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Maruyama, T. and Sumalee, A. (2007) Efficiency and equity comparison of cordon- and area-based road pricing schemes using a trip-chain equilibrium model, *Transportation Research Part A*, 41(7), pp. 655-671. doi:10.1016/j.tra.2006.06.002.

# Efficiency and Equity Comparison of Cordon and Area based Road Pricing Schemes Using a Trip-chain Equilibrium Model

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## Abstract

This paper compares performances of cordon and area road pricing regimes on their social welfare benefit and equity impact. The key difference between the two systems is that the cordon charges travellers per crossing whereas the area scheme charges the travellers for an entry permit (e.g. per day). For the area licensing scheme, travellers may decide to pay or not to pay the toll depending on the proportion between their travel costs for the whole trip-chains during a valid period of the area license and the toll level. A static trip-chain equilibrium based model is adopted in the paper to provide a better evaluation of the area-based tolls on trip-chain demands. The paper proposes a modified Gini coefficient taking in account assumptions of revenue re-distribution to measure the spatial equity impact. The model is tested with the case study of the Utsunomiya city in Japan. The results demonstrate a higher level of optimal tolls and social welfare benefits of the area-based schemes compared to those of the cordon-based schemes. Different sizes of the charging boundary have significant influences on the scheme benefits. The tests also show an interesting result on the non-effect of the boundary design (for both charging types) on their equity impacts. However, when comparing between charging regimes it is clear that the area schemes generate more inequitable results.

Key words: road pricing, cordon pricing, area licensing scheme, trip-chain model, equity impact

## 1. Introduction

Road pricing or congestion charging has been proposed as one of the most effective policies to curb down traffic congestion in cities (Ministry of Transport 1964). By internalising externalities of congestion to travellers in a form of tolls, it is believed that additional social welfare improvement can be gained (Vickrey 1955). The idea has been gradually becoming a real practice throughout the world (May and Sumalee 2003). In practice, the tolls can be imposed upon travellers per crossing of designated toll points or cordon line (cordon-based scheme), per time or distance of travel, or per day as similar to area entry charge (area-based scheme). Theoretically, it is still uncertain on the type of charging regime which performs best. May and Milne (2000) compared performances of cordon-based, distance-based, time-based, and delay-based charging schemes using a static assignment model and concluded that the delay-based charging scheme performed best in reducing the congestion level in the network whereas the cordon-based scheme was the least effective. They did not, however, consider the case of area-based scheme nor discuss the impact on social welfare and equity of different charging systems.

Despite the possible lower benefit of the cordon-based scheme, the cordon-based system has been the most widely studied charging regime in literature. This is possibly due to the high potential of this system to be actually implemented in practice (May *et al* 2002). It is also relatively simple to apply this charging system into transport models (in most modelling studies the tolls were generally treated as additional delays converted by the concept of value of time imposed on the drivers travelling on tolled roads) (Milne 1997). From the fruitful research into the cordon-based pricing, several modelling studies paid attentions on the evaluation of performances of the scheme with different designs (May *et al* 2002; Santos 2004; de Palma *et al* 2005), equity impact of the scheme (Giuliano 1992; Yang and Zhang 2002; Sumalee 2003; Santos and Rojey 2004), optimal toll level (McDonald 1995; Verhoef 2002; Shepherd and Sumalee 2004; Sumalee *et al* 2006), and optimal design of the scheme (Mun *et al* 2003; Sumalee 2004a; Zhang and Yang 2004; Ho *et al* 2005).

However, apart from the current scheme in Singapore and a relatively small scheme in Durham, UK, there has not been any real world implementation of an urban cordon-based road pricing system. Furthermore, several so called 'charging cordon' schemes and proposals may in practice operate differently from the original concept of the cordon system (charged per crossing). For instance, the Norwegian toll ring schemes in fact allow substantial use of multi-trip passes or the failed Edinburgh double cordon scheme proposed to charge only for the first cordon crossed on each day. Thus simply modelling these schemes as typical cordon schemes may not well represent the impact of the tolls.

Interestingly, the area-based charging scheme which is equally practical to implement as the cordon-based scheme has not received much attention from the studies both from theoretical and practical perspectives. The area-based scheme was actually the first one to be implemented in the original Singapore Area Licensing Scheme (ALS) (Holland and Watson 1978) and is the basis of the current congestion charging system in London. The area-based scheme operates differently from the cordon-based in which the travellers are charged for a permit to enter a designated area for a certain period (typically one day). After charged, the drivers can travel inside and enter freely to the charged area without additional charge. In modelling terms, this is a path cost which is not equal to a linear addition of all relevant link costs (Gabriel and Bernstein 2000). Thus, analysing the performance and effect of the area-based charging scheme using a transport model is not as straightforward as the case of the cordon-based system.

In addition, the effect on travel behaviour between the area-based and cordon-based schemes can be rather different due to the different level of charges imposed upon a traveller. In particular, if one considers a simple return journey by a car between work and home (assuming the trip crosses the charges zone boundary only once on each leg) the toll imposed on the trip will be twice as high in the case of cordon-based scheme compared to that of the area-based system. In particular, the longer the path the lower the proportion between the toll imposed on the traveller and the total travel cost/time of the path which may result in a different level of demand suppression under the area-based scheme as compared to the cordon-based one. In a more complex framework, travellers may even decide to re-schedule their overall trips over a certain period (e.g. week) to a single path (e.g. in a day) to maximise the net utilities (benefit of accessing the activities net the tolls paid) of that path.

Therefore, despite having similar topological structures the wide range of research results on the cordon-based system may not be transferable to the case of an area-based scheme. It is also of interest from policy perspective to compare the performances of these two systems.

Thus far, none of the studies have addressed this issue. This is indeed the main objective of this paper in which we aim to compare the performances in terms of social welfare improvement and equity impact of the cordon-based and area-based road pricing systems with different toll levels and sizes of the charged area.

In order to analyse the impact of the area-based pricing scheme, the modelling framework must be capable of evaluating the actual proportion between the actual cost normally experienced by the traveller and the additional tolls imposed upon him or her. To this end, for the case of an area-based pricing scheme there is a need to analyse the whole path of a traveller in which that path is associated with a certain level of toll which is not related to the number of times (s)he crossing the boundary of the charged area. In path models, travel demand should be associated with trip-chain, i.e. activity-based demand.

There has been a fast growing list of literature on the activity-based model in which it is widely agreed that transport demand is a derived demand from a need or desire to access certain activities (see Kitamura 1984; Golob 2000; Kockelman 2001; Zhang *et al* 2005). In network modelling, Recker (1995, 2001) proposed a general framework of time-space model with a maximising utility decision of a household or individual on their activities and path under different time-window or vehicle access constraints. However, this framework does not integrate the actual decision on the travel path nor the flow dependent congestion condition. Lam and Yin (2001) proposed an integrated route choice and sequential activity choice under the congested condition. The model is framed as a dynamic user equilibrium model. The representation of path in their formulation, however, is not related to the predefined set of necessary activities. It is also practically cumbersome to solve such a fully dynamic model for a realistic case.

The approach adopted in this paper following Maruyama and Harata (2006) is based on a static model with an explicit representation of trip-chains. The model is developed based on the well-known Wardrop's user equilibrium principle (Wardrop 1952) and can be formulated as an equivalent optimisation problem, and hence enables its application to a large scale network. In this model, the demand is defined following different patterns of trip-chains defined by ordered sets of activities. The travellers in this model will then choose the path (route choice) so as to minimise their dis-utilities of travel by ensuring that the trip visits all activities nodes defined *as priori*. The cordon-based and area-based road pricing schemes can be applied directly into this modelling framework in which for the area-based scheme a path passing a part of a charged area will be tolled only once (whereas the cordon-based scheme still charge users per crossing). This equilibrium based model will be used in this paper to analyse the performances of the cordon-based and area-based road pricing schemes in terms of their social welfare benefit and equity impact.

This paper is organised into five further sections. The next section introduces the static path equilibrium based model, and then the third section explains the solution algorithm for solving this model. Section four discusses formulations of social welfare and equity impact which will be then used as measures to compare the performances of the cordon-based and area-based schemes in section five. The case study in section five is the case of Utsunomiya city in Japan. The final section concludes the paper and discusses future research.

## **2. Trip-chain equilibrium based model formulation**

Consider a directed graph  $G(A, N)$  representing a traffic network with a set of links  $a \in A$  and nodes  $n \in N$ . A subset of nodes  $R \subset N$  defines the origin of a trip. Each trip in this network is associated with a trip-chain. The trip-chain,  $h \in H$ , is defined by an ordered set of nodes representing activities, i.e.  $\Omega_h = \{n_1, \dots, n_k\}$ , in which  $n_1 \in R$  is the origin node of this trip-chain. Let  $d_h$  and  $D_h(C_h)$  denote the demand and the demand function for trip-chain  $h$  respectively where  $C_h$  is the trip-chain generalised travel cost which will be defined later on. Each link in the network is associated with a travel time function,  $t_a(v_a)$ , where  $v_a$  is the total link flow. Figure 1 shows two different patterns of trip-chains, i.e. home  $\rightarrow$  work  $\rightarrow$  home, and home  $\rightarrow$  work  $\rightarrow$  shopping  $\rightarrow$  home.

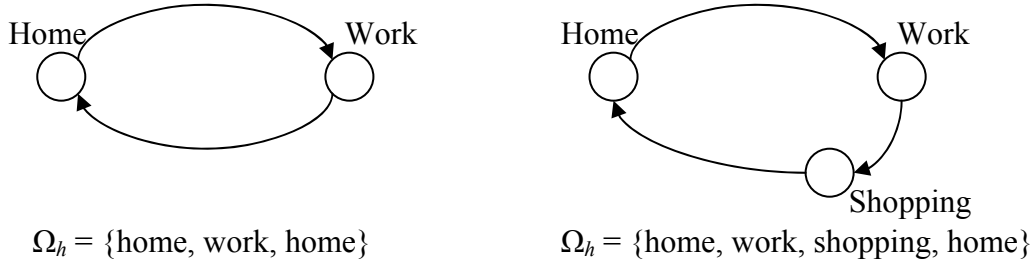


Figure 1: Illustration of trip-chains defined by a series of activity nodes

For each consecutive ordered activity node in  $\Omega_h$ , let  $p \in \Psi(n_i, n_{i+1})$  be a path connecting between node  $n_i$  and  $n_{i+1}$  (i.e. consider node  $n_i$  and  $n_{i+1}$  as a pair of origin and destination nodes) and  $\Psi(n_i, n_{i+1})$  is the set of all possible paths between node  $n_i$  and  $n_{i+1}$ . Thus, a path  $l$  for trip-chain  $h$  can be defined as a possible combination between different possible paths between each consecutive ordered nodes:  $\theta_h^l = \{p \in \Psi(n_1, n_2), \dots, p \in \Psi(n_{k-1}, n_k)\}$ , where  $k = |\Omega_h|$  and let  $\Theta_h$  be the set of all possible paths for trip-chains. The number of possible paths for each trip-chain is  $\prod_{1 \leq i \leq k-1} |\Psi_{n_i, n_{i+1}}|$ . Figure 2 illustrates the concept of a path for a trip-chain.

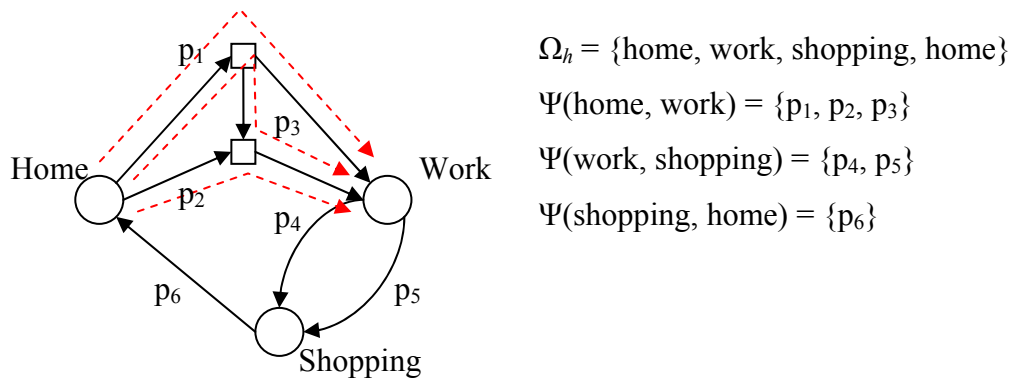


Figure 2: Illustration of a path for a trip-chain

From this figure, the trip-chain of interest is from home to work, work to shopping, and then return home. The possible paths between home to work are paths 1, 2, and 3. The possible paths between work and shopping are paths 4 and 5, and for shopping and home is path 6. Thus, in this example there are totally six possible paths for this trip chain which are paths  $p_1 \rightarrow p_4 \rightarrow p_6$ ,  $p_1 \rightarrow p_5 \rightarrow p_6$ ,  $p_2 \rightarrow p_4 \rightarrow p_6$ ,  $p_2 \rightarrow p_5 \rightarrow p_6$ ,  $p_3 \rightarrow p_4 \rightarrow p_6$ , and  $p_3 \rightarrow p_5 \rightarrow p_6$ .

For path  $p$ , if link  $a$  is related to path  $p$ , then  $\delta_{a,p}$  is 1 and 0 otherwise. The travel cost of path  $p$  can then be defined as  $C_p = \sum_{\forall a} \delta_{a,p} t_a(v_a)$ , where  $v_a = \sum_{\forall k} \delta_{a,k} f_k$  in which  $f_k$  is the path flow on path  $k$ .  $g_h^l$  is defined to be the flow on path  $l$  related to trip-chain  $h$ . Note that for all  $p \in \theta_h^l$  we must have  $f_p = g_h^l$ . Then for path  $l$ , we can define the path travel time as:  $C_h^l = \sum_{\forall p \in \theta_h^l} C_p$ .

Let now consider a pattern of a toll charging regime. For each toll pattern, let  $\tau_h^l$  define the amount of tolls imposed upon path  $l$  associated with trip-chain  $h$ . Under the cordon-based scheme,  $\tau_h^l$  is simply equal to  $\sum_{p \in \theta_h^l} \sum_{\forall a} \delta_{a,p} \tau_a$ , where  $\tau_a$  is the toll per crossing on link  $a$ . This is

indeed the case of the path cost with additive-link costs. For the case with a simple area-based scheme which imposes a uniform toll level of  $\tau$  on all trips accessing the charged area, the tolls imposed on the path  $l$  can be defined such that  $\tau_h^l = \tau$  if  $\sum_{p \in \theta_h^l} \sum_{\forall a} \delta_{a,p} \varepsilon_a > 0$  and 0

otherwise, where  $\varepsilon_a = 1$  if link  $a$  is in the charged area and 0 otherwise. It is also possible to calculate the toll levels for each path for a more complex area-based pricing system. Thus the total generalised travel costs on path  $l$  can be defined as  $C_h^l = \sum_{\forall p \in \theta_h^l} C_p + \tau_h^l$ .

The traveller in this model is assumed to choose the path following the Wardrop's user equilibrium principle which can be defined as:

$$\begin{aligned} g_h^l > 0 &\Rightarrow C_h^l = D_h^{-1}(T_h) \quad \forall l, \forall h \\ g_h^l = 0 &\Rightarrow C_h^l \geq D_h^{-1}(T_h) \quad \forall l, \forall h \end{aligned}$$

where  $T_h = \sum_{\forall l \in \Theta_h} g_h^l$ .

This can be summarised as a variational inequality (VI) where  $(\mathbf{g}^*, \mathbf{T}^*)$  are equilibrium vectors of path and trip-chain flows if:

$$\sum_{\forall h} \sum_{\forall l \in \Theta_h} \{C_h^l(\mathbf{g}^*) \cdot (g_h^l - g_h^{l*})\} - \sum_{\forall h} \{D_h^{-1}(T_h^*)(T_h - T_h^*)\} \geq 0 \quad \forall (\mathbf{g}, \mathbf{T}) \in \Phi$$

where  $\Phi$  is the feasible set of path and trip-chain flows which will be defined later on. This condition can then be reformulated as:

$$\sum_{\forall a} t_a(v_a^*) \cdot (v_a - v_a^*) + \sum_{\forall h} \sum_{\forall l \in \Theta_h} \tau_h^l \cdot (g_h^l - g_h^{l*}) - \sum_{\forall h} \{D_h^{-1}(T_h^*)(T_h - T_h^*)\} \geq 0 \quad \forall (\mathbf{v}, \mathbf{g}, \mathbf{T}) \in \Phi \quad (1)$$

This can be proven as follows. From the relationship between the trip-chain cost and path cost, we can define:

$$\sum_{\forall h} \sum_{\forall l \in \Theta_h} \{C_h^l(\mathbf{g}^*) \cdot (g_h^l - g_h^{l*})\} = \sum_{\forall h} \sum_{\forall l \in \Theta_h} \sum_{\forall p \in \theta_h^l} \{C_p(\mathbf{f}^*) \cdot (f_p - f_p^*)\} + \sum_{\forall h} \sum_{\forall l \in \Theta_h} \tau_h^l \cdot (g_h^l - g_h^{l*})$$

and then from the relationship between the path cost and the link cost we can define:

$$\sum_{\forall h} \sum_{\forall l \in \Theta_h} \sum_{\forall p \in \theta_h^l} \{C_p(\mathbf{f}^*) \cdot (f_p - f_p^*)\} = \sum_{\forall h} \sum_{\forall l \in \Theta_h} \sum_{\forall p \in \theta_h^l} \sum_{\forall a} \{\delta_{a,p} \cdot t_a(v_a^*) \cdot (f_p - f_p^*)\}.$$

Then, by multiplying  $\delta_{a,p}$  to  $(f_p - f_p^*)$  we then get:

$$\sum_{\forall h} \sum_{\forall l \in \Theta_h} \sum_{\forall p \in \theta_h^l} \sum_{\forall a} \{\delta_{a,p} \cdot t_a(v_a^*) \cdot (f_p - f_p^*)\} = \sum_{\forall a} t_a(v_a^*) \cdot (v_a - v_a^*).$$

The feasible region,  $\Phi$ , can be defined by a set of equality equations ensuring the flow conservations in the network:

$$\begin{aligned} T_h &= \sum_{\forall l \in \Theta_h} g_h^l, \forall h \\ f_p &= g_h^l, \forall h, \forall l \in \Theta_p, \forall p \in \theta_h^l \\ v_a &= \sum_h \sum_{l \in \Theta_p} \sum_{p \in \theta_h^l} \delta_{a,p} f_p, \forall a \\ T_h, g_h^l, f_p, v_a &\geq 0 \quad \forall h, \forall l, \forall p, \forall a \end{aligned}$$

From the theorem of VI (see Nagurney 1993), under the condition of separable link cost functions the VI expressed in (1) can be reformulated as an equivalent optimisation program:

$$\begin{aligned} \min Z &= \sum_a \int_0^{v_a} t_a(x) dx + \sum_h \sum_{l \in \Theta_h} \tau_h^l \cdot g_h^l - \sum_h \int_0^{T_h} D_h^{-1}(y) dy \\ \text{s.t.} & \\ T_h &= \sum_{\forall l \in \Theta_h} g_h^l, \forall h \\ f_p &= g_h^l, \forall h, \forall l \in \Theta_p, \forall p \in \theta_h^l \\ v_a &= \sum_h \sum_{l \in \Theta_p} \sum_{p \in \theta_h^l} \delta_{a,p} f_p, \forall a \\ T_h, g_h^l, f_p, v_a &\geq 0 \quad \forall h, \forall l, \forall p, \forall a \end{aligned} \tag{2}$$

Note that  $\tau_h^l$  can be calculated following the earlier discussion in the previous section. The objective function of (2) is strictly convex in  $\mathbf{v}$  (due to the first term with an assumption of monotone link cost function) and  $\mathbf{T}$  (due to the assumption of strictly monotone of the inverse demand function) and the feasible region is also a convex set. Thus, we can conclude that the solution of (2) in the vectors of  $\mathbf{v}$  and  $\mathbf{T}$  exist and unique. For the solution in the vector of  $\mathbf{g}$ , the existence follows the proof for the existence of  $\mathbf{v}$  and  $\mathbf{T}$ . However, its uniqueness is not guaranteed due to the non-strictly convex of the second-term of the objective function in (2). However, the revenue from the solution of (2) is unique due to the fact that the objective function of (2) is unique and the first and last term of the objective of (2) is unique, and thus the second term must also be unique.

Noteworthy, the formulation in (2) naturally reduces to the equivalent optimisation problem with an objective being a function of only the link flows and the trip-chain demand (the second term of the objective function in (2) reduces to a function with link flows in the case of cordon-based pricing). Therefore, one can apply any standard algorithm for the traffic assignment problem to solve (2) with the cordon-based pricing. However, for the case with the area-based pricing one needs to modify existing algorithms to take account of the non-additive cost in the second term of the objective function. This is discussed in the next section.

### 3. Solution algorithm for the equilibrium path model

Although the objective function of (2) firstly appears as a different formulation from the standard objective function of the traffic assignment model, it is still possible to adopt the method of partial linearization (or the double stage algorithm) to solve the problem. The algorithm can be defined as follows:

**Step 0:** Initialization. Set  $k = 1$ ; Find a set of feasible flows  $\mathbf{X}^{(k)} = (\mathbf{v}^{(k)}, \mathbf{g}^{(k)}, \mathbf{T}^{(k)})$

**Step 1: Travel-time update.** Calculate  $t_a = t_a(v_a), \forall a$ .

**Step 2: Direction finding.** Solve partial linear approximation problem to obtain auxiliary solution  $\mathbf{Y} = (\tilde{\mathbf{v}}, \tilde{\mathbf{g}}, \tilde{\mathbf{T}})$ .

**Step 3: Move-size determination.** Find  $0 \leq \alpha \leq 1$  that solves  $\min_{0 \leq \alpha \leq 1} Z(\mathbf{X}^{(k)} + \alpha(\mathbf{Y} - \mathbf{X}^{(k)}))$ .

**Step 4: Flow update.** Set  $\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} + \alpha(\mathbf{Y} - \mathbf{X}^{(k)})$ .

**Step 5: Convergence test.** If convergence is not achieved, set  $k := k+1$  and return to **Step 1**. Otherwise, terminate.

This is a standard partial linearization method (see Patriksson 1994 for more detail). As discussed in the previous section, the algorithm above can be applied directly to the problem in (2) for the case with the cordon-based pricing scheme as in the standard traffic assignment problem. However, for the case with the area-based charging scheme there is a need for a slight modification in **Step 2** of the algorithm above. We need to divide the paths into  $P$  subsets,  $M_1, \dots, M_P$  in which the paths belonging to each set of  $M_P$  have the same level of  $\tau_h^l$  (denoted by  $\tau_P$ ).

For instance, with a simple uniform area-based system with a toll level of  $\tau$  two set of paths can be defined. The first set is associated with those trips with the toll level of  $\tau$  in which the paths in this category passes the charged area at least once and the second set is those trips without any tolls imposed. Note that associating with each  $M_P$ , there is a set of links which can be used and cannot be used which will ensure that all paths generated under this set are associated with the corresponding toll pattern. For example, in the simple area-based system discussed earlier, under the subset of tolled paths all links in the network can be used. On the other hand, for the subset of un-tolled paths only those links outside the charged area can be used. This is a particular useful property which can be used to find the shortest path without enumerating all possible paths.

The objective function of the partial linear approximation problem in **Step 2** can be defined as:

$$\min_{(\tilde{\mathbf{g}}, \tilde{\mathbf{T}})} \tilde{Z} = \sum_h \sum_{l \in \Theta_h} C_h^l(\mathbf{X}^{(k)}) \tilde{g}_h^l - \sum_h \int_0^{\tilde{T}_h} D_h^{-1}(y) dy \quad (3)$$

Thus, finding the optimal solution to this partial linearization problem in (3) requires an assignment of the demand for trip-chain  $h$ ,  $\tilde{T}_h$ , to the path with the smallest cost (let  $\mu_h$  be the cost of the cheapest path for trip-chain  $h$ ), where  $\tilde{T}_h = D_h(\mu_h)$ . This allocation procedure involves an algorithm for finding the shortest path for each trip-chain (note that in the standard traffic assignment problem this is equivalent to the shortest-path problem). However, since the path cost is not link-additive due to the area toll, a slight modification of the standard assignment procedure is required. To explain this, **Step 2** is summarized in a more detail below.

**Step 2.1:** For each  $M_P$ , set  $t_a = \infty$  or sufficiently large value for link  $a$  which cannot be used under the toll pattern  $P$  and set  $t_a = t_a(v_a^k)$  otherwise. Then, for each trip-chain,  $h$ , find the cheapest path as follows. For each consecutive nodes  $n_i$  and  $n_{i+1}$  in the set of  $\Omega_h$ , find the shortest path between  $n_i$  and  $n_{i+1}$  based on link travel time ( $t_a$ ). This provides the shortest travel time  $C_{n_i, n_{i+1}}$  between each nodes of  $n_i$  and  $n_{i+1}$  and let  $\mu_h^P$  be the



summation of all  $C_{n_i, n_{i+1}}$  which is the shortest path travel time. Then the cost of the cheapest path for trip-chain  $h$  under  $M_P$  can be defined as  $\mu_h^P + \tau_P$ .

**Step 2.2:** Then for each trip-chain,  $h$ , find the cheapest path under different  $M_P$  (let  $w_h$  be the index of the cheapest path for the trip-chain  $h$ ). Let  $\mu_h$  be the generalised cost of this cheapest path (including travel times and tolls) for the trip-chain  $h$ .

**Step 2.3:** For each trip-chain  $h$ , set  $\tilde{T}_h = D_h(\mu_h)$  and set  $\tilde{g}_h^{w_h} = \tilde{T}_h$  and  $\tilde{g}_h^l = 0$  for all  $l \neq w_h$ .  $\tilde{v}_a$  can be found by summing all related auxiliary path flows. This procedure gives auxiliary flows  $\tilde{v}$ ,  $\tilde{g}$ , and  $\tilde{\mathbf{T}}$ . Note that for each path, we also can get the tolls related to that path which is  $\tau_P$  depending on which category of toll pattern the cheapest path is found for each trip-chain. This is required for the formulation of the objective function in **Step 3** to find the optimal step-length.

It should be noted that the direction selected in **Step 2** is a descent direction of the original problem and the algorithm does converge to the desired solution (see Patriksson 1994 for a general proof). Note that in **Step 2.1**, we can also save the computational time by keeping a record of the shortest path between each pair of nodes. Then, if the same pair of nodes appears again in a different trip-chain, the algorithm can skip the calculation of the shortest path for this same pair of nodes.

#### 4. Social welfare and equity impact evaluation

The modelling approach developed in the previous section can be used to evaluate the performances of a road pricing scheme. In this section, two measures to evaluate the social welfare improvement and equity impact are formulated mathematically.

For the social welfare improvement, using the Marshallian rule social welfare can be defined as:

$$SW = \sum_h \int_0^{T_h} D_h^{-1}(x) dx - \sum_a t_a(v_a) v_a \quad (4)$$

which is the area between the curves of the inverse demand and the total travel cost in the network. Note that the second-term is the total travel cost excluding the tolls. The revenue can be defined as:

$$REV = \sum_h \sum_{l \in \Theta_h} g_h^l \tau_h^l \quad (5)$$

As mentioned earlier, despite the non-uniqueness of the equilibrium path flows, the uniqueness of the total revenue can be guaranteed.

From the social welfare and the revenue, the user benefit can be defined as:

$$UB = SW - REV = \sum_h \int_0^{T_h} D_h^{-1}(x) dx - \sum_a t_a(v_a) v_a - \sum_h \sum_{l \in \Theta_h} g_h^l \tau_h^l \quad (6)$$

Road pricing has been promoted as a policy to raise social welfare in society. However, its side-effects cannot be neglected either, especially the effect on equity in society (Jones 1998; Viegas 2001). The equity impact of road pricing has been a focus for concern over the regressive side of this policy for some considerable time (Small 1983; Else 1986; Cohen 1987). Briefly, equity can be defined as a fair or equal distribution of the benefits obtained from accessing the transportation network by different groups of travellers. The category or

group of travellers has been defined in various ways in the literature but the main threads are based on the vertical and horizontal dimensions of user groups. The vertical dimension is typically related with different user classes in terms of income, travel flexibility, access to car, age, sex, etc. Several researchers have studied the vertical equity impact in the context of road pricing (see for instance Gomez-Ibanez 1992; Giuliano 1994; Anderson and Mohring 1995; Fridstrom *et al* 2000). The horizontal dimension is, on the other hand, linked with the analysis of the distribution of benefit/impact across different spatial locations or trip movements.

In the context of the model proposed in the previous section, the main analysis of the equity impact of a pricing scheme will be based on the horizontal dimension which will be related mainly to the impacts on different spatial movements. Unlike the case with the social welfare evaluation, there has not been any common agreement on the most appropriate measure for equity impact. Mayeres and Proost (2001) proposed a weighted social welfare approach to reconcile the equity impact of a road pricing policy by giving a higher weight to a group considered as a disadvantage group in the society (e.g. low-income group). Santos and Rojey (2004) analyses the potential equity impact on different parts of a town using traffic assignment to estimate the percentage of residents crossing and not crossing the cordon line. However, they did not provide a measurable index for a global level of the equity impact.

From traditional economics theory, there exists a notion of an index for income-distribution which measures the distribution of income amongst the population. Several indices have been proposed in the literature (see Cowell 1995). In transport, these measures have been occasionally adopted in analysis. Sumalee (2003) adopted the Gini coefficient to evaluate the spatial equity impact from different charging cordon designs. Vold (2005) measured the spatial equity impact of different transport policy packages using Kolm's measure (Kolm 1976). In this paper, the Gini coefficient will be used as a measure of equity impact.

Given the structure of the trip-chain based model, it is rather more appropriate to categorise the travellers by their trip-chains. This can be considered as an extension of the spatial equity analysis in Sumalee (2003) in which the travellers are grouped by their origin-destination nodes. The original definition of the Gini coefficient can be explained by using figure 3.

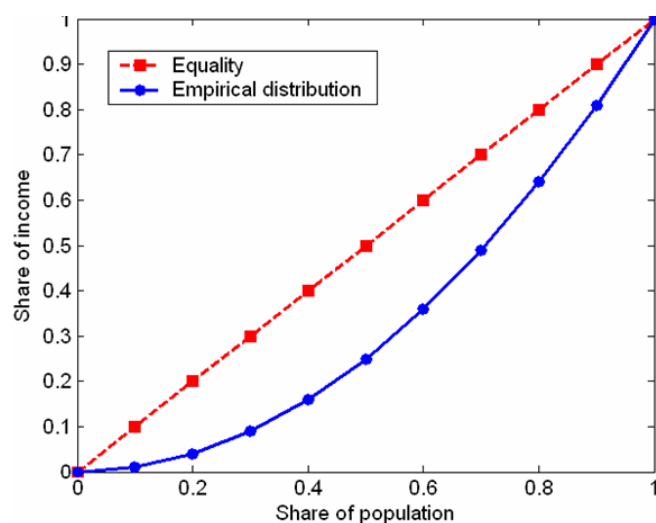


Figure 3: Lorenz curve for measuring the Gini coefficient

From figure 3, a Lorenz curve (empirical distribution line in figure 3) is constructed by organising a population in an increasing order of their incomes (wealth or any other vector) and by assigning to each individual the proportion of total income earned by that individual. The Gini coefficient may then be defined as the proportion between the area between the equality and Lorenz curves and the area under the equality curve. The Gini coefficient thus takes the value between 0 and 1, in which  $Gini = 0$  is the case with the perfect equitable situation. Mathematically, following Dixon *et al* (1987) this can be defined as:

$$Gini = \frac{\sum_j \sum_i |x_i - x_j|}{2n^2 \mu} \quad (7)$$

where  $x_i$  is the income level of individual  $i$ ,  $n$  is the total number of individuals in the population, and  $\mu$  is the mean of the income in the population.

To define the Gini coefficient for our analysis, we need to first define the representation of the income level in this analysis. In the case of road pricing, the main outcome of the policy is the total user benefit improvement after revenue recycling in which this can be disaggregated by different trip-chain patterns. However, with the calculation of the disaggregated user benefit improvement, an assumption on the revenue recycling process must be made (Sumalee 2004b). This is indeed a rather difficult issue to tackle both theoretically and practically. For the purpose of our analysis, we will simply assume a proportion of the total revenue (denoted by  $\rho_h$ ) which will be recycled back to the travellers in trip chain  $h$  (without consideration of the recycling mechanism). Note that  $\rho_h$  should be between 0 and 1 and  $\sum_h \rho_h \leq 1$  (not all revenues have to be transferred back to the travellers).

From this assumption, based on (6) the disaggregated total user benefit for each trip-chain can be defined as:

$$UB_h = \int_0^{T_h} D_h^{-1}(x) dx - \sum_{l \in \Theta_h} C_h^l g_h^l + \rho_h \sum_h \sum_{l \in \Theta_h} \tau_h^l g_h^l$$

where  $C_h^l$  is the generalised travel cost for path  $l$  (including tolls). From this expression, we can then define the average user benefit for each traveller belonging to the group of trip-chain  $h$  as:

$$ub_h = \frac{1}{T_h} \left[ \int_0^{T_h} D_h^{-1}(x) dx - \sum_{l \in \Theta_h} C_h^l g_h^l + \rho_h \sum_h \sum_{l \in \Theta_h} \tau_h^l g_h^l \right] \quad (8)$$

Then based on (7) and (8), the Gini coefficient for the distribution of user benefit (after revenue recycling) can be defined as:

$$Gini = \frac{\sum_h \sum_{h'} T_h T_{h'} |ub_h - ub_{h'}|}{2 \left( \sum_h T_h \right)^2 \overline{ub}} \quad (9)$$

where  $\overline{ub}$  is the average of average user benefit after revenue recycling for all travellers.

## 5. Case study of Utsunomiya area

### 5.1 Description of the case study

The case study adopted in this paper is based on a network representing Utsunomiya city in Japan which is situated just to the north of Tokyo. The city has a population of around 450,000. Figure 4 shows the arterial road network in the area of the Utsunomiya city. The network has three main ring roads (outer, inner, and downtown ring roads) which provided us with natural boundaries for the charging schemes which will be defined later. In the model, the network has the total of 1,345 links and 84 zones (see figure 4b for the zoning system).

For the road links, following JSCE (2003) the standard BPR function is employed to represent the relationship between delays and traffic volumes:

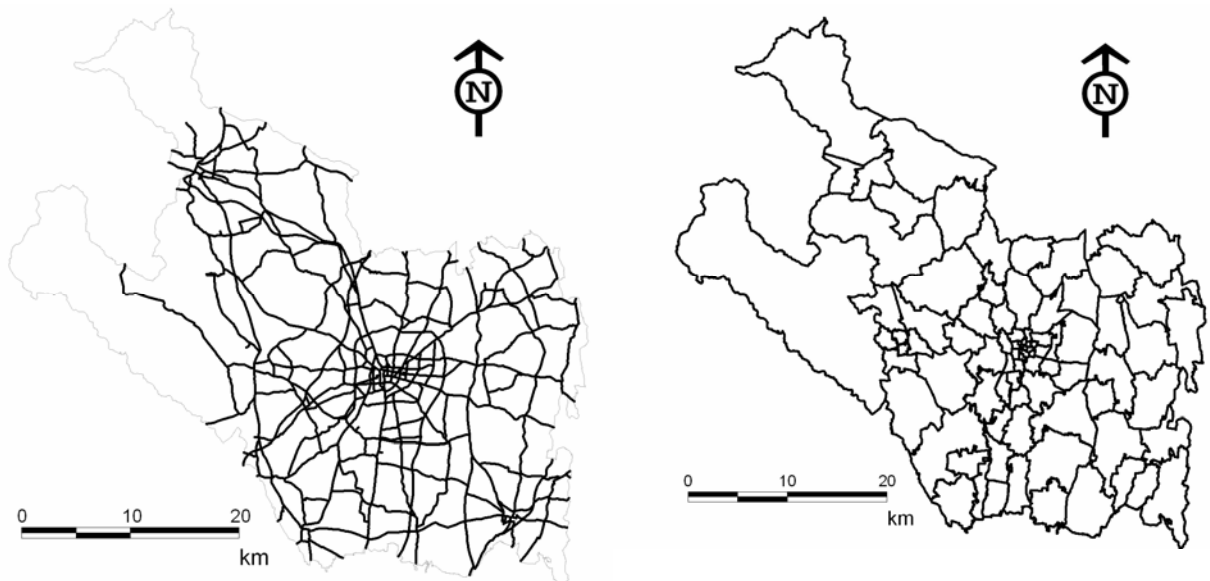
$$t_a(v_a) = t_a^0 \left\{ 1 + 0.48(v_a/c_a)^{2.82} \right\}$$

where  $t_a^0$  and  $c_a$  are free-flow travel time and capacity of link  $a$  respectively. The value of time of 60 Japanese Yens (JPY)/minute (around 0.50 US\$/min) is used. The exponential demand function is defined as:

$$D_h(C_h) = D_h^0 \exp\left\{ \sigma \left( 1 - C_h/C_h^0 \right) \right\}$$

where  $D_h^0$  and  $C_h^0$  are the demand and minimum generalised travel cost for trip-chain  $h$  in the base case (do-nothing scenario) respectively, and  $\sigma$  is the price elasticity for the demand which is set as 0.5.

For the tests, six charging boundaries are defined based on the locations of the three main ring roads of the city. In figure 5a, three hypothetical charging boundaries A, B, and C are defined based on the outer, inner, and downtown ring roads respectively. In addition, in order to investigate the effect of coverage size three variants of the boundaries, D, E, and F, (see figure 5b) are considered. The boundary D is defined approximately inside another downtown ring road. E and F are based on another inner ring road covering the whole area of B. All six boundaries surround the city centre of Utsunomiya which can be considered as natural alternatives for the actual plan of the road pricing in this city. For each of the boundaries, an area-based and cordon-based charging scheme is defined (named scheme area-A, cordon-A, area-B, cordon-B, so forth and so on). We assume that a uniform toll is imposed over the whole day to provide a comparison between the effect of the toll from the cordon-based and area-based pricing schemes on the daily trip-chain demands.

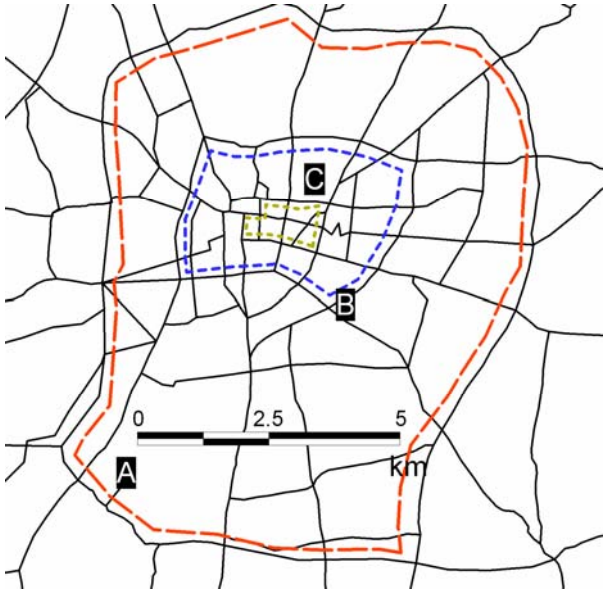


(4a) Arterial road network

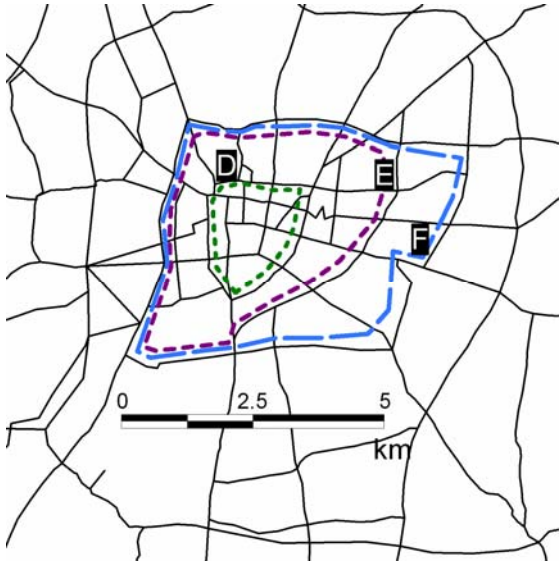
(4b) Zoning system

Figure 4: Arterial road network and zoning system in the Utsunomiya network

The base trip-chain demand is retrieved from the data of the Utsunomiya Metropolitan Area Person Trip (PT) survey in 1992. The master data from the Utsunomiya 1992 PT survey consists of individual answer sheets in the form of a questionnaire with the questions related to the respondent's daily trip activities. The base demand for each trip-chain activity adopted in the model is calibrated by sampling different daily trip-chains from the set of individuals in each zone to get an average frequency of the trip-chains. Then, these frequencies are factored up with the number of total population in that zone to get the total potential demand for different trip-chain patterns  $D_h^0$ .



5(a) Charging boundaries A, B, C



5(a) Charging boundaries D, E, F

Figure 5: Definition of the six charging boundaries in the case study

**5.2 Comparison between cordon-based and area-based schemes: Social welfare**

This section focuses on the comparison of the social welfare benefits from each of the scheme designs. To find an optimal uniform toll, each scheme is tested with different toll levels from 10 to 300 JPY. Figures 6 and 7 show the social welfare benefits for all schemes with different toll levels.

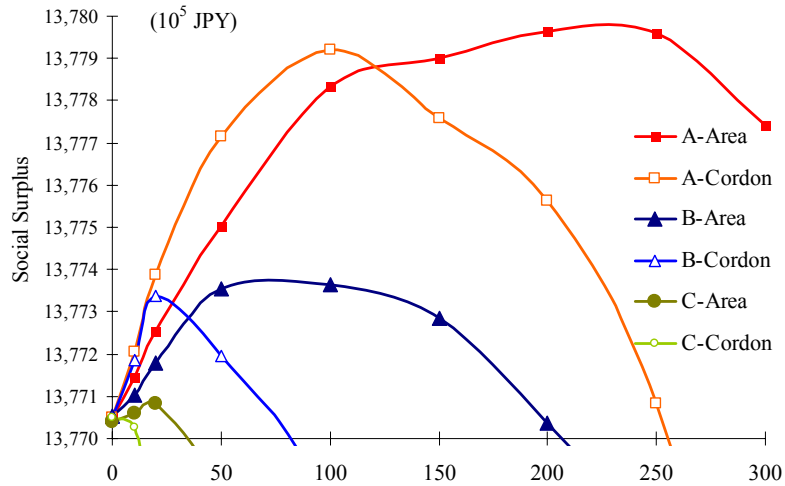


Figure 6: Social surplus (in ten thousands JPY) for area and cordon scheme A, B, and C

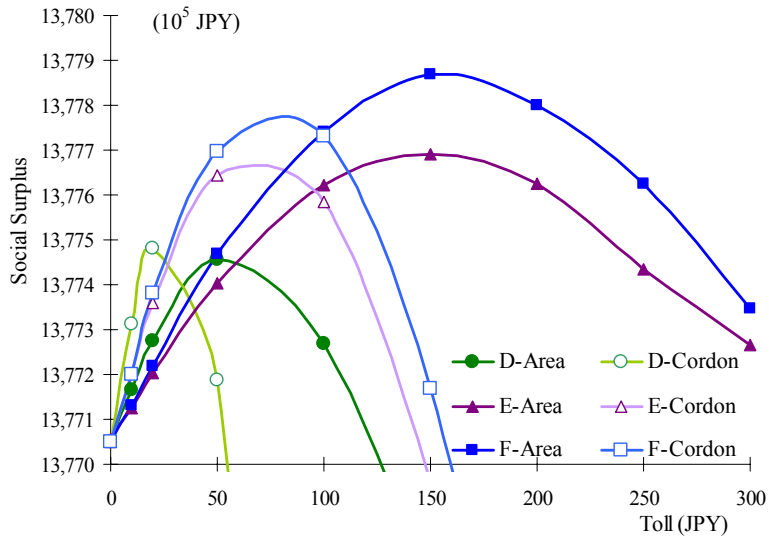


Figure 7: Social surplus (in ten thousands JPY) for area and cordon scheme D, E, and F

From figures 6 and 7, when comparing all twelve charging schemes Area-A produces the highest social surplus (or social welfare benefit) of around 1,378 million JPY with the optimal toll of 200 JPY. For the cordon based schemes, Cordon-A performs best with the benefit very close to that of the Area-A scheme (only around 44,000 JPY lower). Table 1 summarises the optimal benefit and toll for each scheme. Note that the social welfare in the do-nothing scenario (no toll) is around 1,377 million JPY. In the last column of Table 1, the social welfare improvement is the net gain of the social surplus as compared to the social welfare in the do-nothing scenario.

Table 1: Summary of the benefit and optimal toll for each charging schemes

Scheme	Area-coverage (sq-km)	Optimal toll (JPY)	Social welfare (JPY)	Social welfare improvement (JPY)
Area-A	66.0	200	1,377,964,100	914,100
Area-B	9.0	100	1,377,363,687	313,687
Area-C	1.0	10	1,377,081,280	31,280
Area-D	2.5	50	1,377,457,273	407,273
Area-E	12.9	150	1,377,691,874	641,874
Area-F	19.3	150	1,377,869,603	819,603
Cordon-A	66.0	100	1,377,919,857	869,857
Cordon-B	9.0	20	1,377,336,168	286,168
Cordon-C	1.0	0	1,377,050,000	0
Cordon-D	2.5	20	1,377,481,656	431,656
Cordon-E	12.9	50	1,377,642,950	592,950
Cordon-F	19.3	100	1,377,730,178	680,178

From the result, it can be concluded that in this case study the area-based schemes outperformed their counterpart cordon-based schemes with an exception of the charging boundary D in which the cordon-based scheme produced a slightly higher benefit. Figure 8 shows the relationship between the coverage area of the charging scheme and its optimal social welfare. Interestingly, for the area-based scheme (which also charges the internal trips) the higher the coverage area the higher the social welfare improvement. Notice that after a certain level of coverage is reached (20 sq-km covered by the Area-F scheme) the margin of the increase became trivial. For both scheme types, there are some drops in the social welfare benefit with the scheme Cordon-B and Area-B (coverage area of around 10 sq-km). This trend may depend largely on the traffic condition of the case study. Nevertheless, from this test a simple illustration of the potential increasing level of the social welfare for a wider scheme, which may be due to the higher number of trips being tolled, can be made. However, for the cordon scheme a certain size of a charging cordon may allow too many internal trips (which are not charged) and may not be large enough to minimise the possible diversion routes. Thus, the potential control over the demand in the network may not be increasing as the size increases. A similar result was demonstrated in Sumalee (2004a).

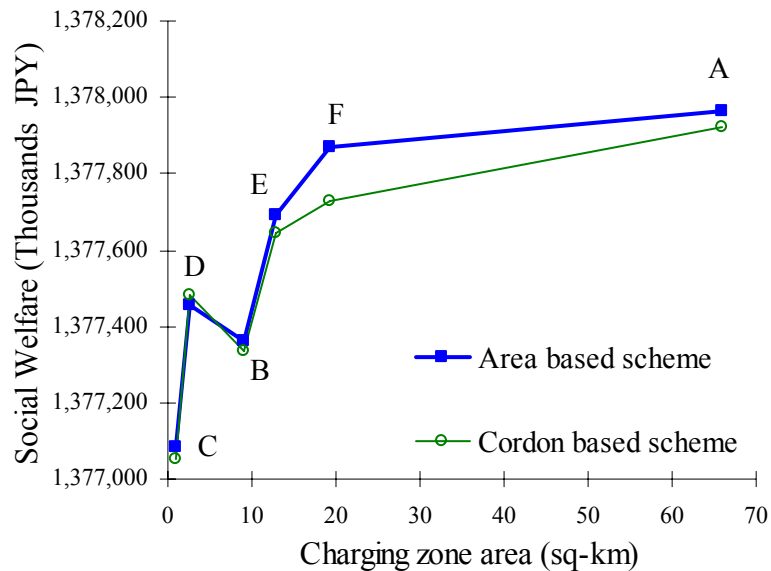


Figure 8: Area coverage of the charging zone and the optimal scheme benefit

Figure 9 below shows the total demand in the network (at its optimal toll) as the size of the scheme increases. This figure demonstrates clearly higher levels of demand depression under the area-based schemes. In addition, it also suggests that the wider the coverage area the lower the travel demand in the network (due to the higher number of trips affected by a wider coverage area). This explains the results in figure 8 in which a wider area-scheme covers a higher level of demand and hence can internalise the externalities to more trips. On the other hand, the total travel demand for the cases with the Cordon-B, Cordon-C, and Cordon-D are not significantly different. This, as discussed earlier, may be due to the trade-off between the interception of the trips crossing the cordon line and the amount of internal traffic allowed to travel for free. For Cordon-A, Cordon-E, and Cordon-F the levels of demand depression are obviously lower than those from the Cordon-B, C, and D. In relation to this result, this may also explain the higher benefits of the area-based scheme.

Figure 10 plots the relationship between the optimal toll level of the scheme and the coverage area of the charging boundary. One clear result is that the wider the coverage area the higher the optimal toll level for both area and cordon based schemes. This is possibly explained by the fact that a wider boundary may impose the toll on longer trips and hence the higher toll is required (proportionally to the longer trip travel time). Observe the cases between the boundaries A and F which are relatively different in terms of their coverage sizes but very similar in the coverage areas. The optimal toll required for the two cases of the area-based scheme is significantly different. On the contrary, for the cordon based scheme the optimal tolls for Cordon-A and F schemes are relatively similar. Thus the similarity of the coverage area for the case of cordon based scheme may underpin the similar optimal toll level, since no toll is imposed on internal trips but only crossing trips. On the other hand, for the area-based scheme the coverage area is an important factor since the toll is imposed on all trips.



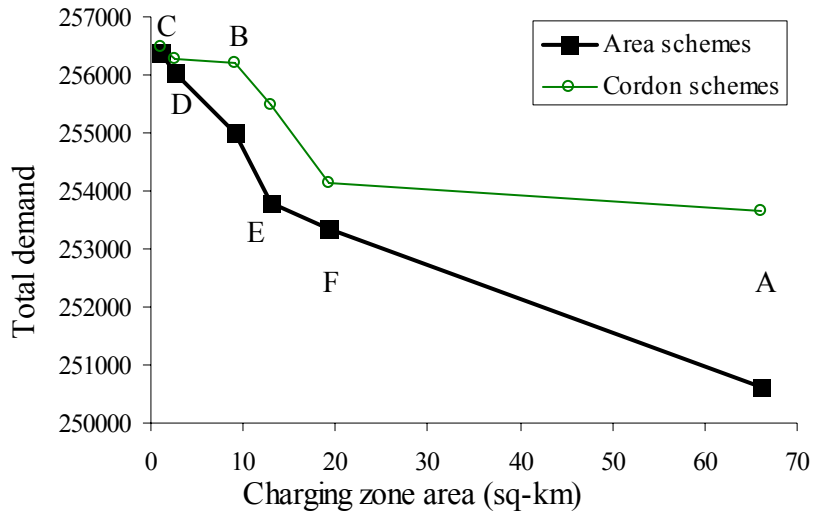


Figure 9: Average coverage of the charging zone and the total travel demand

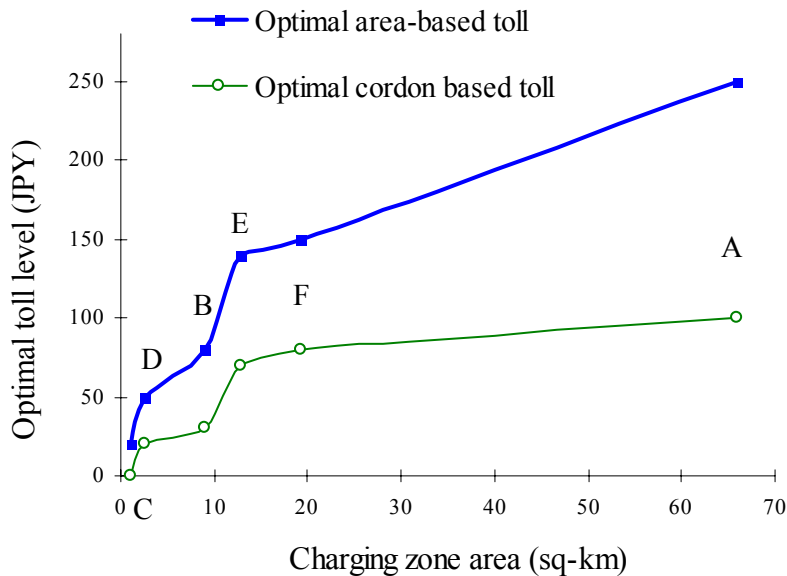


Figure 10: Area coverage of the charging zone and the optimal toll level

The other interesting finding is that the optimal toll level for the area-based scheme is almost twice that of the cordon-based scheme (under the same charging boundary). This may be caused by the difference in the level of tolls imposed on a trip-chain. If the trip-chain crosses the charging boundary twice, the proportion between the toll and the trip travel time for the charging cordon scheme will be twice of the area based scheme (since the traveller only has to pay once under the area based scheme). Under this scenario, the toll level of area-based schemes should be raised higher (twice higher) to achieve the same proportion of the toll and the trip-chain time. However, note that the results are more complex in a general case since the toll of the area-based scheme will also be imposed on the internal trips which are not tolled by the cordon scheme.

### 5.3 Comparison between cordon-based and area-based schemes: Equity impact

This section turns the focus to the issue of the spatial equity impact of the pricing schemes. The Gini coefficient as defined in Section 4 is adopted to measure the equity impact. In this section, for simplicity we only analyse the case without any revenue re-distribution,  $\rho_h = 0$  for all trip-chains. In other words, the Gini coefficient is adopted to measure the distribution of the user benefits of each trip-chain pattern. Recall that Gini value of 0 and 1 represents the perfect equality condition and the most inequitable situation respectively.

Figures 11 and 12 depict the Gini coefficients for different area and cordon based charging schemes with different toll levels respectively.

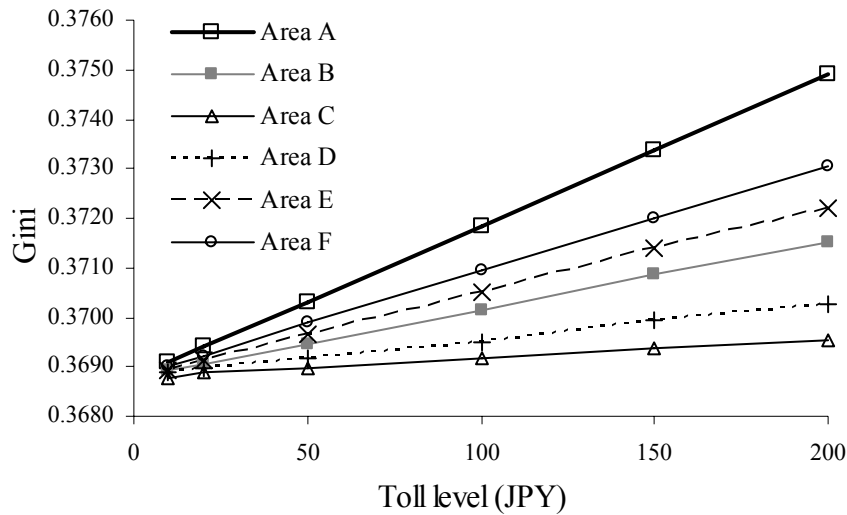


Figure 11: Gini coefficient level for each area-based charging scheme with different tolls

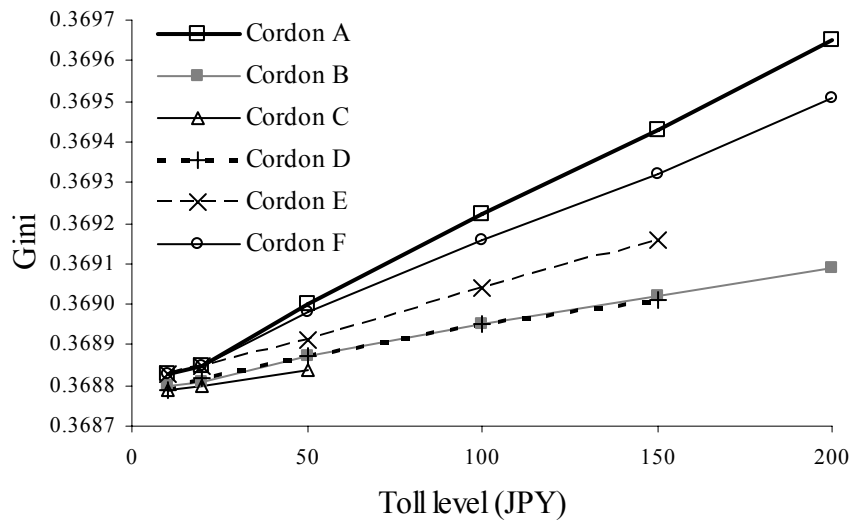


Figure 12: Gini coefficient level for each cordon-based charging scheme with different tolls

The results in these two figures shows the clear increasing level of spatial inequity as the toll increases in both the area and cordon based schemes. However, the relative differences between the Gini coefficients between different scheme designs are relatively small which is in contrast to the result reported in Sumalee *et al* (2005). The reason is due to the similarity of the charging schemes tested in this paper (in terms of their coverage areas) whereas in

Sumalee *et al* (2005), the optimal charging cordon scheme which generated a substantially different level of Gini coefficient was largely different from other judgmental cordon schemes.

From the figures, we can also observe that in general the area-based schemes generated a slightly higher level of spatial inequity compared to the cordon schemes. The reason could be that the area-based scheme favours a longer trip-chain. The longer the trip-chain the lower the proportion of the toll and its total travel cost, which may result in a higher welfare improvement as compared to a shorter trip-chain. In addition, the area-based scheme imposes the toll on a higher number of trips as compared to the cordon schemes and hence more travellers are affected by the tolling scheme.

Figure 13 below shows the comparison between the charging zone coverage area and the Gini coefficient (evaluated at the optimal toll level). In both cases of cordon and area based schemes, the wider the coverage area the higher the Gini coefficient implying a higher level of spatial inequity. The reason could also be based on the results shown in figure 9 implying the level of traffic affected by the tolls. For the area based scheme, the wider the scheme the higher the number of trips charged and hence the higher the level of inequity (the wider the charging zone, the higher the benefit for the longer trip-chains due to the reduction in congestion level inside the charged zone). On the other hand, similar to the case of social welfare the trend of the Gini coefficient for the cordon based scheme is not increasing at the scheme Cordon-B. Again, this can be explained by the similar levels of the travel demand in the network for the cases of Cordon-C, D, and B implying a similar level of trips affected by the tolls despite the differences in their coverage areas.

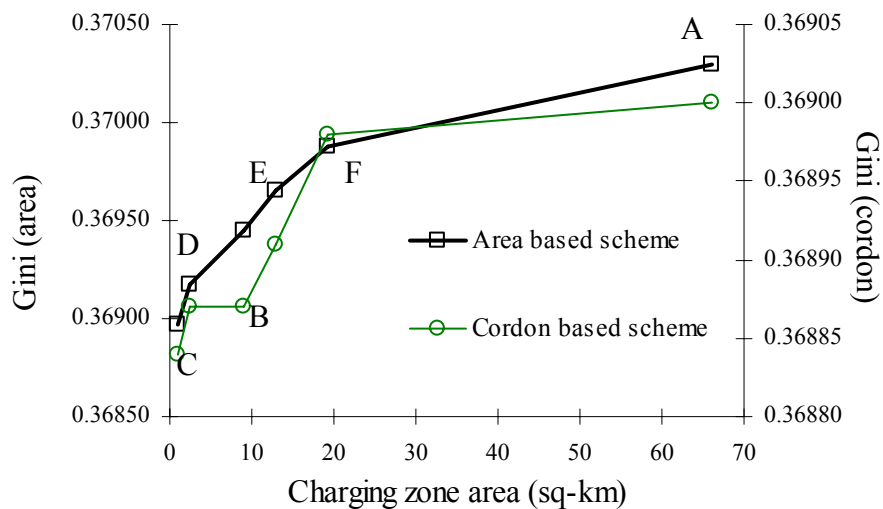


Figure 13: Relationship between the charging zone coverage area and the Gini coefficients

## 6. Conclusions

The key difference between the cordon and area based road pricing schemes is that the cordon scheme tolls travellers per crossing but the area scheme tolls users for a license (for a period of, say, one day) to enter or travel inside the area. Many previous researches have focused only on the analysis of cordon based schemes and have not paid enough attention to the area based schemes despite their similar levels of popularity (in practice) and potential for implementation.

The paper compared the performances of the cordon and area based schemes using the trip-chain equilibrium based model. The trip-chain representation is important for a more realistic response of travellers to the area toll (or license scheme). The paper presented the test results with the case of Utsunomiya city in Japan. Six different charging boundaries were defined based on the locations of existing ring roads in the network. The total of twelve charging schemes, with six of each area and cordon schemes, were defined based on the six boundaries.

The results showed that in general the area-based schemes performed better than the cordon-based schemes in terms of social welfare improvement. The main reason was that the area-based scheme affects a higher volume of demands in the network as compared to the cordon-based scheme with the same boundary. On the other hand, the area-based scheme also generated a higher level of spatial equity impact as measured by the Gini coefficient. The reason for this is similar to that of the social welfare comparison which is related to the level of demands affected by the toll scheme.

On the topology of the scheme, for the area-based scheme it was found that the wider the coverage area, the higher the social welfare, optimal toll level, traffic demand depression, and equity impact. The reason for a higher level of welfare and traffic demand depression is obviously related to the number of trips affected by the toll. For the case of the optimal toll level and equity impact, the wider charged area may charge a higher number of long trip-chains. Thus, the area-based scheme requires a higher toll level to ensure an optimal proportion between the total trip-chain travel time and the toll imposed. The area-based scheme, by its nature, favours a longer trip-chain (since both short and long trip-chains are charged by the same toll although the longer trip-chain may cross the charged area more than the shorter trip-chain). Therefore, the wider charged area may create a higher difference between the benefits of a long and short trip.

The result on the effect of the coverage area of the charging scheme is slightly different for the case of the cordon-based scheme. For the cordon-based scheme, the wider cordon does not guarantee the higher level of demand affected by the toll. This will involve trade-off between the non-tolled trips inside the cordon and the number of tolled trips crossing the cordon line.

Given the initial result showing the different effect of the area and cordon based schemes, it is important to allow the traveller to shift to an arrangement with a longer trip-chain. This is one of our main future research issues. It is also important to develop a model representing the different levels of congestion by different time periods since the trip-chain normally involves the whole day trip. Future research will also focus on the issues of different revenue-redistribution schemes, and their effects on the equity impact and the optimal design of the area-based scheme.

## **Acknowledgements**

This research was conducted while the first author was working at Department of Urban Engineering, University of Tokyo and the second author was visiting Kuwahara laboratory at the Institute of Industrial Science, University of Tokyo. The second author would like to acknowledge his host, Prof. Masao Kuwahara, and the support from UK EPSRC funded Platform Grant project, "Towards a unified theoretical framework for transport network and choice modelling". However, note that this paper should not represent the conclusive view of the project. The authors also would like to thank the two anonymous referees for their constructive comments which helped improving this paper significantly.

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