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# Simulation of Two-Rate Neural Network Control for Stochastic Model of Missile Autopilot

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*Abstract:* - This paper describes a two-rate stochastic control system as state-space (SS) type decomposed and discretized models of stochastic subsystems with the "fast" and "slow" artificial neural networks (NNs). These NNs are used as the dynamic subsystems controllers. This is because such neuromorphic controllers are especially suitable to control complex systems. An illustrative example – two-rate NN hybrid control of decomposed stochastic model of a rigid guided missile over different operating conditions – was carried out using the proposed two-rate SS decomposition technique. This example demonstrates that this research technique results in simplified low-order autonomous control subsystems with various discretization periods and with various speeds of actuation, and shows the quality of the proposed technique. The obtained results show that the control tasks for the autonomous subsystems can be solved more qualitatively than for the original system. The simulation and animation results with use of software package Simulink demonstrate that this research technique would work for real-time stochastic systems.

*Key-Words:* - Control systems, guided missile, modelling, multirate systems, neural networks, simulation, stochastic systems.

## 1 Introduction

During the work stochastic systems can be subjected with unknown beforehand perturbations in some beforehand known limits. In such cases the attempts to derive exact solutions are generally unjustified and poorly useful. Therefore, at a research of such complicated systems we usually resort to their simplification by decomposition on isolated autonomous subsystems.

The nature of the multirate dynamics of the process makes it very attractive for these practical control applications, since the multirate research technique is able to decompose the complex stochastic models of physical systems.

Sandell, Varaiya, Athans, and Safonov [1] reviewed the previous research and divided it into four categories: Model Simplification, Interconnected Systems, Decentralized Control, and Hierarchical Control. In opinion of these authors, we do not believe that the existing mathematical tools are powerful enough to define a preferable structure for decentralized and/or hierarchical control. They claimed that with respect to designing decentralized controllers for many physical large-scale systems a good combination of engineering judgement and analysis can be used to define in a reasonable way a special structure for the dynamic system.

Kokotovic et al. [2]-[3] showed that the singular perturbation theory for difference equations involves a list of ingredients-order reduction, separation of time scales, and boundary layer phenomena. Sufficient conditions are given under which the solution of the original problem tends to the solution of a low-order problem.

Er and Mastorakis [4] presented a review of the classical as well as the modern approach towards multirate control of linear systems. Besides the five types of control strategies for these approaches being discussed, and advantages and disadvantages of these control strategies are highlighted in [5], there exist many other types from which some are modifications from the existing strategies.

The lifting method (Li, Shah, Chen, and Patwardhan [5]) is used to analyze the multirate system in the SS framework. From the discussion in [5] it is clear that the fast sampled model can be easily identified from the input excitation of an open-loop process running under a multirate-sampling scheme with slow output sampling and fast control.

Gu and Tomizuka [6] described a multirate feedback/feedforward system applicable to the design and analysis of a tracking control system under measurement constraints. In this system, the feedback controller is updated at the slow rate, while the feedforward controller is updated at the fast rate with a desired sequence.

Tornero, Gu, and Tomizuka [7] studied the multirate controller, which updates the controller output faster than the measurement sampling frequency by the factor of $N$. The simplest case of $N=2$ has been examined in some details. The controlled system is continuous, and the discrete-time controller is obtained as its zero order hold equivalent. After deriving a set of equations necessary for the analysis and design of multirate systems, the open loop characteristics of the multirate controller and the closed loop characteristics are examined.

In [8] a new method for decomposition of SS models of multi-input/multi-output (MIMO) stochastic discrete-time original systems on two-rate discrete-time aggregative models of subsystems is described. An illustrative example – NN and fuzzy logic hybrid control of two-rate decomposed stochastic model of a tracking system for an experimental aircraft – was carried out using the proposed two-rate SS decomposition technique. This example shows the quality of the proposed technique.

This paper expands the basic ideas presented in [8], but does not repeat [8]. The original stochastic continuous-time system is used to create two-rate stochastic discrete-time subsystems. First, the original continuous-time system is decomposed on lower-order two-rate autonomous continuous-time subsystems. Next, each of the obtained subsystems is discretized with own discretization period.

The goal of this paper is to show the applicability of the proposed two-rate NN control technique for the model of a rigid guided missile [9] to control the vertical acceleration over different operating conditions.

The contribution of the paper is twofold: to develop new decomposition schemes appropriate for real-time multirate NN control applications, and to present the results of two-rate hybrid control for any chosen SS type of MIMO stochastic model in simulation form.

## 2 Decomposition of Stochastic Continuous-Time Systems

Consider the stochastic continuous-time multivariable system

$$\dot{x}(\tau) = Ax(\tau) + Bu(\tau) + v(\tau) \tag{1}$$

$$y(\tau) = Cx(\tau) + w(\tau) \tag{2}$$

where

$$x(\tau)\in R^n, u(\tau)\in R^m, y(\tau)\in R^p, v(\tau)\in R^n, w(\tau)\in R^P$$

are the state, control input, output, noise of excitation of state and noise of measurement vectors, respectively.

Setting $q(\tau) = Tx(\tau)$, where $T$ is a nonsingular matrix, we see that (1)-(2) are transformed into the equations

$$\dot{z}_1(\tau) = \Lambda_1 z_1(\tau) + B_1 u(\tau) + T_1 v(\tau) \tag{3}$$

$$\dot{z}_2(\tau) = \Lambda_2 z_2(\tau) + B_2 u(\tau) + T_2 v(\tau) \tag{4}$$

$$y(\tau) = C_1 z_1(\tau) + C_2 z_2(\tau) + w(\tau) \tag{5}$$

where

$$|\lambda(\Lambda_1)| \langle \gamma_{s_{\max}}, |\lambda(\Lambda_2)| \rangle \gamma_{f_{\min}} .$$

Definition 1: A function with a large derivative, which is quickly decreasing, is said to be the "fast" function, a function with a small derivative, which is slowly decreasing, is said to be the "slow" function.

Consider the first time interval $0 \langle \tau_f \le \tau_{fs}$. According to Definition 1, the variable $z_1$ can be considered as a "slow" function on that interval. Hence, assuming that $\dot{z}_1(\tau_f) = 0$, from (3), we see that

$$z_1(\tau_f) = -\Lambda_1^{-1} B_1 u(\tau_f) - \Lambda_1^{-1} T_1 v(\tau_f). \tag{6}$$

From (4)-(6), we find that the state equations for a "fast" subsystem may be written as

$$\dot{z}_f(\tau_f) = A_f z_f(\tau_f) + B_f u_f(\tau_f) + T_f v_f(\tau_f) \tag{7}$$

$$y_f(\tau_f) = C_f z_f(\tau_f) + D_f u_f(\tau_f) + w_f(\tau_f) \tag{8}$$

where

$$A_f = \Lambda_2, B_f = B_2, T_f = T_2, C_f = C_2,$$

$$D_f = -C_1 \Lambda_1^{-1} B_1, V_f = -C_1 \Lambda_1^{-1} T_1,$$

$$z_f(\tau_f) = z_2(\tau_f), u_f(\tau_f) = u(\tau_f), v_f(\tau_f) = v(\tau_f),$$

$$y_f(\tau_f) = y(\tau_f), w_f(\tau_f) = w(\tau_f) + V_f v(\tau_f).$$

Consider the second time interval $\tau_s \rangle \tau_{fs}$. According to Definition 1, the variable $z_2$ can be considered as a "fast" function of time, achieving on this interval a steady meaning. Hence, assuming that $\dot{z}_2(\tau_s) = 0$, from (4), we find

$$z_2(\tau_s) = -\Lambda_2^{-1} B_2 u(\tau_s) - \Lambda_2^{-1} T_2 v(\tau_s) \qquad (9)$$

From (3), (5) and (9), we find that the state equations for a "slow" subsystem may be written as

$$\dot{z}_s(\tau_s) = A_s z_s(\tau_s) + B_s u_s(\tau_s) + T_s v_s(\tau_s) \qquad (10)$$

$$y_s(\tau_s) = C_s z_s(\tau_s) + D_s u_s(\tau_s) + w_s(\tau_s) \qquad (11)$$

where

$$A_s = \Lambda_1, B_s = B_1, T_s = T_1, C_s = C_1,$$

$$D_s = -C_2 \Lambda_2^{-1} B_2, V_s = -C_2 \Lambda_2^{-1} T_2,$$

$$z_s(\tau_s) = z_1(\tau_s), u_s(\tau_s) = u(\tau_s), v_s(\tau_s) = v(\tau_s),$$

$$y_s(\tau_s) = y(\tau_s), w_s(\tau_s) = w(\tau_s) + V_s v(\tau_s).$$

## 3 State Equations for Two-Rate Subsystems

We note that the state equations for the discretized "fast" subsystem, which was obtained from (7)-(8), may be written as

$$z_f[(t+1)\Delta_f] = F_f z_f(t\Delta_f) + G_f u_f(t\Delta_f) + T_{d_f} \nu_f(t\Delta_f) \qquad (12)$$

$$y_f(t\Delta_f) = H_f z_f(t\Delta_f) + E_f u_f(t\Delta_f) + w_f(t\Delta_f), t = 0,1,2,... \qquad (13)$$

where

$$F_f = \exp(A_f \Delta_f), G_f = \left[\int_0^{\Delta_f} \exp(A_f q) dq\right] B_f,$$

$$T_{d_f} = \left[\int_0^{\Delta_f} \exp(A_f q) dq\right] T_f, H_f = C_f, E_f = D_f.$$

Further, from (10)-(11), we find that the state equations for the discretized "slow" subsystem may be written as

$$z_s[(t+1)\Delta_s] = F_s z_s(t\Delta_s) + G_s u_s(t\Delta_s) + T_{d_{s1}} \nu_s(t\Delta_s) \qquad (14)$$

$$y_s(t\Delta_s) = H_s z_s(t\Delta_s) + E_s u_s(t\Delta_s) + w_s(t\Delta_s), t = 0,1,2,... \qquad (15)$$

where

$$F_s = \exp(A_s \Delta_s), G_s = \left[\int_0^{\Delta_s} \exp(A_s q) dq\right] B_s,$$

$$T_{d_s} = \left[\int_0^{\Delta_s} \exp(A_s q) dq\right] T_s, H_s = C_s, E_s = D_s.$$

## 4 Mathematical Model of a Rigid Guided Missile

The model of a rigid guided missile over different operating conditions is given in [9]. The variables of this model are: $x_1$ = pitch rate (radians/sec), $x_2$ = angle of attack (radians), $x_3$ = elevator deflection angle (radians), $u$ = elevator command (radians), $y$ = vertical acceleration (m/ sec$^2$).

The matrix structure of $A, B, C$ for the SS model of the original system in form (1)-(2) is given by

$$A = \begin{bmatrix} -1.364 & -92.82 & -128.46 \\ 1 & -4.68 & -0.087 \\ 0 & 0 & -190 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 190 \end{bmatrix},$$

$$C = \begin{bmatrix} 1.36 & -184.26 & 76.43 \end{bmatrix}$$

where the constant parameters of operating conditions are: altitude = 12000 m, Mach number = 2 .

## 5 Mathematical Model of Two-Rate Subsystems

The constant matrices of the "fast" subsystem (7)-(8) are given by

$$A_f = [-190], B_f = [229.7055],$$

$$C_f = [64.4703], D_f = [229.177],$$

$$T_f = [0 \quad 0 \quad 1.209],$$

$$V_f = [-1.7932 \quad -3.806 \quad 1.2062].$$

In terms of (10)-(11), the "slow"-subsystem matrices are

$$A_s = \begin{bmatrix} -3.022 & 9.4906 \\ -9.4906 & -3.022 \end{bmatrix}, B_s = \begin{bmatrix} -129.7835 \\ -28.6454 \end{bmatrix},$$

$$C_s = [-1.9210 \quad 18.7394], D_s = [77.943],$$

$$T_s = \begin{bmatrix} 1.0054 & 0 & -0.6831 \\ 0.1756 & -9.8328 & -0.1508 \end{bmatrix},$$

$$V_s = [0 \quad 0 \quad 0.4102].$$

The discretized "fast" subsystem is arranged in the form of (12)-(13) with

$$F_f = [0.3867], G_f = [0.7414],$$

$$H_f = [64.4703], E_f = [229.177],$$

$$T_{df} = [0 \quad 0 \quad 0.0039].$$

The discretized "slow" subsystem (14)-(15) is specified by the following SS matrices

$$F_s = \begin{bmatrix} 0.7974 & 0.3583 \\ -0.3583 & 0.7974 \end{bmatrix}, G_s = \begin{bmatrix} -5.4926 \\ -0.0594 \end{bmatrix},$$

$$H_s = [-1.9210 \quad 18.7394], E_s = [77.943],$$

$$T_{ds} = \begin{bmatrix} 0.0422 & -0.0833 & -0.0289 \\ -0.0014 & -0.3978 & -0.0003 \end{bmatrix}.$$

## 6 Simulation Results

Simulation results for the case of the offered block scheme (see Fig. 1) are given in Figs. 3-4.

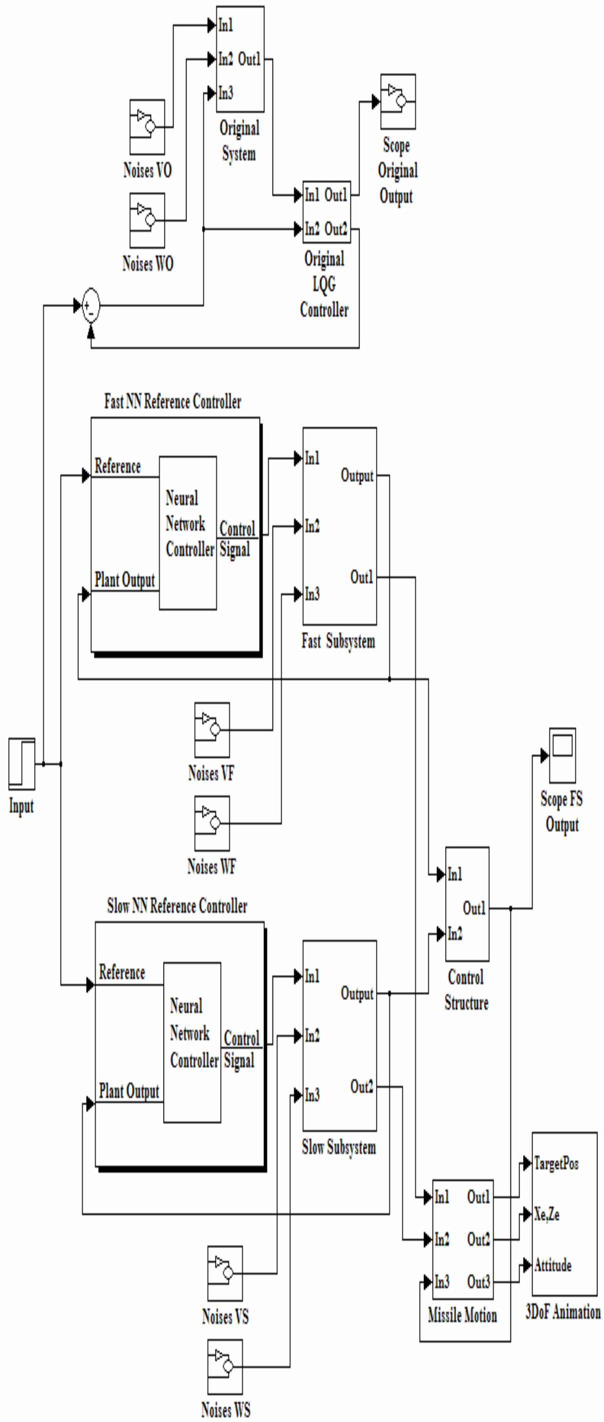


Fig. 1. The internal structures of the original and two-rate control systems.

It can be seen that the real number $N = 8.9$ (where $\Delta_f = 0.005, \Delta_s = 0.0445$ seconds) is chosen as a relation of discretization periods for the subsystems.

The neural model reference control architecture uses two NNs: a controller network and a plant model network, as shown in Fig. 2.

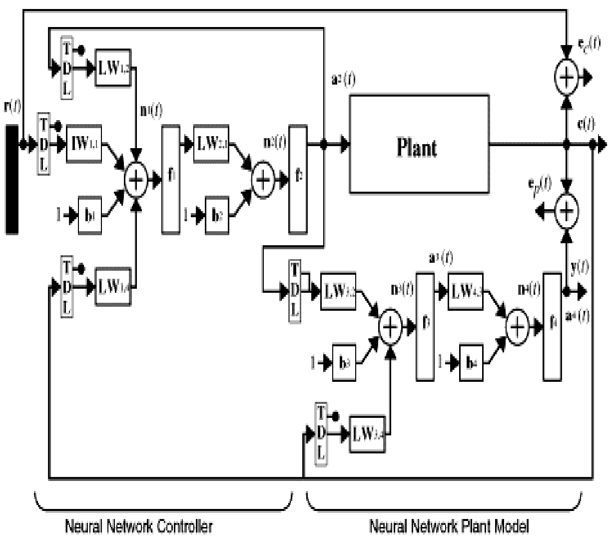


Fig. 2. Block diagrams of the NN controller and NN plant model.

A possible representative view of a rigid guided missile motion by two-rate control system is shown in Fig. 5. The block of animation generates a three dimensional display of a three degrees of freedom trajectory, with the missile's orientation displayed during flight from a viewer position, including a target displayed for this missile.

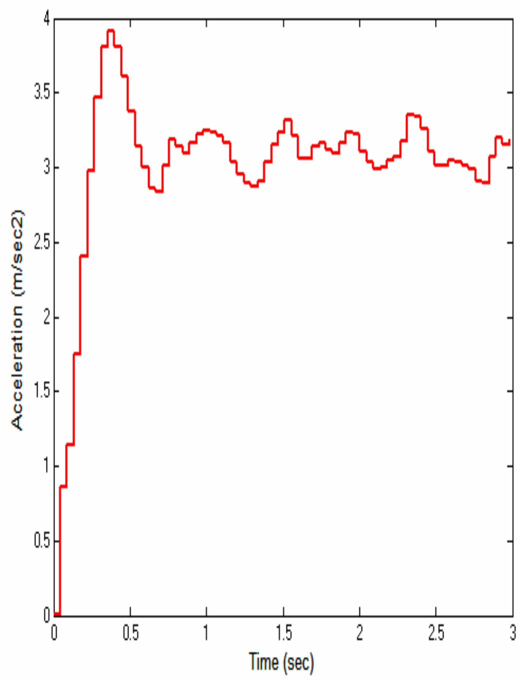


Fig. 3. Response to the output of the original control system.

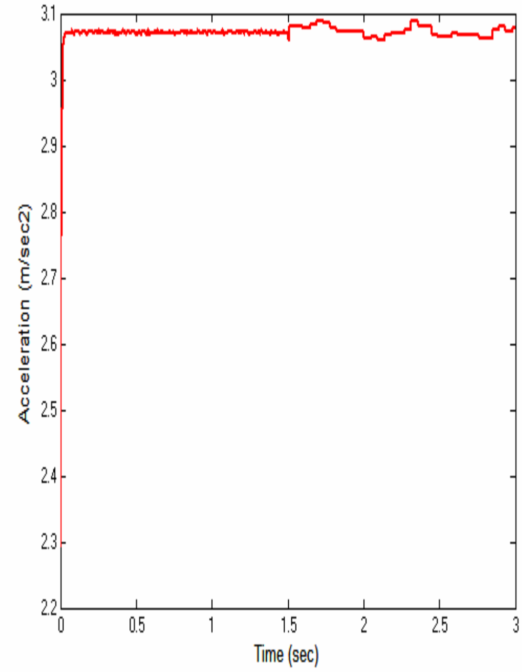


Fig. 4. Response to the output of the two-rate control system.

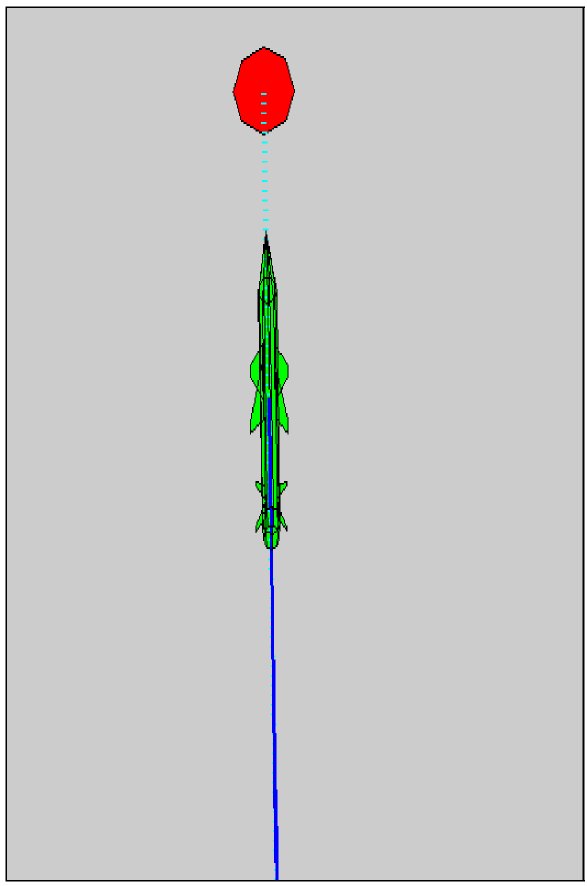

Fig. 5. The visual display of a missile motion.

Note that good results were obtained using only two simple NN reference controllers for the subsystems. These results support the theoretical predictions well and demonstrate that this research technique would work for real systems.

## 7 Conclusions

In this paper a two-rate SS decomposition technique is developed. The obtained autonomous subsystems not only have reduced dimensions of SS matrices and various discretization periods (small and large), but also various speeds of actuation (fast and long response times).

In example of this paper, the original system of third order (the model of a rigid guided missile over certain operating conditions) is partitioned on the "fast" subsystem of first order and on the "slow" subsystem of second order.

The block diagrams of functioning both the original control system and decomposed control subsystems are offered. It is clear that the autonomous subsystems, which are controlled by local NNs, give the best parameters of quality for process of regulation than the original system, which is controlled by one linear-quadratic-Gaussian (LQG) controller.

The discrete-time process simulations with animation were carried out in a MATLAB/Simulink environment. The three dimensional display forms give a researcher an immediate view of a rigid guided missile motion with a range of parameters. This allows us to investigate the sensitivity of the control system, providing a medium for such development and evaluation and enhancing the researcher's understanding of various manoeuvres.

Potential applications are evaluated by control exploration and by computer simulations, using SS type of MIMO continuous-time stochastic model of the original system with multirate phenomena.

In the future, we will extend the proposed control system to other types of noise and different Mach numbers, by modelling the missile motion using the MATLAB/Simulink software.

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