 2007; Pratkanis & Farquhar, 1992). This effect occurs when the decoy is positioned so that it dominates the focal option and the removal of the dominating decoy from the choice set increases the preference for the initially dominated focal alternative. The MLBA can produce the phantom decoy effect if the model is extended to include an attribute bias parameter, as discussed in the fits to the perceptual experiments. In this case, the model produces the phantom decoy by assuming that the phantom decoy influences the evaluation of the remaining options by increasing the weight placed on the phantom decoy's most favorable attribute.

is located at (2, 8), the relative choice share for $X = .62$.[5] The attraction effect can also be seen in the figure. For the range decoy at (3.4, 6), the relative choice share for X is .58. The range–frequency decoy at (3.7, 5.7) produces a smaller effect, with the relative choice share for X being .52. The frequency decoy at (4, 5.4) does not produce an attraction effect with this set of parameters (i.e., relative choice share for X is .45). However, the model can produce the attraction effect for all three decoys with other parameter values, as indicated by the results of the grid search. Note that the ordering of the magnitude of the attraction effect predicted by MLBA (range followed by range–frequency followed by frequency) has been demonstrated empirically (Huber et al., 1982; Trueblood et al., 2013). However, these results are dependent on the model parameters and may differ with other parameter values. Going away from $X = (4, 6)$ toward (1, 3), the range–frequency decoy becomes increasingly dominated. Along this diagonal line, the magnitude of the attraction effect increases for a while and then decreases (e.g., (2, 4) produces a smaller effect than (3, 5)). Recently, Soltani, De Martino, and Camerer (2012) found that the attraction effect increases as the distance between the focal option and decoy increases, but the opposite result has also been shown (Wedell, 1991; Wedell & Pettibone, 1996). Perhaps, as predicted by MLBA, the attraction effect increases for a while as the distance between the focal and decoy options increases, but when the decoy is extremely dominated, the effect lessens. Future work could investigate these possibilities in more detail.

Predictions for Usher et al.'s (2008) data

MDFT has difficulty accounting for the results of a two-stage compromise experiment reported by Usher et al. (2008). In this experiment, participants were asked to select one of three options, from a set in which one option was a compromise between the other two. Following a participant's selection of an extreme option, the option was sometimes announced as unavailable. Participants were then asked to select a second choice between the remaining two options. In this situation, participants mostly selected the compromise option as the second choice. In order to allow a compromise option to be selected as the second choice after an initial selection of an extreme, MDFT uses asymmetric attention weighting.

We show that the MLBA model can account for this finding across a wide range of parameters, without extra assumptions. For this demonstration, we used the compromise stimuli (Set 1) given in Table 3 and searched across the ranges $.5 \leq m \leq 5$, $.1 \leq \lambda_1 \leq .8$, $.1 \leq \lambda_2 \leq .8$, and $5 \leq I_0 \leq 10$. We also set $A = 1$, $\chi = 2$, and $s = 1$ as before. To test the effect, we examined the number of instances the model predicted the compromise option as the second choice after an initial selection of an extreme. Out of the 3,840 parameter combinations examined, 1,938 resulted in an extreme option being selected first. Of these 1,938 parameter sets, the model predicted the compromise option would be the second choice 61.6% of the time. This pattern fits with Usher et al.'s (2008) findings that the compromise option is favored as the second choice.

The MLBA model can produce this result across a range of different parameter values. For example, it can produce the result when the subjective value function is both convex and concave (i.e., $m < 1$ and $m > 1$). When $m = 1$, all three options are equally

likely because the differences in subjective values are 0. The result also holds for both $\lambda_1 < \lambda_2$ and $\lambda_2 < \lambda_1$. When the two decay constants are equal, the compromise option is always the first choice. In this case, positive and negative differences between the attributes of the compromise and extremes are weighted equally. Because the differences are smaller between the compromise and extremes, the weights are larger and the compromise is preferred. If the decay constants are unequal, then it is possible for one of the extremes to emerge as the first choice.

Violations of Regularity

When comparing binary and ternary choice sets, the attraction effect produces a violation of the regularity principle. Informally, the regularity principle asserts that adding extra choice options to a set cannot increase the probability of selecting an existing option. Mathematically, if there is a larger choice set U that nests a restricted choice set, W, then $Pr[X \mid W] \geq Pr[X \mid U]$. The attraction effect violates the regularity principle because the inclusion of an inferior decoy similar to one of the alternatives increases the probability of choosing the dominating alternative. Our parameter search reported above does not address this effect, as we used only ternary choices in those simulations (as in our experiments).

We tested whether the MLBA model can produce a violation of the regularity principle using the same stimulus sets defined in Table 3. Specifically, we used two options: $X = (4, 6)$ and $Y = (6, 4)$. We also used range decoys $R_X = (3.4, 6)$ and $R_Y = (6, 3.4)$. With these stimuli, the MLBA model always predicts $Pr[X \mid \{X, Y\}] = Pr[Y \mid \{X, Y\}] = .5$.[6] To show that the model produces a violation of regularity, the probability of choosing X when R_X is also included in the choice set must be greater than .5. Likewise, the probability of Y when R_Y is included in the choice set must be greater than .5.

To examine violations of the regularity principle, we performed a grid search across the parameters for the following ranges: $.5 \leq m \leq 5$, $.1 \leq \lambda_1 \leq .8$, $.1 \leq \lambda_2 \leq .8$, and $5 \leq I_0 \leq 10$. A total of 3,840 parameter sets were examined, and 1,914 of these sets yielded both $Pr[X \mid \{X, Y, R_X\}] > .5$ and $Pr[Y \mid \{X, Y, R_Y\}] > .5$. Even more impressively, the MLBA can produce a violation of regularity and all three context effects simultaneously with the same set of parameters (e.g., $m = 5$, $\lambda_1 = .2$, $\lambda_2 = .4$, and $I_0 = 5$ for the stimuli in Table 3).

Quantitative Modeling Results

In the following sections, we report quantitative analyses of MDFT and the MLBA model. We begin by fitting MDFT and the MLBA model to the data from the inference experiments discussed above. Then, we apply the generalization criterion methodology to examine the model fits adjusted for complexity (Busemeyer & Wang, 2000). For this method, we conducted a new experiment

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[5] The MLBA model can also produce a compromise effect when the compromise option is not the option with the least amount of attribute dispersion. For example, if $X = (3, 7)$ is a compromise between $C_X = (1, 9)$ and $Y = (5, 5)$, the model will produce a compromise effect for parameter values $m = 5$, $\lambda_1 = .3$, $\lambda_2 = .3$, and $I_0 = 5$.

[6] To produce unequal preference for symmetric alternatives, the model would need to be extended to include an attribute bias parameter, as discussed in the fits to the perceptual experiments.

testing all three context effects simultaneously in the inference domain. Following the model comparison, we provide a discussion of model parameters including a prediction from the MLBA model for a general preference for intermediate options (i.e., those with less attribute dispersion), which we confirm experimentally. We conclude by fitting both MDFT and the MLBA model to individual data from the perceptual experiments. We did not fit the LCA model because it requires computationally intensive simulations. Although the issue also prevented us from fitting the MDFT model for internally controlled stopping times, we were able to fit the MDFT model with an externally controlled stopping time, since there are analytical solutions for this situation.

Model Fits to Inference Experiments

For the inference experiments (Trueblood, 2012), we fit the average choice probabilities across subjects because there were not enough data to analyze each subject individually. For example, in the inference attraction experiment, there were only 20 trials for each choice set, whereas for the perceptual attraction experiment (Trueblood et al., 2013) there were 90 trials for each set. We discuss individual fits to the perceptual data in a later section.

We fit the three inference experiments separately, obtaining a different set of parameter estimates for each effect. The experiments were fit separately rather than combined because there were different groups of subjects in each experiment, and it is reasonable to assume that they used different strategies or settings in different conditions. In a later section, we fit the three context effects simultaneously using data from a combined experiment.

We fit the MLBA model and MDFT to choice probabilities only rather than choice probabilities and response times. In the multi-alternative choice literature, MDFT and the LCA model have only been analyzed with respect to choice probabilities. Future work could use response time measures to further test the models. The MLBA model was fit by numerically integrating over decision times as discussed in Hawkins et al. (2013). For the MLBA model, we allowed the four parameters used to define the mean drift rates to vary freely but fixed the starting point parameter to $A = 1$, the threshold parameter to $\chi = 2$, and the drift rate noise parameter to $s = 1$. The boundary χ plays a role in response bias and overall response speed, but without response times, response bias can be absorbed into the drift rates and speed is irrelevant. The relationship between A and χ is important in speed–accuracy trade-off manipulations, but Hawkins et al. showed that different values of A and χ produce almost equivalent results when integrating over decision times.

We fit MDFT in two ways. First, we fit the model by fixing the variance parameter $\sigma^2 = 1$ and allowing four parameters to vary freely: the attention weight $p(w_P)$, the dominance dimension weight β, and the two parameters ϕ_1 and ϕ_2 associated with the Gaussian mapping used to determine the strength of lateral inhibition (see Equation A8). We then refit the model, allowing σ^2 to be a free parameter along with the other four parameters. Hotaling et al. (2010) argued that for externally controlled stopping times, the variance parameter should be allowed to vary for different effects. For both ways of fitting the model, we fixed the decision time to $t = 1,001$, as in Hotaling et al. In previous simulations, the subjective values in the M matrix of MDFT have been between 0 and 10 (Hotaling et al., 2010). Because the attribute values for the

inference experiments were associated with eyewitness testimony strengths ranging from 0 to 100, we used the attribute values divided by 10 in both versions of MDFT and the MLBA model. (Note that with the externally controlled stopping time, MDFT does not use a response boundary.)

For the attraction effect experiment, we fit 18 choice probabilities. These probabilities arise from the six ternary choice sets used to test range, frequency, and range-frequency decoys. For each decoy, two choice sets were used to examine the effect: one choice set with the decoy favoring one alternative and another choice set with the decoy favoring the other alternative. For the similarity effect experiment, we fit 12 choice probabilities, from four ternary choice sets: $\{X, Y, S_X\}$, $\{X, Y, S_{Y1}\}$, $\{Y, Z, S_Z\}$, $\{Y, Z, S_{Y2}\}$. For the compromise effect experiment, we fit nine choice probabilities from three ternary choice sets: $\{C_X, X, Y\}$, $\{X, Y, Z\}$, $\{Y, Z, C_Z\}$. We fit both the MLBA model and MDFT by minimizing the sum of squared error between the model predictions and the data. When fitting mean probabilities, parameters that minimize sum of squared error will approximate the maximum likelihood, as long as the probabilities are not too close to 0 or 1 (as here).

The mean square error and R^2 values for the MLBA model and two versions of MDFT are given in Table 4. Figure 8 shows the observed and fitted mean choice probabilities for the MLBA model and both versions of MDFT. For all three inference experiments, the MLBA model is able to account for about 85%–95% of the variability in the choice proportions. MDFT is able to account for a high proportion of the variability in the attraction effect data, but fails to account for the variability in the similarity and compromise data. The R^2 values for the similarity and compromise effects improve significantly when σ^2 is allowed to vary freely, but they are still substantially poorer than the MLBA's fits. It is unlikely that the MDFT's poor fits are due to the externally controlled stopping time procedure, as Hotaling et al. (2010) found that the externally controlled stopping time model produced essentially the same results as long internally controlled stopping times.

Generalization Criterion Methodology

The model fits above consider only goodness of fit and do not take into account the relative complexity of the models. Overly complex models can sometimes "overfit," generating very good agreement with particular data sets by describing random noise rather than systematic patterns. To examine the fit of the MLBA model and MDFT adjusted for complexity, we used the generalization criterion methodology (GCM) formalized by Busemeyer and Wang (2000). The GCM is an extension of cross-validation, using two statistically independent samples, where one of the samples is used to calibrate the models and the other is used to test the generalizability of the models. Each model is fit to the calibration data by minimizing the discrepancy between the observed data and the model predictions. For our purposes, we define the discrepancy as the mean square error. The best fitting parameters from the calibration stage are then used to compute new predictions for the generalization data. Thus, each model makes a priori predictions for the new data set. The discrepancy between these predictions and the data is computed for each model. The models are then compared by choosing the model with the smallest discrepancy in the generalization data set. The GCM provides a

Table 4

Mean Square Error and R^2 Values for the Multiattribute Linear Ballistic Accumulator (MLBA) Model and Multialternative Decision Field Theory (MDFT) for the Inference and Perceptual Experiments

| | MLBA | | | | MDFT | | | |
| | $\beta = 1$ | | β free | | $\sigma^2 = 1$ | | σ^2 free | |
Experiment	MSE	R^2	MSE	R^2	MSE	R^2	MSE	R^2
Attraction inference	.003	.92			.002	.94	.002	.94
Similarity inference	.004	.89			.023	.29	.018	.43
Compromise inference	.004	.94			.038	.34	.031	.47
Combined inference	.007	.86			.037	.25	.034	.32
Attraction perceptual	.027 (.026)	.62 (.17)	.011 (.007)	.83 (.06)	.005 (.003)	.90 (.06)	.002 (.002)	.95 (.04)
Similarity perceptual	.009 (.014)	.49 (.31)	.007 (.013)	.65 (.27)	.016 (.021)	.21 (.23)	.014 (.018)	.30 (.25)
Compromise perceptual	.007 (.013)	.75 (.31)	.005 (.010)	.86 (.21)	.028 (.032)	.56 (.24)	.027 (.033)	.63 (.26)

Note. Standard deviations are given in parenthesis for the perceptual experiments.

measure of fit adjusted for complexity because overfitting in the calibration stage is punished by poor generalization.

For the calibration stage, we used the results from a combined experiment testing all three context effects simultaneously in the inference paradigm developed by Trueblood (2012). As in Trueblood, participants were asked to make choices about criminal suspects based on eyewitness testimony strengths. However, the numerical values of these strengths differed from those of Trueblood. In pilot experiments, it was found that the effects interfered with one another so that all three effects were not demonstrated simultaneously. In particular, we found a strong similarity effect, a weak attraction effect, and no compromise effect. Berkowitsch, Scheibehenne, and Rieskamp (2013) found similar results in a combined experiment with consumer products. However, in their experiment, the attraction and compromise effects were strong and the similarity effect was weak. To help remove any possible interference effects, we used different regions of the attribute space for different effects. For the similarity and attraction effects, the values of the eyewitness testimony strengths were below 45. For the compromise effect, the strengths ranged from 35 to 75. Example stimuli for this experiment are given in the top three rows of Table 5. Example stimuli for the separate experiments discussed in Trueblood are given in the bottom three rows of the Table 5. For the separate experiments, the stimuli for the attraction and similarity effects were in the same range as the stimuli for the compromise effect as shown in the table.

Method

Sixty-eight undergraduate students from Indiana University participated for course credit. Participants were told they would see three suspects of a crime on each trial and were instructed to select the suspect that seemed most likely to have committed the crime based on the strengths of two eyewitness testimonies. Participants were also told that the testimonies of both eyewitnesses were equally valid and important and that the strengths of the testimonies were equated. The suspects and eyewitness strengths were presented in a table format with different suspects in different rows. Participants did not receive feedback.

Each participant completed 240 trials that were divided into three blocks of 80 trials. The three blocks were used to test the three effects and were presented in randomized order across par-

ticipants. Within each block, participants saw 40 trials testing one of the effects and 40 filler trials. The presentation order of the trials within each block was also randomized. Filler trials where one alternative was clearly superior were used to assess accuracy throughout the experiment. The trials used to test for context effects were subdivided so that the decoy was placed near one alternative for some trials and near the other alternative for other trials. The similarity effect was tested using four ternary choice sets, as in Trueblood (2012).

Results

Data from three participants were not analyzed because their accuracy was 2 standard deviations lower than the average accuracy on the filler trials. Figure 9 shows the mean choice probabilities for focal and nonfocal alternatives in the attraction, similarity, and compromise effect trials. For the attraction effect trials, the choice probability for the focal alternative ($M = .55$) was significantly larger than the choice probability for the nonfocal alternative ($M = .42$), $t(64) = 3.14$, $p = .003$. The similarity trials also showed that across all four choice sets the choice probabilities were significantly larger for focal options ($M = .43$) than nonfocal options ($M = .36$), $t(64) = 2.58$, $p = .012$. For the compromise effect, the choice probability for compromise alternatives ($M = .47$) was significantly larger than the choice probability for extreme alternatives ($M = .41$), $t(64) = 2.17$, $p = .034$. We also examined how frequently multiple effects occurred within a single participant. Fifty-seven percent of participants demonstrated at least two effects (11% showed all three effects, 17% showed the attraction and similarity effects, 14% showed the attraction and compromise effects, and 15% showed the similarity and compromise effects).

Model Comparison

For the calibration stage of the GCM, we fit both versions of MDFT (with σ^2 fixed and free) and the MLBA model to the average choice probabilities across subjects from the combined inference experiment discussed above. We did not fit individual choice probabilities because there were not enough data from each subject. The models were fit to choice probabilities only rather than choice probabilities and response times.
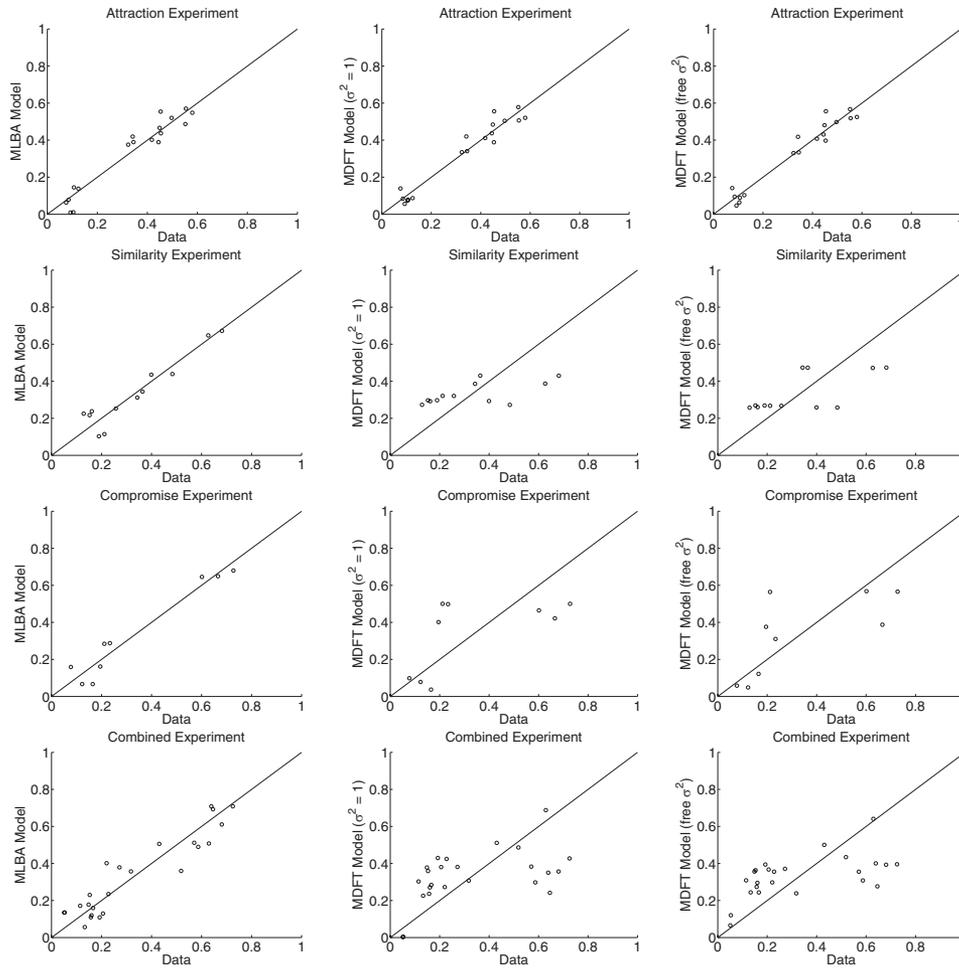
Figure 8. Observed and fitted mean choice probabilities for the inference experiments. The first column shows the fits of the multiattribute linear ballistic accumulator (MLBA), the second column shows the fits of multialternative decision field theory (MDFT) with fixed σ^2, and the third column shows the fits of MDFT when σ^2 was allowed to vary freely. The first row shows the fits to the attraction data (18 mean choice probabilities), the second row shows the fits to the similarity data (12 mean choice probabilities), the third row shows the fits to the compromise data (nine mean choice probabilities), and the fourth row shows the fits to the combined inference experiment (24 mean choice probabilities).

We fit 24 choice probabilities arising from the eight ternary choice sets used in the experiment. The models were fit by minimizing the sum of squared error between the model predictions and the data as before. The mean square error and R^2 values for the MLBA model and two versions of MDFT are given in Table 4. Figure 8 (fourth row) shows the observed and fitted mean choice probabilities for the MLBA model and both versions of MDFT. The MATLAB programs for fitting the MLBA model and choice probability data are available in the supplemental materials.

For the generalization stage, we used the best fitting parameters for each model from the calibration stage (data from the combined experiment) to predict the data from the separate inference experiments presented in Trueblood (2012). We then calculated the discrepancy for each model in the generalization data (separate experiments). A comparison of MDFT with σ^2 fixed versus σ^2 free found that the σ^2 fixed version was preferred. After this,

we compared the discrepancy of the MLBA model to that of the MDFT model with σ^2 fixed and found that the MLBA had a smaller discrepancy (mean square error smaller by .08), implying that the MLBA model fits best, even after accounting for model complexity.

Parameter Values

Table 6 gives the best fitting parameters for the MLBA model for the inference experiments. For all of the experiments, the *m* parameter is greater than 1, indicating that options with less attribute dispersion are preferred. The *m* parameter is much smaller for the attraction experiment than the remaining three experiments because there were no intermediate options in this experiment. Further, all options had similar degrees of attribute dispersion (i.e., (35, 65) and (65, 35)). The similarity, compro-

Table 5
Example Stimuli From the Combined and Separate Inference Experiments

Choice set	Option 1	Option 2	Option 3
Combined			
Attraction	(25, 45)	(45, 25)	(17, 45)
Similarity	(25, 45)	(45, 25)	(24, 46)
Compromise	(75, 35)	(55, 55)	(45, 65)
Separate			
Attraction	(35, 65)	(65, 35)	(28, 65)
Similarity	(35, 75)	(55, 55)	(33, 77)
Compromise	(75, 35)	(65, 45)	(55, 55)

mise, and combined experiments included intermediate options such as (55, 55) (see Table 5 for details). For $m > 1$, the model predicts a general preference for intermediate options. We confirmed this preference experimentally with a new study examining binary choice sets in the inference domain, discussed below.

Table 7 gives the best fitting parameter values of MDFT (σ^2 free) to the inference experiments. The parameter values for the four experiments are generally quite different, although the attention weight is similar for the experiments. We do not provide an in-depth discussion of the MDFT parameters, as this has been done previously (Hotaling et al., 2010; Roe et al., 2001). However, it is interesting to note that the σ^2 parameter differs greatly across the experiments. In particular, σ^2 is largest for the attraction and combined experiments and smallest for the similarity experiment. This follows the intuition of Hotaling et al. (2010) that the variance of the noise is greater for the attraction and compromise effects than for the similarity effect.

Binary Inference Experiment

Results from the quantitative fitting of the MLBA model suggest that individuals show extremeness aversion (i.e., mediocre options are preferred to extreme options), as seen by values of the *m* parameter that are greater than 1. To test the generality of this preference, we ran a new experiment comparing mediocre and extreme options in binary choice. For this experiment we used the inference paradigm with stimuli similar to those used in the compromise experiment.

Method

Fifty-one undergraduate students from the University of California, Irvine, participated for course credit, completing the computer-based experiment online at a time of their choosing. The experiment used the same eyewitness testimony procedure as described above. Unlike the previous inference experiments, the choice sets in this experiment only contained two options. Each participant completed four blocks of 34 randomized trials, which were divided into 24 trials with one mediocre and one extreme option and 10 filler trials. An option was defined as mediocre if it was the alternative in the choice set with eyewitness strengths closest to 50. The different choice sets used for testing extremeness aversion are given in Table 8. Noise was added to the attribute values to introduce variation to the task similar to the previous

inference experiment. For example, on a trial using Choice Set 1 from the table, participants might see (74, 36) and (67, 43). The filler trials contained one option that was clearly superior and were used to assess accuracy as in previous experiments.

Results and Discussion

One subject was excluded from the analysis because his or her accuracy on the filler trials was 2 standard deviations lower than the average. The mean choice probability for mediocre options ($M = .58$) was significantly larger than the mean choice probability for extreme options ($M = .42$), $t(49) = 2.25$, $p = .029$. The choice probabilities for the different choice sets listed in Table 8 are shown in Figure 10. Choice sets that contained symmetric options such as (35, 75) and (75, 35) were combined for the figure (e.g., Choice Set 1 was combined with Choice Set 4). This result indicates that participants had a general preference for mediocre alternatives, as predicted by the MLBA model when $m > 1$.[7]

This result also confirms a previous finding by Usher et al. (2008) for greater preference for compromise options in both binary and ternary choice. In their experiment, participants were given either binary or ternary choice sets containing consumer products such as laptops and asked to select the one they preferred. The results indicated that in both types of choice sets, the midrange options were selected more often. Usher et al. interpreted this finding as support for the LCA model because the value function used in the model produces equal preference curves where midrange options have higher utility than extreme options.

Model Fits to Perceptual Experiments

In this section, we provide fits for both MDFT and the MLBA model to the perceptual experiments presented in Trueblood et al. (2013). For these experiments, we introduce an extended version of MLBA that allows for perceptual biases. Holmberg and Holmberg (1969) found that as the height-to-width ratio of rectangles increased, so did their apparent area. This "elongation effect" suggests that height might play a more important role in area judgment than width. We model attribute biases by allowing a bias parameter β to mediate the attention weights:

$$
\begin{aligned}
w_{Pij} &= \exp(-\lambda \,|\, u_{Pi} - u_{Pj} \,|) \\
w_{Qij} &= \exp(-\lambda\beta \,|\, u_{Qi} - u_{Qj} \,|),
\end{aligned}
\tag{5}
$$

where $\beta > 0$. There is a bias toward attribute Q when $\beta > 1$ and a bias toward attribute P when $\beta < 1$. When $\beta = 1$, there is no bias. For the inference experiments, we had no reason to believe participants would be biased toward one eyewitness over the other, since they were instructed to treat them equally. However, one might find attribute biases in other high-level domains. For example, in a choice among consumer products, price might receive more weight than quality.

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[7] Extremeness aversion was more pronounced in the compromise experiment than in the binary experiment. In the compromise experiment, the midrange option with attribute values (55, 55) was selected 60%–70% of the time. As a result, the best fit *m* parameter was quite large. Extremeness aversion might be weaker in the binary experiment because it is easier to discern the additive indifference rule with fewer options.
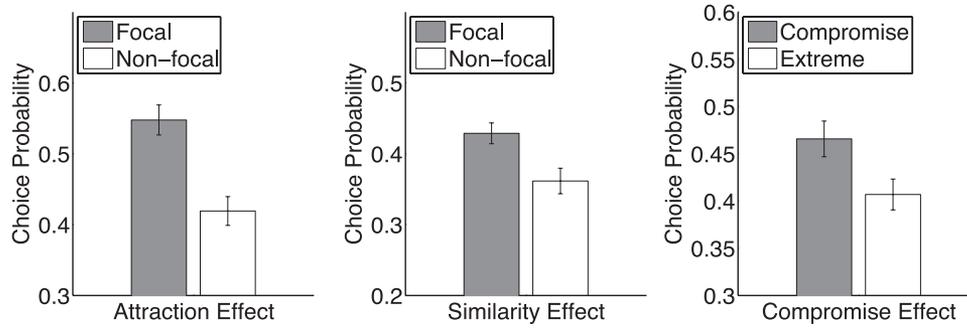
Figure 9. Mean choice probabilities with error bars showing the standard error of the mean for the attraction, similarity, and compromise effects from the combined inference experiment.

For these experiments, we used the same fitting methods as for the inference experiments, but applied them to the individual choice probabilities, which was possible because we had a lot of data per subject. We fit two versions of MLBA (e.g., with and without fixed β) and two versions of MDFT (e.g., with and without fixed σ²). We converted attribute values to log coordinates to allow objectively indifferent options (i.e., rectangles with the same area) to follow an additive rule rather than a multiplicative rule. Specifically, the attribute dimensions were height and width, and a pair of options were experimentally defined as indifferent if they had equal area, that is, $P_1 \cdot Q_1 = P_2 \cdot Q_2$ or equivalently $\log(P_1) + \log(Q_1) = \log(P_2) + \log(Q_2)$.

For the attraction experiment, we fit 18 choice probabilities arising from the six ternary choice sets used in the experiment. Four subjects were removed from the model fitting because their accuracy on the filler trials (which have a clear correct response) was 2 standard deviations lower than the average. These subjects were also removed from the data analyses in Trueblood et al. (2013). For the similarity experiment, we fit 12 choice probabilities arising from the four ternary choice sets used in the experiment. No subjects were removed from the model fitting. For the compromise experiment, we fit six choice probabilities arising from the two ternary choice sets used in the experiment. These choice sets were {X, Y, Z} and {Y, Z, C_Z}. Four subjects were removed from the model fitting because their filler accuracy was 2 standard deviations lower than the average. These subjects were also removed from the data analyses in Trueblood et al.

The mean square error and R^2 values for the two versions of MLBA and MDFT are given in Table 4. Figure 11 gives the observed and fitted mean choice probabilities for the MLBA model with free β and two versions of MDFT. The two versions

of MDFT were able to account for almost all of the variability in the attraction effect data as indicated by R^2 values above .9. The MLBA model with β free provided good fits to the attraction and compromise data and accounted for more than 80% of the variability in the choice proportions. The MLBA fits to the similarity data were poorer, probably reflecting closer-to-random choices made by participants in these conditions, as seen by choice probabilities near .33 in Figure 11. The average R^2 values for the similarity and compromise data were much lower for both versions of MDFT. These fits were also worse than those of the MLBA model with no perceptual bias (i.e., β = 1).

From Table 4, the MLBA model is able to provide a consistent account of the variability in the data across the different context effects and experimental domains. On the other hand, MDFT provides an excellent account of the attraction effect data in both the inference and perceptual experiments, but has difficulty accounting for the compromise data in the inference experiment and the similarity data in both the inference and perceptual experiments.

General Discussion

MDFT and the LCA model have provided great insight into multialternative choice behavior. However, these models have some drawbacks. The LCA model is difficult to fit to data because it requires computationally intensive simulations for both internally and externally controlled stopping rules. Further, the model assumes an asymmetry in the treatment of losses and gains, which has an uncertain ability to generalize to paradigms where the definition of loss is arbitrary (such as perceptual choices). Unlike the LCA model, MDFT has an analytical solution for externally

Table 6
Best Fitting Parameters for the Multiattribute Linear Ballistic Accumulator Model for the Inference Experiments

Experiment	m	λ_1	λ_2	I_0
Attraction	1.68	.024	.001	7.73
Similarity	32.02	.325	.897	0.31
Compromise	29.48	.234	.204	15.21
Combined	22.30	.193	.278	13.39

Table 7
Best Fitting Parameters for Multialternative Decision Field Theory (σ² Free) for the Inference Experiments

Experiment	$p(w_P)$	β	ϕ_1	ϕ_2	σ^2
Attraction	.52	6.16	.50	.11	2.00
Similarity	.50	1.00	.001	.25	0.0001
Compromise	.61	1.00	.007	.50	0.001
Combined	.53	1.00	.001	.04	15.18

Table 8
Choice Sets Used in the Binary Inference Experiment to Test Preferences for Extreme Versus Mediocre Options

Choice set	Extreme	Mediocre
1	(75, 35)	(65, 45)
2	(75, 35)	(55, 55)
3	(65, 45)	(55, 55)
4	(35, 75)	(45, 65)
5	(35, 75)	(55, 55)
6	(45, 65)	(55, 55)

controlled stopping times and does not assume loss aversion as a primitive. However, this model requires simulations for tasks with internally controlled stopping times and has some stability problems. The MLBA model is similar to MDFT and the LCA model in many regards, but it is simpler and more tractable. It has analytical solutions for both internally and externally controlled stopping times and, unlike the LCA model, does not assume loss aversion.

The MLBA model passes the tests that have been applied to previous dynamic models of multialternative choice (e.g., MDFT and the LCA model), including accounting for all three context effects with a single set of parameters. It also passes more difficult tests such as accounting for the data from Usher et al. (2008) involving unavailable options in the compromise effect, and naturally predicts previous results for the attraction and compromise effects under time pressure. Going beyond MDFT and the LCA model, the MLBA model makes a new prediction about the influence of time pressure on the similarity effect, which we confirmed experimentally.

All the above, and previous work, has focused on qualitative analyses of the models; that is, does the model produce the expected ordering of some probability measures? We took this a step further in applying the MLBA to data from inference and perceptual decision tasks, by examining the model's quantitative fit—at an individual-subject level, where possible. These quantitative analyses represent one of the first attempts at fitting a dynamic model to empirical data for all three context effects (see Berkowitsch et al., 2013, for a comparison of MDFT and utility models for context effects in consumer choice). The quantitative analyses showed that the MLBA model provided a better description of the data than MDFT. Further, the generalization criterion methodology confirmed that the MLBA model's superior fit to data was not due to greater model complexity.

The Front-End Process

The MLBA model consists of two components: a front-end process that compares options to produce response tendencies (drift rates) and a back-end process that determines the probability that a particular option will be selected and the time it takes to make that selection. The back-end process is the LBA model developed by Brown and Heathcote (2008). This article develops the front-end attribute processing component. The coupling of front-end and back-end processes is not new. For example, the SAMBA model (Brown, Marley, Donkin, & Heathcote, 2008) of choice and response times in absolute identification tasks proposes a front end to the ballistic accumulator model (Brown & Heath-

cote, 2005). In this model, physical magnitudes are used to set up the front-end process that feeds into the ballistic accumulator back end. As in the SAMBA model, in the MLBA model we assumed that the front-end process completes in a fixed time, with variability in deliberation time explained by the time taken by the back-end process to integrate information.

The coupling of front-end and back-end processes to produce context effects maps to a common idea in the neurophysiological literature, where the front-end process modulates action selection in the back-end process. For example, Frank (2005) and O'Reilly and Frank (2006) developed a model in which the striatum modulates motor actions and working memory updating in frontal cortex. We do not argue for a particular mapping between the processes in our model and specific brain regions, but speculate that the gating of actions by a front-end process could have a neural explanation. More generally, the LBA model has been successfully used in model-based neuroimaging (Forstmann et al., 2008, 2010). This approach uses a cognitive model to isolate and quantify the latent cognitive processes of interest to more effectively associate them to brain measurements. The MLBA model could play a similar role in future research.

Elements of the front-end process could also be incorporated into MDFT. In particular, MDFT might benefit from the subjective value function in Equation 2 that is used to map physical stimuli into psychological magnitudes. By introducing such a function, MDFT might provide better fits to data. Decision field theory has been shown to provide an excellent account for binary choice data (Busemeyer & Townsend, 1993), so it seems likely that the fits of MDFT to multialternative choice data could be significantly improved by making such changes.

Recent Alternative Models

Bhatia (2013) proposed a model of multialternative, multiattribute choice called the associative accumulation model that assumes that preferences are determined by the accessibility of attributes. In this model, the preference for an alternative is a weighted sum of its attribute values where the greater the amount of an attribute, the
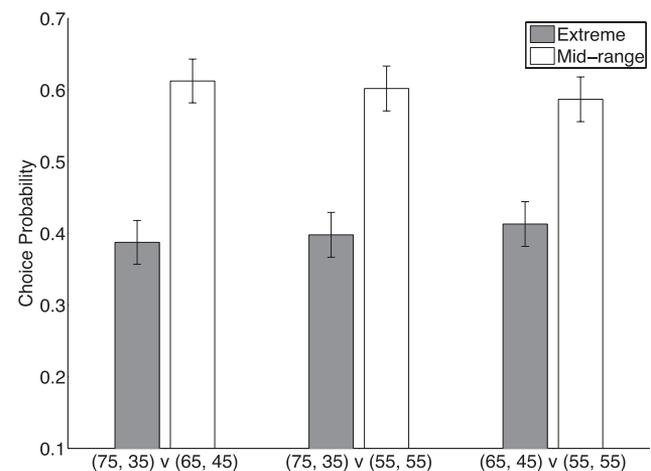

Figure 10. Mean choice probabilities with error bars showing the standard error of the mean for the binary inference experiment. In general, participants preferred midrange options to extremes.
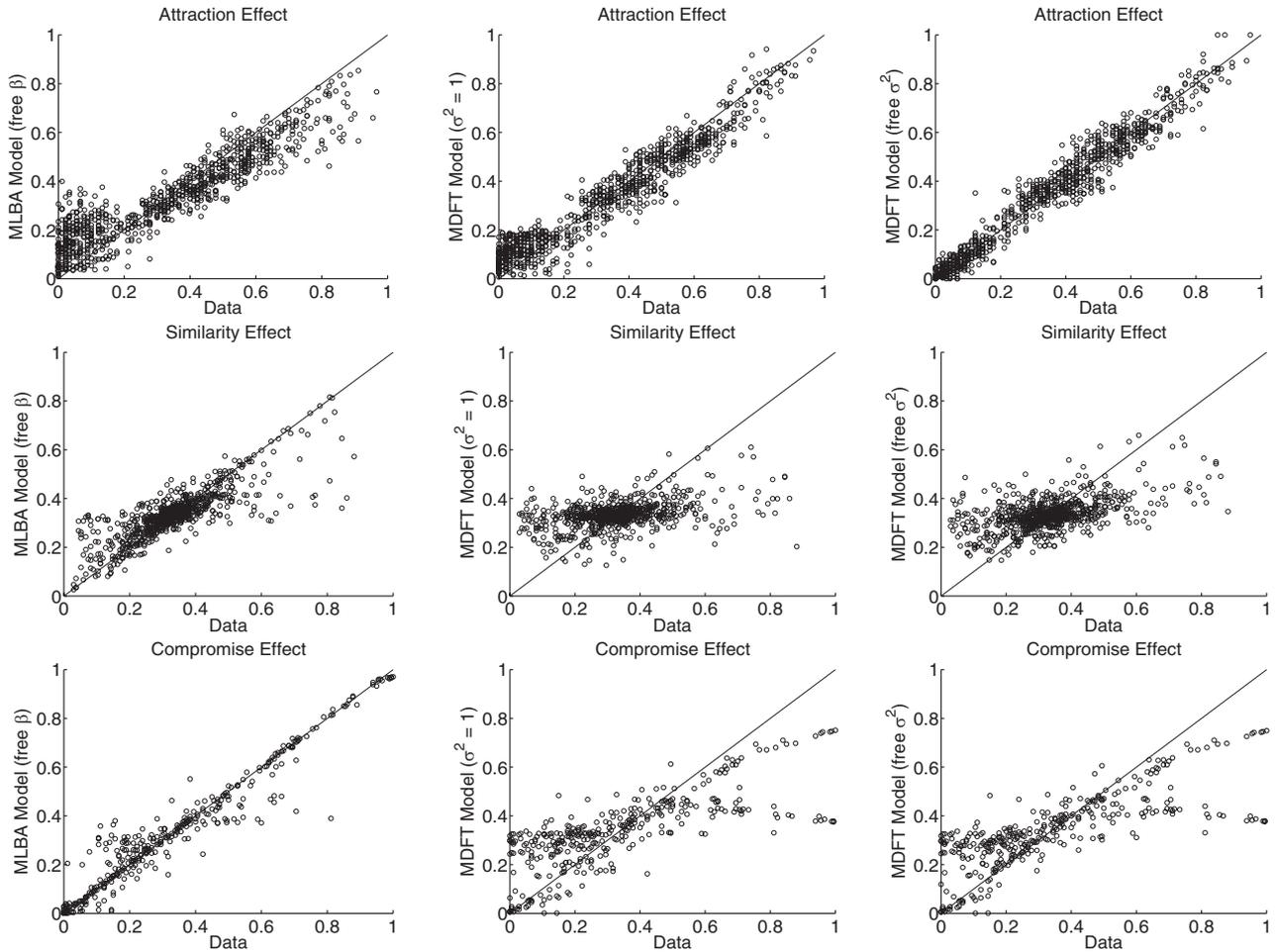
Figure 11. Observed and fitted mean choice probabilities for the perceptual experiments. The first column shows the fits of the multiattribute linear ballistic accumulator (MLBA) when β was allowed to vary freely, the second column shows the fits of multialternative decision field theory (MDFT) with fixed σ^2, and the third column shows the fits of MDFT when σ^2 was allowed to vary freely. The first row shows the fits to the individual choice probabilities in the attraction experiment, the second row shows the fits to the individual choice probabilities in the similarity experiment, the third row shows the fits to the individual choice probabilities in the compromise experiment.

more heavily that attribute is weighted. The model can produce all three contexts effects, but similar to MDFT and the LCA model, computationally intensive simulations are needed to do so. The model can also produce a bias toward midrange options similar to the MLBA and LCA models, but can only do so if each attribute is assumed to have a high level of initial activation. Without this initial activation of attributes, the model produces an extremeness bias contrary to the experiments presented here and in Usher et al. (2008).

Wollschläger and Diederich (2012) developed the 2*N*-ary choice tree model of preferential choice that can also produce the three context effects. The model implements pairwise comparisons of alternatives on weighted attributes through an information sampling process. Similar to the MLBA model, the 2*N*-ary choice tree has closed form solutions for choice probabilities and response deadlines. Also, like the MLBA model, the 2*N*-ary choice tree does not use inhibition or loss aversion to account for the effects.

However, the model requires unequal attribute weights in order to account for the attraction effect, and it is unclear whether the model can account for all three effects with the same set of parameters.

Soltani et al. (2012) developed the range-normalization model to explain the attraction, similarity, and asymmetrically dominant decoy effects. This model is neurally inspired and accounts for context effects by adjusting the neural representations of available options on a trial-by-trial basis. The model suggests that context effects are the natural result of biophysical limits on neural representations in the brain. The model has yet to be applied to the compromise effect.

One challenge for the MLBA model is accounting for coherence effects, that is, the finding that attribute weights (Simon, Krawczyk, & Holyoak, 2004) and the subjectively perceived validity of cues in probabilistic inference (Holyoak & Simon, 1999) are changed in the decision process to favor the option that is ulti-

mately selected. These effects suggest bidirectional reasoning from the attributes to the evaluation of options, and vice versa. Coherence effects can be explained by parallel constraint satisfaction network models such as the one in Glöckner and Betsch (2008, 2012), which has been successful in predicting choice, decision time, and confidence in probabilistic inference tasks. Future work could generalize the MLBA model to allow the drift rates to change within a trial. Changes in drift rates across a trial could correspond to changes in attention. This type of generalization might allow the MLBA model to account for coherence effects. However, it is difficult to imagine that such a model could have closed formed solutions for choice and response times.

Concluding Comments

The MLBA model provides a dynamic account of multialternative, multiattribute decision making. The model postulates a new psychological theory of how a front-end attribute processing component gates choices and response times in a back-end process. MLBA makes testable new predictions, such as about the influence of time pressure on context effects, which we confirmed experimentally. At the quantitative level, the model is easily fit to individual subject data from the perceptual experiments. These quantitative analyses go beyond the standard qualitative tests and set a new bar for evaluating models of context effects.

Past research has shown that context effects play a significant role in choice behavior and can impact real-life decisions. However, this research has mainly focused on the presence of context effects in high-level tasks where options have hedonic values. The inference and perceptual experiments discussed offer evidence suggesting that context effects are not confined to choices among options that have affective value, such as consumer products. These results provide strong support for the conclusion that context effects are a general feature of choice behavior and are fundamental to decision making. This is an important result that has consequences for not only decision making but other fields such as memory, categorization, and signal detection that use Luce's (1959) ratio of strengths model.

The experiments also offer the first evidence that all three effects can occur within the same nonhedonic experimental paradigm, using simple perceptual decisions. All three models discussed in this article—MDFT, the LCA model, and the new MLBA model—assume that a single set of cognitive principles can be used to account for all three effects. The inference and perceptual experiments provide a crucial test of this assumption. Moreover, the combined inference experiment shows that all three context effects can be obtained within a single subject in a single experiment.

In the perceptual experiments, the magnitude of the context effects was much smaller than in the inference experiments. We hypothesize that this is due to short response times in the perceptual domain. As shown in the experiments by Pettibone (2012) and the new experiments discussed above, all three effects grow with longer deliberation time, as predicted by the MLBA model. Future experiments could be designed to examine the relationship between effect magnitude and response times in perception. We anticipate that such experiments will confirm the previous experimental evidence for the positive relationship between deliberation time and the magnitude of the effects.

Dynamic models were developed to explain context effects because these effects violate the assumptions of static choice rules such as Luce's (1959) ratio of strengths model. However, the extra flexibility introduced by the inclusion of dynamics needs to be justified. Response time measures provide one way to test the dynamic assumptions of these models. Further, it might be possible to distinguish the three models on the basis of response time data as described by Tsetsos et al. (2010). Perceptual choice is one possible domain for exploring the relationship between preference and response time because choices are made quickly and response time measurement is easy. Future experiments could exploit this characteristic of perceptual choices to further understanding of the dynamic nature of context effects.

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Appendix A

Multialternative Decision Field Theory

Multialternative decision field theory models preferences by associating each alternative with a valence value that represents the advantages or disadvantages of the alternative at each moment in time. For a set of options such as $\{X, Y, Z\}$, the valences can be described by the vector $V(t) = [v_X(t), v_Y(t), v_Z(t)]'$, where $V(t)$ is defined by three components.

The first component is a subjective evaluation of each alternative on each attribute value. This subjective value is meant to capture personal assessment of the options. In the case where there are two attributes, let P_i and Q_i denote the actual value of option i on the attributes. In the model, the values m_{P_i} and m_{Q_i} represent the subjective value of option i on attributes P and Q.

The collection of all the subjective evaluations can be written as an $n \times m$ matrix, where n is the number of options and m is the number of attributes. In the case of three options with two attributes, the matrix form is given by Equation A1:

$$M = \begin{bmatrix} m_{P1} & m_{Q1} \\ m_{P2} & m_{Q2} \\ m_{P3} & m_{Q3} \end{bmatrix}. \qquad (A1)$$

The second component of the valence vector is an attention weight for each attribute. It is assumed a decision maker allocates a certain amount of attention to each attribute at each moment in time. Let

(*Appendices continue*)

$w_j(t)$ denote the attention allocated to attribute j at time t. It is assumed the attention weights change across time to reflect an individual's shifts in attention. For example, in the inference experiments, an individual might pay more attention to one of the eyewitnesses early in the trial but switch attention to the other eyewitness later in the trial. This attention switching mechanism was motivated by the elimination by aspects heuristic (Tversky, 1972).

In the case where there are only two attributes, let $w_P(t)$ be the attention weight for attribute P and $w_Q(t)$ be the attention weight for attribute Q. In Roe et al. (2001), it is assumed that attention changes across time in an all-or-none fashion so that $w_P(t) = 1$ and $w_Q(t) = 0$ at one moment and $w_P(t + 1) = 0$ and $w_Q(t + 1) = 1$ at the next moment. The shifts in attention are determined by fixed probabilities $p(w_P)$ and $p(w_Q) = 1 - p(w_P)$. This scanning of attributes is implemented according to a Bernoulli process. More complex models of attention switching have also been proposed, such as models with the Markov property (Diederich, 1997). The attention weights for both attributes can be written as the following vector:

$$W(t) = \begin{bmatrix} w_P(t) & w_Q(t) \end{bmatrix}'. \tag{A2}$$

The subjective evaluations given by the M matrix can be combined with the attention weights given by the $W(t)$ matrix to produce a weighted value of each option on each attribute at every moment in time: $MW(t)$. The entries in the matrix take the form $w_P(t) \cdot m_{Pi} + w_Q(t) \cdot m_{Qi}$. This is similar to a classical weighted utility model; however, the sum is stochastic because of the changes in attention weights over time.

The third component of the valence vector is a comparison mechanism that contrasts the weighted evaluation of the alternatives. This mechanism is used to determine the relative advantages and disadvantages for each option in the choice set. The valence for each option is defined as the difference of the weighted value of one alternative against the average of all the others. For n alternatives with two attributes, the valence of option i can be written as

$$v_i(t) = (w_P(t) \cdot m_{Pi} + w_Q(t) \cdot m_{Qi}) - \left(\frac{1}{n-1}\right)\sum_{j \neq i}^{n-1} (w_P(t) \cdot m_{Pj}$$
$$+ w_Q(t) \cdot m_{Qj}). \tag{A3}$$

For three options, this comparison process can also be described by a matrix operation by first defining the contrast matrix:

$$C = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}. \tag{A4}$$

With this matrix, the valence vector can be written as the matrix product

$$V(t) = CMW(t), \tag{A5}$$

where each row corresponds to a weighted comparison of the options as defined in Equation A3.

In the model, each alternative is associated with a preference state $P_i(t)$ that fluctuates across time. This preference state is determined by integrating all of the valences for option i up until time t. The preference states for all of the options can be written as a vector. For example, for three options $\{X, Y, Z\}$, the preference state vector is $P(t) = [P_X(t), P_Y(t), P_Z(t)]'$. The preference state at time $t + 1$ is determined by the previous preference state and the current valence:

$$P(t + 1) = SP(t) + V(t + 1). \tag{A6}$$

It is assumed that the initial preference state is $P(0) = 0$. The feedback matrix S allows for cross-talk among the options. Specifically, the S matrix contains positive self-connections and negative interconnections among the options.

The self-connections in the S matrix determine an individual's memory of previous preference states. If these self-connections are set to 1, then the individual has perfect memory of the preference states. If the self-connections are set to 0, then the individual has no memory for previous preference states. Values between 1 and 0 correspond to decay in memory across time. The interconnections in the S matrix determine how options influence one another and these values, which are typically negative, can be interpreted as implementing lateral inhibition. The effects of lateral inhibition are typically assumed to be distance dependent. Specifically, the strength of the lateral inhibition is determined by the distance between two options in an "indifference/dominance" space.

Consider a pair of options (P_i, Q_i) and (P_j, Q_j). Define the distance between these two options as $(\Delta P, \Delta Q) = (P_i - P_j, Q_i - Q_j)$. These distances are then mapped to the corresponding coordinates in the indifference and dominance space: $(\Delta I, \Delta D) = 1\sqrt{2} \cdot ((\Delta Q - \Delta P), (\Delta Q + \Delta P))$, where ΔI is the difference along the indifference dimension and ΔD is the difference along the dominance dimension. With these coordinates, the distance function that weights changes more in the dominance dimension than the indifference dimension is defined as

$$Dist_{ij} = \sqrt{(\Delta I)^2 + \beta \cdot (\Delta D)^2}. \tag{A7}$$

These distances between the options determine the feedback matrix, S, via the Gaussian function:

$$S_{ij} = \begin{cases} 1 - \phi_2, & \text{if } i = j \\ -\phi_2 \cdot exp(-\phi_1 \cdot Dist_{ij}^2), & \text{if } i \neq j. \end{cases} \tag{A8}$$

(Appendices continue)

Appendix B

The Leaky Competing Accumulators Model

In the leaky competing accumulators (LCA) model, each option is associated with an activation value determined by an input value $I_i(t)$ and lateral inhibition. The input for option i in a choice set with n alternatives is given by

$$I_i(t) = I_0 + \sum_{j \neq i}^{n-1} V(d_{ij}(t)), \qquad (B1)$$

where I_0 is a positive constant and $d_{ij}(t)$ is the advantage or disadvantage of option i as compared to option j at time t. In the case of three alternatives $\{X, Y, Z\}$, the input values can be written as the vector $I(t) = [I_X(t), I_Y(t), I_Z(t)]'$.

At each moment in time, attention is allocated to one attribute dimension or the other in an all-or-none fashion where shifts in attention are determined by fixed probabilities $p(w_P)$ and $p(w_Q) = 1 - p(w_P)$. This attention switching procedure is the same as in multialternative decision field theory (MDFT), again motivated by the elimination by aspects heuristic (Tversky, 1972). If attribute P is selected at time t, then $d_{ij}(t)$ is the difference $P_i - P_j$ and likewise for attribute Q.

The function V in Equation B1 is an asymmetric value function consistent with the Tversky and Kahneman (1991) and Tversky and Simonson (1993) loss aversion function:

$$V(x) = \begin{cases} z(x), & \text{if } x \geq 0 \\ -\{z(|x|) + [z(|x|)]^2\}, & \text{if } x < 0, \end{cases} \qquad (B2)$$

where $z(x) = \log(1 - x)$. As proposed by Tversky and Kahneman, the function $V(x)$ has a steeper slope in the loss domain as compared to the gain domain. Usher and McClelland (2004) claim that this aspect of the model is essential for accounting for the attraction and compromise effects.

In the model, each alternative is associated with an activation state $A_i(t)$ that fluctuates across time. For three options $\{X, Y, Z\}$, the activation states can be written as the vector $A(t) = [A_X(t),$ $A_Y(t), A_Z(t)]'$. The activation state at time $t + 1$ is determined by the previous activation state and input values:

$$A(t + 1) = SA(t) + (1 - \lambda)[I(t) + \epsilon(t)]. \qquad (B3)$$

Negative activations are truncated to 0, and $\epsilon(t)$ is a vector $[\epsilon_1(t), \epsilon_2(t), \epsilon_3(t)]'$ of noise terms. It is assumed that ϵ_i is normally distributed with mean equal to 0 and standard deviation given by σ.

As in MDFT, the feedback matrix S allows for cross-talk among the options and contains positive self-connections and negative interconnections. However, these connections are not determined by an indifference/dominance distance function. Rather, the S matrix in the LCA model is defined as

$$S = \begin{bmatrix} \lambda & -\beta \cdot (1 - \lambda) & -\beta \cdot (1 - \lambda) \\ -\beta \cdot (1 - \lambda) & \lambda & -\beta \cdot (1 - \lambda) \\ -\beta \cdot (1 - \lambda) & -\beta \cdot (1 - \lambda) & \lambda \end{bmatrix}, \qquad (B4)$$

where λ is a decay parameter and β is a lateral inhibition parameter. As in MDFT, it is assumed that memory for a previous state can decay across time. The λ parameter ranges from 0 to 1 with a value of 0 corresponding to no memory of previous activation states and a value of 1 corresponding to perfect memory of previous states. An important difference between MDFT and the LCA model is that off-diagonal elements in the S matrix for the LCA model are constant. In MDFT the magnitude of these elements is determined by the distance function.

In total the LCA model has four free parameters: the attention weight $p(w_P)$, the initial input I_0, the decay parameter λ, and the lateral inhibition parameter β. The model also has a variance parameter σ that determines the noise in the accumulation process and a threshold parameter associated with the decision criterion. These two additional parameters are typically fixed for simulations.

(Appendices continue)

Appendix C

Mapping Objective to Subjective Values

Consider a pair of options (P_1, Q_1) and (P_2, Q_2) objectively defined as indifferent by the additive rule: $P_1 + Q_1 = P_2 + Q_2$. The line connecting these options can be written as $x/a + y/b = 1$, where the x-intercept is $a = P_1 - Q_1(P_2 - P_1)/(Q_2 - Q_1)$ and the y-intercept is $b = Q_1 - P_1(Q_2 - Q_1)/(P_2 - P_1)$. Notice that this line will be in the direction of the unit-length vector $\frac{1}{\sqrt{2}} \cdot [-1, 1]'$ because indifference is defined by an additive rule. The transformation from objective to subjective values maps the line to a curve satisfying the equation $(x/a)^m + (y/b)^m = 1$. There are several ways the mapping can be defined. We use the trigonometric mapping given below because it has an analytical solution and extremeness aversion increases with increasing concavity (i.e., increasing values of m).

Let θ be the angle between the x-axis and the vector $< P_1, Q_1 >$, which can be written as

$$\theta = \arctan\left(\frac{Q_1}{P_1}\right). \tag{C1}$$

Let (u_{P_1}, u_{Q_1}) be the subjective values for Option 1 on the curve. These subjective values are given by

$$u_{P_1} = \frac{b}{\left[\tan^m(\theta) + \left(\frac{b}{a}\right)^m\right]^{\frac{1}{m}}} \tag{C2}$$

and

$$u_{Q_1} = \frac{a \cdot \tan(\theta)}{\left[1 + \left(\frac{a}{b}\right)^m \tan^m(\theta)\right]^{\frac{1}{m}}}. \tag{C3}$$

The mapping of Option 2 proceeds in a similar fashion by first defining the angle θ and then using this angle to calculate the subjective values. If there is a third option in the choice set containing Options 1 and 2 that does not lie on the line $x/a + y/b = 1$, then the x- and y-intercepts for the line passing through Option 3 in the direction of the unit-length vector $\frac{1}{\sqrt{2}} \cdot [-1, 1]'$ are used.