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Abstract

People give information about their personal preferences in many different ways, depending on their background. This paper deals with group decision making problems in which the solution depends on information of a different nature, i.e., assuming that the experts express their preferences with numerical or linguistic values.

The aim of this paper is to present a proposal for this problem. We introduce a fusion operator for numerical and linguistic information. This operator combines linguistic values (assessed in the same label set) with numerical ones (assessed in the interval [0,1]). It is based on two transformation methods between numerical and linguistic values, which are defined using the concept of the characteristic values proposed in this paper. Its application to group decision making problems is illustrated by means of a particular fusion operator guided by fuzzy majority. Considering that the experts express their opinions by means of fuzzy or linguistic preference relations, this operator is used to develop a choice process for the alternatives, allowing solutions to be obtained in line with the majority of the experts' opinions.

Keywords: Linguistic modelling, fusion operators, aggregation operators, fuzzy linguistic quantifier, choice degrees, group decision making.

1 Introduction

Combining large quantities of data is absolutely essential in many sciences (e.g, Biology, Decision Theory, Artificial Intelligence, Fuzzy Sets Theory). It consists of the treatment and the processing of a data set provided by different information sources with a view to obtaining a single elaborated one. In this paper, we address the problem of combining information in Fuzzy Sets Theory applied to Decision Theory. Specifically, we are interested in the study of fusion operators of imprecise information of a different nature, numerical and linguistic, which allow us to solve some Group Decision Making (GDM) problems.

A GDM problem is defined as a decision situation in which (i) there are two or more experts, each of them characterized by his own perceptions, attitudes, motivations,... (ii) who recognize the existence of a common problem, and (iii) attempt to reach a collective decision. Due to the fact that the information provided by the human beings is in fact often vague and imprecise, the modelling of these problems requires adequate representation models of imprecise information and fusion operators of imprecise information.

In a fuzzy context, a GDM problem may be modelled as follows. There is a finite set of alternatives, $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$, as well as a finite set of experts $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geq 2)$. Each expert, $e_k \in E$, provides his preferences on X by means of one of the two following preference relation models:

- Using a fuzzy preference relation, P^k, with a membership function, $\mu_{P^k} : X \text{x} X \rightarrow [0,1]$, where $\mu_{P^k}(x_i, x_j) = p_{ij}^k$ denotes the preference degree of the alternative x_i over x_j [16].

- Using a linguistic preference relation assessed in a pre-established label set, $S = \{s_0, \ldots, s_T\}$, i.e., with a membership function, $\mu_{P^k} : X \text{x} X \rightarrow S$, where p_{ij}^k denotes the preference degree of the alternative x_i over x_j linguistically expressed [12, 14].

Many GDM problems may require the use of both relation types. For example, when the experts come from different working areas, and depending on their specific knowledge, some prefer to give their preferences in a numerical domain, and others in a linguistic one.

In this paper, we deal with such GDM problems. Some experts provide their preferences by means of fuzzy preference relations, and others, by means of linguistic preference relations. We introduce the following new developments to model this GDM problem type:

1. Two transformation functions between numerical and linguistic domains based on the concept of characteristic values.

2. A fusion operator of numerical and linguistic information defined using the transformation functions above.

3. A particular fusion operator based on the Linguistic Ordered Weighted Averaging (LOWA) aggregation operator [11, 14], which is guided by the concept of fuzzy majority.

4. And finally, a choice process for GDM problems based on a choice degree of alternatives and on the proposed fusion operator. Specifically, we use the quantifier guided non-dominance degree defined in [4].

The structure of this paper is the following: Section 2 briefly reviews the linguistic approach considered, Section 3 defines the combination of numerical and linguistic information by means of a fusion operator, Section 4 presents the choice process based on this fusion operator, Section 5 develops an example, and finally, some concluding remarks are made.

2 Linguistic Approach

In this section, we present some basic assumptions about the linguistic approach used to represent the linguistic information in decision making.

When we work with vague or imprecise knowledge, we cannot estimate with an exact numerical value. Then, a more realistic approach may be to use linguistic assessments instead of numerical values, that is, by assuming that the variables which participate in the problem are assessed by means of linguistic terms [21]. This approach is appropriate for a lot of problems, since it allows a representation of the information in a more direct and adequate form if we are unable of expressing it with precision. The following references show some linguistic approaches in decision making [2, 7, 12, 14, 18, 20]

Usually, depending on the problem domain, an appropriate linguistic term set is chosen and used to describe the vague or imprecise knowledge. The elements in the term set will determine the granularity of the uncertainty, that is, the level of distinction among different countings of uncertainty. In [1] the use of term sets with an odd cardinal was studied, representing the mid term a assess of "approximately 0.5", with the rest of the terms being placed symmetrically around it and the limit of granularity being 11 or no more than 13.

The semantics of the elements in the term set is given by fuzzy numbers defined in the [0,1] interval, which are described by membership functions. Because the linguistic assessments are just approximate ones given by the individuals, we can consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments, since it may be impossible or unnecessary to obtain more accurate values. This representation is achieved by the 4-tuple (x_0, x_1, x_2, x_3), x_1 and x_2 indicate the interval in which the membership function value is 1, and x_0 and x_3 are the left and right limits of the definition domain of trapezoidal membership function.

The first priority ought to be to establish what kind of label set to use. Then, let $S = \{s_i\}, i \in H = \{0, \ldots, T\}$, be a finite and totally ordered term set in [0,1] in the usual sense [1]. Any label s_i represents a possible value for a linguistic real variable, that is, a vague property or constraint in [0,1]. We consider a term set with odd cardinal, where the middle label represents an uncertainty of "approximately 0.5" and the remaining terms are placed symmetrically around it. Moreover, the term set must have the following characteristics:

1) The set presents a total order: $s_i \geq s_j$ if $i \geq$ j.

2) There is the negation operator: Neg$(s_i) = s_j$ such that $j = T$-i.

3) Maximization operator: MAX$(s_i, s_j) = s_i$ if $s_i \geq s_j$.

4) Minimization operator: MIN$(s_i, s_j) = s_i$ if $s_i \leq s_j$.

For example, this is the case of the following set of the nine labels with its semantic associated [1]:

C	$Certain$	$(1, 1, 1, 1)$
EL	$Extremely\_likely$	$(.93, .98, .99, 1)$
ML	$Most\_likely$	$(.72, .78, .92, .97)$
MC	$Meaningful\_chance$	$(.58, .63, .80, .86)$
IM	It_may	$(.32, .41, .58, .65)$
SC	$Small\_chance$	$(.17, .22, .36, .42)$
VLC	$Very\_low\_chance$	$(.04, .1, .18, .23)$
EU	$Extremely\_unlikely$	$(0, .01, .02, .07)$
I	$Impossible$	$(0, 0, 0, 0)$.

Formally speaking, it seems difficult to accept that all experts should agree on the same membership function associated to linguistic terms, and therefore, there are no universality distribution concepts. On the other hand, we should point out that the experts cannot be ready to give the membership functions associated to labels. Therefore, in our context, we consider an environment where experts can discriminate perfectly the same term set under a similar conception, taking into account that the concept of a linguistic variable serves the purpose of providing a means of approximated characterization of imprecise preference information. We make the experts' activity easy by giving them some more used term sets, e.g., the aforementioned set of nine labels.

3 Combining Numerical and Linguistic Information

We focus on the design of fusion operators of quantitative and qualitative information, i.e., we provide a method for combining numerical and linguistic information. We assume that the information is provided using absolute and compatible scales, i.e., all users use the same numerical domain (specifically the unit interval [0,1]) to provide the quantitative assessments and the same term set (labels and semantics) to provide the qualitative ones. The problem of combining information when different and incompatible scales are used is not addressed here.

We define a fusion operator which acts as follows: it transforms all numerical and linguistic input information given by different users to an intermediate expression domain, aggregates the information in that domain, and finally, transforms the elaborated information into output information depending on the user's initial domain. Therefore, to define this fusion operator type, we have to answer the following three questions:

- how does it transform the information among different domains?,
- what is the intermediate expression domain?, and
- how does it combine the information in the intermediate domain?.

These three questions are analyzed in the following subsections. After that we present the fusion operator.

3.1 Transformation Methods

In this subsection, we shall characterize some transformation functions between the linguistic and numerical expression domains. As was mentioned earlier, any linguistic label has its associated fuzzy number, and thus, before defining these transformation functions, we introduce the concept of the *characteristic values* associated to a fuzzy number.

Let $F(\mathcal{R})$ be the set of fuzzy numbers defined on $\mathcal{R}$. Each fuzzy number, $y_i \in F(\mathcal{R})$, has associated a membership function, $\mu_{y_i} : F(\mathcal{R}) \longrightarrow [0,1]$. Let us consider that for each fuzzy number, y_i, we know a set of characteristic values, $CV_{y_i} = \{C_i^1, C_i^2, \ldots, C_i^z\}$, which are crisp values that summarize the information given by y_i, i.e., they support its meaning. We shall assume that, $C_i^j \in Supp(y_i) = \{r \epsilon \mathcal{R} \mid \mu_{y_i}(r) > 0\}$. Without loss of generality, we can define a set of functions $CF = \{f_j,\ j = 1, \ldots, z,\}$, in such a way that each function f_j associates a characteristic value to each fuzzy number y_i, i.e.,

$$f_j : F(\mathcal{R}) \longrightarrow \mathcal{R},$$

$$f_j(y_i) = C_i^j.$$

Therefore, each set of characteristic values, $\{C_i^j,\ \forall i\}$, symbolizes a particular characteristic function, f_j, for a set of fuzzy numbers, $\{y_i,\ \forall i\}$. Some examples of this function type are: the *deffuzification methods* applied in fuzzy control [5], the *ranking functions* [3, 23], and the *value* of a fuzzy number defined in [8].

3.1.1 Transformation Function from Linguistic Domain to Numerical Domain

Here, we define a *Linguistic-Numerical Transformation Function*, which obtains a numerical value from a given label.

Let s_i be a linguistic label, $s_i \in S$, and suppose that it has associated a set of characteristic values, $CV_{s_i} = \{C_i^1, C_i^2, \ldots, C_i^z\}$, obtained by means of a set of characteristic functions, acting on its associated fuzzy number, $y_{s_i} \in F(\mathcal{R})$, i.e., $C_i^1 = f_1(y_{s_i}),\ C_i^2 = f_2(y_{s_i}), \ldots,\ C_i^z = f_z(y_{s_i})$. In the following, we denote the characteristic value of a label s_i, $f_j(y_{s_i})$, as $G_j(s_i)$.

Definition 1. *The Linguistic-Numerical Transformation Function, ψ^N, is defined according to the following expression:*

$$\psi^N : S \longrightarrow [0,1]$$

$$\psi^N(s_i) = g(G_1(s_i), ..., G_z(s_i)),$$

where g is any aggregation operator verifying:

$$min\{v_1, \ldots, v_z\} \leq g(v_1, \ldots, v_z) \leq max\{v_1, \ldots, v_z\}.$$

Therefore, this function, ψ^N, obtains the real value of a label by means of the aggregation of its respective characteristic values. Clearly, $\psi^N(s_i) \in Supp(y_{s_i})$.

We must denote that there are no scientific bases for the choice of characteristic values (i.e., no defuzzifier is derived for a theretical principle, such as maximization of fuzzy information or entropy). Because we are interested in the aggregation of some of them, one criterion for the choice of a caracteristic value may be the computation simplicity. For an additional discussion on these values, see [3, 5, 8, 23]. Below, we show an example of ψ^N with four characteristic values.

Example 1. Let us consider the set of nine labels introduced in Section 2. Because we have trapezoidal membership functions for representing the labels, we define the characteristic values according to the four parameters used to represent the trapezoidal membership function of a label, s_i, $(x_o^i, x_1^i, x_2^i, x_3^i)$. We consider the following methods: the *value* of a fuzzy number [8], the *maximum value* and the *center of gravity* [5].

- *Value.* The characteristic *value* of a label s_i, $G_1(s_i)$, is:

$$G_1(s_i) = \int_0^1 s(r)(L_{y_{s_i}}(r) + R_{y_{s_i}}(r))dr$$

  where $L_{y_{s_i}}(\cdot)$ and $R_{y_{s_i}}(\cdot)$ are the r-cut representations of y_{s_i} and $s(r)$ is a reducing function [8]. $G_1(s_i)$ may be seen as a central value that represents, from a global point of view, the value of the (ill-defined) magnitude that the fuzzy number (associated to the label) represents [8]. Its expression using the trapezoidal membership functions and $s(r) = r$ is:

$$G_1(s_i) = \frac{(x_1^i + x_2)}{2} + \frac{[(x_3^i - x_2^i) - (x_1^i - x_0^i)]}{6} = \frac{2x_1^i + 2x_2^i + x_3^i + x_0^i}{6}.$$

- *Maximum Value.* Given a label, s_i, with a membership function, $\mu_{y_{s_i}}(v), v \in V = [0,1]$, its height is defined as

$$height(s_i) = Sup\{\mu_{y_{s_i}}(v),\ \forall v\}.$$

  Therefore, this method may obtain a representative value of a label in different ways [5]. We assume two of these ways, obtaining two characteristic values, $G_2(s_i)$ and $G_3(s_i)$, according to the following expressions:

$$G_2(s_i) = min\{v \mid \mu_{y_{s_i}}(v) = height(s_i)\},$$

$$G_3(s_i) = max\{v \mid \mu_{y_{s_i}}(v) = height(s_i)\}.$$

  Therefore, their representations, based on the trapezoidal notation, are:

$$G_2(s_i) = x_1^i \text{ and } G_3(s_i) = x_2^i.$$

- *Center of Gravity.* This method summarizes the meaning of a label, s_i, into a numerical value as:

$$G_4(s_i) = \frac{\int_V v\mu_{y_{s_i}}(v)dv}{\int_V \mu_{y_{s_i}}(v)dv}$$

  For trapezoidal fuzzy numbers, we obtain:

$$G_4(s_i) = \begin{cases} x_0^i & \text{if } x_0^i = x_1^i = x_2^i = x_3^i \\ \frac{(x_3^i)^2 + (x_2^i)^2 - (x_1^i)^2 - (x_0^i)^2 + x_3^i x_2^i - x_0^i x_1^i}{3(x_3^i + x_2^i - x_1^i - x_0^i)} & \text{otherwise} \end{cases}$$

If a decision maker uses as aggregation function, g, the *mean function,* then, the transformation function, called ψ_1^N, is:

$$\psi_1^N(s_i) = \frac{G_1(s_i) + G_2(s_i) + G_3(s_i) + G_4(s_i)}{4}.$$

and thus,

$$\psi_1^N(s_i) = \begin{cases} x_0^i & \text{if } x_0^i = x_1^i = x_2^i = x_3^i \\ \frac{8(x_1^i + x_2^i)H + (x_3^i + x_0^i)H + 8(H + x_2^i x_3^i - x_1^i x_0^i)}{24H} & \text{otherwise} \end{cases}$$

with $H = x_3^i + x_2^i - x_1^i - x_0^i$.

If we consider the labels, $\{C, EL, IM, SC\}$, then:

$\psi_1^N(C) = 1.$ $\psi_1^N(EL) = \frac{0.9783+0.98+0.99+0.9725}{4} = 0.98.$

$\psi_1^N(IM) = \frac{0.4916+0.41+0.58+0.4894}{4} = 0.49.$ $\psi_1^N(SC) = \frac{0.29167+0.22+0.36+0.2927}{4} = 0.29.$

We should point out that the results of ψ_1^N depend on three factors: (i) the numerical meaning or semantic of the linguistic terms, (ii) the aggregation function g, and (iii) the chosen set of functions, CF. Therefore, the sensitivity of ψ_1^N is conditioned by the decisions made in each factor.

3.1.2 Transformation Function from Numerical Domain to Linguistic Domain

Here, using the aforementioned characteristic values, we define a *Numerical-Linguistic Transformation Function*, which gives a representative label for a given numerical value.

Definition 2. *Let $r \in [0,1]$ be a numerical value. Let s_i be a label verifying that*

$$h(r, s_i) = min\{h(r, s_t) | \forall s_t \in S\},$$

with

$$h(r, s_t) = \begin{cases} z & \text{if } r \notin Supp(s_t) \\ \sum_{j=1}^{z} (r - G_j(s_t))^2 & \text{if } r \in Supp(s_t) \end{cases}$$

where z is the cardinal of the characteristic function set, CF. Then the Numerical-Linguistic Transformation Function, called ψ^L, is defined according to the following expression:

$$\psi^L : [0,1] \longrightarrow S$$

$$\psi^L(r) = s_i.$$

Example 2. Working in the same context as Example 1, if the considered numerical value is $r = 0.73$, then $\psi_1^L(0.73) = MC$, since

$$min\{h(0.73, C), h(0.73, EL), h(0.73, ML), h(0.73, MC), h(0.73, IM), h(0.73, SC), h(0.73, VLC),$$

$$h(0.73, EU), h(0.73, I)\} = min\{4, 4, 0.48, 0.2, 4, 4, 4, 4, 4\} = 0.2 = h(0.73, MC).$$

3.2 On the Intermediate Expression Domain

As we said at the beginning, the numerical expression domain is the unit interval [0,1] and the linguistic one is a label set S. Therefore, the intermediate expression domain could be any one of them. We propose using the linguistic nature intermediate domain. We find it reasonable to work on the more general expression level, and later, to express the results in the specific expression levels on the basis of the following reasons:

- There is a loss of information in both transformations. But, we find the linguistic-numerical transformations to be more appropriate than the numerical-linguistic ones, because the first ones try to determine exactly a numerical value from a linguistic preference given by an expert incapable of providing his preference with the numerical value.

- For an expert who uses a numerical expression domain to provide his preferences, to use a linguistic one should not be (theoretically) a difficult task. However, for an expert who uses a linguistic expression domain, using a numerical one is not easy, because he may have a vague knowledge about his preference and very often is not able to estimate it with an exact numerical value (from the range of possible numerical values that support the meaning of a label).

3.3 Combining Information in the Intermediate Domain

Since we use a linguistic nature intermediate domain, the information will be combined by means of the aggregation operators of linguistic information [6, 11, 14, 18, 20]). We could use any aggregation operator, but here, we propose using *quantifier guided aggregation operators* [11, 14, 20], representing the concept of fuzzy majority in its computation. In this way, and since our application is developed in GDM problems, we find that the final decisions reflect what the majority of experts prefer, as for instance what was done in [10, 11, 12, 14, 15].

Specifically, we propose using an operator with direct computation, the *LOWA operator* [11, 14], which is based on the *OWA operator* defined by Yager [19], and on the *convex combination of linguistic labels* defined by Delgado et al. [6].

Definition 3. [11, 14] *Let $A = \{a_1, \ldots, a_m\}$ be a set of labels to be aggregated, then the LOWA operator, ϕ, is defined as*

$$\phi[a_1, \ldots, a_m] = W \cdot B^T = \mathbf{C}^m\{w_k, b_k, k = 1, \ldots, m\} =$$

$$= w_1 \odot b_1 \oplus (1 - w_1) \odot \mathbf{C}^{m-1}\{\beta_h, b_h, h = 2, \ldots, m\}$$

where $W = [w_1, \ldots, w_m]$ is a weighting vector, such that, (i) $w_i \in [0, 1]$, and (ii) $\Sigma_i w_i = 1$; and $B = \{b_1, \ldots, b_m\}$ is a vector associated to A, such that, $B = \sigma(A) = \{a_{\sigma(1)}, \ldots, a_{\sigma(n)}\}$, where, $a_{\sigma(j)} \leq a_{\sigma(i)}\ \forall\ i \leq j$, with σ being a permutation over the set of values A. $\beta_h = w_h / \Sigma_2^m w_k, h = 2, \ldots, m$; and $\mathbf{C}^m$ is the convex combination operator of m labels [6]. If m=2, then it is defined as

$$\mathbf{C}^2\{w_i, b_i, i = 1, 2\} = w_1 \odot s_j \oplus (1 - w_1) \odot s_i = s_k, \quad s_j,\ s_i\ \in\ S,\ (j \geq i)$$

such that,

$$k\ =\ MIN\{T, i\ +\ round(w_1 \cdot (j - i))\},$$

where "round" is the usual round operation, and $b_1 = s_j,\ b_2 = s_i$. If $w_j = 1$ and $w_i = 0$ with $i \neq j\ \forall i$, then the convex combination is defined as

$$\mathbf{C}^m\{w_i, b_i, i = 1, \ldots, m\} = b_j.$$

Several arguments (axioms and properties) for its rational aggregation way were given in [14].

Given that we are interested in the area of quantifier guided aggregations, following Yager's method [19], we may calculate weights of the OWA operator using fuzzy linguistic quantifiers [22], representing the fuzzy majority. For a non-decreasing relative quantifier, Q, the weights are obtained as

$$w_i = Q(i/m) - Q((i-1)/m),\ i = 1, \ldots, m.$$

where the non-decreasing relative quantifier, Q, is defined as [22]

$$Q(y) = \begin{cases} 0 & \text{if } y < a \\ \frac{y-a}{b-a} & \text{if } a \leq y \leq b \\ 1 & \text{if } y > b \end{cases}$$

with $a, b, y \in [0,1]$, and $Q(y)$ indicating the degree to which the proportion y is compatible with the meaning of the quantifier it represents. Some examples of relative quantifiers are "most" $(0.3, 0.8)$, "at least half" $(0, 0.5)$ and "as many as possible" $(0.5, 1)$. In the following, ϕ_Q denotes the LOWA operator whose weights are computed using a linguistic quantifier, Q.

3.4 Fusion Operator of Numerical and Linguistic Information

This operator acts on three steps:

1. It transforms all inputs into a usual linguistic intermediate domain by means of a particular numerical-linguistic transformation function,

2. the transformed information is aggregated by means of a concrete linguistic aggregation operator, and finally,

3. the output information is expressed in each user's expression domain, using an appropriate linguistic-numerical transformation function.

This idea is shown in Figure 1, and characterized in the following definition.

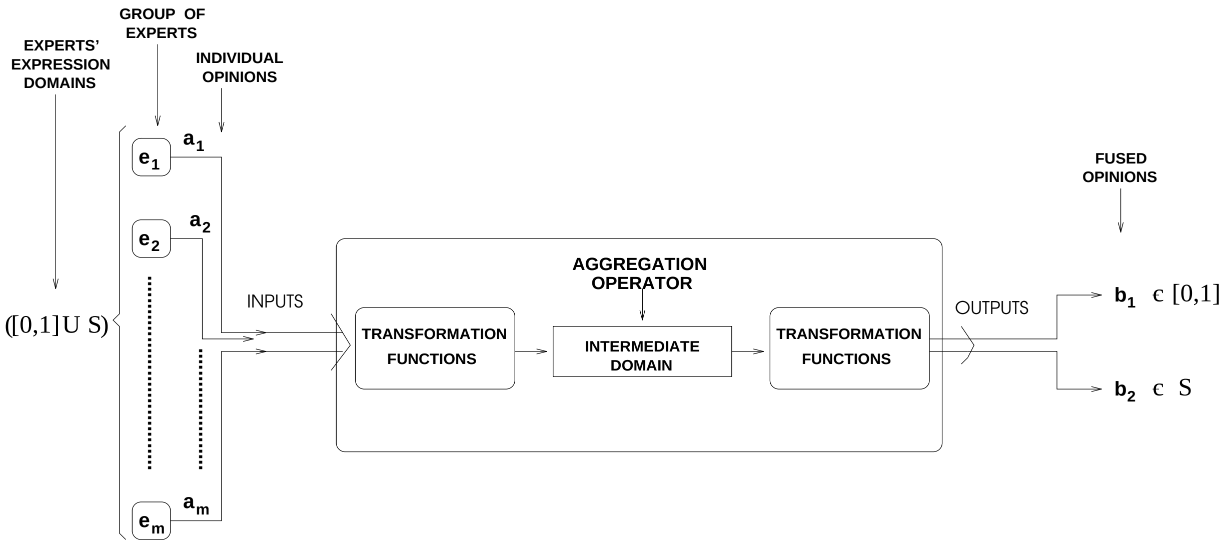

Figure 1: Schema of the Fusion Operator

Definition 4. *Let* $E = \{e_i,\ i = 1, \ldots, m\}$ *be a group of experts, and let* $A = \{(a_i, c_i), i = 1, ..., m\}$ *be their respective opinions to be combined, such that,* $c_i \in \{0, 1\}$*, and if* $c_i = 1$ *then* $a_i \in S$ *and if* $c_i = 0$ *then* $a_i \in [0, 1]$*. A fusion operator of linguistic and numerical information,* ω*, is defined according to:*

$$\omega : (([0,1] \ \cup \ S)\text{x}\{0,1\})^m \longrightarrow (S \text{ x } [0,1])$$

$$\omega[(a_1, c_1), (a_2, c_2), \ldots, (a_m, c_m)] = (b_1, b_2),$$

such that, $b_1 \in S$ *is a linguistic output given by*

$$b_1 = ?^L[\lambda(a_1, c_1), \lambda(a_2, c_2), \ldots, \lambda(a_m, c_m)],$$

$$\lambda(a_j, c_j) = \begin{cases} a_j & \text{if } c_j = 1 \\ \psi^L(a_j) & \text{otherwise} \end{cases}$$

with $?^L$ *an aggregation operator of linguistic information, and* $b_2 \in [0, 1]$ *is a numerical output obtained as* $b_2 = \psi^N(b_1)$.

As we mentioned above, regarding the application of fusion operators in GDM problems, we use a particular fusion operator based on the LOWA operator guided by fuzzy majority (i.e., $?^L = \phi_Q$), symbolized by ω_Q^{LOWA}.

4 A GDM Process under Numerical and Linguistic Assessments

Here, we present a direct choice process developed from the fuzzy and linguistic preference relations provided by the experts, called *Non-Dominance Based Choice Process*. It is based on a quantifier guided choice degree of alternatives, i.e., the *non-dominance* property guided by a fuzzy linguistic quantifier, as in [4].

A direct process is developed along three steps, as it is shown in Figure 3 [15].

1. *Exploitation State.* The goal of this state is to calculate the non-dominance degree of each alternative according to each individual preference relation.

2. *Aggregation State.* The goal of this state is to aggregate individual non-dominance degrees obtained in the above step with view to calculate the non-dominance degree of each alternative according to the global opinion of group of experts. To do that, we apply the proposed fusion operator based on the LOWA operator, ω_Q^{LOWA}.

3. *Selection State.* The goal of this state is to find the solution. We choose those alternatives with global maximum non-dominance degree.

We should point out that in the exploitation state, as well as in the aggregation state, the concept of fuzzy majority is used, but with a different meaning. In the first one because the individual degrees are calculated, the *fuzzy majority of alternatives (of non-dominance)* is used [14]. In the second one, since individual degrees of different experts are aggregated, the *fuzzy majority of experts* is used [14]. Therefore, we can use different fuzzy linguistic quantifiers in each state.

Assuming that we have a label set, S, two transformation functions $\{\psi^N, \psi^L\}$ with their characteristic functions $CF = \{f_j,\ j = 1, ..., z\}$, and the concepts of fuzzy majority of non-dominance and fuzzy majority of experts represented by means of the two fuzzy linguistic quantifiers, Q_1 and Q_2, respectively, the choice process is described in the following steps:

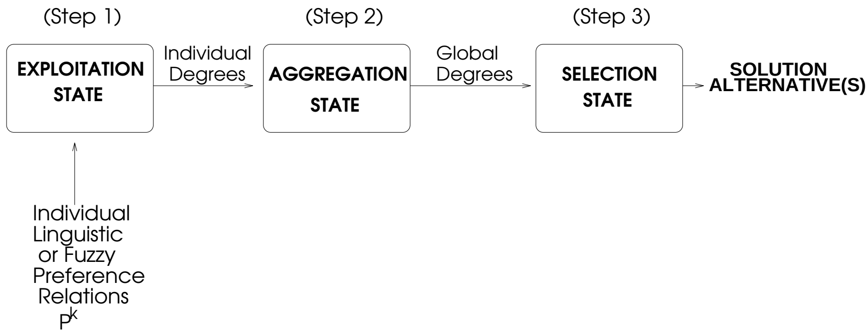

Figure 2: Three Steps of a Direct Choice Process

1. Exploitation State.

In this state we have to calculate the quantifier guided non-dominance degree of each alternative according to the preference relation of each expert, P^k, called *individual quantifier guided non-dominance degree*. It quantifies the degree to which each alternative is not dominated by the fuzzy majority of the remaining ones. It is calculated on the basis of concept of non-dominated alternatives defined by Orlovski [17] as follows:

Definition 5. [19] *Let $A = \{a_1, \ldots, a_m\}$, be a set of numerical values to be aggregated, then the OWA operator, F, is defined as*

$$F[a_1, \ldots, a_m] = W \cdot B^T = \Sigma_{i=1}^{m} w_i \cdot b_i,$$

where W and B are like in Definition 3.

Definition 6. *Given an alternative, $x_i \in X$, the Individual quantifier guided Non-Dominance Degree of x_i, $INDD_i^k$, is defined:*

- *from a fuzzy preference relation, P^k ($p_{ji}^k \in [0,1]$), provided by the expert, e_k, according to the following expression [4]:*

$$INDD_i^k = F_{Q_1}[(1 - p_{ji}^{s,k}), j = 1, ..., n, j \neq i],$$

  where F_{Q_1} is the OWA operator guided by fuzzy majority, and $p_{ji}^{s,k}$ represents the degree to which x_i is strictly dominated by x_j, and it is obtained as $p_{ji}^{s,k} = max\{p_{ji}^k - p_{ij}^k, 0\}$, $\forall i, j$;

- *from a linguistic preference relation, P^k ($p_{ji}^k \in S$), according to the following expression [15]:*

$$INDD_i^k = \phi_{Q_1}[Neg(p_{ji}^{s,k}), j = 1, ..., n, j \neq i],$$

  where ϕ_{Q_1} is the LOWA operator guided by fuzzy majority and

$$p_{ji}^{s,k} = s_0 \ if \ p_{ij}^k > p_{ji}^k,$$

$$or \ p_{ji}^{s,k} = s_h \in S \ if \ p_{ji}^k \geq p_{ij}^k \ with \ p_{ji}^k = s_l, \ p_{ij}^k = s_t \ and \ l = t + h.$$

More specifically, $INDD_i^k$ expresses the degree to which an alternative, x_i, is not dominated by the fuzzy majority of the remaining alternatives according to one expert's opinions, e_k.

2. Aggregation State.

From the sets of individual quantifier guided non-dominance degrees obtained for each alternative, x_i, $\{INDD_i^k, \forall k\}$, and by means of the fusion operator, $\omega_{Q_2}^{LOWA}$, we calculate the *Global quantifier guided Non-Dominance Degrees* for each alternative. It is formed by two components, the first one, $GNND_i^L$, has a linguistic nature, whereas the second one, $GNND_i^N$, is purely numerical. In this way, we obtain the degree to which an alternative, x_i, is not dominated by the fuzzy majority of the remaining alternatives according to all the experts' opinions. It is defined as follows:

$$(GNDD_i^L, GNDD_i^N) = \omega_{Q_2}^{LOWA}[(INDD_i^k, c_k),\ k = 1, ..., m].$$

3. Selection State.

Finally, when the choice degrees of alternatives, $(GNDD_i^L, GNDD_i^N)$, are calculated we obtain the set of solution alternatives, X_{max}^{nd}, as follows:

$$X_{max}^{nd} = \{x_i \in X / GNDD_i^L = MAX_j\{GNDD_j^L,\ j = 1, \ldots, n\}\}$$

which is formed by the alternatives with maximum linguistic global quantifier guided non-dominance degree. Then, the solution is shown to each expert in his respective expression domain using the linguistic or numerical component.

We should point out that if all the alternatives have the same maximum non-dominated degree or this maximum is zero, we need either that the experts provide more information to decide among them, or the development of a negotiation and consensus process among the experts, which allows them to exchange information to update their preferences [2, 13]. On the other hand, if the choice procedure leads to an undesired solution we need either a method to include the experts' undesired degrees in the choice process or a negotiation process.

5 Example

Let's suppose an investment company, which has an amount of money to invest. There are four possible options to invest an amount of money, $\{x_1, x_2, x_3, x_4\}$: a car factory, a food company, an atomic weapons factory, and a computer company, respectively. In the company, all the decisions are made according to the opinions provided by the managers of four departments, $\{e_1, e_2, e_3, e_4\}$: business department, social-policy department, risk analysis department and the environment department. Given that these experts come from different areas of knowledge such as economics, biology, law, ... some may have more facility to express their opinions with numbers, while others may prefer to express their opinions by means of linguistic assessments. Assuming that the experts, $\{e_1, e_3\}$, use the numerical domain, $[0, 1]$, and the remaining ones the linguistic domain, S, given in Section 2, i.e., a set of nine labels.

Without loss of generality, let us assume that we work with reciprocal preference relations, which, in the case of fuzzy preference relations, implies (i) $p_{ij}^k + p_{ji}^k = 1$, and $p_{ii} = undefined(-)$; and, in the case of linguistic preference relations, implies (i) $p_{ij}^k = Neg(p_{ji}^k)$, and (ii) $p_{ii} = undefined(-)$. Consider that preference relations provided by the experts are:

$$P^1 = \begin{bmatrix} - & 0.3 & 0.7 & 0.1 \\ 0.7 & - & 0.6 & 0.6 \\ 0.3 & 0.4 & - & 0.2 \\ 0.9 & 0.4 & 0.8 & - \end{bmatrix} \quad P^2 = \begin{bmatrix} - & IM & C & EU \\ IM & - & EU & C \\ I & EL & - & VLC \\ EL & I & ML & - \end{bmatrix}$$

$$P^3 = \begin{bmatrix} - & 0.5 & 0.7 & 0 \\ 0.5 & - & 0.8 & 0.4 \\ 0.3 & 0.2 & - & 0.2 \\ 1 & 0.6 & 0.8 & - \end{bmatrix} P^4 = \begin{bmatrix} - & IM & EL & I \\ IM & - & I & EL \\ EU & C & - & VLC \\ C & EU & ML & - \end{bmatrix}$$

Considering that both Q_1 and Q_2 are the fuzzy linguistic quantifier *"as many as possible"* with the pair, (0.5,1), assuming the transformation functions $\{\psi_1^N, \psi_1^L\}$ presented in Examples 1 and 2, and using the fusion operator, $\omega_{Q_2}^{LOWA}$, the choice process is applied as follows:

1. Exploitation State.

From the aforementioned preference relations, $\{P^1, P^2, P^3, P^4\}$, we obtain the respective strict preference relations:

$$P^{s,1} = \begin{bmatrix} - & 0 & 0.4 & 0 \\ 0.4 & - & 0.2 & 0.2 \\ 0 & 0 & - & 0 \\ 0.8 & 0 & 0.6 & - \end{bmatrix} P^{s,2} = \begin{bmatrix} - & I & C & I \\ I & - & I & C \\ I & ML & - & I \\ ML & I & IM & - \end{bmatrix}$$

$$P^{s,3} = \begin{bmatrix} - & 0 & 0.4 & 0 \\ 0 & - & 0.6 & 0 \\ 0 & 0 & - & 0 \\ 1 & 0.2 & 0.6 & - \end{bmatrix} P^{s,4} = \begin{bmatrix} - & I & ML & I \\ I & - & I & ML \\ I & C & - & I \\ C & I & IM & - \end{bmatrix}$$

We calculate the individual quantifier guided dominance degrees, $INDD_i^k$, by means of the OWA and LOWA operators, F_{Q_1} and ϕ_{Q_1}, with the weighting vector, $W = [0, 0.334, 0.666]$. The result is shown in Table 1.

$INDD_i^k$	e_1	e_2	e_3	e_4
x_1	0.336	MC	0.336	SC
x_2	1	MC	0.8668	SC
x_3	0.4668	EU	0.4	SC
x_4	0.8668	SC	1	MC

Table 1: Individual Quantifier Guided Non-Dominance Degrees

2. Aggregation State.

Assuming that the linguistic intermediate domain is the label set, S, and aggregating the said individual degrees by means of the fusion operator, $\omega_{Q_2}^{LOWA}$, but now with the weighting vector, $W = [0, 0, 0.5, 0.5]$, we obtain the global quantifier guided non-dominance degrees for each alternative, $(GNDD_i^L, GNDD_i^N)$, as shown in Table 2.

	x_1	x_2	x_3	x_4
$(GNDD_i^L, GNDD_i^N)$	(SC, 0.29)	(IM, 0.49)	(VLC, 0.138)	(IM, 0.49)

Table 2: Global Quantifier Guided Non-Dominance Degrees

3. Selection State.

Finally, we find the set of solution alternatives $X^{nd}_{max} = \{x_2, x_4\}$, since $GNDD^L_2 = GNDD^L_4 = IM$ and $IM = MAX_j\{GNDD^L_j\}$.

Therefore, according to the different experts' opinions, the food and computer companies are the best options to invest the money. Then, the experts receive that information in the following way:

- experts e_1 and e_3: $\{(x_2, 0.49), (x_4, 0.49)\}$, and
- experts e_2 and e_4: $\{(x_2, IM), (x_4, IM)\}$.

6 Concluding Remarks

Depending on their background, people give information about their personal preferences in many different ways. Particularly, we have shown that it is possible to combine linguistic and numerical information. We have studied the case in which experts provide their opinions by means of numerical or linguistic assessments. We have proposed a fusion operator of numerical and linguistic information, which allows us to combine numerical values assessed in [0,1] and linguistic values assessed in a label set S. To build this fusion operator we have designed two transformation methods between the numerical and linguistic domains based on the concept of characteristic values. Later, we have shown the application of this fusion operator in a GDM problem in which the experts provide their preferences by means of fuzzy and linguistic preference relations.

In the future, we plan to study the case in which the experts provide their opinions by means of linguistic assessments with multi-granularity term sets.

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