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Uncertainty associated with model predictions of surface and crown fire rates of spread

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ABSTRACT

The degree of accuracy in model predictions of rate of spread in wildland fires is dependent on the model's applicability to a given situation, the validity of the model's relationships, and the reliability of the model input data. On the basis of a compilation of 49 fire spread model evaluation datasets involving 1278 observations in seven different fuel type groups, the limits on the predictability of current operational models are examined. Only 3% of the predictions (i.e. 35 out of 1278) were considered to be exact predictions according to the criteria used in this study. Mean percent error varied between 20 and 310% and was homogeneous across fuel type groups. Slightly more than half of the evaluation datasets had mean errors between 51 and 75%. Under-prediction bias was prevalent in 75% of the 49 datasets analysed. A case is made for suggesting that a ±35% error interval (i.e. approximately one standard deviation) would constitute a reasonable standard for model performance in predicting a wildland fire's forward or heading rate of spread. We also found that empirical-based fire behaviour models developed from a solid foundation of field observations and well accepted functional forms adequately predicted rates of fire spread far outside of the bounds of the original dataset used in their development.

1. Introduction

Wildland fire behaviour is broadly defined as the manner in which fuel ignites, flame develops, fire spreads and exhibits other related phenomena as determined by the interactions of fire with its environment – i.e. fuels, weather and topography. The immediate needs of fire operations personnel with respect to fire behaviour information can be decidedly different from the interests of fire researchers. Nevertheless, as Van Wagner (1985) has stated, "If one could boil down the whole science of fire behaviour to its practical essence, it might be to put in the hands of the fire boss a decent estimate of how fast his newly-reported fire will advance" (Fig. 1). In this respect, the knowledge of a free-burning fire's rate of spread (Albini, 1984) is often central to being able to compute or estimate other fire behaviour characteristics (Fig. 2).

Models for predicting rate of fire spread and other characteristics of behaviour are typically distinguished on the basis of three broad categories: (i) physical, (ii) empirical or (iii) semi-empirical models (Sullivan, 2009a,b). Physical or process-based models are mostly developed with theoretical purposes in mind, aiming to better understand the physical and chemical processes controlling fire propagation. The justification for empirical or semi-empirical models is largely to support a decision making process. Emphasis on the purpose and perfection of the process description is not necessarily sought (Alexandrov et al., 2011).

Irrespective of the model approach taken, a pertinent question facing any wildland fire behaviour modeller is: how accurately can one expect to predict the spread rate of a wildland fire with currently available models? The aim of this study was to address this question by examining error statistics associated with studies that have used independent datasets derived from field observations as means of evaluating the performance of models used in the prediction of surface and crown fires rates of spread for operational decision-making or as planning and research tools. Given the existing evidence we also wished to determine what should be considered an acceptable error.

2. Background information

2.1. Predicting wildland fire rate of spread

When observed closely, a free-burning fire spreads through highly variable and chaotic motions, although if one considers the

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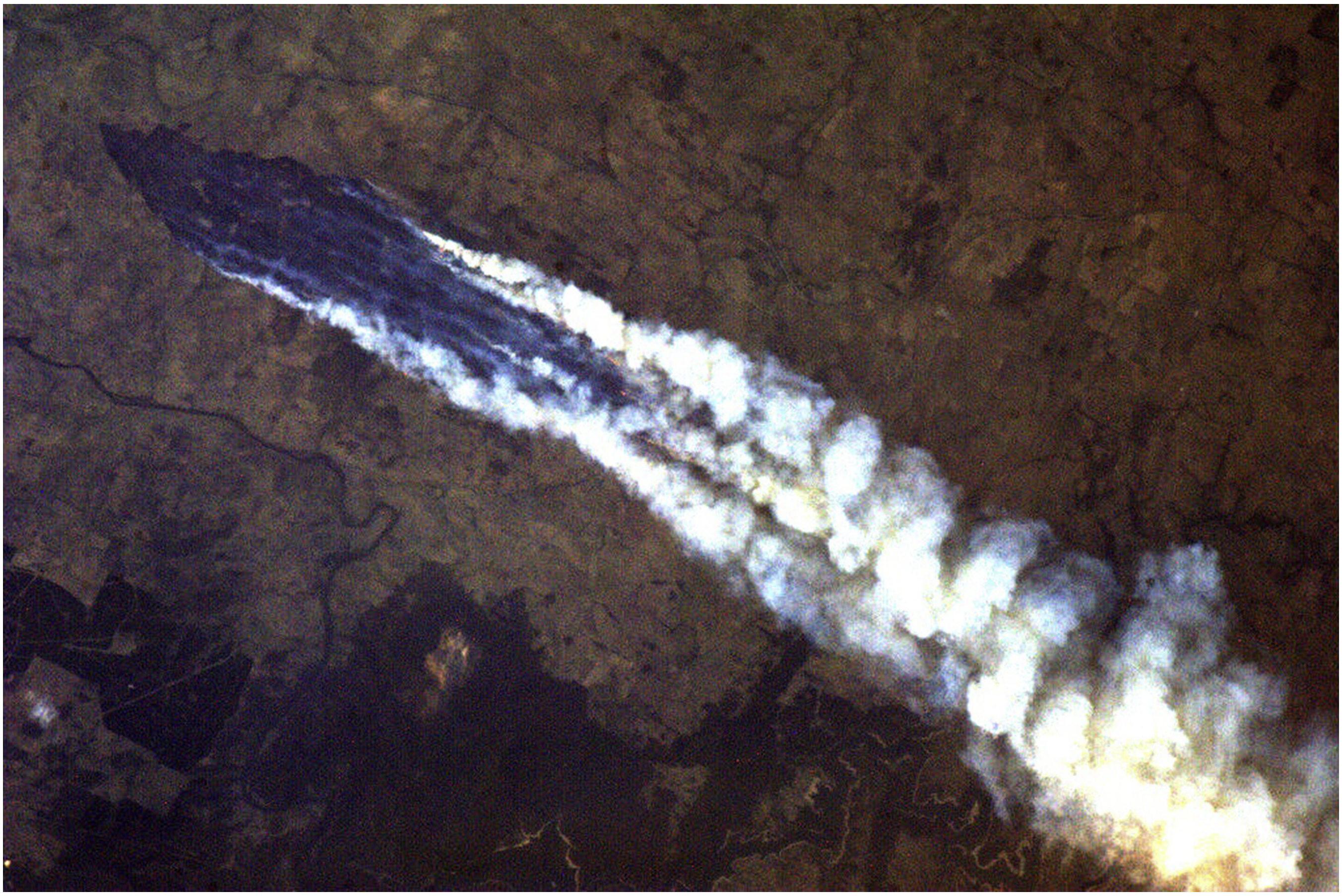

Fig. 1. The view from space of the Cobbler Road Fire near Yass, New South Wales, Australia, spreading through fully-cured grasslands on the afternoon of 8 January 2013 under the influence of exceptionally strong average winds (~50 km/h). Wind-driven fires typically exhibit very elongated, elliptical shapes in such situations. Photo credit: Chris Hadfield/NASA.

time scales of practical interest, spread can be taken as effectively continuous, giving rise to the concept of a 'pseudo-steady' fire propagation state. In near-real time forecasting of wildland fire behaviour, presently the objective is to be able to predict the spread rate of a fire propagating at this pseudo-steady state over time intervals of 30 min or longer (Rothermel, 1983, 1991; Andrews et al., 2007; Cheney and Sullivan, 2008).

Empirical-based models developed from experimental fires carried out under field conditions and covering a broad range of fuel complexes (from open grasslands to conifer and eucalypt forests) and weather conditions are typically but not always (e.g. Burrows et al., 2009) able to predict the source dataset with mean absolute percent errors between 20 and 40% (Fig. 3). The main sources of error in model predictions of wildland fire behaviour are considered to be a lack of model applicability, internal inaccuracy, and data input errors (Albini, 1976; Alexander and Cruz, 2013b). The error is expected to be higher when the models are applied to predict fire spread rates in an operational setting due to the natural variability in fuels and uncertainties in forecasted weather conditions over broad spatial and temporal scales (Rothermel, 1983). It is also expected that in general terms, fire behaviour data collected in an operational setting has a higher degree of uncertainty due to the logistical and time constraints to set up measuring equipment and directly observe fire behaviour.

2.2. Variability in rate of fire spread

The above mentioned wide variability in fire behaviour in time and space, in even the most homogenous environments, led Rothermel (1983) to point out that it is quite unlikely that the minute-by-minute movement of a fire will ever be accurately predictable with any degree of certainty in the foreseeable future. This is largely due to the capricious nature of the prevailing surface winds (Albini, 1982; Cheney et al., 1993; Sullivan and Knight, 2001), horizontal and vertical fuel heterogeneity (Hiers et al., 2009), chaotic nature of turbulent flow driving the fire propagation processes (Clark et al., 1999), and the dynamic feedback mechanisms associated with the fire and the surrounding environment (Nelson et al., 2012). These fine scale variations in the drivers of fire propagation are the cause of an apparent paradoxical phenomena where more accurate predictions are made in forecasting the spread of 15–30 min fire runs with average wind speeds then making predictions for short spread durations (i.e. 1–3 min) based on the nearby measured wind speed (Cheney et al., 1993).

Detailed measurements of rate of spread in experimental fires have revealed unsteady fire behaviour and high variability over short time periods but consistency over longer time periods (Fig. 4). Several authors have described how fluctuations in wind speed and direction (Crosby and Chandler, 1966), and subtle changes in fuel structure can lead to dramatic changes in fire spread (Anderson et al., 1982; Cheney and Gould, 1995; Fernandes et al., 2000, 2004). Cruz et al. (2013), for example, quantified fire spread variability in 200–400 m long experimental fire runs in shrublands, finding maximum rates of spread to be 1.8 to 5.9 faster than the average, and the coefficient of variation (i.e. the ratio of the standard deviation to the mean expressed as a percentage) varying between 56 and 167. Similarly, Taylor et al. (2004) found maximum rates of spread to be 1.6–2.9 times the average in 75–

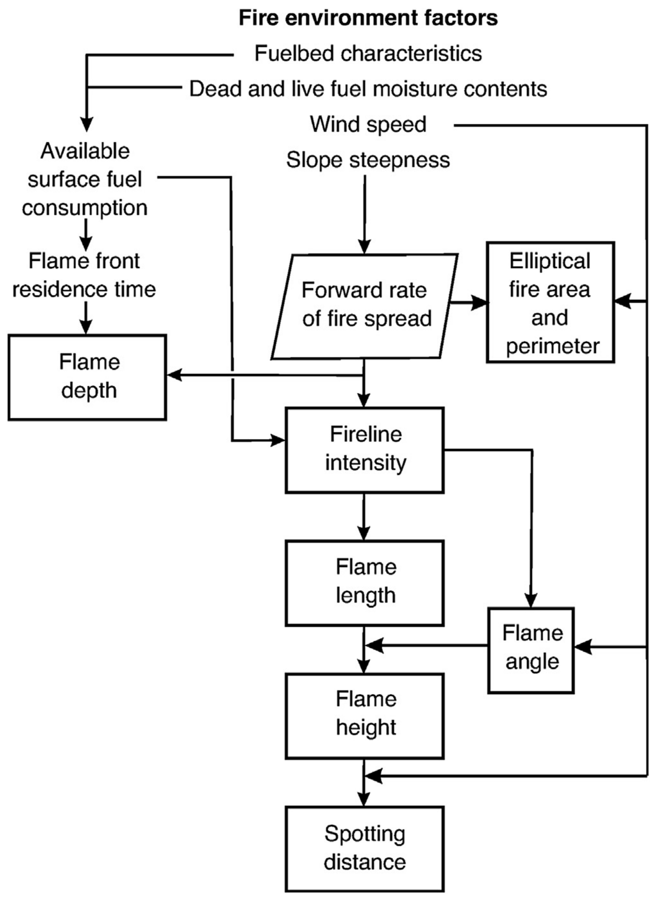

Fig. 2. Flow chart illustrating the linkages that forward rate of fire spread has to the flame front dimensions and other characteristics of surface fire behaviour. The same flow processes apply to crown fires but with the addition of available canopy fuel consumption to the determination of fireline intensity.

150 m crown fire runs in conifer forests. A measure of the inherent fine-scale variability in wildland fires is also observable in laboratory fires, where even under constant environment conditions with replicated fuelbeds, there can be as much as ±20% unexplained variation in observed spread rates (Catchpole et al., 1993, 1998).

Given the variability in rate of fire spread associated with variations in winds, fuels, and fire – wind field interactions, an experimental fire run that does not extend over at least three or four gust–lull cycles might not be representative of the average wind speed measured nearby. As a general rule of thumb, Canadian fire behaviour researchers have ideally wanted experimental fires to burn for at least 10 min (Alexander and Quintilio, 1990). Linn et al. (2012) used a physics-based numerical model and data from experimental crown fires (Stocks et al., 2004) to highlight these time and space scale issues surrounding the effects of transient wind and the required length of time to adequately sample an experimental fire run.

3. Methods

Data were compiled for as many fire behaviour model performance studies as possible. This relied upon the authors' combined knowledge of the relevant literature and their personal reference collections. The resultant sources included scientific peer-reviewed journal articles, conference and workshop papers, technical reports from government agencies, and postgraduate university theses.

The principal requirement for inclusion of a given study was that the evaluation data, collected on outdoor experimental fires, operational prescribed fires and (or) wildfires, involved a single head-fire line source ignition pattern similar to a free-burning wildfire (Fig. 1), as opposed to a point-source fire(s) or strip-head fires (Wade and Lunsford, 1989). Furthermore, the dataset could not have been used in the model development (i.e. the predictions are being compared against completely independent observational data).

Experimental fires carried out in fuelbeds or with head-fire widths judged too narrow to yield realistic pseudo-steady state rates of spread (Anderson, 1968; Cheney et al., 1993; Wotton et al., 1999) were excluded (e.g. Brown, 1972; Lindenmuth and Davis, 1973; Davies et al., 2009), including laboratory studies (e.g. McAlpine and Xanthopolous, 1989; Catchpole et al., 1993; Weise et al., 2010). To ensure that each study included in the analysis had sufficient data to discern model adequacy for the particular fuel type and burning conditions, we restricted the analysis to studies with more than five paired observations. Thus, the studies by Butler and Reynolds (1997), Hély et al. (2001), Streeks et al. (2005), and Stephens et al. (2008) for example were excluded from the analysis. Finally, studies where model outputs were not fully independent, as a result of authors fine tuning model predictions in relation to the observed rates of fire spread (e.g. Norum, 1982), were also not selected for the analysis.

The calculated error statistics for rate of fire spread were the root mean square error (RMSE), the mean absolute error (MAE), the mean absolute percent error (MAPE), and the mean bias error (MBE) (Appendix A). In most cases, these statistics were not reported so we computed them from the data contained in the associated publication or by contacting the investigators directly for the data.

The percentages of under-, exact, and over-predictions were also calculated. To our knowledge, no definition of an exact prediction exists. For the purposes of this study, an exact prediction was arbitrarily defined as one where the error was less than ±2.5% of the observed rate of fire spread (i.e., a 5% error band around an observed value).

The methods used to measure environmental and fire behaviour variables varied considerably between studies. As a result, it is expected that the data reliability or quality used in the evaluation of models in the various studies will vary accordingly. The largest differences in data reliability are expected to be found between data collected in experimental fire programs and wildfire situations. Experimental studies are characterized by detailed sampling of fuel structure, weather and fire behaviour (e.g. Gould, 1994; Stocks et al., 2004; Cheney et al., 2012) whereas wildfire case studies tend to rely on broad assumptions regarding fuels and representativeness of weather data (e.g. Alexander and Cruz, 2006; Athanasiou and Xanthopoulos, 2010). An excellent discussion regarding the reliability of wildfire behaviour data can be found in Gould et al. (2011) and Cheney et al. (2012). Even within experimental fire studies it is expected to find wide variation in data quality arising from the sampling intensity of environmental variables, methods used to measure rate of fire spread (e.g. visual observation, video recording, tags, thermocouple grids, infrared imagery) and the length of the fire run. The paucity of information on matters of data quality included in the published studies did not allow for inclusion of qualitative or quantitative variables describing data reliability in our analysis as Cheney et al. (2012) was able to undertake for example.

We tested for significant differences in over- and under-prediction errors among (i) type of data (experimental fire, prescribed fire and wildfire) and (ii) fuel type using analysis of variance (ANOVA). If statistically significant ($p < 0.05$) ANOVA results were found, the Scheffé S multiple comparison test (Scheffé, 1959) was used to determine which particular means differ.

4. Results

Our search for fire spread model performance studies resulted in the identification of 49 suitable studies for a total of 1278 individual rate of spread observation–prediction pairs. Each individual data pair comprises the rate of spread measured for an individual fire and a model prediction associated with the prevailing environmental conditions during the period of fire spread. Most studies involved comparison of observed spread rate with a single predictive model, whereas others evaluated multiple models (e.g. Burrows, 1994; McCaw et al., 2008). The data covered a wide range of fuel complexes and fire propagation regime types (i.e. surface fire, passive crown fire, active crown fire). The broad fuel type groups consisted of grasslands ($n = 6$), shrublands ($n = 9$), logging slash ($n = 3$), conifer forest ($n = 17$), hardwood forest ($n = 3$), mixedwood forest ($n = 2$), and eucalypt forest ($n = 9$). The studies were geographically distributed as follows: Africa ($n = 3$), Australia ($n = 14$), Europe ($n = 3$), and North America (i.e. Canada and US; $n = 29$).

The error statistics associated with the comparison of model predictions of rate of spread versus observations as found in the various studies are presented in Table 1, with the results being stratified by MAPE. A selection of the studies reported in Table 1 is

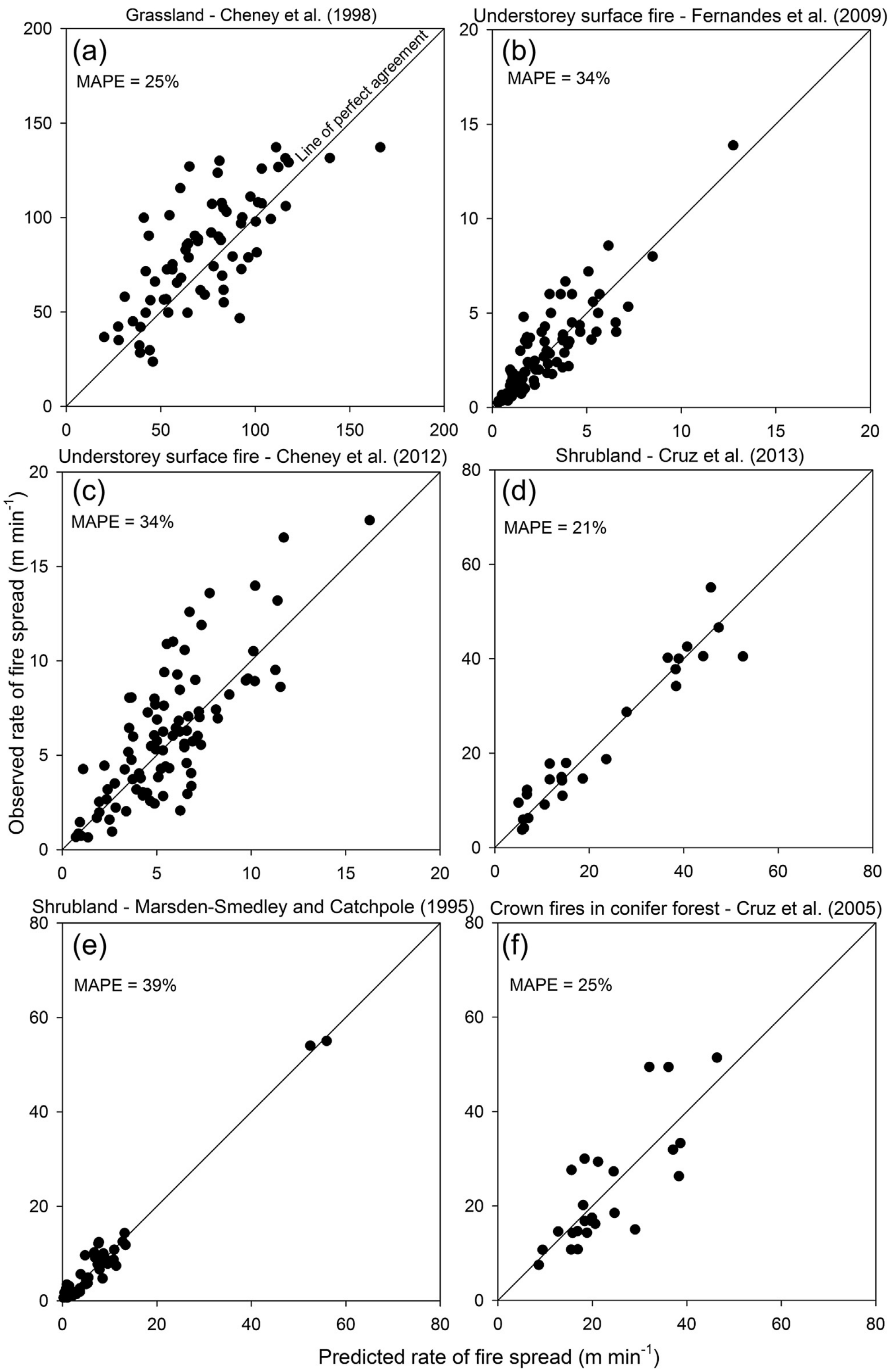

Fig. 3. Observed rates of spread used in model development versus model predictions for experimental surface fires in (a) grasslands fires (Cheney et al., 1998), (b) surface fire in maritime pine forest (Fernandes et al., 2009), (c) surface fire in dry eucalypt forest (Cheney et al., 2012), (d) surface and crown fires in mallee-heath shrublands (Cruz et al., 2013), (e) buttongrass moorlands (Marsden-Smedley and Catchpole, 1995), and (f) crown fires in conifer forests (Cruz et al., 2005). MAPE = mean absolute percent error.

also graphically portrayed in Figs. 5 and 6. All of the studies associated with Table 1 deal with empirical or semi-empirical models. A brief description of the 13 rate of fire spread models associated with Table 1 is given in Table 2. Comparisons involving Rothermel's (1972) surface fire spread model figured into 30 of the 49 comparisons. The omission of any physics-based fire behaviour model comparisons from Table 1 simply reflects the fact that there has been minimal evaluation against empirical data to date (Alexander and Cruz 2013a).

The lowest MAPE values given in Table 1 varied between 20 and 30%, involving seven comparison studies. These errors were associated with experimental fire and prescribed fire studies where all

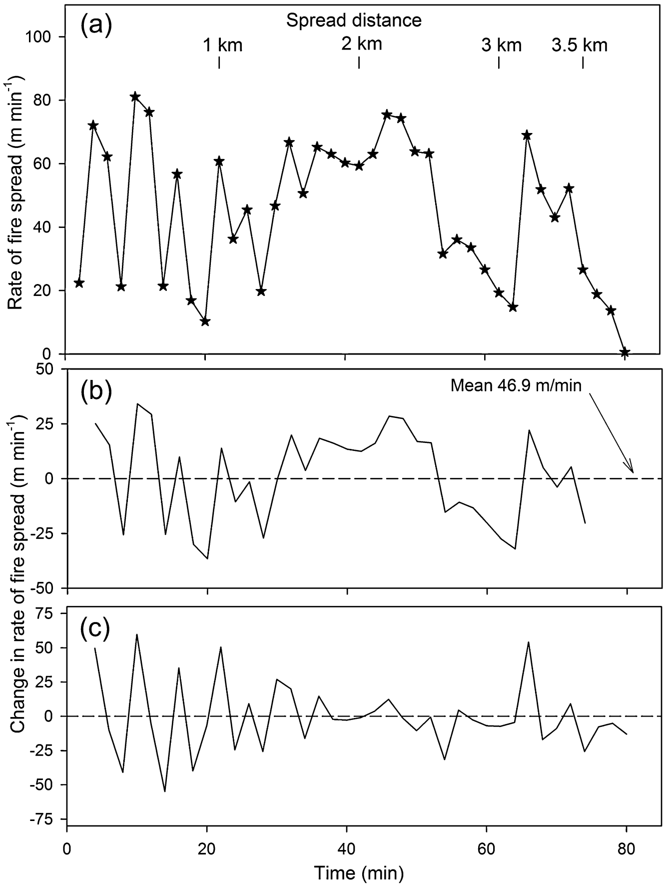

Fig. 4. Variability in head fire rate of spread at 2-min intervals versus elapsed time since ignition for an experimental fire in grassland (spinifex) fuels in Western Australia (adapted from data presented in Burrows et al., 1991): (a) rate of fire spread as a function of time from ignition to extinction; (b) changes in rate of fire spread about the mean (46.9 m min^{-1}); and (c) change in rate of spread from the previous 2-min time interval.

the fuel and weather variables were measured on site. Approximately half of the studies listed in Table 1 (i.e. 25 out of the total of 49) had a MAPE between 51 and 75%. For comparisons dealing exclusively with wildfire data, the MAPE values varied between 33 and 59%.

Considering the overall dataset, no significant differences were found for under- and over-prediction errors by the type of data source (i.e. experimental fire, prescribed fire, or wildfire) (Table 3). Only two models associated with Table 1, Cruz et al. (2005) and Cheney et al. (2012), were separately evaluated against experimental fire and wildfire datasets. The models predicted lower average errors for the experimental datasets (i.e. 26 and 35% MAPE respectively) than for the wildfire datasets (i.e. 46 and 52% MAPE for Cruz et al., 2005; 54% MAPE for Cheney et al., 2012). This increase in mean error is expected given the uncertain nature of the exact environmental conditions driving the propagation of the various wildfires. However, this is not identifiable in the overall analysis between the types of fires presented in Table 3 because of the wide range of models tested and the variability in the quality of data involved. Noteworthy, in this evaluation against wildfire data both models were extended beyond the bounds of dead fuel moisture content, wind speed, and rate of fire spread present in the datasets used in their development. As an example, the maximum rates of fire spread in the model development datasets used in Cruz et al. (2005) and Cheney et al. (2012) were 51.4 and 22.7 m min^{-1} respectively. Fig. 6d and f does not show evidence of bias or loss of predictability above these values for the Cruz et al. (2005) and Cheney et al. (2012) models, respectively. This suggests that the underlying functional relationships for the effect of dead fuel moisture content and wind speed in these models are valid for more severe burning conditions (i.e. drier and windier), than the ones used in the model development.

Analysis of variance identified significant differences for mean under- and over- prediction errors among fuel types. Table 4 provides the mean and standard deviation for under- and over-prediction errors by fuel type. Although one can observe marked differences in the number of fires and the range in rate of spread, the error statistics can be considered somewhat similar. For the under-prediction cases, the multiple comparison method only found differences ($p < 0.05$) between the grasslands and logging slash subsets. For the over-prediction cases, the multiple comparison method showed the mean error for the grassland fires to be significantly higher ($p < 0.05$) from the shrubland and eucalypt forest fuel types. No significant differences were observed for all other possible fuel type group combinations. The highest error found for the grassland fires was the result of the Burrows et al. (1991) evaluation of the Griffin and Allan (1984) model, where the average under-prediction error was found to be 217%.

Another important error component to consider with respect to model performance is its inherent bias. An interesting outcome of our analysis is that models are more likely to under-predict than over-predict field observations (Fig. 7; Tables 3 and 4). Of the 1278 individual model predictions versus observations, under-prediction occurred in 64% of the cases. Mean under-prediction was 49% (standard deviation of 24%) of the observed rate of fire spread. Mean over-prediction was 89% (standard deviation of 125%) above the observed value. Average under-predictions are lower than over-predictions because the maximum under-prediction is bound to 100% of the observed value, whereas the over-prediction can be several times the observed value. When analysing under- and over-prediction trends at the study level, under-predictions were prevalent in 76% (i.e. 37 out of 49) of the model comparisons given in Table 1. Noteworthy, the above mentioned model evaluation against wildfire datasets did not produce an under-prediction bias (Fig. 6d and f). Although the range in rate of spread in the independent wildfire datasets was much higher than in the datasets used in model development (e.g. Fig. 3c v. Fig. 6f and Fig. 3f v. Fig. 6d), the model structure allowed for consistent predictions over the full range of observed fire behaviour.

In most of the cases, the under-prediction bias was small and not easily discernible. However, a careful analysis revealed that there were combinations of fire spread model – fuel type group that result in a predominant, if not total, under-prediction bias (e.g. Figs. 5b, 6b and 6e).

Further examination of the results reported in Table 1 show that the concept of an exact prediction of rate of fire spread is an elusive one. One could argue that perhaps the only certainty about wildland fire behaviour predictions is that it is extremely unlikely that a prediction will exactly match the observed fire behaviour characteristic as readily evident from the information presented in Table 1. Only 3% of the predictions (i.e. 35 out of 1278) were considered to be exact predictions according to the criteria used in this study.

5. Discussion

There are at least four implications for fire research and management emanating from the results of this study that are worthy of discussion. These can be posed as questions to be addressed: Is the spread rate of crown fires more difficult to predict than for surface fires? What would constitute an acceptable error for evaluating

Table 1
Summary of statistics and related information associated with studies that have relied upon independent data derived from field observations in order to evaluate the performance of models used to predict the rate of spread (ROS) of wildland fires. ROS models: AGM62 = McArthur (1962); CAW05 = Cruz et al. (2005); CGMA12 = Cheney et al. (2012); CMAG13 = Cruz et al. (2013); FCCS = Schaaf et al. (2007); FFBT = Sneejuwagt and Peet (1998) Forest Fire Behaviour Tables; FFDM = McArthur (1967) Forest Fire Danger Meter; FBPS = Forestry Canada Fire Danger Group (1992) Fuel Type C-4 (Immature Jack or Lodgepole Pine); GA84 = Griffin and Allan (1984); GFDM = McArthur (1966) Grassland Fire Danger Meter; MSC95 = Marsden-Smedley and Catchpole (1995); RCR72 = Rothermel (1972); and RCR91 = Rothermel (1991). Type of data: E = experimental fire; P = prescribed fire; and W = wildfire. Type of fire: S = surface fire; PC = passive crown fire; and AC = active crown fire. Fuel type group: G = grassland; S = shrubland; LS = logging slash; CF = conifer forest; HF = hardwood forest; MF = mixedwood forest; and EF = eucalypt forest. Error statistics: RMSE = root mean square error; MAE = mean absolute error; MAPE = mean absolute percentage error; and MBE = mean bias error (see Appendix A). U − E − O predictions: U = Under-prediction; E = Exact prediction (defined as a prediction with an error of less than ±2.5%); and O = Over-prediction.

Reference	ROS model	Type Of Data	Type of fire	Fuel type group	No. of fires	ROS Range (m min^{-1})	RMSE	MAE (m min^{-1})	MAPE (%)	MBE	U − E − O predictions (%)
MAPE 30% or less											
McCaw (1997)	RCR72	E	S, AC	S	13	7.68–66	6.58	5.18	20	−1.8	54 − 0 − 46
Brose (1997)	RCR72	E	S	HF	15	0.1–2.5	0.12	0.09	20	0.03	40 − 33 − 27
P. Hefner −Rothermel and Rinehart (1983)	RCR72	P	S	G	15	0.9–150.9	17.9	13	22	−0.10	53 − 14 − 33
Hough and Albini (1978)	RCR72	E	S	CF	31	1.9–14.2	1.78	1.5	26	0.9	71 −0 − 29
Cruz et al. (2005)	CAW05	E	AC	CF	10	22.3–70.1	14.5	11.4	26	7.7	70 − 10 − 20
Marsden-Smedley and Catchpole (1995)	MSC95	E, W	S	S	9	1.1–8.7	0.93	0.79	27	0.2	56 − 0 − 44
van Wilgen et al. (1985)	RCR72	E	S	S	10	2.4–53.4	7.18	6.17	30	−2.1	43 − 0 − 57
MAPE between 31 and 50%											
Cheney et al. (2012)	CGMA12	E	S, PC, AC	EF	16	2.5–16	2.86	2.16	35	0.03	50 − 0 − 50
Everson et al. (1988)	RCR72	E	S	G	40	1.8–93.6	14.35	8.44	39	2.5	50–13 − 37
Cruz and Fernandes (2008)	RCR72	E	S	CF	17	0.8–6.1	1.13	0.75	42	0.4	65 − 0 − 35
Schaaf et al. (2007)	FCCS	W	AC	CF	15	10.7–90	22.2	15.2	42	15.0	93–0 − 7
Gould (1991)	RCR72	E	S	G	75	19.2–124.2	35.4	29.5	44	11.0	71 − 3 −26
Athanasiou and Xanthopoulos (2010)	RCR72	W	S	S	27	1.7–55.2	8.54	6.15	44	−1.7	33 − 8 − 59
Gould (1994)	AGM62	P	S	EF	37	0.43–3.91	0.84	0.61	45	0.6	89 − 0 − 11
Cronan and Jandt (2008)	CAW05	W	PC, AC	CF	5	4.8–43.1	9.27	7.1	46	−6.3	20 − 0 − 80
Tolhurst et al. (1992)	AGM62	P	S	EF	41	0.1–2.5	0.41	0.26	48	0.12	59 − 2 − 39
MAPE between 51 and 75%											
Masters and Engle (1994)	RCR72	P	S	MF	6	8.3–20	8.9	7.87	52	7.9	100 − 0 − 0
Alexander and Cruz (2006)	CAW05	W	AC	CF	57	10.7–107	18.9	14.9	52	−6.6	30 − 0 − 70
Sneeuwjagt and Frandsen (1977)	RCR72	E, W	S	G	42	0.2–61.0	10.8	3.4	53	−2.1	55 − 7 − 38
Cruz et al. (2013)	CMAG13	E, P, W	S, AC	S	12	7.5–125	22.3	18.2	53	3.1	75 − 0 − 25
van Wagtendonk and Botti (1984) − f[a]	RCR72	P	S	S	16	2.6–36	10.46	6.59	53	4.8	56 − 6 − 38
Grabner (1996);Grabner et al. (1999)	RCR72	E	S	HF	28	0.3–10.1	2.06	1.38	53	0.6	57 − 0 − 43
Cruz et al. (2005)	RCR91	E	PC	CF	14	3.35–15.8	5.2	3.96	53	1.2	50 − 7 − 43
Burrows (1994,1999)	FFBT	E	S	EF	35	0.25–9.9	1.27	0.84	54	0.4	51 − 0 − 49
Stocks et al. (2004)	FBPS	E	PC, AC	CF	11	15.8–69.8	26.5	23.4	54	23.4	100 − 0 − 0
Cheney et al. (2012)	CGMA12	W	PC, AC	EF	25	10–175	41.05	26.4	54	−6.8	32 − 0 − 68
McCaw et al. (2008)	FFBT	E	S, PC	EF	97	0.24–19.4	4.94	3.78	55	3.6	85 − 2 − 13
Burrows (1994,1999)	FFDM	E	S	EF	35	0.25–9.9	1.78	1.13	56	0.89	66 − 0 − 34
Bevins (1976)	RCR72	E	S	LS	9	0.2–4.5	0.903	0.59	57	0.3	45 − 22 − 33
van Wagtendonk and Botti (1984) − b[a]	RCR72	P	S	CF	18	1.0–4.3	1.54	1.23	57	1.2	94 − 0 − 6
Lawson (1972)	RCR72	E	S	CF	8	0.86–2.0	0.76	0.69	58	0.7	100 − 0 − 0
Alexander and Cruz (2006)	RCR91	W	AC	CF	54	10.7–107	30.7	25.3	59	25.3	100 − 0 − 0
Bushey (1985)	RCR72	E	S	S	6	2.0–32.9	4.27	3.37	60	0.6	67 − 0 − 33
van Wagtendonk and Botti (1984) − e[a]	RCR72	P	S	CF	5	0.3–1.53	0.6	0.47	60	0.4	80 − 0 − 20
Cuiñas et al. (1996)	RCR72	E	S	S	51	0.28–16.0	3.85	2.66	60	2.2	84 − 0 − 16
Quintilio (1972)[b]	RCR72	E	S	LS	20	0.6–19.8	5.74	4.22	63	3.8	75 − 0 − 25
van Wagtendonk and Botti (1984) − d[a]	RCR72	P	S	CF	11	0.35–1.5	0.63	0.51	63	0.5	82 − 0 − 18
Stocks and Walker (1972)[b]	RCR72	E	S	LS	24	1.8–37.7	10.47	7.20	64	6.9	92 − 0 − 8
McCaw et al. (2008)	FFDM	E	S, PC	EF	97	0.24–19.4	5.31	3.98	65	3.6	81 − 1 − 18
Grabner (1996);Grabner et al. (1999)	RCR72	E	S	HF	30	0.3–13.4	4.41	3.04	67	3.0	87–13 − 0
Cruz et al. (2005)	RCR91	E	AC	CF	25	7.5–51.4	18.9	16.1	68	16.1	100 − 0 − 0
Dell'Orfano (1996)	RCR72	E	S	MF	9	0.3–7.9	2.41	1.68	71	0.9	56 − 11 − 33
MAPE 76% and greater											
Cruz et al. (2005)	CAW05	E	PC	CF	14	3.35–15.8	5.9	5.2	79	−2.5	27 − 0 − 73
van Wagtendonk and Botti (1984) − c[a]	RCR72	P	S	CF	18	0.3–3	0.91	0.65	83	−0.1	22 − 28 − 50
van Wilgen and Wills (1988)	RCR72	E	S	G	10	2.5–60.1	10.1	8.4	86	−3.5	40 − 0 − 60
Burrows (1994,1999)	RCR72	E	S	EF	35	0.25–9.9	1.15	0.94	95	−0.11	40 − 0 − 60
Gelobter et al. (1998)	RCR72	W	S	S	12	1–87	21.1	13.8	125	5.5	50 − 0 − 50
Burrows et al. (1991)	GA84	E	S	G	58	4.3–66.6	49.3	43.4	217	−43.4	100 − 0 − 0
van Wagtendonk and Botti (1984) − a[a]	RCR72	P	S	CF	8	0.3–15.2	3.78	2.17	310	−0.4	13 − 0 − 87

[a] The six fuel types of van Wagtendonk and Botti (1984) are: a – bear clover, b – ponderosa pine, c – mixed conifer-pine, d – mixed conifer-fir, e – true fir, and f – montane chaparral.

[b] The statistics describing the performance of the Rothermel (1972) model against the experimental fires in logging slash documented by Quintilio (1972) and Stocks and Walker (1972) were calculated as part of the present study. Fine dead fuel moisture was estimated as per Rothermel (1983). A wind adjustment factor of 0.4 was assumed (Andrews, 2012). Fuel Model 12 – Medium Logging Slash (Anderson, 1982) was used for all of the rate of spread predictions.

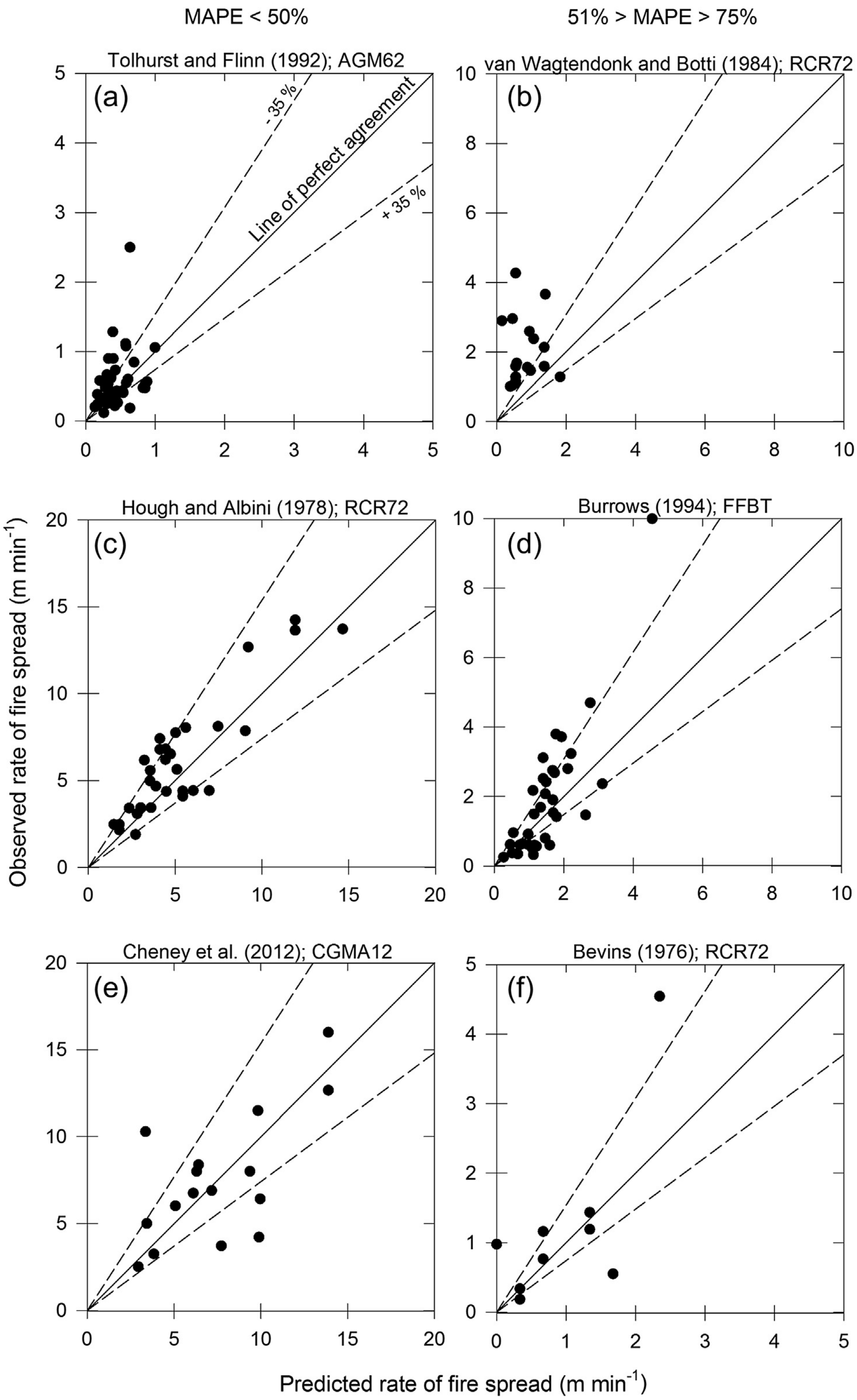

Fig. 5. Observed rates of spread versus model predictions for a selection of studies presented in Table 1 featuring 'moderately slow' spreading fires (i.e. less than 20 m min^{-1}). The dashed lines around the line of perfect agreement indicate the ±35% error interval. Refer to Table 2 for the meaning of the model abbreviations (e.g. AGM62 = McArthur, 1962). MAPE = mean absolute percent error.

model performance? What are the consequences of model error in rate of spread predictions? And given the realities of model predictions and the uncertainties associated with predicting wildland fire behaviour what, if anything, can be done operationally to reduce the average error?

5.1. Surface fire versus crown fire rates of spread prediction

There is a perception amongst some fire researchers that the behaviour of crown fires is more unpredictable than that of surface fires (e.g. McAllister et al., 2012; Finney et al., 2013). This is

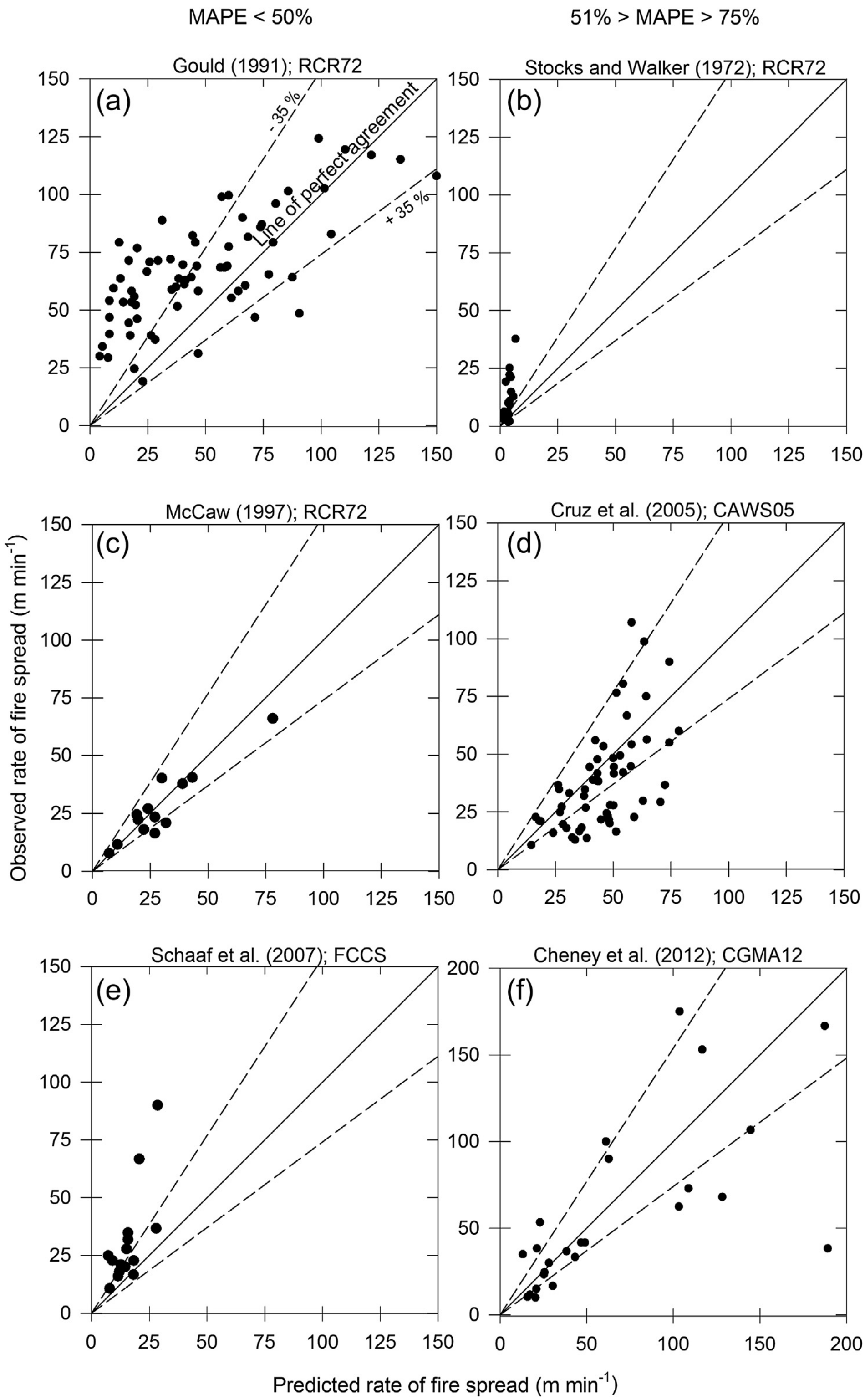

Fig. 6. Observed rates of spread versus model predictions for a selection of studies presented in Table 1 featuring both 'moderately slow' and 'exceedingly fast' spreading fires (i.e. upwards of ~150 m min^{-1}). The dashed lines around the line of perfect agreement indicate the ±35% error interval. Refer to Table 2 for the mean of the model abbreviations (e.g. RCR72 = Rothermel, 1972). MAPE = mean absolute percent error.

partially due to the high-energy release rates, strong interaction between the fire and the local wind field and small-scale chaotic dynamics. However, examination of the error metrics obtained for surface and crown fires in this study, shows no differences in predictability. The fact that the highest MAPE were obtained for surface fires lends some credence to the contention of Van Wagner (1979) that "the prediction of surface fire behaviour is, in fact, probably more difficult than the prediction of crowning potential, because of the multiplicity of possible forest floor and understory fuel complexes".

Table 2
List and brief description of the wildland fire rate of spread (ROS) models associated with Table 1.

Rate of fire spread Model	Principal reference	Technical basis	Primary fuel type(s) and geographical area of origin	Fire propagation regime type	Fire management application
AGM62	McArthur (1962)	Empirical	Dry sclerophyll eucalypt forest (Australia)	Surface fire	Prescribed fire
GFDM	McArthur (1966)	Empirical	Grassland (Australia)	Surface fire	Wildfire
FFDM	McArthur (1967)	Empirical	Dry sclerophyll eucalypt forest (Australia)	Surface and crown fires	Wildfire
RCR72	Rothermel (1972)	Semi-empirical	Grass, shrub, litter and slash (US)[a]	Surface fire	Wildfire
GA84	Griffin and Allan (1984)	Empirical	Semi-arid spinifex grasslands (Australia)	Surface fire	Prescribed fire
FFBT	Sneeuwjagt and Peet (1998)	Empirical	Dry sclerophyll eucalypt forest (Western Australia, Australia)	Surface and crown fire	Prescribed fire and wildfire
RCR91	Rothermel (1991)	Empirical	Conifer forests (Northern Rocky Mountains, US)	Crown fire	Wildfire
FBPS	Forestry Canada Fire Danger Group (1992)	Empirical	Conifer, mixedwood and deciduous forests, slash, grasslands (Canada)	Surface and crown fires	Wildfire
MSC95	Marsden-Smedley and Catchpole (1995)	Empirical	Buttongrass moorlands (Tasmania, Australia)	Surface fire	Prescribed fire and wildfire
CAW05	Cruz et al. (2005)	Empirical	Conifer forests (Canada)	Crown fire	Wildfire
FCCS	Schaaf et al. (2007)	Semi-empirical	Conifer forests (US)	Crown fire	Wildfire
CGMA12	Cheney et al. (2012)	Empirical	Dry sclerophyll eucalypt forest (Australia)	Surface fire	Wildfire
CMAG13	Cruz et al. (2013)	Empirical	Semi-arid mallee-heath shrublands (Australia)	Surface and crown fires	Prescribed fire and wildfire

[a] The Rothermel (1972) model has received world-wide use.

The predictability of the spread rate of crown fires is partially due to the fact that after a crown fire is established, its sustained propagation is to a large degree a function of the wind speed and fuel moisture content (Rothermel, 1991; Cruz et al., 2005), and the heat release rates, while very high, occur over a relative narrow range (i.e. within an order of magnitude). For a surface fire spread model to be successful, it needs to be able to describe spread rates and fireline intensities spanning 3 or 4 orders of magnitude. They also need to be able to adequately describe fire propagation over distinct entrainment and combustion regimes (Nelson et al., 2012), and capture the effect of small-scale variation in fuel structure under marginal burning conditions (i.e. high fuel moisture contents). The level of detail required to satisfactorily describe the full range of surface fire behaviour in wildland fuels is not yet present in currently available empirical and physical-based fire behaviour models.

5.2. *Acceptable error for evaluation of model performance*

No universally agreed upon standard currently exists within the wildland fire community in regards to fire behaviour model performance, and in particular what constitutes an acceptable error in a fire rate of spread prediction. Andrews (1980) and Alexander and Cruz (2006) point out that the definition of such quantity would depend to a large degree on the values-at-risk, user requirements, and human psychology. Given the inherent variation in wildland fire behaviour, Albini (1976) suggested that model builders consider models successful if the relationships predict fire behaviour within a factor of two or three over a range of two or three orders of magnitude. McArthur (1977) felt that the forest and grassland fire danger meters that he developed for Australia could predict rate of spread and other fire characteristics to within ±20% of the actual observed fire behaviour (e.g. if the predicted rate of spread was 15 m min^{-1} then the observed rate of spread should vary from 12 to 18 m min^{-1}). On the basis of what has been reported here, an assertion such as McArthur's (1977) would be considered as quite optimistic.

The 1984 interim edition of the Canadian Forest Fire Behaviour Prediction System presented graphs associated with head fire rate of spread – Initial Spread Index models with confidence intervals where observed spread rates could be expected to fall within about 70% of the time (Lawson et al. (1985). Similarly, Rothermel (1991) presented nomographs for predicting crown fire rate of spread with 75% confidence intervals although no rationale was provided for the definition of the width of these confidence intervals.

Alexander and Cruz (2006) had suggested that a ±25% error for individual predictions would represent an excellent level of prediction accuracy for rate of fire spread based on the performance of a crown fire model against wildfire observations (Fig. 6d). Considering average error, the degree of inherent unexplained variation evident from the results obtained to date from experimental fire and wildfire observations (Table 1) suggest that higher errors are to be expected.

What would a reasonable standard for fire spread rate model performance be? In this study, only 3 out of 49 studies (i.e. 6.3%) had MAPE values below 25%. Considering that at an alpha level of 0.05, one would find by chance one study out of every twenty with an error lower than 25%, one would conclude that predictions with an average error lower than this value are unlikely.

Table 3
Sample size, range in rate of fire spread (ROS), and summary of error statistics by type of data for the studies listed in Table 1.

Type of data	No. of fires	ROS range (m min^{-1})	Exact predictions	Under-predictions			Over-predictions		
				No. of fires	Mean MAPE (%)	Standard deviation	No. of fires	Mean MAPE (%)	Standard deviation
Prescribed fire	182	0.1–150.9	7	121	49%	21%	54	102%	164%
Experimental fire	891	0.1–124.2	25	575	49%	24%	291	101%	133%
Wildfire	204	1.65–175	3	112	50%	22%	89	63%	60%

Table 4
Sample size, range in rate of fire spread (ROS), and summary of error statistics per fuel type for the studies listed in Table 1.

Fuel type group	No. of fires	ROS range (m min^{-1})	Exact predictions	Under-predictions			Over-predictions		
				No. of fires	Mean MAPE (%)	Standard deviation	No. of fires	Mean MAPE (%)	Standard deviation
Grasslands	240	0.2–124.2	12	106	42%	25%	122	140%	174%
Shrublands	161	0.3–150.9	3	98	46%	29%	60	49%	45%
Hardwood forest	73	0.1–13.4	8	49	54%	30%	16	72%	93%
Conifer forest	318	0.3–107	6	214	50%	22%	98	86%	126%
Eucalypt forest	418	0.1–175	4	289	49%	21%	125	80%	95%
Mixedwood forest	15	0.3–20	1	11	62%	17%	3	89%	58%
Logging slash	53	0.2–37.7	1	42	58%	20%	10	86%	67%

Admittedly, it may not be possible to directly extract absolute guidance from Table 1 as to what could constitute an acceptable error for fire spread model performance. Alternatively, one might consider that a ±1 standard deviation as a measure of acceptable error. Assuming a normal distribution, one standard deviation corresponds to 34.1% departure from the mean. For practical purposes this number can be rounded to 35%. Eight of the model comparisons in Table 1 (i.e. 17%) had a MAPE equal or lower than 35%, suggesting that this could constitute a more realistic benchmark by which to judge good model performance when accurate input data is available. Such a benchmark would be deemed as only applicable to research studies. In operational practice, the uncertainty in the estimation of the input data, often times involving large spatial (e.g. > 1000 ha) and temporal (i.e. from one to several hours) scales, would understandably result in wider error intervals.

On the basis of (i) the average errors associated with the development of empirical fire behaviour models (Fig. 3), (ii) previously reported suggestions in the wildland fire behaviour science literature regarding a model performance error threshold as described above, (iii) the results obtained from an analysis of 49 model evaluation datasets and (iv) other considerations described in the preceding paragraph, it would appear that a ±35% error would constitute a reasonable and conservative standard for fire spread rate model performance.

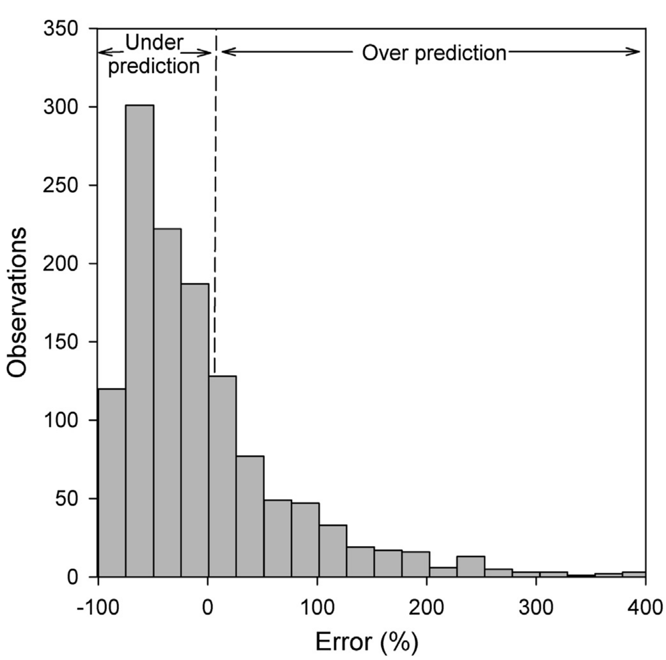

Fig. 7. . Distribution of under and over-prediction expressed as a percent errors for the fires associated with Table 1.

5.3. *Error propagation in model predictions of wildland fire behaviour*

This study has focused on the rate of spread characteristic associated with wildland fire behaviour. However, it should be clear that given the inter-relationships between rate of fire spread and other fire characteristics (Fig. 2) in fire behaviour prediction systems, that there is considerable potential for error propagation. For example, predictions of rate of fire spread and fuel consumed are used to calculate fireline intensity (Byram, 1959; Catchpole et al., 1992) and various flame front dimensions such as length, height and depth (Fons et al., 1963; Alexander and Cruz, 2012) on either a linear or areal basis, which are in turn used to gauge the onset of crowning (Van Wagner, 1977), firebreak breaching (Byram, 1959; Wilson, 1988), and maximum spotting distance (Albini et al., 2012), as well certain fire impacts like crown scorch height (Van Wagner, 1973) and coarse woody fuel consumption (Hollis et al., 2011a).

As models become more complex to better describe fuel and fire dynamics, a higher number of intermediate calculations are required. Cruz et al. (2004) have shown that the compounding errors arising from the choice of fuel model and fuel availability for flaming combustion can result in a five-fold under-prediction bias in the threshold for crowning when linking Rothermel's (1972) rate of fire spread model with Van Wagner's (1977) crown fire initiation model. Simpler models that directly relate the output (e.g. onset of crowning and crown fire rate of spread) to the main controlling environmental variables and thereby limit error propagation are perhaps best for fire management applications (Cruz et al., 2004; Hollis et al., 2011b). Setting aside slope steepness for the moment, for a given fuel type most fire behaviour characteristics are determined principally by fuel moisture and wind speed (e.g. Forestry Canada Fire Danger Group, 1992; Fernandes et al., 2009; Cheney et al., 2012).

5.4. *On dealing operationally with uncertainty in model predictions*

Traditionally, the operational use of fire behaviour models has largely followed a deterministic approach to a simulation or prediction based on the notion of providing the best possible estimates of the input variables (Rothermel, 1983). One of the limitations of this approach is that it fails to provide any indication of the conditions of uncertainty surrounding the model predictions (Gill, 2001). Several authors have previously suggested that a prediction of the likely distributions of rate of fire spread and fireline intensity would be preferable to a deterministic approach (e.g. Salazar and Bradshaw, 1986). The application of multiple prediction, or ensemble, and data assimilation methods has the potential to reduce the users grasp of the degree of uncertainty associated with a fire behaviour forecast.

Ensemble methods are commonly used to quantify errors in forecasting the behaviour of phenomena with significant uncertainty, such as in meteorology (Zhu, 2005), hydrology (Cloke and Pappenberger, 2009) and climatology (Benestad, 2004). The potential for the operational use of ensemble methods in fire behaviour prediction has not yet been fully realized. Kourtz (1972) suggested the application of ensemble methods to improve the interpretation of fire danger rating forecasts. Anderson et al. (2007) and Finney et al. (2010) have recently applied ensemble methods in spatially explicit fire growth simulators in order to quantify the effect of perturbations in weather variables in overall fire development. Cruz (2010) demonstrated the application of a simple Monte Carlo-based ensemble method to incorporate weather input uncertainty into the prediction of grassland rate of fire spread. In this case, the modelled outputs did not improve the general fit statistics but provided complementary information, such as error bounds and probabilistic outcomes, which extended the range of questions that can be answered by fire behaviour models.

Data assimilation is a computational technique that incorporates observed data into a running model to recursively update the model state and conduct further forecasts. Beer (1991) showed the advantage of data assimilation methods to improve forecasts of fire spread although such approach has not been considered by fire modellers until recently. Mandel et al. (2009, 2012) present a dynamic data-driven fire propagation modelling system and discusses some of the challenges of applying data assimilation methods to wildfire situations, namely the difficulty in establishing an accurate location of the active fire perimeter during fire runs and the appropriateness of Ensemble Kalman Filters to fire spread scenarios.

6. Concluding comments

A comprehensive survey of the error statistics associated with rate of fire spread model evaluation studies was undertaken in order to gauge the general predictive ability of such models. This has led to new insights into some of the uncertainties associated with model predictions of free-burning wildland fire behaviour. The analysis also supports in part a suggestion that an error threshold of 35% would constitute an acceptable error for model predictions of rate of fire spread, a point that appeals to prominent figures in the field of wildland fire behaviour modelling such as R.C. Rothermel (USDA Forest Service – retired, personal communication, 2012).

Lindenmuth and Davis (1973) considered that one of the important tasks of wildland fire research was to evaluate physical and theoretical predictive models in a controlled environment to determine how well they performed over a range of burning conditions. This paper has accordingly highlighted the value of model evaluation based on independent datasets. The present study has also clearly shown that empirical-based fire behaviour models founded on solid field observations and well accepted functional forms, are quite adequate for predicting rates of fire spread outside of the bounds of the original dataset used in their development.

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Appendix A. Error statistics used to judge model performance

Table A1
Equations and explanation of deviation statistics used to quantify rate of fire spread model performance in Table 1. RMSE is the root mean square error, MAE is the mean absolute error, MAPE is the mean absolute percent error, MBE is the mean bias error, y_i is the observed rate of fire spread, $\hat{y}_i$ is the predicted value, and n is the sample size. See Willmott (1982) for further information on the individual statistics.

$\mathrm{RMSE} = \sqrt{\frac{\sum(\hat{y}_i - y_i)^2}{n}}$	The RMSE is a useful overall measure of model performance. The RMSE provides a measure of the precision of the estimates in the same units as the dependent variable (e.g., rate of fire spread, m min^{-1}). A "good" model will provide low values of the RMSE. Because large errors are weighted heavily, this can result in a large RMSE even though the errors may be otherwise small.
$\mathrm{MAE} = \frac{\sum\lvert\hat{y}_i - y_i\rvert}{n}$	MAE, which like the RMSE is expressed in the same units as the original data, is a quantity used to measure how close predictions are to observed value. As the name suggests, the MAE is an average of the absolute error. The MAE is similar to the RMSE but is less sensitive to large errors.
$MAPE = \frac{\sum\left(\frac{\lvert\hat{y}_i - y_i\rvert}{y_i}\right)}{n}100$	The MAPE is a very popular measure of the accuracy of a predictive model or system. It represents the summed differences between the individual predicted versus observed values divided by the observed value; multiplying it by 100 makes it a percentage error. If a perfect fit is obtained then the MAPE is zero. A MAPE of 10% is considered a very good result. A MAPE in the range of 20–30% or even higher is quite common.
$\mathrm{MBE} = \frac{\sum(\hat{y}_i - y_i)}{n}$	The MBE describes the dispersion or spread of the residual distribution about the estimate of the mean. A positive value indicates an over-prediction trend while a negative is an indication of an under-prediction trend.

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