Power allocations for multilevel coding with sigma mapping

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A recursive power-allocation method for multilevel coding systems with sigma mapping is proposed. With the proposed power allocation, it is possible to design iteratively decodable capacity-approaching codes with high bandwidth efficiency by using binary turbo-like codes.

Introduction: The multilevel coding (MLC) scheme was proposed by Imai and Hirakawa [1] to implement bandwidth-efficient coded modulation for additive white Gaussian noise (AWGN) channels. The MLC consists of multiple levels whereby each level is protected by an error-correcting code (ECC). Traditionally, multiple coded bits from different levels are mapped to a signal point according to the so-called 'mapping by set partitioning' [2]. In 1997, Duan, Rimoldi and Urbanke [3] proposed a capacity-approaching mapping method, which we call sigma mapping. With sigma mapping, the MLC system can be treated as a multi-user system (i.e. one level as one user). So it is not surprising that the decoding/demapping algorithm can be implemented in an iterative manner as described in [4]. In addition, since co-operation among different 'users' is perfect, we are able to play more at the transmitter. For example, we may utilise unequal power allocations to facilitate the iterative decoding/demapping process. An interesting question is: how much power should we allocate at each level? In this Letter, we show (by simulations) that, with appropriate power allocations, powerful binary codes with simple iterative decoding/demapping algorithms can be utilised to approach the channel capacities for a wide range of signal-to-noise ratios (SNRs).

Encoding and decoding: As shown in Fig. 1, a binary data sequence $\underline{u}$ of length K is partitioned into m subsequences $\underline{u}^{(i)}$ of length $K^{(i)}$ for $0 \le i \le m-1$. The ith subsequence $\underline{u}^{(i)}$ is encoded by a binary component code at the ith level, resulting in a sequence $\underline{c}^{(i)}$ of length N. The randomly interleaved version $\underline{v}^{(i)}$ of $\underline{c}^{(i)}$ for all i are then mapped to a signal sequence $\underline{x}$ by a linear superposition (called sigma mapping) with parameters $(E_0, E_1, \ldots, E_{m-1})$, where E_i is the symbol energy at the ith level. The signal sequence is then transmitted over an AWGN channel. At time t, the received signal is $y_t = x_t + w_t$, where w_t is assumed to be a Gaussian random variable with mean 0 and variance σ^2. After receiving the whole sequence $\underline{y}$, an iterative decoding/demapping algorithm similar to that described in [4] can be employed to estimate the data sequence $\underline{u}$.

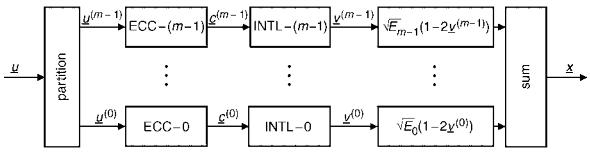

Fig. 1 *Multilevel coding system with sigma-mapping parameterised by symbol energies* $(E_0, E_1, \ldots, E_{m-1})$

Power allocation problem: The coding rate is $R = R_0 + R_1 + \cdots + R_{m-1}$ bits per dimension, where $R_i = K^{(i)}/N$. According to the channel coding theorem, the total energy per dimension $E = E_0 + E_1 + \cdots + E_{m-1}$ must satisfy $E/\sigma^2 \ge 2^{2R} - 1$ in order that the bit error rate (BER) is arbitrarily small. The problem is, for a given BER, how to choose $(E_0, E_1, \ldots, E_{m-1})$ such that the total energy E is minimised. Some clues have been mentioned for multi-access systems: see [5] for theoretical analysis and [6] for practical considerations. But for a general MLC system, this is still an open problem. We next describe three methods.

Method 1: Power allocation for ideal coding and successive decoding: It is well-known (e.g. see [7]) that we can choose $(E_0, E_1, \ldots, E_{m-1})$ such that:

$$\frac{E_i}{\sigma^2 + \sum_{j<i} E_j} = 2^{2R_i} - 1 \text{ for } i = 0, 1, 2, \ldots, m-1$$

Theoretically, if the code length N is large enough and the component codes are capacity-achieving codes (in the single level case) with Gaussian-like coded symbols, we can approach the capacity $C = (1/2)\log_2(1 + E/\sigma^2)$ by using a simple successive cancellation procedure. For example, consider a two-level system. We can first decode the second level by treating the first level as additive noise. The noise power seen by the second level is $\sigma^2 + E_1$. Suppose that this is successful. We can then strip off the second level signals from the received signals. The remaining signals have SNR equal to E_1/σ^2. Assume that both levels are protected by capacity-achieving codes. Then the (asymptotic) aggregate rate is:

$$\frac{1}{2}\log\left(1 + \frac{E_1}{\sigma^2}\right) + \frac{1}{2}\log\left(1 + \frac{E_2}{\sigma^2 + E_1}\right) = \frac{1}{2}\log\left(1 + \frac{E_1 + E_2}{\sigma^2}\right) \quad (1)$$

Note that the right-hand side of (1) is exactly the channel capacity corresponding to the total transmission energy $E = E_0 + E_1$. This reasoning can be easily generalised to systems with more levels.

Method 2. Power allocation for practical coding and successive decoding: If the codes for individual levels are not capacity-achieving, the power required by each level should be higher than that obtained from Method 1. Assume that all component codes are binary and fixed. Let SNR_i be the required SNR at a designated BER (say, 10^{-5}) for the ith component code over a single-level AWGN channel. For example, the required SNR for the original turbo code [8] to achieve BER of 10^{-5} is about 1.17 (0.7 dB). The following simple power-allocation strategy can ensure the success of the successive decoding procedures provided that the parameter α is chosen to be large enough:

$$\frac{E_i}{\sigma^2 + \sum_{j<i} E_i} = \alpha \times SNR_i \quad \text{for } i = 0, 1, 2, \ldots, m-1 \quad (2)$$

Note that if all the component codes are identical, (2) can be reformulated as $E_i/\sigma^2 = E_0/\sigma^2(1 + E_0/\sigma^2)^{i-1}$, as derived in [5, 6].

Method 3. Power allocation for practical coding and iterative decoding: It is interesting to note that the total energy can be further reduced if the iterative decoding/demapping algorithm is employed. Unlike Methods 1 and 2, we do not have closed-form power allocations in this case. Instead, we propose the following simulation-based recursive search algorithm, which is conceptually simple. Assume that a solution has been found for a k-level system, denoted by $(E_0, E_1, \ldots, E_{k-1})$. We choose $(E_0, E_1, \ldots, E_{k-1}, E_k)$ as the power allocation for the $(k+1)$-level system, where E_k is determined by simulations such that the BER of the $(k+1)$-level system is at the designated BER.

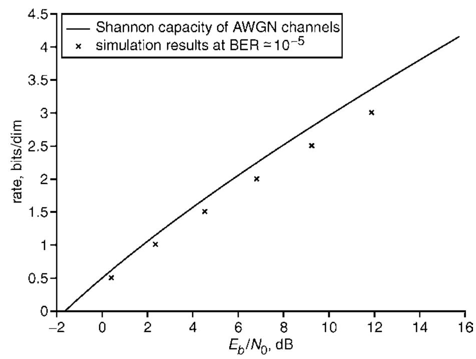

Fig. 2 *Simulation results at BER around* 10^{-5} *by using proposed power-allocation Method 3, where number of levels varies from one to six*

Simulation results: We take the doped code of length 2×10^5 proposed in [9] as the component code at each level. The designated BER is set to be 10^{-5}. The SNRs resulting from Method 3 are shown in Fig. 2 for systems with from one to six levels. For comparison, the capacity curve is also plotted in Fig. 2. The simulation results are quite close to the capacity, although the gap gets wider when the coding rate increases (equivalently, when the number of levels increases). At rate of 3 bits/dim (the six-level system), the gap is around 1.7 dB, which can be further narrowed if an *a posteriori* probability (APP) demapping algorithm rather than the elementary signal estimator (ESE) in [4] is implemented during the iterative process. For the six-level system in this example, Method 3 leads to slightly better performance (about 0.1 dB) than Method 2 at BER around 10^{-5}. It is worth pointing out that such benefits can be significant when 'poor' codes are used as component codes. To illustrate this, we have included simulation results in Fig. 3 for another six-level MLC system whereby the (7, 5) convolutional code with length 4096 is taken as the component code. It can be seen that the gap is about 13.5 dB at BER around 10^{-5}. At BER around 10^{-5}, the parameters obtained by using Method 3 are $\{E_i\} = (3.80, 4.14, 12.00, 30.16, 62.08, 169.64)$, while the parameters obtained by using Method 2 are $\{E_i\} = (3.30, 14.18, 60.96, 262.05, 1126.50, 4842.58)$. Here the noise variance is normalised to one.

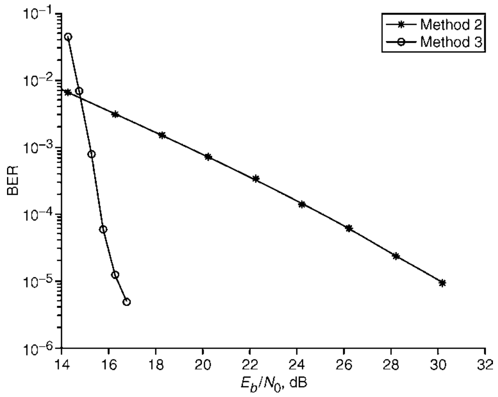

Fig. 3 *Performance comparison between two different power allocations for six-level-coding/sigma-mapping systems*

Conclusions: We have proposed a recursive power-allocation method for multilevel coding/sigma-mapping systems. Simulation results indicate that, based on power allocations, we are able to design practical power-limited/bandwidth-limited MLC schemes with performance close to the Shannon limits.

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References

1 Imai, H., and Hirakawa, S.: 'A new multilevel coding method using error-correcting codes', *IEEE Trans. Inf. Theory*, 1977, **IT-23**, pp. 371–377
2 Ungerboeck, G.: 'Channel coding with multilevel/phase signals', *IEEE Trans. Inf. Theory*, 1982, **IT-28**, pp. 55–67
3 Duan, L., Rimoldi, B., and Urbanke, R.: 'Approaching the AWGN channel capacity without active shaping'. Proc. IEEE Int. Symp. Information Theory, 1997, Ulm, Germany, p. 374
4 Ping, L., Liu, L., Wu, K.Y., and Leung, W.K.: 'Approaching the capacity of multiple access channels using interleaved low-rate codes', *IEEE Commun. Lett.*, 2004, **8**, (1), pp. 4–6
5 Viterbi, A.J.: 'Very low rate convolutional codes for maximum theoretical performance of spread-spectrum multiple-access channels', *IEEE J. Sel. Areas Commun.*, 1990, **8**, pp. 641–649
6 Chayat, N., and Shamai (Shitz), S.: 'Iterative soft onion peeling for multiple-access and broadcast channels'. Proc. 9th IEEE Int. Symp. on Personal, Indoor and Mobile Radio Communications, 1998, Boston, MA, USA, Vol. 3, pp. 1385–1390
7 Cover, M.C., and Thomas, J.A.: 'Elements of information theory' (Wiley, New York, 1991)
8 Berrou, C., Glavieux, A., and Thitimajshima, P.: 'Near Shannon limit error-correcting coding and decoding: turbo codes'. Proc. IEEE Int. Conf. on Communications, 1993, Geneva, Switzerland, Vol. 2, pp. 1064–1070
9 Ten Brink, S.: 'Rate one-half code for approaching the Shannon limit by 0.1 dB', *Electron. Lett.*, 2000, **36**, (15), pp. 1293–1294