On Quantizer Design for Soft Values in the Multiple-Access Relay Channel

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***Abstract*—A network with two sources, one relay, and one destination is considered. Under the assumption of noisy source-relay links causing the relay to be unable to decode without error, we propose a quantizer design framework where the quantizer jointly compresses the soft information available for both sources at the relay. The quantizer design is based on the information bottleneck method using the notion of relevant information as an optimization criterion.**

I. INTRODUCTION

The designated purpose of a relay in wireless networks is to facilitate the transmission of other users within the same cell, where one important benefit of doing so is cooperative diversity [1]. While the concept of network coding [2] was originally developed to increase throughput in wireline networks, the application of network coding to wireless networks has been shown to effectively combat the effects of the fading channels [3]–[6], thereby providing cooperative diversity. Common to this work is the assumption that the relay node can recover the source messages perfectly, thus restricting the investigation to relaying protocols based on the decode-and-forward (DF) [7]–[9] strategy, or to the strategy that the relay does not transmit at all if residual errors remain after decoding, provided there exists an error detection scheme at the relay node. Since the information obtained at the relay can still be helpful for decoding at the destination in cases where the source-relay (SR) links are not good enough to support error-free recovery of the source messages, we propose in this paper a practical way of forwarding that information to the decoder.

Throughout, we consider the multiple-access relay channel (MARC) shown in Fig. 1 with two sources s_1, s_2, one relay r, and the destination d. The network geometry is assumed to be such that the relay is closer to the destination than to the sources, so that the SR channel quality is too low to permit reliable decoding at the relay. The relay-destination (RD) link, however, can support a higher rate due to its proximity to the destination. For such a scenario, it was shown [10] that having the relay form the log-likelihood ratios (LLRs) of the network coded message $x_1 \oplus x_2$ and transmitting those LLRs in an analog manner to the destination does significantly increase the receiver performance, which can be further improved upon by appropriately quantizing the LLRs at the relay [11]. However, if the restriction to SR channels of equal signal-to-noise ratio (SNR) as in [10], [11] is lifted, the reliability of the LLR of the network coded message is undesirably dominated by the user with the worse SR channel. To overcome that drawback, we propose in this work a quantizer design without going through the intermediate step of computing the LLR of $x_1 \oplus x_2$, enabling the relay to handle different SR channel qualities efficiently. Throughout, we assume the relay node to only have channel state information about the SR links and the RD link, but not about the outer source-destination links, which renders source coding with side information at the destination [12] impossible, as suggested for the compress-and-forward [7] relaying strategy.

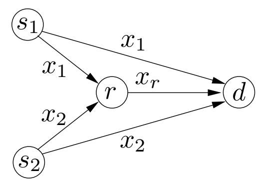

Fig. 1. The multiple-access relay channel.

For clarity of exposure, the system model and quantizer design using the LLRs of the network coded message (for symmetric SR channels) will be summarized below in Section II before addressing the general case and the main result of this paper in Section III, followed by simulation results in Section IV.

II. THE SYMMETRIC CASE

A. System Model

At each source s_i, $i = 1, 2$, a block of information bits $\boldsymbol{u}_i \in \{0,1\}^k$ is encoded to a block of code bits $\boldsymbol{c}_i \in \{0,1\}^n$ with a channel code of rate $R = k/n$. Throughout, we assume BPSK modulation at the sources, so that the transmit block at source i is $\boldsymbol{x}_i \in \{+1,-1\}^n$. Although being suboptimal, for ease of implementation, communication is presumed to take place on channels orthogonalized either in time, frequency or code. Without loss of generality, we consider a time-division network with three phases, where the two sources transmit in time slots 1 and 2, respectively, and the relay transmits its codeword $\boldsymbol{x}_r$ in the third time phase. Then, the received

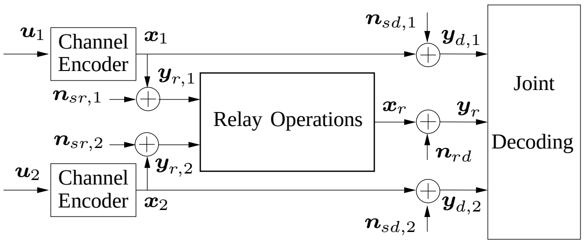

Fig. 2. System model.

signals at the relay and the destination are given by

$$\begin{aligned} \boldsymbol{y}_{r,i} &= \boldsymbol{x}_i + \boldsymbol{n}_{sr,i} \\ \boldsymbol{y}_{d,i} &= \boldsymbol{x}_i + \boldsymbol{n}_{sd,i} \\ \boldsymbol{y}_r &= \boldsymbol{x}_r + \boldsymbol{n}_{rd}, \end{aligned}$$

where the zero-mean noise variables follow a circular symmetric complex normal distribution with variances $\sigma^2_{sr,i}, \sigma^2_{sd,i}$ and σ^2_{rd}, respectively. The model is shown in Fig. 2.

Upon receiving $\boldsymbol{y}_{r,1}$ and $\boldsymbol{y}_{r,2}$, the relay invokes soft decoders to compute the LLRs $\boldsymbol{L}_1, \boldsymbol{L}_2 \in \mathbb{R}^n$ about the coded bits $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$, cf. Fig. 3. Since error-free decoding at the relay is not possible, the relay cannot re-encode the source messages for transmission, but rather chooses to transmit a compressed version of its estimates $\boldsymbol{L}_1$ and $\boldsymbol{L}'_2$, which is the interleaved version of $\boldsymbol{L}_2$. Following [10], the relay then computes the LLRs $\boldsymbol{L}_r \in \mathbb{R}^n$ of the network coded block of code bits $\boldsymbol{x}_1 \oplus \boldsymbol{x}'_2$, whose m-th element is given by

$$\begin{aligned} L_{r,m} &= \ln\left(\frac{1+\mathrm{e}^{L_{1,m}+L'_{2,m}}}{\mathrm{e}^{L_{1,m}}+\mathrm{e}^{L'_{2,m}}}\right) \\ &= L_{1,m} \boxplus L'_{2,m} \\ &\approx \operatorname{sign}(L_{1,m})\operatorname{sign}(L'_{2,m}) \min\left\{|L_{1,m}|, |L'_{2,m}|\right\}, \end{aligned}$$

taking the $\boxplus$ notation from [13]. Then, the relay forms a quantized version $\boldsymbol{Z} \in \mathcal{Z}^n$ of $\boldsymbol{L}_r$, where $\mathcal{Z}$ is the quantizer index set, source encodes and channel encodes the quantizer output yielding the relay codeword $\boldsymbol{x}_r \in \mathbb{M}^n$, with $\mathbb{M}$ being the modulation alphabet used at the relay. Note that we restrict the relay to sending n (complex) symbols from $\mathbb{M}$. The quantizer design will be topic of the following two subsections.

The destination uses the iterative decoder structure shown in Fig. 4 which is strongly reminiscent of a turbo decoder with two soft decoders acting as component decoders. These component decoders exchange soft information about the coded bits, however, the exchange of information between the decoders is limited by the *relay check nodes*, which use the received word $\boldsymbol{y}_r$ from the relay to compute the estimate $\hat{\boldsymbol{L}}_r$ of $\boldsymbol{L}_r$ at the destination before calculating the *a priori* information $L'^{(2)}_{\mathrm{A},m} = L^{(1)}_{\mathrm{E},m} \boxplus \hat{L}_{r,m}$ and $L^{(1)}_{\mathrm{A},m} = L'^{(2)}_{\mathrm{E},m} \boxplus \hat{L}_{r,m}$ for the component decoders. Now, if the information obtained from the relay about $\boldsymbol{x}_1 \oplus \boldsymbol{x}'_2$ is very reliable, the check nodes barely limit the exchange of extrinsic information, and the overall decoder looks like a turbo decoder, where however, all the *code bits* are coupled by $\boldsymbol{x}_1 \oplus \boldsymbol{x}'_2$. In contrast, if no information is received from the relay, then the relay check

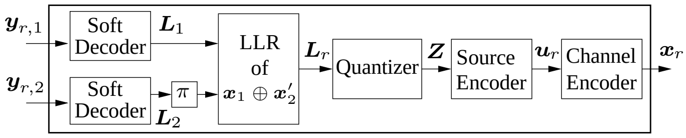

Fig. 3. Relay operations in the symmetric case.

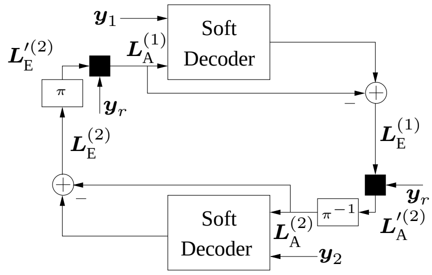

Fig. 4. Iterative decoder.

node completely prevents any information exchange between the decoders, resulting in two separate soft decoders.

Given the aforementioned close relation of the decoder structure with a turbo decoder, the Extrinsic Information Transfer (EXIT) chart [14] serves as an helpful tool to analyze the decoding process. It turns out [10], [11] that the EXIT curve of the relay check node is almost a straight line never exceeding the diagonal of the EXIT chart with $I_{\mathrm{out}} = I(X;Z)$ for $I_{\mathrm{in}} = 1$, where $X = X_1 \oplus X_2$. See Fig. 5 for an example. Note that due to the symmetry in the setup, the EXIT curve for the two check nodes is the same, where $I_{\mathrm{in}} = I(X_i; L^{(i)}_{\mathrm{E}})$ and $I_{\mathrm{out}} = I(X_j; L^{(j)}_{\mathrm{A}})$, $i,j \in \{1,2\}, i \neq j$. In order for the iterative decoder to converge, it is desirable to choose a quantizer at the relay that makes the slope of the relay check node EXIT curve as large as possible. For a given quality on the SR links, this is equivalent to maximizing $I(X;Z)$ while minimizing the rate required on the RD link. This can be achieved using the information bottleneck method (IBM), summarized below.

B. The Information Bottleneck Method

The IBM [15] provides a framework for quantization of a random variable $L_r \in \mathcal{L}$ related to another variable X by their

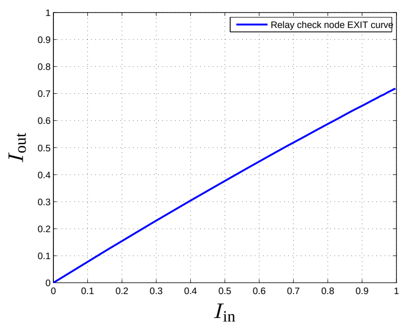

Fig. 5. EXIT curve of relay check node, $[37_8, 21_8]$ convolutional code, $\mathrm{SNR}_{sr,1} = \mathrm{SNR}_{sr,2} = -2\,\mathrm{dB}$, $\mathrm{SNR}_{rd} = 2.3\,\mathrm{dB}$, three quantization regions.

joint distribution $p(x, \ell)$. The goal is to preserve the *relevant* information about X in the quantization $Z \in \mathcal{Z}$, leading to the minimization problem

$$\min_{p(z|\ell)} I(L_r; Z) \quad \text{s.t. } I(X; Z) \geq \tilde{D} \tag{1}$$

for some $\tilde{D} > 0$. By choice of a Lagrangian multiplier $\beta > 0$, the optimization problem (1) can be rewritten as

$$\min_{p(z|\ell)} I(L_r; Z) - \beta I(X; Z). \tag{2}$$

Further, there is an iterative optimization algorithm similar to the Blahut-Arimoto algorithm [16] shown to converge to a local minimum of (2).

C. Quantizer Design

We can use the IBM to compute a quantizer which, for a given SR SNR, maximizes the relevant information $I(X; Z)$ subject to a constraint on the rate on the RD link. To accomplish this, the parameter β in the optimization algorithm has to be greatly larger than one to yield a deterministic mapping $p(z|\ell)$, and the distribution $p(x, \ell)$ is obtained by means of simulation for the particular channel code used. Summarizing, by designing the quantizer at the relay with the IBM, one can maximize the slope of the EXIT curve of the relay check node, thereby optimizing the convergence properties of the iterative decoder. Throughout, we assume the rate of the quantizer chosen at the relay to be such that the quantizer output $\boldsymbol{Z}$ can be communicated reliably to the destination. In the following, we will refer to the quantizer design on $\boldsymbol{L}_r$ as the XOR-solution to the compression problem at the relay.

The above investigations are limited to the case where the SR links are symmetric with respect to the SNR. If, however, the SR links are of different channel quality, then the reliability of $\boldsymbol{L}_r$ about the network coded message will be dominated by the user with the weaker SR channel. Therefore, in order for the relay to be able to handle asymmetric SR links effectively, the quantizer should be adapted to operate on $\boldsymbol{L}_1$ and $\boldsymbol{L}_2'$ directly, and accordingly, the quantizer design algorithm as well.

III. The General Case

A. Quantizer Design

As in the symmetric case, the framework for the quantizer design will be provided by the IBM, but with a different expression as relevant information, whose choice will be motivated in the following. As mentioned above, the EXIT curve of the relay check node is almost a straight line with $I_{\text{out}} = I(X; Z)$ for $I_{\text{in}} = 1$, and for the symmetric case considered above, the curves are the same for both check nodes in the decoder. In general, however, these curves will be different. During the iterative decoding process, the component decoders produce random variables $\boldsymbol{L}_{\text{E}}^{(1)}$ and $\boldsymbol{L}_{\text{E}}^{(2)}$ with some mutual information $I(X_i; L_{\text{E}}^{(i)})$, $i = 1, 2$, which is the input information to the corresponding relay check node. At this point, again assuming error-free transmission of the quantizer output $\boldsymbol{Z}$, we note that the relay check node in the receiver processes $\boldsymbol{Z}$ and $\boldsymbol{L}_{\text{E}}^{(i)}$ to produce *a-priori* information for the corresponding component decoder. Therefore, we would like the quantizer at the relay to be such that $I(X_i; Z, L_{\text{E}}^{(j)})$, $i, j \in \{1, 2\}$, $i \neq j$, is maximal, both for the information exchange from decoder 1 to decoder 2, and the information exchange from decoder 2 to decoder 1. Since

$$I(X_i; Z, L_{\text{E}}^{(j)}) = I(X_i; L_{\text{E}}^{(j)}) + I(X_i; Z | L_{\text{E}}^{(j)}) = I(X_i; Z | L_{\text{E}}^{(j)}),$$

we are left with maximizing $I(X_i; Z | L_{\text{E}}^{(j)})$. Due to the characteristic property of the relay check node of being almost a straight line, the problem simplifies to maximizing the output information for perfect input information $I_{\text{in}} = 1$. Now, perfect input information means that $I(X_i; L_{\text{E}}^{(i)}) = 1$ for $i = 1, 2$, so that X_i is known at the decoder output. Consequently, to allow maximal information transfer from decoder 1 to decoder 2, $I(X_2; Z|X_1)$ should be maximized, and analogously, for decoder 1 to receive maximal information from decoder 2, $I(X_1; Z|X_2)$ should be as large as possible. Various combinations of these information expressions can be taken to form the relevant information term for the IBM. For example, choosing $I_{\text{rel}} = \min\{I(X_1; Z|X_2), I(X_2; Z|X_1)\}$ as the relevant information expression will aim for keeping the turbo loop in the iterative decoder running as long as possible. However, if one user has a very bad SR link, that choice of I_{rel} also claims much of the rate of the RD link for communicating very unreliable information about that users data. Therefore, we propose to take the average of $I(X_1; Z|X_2)$ and $I(X_2; Z|X_1)$ as the relevant information term, so that the relay can opportunistically allocate more of its rate to the user with the better SR channel. In terms of the IBM, to design a quantizer at the relay, we solve

$$\min_{p(z|\ell_1, \ell_2)} I(L_1, L_2; Z) \text{ s.t. } I(X_1; Z|X_2) + I(X_2; Z|X_1) \geq \tilde{D}, \tag{3}$$

where now, the relevant information is $I_{\text{rel}} = I(X_1; Z|X_2) + I(X_2; Z|X_1)$. For some multiplier $\beta > 0$, the implicit solution to this problem can be shown to be

$$\begin{aligned} &p(z|\ell_1, \ell_2) = \\ &\frac{p(z)}{N(\ell_1, \ell_2, \beta)} \exp\Big\{ -2\beta D\big(p(x_1, x_2|\ell_1, \ell_2) || p(x_1, x_2|z)\big) \\ &\quad + \beta D\big(p(x_1|\ell_1) || p(x_1|z)\big) + \beta D\big(p(x_2|\ell_2) || p(x_2|z)\big) \Big\}, \end{aligned}$$

where $D(p||q)$ is the relative entropy between p and q, and $N(\ell_1, \ell_2, \beta)$ is a normalizing term ensuring that $p(z|\ell_1, \ell_2)$ is a valid probability distribution. To compute a locally optimal solution to (3), we can use an appropriately modified version of the iterative information bottleneck algorithm [15], [11], restricting ourselves to $\beta \gg 1$ to obtain a two-dimensional vector quantizer using the deterministic mapping $p(z|\ell_1, \ell_2)$.

B. Decoding

The operations of the relay check node are summarized in Fig. 6. In case that residual errors are detected in $\hat{\boldsymbol{u}}_r$ by, e.g., a cyclic redundancy check (CRC), all the information from the relay is discarded to avoid catastrophic error propagation

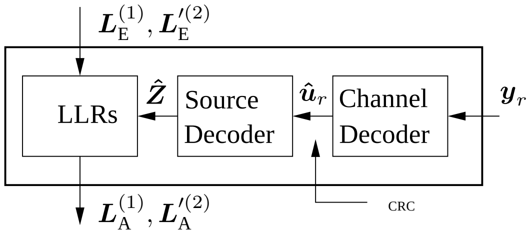

Fig. 6. Operations of the relay check node.

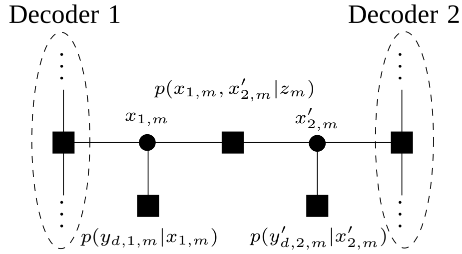

Fig. 7. One section of the factor graph of the decoder.

through the source decoder. We will now describe the function of the block labeled "LLRs" in Fig. 6. To do so, we will take a slightly different view on the decoding algorithm, and derive the function of that block on the factor graph of the decoder, one section of which is shown in Fig. 7. The nodes labeled $x_{1,m}$ and $x'_{2,m}$ are two variable nodes of the two different component channel codes coupled by the quantization at the relay. The direct observations from the two sources are the function nodes $p(y_{d,1,m}|x_{1,m})$ and $p(y'_{d,2,m}|x'_{2,m})$, whereas the coupling through the quantization at the relay is expressed in the function node $p(x_{1,m}, x'_{2,m}|z_m)$. Since $\boldsymbol{Z}$ is available perfectly at the receiver, the destination exploits its knowledge of the quantizer chosen by the relay to obtain $p(x_1, x_2|z)$, which is among the output of the iterative optimization algorithm. Note that the function node $p(x_1, x_2|z)$ is the message passing equivalent of the LLR block in Fig. 6. To find processing rules for the LLRs $\boldsymbol{L}_E^{(1)}$ and $\boldsymbol{L}_E'^{(2)}$, we apply the message passing rules for function nodes to $p(x_1, x_2|z)$. Using the definitions

$$
\begin{aligned}
L(x_{1,m}, x'_{2,m}\!=\!1|z_m) &= \ln\left(\frac{p(x_{1,m}\!=\!1, x'_{2,m}\!=\!1|z_m)}{p(x_{1,m}\!=\!-1, x'_{2,m}\!=\!1|z_m)}\right) \\
L(x_{1,m}\!=\!1, x'_{2,m}|z_m) &= \ln\left(\frac{p(x_{1,m}\!=\!1, x'_{2,m}\!=\!1|z_m)}{p(x_{1,m}\!=\!1, x'_{2,m}\!=\!-1|z_m)}\right) \\
L(x_{1,m}, x'_{2,m}\!=\!-1|z_m) &= \ln\left(\frac{p(x_{1,m}\!=\!1, x'_{2,m}\!=\!-1|z_m)}{p(x_{1,m}\!=\!-1, x'_{2,m}\!=\!-1|z_m)}\right) \\
L(x_{1,m}\!=\!-1, x'_{2,m}|z_m) &= \ln\left(\frac{p(x_{1,m}\!=\!-1, x'_{2,m}\!=\!1|z_m)}{p(x_{1,m}\!=\!-1, x'_{2,m}\!=\!-1|z_m)}\right) \\
L(x_{1,m}, x'_{2,m}|z_m) &= \ln\left(\frac{p(x_{1,m}\!=\!1, x'_{2,m}\!=\!-1|z_m)}{p(x_{1,m}\!=\!-1, x'_{2,m}\!=\!1|z_m)}\right),
\end{aligned}
$$

one obtains that

$$
\begin{aligned}
L_{A,m}^{'(2)} &= \ln\left(\frac{1 + \mathrm{e}^{L_{E,m}^{(1)}}\mathrm{e}^{L(x_{1,m}, x'_{2,m}=1|z_m)}}{\mathrm{e}^{L_{E,m}^{(1)}}\mathrm{e}^{L(x_{1,m}, x'_{2,m}|z_m)} + \mathrm{e}^{-L(x_{1,m}=-1, x'_{2,m}|z_m)}}\right) \\
L_{A,m}^{(1)} &= \ln\left(\frac{1 + \mathrm{e}^{L_{E,m}^{'(2)}}\mathrm{e}^{L_m(x_{1,m}=1, x'_{2,m}|z_m)}}{\mathrm{e}^{L_{E,m}^{'(2)}}\mathrm{e}^{-L(x_{1,m}, x'_{2,m}|z_m)} + \mathrm{e}^{-L(x_{1,m}, x'_{2,m}=-1|z_m)}}\right),
\end{aligned}
$$

which are just a realization of the message passing rules in factor graphs.

C. Interpretation

The expressions above are closely related to the boxplus computation in the symmetric case. Note that if

$$L(x_{1,m}, x'_{2,m}|z_m) = 0 \tag{4}$$
$$-L(x_{1,m}\!=\!-1, x'_{2,m}|z_m) = L(x_{1,m}, x'_{2,m}\!=\!1|z_m) \tag{5}$$
$$-L(x_{1,m}, x'_{2,m}\!=\!-1|z_m) = L(x_{1,m}\!=\!1, x'_{2,m}|z_m), \tag{6}$$

they simplify to

$$
\begin{aligned}
L_{A,m}^{'(2)} &= L_{E,m}^{(1)} \boxplus L(x_{1,m}, x'_{2,m}\!=\!1|z_m) \\
L_{A,m}^{(1)} &= L_{E,m}^{'(2)} \boxplus L_m(x_{1,m}\!=\!1, x'_{2,m}|z_m).
\end{aligned}
$$

It turns out that application of the general quantizer design algorithm to symmetric SR channels can result in a distribution $p(x_1, x_2|z)$ such that Eqs. (4)-(6) are fulfilled, so that the XOR-solution is recovered in those situations without computing $\boldsymbol{L}_r$ first.

IV. Simulations

In this section, we provide some simulation results for the quantizer design algorithm. Throughout, the underlying channel codes used at the sources are rate 1/2 recursive convolutional codes with generator

$$G(D) = \left(1, \frac{1 + D^4}{1 + D + D^2 + D^3 + D^4}\right).$$

Source coding at the relay is performed using an arithmetic code, and the channel code on the RD link is the turbo code specified in the UMTS standard [17].

The first two examples, shown in Figs. 8 (a) and 8 (b), depict the partitioning of the (L_1, L_2)-plane into quantization regions as obtained by the iterative optimization algorithm. Each of the resulting regions is color coded, with each color corresponding to one symbol of the quantizer alphabet $\mathcal{Z}$. Using $N = |\mathcal{Z}| = 3$ regions, the quantizer is shown in Fig. 8 (a) for symmetric SR links at $\mathrm{SNR}_{sr,1} = \mathrm{SNR}_{sr,2} = -2\,\mathrm{dB}$. Note that this partition effectively mimics the XOR operation at the relay, and since $H(Z) = 1.44$, a minimum SNR on the RD link of $2.33\,\mathrm{dB}$ is required for error-free transmission. To achieve a word error probability of about 10^{-4} with the UMTS turbo code, simulations suggest that 1.5 dB have to be added to the Shannon limit SNR, so that $\mathrm{SNR}_{rd} = 3.83\,\mathrm{dB}$ is required. In contrast, if channel conditions on the SR links are profoundly different, then the relay should preferably allocate more of the rate available on the RD channel to the

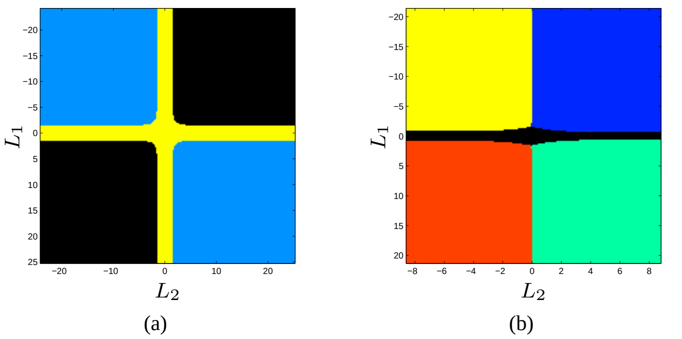

Fig. 8. Quantizer obtained for (a)$\text{SNR}_{sr,1} = \text{SNR}_{sr,2} = -2\,\text{dB}$ and $N = 3$ quantization regions, (b) $\text{SNR}_{sr,1} = -3\,\text{dB}$, $\text{SNR}_{sr,2} = -8\,\text{dB}$, and $N = 5$ quantization regions.

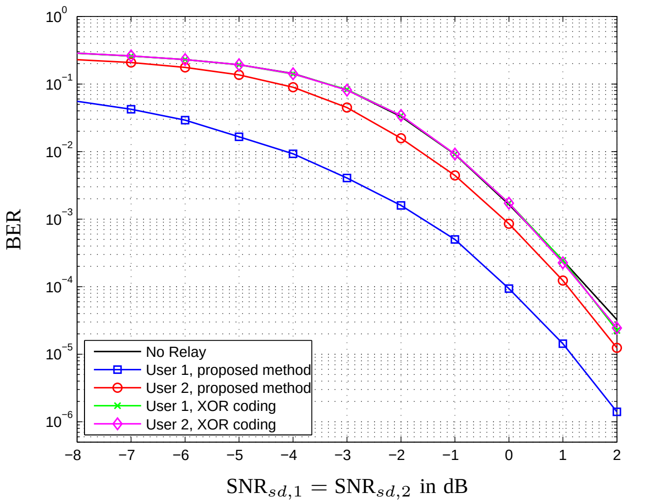

Fig. 9. BER obtained using the quantizer in Fig. 8 (b) at $\text{SNR}_{sr,1} = -3\,\text{dB}$, $\text{SNR}_{sr,2} = -8\,\text{dB}$, and $N = 5$ quantization regions.

stronger user, and this is exactly achieved with the general formulation of the quantization problem at the relay, as shown in Fig. 8 (b) for $N = 5$ regions, where $\text{SNR}_{sr,1} = -3\,\text{dB}$ and $\text{SNR}_{sr,2} = -8\,\text{dB}$. Since $H(Z) = 2.30$, the RD link SNR has to be about $7.45\,\text{dB}$ for reliable transmission using the UMTS turbo code as channel code.

Finally, we plot the bit-error-rate (BER) in Fig. 9 when the quantizer described above for $\text{SNR}_{sr,1} = -3\,\text{dB}$, $\text{SNR}_{sr,2} = -8\,\text{dB}$, and $N = 5$ is used at the relay. The modulation scheme at the relay is 16-QAM, and $\text{SNR}_{rd} = 7.45\,\text{dB}$. As expected, user 1 with the stronger relay channel shows a superior BER performance than user 2, which is performing slightly better than a user that could not exploit the relay at all. In the same figure, we also plot the resulting BER curves when the relay computes the LLRs $\boldsymbol{L}_r$ of the network coded block of code bits $\boldsymbol{x}_1 \oplus \boldsymbol{x}_2'$ before quantization as in [11], again for a quantizer with $N = 5$ regions. As expected, the performance of that method is poor here due to the asymmetry in the channel quality.

V. Conclusion

In this paper, we considered the multiple-access relay channel where residual errors are assumed to remain after decoding at the relay. In order to forward the soft information at the relay in a bandwidth efficient manner, a quantizer design framework is proposed for the LLRs at the output of the soft decoders at the relay. Motivated by the EXIT chart, the quantizer design is such that it codes the LLRs of the two users in a "soft" equivalent to network coding if the SR links are of equal quality. In case of SR links of different quality, however, more resources of the RD link are opportunistically allocated to the stronger user. In addition to examples visualizing the quantizer, BER simulations are provided underlining the significance of proper quantizer design for asymmetric SR channels.

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