Image Complexity Metrics for Automatic Target Recognizers

Richard Alan Peters II
Department of Electrical
and Computer Engineering
Vanderbilt University
Nashville, TN 37235

Robin N. Strickland
Department of Electrical
and Computer Engineering
University of Arizona
Tucson, AZ 85721

1990 Automatic Target Recognizer System and Technology Conference,
Naval Surface Warfare Center, Silver Spring, MD, 30-31 October 1990.

Abstract

Designers of automatic target recognizers (ATR) need measures of image complexity to compare the performance of different ATRs. An image complexity metric should provide an *a priori* estimate of the difficulty of locating a true target in an image. An ideal image metric is a mapping from the set of all images to a finite real interval. The extrema of the interval indicate extrema in difficulty. The mapping must be monotonic in probability. An ideal image complexity metric is independent of specific ATRs and targets.

This is, of course, an impossible ideal. In the context of ATR design, complexity must be linked to the difficulty of the task. But, tasks that are difficult for one ATR may be easy for another and vice-versa. Therefore, there can be no completely ATR-independent complexity measure. It is possible, however, to define a class of ATRs based on the similarity of the image features they use for detection. Within such a class, it could be possible to define a nearly ideal metric, since the image characteristics which frustrate one ATR present similar difficulties to the others. This metric would be a measure of *image* features. But, its computational definition would derive from the definitive attributes of the ATR class.

In this paper, we review recent ideas about complexity in general, and we review some measures of image complexity in particular. We demonstrate that a significant number of ATR algorithms in the public domain literature share similar computational attributes. We use these attributes to define an ATR class. We then propose an image complexity metric for the class.

1 Introduction

The development of Automatic Target Reconizer (ATR) technology requires objective measures of ATR performance [1, 2, 3, 5, 19]. Some of these measures would relate successful recognition to the characteristics of the input. In particular, it would be useful to know how the probability of detection varies with the complexity of a scene. This requires an *independent* measure of image complexity. (We refer to such a measure as an "ATR image complexity metric", or simply "ATR metric," "image metric," or "complexity metric.")

Although much debated, there is no precise definition of complexity. There is among mathematicians, physicists, and computer scientists a general consensus that the complexity of an object or a system is a measure of the inherent difficulty of performing the tasks associated with it. "Difficulty" is a subjective concept; a precise definition is necessarily limited. In section 2 we review some of the ideas about complexity as a general phenomenon.

Image complexity, in the context of ATR performance, is a measure of the inherent difficulty of finding a true target in a given image. The difficulty of an ATR's task depends not only on its input, but also on the type of information it extracts and its method of extracting it. What is difficult for one ATR may not be for another. Therefore, a measure of image complexity, entirely independent of all ATRs, is not possible. Many image metrics have been proposed, however, because of their potential usefulness. We list and discuss briefly in section 3 as many of these as we have been able to find.

Many of the ATRs in the public-domain literature share common characteristics. In section 4 we show that a number of ATRs select as possible targets, the image regions that have the highest contrast or edge strength. These ATRs form a class that we call *contrast-edge* ATRs.

Since all contrast-edge ATRs perform a similar task, and because that task is quantifiable, it is possible to estimate the inherent difficulty of the task in a given image. Therefore, it is possible to devise a complexity metric that is apparently independent of individual ATRs in the class [26]. We do this in section 5.

In the conclusion, section 6, we discuss some of the limitations of our metric and suggest how it may be extended to other classes of ATRs.

2 Complexity

At first thought, complexity seems like a simple idea. But, it is difficult to define. ("I don't know how to define it, but I know it when I see it.") Grassberger [16] has written a concise, readable introduction to complexity. The ideas in the next two paragraphs are from that work.

Complexity is not the same as simple unpredictability or randomness. A pure gaussian white noise field is both highly random and visually very simple. The digits 3141592653589793... pass tests of randomness with flying colors even though they express a simple ratio of two lengths. Complexity is not the same as information in the sense of Claude Shannon. A message may require a large bandwidth communication channel or a lot of time to transmit which implies that its information content is high. But the message could be very simple in meaning. So, is complexity a measure of meaning? Perhaps, but meaning is, at best, an equally vague term.

Computer scientists have long dealt with the complexity of computation. Computational complexity is related to the length of the shortest program able to perform some computation or the time required to do it. For example, Bennett's logical depth of a string [4] is the time needed to run the shortest program that generates the string. This leads Grassberger to state [16]

> The complexity of an object (pattern, string, machine, algorithm, ...) is the difficulty of the most important task associated with this object .... But complex situations are characterized by not having a single most relevant task associated with them....

If so, then any notion of complexity is task-dependent and "one should not expect a universal notion of complexity applicable to all situations."

Much in the same spirit, Traub and Woźniakowski [34, 38] have defined a paradigm they call "information-based complexity." This is "the study of the intrinsic difficulty of solving problems for which the information is partial, contaminated, and priced" [38]. They distinguish this from "combinatorial complexity" wherein the information is complete, exact, and free. Information-based complexity is characterized by the ϵ-complexity which is the minimal cost required to compute approximate solutions to within ϵ of the true solution. The ϵ-complexity itself can almost never be computed exactly. Instead, the complexity of a problem is defined by upper and lower bounds on the ϵ-complexity.

The ideas of Grassberger, Traub, and Woźniakowski presented here are representative of modern theories of complexity. They indicate that complexity of a phenomenon cannot be divorced from the tasks involved in understanding the phenomenon.

3 Image Complexity

The need for an ATR image complexity metric that is independent of specific ATRs or targets is well documented. (See [1, 2, 5, 3, 19].) An ATR image complexity metric would facilitate the evaluation of different ATRs by setting a standard for comparison. Such a metric must predict the performance of a large class of ATRs on diverse imagery, without advance knowledge of targets. Many ATR metrics have been proposed in the literature. Yet, at this date, none has been shown to meet the above goal. In part, this may be because almost none have directly addressed the computational problems involved in target recognition.

The ATR metrics in the current literature can be classified in terms of their functional dependencies (see Table 1). ATR metrics depend on either global image statistics – those derived from the set of all pixels in the image – or regional statistics – those derived from the individual regions of a segmented image.

Table 1: A Taxonomy of ATR Metrics

	Target Independent		Target Dependent
Gray-level	global	regional	regional
Edge	global	regional	regional
Size/Shape			regional

Table 2: Global Metrics

	Gray-level dependent		Ref.
2.1	Image gray-level standard deviation		[3]
2.2	Image gray-level entropy		[3]
2.3	$U = -\sum_x \sum_y [f(x,y) - \bar{f}(x,y)]^2$	(gray-level uniformity)	[5]
2.4	$\mathrm{SNR}_{IN} = \mid f(x,y) - \mu_{\mathrm{B}} \mid / \sigma_{\mathrm{B}}$	(input signal-to-noise ratio)	[9]
2.5	$c_1 = \frac{1}{Z} \sum_k \frac{N_k Z}{N} \ln \frac{N_k Z}{N}$	(conspicuousness 1)	[37]
2.6	$c_2 = \frac{1}{Z} \sum_k N_k \ln N_k$	(conspicuousness 2)	[37]
2.7	$N'_k = [\frac{1}{8} \sum_k (N_k^{(k)} - N_k)^2]^{1/2}$	(conspicuousness difference)	[37]
2.8	The spread of the main diagonal of the co-occurrence matrix		[17]

	Edge dependent	Ref.
2.9	The number of edges per unit area in the image	[5]
2.10	$I = -\log_2 P$ (P is no. of possible images with given no. of edge points)	[5]
2.11	Pixel-to-neighborhood edge strength ratio	[13]

The statistics are gathered either from the gray-level (light intensity) image directly or from an edge-map of the image. Some ATR metrics depend on *a priori* information about actual targets in an image to be characterized. Most of these must be told the exact pixel sets containing targets. Others need only typical target statistics. Target dependent ATR metrics are necessarily regional. Some depend on the size or shape of regions as well as gray-level or edge information. Target independent ATR metrics may be either global or regional but depend only on gray-level or edge statistics.

In the next four sections, we review some 50 image metrics. Sections 3.1-3.3 concern target-independent metrics and section 3.4 concerns target dependent metrics.

3.1 Global metrics

Global metrics are functions of all the pixels in an image. Of the ones we have found, some are dependent on all pixel values and others are dependent on edge pixels alone. Table 2 lists the global metrics.

3.1.1 Gray-level dependent

If one assumes that an ATR performs best when presented with a highly contrasting target against a uniform, untextured background, then a metric for such qualities might indicate ATR performance. Most of the global gray-level metrics in the literature seem to be designed with that in mind. They are measures of contrast and uniformity. The standard deviation of an image and the entropy of its histogram are two of the simplest measures of the contrast in an image. Bhanu [5] defines the gray-level uniformity, U, to be a metric that is a global average of local gray-level homogeneity.

$$U = -\sum_x \sum_y [f(x,y) - \bar{f}(x,y)]^2 \tag{1}$$

where $f(x,y)$ is the gray-level at pixel (x,y) and $\bar{f}(x,y)$ is the average gray-level in a 3×3 window centered at (x,y).

As ATR algorithms are often sensitive to noise, signal to noise ratios (SNR) have been suggested as metrics. Burton and Benning [9] propose

$$\mathrm{SNR}_{IN} = \mid f(x,y) - \mu_\mathrm{B} \mid / \sigma_\mathrm{B} \tag{2}$$

where $f(x,y)$ is the gray-level at pixel (x,y) and μ_B and σ_B are the mean and standard deviation of a 20×30 neighborhood centered at (x,y) that excludes a target-sized region likewise centered at (x,y).

Winkler and Vattdrodt [37] define two "conspicuousness" metrics which they state are "pure measures of contrast" (p. 364). They define a picture as a collection of N "spots" distributed over Z "cells". The total number of pictures that can be made this way is

$$V = N!/\prod_{k=1}^{Z} N_k \tag{3}$$

where N_k is the number of spots in cell k. Based on this, they define a measure of conspicuousness as

$$c = 1/q \sum_{k=1}^{Z} ln(ZN_k/q) \tag{4}$$

where q is a parameter. Taking $q = N$ yields

$$c_1 = \frac{1}{Z}\sum_{k=1}^{Z} \frac{N_k Z}{N} \ln \frac{N_k Z}{N}. \tag{5}$$

Taking $q = Z$ yields

$$c_2 = \frac{1}{Z}\sum_{k=1}^{Z} N_k \ln N_k. \tag{6}$$

To account for the relative positions of cells, they consider the average difference in cell content between a cell and its 8-neighbors, given by

$$N'_k = [\frac{1}{8}\sum_{k=1}^{Z}(N_k^{(k)} - N_k)^2]^{1/2}. \tag{7}$$

They substitute N'_k for N_k in c_2 and claim the result, c'_2 has the following properties:

1. c'_2 is always nonnegative;
2. c'_2 is zero if and only if the N_k are distributed uniformly;
3. c'_2 is maximum if and only if all dots are concentrated in one cell;
4. c'_2 is invariant under adding a constant number of dots to all cells;
5. c'_2 is invariant under "switching contrast from positive to negative picture" (p.366).

Another global gray-level metric uses the co-occurrence matrix, M. M is an $n \times n$ matrix where n is the number of gray-levels in the image. For any pixel, p, with gray-level i, element m_{ij} of M represents the probability that one of p's 4-neighbors has gray-level j. The main diagonal of M contains the probabilities that a pixel of gray-level i has a 4-neighbor of gray-level i. Therefore, the pixels on and near the main diagonal of the co-occurrence matrix contain information about the gray-level uniformity of the image.

Haller [17] had a group of 24 people rate for complexity a set of 10 aerial photographs of the ground. He showed that the spread of the main diagonal of the co-occurrence matrix matched the human rank ordering of eight of the 10 photographs. Haller's analysis implied that human observers found images with uniform regions of only a few gray-levels to be simpler than images whose uniforms existed at many gray-levels.

Table 3: Region Dependent Metrics

Gray-level dependent			Ref.
3.1	$u_{jG} = A_j \sigma_j^2 / A_G \sigma_{\max}^2$	(region uniformity)	[20]
3.2	$C_{ij} = \lvert\mu_i - \mu_j\rvert / \lvert\mu_i + \mu_j\rvert$	(local contrast)	[20]
3.3	$c_j = \sum_i a_{ij} Cij$	(regional contrast)	[20]
3.4	$c_G = \sum_j v_j c_j / \sum_j v_j$	(global contrast)	[20]
3.5	$R = V + T + W$	(structural entropy)	[15]
Edge dependent			**Ref.**
3.6	$d_j = \lvert F_L - F_R\rvert / \lvert F_L + F_R\rvert$	(line contrast)	[20]
3.7	$h_j = g_j$ if $g_j > 3d_j > 3\epsilon$. otherwise, $h_j = d_j$		[20]
3.8	$H_G = \sum_j w_j h_j / \sum_j w_j$	(regional line contrast)	[20]
3.9	$C(t) = \sum_{\{i \geq t, j < t\}} m_{i,j}$	(busyness of co-occurrence matrix)	[36]

3.1.2 Edge dependent

Any source of thermal energy appears in an infrared image as a bright area. By the second law of thermodynamics, a hot object tends to transfer heat to its cooler surroundings. The quantity of heat transferred to a particular area is a function of the time that the object has been in the area. Consequently, stationary objects, hot or cold, usually have blurred edges in the image. Conversely, a hot moving object will have sharp edges. Since ATRs often look for hot moving objects, many of them are cued to sharp edges. If there is much clutter with sharp edges in an image, an ATR of this type could have a difficult task. Therefore, global edge-dependent metrics measure the amount or intensity of edge activity in an image.

Bhanu [5] states that since targets are usually present in the vicinity of large magnitude edge points, an image can be characterized in terms of the number of edge points whose magnitudes exceed a threshold. Given this, he claims that the number of edges exceeding a threshold per unit area in an image is a reasonable estimator of target like features. This assumes that, in general, highly textured images will present more of a challenge to an ATR than less textured images. Bhanu [5] asserts that the difficulty of an ATR's task is directly related to the amount of variation in an image. There are finite number of images of a particular size that contain a given fixed number of edge points. Let P represent this number. In terms of edge pixels,

$$I = -\log_2 P \tag{8}$$

is the information content of the image; this is a measure of the amount of variation.

A report by ERIM [13] states that a measure of edge clutter is given by the edge strength ratio, the average squared edge-strength as measured by a standard edge operator, normalized to the local background intensity variance. This compares the edge strength at a pixel to the average in a neighborhood.

3.2 Region dependent metrics

The segmentation stage of an ATR partitions the input image into regions. Some of the regions are presumed to be targets; these are examined by the classifier. There have been a number of metrics devised to measure the accuracy of a segmentation. Although most have not been used as ATR metrics, they could be; in fact, we do incorporate functions related to these in our new metric presented in section 5. Table 3 lists region dependent metrics.

3.2.1 Gray-level dependent

Levine and Nazif [20] have introduced a "general purpose performance measurement scheme for image segmentation algorithms" (p. 155). Among the gray-level characteristics the scheme measures are region uniformity, local contrast, region contrast, and global contrast.

Assume region R_j has area A_j and gray-level variance σ_j^2 and is a subset of a larger region, G, that has area A_G. Let $\sigma_{\max}^2$ be one half of the squared difference between the maximum and minimum gray-levels in region G. Then

$$u_{jG} = A_j \sigma_j^2 / A_G \sigma_{\max}^2 \tag{9}$$

is a regional uniformity measure. It is R_j's fraction of the maximum possible variance in G weighted by R_j's proportion of the area of G.

Levine and Nazif use the Weber-Fechner relation to measure the local contrast between adjacent regions R_j and R_i.

$$C_{ij} = |\mu_i - \mu_j| / |\mu_i + \mu_j| \tag{10}$$

where μ_i and μ_j are the mean gray-levels in regions i and j. (The Weber-Fechner relation is a well-known model of human contrast perception.) They define the relative contrast between a region, R_j, and its surrounding neighbors as

$$c_j = \sum_i a_{ij} C_{ij} \tag{11}$$

where the sum is performed over all adjoining regions. a_{ij} is the adjacency of regions R_j and R_i, the fraction of the total outside boundary of R_j occupied by pixels in R_i. The global contrast of area G weights the relative contrasts of all regions in the area by a function, v_j, of region area.

$$c_G = \sum_j v_j c_j / \sum_j v_j \tag{12}$$

where the sum is performed over all regions, R_j, in G. Levine and Nazif derive the weights, v_j, from a curve representing the human perception of apparent contrast as a function of region size.

Gonzalez *et al.* [15] describe an image as a finite collection of regions. Each region has *unary* properties such as average gray-level and *binary* properties such as adjacency to another region. They define the *structural variance* as

$$V = 1 - (S_y^2 / U_y^2) \tag{13}$$

where U_y is the average, and S_y is the variance, of the number of regions per unary property. This value is a maximum when each unary property is held by an equal number of regions. It is at a minimum when all regions have the same property. The *intraset entropy*, T, is defined in terms of disjoint subsets of regions. In this context, a disjoint subset, S_i, of regions is such that any two regions in the subset share a binary property and no region from S_i shares a binary relation with S_j if $i \neq j$. Let N_D be the number of disjoint subsets of regions, let $f_{\mathrm{B}}(S_i)$ be the number of binary relations among the regions in S_i, and let $|S_i|$ be the number of regions in S_i. Then

$$T = \frac{1}{N_D} \sum_{i=1} N_D f_{\mathrm{B}}(S_i) / |S_i|^2 \tag{14}$$

$T = 0$ when there are no binary regions. T is maximum when there is only one disjoint set (i.e. all regions share binary relations). Let $f_U(S_i)$ be the number of unary relations among the regions in S_i. Let $g_U(S_i, S_j) = |f_U(S_i) - f_U(S_j)|$ and let $h_{\mathrm{B}}(S_i, S_j) = |f_{\mathrm{B}}(S_i) - f_{\mathrm{B}}(S_j)|$. Gonzalez *et al.* define the *interset entropy* as

$$W = \frac{1}{N_D} \sum_{i=1} N_D \sum_{j=1} N_D \frac{g_U(S_i, S_j)}{1 + h_{\mathrm{B}}(S_i, S_j)} \tag{15}$$

If all regions have the same number of unary relations, or if all regions share all binary relations, then $W = 0$. The *structural entropy* is

$$R = V + T + W \tag{16}$$

Table 4: Classification Metrics

			Ref.
4.1	The number of target-like objects that are not targets		[5]
4.2	$P_{CD} = N_{TT}/N_{CT}$	(No. true targ. / No. candidate targ.)	[9]
4.3	$\mathrm{SNR}_{OUT} = \mid S_i - \mu_C \mid / \sigma_C$	(output signal-to-noise ratio)	[9]
4.4	$e_{\mathrm{T}} = \frac{1}{MN}[\sum_k \lVert p_{k,mc} - p_{k,cc} \rVert^2]^{1/2}$	(normalized pixel distance error)	[39]

3.2.2 Edge dependent

The segmentation metric of Levine and Nazif [20] includes three line contrast measures. Lines include thin, extended regions and edges between contrasting regions. Let F_{jL} and F_{jR} represent adjacency weighted averages of gray-levels for regions left and right of line j. The line contrast is

$$d_j = |F_L - F_R| / |F_L + F_R| \tag{17}$$

For thin-region lines (as opposed to edges between regions) they define a measure, g_j, of the local gradient divided by the difference between the maximum and minimum gray-levels in the global region containing the line. A hybrid measure of line contrast selects g_j over d_j if d_j is small and g_j is sufficiently larger than d_j. Let

$$h_j = \begin{cases} g_i & \mathrm{if} g_j > 3d_j > 3\epsilon \\ d_i & \text{otherwise} \end{cases} \tag{18}$$

If the region in which the lines exist is G then, a regional measure of line contrast is

$$H_G = \sum_j w_j h_j / \sum_j w_j \tag{19}$$

where the weights, w_j are the lengths of the lines.

Weszka and Rosenfeld [36] use the "busyness" of a co-ocurrence matrix, M. A threshold of the image at t partitions M into 4 rectangular regions. Matrix elements m_{ij} for $i \geq t$ and $j < t$ contain the gray-level co-occurrence probabilites that pixels in the image are on the borders of segments (i.e. on the edges) caused by thresholding. They define the busyness as

$$C(t) = \sum_{\{i \geq t, j < t\}} m_{i,j} \tag{20}$$

. The authors claim:

> If $C(t)$ is relatively high for a given threshold [t] we would expect the thresholded image to contain a large number of noise points and/or jagged edges. Conversely, a relatively low C(t) would indicate that the threshold chosen results in a smooth picture.

3.3 Classification metrics

In the public domain literature, there are few metrics for the evaluation of the classifier stage of an ATR. It could be that most classifiers are proprietary or classified hence unpublished, or it could be that their algorithms are too diverse for uniform characterization. There are, however, a few. All of them are dependent on *a priori* knowledge of the true targets in the image. They are listed in table 4.

Burton and Benning [9] propose a metric based on the input to the classifier. The probability of candidate detection, P_{cd} is the probability that a true target is among the rank-ordered list of target candidates passed out of the detector-segmenter. This is a simple ratio. Let N_{TT} be the number of true targets and let N_{CT} be the number of candidate targets. Then

$$P_{CD} = N_{TT}/N_{CT} \tag{21}$$

In a similar vein, Bhanu [5] suggests that simply the number of target-like objects that are not targets is a measure of the complexity of the input image.

Burton and Benning also define a detector/segmentor output signal to noise ratio.

$$\mathrm{SNR}_{OUT} = \mid S_i - \mu_C \mid / \sigma_C \tag{22}$$

where S_i is the detector's score for a candidate target of rank i and μ_C and σ_C are the mean and standard deviation of the scores of the clutter (non-target) candidates. In their study, Burton and Benning used the top 30 clutter candidates to compute μ_C and σ_C. The target output SNR is a measure of the discernability of the targets relative to the clutter after the detection stage.

Yasnoff *et al.* [39] assume that "the amount of error for a misclassified pixel is related to the distance in the digital image from the misclassified pixel to the nearest pixel that is actually of the misclassified class" (p. 230). They define, e_{T}, the normalized pixel distance error, as the square root of the sum of the squares of the euclidean distances of all misclassified pixels, divided by the number of pixels in the image. For an $M \times N$ image, that is

$$e_{\mathrm{T}} = \frac{1}{MN}[\sum_k \|p_{k,mc} - p_{k,cc}\|^2]^{1/2} \tag{23}$$

Yasnoff *et al.* actually *tested* their metric and found that although e_{T} is a good measure of some types of segmentation errors, it is not especially accurate when meaningful shapes are a factor.

3.4 Target dependent metrics

The majority of ATR image complexity metrics are target dependent. That is, they require explicit information about the location of the true targets in the image. Like the global metrics and region dependent metrics, most target dependent metrics use either gray-level or edge information. In addition to these, however, are metrics based on target size and shape. Table 5 is a list of these.

3.4.1 Gray-level dependent

Of gray-level dependent, known-target metrics there are three kinds: those that measure contrast, those that measure feature distributions and those that compare feature distributions.

Contrast. It seems reasonable to suppose that if a target contrasts highly with its background, it will be easier to find. Consequently, a very simple measure of complexity is the contrast between a target and its immediate background [3].

$$\text{contrast} = |\mu_{\mathrm{T}} - \mu_{\mathrm{B}}| \tag{24}$$

where μ_{T} is the average gray-level of the pixels in the target and μ_{B} is the average gray-level of the pixels adjacent the target.

Lahart *et al.* describe a number of target dependent metrics based on the above assumption. Let μ_{T} and σ_{T} be the mean and standard deviation of the gray-levels inside the minimum covering rectangle of the target. Let μ_{B} and σ_{B} be the mean and standard deviation of the gray-levels inside a rectangular annulus whose inner border coincides with the target rectangle and whose outside dimensions are twice those of the target rectangle. The target interference ratio

$$\mathrm{TIR} = (\mu_{\mathrm{T}} - \mu_{\mathrm{B}})/\sigma_{\mathrm{B}} \tag{25}$$

indicates the separability of a target from its background. Since the metric varies inversely with the background standard deviation, it has smaller values for textured backgrounds. The target-background interference ratio

$$\mathrm{TBIR} = (\mu_{\mathrm{T}} - \mu_{\mathrm{B}})/(\sigma_{\mathrm{T}}\sigma_{\mathrm{B}})^{1/2} \tag{26}$$

favors uniform targets against uniform backgrounds. It varies inversely in the standard deviations of both target and background. (A report by engineers at ERIM [13] favors the squares of these two metrics.) Lahart's new target interference ratio

$$\mathrm{TIR}_{\mathrm{new}} = (\mu_{\mathrm{T}} - \mu_{\mathrm{B}})/(\sigma_{\mathrm{T}} + \sigma_{\mathrm{B}})^{1/2} \tag{27}$$

Table 5: Target Dependent Metrics

Gray-level dependent			Ref.
Contrast			
5.1	$\mathrm{TBC} = \lvert\mu_{\mathrm{T}} - \mu_{\mathrm{B}}\rvert$	(Target to background contrast)	[3]
5.2	$\mathrm{TIR} = (\mu_{\mathrm{T}} - \mu_{\mathrm{B}})/\sigma_{\mathrm{B}}$	(target interference ratio)	[19]
5.3	$\mathrm{TBIR} = (\mu_{\mathrm{T}} - \mu_{\mathrm{B}})/(\sigma_{\mathrm{T}}\sigma_{\mathrm{B}})^{1/2}$	(target background i. r.)	[19]
5.4	$\mathrm{TIR}_{\mathrm{new}} = (\mu_{\mathrm{T}} - \mu_{\mathrm{B}})/(\sigma_{\mathrm{T}} + \sigma_{\mathrm{B}})^{1/2}$	(new target interference ratio)	[19]
5.5	$\mathrm{TIR}^2 = [(\mu_{\mathrm{T}} - \mu_{\mathrm{B}})/\sigma_{\mathrm{B}}]^2$		[13]
5.6	$\mathrm{TBIR}^2 = (\mu_{\mathrm{T}} - \mu_{\mathrm{B}})^2/\sigma_{\mathrm{T}}\sigma_{\mathrm{B}}$		[13]
5.7	$C_A = (s_{\mathrm{crit}}/T_A)B(s_{\mathrm{crit}}, A)$	(Waldman's Clutter Metric)	[35]
Distribution measure			
5.8	Global target prominence (GTP) of target contrast		[3]
5.9	GTP of brightest pixel on target		[3]
5.10	Target entropy		[3]
5.11	Target standard deviation		[3]
Distribution comparison			
5.12	Clutter area		[19]
5.13	$C = \int \cdots \int \min\{f_{\mathrm{T}}(x_1, \ldots, x_n), f_{\mathrm{B}}(x_1, \ldots, x_n)\} \mathrm{d}x_1 \ldots \mathrm{d}x_n$		[10]
5.14	Kolmogorov-Smirnov statistic		[3]
5.15	Target vs. background entropy		[3]

Edge dependent			Ref.
5.16	Average of edge strength over all targets		[3]
5.17	Target edge standard deviation		[3]
5.18	GTP of average target edge strength		[3]
5.19	Target average contour length (ACL)		[3]
5.20	GTP of target ACL		[3]
5.21	Target vs. image ACL		[3]
5.22	χ^2 connectivity		[3]
5.23	$H = -\sum_i \sum_{n=0}^{1} p_i(n) \log p_i(n)$	(edge is target/not-target entropy)	[5]

Shape/size measures		Ref.
5.24	Pixels on target (number of pixels)	[3]
5.25	Largest target size in resolution cells	[19]
5.26	Expected number of resolution cells on target	[13]
5.27	Aspect ratio (target length to width)	[3]
5.28	Target perimeter squared divided by target area	[3]

is similar to the TBIR but is invariant under the addition or multiplication of the image by a constant.

Waldman et al. [35] attempt to create a measure of image clutter "in accord with human intuitive estimates of clutter, being based on the similarity of the background texture to the target in size, shape, and orientation" (p.137). Their metric uses a gray-level cooccurrence matrix for G gray-levels, step size s, and angle A, normalized to form a two-dimensional probability distribution with values $P_{ij}(s, A)$. The spread of the main diagonal is [27]

$$B(s, A) = \sum_{i=0}^{G-1} \sum_{i=0}^{G-1} |i - j| P_{ij}(s, A) \tag{28}$$

The authors claim without proof that $B(s, A)$ increases with the step size until s equals the average texture size and then flattens. Thus, finding s_{crit} the position of the knee on the curve yields the average texture size in the direction A. If T_A is a known target cross-section at angle A, then a measure of clutter is given by

$$C_A = (s_{\text{crit}}/T_A) B(s_{\text{crit}}, A) \tag{29}$$

Distribution Measure. An image feature, such as gray-level, local contrast, or edge strength, that can be measured at each pixel, can be thought of as a random variable. Then a frequency distribution of the feature over the image can be treated as a probability distribution. There are a number of metrics based on probability distributions of this type.

Beard *et al.* [3] describe some metrics that use *global target prominence* (GTP). The GTP of a feature is the probability that the background's feature content is less than that of the target. That is, the GTP is the integral of the feature's probability distribution (in the image) up to the value of the feature possessed by the target. Or in other words, if all values of the feature greater than the target's value define the tail of the distribution, then the GTP is one minus the area under the tail. Two GTP measures based on gray-levels are the GTP of target contrast and the GTP of the brightest pixel on target. Beard *et al.* also claim that the entropy of the target (presumably the entropy of the probability distribution defined by the gray-level histogram of the pixels in the target region) is "a measure of image variation on target ... [that] should be quite sensitive to target signature modality" (p. 9). They also claim that the standard deviation of the target is complementary to the entropy.

Distribution Comparison. These metrics compare the probability distributions of features in the target to those of features in the background. Lahart *et al.* use the target interference ratio (TIR) to define the clutter area as the probability that the TIR measurement of some part of the background exceeds the median target TIR. Carlson *et al.* [10] observe that for a statistical feature-based target recognizer to work well, there must be measurable differences between the feature distributions of the target areas and the background areas. The authors posit that the extent to which the distributions are *not* different, determines the complexity of the task. In other words, the area of overlap in the target and background feature distributions is a probabilistic measure of the complexity. Then, the complexity is

$$C = \int \cdots \int \min\{f_{\text{T}}(x_1, \ldots, x_n), f_{\text{B}}(x_1, \ldots, x_n)\} \mathrm{d}x_1 \ldots \mathrm{d}x_n \tag{30}$$

where f_{T} and f_{B} are the target and background distributions, respectively. Beard *et al.* [3] suggest the use of a Kolmogorov-Smirnov statistic since it is precisely a measure of the difference between two distributions. They also suggest the ratio of target entropy to background entropy since "high entropy targets should be easily distinguishable from low entropy backgrounds" (p. 8).

3.4.2 Edge dependent

Simple measures of target edge strength such as the average edge strength over all targets and the target edge strength standard deviation have been listed as metrics by Beard *et al.* [3]. The global target prominence (see sec. 3.4.1) of the average target edge strength "quantifies the relative amount of information contained in the target region as compared to the rest of the image" (p.7).

Beard *et al.* define a statistic called the average contour length (ACL) which is a measure of the connectivity of edge pixels. The target ACL is a measure of edge information in known targets. The GTP of the

ACL is an alternative to the GTP of the average edge strength that is dependent on edge direction as well as magnitude. The target versus image ACL is a metric that

> is much less sensitive to local structure [than the GTP of target ACL] but still generates an estimate of target prominence while avoiding the difficulty of arbitrarily 'slicing up' the image into subregions which may violate the continuity of local contours (p. 8).

Some target dependent image metrics compare the edge characteristics of known targets to those of the background. Beard *et al.* state that the χ^2 connectivity "is a straight-forward test of similarity in edge content between the target and background regions" (p. 9). The χ^2 connectivity is a test of the hypothesis that the true proportion of i-connected edge pixels in the target region is p versus the probability that it is different from p. Bhanu [5] assumes that the difficulty of an ATR's task is directly related to the amount of variation in the image. Let $p_i(0)$ be the probability that edge point i belongs to a target. Then the probability that edge point i belongs to the background is $p(1) = 1 - p_i(0)$. Bhanu claims that the entropy

$$H = -\sum_i \sum_{n=0}^{1} p_i(n) \log p_i(n) \tag{31}$$

of the distribution of $p_i(n)$ over all edge points is a measure of the image variation.

3.4.3 Shape/size measures

Some of the target dependent metrics depend on the shape or size of targets and non-targets in the image. The simplest size measures are the number of pixels on target (more precisely, pixels *in the* target), or the average number of pixels on target, or the size in pixels of the largest target. The simplest shape measure is the aspect ratio, i.e., the ratio of target height (or length) to width. Another is the square of the number of pixels in the boundary of the target divided by the number of pixels on target. Each of these, if small, presumably would indicate a harder job for the ATR.

Some shape and size metrics make use of *resolution cells* rather than pixels. A resolution cell in an image is essentially the smallest set of pixels that will cover the pointspread function of the image. The pointspread function is a linear filter that approximates any non-motion blur that was introduced into the true image by the optical system or the sensors that produced the image. Thus, the size of a resolution cell is always greater than or equal to the size of a pixel. All of the above metrics could be calculated in terms of resolution cells rather than pixels.

4 Contrast-Edge Automatic Target Recognizers

An image complexity metric for predicting the performance of more than one ATR on an arbitrary image would not be possible if there were not similarities among ATRs. In the course of this research we found descriptions in the public domain literature of 21 ATR algorithms. These 21 algorithms shared common characteristics that made the development of a metric possible.

Most of the ATRs in our sample used expected target size and contrast or edge information to detect potential targets. Many locate all the compact regions of appropriate size which are mostly brighter than their immediate surroundings. Others, rather than using contrast explicitly, distinguish between regions based on their gray-level uniformity. Such schemes use contrast implicitly as the region delimiter. Some detectors look for edges which encircle blob-like regions. Others search for coincidence between edge locations and the boundaries of contrasting regions.

Contrast dependent ATRs include the contrast boxes of McWilliams and Srinath [22], Dudani *et al.* [12], and those of Texas Instruments, Ford Aerospace, and Westinghouse described by Schachter [29]. The pyramid techniques described by Dawson and Treese [11], Schneier [30], Burt *et al.* [8], and Hartley *et al.* [18] all depend on contrast for detection. Narayanan and Rosenfeld [25] devised the "superspike" algorithm, used by Hartley [18] in an ATR, to segment images based on region uniformity. Goehrig and Ledford [14] employ adaptive gray-level threshold segmentors in their ATR. Rubin and Tseng [28] use a linear combination of gray-level statistics from concentric moving windows.

Edge dependent ATRs include the *spoke filter* of Minor and Sklansky [24] and Milgram's *superslice* algorithm [23]. Bhanu and Holben [6] use edge-controlled relaxation segmentors in ATRs. Bhanu's [7] *intelligent automatic target cuing* system looks for target-sized concentrations of edges in the input. Soland and Narenda's PATS ATR [32] looks for hot-spots that coincide with edges.

In our analysis, we found that many of the ATRs in the public-domain literature share three characteristics:

- They operate on infrared imagery.
- They look in the image for bright compact areas with sharp edges.
- Their algorithms employ detection, segmentation, and classification as logically separate steps in a three stage process.

Moreover, there is similarity in the effective results of detection and segmentation by ATRs of this type. These first two steps partition an image and compute for each region in the partition an estimate of the probability that the region is a target. This estimate is often a function of one or both of two local image characteristics: (1) the strength of the luminance edges in the region, or (2) the contrast of region with respect to the surrounding regions. Only regions with probabilities exceeding a threshold are examined by the classifier stage. ATRs with these characteristics form a class that we will refer to as *contrast-edge* ATRs.

5 A Image Complexity Metric for Contrast-Edge ATRs

Many researchers of complex systems hold the opinion that complexity is neither simple unpredictability nor pure information content. The complexity of a system is, instead, the difficulty of the tasks associated with the system. Precisely what "difficulty" means depends on the task. It could be the shortest time required to reach a solution, it could be the length of the shortest algorithm required to reproduce the information, or it could be the minimum amount of energy necessary to produce a result.

5.1 The measurement of difficulty

To devise a measure of image complexity for the characterization of ATR performance, one must analyze the task involved. For the ATRs that we studied, the task is to divide the image into regions and to decide which of the regions is a target. To make this decision, the ATR rates each region according to its similarity to a target. The ATR chooses for its target, the most highly rated region – the region with the greatest *target-similarity.* Typically, the ranking occurs in two stages. The detector/segmentor makes a preliminary selection of candidates. Then the classifier chooses one of the candidates as its target.

If a class of ATRs use common image features and if each member of the class extracts the same set of features, the same way for every image, then the complexity due to feature extraction is constant (i.e. the same for every image). The difficulty of the ATR's ultimate task is an increasing function of the number of target-like regions in an image, since each of these must be analyzed by the classifier and it is the number of target-like regions that will differ from image to image. If the classifier's task is the same for every region it examines, then the complexity of an image is proportional to the number of target-similar regions it contains. If the classifier's task is not the same for each region but instead, is a function of the region's target-similarity measure, then the complexity is a function of the distribution of the measures of the most highly target-similar regions. The task will be more difficult if the target-similarity values of all the candidates are close to one another.

The complexity is evident from the tail of the region target-similarity distribution. If the tail is long and thin, the task is relatively simple; if it is short and fat, the task is more difficult. A complexity metric for ATRs from this class will segment an image into regions that are as target-like as possible, compute the target-similarity of each region, and then measure the tail of the distribution of target similarity values.

5.2 A new metric

With these ideas in mind, we have devised an image complexity metric for the class of contrast-edge automatic target recognizers. These simple ATRs look for compact regions of relatively uniform gray-level that are highly discernible through gray-level contrast or strong edges. Our complexity measurement algorithm segments an image into target-like regions, measures the contrast and edge strength of each region, combines the measurements into a target-similarity score, creates a score distribution, and measures its tail.

In his 1988 Ph.D. dissertation, Peters [26] describes an image segmentation procedure that partitions an image into compact regions of uniform gray-level, no larger than the expected target size. The segmentation is a split-and-merge algorithm. Recursive thresholding determines the splits. Following each split, a morphological operation determine the merges. The operation is a mask-guided, binary close-open transform with a target-sized, disk structuring element [31]. Peters chose these operations because they mimic the effective action of many contrast-edge ATRs.

5.3 Target similarity

After we apply Peters's segmentation procedure, we estimate the target similarity of each region r_i, where $i = 1, \ldots, N$, the number of regions in the image. We define this as the length of a vector, $\vec{v}_i = (c_i, e_i)$, where component c_i is the relative contrast of r_i with respect to its neighbors, and component e_i is the average edge strength of r_i.

To measure c_i, we use the metric proposed by Levine and Nazif [20] with the following modification: We omit the Weber-Fechner relation from our metric since it mimics human perception and does not apply here. The relative contrast is the sum of the magnitude of the the difference in average gray-level between region r_i and its neighbors, $\{r_j \mid j = 1, \ldots, n_i\}$, weighted by the adjacency of r_i and r_j. That is,

$$c_i = \sum_{j=1}^{n_i} \frac{B(r_i, r_j)}{B(r_i)} \frac{\mid G(r_i) - G(r_j) \mid}{G_{\max}} \tag{32}$$

where n_i is the number of regions, r_j, adjacent to region r_i in a four connected grid topology; $B(r_i, r_j)$ is the number of pixels in r_j that adjoin r_i; $B(r_i) = \sum_{j=1}^{n_i} B(r_i, r_j)$ is the number of pixels in the outside boundary of r_j; $G(r_i)$ is the mean gray-level of r_i; and $G_{\max}$ is the maximum gray-level difference possible in the image. This definition causes a large gray-level difference at a few pixels to be equivalent to a lesser difference over more pixels. For each region, c_i is always less than one with equality for a region that is completely surrounded by another and whose gray-level difference with the surrounding region is as large as possible.

We need an edge measure to complement our contrast measure because it is possible for an image to have high contrast, yet have no highly discernible objects. High contrast is the result of large differences in average gray-level between regions. If these regions merge into one another gradually over relatively large distances, it is difficult to tell where one region ends and the other begins; they are indistinct. In this situation, the edges have low intensity and an edge-based ATR presumably would have trouble identifying objects.

The regions isolated by Peters's morphological split-merge segmenter have relatively uniform gray-level. It is likely that there will be edges near the border of regions since the presence of a region boundary indicates a local change in average image brightness. If there is an edge near the boundary, its intensity is a measure of the abruptness of the change. Thus, the average intensity of the edges closest to the boundary is a measure of the distinctness of the region.

To compute the average edge strength, e_i of region r_i we first create an edgemap for the entire image. (We have used a Marr-Hildreth [21] edge detector because of its well-known properties including relative insensitivity to noise.) We compute the average edge strength, e_i, of r_i by finding the edge pixel closest (within limits) to each inside boundary pixel of r_i, summing their intensities and dividing by the length of the inside boundary of r_i [26].

The length, $v(r_i)$, of vector $\vec{v}_i$

$$v(r_i) = \|\vec{v}_i\| = \sqrt{c_i^2 + e_i^2} \tag{33}$$

is a measure of the relative discernibility of region r_i with respect to its adjacent neighbors. Since the contrast-edge ATRs select regions based on their contrast or edge-strength, we call $v(r)$ the *target-similarity* of region r. We consider $v(r)$ as a positive, real random variable. We assume that the $v(r_i)$ are identically distributed over i.

Let I represent an image and let $\{r_i\}_{i=1}^N$ be the set of all regions in the image. This set partitions I. That is,

$$I = \bigcup_{i=1}^{N} r_i \quad \text{and} \quad r_i \cap r_j = \emptyset \text{ if } i \neq j \tag{34}$$

Let z be a real number and let $\mathcal{S}_z$ be the set of all regions in I with target-similarity no greater than z. That is,

$$\mathcal{S}_z = \{r_i \in I \mid v(r_i) \leq z\} \tag{35}$$

We define the probability that some region has a target-similarity less than z by

$$P_v(z) = \text{Prob}\{v \leq z\} = \frac{1}{N} \times \text{ number of regions in } \mathcal{S}_z \tag{36}$$

Let $P_v(z^+)$ and $P_v(z^-)$ represent the limits of $P_v(x)$ as x approaches z from above and below, respectively. Then, with the assumption that the $v(r_i)$ are identically distributed,

$$p_v(z) = \text{Prob}\{v = z\} = P_v(z^+) - P_v(z^-) \tag{37}$$

is the probability distribution of $v(r_i)$.

Let $v_{\max}$ be the largest target-similarity measure of a region from image I. Let $\mathcal{L}_z$ be the rank-ordered complement of $\mathcal{S}_z$ in I. That is,

$$\mathcal{L}_z = \{r_i \in I,\ i = 1, \ldots, k \mid v(r_i) > z,\ v(r_1) = v_{\max}, \text{ and } v(r_i) \leq v(r_j) \text{ for } i > j\} \tag{38}$$

Then $\mathcal{L}_z$ is the set of k regions in I whose target-similarity exceeds z. If $\mathcal{L}_z$ contains only a few regions when $z \ll v_{\max}$ and if $v(r_j) \ll v(r_i)$ for all $v(r_i) \in \mathcal{L}_z$ and $v(r_j) \notin \mathcal{L}_z$, then I is a relatively simple image. In this case, the distribution indicates that I has a few regions of target size with high contrast and sharp edges against a background of low-contrast or soft-edged regions. If, on the other hand, there are many regions with target-similarity near maximum, then there are many target-sized regions with similar contrast and edge strength; the image is relatively complex. Therefore, it is the tail of $p_v(z)$ that shows the complexity of the image. Possible metrics include

$$m_1(z) = 1/(\text{the number of regions in } \mathcal{L}_z) \tag{39}$$

for $z < v_{\max}$. Metric $m_1(z)$ is simply the inverse of the number of regions whose target-similarity exceeds z. Let

$$m_2(p) = 1 - \frac{v_p}{v_{\max}} \tag{40}$$

where v_p is the pth percentile $v(r)$ for regions in I. Metric m_2 is a measure of the spread of the tail or the distance from the body of the distribution to the most significant outlier. Another metric is the weighted average of the distances to all outliers

$$m_3(p) = \sum_{i=1}^{k} \alpha_i (v(r_i) - v_p) \text{ for all } r_i \in \mathcal{L}_{v_p} \tag{41}$$

where $\{\alpha_i\}_{i=1}^k$ are a set of weights such that $\alpha_i \leq \alpha_j$ for $i > j$.

These metrics depend implicitly on the image segmentation. In particular they indicate complexity on one scale: the size of the structuring element used for the morphological segmentation procedure. Image complexity must be a function of scale. Therefore, if the size of true targets is not known a priori, then the segmentation must be performed for a number of scales and the metric applied to each. Peters's segmentation procedure computes smaller segmentation maps from the larger ones. So the computational complexity of the segmentation does not increase with finer resolutions.

6 Conclusion

Beard *et al.* [3] employ five criteria for the selection of ATR metrics. A metric must be:

1. monotonically related to ATR task difficulty
2. significant for ATR performance
3. descriptive of scene parametric variation
4. algorithmically uncomplicated
5. relatively easy to implement

The first criterion links complexity to the inherent difficulty of the task. In light of the discussion of complexity in section 2, criteria 2 and 3 appear not to be independent of the first criterion; any image metric that meets 1 must meet 2 and 3. The final two criteria are ideals. However, we find it unlikely that a metric that satisfies criterion 1 could be any simpler algorithmically than the class of ATRs it is designed for.

In the current literature, there is no complexity metric that meets Beard's criteria. This is not due, in our opinion, to the strictness of the criteria nor to any deficiency among them. Rather, it is the result of trying to produce metrics that are independent of all ATRs. A metric that is completely independent of the computational task cannot be a complexity metric.

We have proposed three metrics that should indicate the amount of work a classifier must perform to recognize a target from among the list of candidates proposed by the detector/segmentor stages of a contrast-edge ATR. Our metrics are measures of the target-similarity distribution of regions in a segmented image.

Using contrast and edge information alone, these metrics will not predict the performance of ATRs that use other information. We believe, however, that the idea of measuring a region target-similarity distribution could be a universal ATR performance predictor. For ATRs other than the simple contrast-edge types, if one can partition an image so that each region exhibits the features needed by the ATRs, then one can create a target similarity distribution and apply the above metrics. The three metrics we have proposed simply indicate how much information there is in an given image that will have to be analyzed by the classifier and they indicate how distinct the information is. These metrics were designed to be independent of a specific ATR and therefore may not be an accurate estimators of a specific ATRs response to different images.

We cannot be sure that these metrics are useful until they are tested extensively. That remains to be done. Moreover, we do not know if the metrics are robust in the segmentation. That is, if slight changes the segmentation greatly affect the metric then the metrics may not be very useful. We intend to test the metrics on a large number of images and compare the results of using different segmentations.

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