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Some New Parallel Mechanisms Containing the Planar Four-Bar Parallelogram

Abstract

A parallelogram allows the output link to remain at a fixed orientation with respect to an input link, for which it acts as a unique role in the design of parallel mechanisms. In this paper, the unique role of a parallelogram is used completely to design some new parallel mechanisms with two to six degrees of freedom (DoFs). In these mechanisms, some with three DoFs possess the advantage of very high rotational capability and some with two DoFs have the translational output of a rigid body. More than that, the design concept is also applied first to some parallel mechanisms to improve the systems' rotational capability. The parallel mechanisms proposed in this paper have wide applications in industrial robots, simulators, micromanipulators, parallel kinematics machines, and any other manipulation devices in which high rotational capability and stiffness are needed. Especially, the paper provides new concepts of the design of novel parallel mechanisms and the improvement of rotational capability for such systems.

KEY WORDS—parallel mechanism, degrees of freedom, rotational capability, mechanical design, parallelogram

1. Introduction

A mechanism is a device that transforms motion to some desired pattern. Nowadays, mechanisms play more important roles in the human world, from light to heavy industries. Of these mechanisms, the four-bar mechanism is the most familiar and important one. We can find it everywhere, e.g., the open-close mechanism of a bus door in our daily life, the fixture or switch in most machines, the backhoes in agriculture, linkage-driven exercise mechanisms for amusement, and digging machines in heavy industry. Especially, in the last few decades we have witnessed an important development in the use of serial robots in the industrial world, mainly due to their flexibility. However, the mechanical architecture of the most common robots does not seem adaptable to certain tasks because of an accumulation of link position errors, and poor stiffness and inertia. As the increasing demand on accuracy, high payload capability and low cost, multi-closed-loop mechanisms have therefore been studied recently, and are being more and more regularly used within the industrial world. This is so for the parallel mechanism.

A parallel mechanism is typically made of two rigid bodies, one movable and the other one fixed, connected to each other by at least two kinematic chains. Such a mechanism possesses advantages in terms of high mechanical stiffness, wide bandwidth, and excellent load/weight ratio, for which they are very popular in industrial applications. For example, in 1965, a Stewart platform was proposed for the design of a motion simulator (Stewart 1965), where six spatial degrees of freedom (DoFs) are needed. After this, many interesting parallel mechanical systems with a specified type and number of DoFs have been extensively studied. They have been applied to various fields, e.g., aircraft simulators (Pouliot, Gosselin, and Nahon 1998), adjustable articulated trusses (Jain and Kramer 1990), wrists (Agrawal, Desmier, and Li 1995), high-performance camera-orienting devices (Gosselin and St-Pierre 1997), industrial robots (Cleary and Brooks 1993), force/torque sensors (Kerr 1989), micromanipulators (Reboulet and Pigeyre 1990; Liu et al. 2001a), haptic devices (Stocco, Salcudean, and Sassani 2001), wire robots (Albus, Bostelman, and Dagalakis 1993), mining machines (Arai et al. 1991), earthquake devices (Ceccarelli et al. 1999), medical devices (Brandt et al. 1999), and, especially, parallel kinematics machines (Valenti 1995). In these designs, Delta (Clavel 1988) is definitely a success, thanks to its smart and simple design. It gives the typical story of successful application of parallel mechanisms (Bonev 2001). Now Delta can be extensively found in pick-and-place applications and machine tools.

But it is clear today that most efforts have contributed to 3-DoF and 6-DoF parallel mechanisms and few to those

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with two, four and five DoFs. In the case of spatial 3-DoF parallel mechanisms, the output is limited to pure translations (Clavel 1988; Hervé 1992), pure rotations (Cox and Tesar 1989; Gosselin and Angeles 1989; Liu, Jin, and Gao 2000a; Gregorio 2002; Kong and Gosselin 2002), and complex motion[1] (Huang, Wang, and Fang 2002). There are few mechanisms with three DoFs that can combine the spatial rotational and translational DoFs and be further with definite motion (be opposite to complex motion). All these parallel mechanisms (both spatial 3-DoF and 6-DoF mechanisms with parallel chains) have one disadvantage in common, i.e., the limited rotational capability except for the architectures with redundant actuators (Kim et al. 2002). Rotational capability is undoubtedly one of the most important performances in industrial applications. Then, the proposal of parallel mechanisms with high rotational capability is still the research objective in this field. What is more, 4-DoF and 5-DoF parallel mechanisms are still welcome in the industrial world, e.g., the machine tool field (Pierrot et al. 2001).

The planar four-bar parallelogram is well known in the balance of planar and spatial mechanisms (Streit and Shin 1990; Wang and Gosselin 1999). On the other hand, a parallelogram allows the output link to remain at a fixed orientation with respect to an input link, for which it acts as a unique role in the design of parallel mechanisms as well. In 1992, the concept of planar parallelogram was first used in the design of a Star-Like robot (Hervé 1992), which has three translational DoFs. Since then, the concept of the parallelogram has attracted researchers' attention, e.g., a spatial 3-DoF parallel manipulator, ***HALF***, was proposed in 2001 (Liu et al. 2001b), in which the third leg consists of a parallelogram which guarantees a rotational DoF output. In these designs, the parallelogram acts as the role of guarantee of desired output, e.g., no rotational DoFs for Star-Like, and one rotation for ***HALF***. In this paper, we study such parallel mechanisms with the planar four-bar parallelogram.

Moreover, a parallelogram allows the output link to remain at a fixed orientation with respect to an input link, in such a way that one end of a socket-ball joint is attached to the output link and can then relatively increase the swing range of the joint. This will eventually improve the rotational capability of the moving platform. Based on such a concept, some parallel mechanisms are presented in virtue of the planar four-bar parallelogram in this paper.

The aim of this paper is then clear. Some new fully parallel mechanisms with two to six DoFs are proposed. The novelty of these mechanisms is that at least one leg of all enclosed parallel mechanisms consists of a planar four-bar parallelogram. These parallel mechanisms are intended for pure translation in planar, high or improved rotational capability and better stiffness. These mechanisms will have greatly potential applications in the industrial world.

1. Complex motion is defined as a simultaneous combination of rotation and translation. Any reference line drawn on the body will change both its linear position and its angular orientation. Points on the body will travel nonparallel paths, and there will be, at every instant, a center of rotation, will continuously change location (Norton 1999).

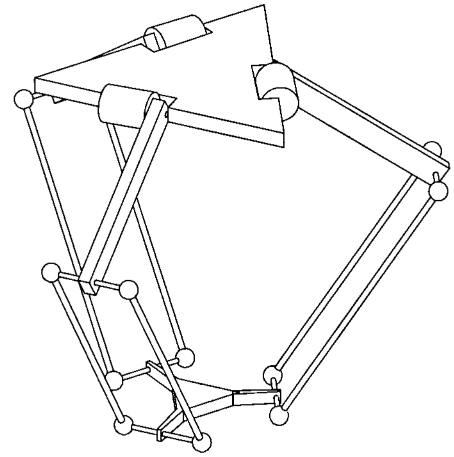


Fig. 1. The Delta robot.

2. Concept of the Parallelogram

2.1 Description

The well-known Delta robot (Clavel 1988) is shown in Figure 1, where each of the three legs consists of a spatial four-bar mechanism. The four bars are connected end-to-top in turn by spherical joints. Although it is a spatial mechanism, each two of the four bars should be parallel to each other at every instant, for which Delta has three translational DoFs. The design concept was advanced to a new family of 4-DoF parallel robots, H4, in 1999 (Pierrot and Company 1999). Such a design concept is very important to the parallel mechanism design using a planar four-bar parallelogram.

The planar four-bar parallelogram is a mechanism in which four bars are connected end-to-top in turn by revolute joints. As shown in Figure 2, links 1 and 3 are identical with each other in the link length, and so are links 2 and 4. Then, links 1 and 2 will have identical orientations with links 3 and 4, respectively. The parallelogram was first applied to the design of the Star-Like robot in 1992 by Hervé (1992), which also has three translational DoFs. Later, the design concept was used in the design of another three translational DoFs parallel mechanism by Tsai and Stamper (1996), which was the first design to resolve the puzzled problem of UU ("U" stands for universal joint) chains as well. From then on, the concept of the parallelogram attracted the attention of many researcher, e.g., TURIN (Sorli and Ferraresi 1997) with six DoFs and CaPaMan (Ceccarelli et al. 1999) with three DoFs. In 2001, a spatial 3-DoF (two translational DoFs and one rotational DoF) parallel manipulator, ***HALF***, was proposed (Liu 2001;

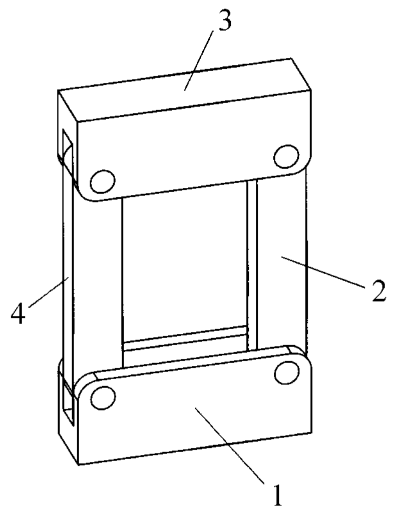


Fig. 2. The planar parallelogram.

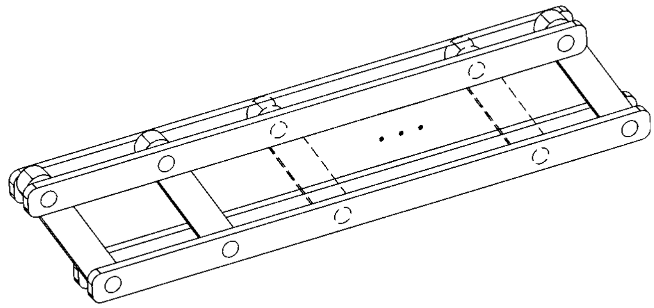


Fig. 3. Design example of the parallelogram with redundant constraints.

Liu et al. 2001b), in which the third leg consists of a parallelogram which guarantees a rotational DoF output. The special advantage of the manipulator is its high rotational capability, e.g., ±45, thanks to all single joints involved in the rotational DoF.

2.2 Advantages

Although the locus of any point on the output link is a circle, a parallelogram allows an output link to remain at a fixed orientation with respect to an input link. This is the reason why those parallel mechanisms use the concept of parallelogram; especially, it is very useful to deal with the UU chain. The parallelogram can act as the role of guarantee of desired output, e.g., no rotational DoFs for Star-Like and Tsai's manipulator (Tsai 1998), and one rotation for ***HALF***. In those designs, the parallelogram acting as the constant link in each leg can also increase the leg's stiffness greatly. Needless to say, a parallelogram has higher stiffness with respect to a single bar. Moreover, the leg stiffness can be improved further by increasing redundant constraints. For example, the parallelogram can be designed as shown in Figure 3, where the number of parallelograms is increased. Undoubtedly, the more parallelograms the leg has, the higher the stiffness the manipulator can obtain, and, in relative terms, the higher the fabricate accuracy it needs for the reason of redundant constraints.

For most parallel mechanisms, the rotational capability reaches its maximum at the original point of the workspace. Also, the maximum is usually limited for some parallel mechanisms with spherical joints because of the limited swing angle of such joints. What is more, the titling angle of the moving platform will be smaller and smaller from the point to the boundary. Because a parallelogram allows the output link to remain at a fixed orientation with respect to an input link, in such a way one end of a socket-ball joint is attached to the output link and can then relatively increase the swing range of the joint. The method will eventually improve the rotational capability of the moving platform. For example, as shown in Figure 4, suppose that, in some kind of parallel mechanism, the combination of actuations M_1 and M_2 will enable the moving platform to rotate about point A. In Figure 4(a), each of the two legs is a traditional one, which consists of a constant link. The link is connected to the moving platform by a socket-ball joint. In Figure 4(b), each leg consists of a planar four-bar parallelogram, which is connected to the moving platform by a socket-ball joint as well. The socket is fixed to the output link of each parallelogram. In Figure 4, line a is the reference line, from which the tilting angle for the joints is limited to $\pm 45^\circ$. Let each moving platform rotate 15° about original point A from the zero orientation. From Figure 4, we can see that the orientation of line a in Figure 4(a) is variable according to the change of that of the link, but it retains its orientation for line a in Figure 4(b). For this reason, in the orientation 15°, the socket-ball joint in Figure 4(a) exceeds the tilting limit because $54.2^\circ > 45^\circ$ already. But in Figure 4(b), the joint does not reach its limit yet. Therefore, the moving platform in Figure 4(b) can obtain higher rotational capability than that in Figure 4(a), for which we can conclude that a parallel mechanism with a parallelogram in its leg can have higher rotational capability than the traditional one. Moreover, because the socket will hold its original orientation at every instant, the rotational capability at the point near the original point will not be lower.

2.3 Disadvantages

Now, the advantages of using the parallelogram in the design of a parallel mechanism are clear, e.g., desired DoF output, higher stiffness and improved rotational capability (in some cases, high rotational capability). Then, what about the disadvantages? It is obvious that a parallelogram is completely kinematically equivalent to the combination of a revolute joint

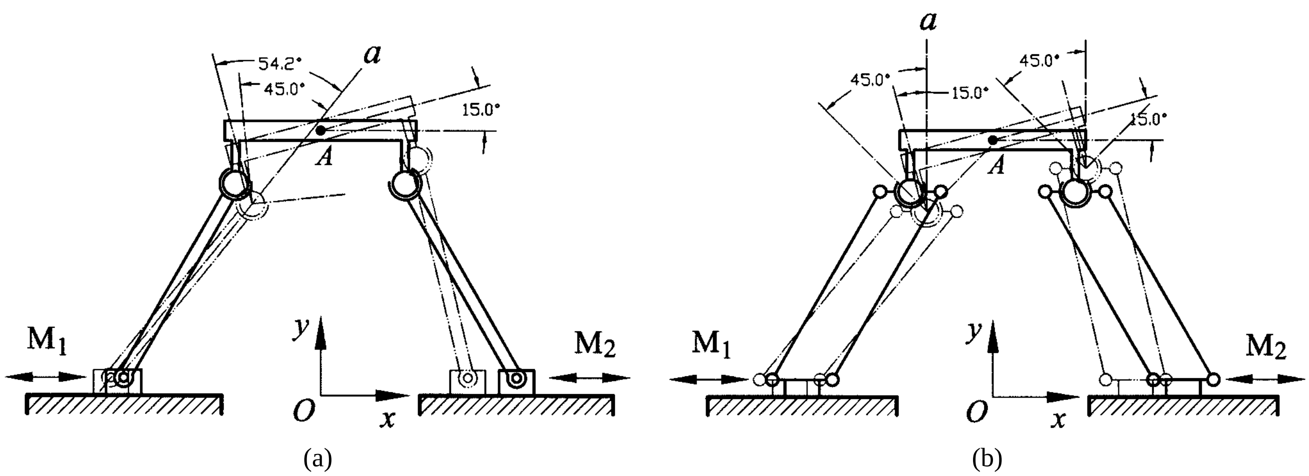


Fig. 4. Rotational capability comparison between two kinds of leg mechanisms.

and a fixed-length link. Compared with this combination, the disadvantage comes from the complex structure. In a parallelogram, there are four revolute joints and four links. The axes of all joints should be parallel to each other, and the lengths for each two of the four links should be equal to each other. That is to say, the fabrication accuracy should be high enough, otherwise it will lead to the accumulation of errors, which will eventually result in poorer accuracy of the device. But because the joints are all single-DoF joints, it will not be a critical problem to fabricate it with high accuracy. The complex structure will also result in interference between the moving platform and legs, and also the interference between links in itself. To avoid this problem, the designer should consider it with great care. Considering the advantages that the parallelogram has, the complex structure could be acceptable. All in all, it is well worth designing a parallel mechanism with the planar four-bar parallelogram.

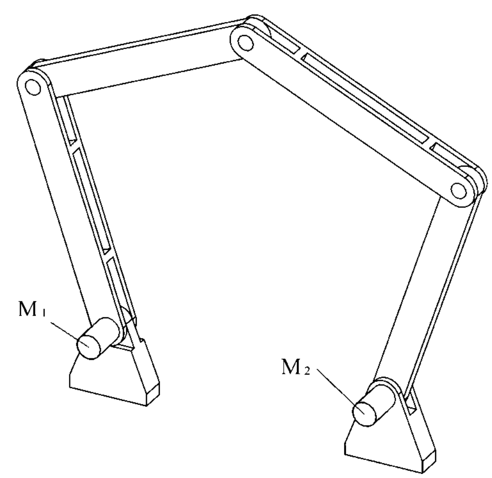


Fig. 5. A planar 2-DoF parallel mechanism.

3. The New Parallel Mechanisms

Based on the concept of the parallelogram, some new parallel mechanisms are presented in virtue of the planar four-bar parallelogram. In these designs, a parallelogram will play the role in desired DoF output and improved rotational capability and stiffness.

3.1 Two DoFs

The most planar 2-DoF parallel mechanisms (Asada and Kanade 1983; McCloy 1990; Gao, Liu, and Gruver 1998) are the well-known five-bar mechanisms with prismatic actuators or revolute actuators. In the case of the mechanism with revolute actuators, the mechanism consists of five revolute pairs and the two joints fixed to the base are actuated, as shown in Figure 5. The output of the mechanism is a planar 2-DoF motion of a point on the end effector. A different design is shown in Figure 6, which consists of two R(Pa)R chains (in this paper, “R” denotes a revolute joint, and “Pa” denotes a parallelogram with four revolute joints). In this design, we can see that, if the revolute actuator M_1 is locked, the mechanism is equivalent to a planar four-bar mechanism with the actuator M_2. If M_1 is active but M_2 is locked, the mechanism will have a translation along the axis, at the same time there is a rotation about the x-axis, which is actually an associated movement. Then the output is the translation along the x-axis and another translation in the O-yz plane or a rotation about the x-axis.

As mentioned above, the output of most planar 2-DoF parallel mechanisms is the planar motion of a point, while the orientation will change instantly. In some applications, an object

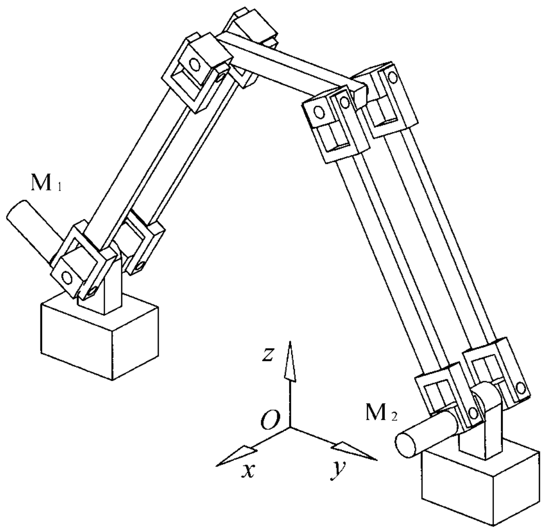


Fig. 6. A type of 2-DoF parallel mechanism.

should be transferred with fixed orientation. In such cases, the translational motion of a rigid body with retained orientation is needed and the above-mentioned 2-DoF parallel mechanisms are unavailable. The design based on a parallelogram can undertake the job. Such a parallel mechanism with 2-P(Pa) (where "P" denotes a prismatic joint) chain is shown in Figure 7(a), where the two translational DoFs can be reached if the prismatic joints are active. What we should notice is that, to obtain two DoFs of a rigid body in this system, the kinematic chain P(Pa)PRR shown in Figure 7(b) is enough to guarantee the mechanism with two translational DoFs. The reason for using two planar four-bar parallelograms is to increase the system's stiffness and make the system symmetric. The parallel mechanism has been applied to the development of a five-axis hybrid machine tool (Liu 2001; Liu, Tang and Wang 2003), which is now being used to machine a kind of impeller.

Based on the above design concept, some two-DoF parallel mechanisms are listed in Table 1.

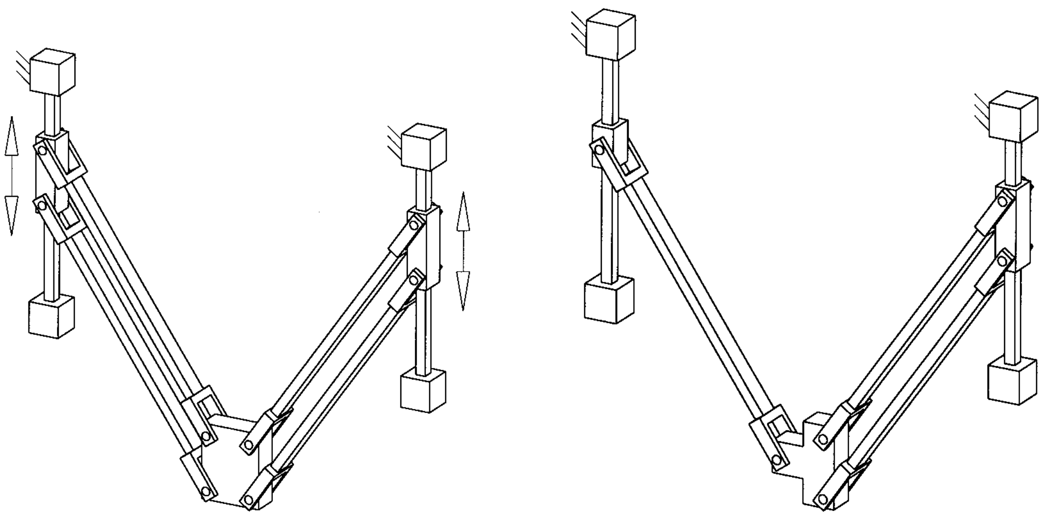
3.2 Three DoFs

There are many parallel mechanisms with three DoFs. One is a planar 3-RRR parallel mechanism (Liu, Wang, and Gao 2000b; Gosselin and Angeles 1988), as shown in Figure 8. The moving platform has three planar DoFs, which are two translations along the x- and y-axes and one rotation around the axis perpendicular to the O-xy plane. Another example is the spherical 3-RRR parallel mechanism (Liu et al. 2000a; Gosselin and Angeles 1989) as shown in Figure 9; in this design all the joint axes intersect at a common point. The motion of any point in the mechanism is the rotation about the point. The moving platform has only orientational DoFs with respect to the base. One of the parallel mechanisms presented by Hunt (1978) is the 3-RPS ("S" denotes a spherical joint) parallel mechanism, which has complex motion, although it does have three DoFs. The most famous robot with three translations is Delta (Clavel 1988), which was proposed by Clavel and marketed by the Demaurex Company and ABB under the name IRB 340 *FlexPicker*. Now Delta has been very attractive in industry. There is another type of 3-DoF parallel mechanism, in which the moving platform is connected to the base through four legs, where the fourth leg is a passive one and is also the leading leg, which means that the leg determines the motion of the moving platform, e.g., the spherical coordinate parallel mechanism, which was used to the design of a machine tool design in IFW of the University of Hannover (Tonshoff, Grendel, and Kaak 1999).

Figure 10 shows four types of spatial 3-DoF parallel mechanisms, where each leg consists of a parallelogram. These mechanisms have a common characteristic in output, i.e., complex motion, which are two rotations and one translation just as the 3-RPS mechanism has. Compared with the 3-RPS and 3-PRS mechanisms, they possess higher rotational capability and stiffness.

Figure 11 shows another spatial 3-DoF parallel mechanism with 1-R(Pa)R-2-RR(Pa)R chain, from which we can see that the action of actuator M_1 will lead to the translation along the z-axis. At the same time there is a rotation about the z-axis, which is actually an associated movement. If actuator M_1 is locked, the mechanism is equivalent to a planar 2-DoF mechanism with the actuators M_2 and M_3 (Gosselin 1996). Then, the output is the translation along the z-axis and two planar motions in the O-xy plane.

Although there are so many 3-DoF parallel mechanisms, it is not difficult to find that the output is just limited to planar DoFs, pure translations, pure rotations, or complex motion, except for two rotations and one translation of the structure like the spherical coordinate parallel mechanism (Tonshoff, Grendel, and Kaak 1999). Parallel mechanisms with high rotational capability are just those with planar kinematics. Spatial 3-DoF parallel mechanisms with high rotational capability and definite motion combining spatial translations and rotations are still the research objective. In 2001, such a parallel mechanism, ***HALF*** (Liu et al. 2001b), was proposed as shown in Figure 12(a), where the moving platform is connected to the base by three non-identical legs, a PRU chain for the first and second legs and a PR(Pa)R chain for the third leg. The mechanism has two translations in the O-yz plane and one rotation about the y-axis. Moreover, the rotational capability can be very high, as much as $\pm 45^\circ$ (Liu et al. 2001b). The ***HALF*** mechanism with revolute actuators (Wang and Liu 2003) is shown in Figure 12(b), where the legs are with two RRU chains and one RR(Pa)R chain. In these designs the first and second legs should be in the same plane. In these two legs, the axes for the revolute joints in the U joints that are connected to the moving platform should be collinear. Also, the axis of the revolute joint in the third leg linked to the



(a) (b)

Fig. 7. The new parallel mechanism with two translational DoFs.

Table 1. Some 2-DoF Parallel Mechanisms

No.	Leg Chains: First Leg	Leg Chains: Second Leg	Mechanism Chains	Remarks
1	R(Pa)R	R(Pa)R	2-R(Pa)R	As shown in Figure 6.
2	P(Pa)	P(Pa) or PRR	P(Pa)(Pa)P or P(Pa)PRR)	There are two cases; the actuated P joint can be vertical or horizontal. The vertical case is shown in Figure 7.
3	(Pa)(Pa)	(Pa)(Pa) or RRR	(Pa)(Pa)(Pa)(Pa) or (Pa)(Pa)RRR	In each leg, a revolute joint in the input link of the parallelogram that is attached to the base is actuated.

Note. P, R, and (Pa) denote prismatic joint, revolute joint, and parallelogram, respectively.

moving platform should be parallel to the two axes, as shown in Figure 12. Moreover, in ***HALF***, the universal joints can also be replaced by spherical joints because of the unique third leg. In the first and second legs of ***HALF***, because two revolute joints connected to the moving platform should have the same axis, the joints can be simplified to one revolute joint, i.e., the two U joints can be replaced by three revolute joints.

Figure 13 shows a type of parallel mechanism, called ***HANA***. Figure 13(a) is the mechanism with prismatic actuators and Figure 13(b) is that with revolute actuators. As shown in Figure 13(a), the moving platform is connected to the base by a PRU chain and two PR(Pa)R chains. The mechanism is a general manipulation device that must have three DoFs when the prismatic actuators are active. Due to the arrangement of links and joints of the mechanism, the combination of the three legs constrains the rotation of the moving platform with respect to the x- and z-axes and the translation along the x-axis. This leaves the mechanism with two translational degrees in the O-yz plane and one rotational degree of freedom about the y-axis. Table 2 shows the mechanism capability description. We can see that ***HALF*** and ***HANA*** have same output, but there is an outstanding difference between these two mechanisms in terms of the rotational DoF, i.e., ***HANA*** is actuation redundant for the DoF but ***HALF*** is not. The rotational DoF of ***HANA*** is reached with the combination of the first and second legs with PR(Pa)R chains. This situation is the same as ***HANA*** with revolute actuators shown in Figure 13(b).

Now let us look at another spatial 3-DoF parallel mechanism with high rotational capability, ***HALF***-II, as shown in Figure 14. The mechanism is with a 2-P(Pa)R-1-PR(Pa)R chain. Table 3 gives the mechanism capability in detail, from which we can see that the first leg itself can constrain the moving platform with the translation along the x-axis and rotations about the z- and x-axes. The second leg chain can be identical with or different from the first leg, e.g., the second leg can be with a $P_yR_xU_{xy}$ chain. Moreover, the third leg can

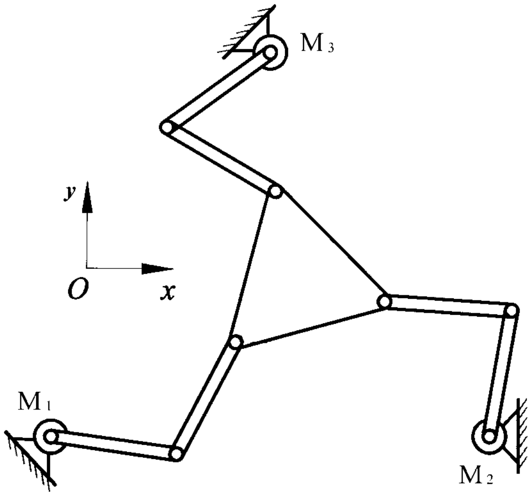


Fig. 8. A planar 3-DoF parallel mechanism.

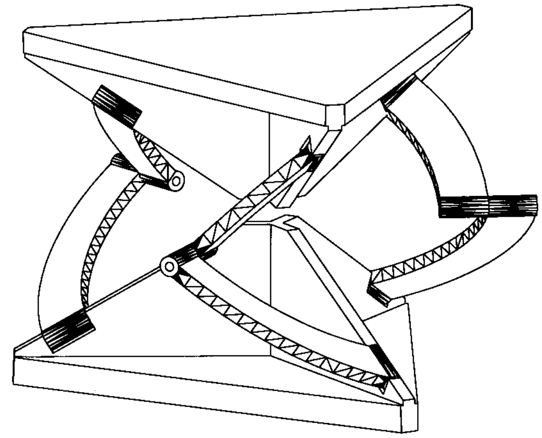


Fig. 9. A spherical 3-RRR parallel mechanism.

Table 2. The Constraint and DoFs of *HANA* Mechanism with Prismatic Actuators

Single Leg			Combination of Three Legs	
No.	Chain Type	Constraints	Constraints	Remained DoFs
1	$P_zR_y(Pa)_yR_y$	$\{RO_x, RO_z\}$		
2	$P_zR_y(Pa)_yR_y$	$\{RO_x, RO_z\}$	$\{T_x, RO_x, RO_z\}$	$\{T_y, T_z, RO_y\}$
3	$P_zR_xU_{x,y}$	$\{T_x, RO_z\}$		

Note. P, R, U, (Pa), T, and RO denote prismatic joint, revolute joint, universal joint, parallelogram, translation, and rotation, respectively. In each case, the subscript denotes the DoF.

Table 3. The Constraint and DoFs of *HALF*-II

Single Leg			Combination of Three Legs	
No.	Chain Type	Constraints	Constraints	Remained DoFs
1	$P_y(Pa)_zR_y$	$\{RO_x, RO_z, T_x\}$		
2	$P_y(Pa)_zR_y$	$\{RO_x, RO_z, T_x\}$	$\{T_x, RO_x, RO_z\}$	$\{T_y, T_z, RO_y\}$
3	$P_xR_y(Pa)_yR_y$	$\{RO_x, RO_z\}$		

Note. P, R, (Pa), T, and RO denote prismatic joint, revolute joint, parallelogram, translation, and rotation, respectively. In each case, the subscript denotes the DoF.

then be a 4-DoF, 5-DoF or 6-DoF chain, e.g., a P_xUU or P_xUS. It can also be a traditional P_xSS chain. The possible chains for the three legs and the mechanism are shown in Table 4, from which we can see that there are six types of mechanisms with different leg chains, e.g., mechanisms with $P_y(Pa)_zR_y$ – $P_yR_xU_{xy}$ – $P_xR_y(Pa)_yR_y$ and $P_y(Pa)_zR_y$ – $P_y(Pa)_zR_y$ – P_xUU chains are shown in Figures 15(a) and (b), respectively. What we should notice is that, in the P_xUU chain, the axis of the revolute joint in the first U joint attached to the P joint must be parallel to the y-axis, and the axis of the revolute joint in the second U joint connected to the moving platform must also be parallel to the y-axis. Then the mechanisms can also have three DoFs, which are two translational DoFs in the O-yz plane and one rotational DoF about the y-axis. Additionally, for the P_xUS chain in Table 4, the axis of the revolute joint in the first U joint attached to the P joint should be parallel to the y-axis as well. Based on such a concept, there are also many topology architectures for ***HALF*** and ***HANA***, which will not be described in detail in this paper.

We can see that ***HALF***-II and ***HALF*** have identical DoFs, even though there is a difference between these two mechanisms. In ***HALF***, the first and second legs are PRU chains,

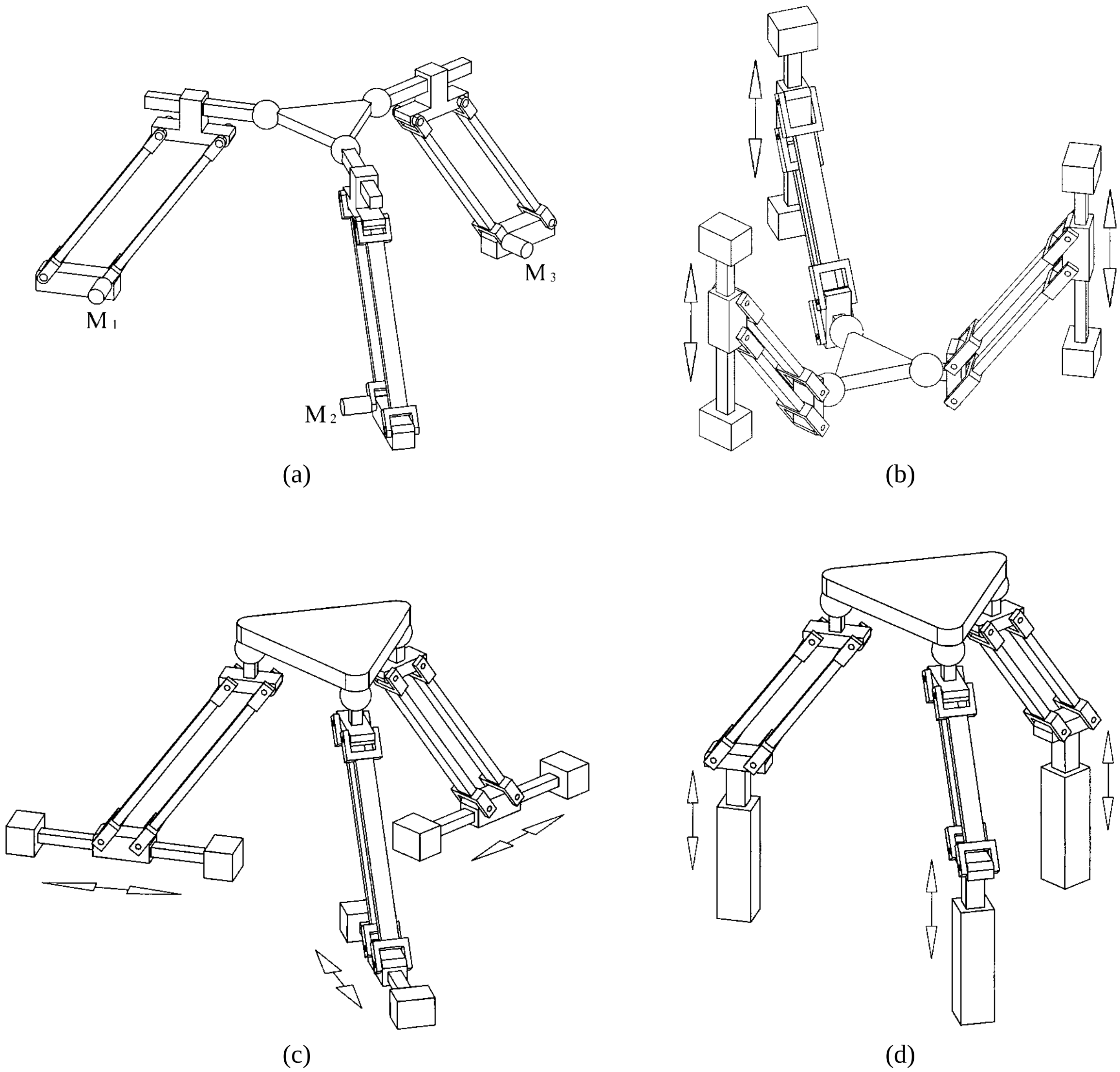


Fig. 10. Four types of spatial 3-DoF parallel mechanisms with complex motion.

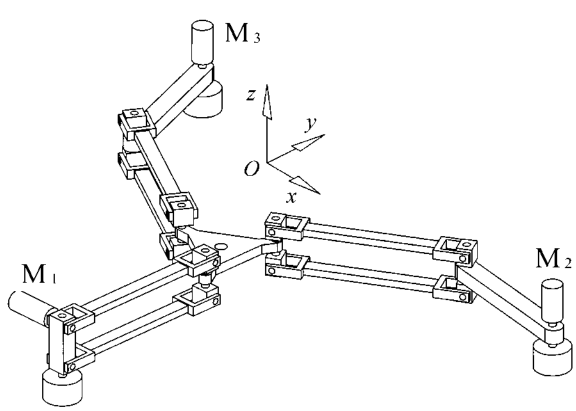


Fig. 11. A new spatial 3-DoF parallel mechanism.

which can constrain the moving platform with the translation along the x-axis and the rotation about the z-axis. The combination of the two legs makes the moving platform rotate freely about the x-axis. Because this is an undesirable output for ***HALF***, the rotational DoF should be constrained by the third leg. In such a situation, the stiffness of the third leg should be high enough to undertake the inner torque of the moving platform. In ***HALF***-II, because the first and second legs themselves can constrain the rotation about the x-axis, there is no additional requirement on the stiffness of the third leg.

Figure 16 shows another concept of spatial 3-DoF parallel mechanisms. For such a mechanism with n DoFs, it usually consists of n identical actuated legs with six DoFs and one

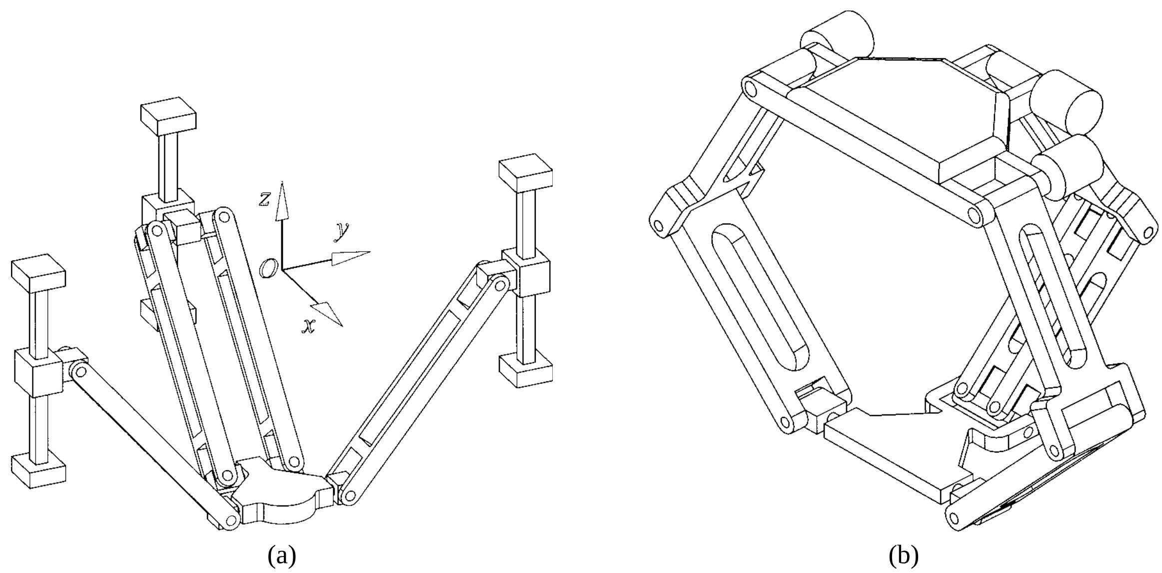


Fig. 12. ***HALF*** parallel mechanisms.

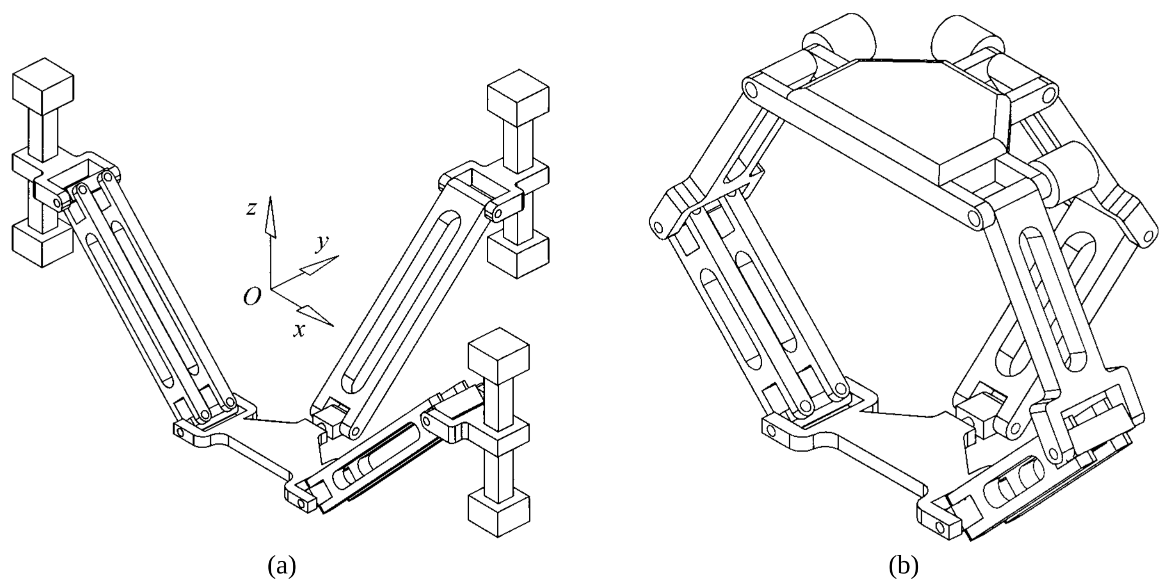


Fig. 13. ***HANA*** parallel mechanisms.

passive leg with n DoFs connecting the platform and the base, i.e., the DoF of the mechanism is dependent on the passive leg's DoF. We can improve the rigidity of this type of mechanism through optimization of the link rigidities to reach a maximal global stiffness and precision (Zhang and Gosselin 2002). For the mechanism shown in Figure 16, the passive leg is with a (Pa)U chain, which leads to one translational DoF and two rotational DoFs of the mechanism. Table 5 lists 11 types of 3-DoF parallel mechanisms.

3.3 Four DoFs

A fully parallel mechanism is defined as such a parallel mechanism, in which the number of chains is strictly equal to the number of DoFs of the moving platform (Merlet 2000). According to this definition, there is actually no 4-DoF fully parallel mechanism with fully symmetric structure, i.e., all legs are identical with each other. The mechanisms with four DoFs proposed earlier are usually not such mechanisms, e.g.,

Table 4. Possible Chains for Legs and *HALF*-II

Leg Chain			
First Leg	Second Leg	Third Leg	Mechanism Chains
$P_y(Pa)_zR_y$	$P_y(Pa)_zR_y$	$P_xR_y(Pa)_yR_y$	$P_y(Pa)_zR_y – P_y(Pa)_zR_y – P_xR_y(Pa)_yR_y$
			$P_y(Pa)_zR_y – P_yR_xU_{xy} – P_xR_y(Pa)_yR_y$
		P_xUU	$P_y(Pa)_zR_y$- $P_y(Pa)_zR_y$- P_xUU
	$P_yR_xU_{xy}$		$P_y(Pa)_zR_y – P_yR_xU_{xy} – P_xUU$
		P_xUS (or P_xSS)	$P_y(Pa)_zR_y – P_y(Pa)_zR_y – P_xUS$
			$P_y(Pa)_zR_y – P_yR_xU_{xy} – P_xUS$

Note. P, R, U, (Pa), and S denote prismatic joint, revolute joint, universal joint, parallelogram, and spherical joint, respectively. In each case, the subscript denotes the DoF.

Table 5. Some 3-DoF Parallel Mechanisms

	Leg Chains			Mechanism	
No.	First Leg	Second Leg	Third Leg	Chains	Remarks
1	(Pa)PS	(Pa)PS	(Pa)PS	3- (Pa)PS	As shown in Figure 10(a)
2	P(Pa)S	P(Pa)S	P(Pa)S	3- P(Pa)S	As shown in Figures 10(b)-(d)
3	R(Pa)R	RR(Pa)R	RR(Pa)R	1-R(Pa)R-2-RR(Pa)R	As shown in Figure 11
4	PRU	PRU	PR(Pa)R	2-PRU-1-PR(Pa)R	As shown in Figure 12(a)
5	RRU	RRU	RR(Pa)R	2-RRU-1-RR(Pa)R	As shown in Figure 12(b)
6	PR(Pa)R	PR(Pa)R	PRU	2- PR(Pa)R -1- PRU	As shown in Figure 13(a)
7	RR(Pa)R	RR(Pa)R	RRU	2- RR(Pa)R -1- RRU	As shown in Figure 13(b)
8	P(Pa)R	P(Pa)R	PR(Pa)R	2-P(Pa)U-1-PR(Pa)R	Figure 14; please see Table 4 for other topology architectures
	The first, second and third legs	The fourth (passive) leg			
9	UPS (PUS or RUS)	(Pa)U		3-UPS-1-(Pa)U	As shown in Figure 16
10		(Pa)PR		3-UPS-1-(Pa)PR	
11		(Pa)RP		3-UPS-1-(Pa)RP	

Note. P, R, U, (Pa), and S denote prismatic joint, revolute joint, universal joint, parallelogram, and spherical joint, respectively.

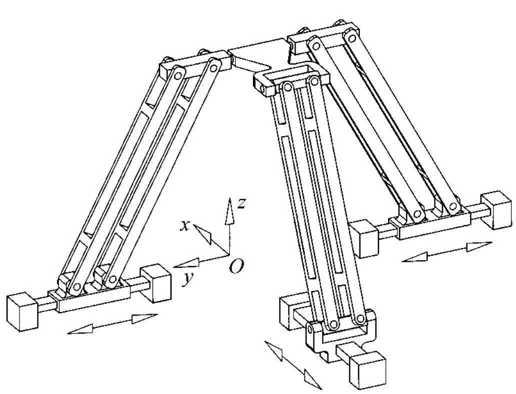
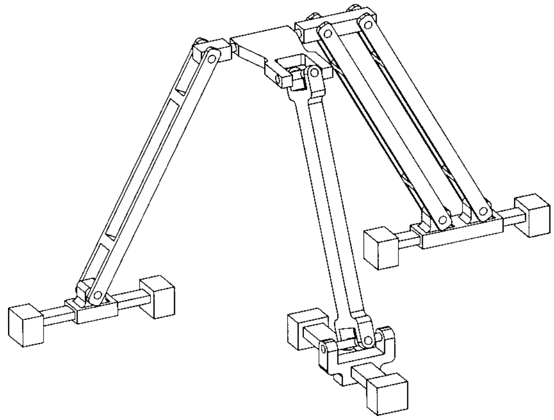


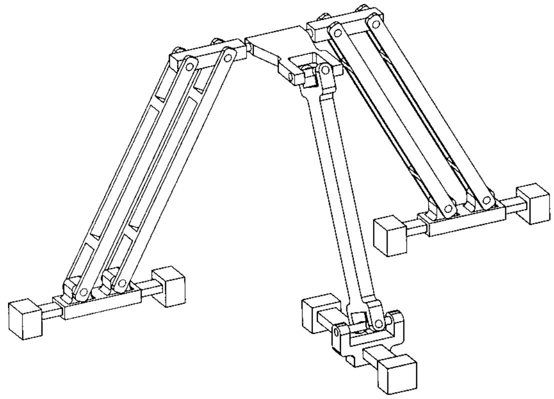
Fig. 14. Kinematic structure of ***HALF***-II.

a mechanism presented in Tanev (1998), where there are two actuators in one of its three legs. In Pierrot and Company (1999), a new family of 4-DoF parallel robots, H4, was proposed. Two mechanisms in this family are with 2-$[R(U\text{-}S)_2]_2R$ and 2-$(PUU)_2R$ chains, respectively. Clavel (2002) presented another 4-DoF parallel mechanism, HITA-STT, which can enable five sides of a cube-shaped object. Chen and Zhao (2002) introduced a 4-DoF parallel mechanism with a 2-$(PR)_2S$-2-PUS chain, the output of which is two translational DoFs and two rotational DoFs. Huang and Li (2002) proposed two types of 4-DoF parallel mechanism, which are with 4-RRR(RR) and 4-RPR(RR) chains, respectively. Both of these have the output of three translations and one rotation.

Based on the concept of the four-bar parallelogram, several parallel mechanisms with four DoFs are proposed in this



(a)



(b)

Fig. 15. Two topology architectures of ***HALF***-II.

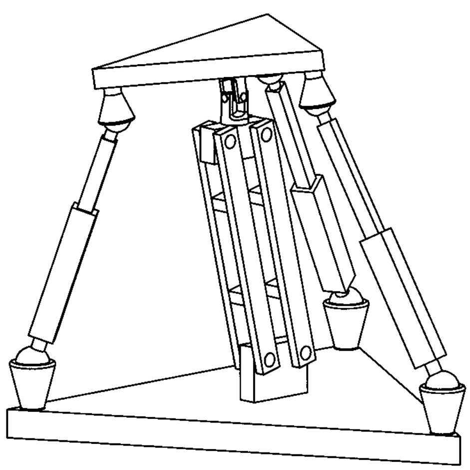


Fig. 16. A spatial 3-DoF parallel mechanism with a passive leg.

paper. For example, Figure 17(a) shows a 4-DoF parallel mechanism with a 2-PR(Pa)U-2-PR(Pa)R chain, and in Figure 17(b) the mechanism is with a 2-PUU-2-PR(Pa)R chain. The moving platform for both mechanisms has four DoFs, which are three translations and one rotation about the y-axis with respect to the base. Even then, we should notice that, if the quadrangle of the moving platform is similar to that of the base, the mechanisms will be in their singular configurations at the original pose.

Figure 18 shows three types of 4-DoF parallel mechanisms with the passive leg. In each of these designs, there are four legs with UPS (or SPS) chains, where the P joint is actuated. The fifth (passive) leg in the mechanism of Figure 18(a) is with a (Pa)PU chain, a (Pa)S chain for that of Figure 18(b), and a P(Pa)U chain for the mechanism in Figure 18(c).

Other parallel mechanisms with 4 DoFs can be found in Table 6.

3.4 Five DoFs

According to the definition of a fully parallel mechanism, similarly there is no 5-DoF fully parallel mechanism with fully symmetric structure as well. A 5-DoF parallel mechanism proposed by Austad (1987) consists of two parallel mechanisms. Since then, there has been little work on parallel mechanisms with five DoFs, apart from the mechanisms with 3-RR(RRR) and 3-RRR(RR) chains, presented by Huang and Li (2002). It also seems difficult to design a 5-DoF parallel mechanism with a fully symmetric structure using the parallelogram except for those as shown in Figure 19, where two types of 5-DoF mechanisms are illustrated. In these two designs, the fifth leg is actuated and is also the leading leg, i.e., the output of the mechanism is dependent on the leg. The other four legs are with UPS (or SPS) chains, where the P joint is actuated. In Figure 19(a), the kinematic chain for the fifth leg is P(Pa)S, and P(Pa)PU for that in Figure 19(b), where the prismatic joints attached to the base are actuated. The two mechanisms are with two translations and three rotations, and three translations and two rotations, respectively. We can find other parallel mechanisms with five DoFs from Table 7.

3.5 Six DoFs

In the past few decades, 6-DoF parallel mechanisms have been the most studied mechanisms. Such a mechanism can provide us with entire DoFs that a rigid body can have in a three-dimensional space, for which this type of parallel mechanism has been attracting more attention in the field. Generally, each leg of such a mechanism is a typical serial chain with six DoFs. The fully parallel mechanism is then usually a mechanism including at least six such legs. Theoretically, these six legs can be arranged arbitrarily, e.g., 6-6 (six joints on the base and six on the moving platform) (Sreenivasan, Waldron, and Nanua 1994), 6-3 (Hunt 1983), 5-5 (Hunt and

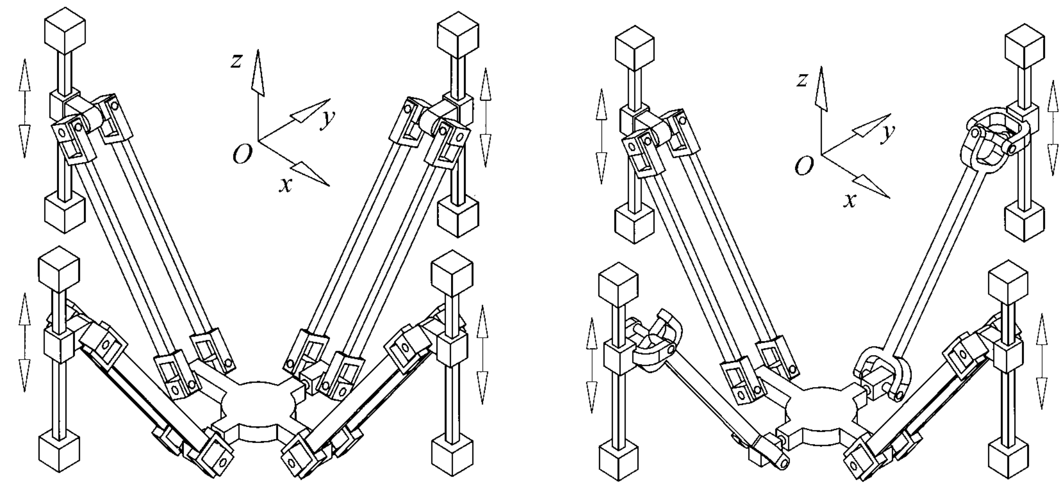


Fig. 17. Two new 4-DoF parallel mechanisms.

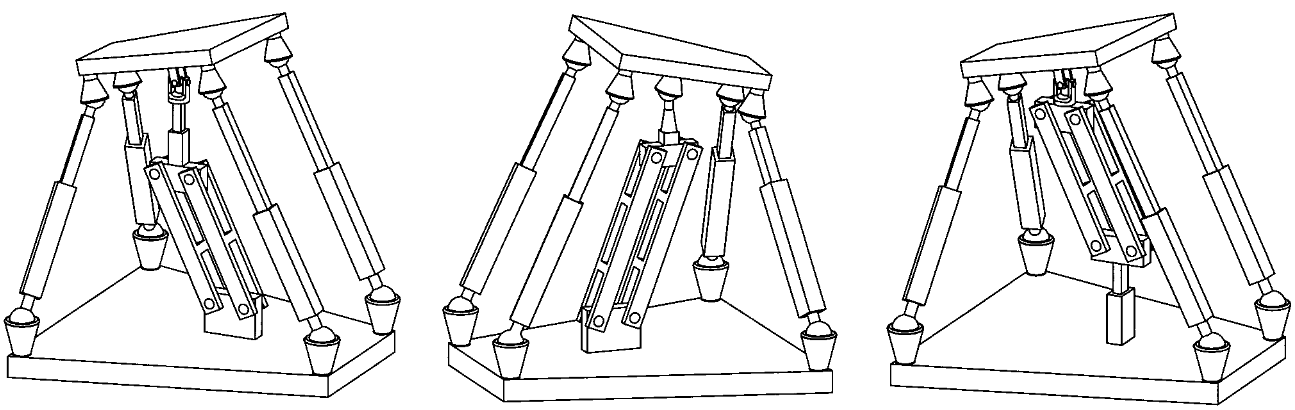


Fig. 18. Three types of 4-DoF parallel mechanisms with a passive leg.

Primrose 1993), 5-4 (Innocenti and Parenti-Castelli 1993), 4-4 (Lin, Duffy, and Griffis 1992), 3-2-1 (Bruyninckx 1997), or a cubic type (each two of the six legs are settled in the side of a cube) (Dafaoui et al. 1998). They, undoubtedly, possess the advantages of high stiffness and high payload capability. At the same time, they also suffer problems of complex forward kinematics, relatively small useful workspace, especially limited rotational capability for the reason of at least six kinematic chains and more multi-DoF joints. There are also some 6-DoF parallel mechanisms with three legs, in each of which two actuators are attached, e.g., the 3-PRPS mechanism (Behi 1988; Alizade and Tagiyev 1994), 3-PPSP (Byun and Cho 1997), 3-RSPR (Sima'an, Glozman, and Shoham 1998), and, especially, a mechanism with the 2-DoF planar actuator (Ben-Horin and Shoham 1996). Compared with a 6-DoF mechanism with six legs, such a mechanism has a much larger workspace, very simple forward and inverse kinematic solutions, fewer moving parts and joints. The concept of parallelogram is also used in the design of such mechanisms, e.g., TURIN (Sorli and Ferraresi 1997) with a 3-(Pa)(Pa)PS chain and a mechanism with a 3-R(Pa)S chain (Ebert-Uphoff and Gosselin 1998).

A new 6-DoF mechanism is shown in Figure 20, which consists of three identical legs, i.e., PP(Pa)S chains, where the two prismatic joints are actuated. Moreover, in each leg, one of the two P joints can be vertical and the two P joints can be a 2-DoF planar actuator. Please see Table 8. Compared with a 3-PPRS mechanism and others with planar actuators,

Table 6. Some Parallel Mechanisms with Four DoFs

No.	Leg Chains: First Leg	Leg Chains: Second Leg	Leg Chains: Third Leg	Leg Chains: Fourth Leg	Mechanism Chains	Remarks
1	PR(Pa)U	PR(Pa)R	PR(Pa)U	PR(Pa)R	2-PR(Pa)U-2-PR(Pa)R	As shown in Figure 17(a)
2	PUU	PR(Pa)R	PUU	PR(Pa)R	2-PUU-2-PR(Pa)R	As shown in Figure 17(b)
	The first, second, third and fourth legs		The fifth (passive) leg			
3	UPS (PUS or RUS)		(Pa)PU		4-UPS-1-(Pa)PU	As shown in Figure 18(a)
4			(Pa)RU		4-UPS-1-(Pa)RU	
5			(Pa)S		4-UPS-1-(Pa)S	As shown in Figure 18(b)
6			P(Pa)U		4-UPS-1-P(Pa)U	As shown in Figure 18(c)
7			(Pa)UP		4-UPS-1-(Pa)UP	
8			R(Pa)U		4-UPS-1-R(Pa)R	

Note. P, R, U, (Pa), and S denote prismatic joint, revolute joint, universal joint, parallelogram, and spherical joint, respectively.

Table 7. Some 5-DoF Parallel Mechanisms

No.	Leg Chains: The First, Second, Third and Fourth Legs	Leg Chains: The Fifth Leg	Mechanism Chains	Remarks
1	UPS (PUS or RUS)	P(Pa)S	4-UPS-1-P(Pa)S	There are two cases; the actuated P joint can be vertical or horizontal. The vertical case is shown in Figure 19(a)
2		P(Pa)PU	4-UPS-1-P(Pa)PU	There are two cases; the actuated P joint can be vertical or horizontal. The vertical case is shown in Figure 19(b)
3		P(Pa)RU	4-UPS-1-P(Pa)RU	In the fifth leg, the P joint is actuated.
4		P(Pa)UP	4-UPS-1-P(Pa)UP	In the fifth leg, the first P joint is actuated.
5		(Pa)PS	4-UPS-1-P(Pa)S	In the fifth leg, the P joint is actuated.
6		(Pa)RS	4-UPS-1-(Pa)RS	In the fifth leg, one of the four revolute joints in the parallelogram is actuated.
7		R(Pa)S	4-UPS-1-R(Pa)S	In the fifth leg, the R joint is actuated.

Note. P, R, U, (Pa), and S denote prismatic joint, revolute joint, universal joint, parallelogram, and spherical joint, respectively.

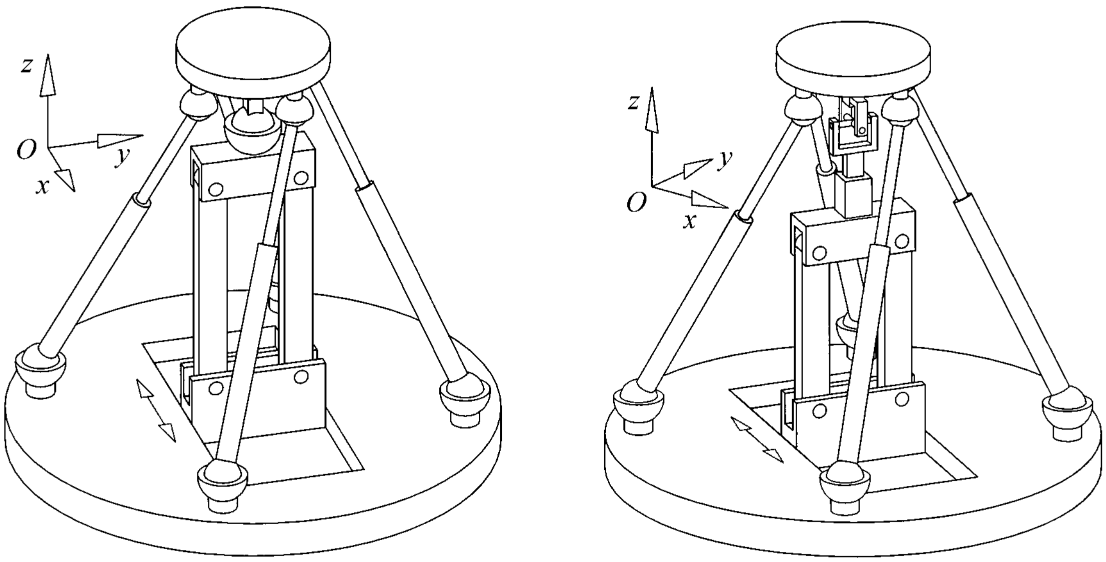
the parallel mechanism has a higher rotational capability and stiffness.

4. Conclusions

The four-bar parallelogram allows the output link to remain at a fixed orientation with respect to an input link. Applied to the design of a parallel mechanism, it can play unique roles in terms of desired DoF output, higher stiffness, and improved rotational capability. In this paper, such advantages of the four-bar parallelogram are used completely in the design of some parallel mechanisms. For the first investigation, 30 types of new parallel mechanisms with two to six DoFs are presented.

In these new parallel mechanisms, especially, some mechanisms with three DoFs, e.g., ***HALF*** and ***HANA*** mechanisms, possess the advantage of very high rotational capability, e.g., as much as $\pm 45°$, and some mechanisms with two DoFs have the translational output of a rigid body with a retained orientation.

The new parallel mechanisms have wide applications in industrial robots, simulators, micromanipulators, parallel kinematics machines, and any other manipulation devices in which



(a) (b)

Fig. 19. Two types of new 5-DoF parallel mechanisms.

Table 8. Some 6-DoF Parallel Mechanisms

No.	Leg Chains: First Leg	Leg Chains: Second Leg	Leg Chains: Third Leg	Mechanism Chains	Remarks
1	PP(Pa)S	PP(Pa)S	PP(Pa)S	3-PP(Pa)S	One example is shown in Figure 20. In each leg, one P joint can be along vertical direction, and the double P joints can replaced by a 2-DoF planar actuator.

Note. P, (Pa), and S denote prismatic joint, parallelogram, and spherical joint, respectively.

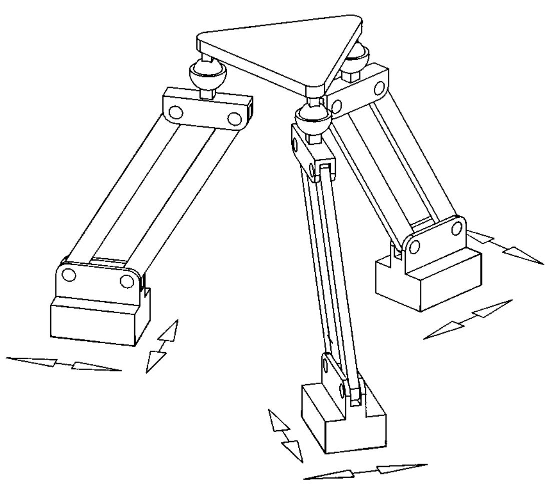


Fig. 20. A new 6-DoF parallel mechanism.

high rotational capability and stiffness are needed. Especially, the paper provides new concepts for the design of novel parallel mechanisms and the improvement of stiffness and rotational capability for such systems. Based on the design concept of this paper, more and more new parallel mechanisms can be reached.

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