AN ANALOG INTERPRETATION OF COMPRESSION FOR DIGITAL COMMUNICATION SYSTEMS

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ABSTRACT

The combination of a source coder and a digital modulator is in this article viewed as an analog-to-analog converter with signal compression ability. A reference channel with defined available bandwidth, total received power, and noise statistics, can transmit one analog source signal. Compression is here defined as the number of such source signals that the digital system can transmit over the reference channel, and reconstruct with the same fidelity as the transmitted analog source signal. An image transmission example is provided where, for a given fidelity on a reconstructed image, the obtainable compression ratio is found for a digital compression system based on subband coding and pulse amplitude modulation.

1. INTRODUCTION

Digital compression of analog signals is used to facilitate signal transmission or storage. All transmission and storage media are, however, in nature analog and the compressed digital signals must therefore be adapted to the media through some form of digital modulation. In essence, the overall signal chain from the analog source through digitization, compression, and digital modulation represents an analog-to-analog conversion, hopefully with compression and other useful properties. The interesting question is, "how can we make fair comparisons between such a system and an alternative analog transmission system, e.g. in terms of bandwidth efficiency and power requirements?"

Traditionally, digital compression and digital modulation systems have been designed separately, partly for the good reason that the transmission chain should be able to carry any signal. However, for closed systems it is more optimal to design the whole system as one entity. Examples of such systems are the future high-definition digital television and cellular telephone communications. Even such integrated systems can be prepared to transmit different signals with error rates fit to various applications. Approaches to joint system design where artificial sources are used may be found in [1, 2, 3, 4, 5].

Traditional digital compression definitions typically measure the reduction in source coder bit rate. Bit rate reduction is, however, problematic in the sense that we need a reference system with a finite rate to measure compression. Any analog signal has infinite rate. Thus, digital compression ratios will always be arbitrary.

We try to make a practical definition of compression in terms of the advantage of using digital compression/modulation over analog transmission for a given channel. The advantage of the digital system over the pure analog system can be defined in terms of an analog compression ratio according to:

> *Given a channel with limited bandwidth, maximum available power at the receiving end, and noise statistics which can transmit one source signal with a certain fidelity in a pure analog mode. How many similar source signals can be transmitted over the same channel with the digital compression/modulation system when requiring reconstruction with the same fidelity?*

The number obtained from this definition is a practical measure for compression efficiency of the complete signal chain.

This paper will motivate this new interpretation of compression. The principle will be exemplified by an image transmission system where a subband coder is combined with pulse amplitude modulation (PAM) signaling.

2. PRINCIPLES

The analog reference signal is assumed to be sampled, i.e. discrete in time/space, but continuous (analog) in amplitude. The reference, analog mode is defined as the transmission scheme where the reference signal is transmitted as an analog PAM signal without any further analog or digital processing. According to Nyquist's sampling theorem, assuming bandwidth B of the analog reference signal, a minimum transmission rate of $R_s = 2B$ symbols/s is required to represent the signal. Compression is obtained by reducing the channel symbol rate, while keeping the signal fidelity and channel signal-to-noise ratio constant. A reduction in the symbol rate by a factor of η as compared to the analog system rate, while keeping the other factors constant, corresponds to a compression efficiency of η.

To illustrate this compression mechanisms, let us consider a white noise channel with a one-sided power spectral density N_0, maximum available power at the receiver $S = S_{max}$, and bandwidth B, which can carry one uncompressed analog source signal. To fully utilize the channel, the power of the received analog source signal is set to $S = S_{max}$, implying a channel signal-to-noise ratio of

$$\frac{S}{N} = \frac{S_{max}}{B \cdot N_0} = \nu, \qquad (1)$$

where N is the channel noise power. Defining E_s as the symbol energy, the power can be expressed as $S = R_s \cdot E_s$.

Assume that the digital compression/modulation scheme can reduce the required bandwidth for a signal by a factor of η and still obtain the same fidelity at the given channel signal-to-noise-ratio, ν. Then a total of η compressed signals can be transmitted within the original bandwidth B and with the same total power S_{max}. This situation is illustrated in Figure 1. It is assumed that the reconstructed signals have the same fidelity in the digital and the analog reference systems. However, in the digital system case it is the aggregate effect of quantization noise and channel noise that causes the reduction in signal quality, whereas in the pure analog case the signal degradations are only caused by transmission errors.

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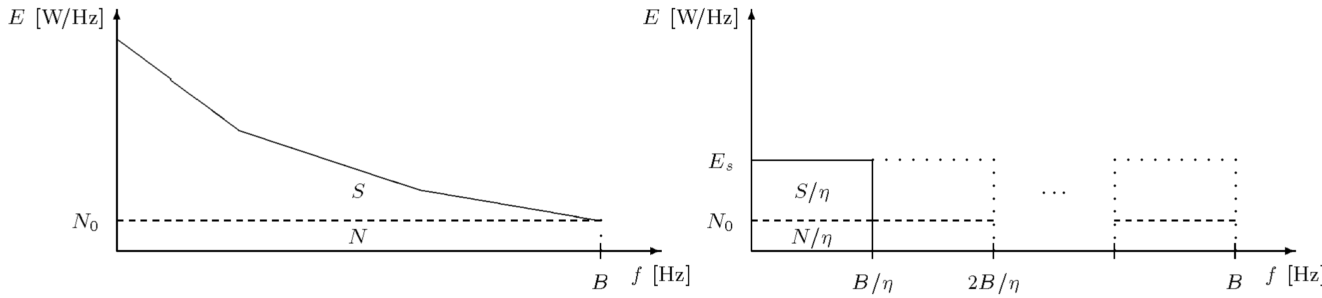

Figure 1. Transmission of analog signal (left) and signal compressed with a factor of η (right).

It is important to recognize that the compression ratio, η, that can be achieved for a given channel condition and image fidelity, depends on the digital compression technique, the mapping from source coder symbols to the modulation signal set, and the size of the modulator symbol alphabet.

How do we obtain a reduced signal bandwidth? Basically this is achieved by first removing signal redundancies and irrelevances from the signal, and then combining symbols which are mapped intelligently into a modulation signal constellation. Broadly speaking, the bandwidth reduction is obtained by lowering the sample rate through the symbol combination. As we shall see in the next section, it is important that source vectors that are close, in sense of the specified distortion measure, are mapped into symbols that are close in the modulation signal space. Transmission errors should ideally lead to reconstruction errors with the same visual character as does the parameter quantization error.

An example of reduction of the number of symbols is vector quantization where L symbols are mapped to an index space. If this index is transmitted as one symbol, we obtain a bandwidth reduction by a factor of L. Furthermore, it is imperative that the index space be arranged to minimize the effect of errors on the whole vector.

To design optimal compression systems it seems reasonable to transmit symbols rather than bits. This means that we may use e.g. PAM or quadrature amplitude modulation (QAM) with a fairly high number of states. A modulation scheme with an M-ary symbol alphabet can transmit $\log_2 M$ bits per symbol. This is well adapted to the system philosophy as high error rates are acceptable if the errors in general lead to transitions to neighboring signal amplitudes. Consequently, the mapping from the compressed source signal amplitudes to the transmission format is crucial, as will be demonstrated in the next section.

The definition of compression efficiency implies a tradeoff between e.g. quantization accuracy, or degree of error protection, and spectral efficiency of the modulation system. E.g. by reducing the quantization noise, i.e. allocate more bits to the quantizers, we increase the internal bit rate in the digital system. This may, however, be compensated for by enlarging the size of the modulator symbol alphabet correspondingly. E.g. by doubling the source coder bitrate, R, a modulator alphabet of twice the original size is needed to keep the bandwidth constant. That is, we can have several quantization strategies with corresponding modulation schemes, which all give the same transmission rate. Assuming that all these equal rate schemes satisfy the given channel conditions, i.e. is transmitted with the same power, the superior scheme is the one that gives the best fidelity of the received signal. Equivalently, this superiority is proved by comparing the digital transmission systems with the analog reference system to obtain the equivalent analog compression efficiencies for each of them. The superior scheme will have the largest compression efficiency as well.

For a given channel description (available bandwidth, received power, and noise level), the analog compression measure specifies the minimum bitrate, i.e. maximum obtainable compression for the digital system, when requiring the same fidelity on the reconstructed image as in the analog system. Better quality of the reconstructed signal can be obtained in two ways, for a given channel condition. Both approaches will, however, result in a reduction in the compression efficiency. By increasing the channel power, less channel errors will occur. This must, however, be compensated for by reducing the number of transmitted signals correspondingly, not to increase the total power above the maximum available, S_{max}. Thus, only a part of the total bandwidth, B, can be utilized. The other way of improving the signal quality is to expand the bandwidth of the compressed signal by increasing the source coder bitrate, or adding explicit error protection bits. Thus, in the first case the channel signal-to-noise ratio is improved at the cost of reducing the available bandwidth, while in the second case the signal-to-noise remains constant, though, at the cost of increasing the bandwidth of each signal. It is emphasized that in both cases the obtained compression is below the maximum obtainable compression ratio η at this specific channel signal-noise-ratio.

As integrated digital system design is becoming more important, this new definition gives a way of comparing the compression efficiency for jointly optimized systems. The definition incorporates essential resources and performance measures in such systems[1]. The compression efficiency can, of course, also be computed in the same manner for complete, separately optimized, digital systems.

3. AN IMAGE TRANSMISSION EXAMPLE

A block diagram for the image transmission system is shown in Figure 2. The subband coder optimized for error-free transmission conditions transforms the input image samples, r, into 8×8 subbands.

The lowpass-lowpass band is quantized with a 5-bit DPCM-encoder with fixed predictor configuration and transmitted without errors. The subband samples, x, from the higher bands are encoded with scalar Laplacian quantizers, based on blockwise, adaptive bit allocation with 6 bit classes. The bit allocation table is transmitted without errors. The quantized subband samples, y_i, are combined to vectors each represented by 6 bits and mapped to indices, s_i, in a 64-point signal space[2].

[1]Two primary communication resources are: available channel bandwidth and received power. Not taking system complexity, delay, etc. into account, the performance optimization criterion is the subjective quality of the received signal, given channel constraints (noise) and resources.

[2]E.g. four quantized subband samples, $y_0, \cdots, y_3$, allocated 2, 2, 1, and 1 bit, respectively, may be combined to a vector and represented by a 6 bit index s_i.

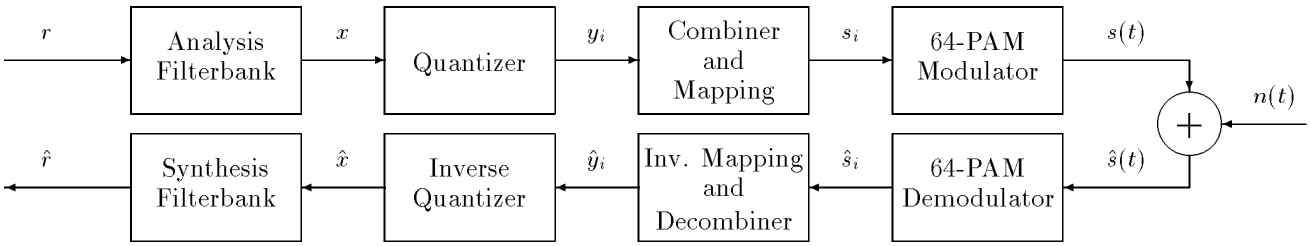

Figure 2. General system block diagram.

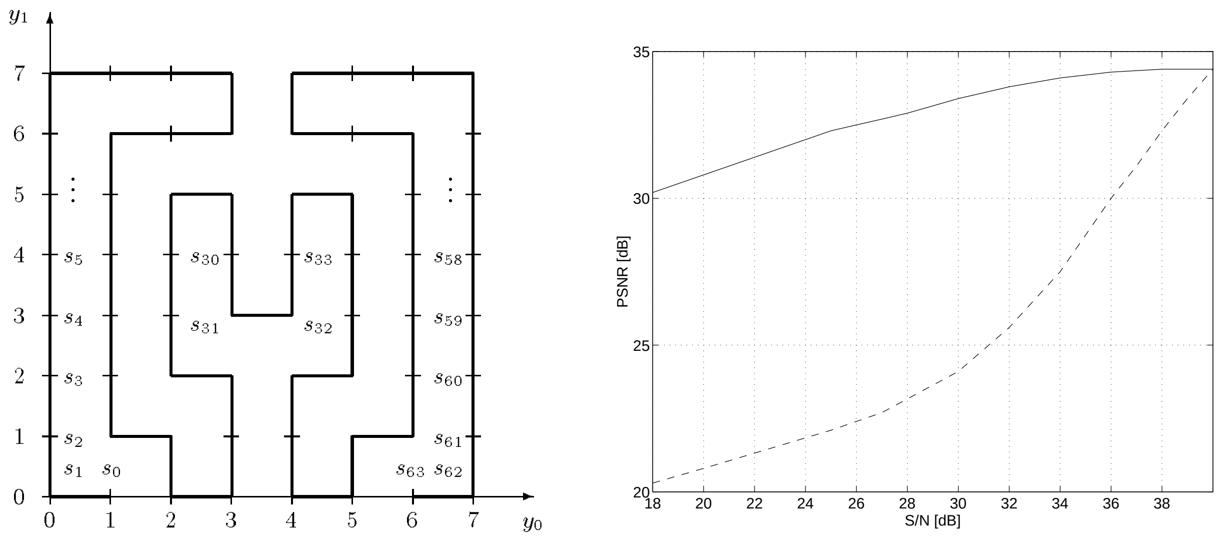

Figure 3. Left: Mapping of two three bit indices y_0 and y_1 to a 64-point signal space. Right: Performance results ("Lena" at 0.5 bit per pixel); random mapping (- - -) and intelligent mapping (—–).

The PAM modulator maps the indices, s_i, into analog signals, $s(t)$, in a uniform 64-PAM symbol alphabet. The PAM-symbols are transmitted over an additive, white Gaussian noise (AWGN) channel. The demodulator performs maximum likelihood detection and the detected samples are mapped back into the index space before inverse quantization and image reconstruction.

Note that the index combining, and decombining at the receiver side, is unambiguously given by the bit allocation table. Thus, no additional side information is required to reconstruct the quantization indices, $\hat{y}_i$, at the receiver.

As mentioned earlier, a crucial design problem is the mapping between the quantized and combined (i.e. compressed) source samples and the PAM format. It is important that the most probable error transitions on the channel result in minimal error in the reconstructed image. This is obtained by optimizing the mappings with respect to minimum transmission power and good neighboring conditions in the channel signal constellation [6, 7]. The mappings have to be optimized for each possible combination of number of bits in the quantized samples. An example of the mapping of two samples y_0 and y_1, each allocated 3 bits, into indices in a signal alphabet of size 64 is shown in Figure 3 (left).

The distortion in the reconstructed image is measured as the ratio of peak signal power to mean square error reconstruction noise power, i.e. peak signal-to-noise ratio (PSNR). The performance results for an intelligent mapping scheme compared to a reference straightforward mapping scheme [6] are also provided in Figure 3 (right). It is emphasized that these mappings are used on an image source signal ("Lena") in contrast to [2, 3, 4, 5] where artificial (Gaussian and Gauss-Markov) sources are used.

In order to calculate the digital transmission system's compression ability, an analog reference image is needed. In lack of continuous valued images we define the original, 512×512 pixel, 8bit, grey tone image "Lena" as the "analog" reference signal. Each pixel value is mapped to one analog PAM symbol and transmitted over an AWGN channel with bandwidth B and signal-to-noise ratio S_{max}/N. The minimum needed bandwidth, B, is given by half the source symbol rate R_s. The experiment was repeated for several noise power levels, $N = N_0 \cdot B$. For this specific image size 512^2 channel symbols are needed per frame. The fidelity of the reconstructed analog image as a function of channel quality is plotted as a reference curve in Figure 4. Due to the two different ways of measuring image and channel quality (peak versus mean signal-to-noise ratio), the curve is drawn with a constant offset at approximately 12.5 dB.

To evaluate the compression efficiency of the digital system, the "Lena" image is processed by the transmission system shown in Fig 2. The image, coded at various bit rates, R, is transmitted over an AWGN channel. The results for various channel signal-to-noise ratios, $\frac{S}{\eta}/\frac{N}{\eta}$, are shown in Fig 4. The compression efficiency is found by noticing where, for a given source coder bit rate, R, the reference curve and the curve for the digital system are crossing each other. At this point the analog reference signal as well as the digitized signal have been received with the same constant signal power per bandwidth $\frac{S_{max}}{\eta}/\frac{N}{\eta} = S_{max}/N = \nu$ and reconstructed with the same fidelity, as measured by the chosen PSNR criterion. The equivalent compression

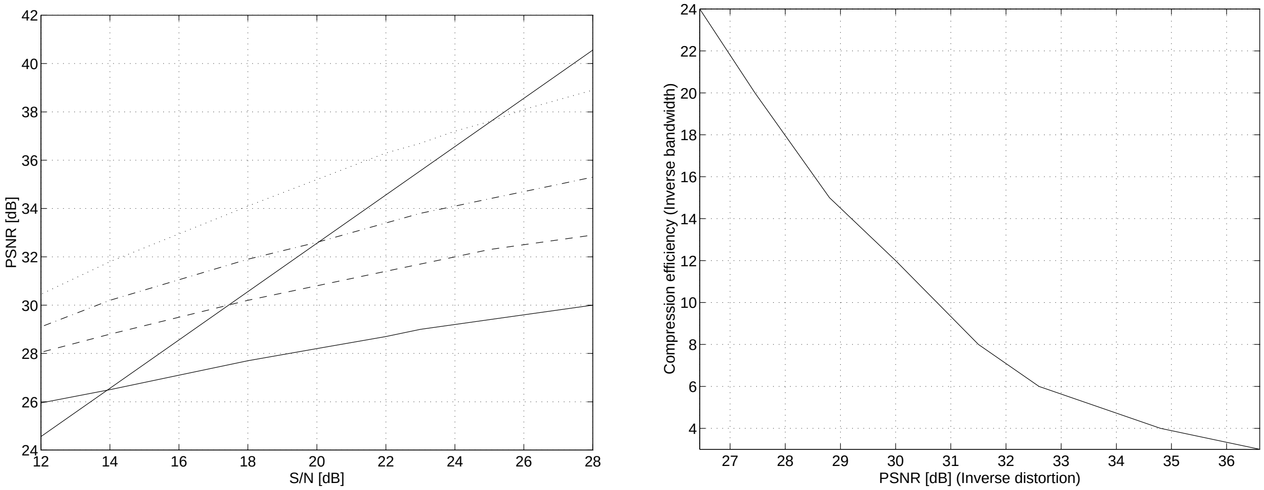

Figure 4. Left: Performance results; analog reference signal (8 bit per pixel) (——) and signal with 2.0 bit per pixel (······), 1.0 bit per pixel (–·–·–), 0.5 bit per pixel (– – –) and 0.25 bit per pixel (——). Right: Compression efficiency as a function of the fidelity of the received image.

efficiency for a given bit rate is found by noticing that $\log_2 64 = 6$ bits per sample is transmitted in a 64-PAM scheme. E.g. for a source coder bit rate of $R = 0.5$ bit/pixel, a compression of $\eta = 6/0.5 = 12$ is obtained. That is, for the same channel conditions, only one twelfth of the pure analog system symbol rate, R_s, is needed for the same channel, or equivalently a bandwidth of $B/12$.

In the right part of Figure 4 the achievable analog compression efficiencies are shown as a function of the fidelity of the received image signal. Each point on the curve represents the obtainable compression ratio for a specific image fidelity. The compression efficiency curve can be interpreted as a function describing the possible reduction in bandwidth for the digital system when the fidelity in the received image and the channel conditions are the same as for the analog reference system. It is emphasized that each point on the curve represents a unique channel quality. An equivalent interpretation is that the curve represents an inverse rate (bandwidth), inverse distortion ($R^{-1}(D^{-1})$) function.

The transmitted and decoded image coded at 1.0 bit per pixel has only slight visual degradations[3] (32.6 dB). We may transmit 6 such signals at the bandwidth B of the original image without increasing the required power consumption. This particular system thereby has a compression efficiency $\eta = 6$ at this image fidelity. As it is assumed that the lowpass-lowpass band is transmitted error-free, an increase in internal bit rate must be expected to protect this particular part of the image signal. The bitrate of the side-information representing either local power levels or bit allocation tables is neither taken into account. All these factors would decrease the compression efficiency of this example. The principle, however, is correct.

Results presented in [7] show that improved performance can be expected, for compression ratios below approximately 10, if 64-PAM is exchanged with 64-QAM.

4. CONCLUSION

A new analog interpretation of digital compression and modulation as a bandwidth reducing device is presented. Integrated system design is becoming more important and this new definition gives a way of measuring the overall compression performance for a given digital system as compared to a pure analog reference system. The definition is tightly connected to the analog channel characteristics, in contrast to traditional compression measures which are based on reduction in source coder bit rate without taking the channel characteristics into account. An image transmission example is presented which illustrates the new definition. Given the channel descriptions and the required fidelity of the reconstructed image, the equivalent analog compression ratio that the overall digital system can provide, is found.

[3] It is, however, emphasized that the characteristics of the noise in the reconstructed image are not quite equal in the two cases because the subband coder introduces artifacts like blurring and ringing, while the pure analog transmission only gives white noise in the resulting image.

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