Forward-Link CDMA Resource Allocation Based on Pricing

Peijuan Liu, Michael L. Honig
ECE Department, Northwestern University
2145 Sheridan Road, Evanston, IL 60208 USA
{peijuan,mh}@ece.nwu.edu

Scott Jordan
ECE Department, University of California, Irvine
544D Engg. Tower, Irvine, CA 92697-2625
sjordan@uci.edu

***Abstract* – This paper studies pricing as a means for resource allocation in a wireless Direct-Sequence (DS)-Code-Division Multiple-Access (CDMA) system. We consider the forward link of a single cell with orthogonal codes and voice traffic. The base station announces a price per unit transmitted power and a price per code, and the users respond according to their individual utilities. The objective is to set prices to maximize either total user utility or total revenue. The solution to the former problem (maximize utility) is presented. To study the latter problem we derive the large system revenue as the number of users and codes tend to infinity with fixed ratio. The large system revenue depends on the distribution of utilities and path loss across the user population, and may not be a unimodal function of the prices. Numerical results based on a simple model for user utility show how the optimal prices and revenue vary with the offered load.**

I. Introduction

This paper presents a framework for studying the use of pricing to allocate resources in a wireless DS-CDMA system. Unlike other approaches to resource allocation, pricing can allocate resources according to perceived user utility, thereby increasing the overall utility of the network. Other attractive properties include the accommodation of a wide range of traffic flows, and potential simplification or elimination of explicit admission control policies.

Here we consider the forward link of a single cell with M orthogonal codes. We assume that only voice users are present, and that the set of user requests is stationary. Although this situation is quite simple, it serves to illustrate the framework for performance analysis, which can be generalized to account for mixed traffic types and multiple cells.

For the case considered, CDMA "resources" are transmitted power and number of codes. Each user has a utility function which is the amount the user is willing to pay for a given Quality of Service (QoS). The base station announces a price per unit transmitted power α_p and a price per code α_c, and each user responds by requesting service to maximize his/her individual surplus (utility minus cost). The goal is to set prices to maximize either total user utility or total revenue subject to the constraint that the number of active users cannot exceed the number of available codes M. Total utility might be maximized if each user supplies the base with a "pre-determined" utility function based on a desired priority or grade of service. In that case, the users need not be charged by the base, and the prices serve as internal network parameters for resource allocation. Total revenue would likely be used if the base does not know the individual utility functions, and the users are directly charged by the base.

We first formulate the preceding constrained optimization problems with a finite fixed user set. The prices which maximize utility are easily obtained. The prices which maximize revenue are more difficult to determine, and we resort to a large system analysis, in which the number of users K and the number of codes M tend to infinity with fixed K/M. Both total utility and revenue can then be expressed in terms of the prices, and distribution functions of user utility and received power. We find that the revenue function is not always unimodal, but give some specific conditions which guarantee unimodality when $\alpha_p = 0$. We also show how the optimal prices and revenue vary with offered load numerically based on a simple model for user utility and received power distribution.

Pricing of resources in communications networks in general has received much attention during the past few years (see, for example [1, 2, 3, 4, 5, 6]). Work related to that presented here on pricing for resource allocation in wireless systems is reported in [7, 8, 9]. In that work, the emphasis is on conserving battery life for the reverse link, and the model is based on a non-cooperative game. We consider a different model based on the forward link, and present a different approach to evaluating performance.

II. Forward Link CDMA Model

We consider a single cell in isolation and assume that each active user is assigned one code from the base. Furthermore, we ignore multiple-access interference at the receiver (mobile), which corresponds to the situation in which the codes are orthogonal and multipath is negligible (or is equalized). The received Signal-to-Interference Plus Noise Ratio (SINR) for user k is then $\gamma_k = h_k P_k / N_0$, where h_k is the attenuation to user k, P_k is the forward link transmitted power to user k, and N_0 is the background noise level. The transmitted power needed to achieve the target SINR γ^* is therefore $P_k^* = N_0 \gamma^* / h_k$.

*This work was supported by the NU-Motorola Center for Communications and by NSF under grant CCR-9903055.

The attenuation h_k depends on the location of the user and on random shadowing. We therefore assume that h_k is chosen from some probability density function ($p.d.f.$) f_h, which in turn determines the transmitted power $p.d.f.$ across users f_P. For the numerical results that follow, we assume that the received power at distance r from the base is given by

$$P_R(r) = P_R(d_0)\left(\frac{d_0}{r}\right)^n \tag{1}$$

where $P_R(d_0)$ is the received power at a close-in reference point d_0 in the far field region of the transmitter antenna, and n is the path loss exponent. We assume that $P_R(d_0)$ is the same as the transmitted power from the base station so that $P_k = P_R(d_0)$, and $\gamma_k = P_k C r^{-n}/N_0$, where $C = d_0^n$ is a constant and $h_k = Cr^{-n}$.

In what follows, we will assume that users are distributed uniformly throughout the cell, and that $n = 4$. Assuming that the SINR requirement is satisfied for each user, the density of transmitted powers over the user population is given by

$$f_P(p) = \frac{C}{2\gamma^* N_0 R^2}\left(\frac{pC}{\gamma^* N_0}\right)^{-1/2}, \quad p \in (0, \gamma^* N_0 R^4/C) \tag{2}$$

where R denotes the cell radius. Of course, other distributions can be derived which account for additional propagation and system effects such as random shadowing and soft hand-off.

Each user is assigned a *utility function*, which reflects the user's willingness to pay in dollars vs. the received QoS. For voice traffic, the QoS is determined by the received SINR. If the received SINR is above a certain threshold, γ^*, we assume that the user gets acceptable service and is indifferent to an increase in received SINR. Conversely, if the received SINR is less than γ^*, then the QoS is unacceptable and the user derives zero utility. For the numerical results which follow, user k is assigned a step utility function $U_k(\gamma) = U_k$, $\gamma > \gamma^*$, $U_k(\gamma) = 0$, $\gamma < \gamma^*$, where γ^* is the target SINR for acceptable voice quality (say, 3 to 7 dB). The utility derived by user k, U_k, is assumed to be chosen from some distribution f_U. Of course, utility functions which correspond to other types of traffic can also be considered within this framework.

The base announces a price per code α_c and a price per unit power α_p. The total charge for service to user k is therefore $\alpha_c + \alpha_p P_k$. Each user responds by maximizing the received surplus (utility minus cost). For a step utility function, this implies that

$$P_k = \begin{cases} N_0\gamma^*/h_k, & U_k \geq \alpha_c + \alpha_p P_k \\ 0, & U_k < \alpha_c + \alpha_p P_k \end{cases} \tag{3}$$

Clearly, as the prices increase, the number of active users decreases. Among the users that have the same path loss h_k, those with the highest utilities remain active. Among the users that have the same utility, those with better channels (less requested transmitted power) are more likely to remain active.

The power radiated by the base causes interference to adjacent cells, and therefore represents an economic *externality*. To account for this, we assume that the base makes a *transfer payment* to the network given by βP_{tot}, where β is a constant and P_{tot} is the total power radiated by the base. Ideally, β should depend on the load in neighboring cells. Note that β can also be interpreted as a Lagrange multiplier which enforces a constraint on P_{tot}.

III. Problem Formulation

The objective of the base is to set the prices α_c and α_p to maximize total utility or revenue minus the total transfer payment, that is,

$$\max_{(\alpha_c,\alpha_p)}\left(U_{tot} = \sum_{k\in S}(U_k - \beta P_k)\right) \tag{4}$$

or

$$\max_{(\alpha_c,\alpha_p)}\left(R_{tot} = \sum_{k\in S}(\alpha_c + \alpha_p P_k - \beta P_k)\right) \tag{5}$$

where

$$S = \{k : U_k \geq \alpha_c + \alpha_p P_k\} \tag{6}$$

is the active user set.

In response to the announced prices, each user maximizes the received surplus,

$$\max_{P_k}[U_k(\gamma) - (\alpha_c + \alpha_p P_k)] \tag{7}$$

where $\gamma = h_k P_k/N_0$. For the step utility function considered, this implies (3).

We constrain (α_c, α_p) to lie in the "*feasible region*"

$$F = \{(\alpha_c, \alpha_p) : |\ S\ | /M \leq 1\} \tag{8}$$

in which the number of active users ($|\ S\ |$) is no greater than the number of codes M (i.e., demand does not exceed supply). Whenever (8) is binding ($|\ S\ | = M$), we say the system is *code limited*. It can be shown that there always exists a maximizing $(\alpha_c, \alpha_p) \in F$.

To maximize U_{tot}, we order the users $k = 1, 2, \cdots, K$ according to decreasing ($U_k - \beta P_k$). Let

$$N = \max\{k : U_k - \beta P_k \geq 0\}$$

and M denote the number of codes. We have the following theorem.

Theorem 1: *To maximize U_{tot}, the base station sets*

$$\alpha_c = U_{M'} - \beta P_{M'}, \;\; \alpha_p = \beta \tag{9}$$

where $M' = \min(N, M)$, so that the first M' users are active and the remaining users (if any) are inactive.

However, the choices of utility maximizing α_c and α_p are not unique. For instance, if

$$\max(0, U_{M'+1} - \beta P_{M'+1}) < \alpha_c \leq U_{M'} - \beta P_{M'}, \;\; \alpha_p = \beta \tag{10}$$

then the same set of users is activated. If $M' = N$, then all users are provided service if their utilities are sufficiently high ($U_k \geq \beta P_k$). In that case, the system is *demand-limited*, i.e., the total utility is limited by the number of users which contribute positive utility to U_{tot}. If $M' = M$, then the system is *code-limited*, i.e., the total utility is limited by the number of available codes (bandwidth).

The prices which maximize revenue are not as easy to determine. As the prices increase, the number of active users decreases, but the revenue per user increases. Optimization becomes difficult with a set of finite users since R_{tot} as a function of α_c and α_p is an irregular surface that has many jumps corresponding to the specific prices at which users become activated or deactivated.

IV. Large System Analysis

To avoid the associated analytical problems with a finite system, we compute the large system revenue and utility by letting the number of users, K, and the number of codes, M, tend to infinity while keeping the load $\rho = K/M$ constant. In the limit, the utility per user U_{tot}/K and revenue per user R_{tot}/K each converge to deterministic values,

$$U(\alpha_c, \alpha_p) = \iint_Q [u - \beta p]\, f_P(p) f_U(u)\, dQ \tag{11}$$

and

$$R(\alpha_c, \alpha_p) = \iint_Q [\alpha_c + (\alpha_p - \beta)p]\, f_P(p) f_U(u)\, dQ \tag{12}$$

where

$$Q = \{(u, p) : p \leq (u - \alpha_c)/\alpha_p\} \tag{13}$$

is the large system limit of the set S given in (6). (We can also view U and R as the expected utility and revenue per user, respectively, where expectation is with respect to the random variables U_k and P_k with $p.d.f.$'s f_U and f_P.)

We wish to select (α_c, α_p) to maximize U or R subject to the feasibility constraint

$$\int_Q f_P f_U dQ \leq 1/\rho \tag{14}$$

which is the large system version of (8). Maximizing utility is equivalent to maximizing the number of active users that satisfy $U_k \geq \beta P_k$.

In analogy with the solution for the finite system, we have the following theorem.

Theorem 2: *To maximize U in a large system, the base station sets $\alpha_p = \beta$. α_c is chosen to satisfy*

$$\int_0^\infty \int_{\alpha_c + \beta p}^\infty f_U(u) f_P(p)\, du\; dp = 1/\rho \tag{15}$$

or $\alpha_c = 0$ if no positive number can satisfy (15).

An immediate inference from Theorem 2 is that when $\rho \leq 1$, $\alpha_c = 0$ is always true. As load increases, the system transits from *demand limited* to *code limited* and the optimal α_c increases whereas α_p remains fixed. The carried load is therefore determined by α_c alone.

To determine the prices that maximize revenue, we can take derivatives of R with respect to α_c and α_p. Setting $\partial R/\partial \alpha_c = 0$ gives

$$\int_{\alpha_c}^\infty f_U(u) \left(\int_0^{(u-\alpha_c)/\alpha_p} f_P(p)\, dp \right) du = \int_0^\infty [\alpha_c + (\alpha_p - \beta)p] f_P(p) f_U(\alpha_c + \alpha_p p)\, dp \tag{16}$$

The right side of (16) represents the gain in revenue due to a marginal increase in α_c, and the left side of (16) represents the corresponding loss in revenue due to users becoming inactive after the price increase.

Similarly, setting $\partial R/\partial \alpha_p = 0$ gives

$$\int_{\alpha_c}^\infty f_U(u) \left(\int_0^{(u-\alpha_c)/\alpha_p} p f_P(p)\, dp \right) du = \int_0^\infty [\alpha_c + (\alpha_p - \beta)p] p f_P(p) f_U(\alpha_c + \alpha_p p)\, dp \tag{17}$$

These terms have the analogous interpretations as those in (16).

When $\alpha_p = 0$ these conditions can be used to obtain the following theorem.

Theorem 3: *For f_P given by (2) and $\alpha_p = 0$, and given any concave utility density function f_U, $R(\alpha_c, 0)$ is a unimodal function of α_c.*

Unimodality is an important property of the revenue objective function, since it implies that simple gradient search techniques can be used to maximize revenue without *a priori* knowledge of the users' utility functions. Although we have not succeeded in proving this for $\alpha_p > 0$, we have observed from numerical results that Theorem 3 remains valid for all concave utility functions f_U considered. We remark that the revenue is not a unimodal function for arbitrary f_U and f_P. For example, the utility distribution $f_U(u) = a_1 \delta(u - U_1) + (1 - a_1)\delta(u - U_2)$ consisting of two impulse masses at U_1 and U_2 typically leads to a non-unimodal revenue function.

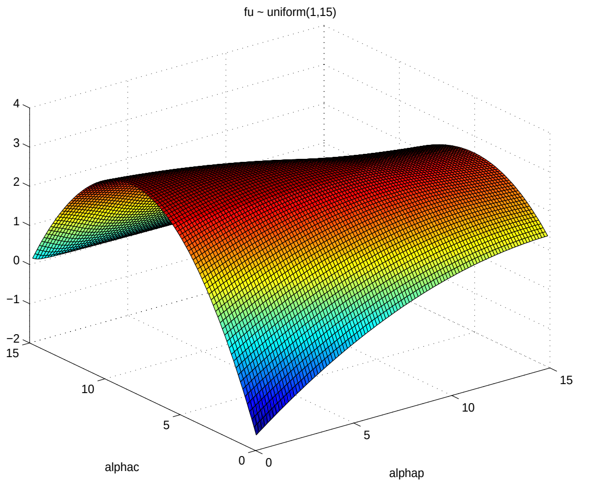

Fig. 1: Revenue vs. (α_c, α_p) for uniform f_U.

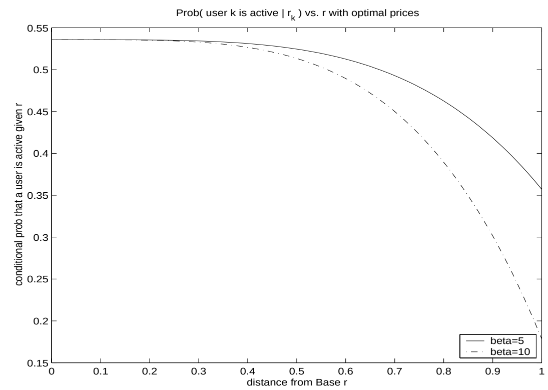

Fig. 2: Conditional prob that a user is active given his distance from base r vs. r with optimal prices and different β's

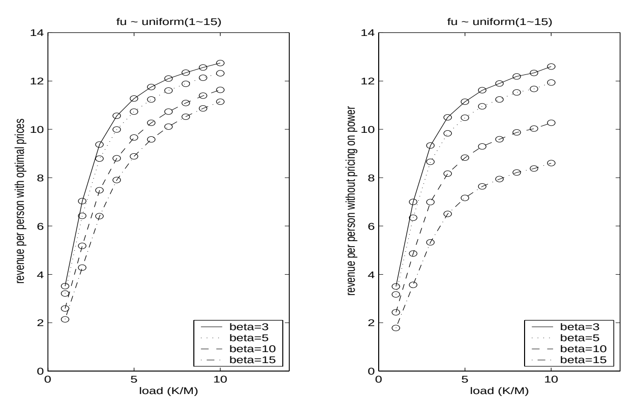

Fig. 3: Revenue vs. load with uniform f_U. The curves on the left correspond to optimal (α_c, α_p), and the curves on the right correspond to $\alpha_p = 0$.

V. NUMERICAL RESULTS

Figure 1 shows revenue vs. (α_c, α_p) for the case where f_U is uniform from $U_1 = 1$ to $U_2 = 15$, $\beta = 5$, $\rho \le 1$ and f_P is given by (2). In this case the revenue cost function is unimodal, and the global maximum occurs at $\alpha_c = U_2/2$ and $\alpha_p = \beta/2$. Based on further numerical results with different parameter sets (U_1, U_2, β), we conjecture that for uniform distribution $f_U(u)$ between U_1 and U_2 and when the load $\rho \le 1$, the optimal (revenue maximizing) prices are:

$$\begin{array}{lll} \alpha_c^* = U_2/2 & \alpha_p^* = \beta/2 & \text{for } U_2 \ge 2U_1 \\ \alpha_c^* = U_1 & \alpha_p^* = 0 & \text{for } U_2 < 2U_1 \end{array}$$

The large system average power per code can be evaluated as

$$P_{av} = \frac{1}{M}\sum_{k=1}^{K} P_k \underset{K\to\infty}{\longrightarrow} \rho \int\int_Q p f_U(u) f_P(p) dQ \qquad (18)$$

and depends on $(\alpha_c, \alpha_p, \beta)$. As an example, for the case $\beta = 15, U_1 = 1, U_2 = 15$, the average power per code with the optimal (α_c, α_p) is 2.3 dB less than if $\alpha_p = 0$ and α_c alone is chosen to maximize revenue. If $\beta = 20$, then this difference increases to 3.6 dB. This is due to the fact that as β increases, the more stringent the power constraint becomes, and the more important α_p is for discriminating against users who have high path loss. Namely, as α_p increases, the distribution of active users becomes skewed more towards the center of the cell, reducing the average power per code. Figure 2 shows the conditional probability that a user is active given its distance from base when $\rho \le 1$. It verifies that as β and/or α_p increase, the distribution of active users indeed becomes skewed more toward the center of the cell.

Figure 3 shows two sets of curves. First, revenue is plotted as a function of load K/M with optimal (revenue maximizing)

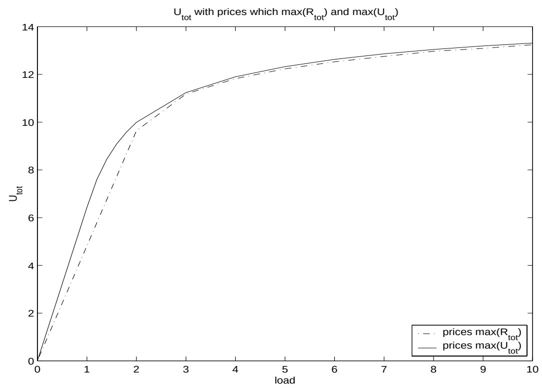

Fig. 4: Utility vs. load with prices that maximizes utility and revenue

α_p and α_c for different transfer payments β. Second, revenue vs. load is shown with optimal α_c, but with $\alpha_p = 0$ (no charge for power). The utility and power distributions are the same as those used to generate Figure 1. As expected, both sets of curves monotonically increase with load. As β increases, corresponding to a smaller constraint on total transmitted power, the difference in revenue between the two cases increases. For fixed β, this difference in revenue also increases with load.

Figure 4 shows utility as a function of load with prices selected to maximize revenue and maximize utility, respectively. The difference in the resulting utility is the greatest when the load is less than 2. Both systems are *demand-limited* when the load is less than 1 and *code-limited* when the load is greater than 2. However, the utility-maximizing system is *code-limited* when the load is between 1 and 2, while the revenue-maximizing system is *demand-limited* in this interval. The two curves converge as the load increases above 2.

These results illustrate that the benefit derived from pricing of power increases as resources (namely, power) become more limited. The fact that revenue increases indicates that pricing selects those users who derive the highest utility from the network. For the model considered, an explicit admission control policy is unnecessary.

VI. Conclusions

We have studied pricing as an approach to forward-link resource allocation in a wireless CDMA network. The base station sets prices for codes and transmitted power, and the users respond by maximizing individual surplus. The system goal is to set prices to maximize either total utility or revenue. Large system optimization problems were formulated in which the large system utility and revenue depend on the distribution of powers and utitilies across the user population. This enables a framework for studying the effect of load and prices on system performance. Whereas the prices that maximize utility can be explicitly determined for arbitrary utility and power distributions, this appears to be difficult for the revenue objective function. Still, the behavior of the revenue as a function of load and prices can be observed numerically.

Our analytical and numerical results indicate that for the step utility function considered, the revenue function is unimodal for a large class of utility distributions. Numerical examples show that pricing for power increases the total utility or revenue, and that this benefit increases with load. Conversely, power pricing can reduce the average transmitted power for a given total utility. Of course, the analytical framework presented here can be used to study the performance with different utility functions, such as those corresponding to data services. Pricing for multi-cell resource allocation is currently being studied.

References

[1] J. MacKie-Mason and H. Varian, “Pricing Congestable Network Resources” *IEEE Journal on Selected Areas in Communications*, Vol.13, No.7, Sept, 1999

[2] A. Lazar, A. Orda and D. Pendarakis, “Virtual Path Bandwidth Allocation in Multiuser Networks” *IEEE/ACM Transaction on Networking*, Vol.1, No.6, Dec, 1997

[3] F. P. Kelly, A. K. Maulloo, and D. K. H. Tan, “Rate Control for Communication Networks: Shadow Prices, Proportional Fairness, and Stability” *Journal of the Operational Research Society*, Vol. 49, pp. 237-252, 1998.

[4] S. Jordan and H. Jiang, “Connection Establishment in High-Speed Networks” *IEEE Journal on Selected Areas in Communications*, Vol.13, No.7, Sept, 1995

[5] H. Jiang and S. Jordan, “A Pricing Model for High Speed Networks with Guaranteed Quality of Service”, *Proceedings Infocom’96*, Vol.2, March 1996

[6] H. Jiang and S. Jordan, “The Role of Pricing in the Connection Establishment Process”, *European Transaction on Telecommunications*, 6(4):421-429 Vol.2, July 1995

[7] D. Famolari and N. Mandayam, “A New Framework for Power Control in Wireless Data Networks: Games, Utility, and Pricing”, *Wireless Multimedia Network Technologies*, Eds: Ganesh, Pahlavan, Zvonar, Kluwer Academic Publishers, pp. 289-310, 1999

[8] N. Feng and N. Mandayam, “Joint Power and Rate Optimization for Wireless Data Services Based on Utility Functions” *Proc. Conference on Information Sciences and Systems*, Johns Hopkins University, Baltimore, MD, March 1999

[9] C. Saraydar and N. Mandayam, “Efficient Power Control via Pricing in Wireless Data Networks” *Proc. IEEE Wireless Communications and Networking Conference*, 1999