Optimal Design Methodology of Plate-Fin Heat Sinks for Electronic Cooling Using Entropy Generation Strategy

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***Abstract*—This paper presents a formal systematic optimization process to plate-fins heat sink design for dissipating the maximum heat generation from electronic component by applying the entropy generation rate to obtain the highest heat transfer efficiency. The design investigations demonstrate the thermal performance with horizontal inlet cooling stream is slightly superior to that with vertical inlet cooling stream. However, the design of vertical inlet stream model can yields to a less structural mass (volume) required than that of horizontal inlet stream model under the same amount of heat dissipation. In this paper, the constrained optimization of plate-fins heat sink design with vertical inlet stream model is developed to achieve enhanced thermal performance. The number of fins and the aspect ratio are the most responsive factors for influencing thermal performances. The heat sink used on AMD Thunderbird 1-GHz processor has been examined and redesigned by presenting optimization methodology. The optimal thermal analysis has a very good agreement to the both of vendors' announced information and using simulation of parabolic hyperbolic or elliptic numerical integration code series (PHOENICS). The optimum design that minimizes entropy generation rate in this paper primarily applied three criteria for plate-fins heat sink optimal design:**

1) formal constrained nonlinear programming to obtain the maximum heat dissipation;
2) prescribed heat dissipation;
3) prescribed surface temperature.

As a result, the thermal performance can be notably improved; both the sink size and structural mass can apparently be reduced through the presented design method and process. This analysis and design methodology can be further applied to other finned type heat sink designs.

***Index Terms*—Engineering design, entropy generation rate, heat transfer, optimal design, plate-fin heat sink.**

NOMENCLATURE

a	Fin height, m.
a/s	Aspect ratio of fin height to fin spacing.
A	Total surface area of heat sink including the fins and exposed other surface, m^2.
A_{Ci}	Cross-sectional area A_c for heat conduction of each fin, m^2.
A_f	Surface area of fins, m^2.
b	Base thickness of the heat sink, m.
d	Fin thickness, m.
D_h	Hydraulic diameter of the channel.
f_{app}	Apparent friction factor for hydro dynamically developing flow.
F_d	Drag force, N.
$f \cdot R_{e_{D_h}}$	Reynolds number group.
h_c	Convective heat transfer coefficient, $\mathrm{W/m^2K}$.
k	Thermal conductivity of the fin material, W/mK.
K_c	Coefficients of a sudden contraction.
K_e	Coefficients of a sudden expansion.
k_f	Fluid thermal conductivity, W/mK.
L	Heat sink length, m.
N_{us}	Nusselt number.
P_r	Prandtl number.
Q	Heat dissipation rate, W.
q_b	Heat transfer of the fin base, W.
q''	Heat transfer rate between each fin body and stream, W/m^2.
s	Spacing between two fins, m.
$R_{e_{D_h}}$	Channel Reynolds number.
R_{es}	Reynolds number.
R^*_{es}	Modified channel Reynolds number.
R_{tot}	Total thermal resistance, K/W.
R_{sin}	Overall thermal resistance of the total finned surface, K/W.
$\dot{S}_{\mathrm{gen}}$	Entropy generation rate, W/k.
T_b	Temperature of the fin base, °C.
T_e	Ambient temperature, °C.
V_{ch}	Channel velocity, m/s.
V_f	Stream velocity, m/s.
W	Heat sink width, m.
x_i	The jth design variable.
β	Fin parameter, (8).
ΔT	Temperature excess, $T_b - T_e$, °K.
η_t	Total surface efficiency.
η_f	Fin efficiency.
ν	Kinematic viscosity of the fluid, $1.6(10^{-5})$ m^2/s.
ρ	Air density, 1.177 kg/m^3.
$\wp$	Surface area per unit length of fins, m.
χ	Fin parameter, βa.

Manuscript received February 1, 2004; revised March 22, 2004. This work was supported by the National Science Council of Taiwan, R.O.C., under Grant NSC 89-2212-E-032-010. This work was recommended for publication by Associate Editor C. D. Patel upon evaluation of the reviewers' comments.

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Digital Object Identifier 10.1109/TCAPT.2004.831812

I. Introduction

THE RAPID advances in microsemiconductor technology have been leading to increase the heat dissipation from microelectronic devices [1]. Various cooling mechanisms have been continuously employed to remove heat from heat sinks; this fuels interests to engineers and researchers for controlling the maximum operating temperature, long-term reliability, and highly efficient performance of electronic components [2]. The use of fins to enhance air-cooling is the simplest and effective heat sink structure under the cost, space and weight constraints [2]. Thus, to develop a systematic air-cooling heat sink design methodology is very important for satisfying the current thermal necessities and successfully removing elevated heat of high-ranking electronic components in the future. Although a considerable amount of the experiments and analysis of air-cooling heat sinks have been published and discussed in the past two decades, however, only a handful of discrete information is available in technical literatures regarding to the practically systematic optimization and design of heat sinks.

During the past two decades, some literatures had contributed the optimal parametric study for enhancing the thermal characteristics of heat sinks. Various literatures documented the study of thermal analysis rather than its geometric design. An early analytical work was presented by Sparrow *et al.* [3] for forced convection cooled and shrouded plate-fin array. Bar-Cohen and Rohsenow [4] developed a composite relation for the variation of heat transfer coefficient along the parallel plate surface to provide the analytic optimizing fins spacing. Wirtz *et al.* [5] presented flow bypass effects on plate-fin heat sinks. Kraus and Bar-Cohen [2] recommended the use of fully developed laminar and turbulent pipe flow correlation for predicting the average heat transfer coefficient, but without any models for developing flow. Teertstra *et al.* [6] presented an analytical forced convection model for the average heat transfer rate from a plate-fin heat sink flow. Bejan and Morega [7] presented the optimal geometry of array of pin-fins and plate-fins by minimizing the thermal resistance between the substrate and the flow forced through the fins. Their work was the pioneer of functioning the optimal geometrical heat sink design with air-cooling fins. Lee [8] introduced a practical guideline for selecting heat sinks and recommended the use of a developing flow correlation with his fitting to parallel-plate data by Kays and London. The study of the least-material optimization by Iyengar and Bar-Cohen [9] pointed out that the triangular-fin array is better in thermal capacity than other fin types; the pin-fin array is better in volumetrically efficient than other fin types. In the recent optimization of heat sink contributed by Culham and Muzychka [10], a parallel-fin heat sink was parametric studied with only the objective function and without design constraints. They combined the necessary optimality and Newton–Raphson method to solve the unconstrained problem. The above literatures clearly shows that the optimum geometrical design regarding heat sinks with electronic cooling of horizontal inlet stream emphasizes the parametric, analytic, and nonformal unconstrained optimization.

The objective of this paper is to present a formal systematic geometric design methodology for popularly used vertical fan-sink assembly, as shown in Fig. 1, mounting on the central microprocessor of a personal computer. The design of horizontal fan-sink assembly is also analyzed and compared with the design of vertical fan-sink assembly. Bejan's [11], [12] theory of overall entropy generation rate has been employed as the design objective function that yields to the minimum thermal resistance and the maximum thermal efficiency. The thermal performance of vendor's heat sink used on AMD Thunderbird 1-GHz processor has been examined and redesigned by applying the presenting optimization methodology and formulation in this paper. Several tasks are resolved from this research work and answered.

Fig. 1. Open AC610D heat sink assembly.

1) How can one perform the optimal design while the heat dissipation is already prescribed?
2) What is the optimal design method as the surface temperature of heat sink is prescribed?
3) What are the significant points of improving the thermal efficiency of the heat sink?
4) How can one redesign an existing heat sink to promote the thermal performance?

Several design examples presented in this paper have been analyzed by using parabolic hyperbolic or elliptic numerical integration code series (PHOENICS) [13] and confirmed by the maximum operating temperature of AMD K-7 posted on the web [14].

II. Design Model and Parameters

The simplest and the most popular used fan-sink assembly is a plate-fin heat sink with vertical inlet cooling stream, shown in Fig. 1, that can be modeled as Fig. 2, where a suitable fan is mounted on the top of the plate-fin sink. The base of the fin is connected to the heat source of an electronic device by a clamp with the appropriate contact pressure. The fundamental dimensions of this fin-sink (L, W, a, b, d, and s) represent the length and the width of the heat sink base, the height of the fin, the base thickness of the sink, the thickness of each fin, and the spacing between two fins, respectively. In the beginning of the design process, these geometrical dimensions are analyzed so that a set of design variables can be selected by the designers. The model

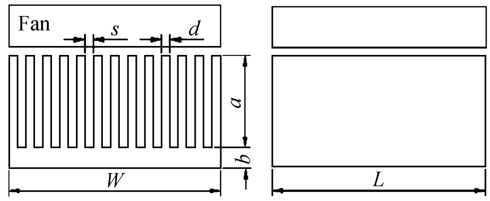

Fig. 2. Geometrical configuration of a plate-fin heat sink with vertical inlet stream.

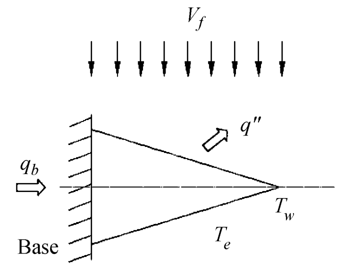

Fig. 3. Schematic of a general fin in convective heat transfer.

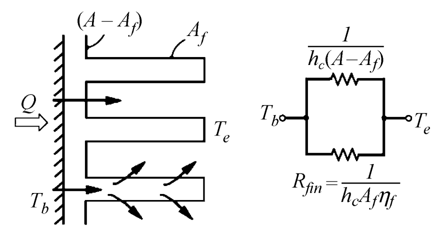

Fig. 4. Finned surface showing the parallel paths of heat loss.

development is subject to the following underlying assumptions [10], no bypassing flow effect, uniform approach flow velocity, constant thermal properties, uniform heat transfer coefficient, and the adiabatic fin tips.

Krans and Bar-Cohen [2] assumes a prescribed heat-transfer coefficient over the length of the fins, however, it is not true. In most heat sink applications, the heat transfer coefficient is a variable introduced by hydrodynamic and thermal-entrance effects. Therefore, the presenting optimization process leads to constant changes in the geometrical size of the fin-sink; as a result, the mean heat transfer coefficient constantly changes. The entropy generation rate associated with heat transfer in the heat sink serves as the capability of transferring heat to the surrounding cooling medium.

The minimization of the total thermal resistance (R_{tot}) is the general criterion for designing a heat sink that can be further decomposed into the junction (between the heat source and the base of the sink-to-case) resistance, the case-to-sink resistance, and the sink-to-ambient resistance. The other criterion for designing a heat sink is to maximize the thermal efficiency. Both criteria have been considered in this paper, resulting in the maximum heat dissipation.

III. Entropy Generation and Thermal Analysis

A fin can generate the entropy associated with the external flow. It also can generate entropy internally because the fin is nonisothermal. The entropy generation rate [11], [12] is defined as

$$\dot{S}_{\text{gen}} = \frac{q_b \Delta T}{T_e^2} + \frac{F_d V_f}{T_e} \tag{1}$$

where the parameter q_b represents the heat transfer of the fin base. The uniform stream with velocity V_f and the absolute temperature T_e passes through a general fin is shown in Fig. 3. The heat transfer rate between each fin body and stream is q''. The local temperature is T_w. Theoretically, a heat sink satisfies

$$\iint q'' dA \cong Q. \tag{2}$$

Thus, the first part of the entropy generation rate in (1) can be written as $Q\Delta T/T_e^2$. There are two parallel heat losses from fin surfaces for rectangular fins as shown in Fig. 4. Perfect bonding between the heat source and the contact area of heat sink base is assumed. The temperature excess (ΔT) of the fin base is represented as $(T_b - T_e)$, where T_b means the absolute temperature of the fin base. Fluid friction manifests itself in the form of the drag force F_d along the direction of V_f. Equation (1) shows that fluid friction and inadequate thermal conduct contribute hand in hand to degrading the thermodynamic performance of the fin. Thus, the optimal thermodynamic size of a fin can be computed by minimizing (1) subjected to necessary design constraints.

For a heat sink set, the temperature excess of ΔT is related to the overall heat sink thermal resistance

$$\Delta T = Q \cdot R_{\text{sin}} \tag{3}$$

where Q is the heat dissipation rate of the heat sink. The entropy generation rate of a heat sink set can be rewritten as

$$\dot{S}_{\text{gen}} = \frac{Q^2 R_{sin}}{T_e^2} + \frac{F_d V_f}{T_e}. \tag{4}$$

The entropy generation rate in (4) indicates a function containing both heat sink resistance and viscous dissipation. The viscous dissipation component is small and can be neglected in the low velocity as buoyancy induced flow. However, the inclusion of the viscous component is essential for determining the optimal flow condition encountered in air-cooling heat sink. The overall thermal resistance of the total finned surface from the fundamental heat transfer [16] is written as

$$R_{\text{sin}} = \frac{1}{h_c A \eta_t}. \tag{5}$$

The total surface efficiency η_t is written as

$$\eta_t = 1 - \frac{A_f}{A}(1 - \eta_f) \tag{6}$$

where A_f is the surface area of fins; A is the total area of heat transfer surface including the fins and other exposed surface. The fin efficiency η_f is the function of a fin parameter χ that can be written in the following compact form:

$$\eta_f = \frac{tanh\chi}{\chi}. \tag{7}$$

The fin parameter χ is equal to βa in which the parameter a represents the length of fins. The parameter β is expressed as

$$\beta = \sqrt{\frac{h_c \wp}{\sum_{i=1}^{N} k A_{c_i}}} \tag{8}$$

where k represents the conductivity of the fin material. The cross-sectional area A_c for heat conduction of each fin is represented as A_{c_i}. The perimeter $\wp$ is the surface area per unit length of fins. h_c is the convective heat transfer coefficient that can be computed using the model developed by Teertstra *et al.* [6]

$$h_c = \frac{N_{\mathrm{us}} k_f}{s} \tag{9}$$

where k_f represents the fluid thermal conductivity. The present paper adopt fully-developed and developing channel flow presented by Teertstra *et al.* [6], thus the Nusselt number can be written in the following [10]:

$$N_{us} = \left[\left(\frac{R_{\mathrm{es}}^* P_r}{2}\right)^{-3} + \left(0.664\sqrt{R_{\mathrm{es}}^*} P_r^{\frac{1}{3}} \sqrt{1 + \frac{3.65}{\sqrt{R_{\mathrm{es}}^*}}}\right)^{-3}\right]^{-\frac{1}{3}} \tag{10}$$

where P_r represents the Prandtl number. The modified channel Reynolds number R_{es}^* is defined as

$$R_{\mathrm{es}}^* = R_{\mathrm{es}} \cdot \frac{s}{L}. \tag{11}$$

The Reynolds number R_{es} is defined as

$$R_{\mathrm{es}} = \frac{s V_{\mathrm{ch}}}{\nu}. \tag{12}$$

The channel velocity V_{ch} related to the free stream velocity V_f that can be derived by the flow continuity

$$V_{\mathrm{ch}} = \frac{V_f L}{2a}. \tag{13}$$

It is noted that the channel velocity V_{ch} of horizontal inlet flow is equal to $V_f(1 + d/s)$, written in [10], and it is adopted later in this paper for thermal analysis and optimization. The total drag force F_d of the heat sink can be obtained by the force balance on the heat sink [17]

$$F_d = \frac{1}{2}\rho V_{\mathrm{ch}}^2 f_{\mathrm{app}} N(2aL + sL) + K_c a W + K_e a W \tag{14}$$

where f_{app} is the apparent friction factor for a hydrodynamically developing flow. This factor of f_{app} for a rectangular channel can be evaluated by using the following laminar flow formulation developed by Muzychka and Yovanovich [18]

$$f_{\mathrm{app}} = \frac{1}{R_{e_{D_h}}}\left[\left(\frac{3.44}{\sqrt{L^*}}\right)^2 + \left(f \cdot R_{e_{D_h}}\right)^2\right]^{\frac{1}{2}} \tag{15}$$

where

$$L^* = \frac{L}{D_h R_{e_{D_h}}} \tag{16}$$

$$R_{e_{D_h}} = \frac{V_{\mathrm{ch}} D_h}{\nu}. \tag{17}$$

The parameter D_h represents the hydraulic diameter of the channel. The parameter $R_{e_{D_h}}$ represents the channel Reynolds number, and ν represents the kinematic viscosity of the fluid. The coefficients of K_c and K_e in (14) for a sudden contraction and expansion, respectively, can be computed using the following simple form:

$$K_c = 0.42\left[1 - \left(1 - \frac{N \cdot d}{W}\right)^2\right] \tag{18}$$

$$K_e = \left[1 - \left(1 - \frac{N \cdot d}{W}\right)^2\right]^2. \tag{19}$$

A Reynolds number group written as $f \cdot R_{e_{D_h}}$ of (15) representing the friction factor of fully developed flow is given by [10]

$$f \cdot R_{e_{D_h}} = 24 - 32.527\left(\frac{s}{a}\right) + 46.721\left(\frac{s}{a}\right)^2 - 40.829\left(\frac{s}{a}\right)^3 + 22.954\left(\frac{s}{a}\right)^4 - 6.089\left(\frac{s}{a}\right)^5. \tag{20}$$

IV. Analysis and Formulation for Optimal Design

In general thermal design of a heat exchanger, one can either minimize total thermal resistance R_{sin} or maximize the total efficiency η_t as the design goal. When one observes (5), the minimization of R_{sin} has the tendency to maximize the convective coefficient h_c, the total fins surface area A, and total thermal efficiency η_t, simultaneously. However, each of the individual design parameters are not being maximized; rather; the product of their values is being minimized. In a general heat sink design problem, the heat dissipation Q and stream temperature T_e can be required to prescribe in advance. Consequently, the minimization of entropy generation rate $\dot{S}_{\mathrm{gen}}$ is equivalent to the minimization of total thermal resistance R_{sin} as well as minimizing $F_d \cdot V_f$ simultaneously. Therefore, the design strategy of minimizing entropy generation rate has the effect of maximizing the thermal efficiency, surface area, and convective coefficients. Additionally, the optimal flow velocity and viscous dissipation also can be found through minimization of entropy generation rate. The heat from heat source transfer to base plate at first, and then transfer to fins. Thus, the thermal performance of base plate including the material and the size are significant. Although the increasing value of L will yield to a wider base surface, it also influences two portions of entropy generation rate. Theoretically, L exists an optimum value through the design process and then engineers can consider the final L by actual situation and requirement. Consequently, those parameters are appropriate to describe as design variables.

TABLE I
UNCONSTRAINED OPTIMAL DESIGN OF THE PLATE-FIN SINK WITH HORIZONTAL INLET AIR FLOW

	N	a(mm)	d(mm)	V_f (m/s)	T_b (°C)	$\dot{S}_{gen}$ (W/K)	s (mm)	a/s	V_{oL}(mm^3)
This paper	20	134	1.61	1.05	32.3	0.00296702	0.9368	143.03	220740
Culham and Muzychka [10]	19.07	122	1.6	1.21	37.2	0.00306207	1.0889	112.04	191123

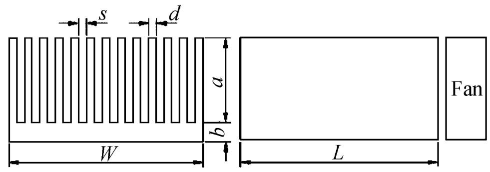

Fig. 5. Geometrical configuration of a plate-fin heat sink with horizontal inlet cooling stream.

Another design concept can be considered: how to optimally design a heat sink yielding to the maximum heat dissipation in which a fin-base temperature T_b is prescribed in advance? For dealing with such a case, one can substitute (21) into (4). The regarding design process has been demonstrated through the following design examples:

$$Q = (T_b - T_e) h_c A \eta_t. \tag{21}$$

V. ILLUSTRATIVE DESIGN EXAMPLES

A. Example 1: Unconstrained Optimal Design With Horizontal Inlet Flow

Fig. 5 shows the geometrical configuration of a plate-fin sink with horizontal inlet cooling flow. Both the base size L and the width W are 50 mm. The total heat dissipation of 30 W is uniformly applied over the base plate of the heat sink with the base thickness b of 2 mm. The thermal conductivity of the heat sink k is 200 W/mK. The ambient air temperature T_e is 25 °C. The conductivity k_f of the air is 0.0267 W/mK, air density ρ is 1.177 kg/m^3, and kinematical viscosity coefficient ν is $1.6(10^{-5})$ m^2/s. The task is to determine the optimal number N of fins, optimum height a of fins, optimum thickness d of each fin, and the optimum flow velocity V_f of cooling flow. The objective function written in (22) in this problem is borrowed from the case (v) in the paper of Culham and Muzychka [10]. The mathematical formulation of optimum design is written as

$$\begin{aligned} &\text{Find} \quad X = [x_1, x_2, x_3, x_4]^{\mathrm{T}} = [N, a, d, V_f]^{\mathrm{T}} \\ &\text{Minimize} \quad \dot{S}_{\text{gen}} = \frac{Q^2 R_{\sin}}{T_e^2} + \frac{F_d x_4}{T_e}. \end{aligned} \tag{22}$$

The design boundary corresponding to each design variable are $2 \le x_1 \le 40$, 25 mm $\le x_2 \le$ 140 mm, 0.2 mm $\le x_3 \le$ 2.5 mm, and $0.5\ \mathrm{m/s} \le x_4 \le 2\ \mathrm{m/s}$. The number of fins must be an integer that can be restricted in the following domain:

$$2 \le N \le \text{Integer}\left[1 + \left(\frac{W-d}{d}\right)\right]. \tag{23}$$

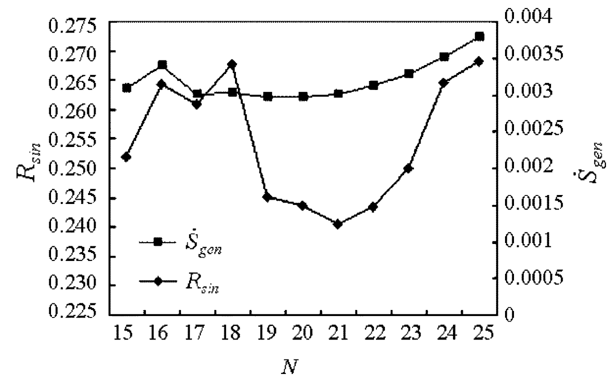

Fig. 6. Relation of $\dot{S}_{\text{gen}}$ and $R_{\sin}$ is the function of fin's number N.

The spacing s between two fins is equal to the following:

$$s = \left(\frac{W-d}{N-1}\right) - d. \tag{24}$$

This problem can be solved by the nonlinear sequential unconstrained minimization technique. In the present work, the augmented Lagrange multiplier (ALM) method is applied that results in Table I. The last column shows the total structural volume of the heat sink indicating as $V_{oL}(\mathrm{mm}^3)$. The larger value of V_{oL} indicates the further structural mass required to construct the heat sink. With a little increasing of the structural volume, the resulting $\dot{S}_{\text{gen}}$ and T_b are slightly lower than the result obtained by Culham *et al.* [10] in which the number of fins is not integer, as shown in Table I.

B. Discussion

To examine the parametric behavior of this problem, we select different N surrounding the optimum value and execute the optimization again. Fig. 6 shows the resulting optimum entropy generation rate $\dot{S}_{\text{gen}}$ and the thermal resistance of heat sink $R_{\sin}$, as the function of chosen fins number N, as well as the relation of $\dot{S}_{\text{gen}}$ and $R_{\sin}$. Fig. 7 shows that the resulting optimum $Q\Delta T/T_e^2$ and $F_d V_f/T_e$ of two portions of entropy generation rate are as the function of chosen number N. These two portions have similar variation tendency while the fins number is more than the optimum value of 20. Fig. 8 shows that the heat sink base temperature T_b and the air drag force F_d is the function of N. For a fin count greater than 20, the relation of T_b and F_d with the number of fins has a similar trend as that shown in Fig. 7. Fig. 9 shows the relation among the optimum design variable a, d and V_f, as the function of N. One can notice that these four variables are influenced jointly in thermal analysis formulation. On the other hand, the optimal values of N, a, d, and V_f obtained in this example are applied to the simulation of PHOENICS that is a general-purpose software package of CFD

TABLE II
UNCONSTRAINED OPTIMAL DESIGN OF THE PLATE-FIN SINK WITH VERTICAL INLET AIR FLOW

	N	a (mm)	d(mm)	V_f (m/s)	T_b (^oC)	$\dot{S}_{gen}$ (WK)	s (mm)	a/s	V_{oL}(mm^3)
This paper	27	39.21	0.50	2.0	41.5	0.00577269	1.4038	27.93	31466

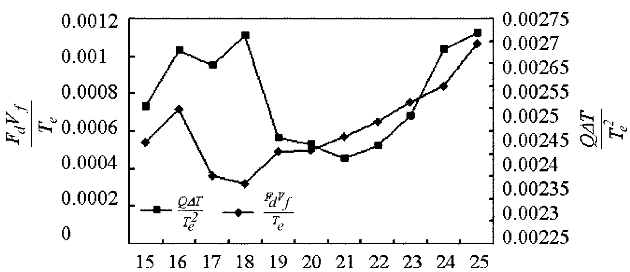

Fig. 7. $(Q\Delta T/T_e^2)$ and $(F_d V_f/T_e)$ is the function of chosen number of N.

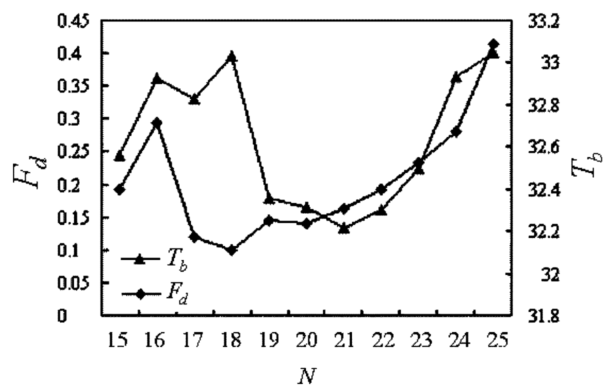

Fig. 8. Base temperature T_b and the air drag force F_d is the function of N.

code; as a result, the highest base temperature T_b is 32.38 °C that is very closed to 32.3 °C computed by the presenting paper.

C. Example 2: Unconstrained Optimal Design With Vertical Inlet Flow

The optimum sizes of a heat sink with vertical inlet cooling flow, as shown in Fig. 2, has been examined and determined in this example. All design conditions and parameters are the same as that in Example 1. The optimum design results are listed in Table II.The optimum results in this example have larger values of $\dot{S}_{\rm gen}$, $R_{\rm sin}$ and T_b, as compared with the results in Example 1. For obtaining the same heat dissipation under the same sink base area, the optimum design in vertical inlet flow yields to additional number of fins, shorter height of fins, lesser thickness of fins, and higher velocity of cooling flow than that in horizontal inlet air flow. Because the geometrical size changes, the total volume of V_{oL} reduces relatively. However, the base temperature is higher than that in Example 1. The heat transfer software package, PHOENICS, again was applied to examine the thermal performance of this example; as a result, the highest base temperature is 41.8 °C which is almost the same as the value of 41.5 °C computed in this example.

For comparing the cooling effect, the fan is relocated to the side surface of the optimum sink obtained herein for generating horizontal inlet flow, as shown in Fig. 5. The geometry of this

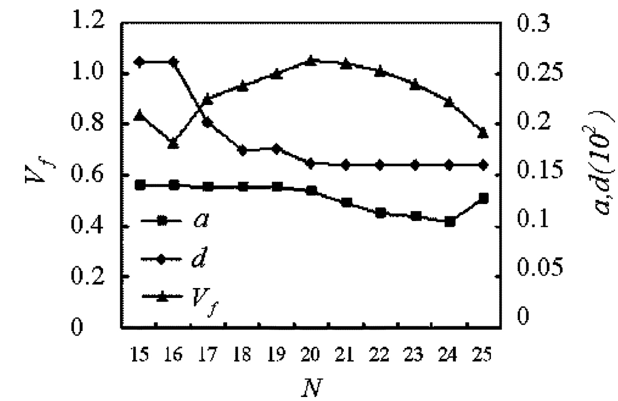

Fig. 9. Relation among the optimum design variables of a, $100d$, V_f and N.

heat sink with horizontal inlet flow is the same size as that with vertical inlet flow described in the preceding paragraph. The simulation of PHOENICS was applied again to this optimum heat sink sizes; as a result, the highest base temperature is found out as 33.55 °C. Although the thermal performance in horizontal inlet flow is slightly effective than that in vertical inlet flow, the total volume of heat sink's design with vertical inlet flow is less than that in horizontal inlet flow. This volume reduction can decrease the cost and fit the limiting assembly space so that the plate-fin heat sink with vertical inlet fan still mainly been adopted by most practical applications.

D. Example 3: Constrained Optimal Design With Vertical Inlet Flow

In this Example, we show a practical air-cooled heat sink design to dissipate the generating heat from the electronic component. By following Example 2, we select N, a, d and L are design variables to redesign the heat sink. Several constrained design conditions are considered in this problem. Thus, the nonlinear constrained mathematical formulation can be written as

$$\text{Find} \quad \mathrm{X} = [x_1, x_2, x_3, x_4]^{\mathrm{T}} = [N, a, d, L]^{\mathrm{T}}$$
$$\text{Min} \quad \dot{S}_{\rm gen} = \frac{Q^2 R_{\rm sin}}{T_e^2} + \frac{F_d V_f}{T_e}. \tag{25}$$

Subject to

$$g_1 : 1 - \left(\frac{W-d}{N-1} - d\right) \le 0 \tag{26}$$
$$g_2 : \left(\frac{W-d}{N-1}\right) - d - 5 \le 0 \tag{27}$$
$$g_3 : 0.01 - \frac{a}{s} \le 0 \tag{28}$$
$$g_4 : \frac{a}{s} - 19.4 \le 0 \tag{29}$$
$$g_5 : 0.0001 - \sqrt{\frac{sV_{\rm ch}}{\nu} \times \frac{s}{L}} \le 0 \tag{30}$$

TABLE III
RESULT OF CONSTRAINED OPTIMAL DESIGN WITH VERTICAL INLET AIR FLOW

	N	a (mm)	d(mm)	L (mm)	T_b (^oC)	$\dot{S}_{gen}$ (W/K)	s (mm)	a/s	V_{oL}(mm^3)
This paper	19	35.89	0.854	175.6	31.89	0.00349924	1.8762	19.13	119820

where

$$s = \frac{W - d}{N - 1} - d. \quad (31)$$

The design bounds of each design variable are the same as that in Example 1. The velocity of cooling air V_f is 2 m/s. The constrained function of g_1 and g_2 indicating the spacing between two fins has to be greater than 1 mm and smaller than 5 mm from manufacturing obligation. The constrained function of g_3 and g_4 indicating the aspect ratio of fin height to fin spacing that must be between 0.01 and 19.4 from current manufacturing capability. The constraint g_5 prevents $\sqrt{R^*_{\mathrm{es}}}$ in (10) from being zero. This constrained optimization problem have been solved by the feasible direction method. The optimum result listed in Table III, as compared with Example 2, generates an improved performance due to obtain the lower surface temperature. The total structural mass is less than the original design written in Table I.

E. Example 4: Optimal Design and Analysis of AOpen AC610D Heat Sink With Prescribed Heat Dissipation

The thermal performance of AOpen AC610D [15] heat sink with air-cooling (Fig. 1) used on AMD Thunderbird 1-GHz CPU has been examined in this example. Further information can be found from the web [14]. This heat sink can dissipate 54 W and is made of aluminum alloy with thermal conductivity of 200 W/mK. The geometrical heat sink base is 63 × 80 × 8 mm containing 21 plate-fins in which each fin is 80 × 1 × 32 mm, as shown in Fig. 10. A fan with 4 m/s air velocity vertically accumulate on the fins' top. The environmental average temperature is 30 °C. Through the simulation of PHOENICS, the highest T_b was found as 65.7 °C that agrees with the announced value of 65.5 °C posted on the web [14]. There are two optimal redesign cases of plate-fin heat sink for AOpen AC610D that is describing as following.

Case 1) Determine the optimum fins number N and fin thickness d to dissipate 54 W under same geometrical sizes of AOpen AC610D. The design formulation and constraints are (25)to (31). The optimum results are listed in Table IV where the second row indicates the associated characteristics computed by the formula presenting in this paper. One can observe that the increasing of both aspect ratio and fins number can promote the thermal performance where the base temperature drops to 50.04 °C from 65.93 °C. One also can see the total mass of heat sink is reduced in this design, as compared with the original one. It is noted that the T_b value of 65.93 °C computed by (3) is very close to the announced value of 65.5 °C on the web. Therefore, this example shows that both using the presented formulation in this paper and using the simulation of PHOENICS agree with the value of T_b announced on the web.

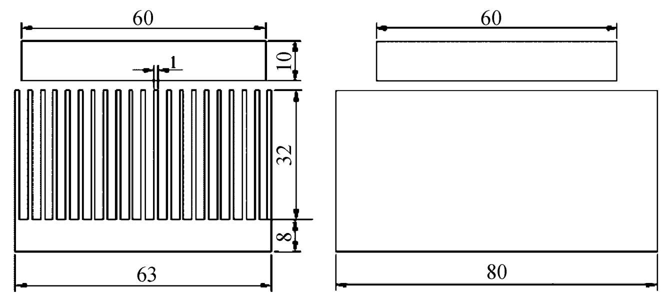

Fig. 10. Geometrical configuration of AOpen AC610D heat sink (length units are in mm).

Case 2) The fin height a and the base length L are allowed to be designed for obtaining the optimal performance of the AC610D heat sink; thus, total design variables are N, a, d and L. The design conditions and parameters are the same as that in Case 1; the optimal results are listed in Table V. The optimum design of the current case shows a superior thermal performance to Case 1 due to a lower base temperature of 35 °C. The primary changes include a longer L and higher a, as compared to original design. The design in this case has fewer fins number N and larger thickness d, as compared to two variables design in Case 1. The aspect ratio in both of Case 1 and Case 2 notably goes to the upper limit. We also examined the thermal characteristics of this case by PHOENICS; as a result, the highest base temperature of 36.4 °C is close to 35.0 °C computed in this paper.

F. Example 5: Optimal Design of AOpen AC610D With Prescribed Surface Temperature for Maximum Heat Dissipation

Two other design cases are constructed here for redesigning the existing heat sink based on the presenting design model.

Case 1) What are the two variables to be optimized to redesign the AOpen AC610D heat sink of Example 4 with an original required temperature T_b of 65 °C? The mathematical formulation is written as

$$\text{Find} \quad \boldsymbol{X} = [x_1, x_2]^{\mathrm{T}} = [N, d]^{\mathrm{T}}$$
$$\text{Minimize} \quad \dot{S}_{\mathrm{gen}} = \frac{[(T_b - T_e)h_c A\eta_t]^2 R_{\sin}}{T_e^2} + \frac{F_d V_f}{T_e} \quad (32)$$

subject to constraints of (26)–(31), where (32) can be obtained by (21) and (22). The optimal results of this problem are listed in Table VI. The optimum design of N and d in this case result in the maximum power dissipation of 73.4 W that is much higher than the original 54 W. Table VI clearly shows that the less thickness of fins and simultaneously increasing the number of fins can considerably enhance the thermal performance. One also see the total volume is less than that in

TABLE IV
TWO VARIABLES OPTIMAL DESIGN OF AC610D HEAT SINK

	N	$d(mm)$	s (mm)	T_b (^oC)	$\dot{S}_{gen}$ (W/K)	$R_{sin}(K/W)$	a/s	F_d (N)	$V_{oL}(mm^3)$
This paper	28	0.6594	1.6554	50.04	0.013611	0.3771	19.33	0.14393	87585
AOpen AC610D	21	1.0	2.0895	65.93	0.022002	0.6654	15.31	0.08317	94080

TABLE V
FOUR VARIABLES OPTIMAL DESIGN OF AC610D HEAT SINK

	N	a (mm)	d (mm)	L (mm)	s (mm)	T_b (^oC)	$\dot{S}_{gen}$ (W/K)	$R_{sin}(K/W)$	a/s	F_d (N)	$V_{oL}(mm^3)$
This paper	18	39.927	1.5571	134.85	2.0572	35.00	0.013098	0.1266	19.40	2.3978	218870
AOpen AC610D	21	32.0	1.0	80.0	2.0895	65.93	0.022002	0.6654	15.31	0.0832	94080

TABLE VI
TWO VARIABLES OPTIMAL DESIGN OF AC610D HEAT SINK WITH PRESCRIBED $T_b = 65$ °C

	N	d (mm)	s (mm)	$\dot{S}_{gen}$ (W/K)	$R_{sin}(K/W)$	a/s	Q (W)	$V_{oL}(mm^3)$
This paper	27	0.5091	1.8999	0.029866	0.4768	16.84	73.4	75508
AOpen AC610D	21	1.0	2.0895	0.022002	0.6654	15.31	54.0	94080

TABLE VII
FOUR VARIABLES OPTIMAL DESIGN OF AC610D HEAT SINK WITH PRESCRIBED $T_b = 65$ °C

	N	a (mm)	d (mm)	L (mm)	s (mm)	$\dot{S}_{gen}$ (W/K)	$R_{sin}(K/W)$	a/s	Q (W)	$V_{oL}(mm^3)$
This paper	24	28.50	1.090	75.80	1.6017	0.029725	0.4616	17.79	83.23	94716
AOpen AC610D	21	32.0	1.0	80.0	2.0895	0.022002	0.6654	15.31	54.0	94080

TABLE VIII
OPTIMAL DESIGN OF AC610D HEAT SINK WITH PRESCRIBED LOW BASE TEMPERATURE

	N	$a(mm)$	$d(mm)$	$L(mm)$	s (mm)	$\dot{S}_{gen}$ (W/K)	$R_{sin}(K/W)$	a/s	Q (W)	$V_{oL}(mm^3)$
This paper	22	21.40	0.856	71.00	2.1032	0.00208282	0.1688	10.176	59.24	64397
AOpen AC610D	21	32.0	1.0	80.0	2.0895	0.022002	0.6654	15.31	54.0	94080

Case 1 of Example 4 and particularly less than the original design.

Case 2) The four design variables of N, d, a, and L are required to be determined by minimizing (32) subject to (26)–(31). The result in Table VII shows that a smaller spacing s between two fins and a slightly high aspect ratio yield to an improved thermal design. The resulting geometrical size of $75.8 \times 63 \times 36.5$ mm is smaller than the original $80 \times 63 \times 40$ mm without the mass changes. The presenting optimum design can promote 54% of the heat dissipation in a smaller heat sink size, as compared with the original one. Once again, we examine the current design by PHOENICS with power loss of 83 W and found out that the highest temperature is 66.31 °C that is nearly closed to the prescribed temperature of 65 °C.

G. Example 6: Optimal Design of AOpen AC610D With Prescribed Low Surface Temperature for Maximum Heat Dissipation

Assume a heat sink design requires low temperature of 40 °C in the circumstances similar to AMD Thunderbird 1-GHz heat source. Determine a plat-fin heat sink size to obtain the maximum heat dissipation where the width of sink base is 63 mm. This problem can be formulated to find design variables N, a, d and L by minimizing $\dot{S}_{\rm gen}$ of (32) subjected to (26)–(31) under the prescribed 40 °C of T_b. The optimal result is listed in Table VIII where the geometrical size of $71.0 \times 63 \times 29.40$ mm is smaller than the original $80 \times 63 \times 40$ mm and the total mass is less than that of original design. This design still can dissipate a larger power of 59.24 W than the original 54 W; however, the original surface temperature 65 °C can be significantly reduced to 40 °C.

VI. CONCLUSION

A systematic optimal design process and parametric study of the plate-fin heat sink by applying the theory of entropy generation rate is developed in this paper. The resultant thermal analysis with presenting method confirms the results from both vendors' announced information and the simulation of package software. Three criteria of optimum heat sink design by minimizing the entropy generation rate are successfully presented.

1) Using formal nonlinear constrained optimization technique, as shown in Example 3.

2) Using prescribed heat dissipation, as shown in Example 4.

3) Using prescribed surface temperature, as shown in Example 5 and 6.

Including the illustrative examples, several concluding remarks can be summarized as follows.

1) The optimal design of a plate-fin heat sink with horizontal inlet flow has a slightly better performance than that with vertical inlet flow. However, the presenting formulation of the vertical inlet flow model using a formal nonlinear constrained optimization technique, by minimizing the entropy generation rate that constructs a lesser amount of mass (volume) design, as compared with a horizontal inlet flow model without a constraint design. The aspect ratio a/s in minimizing the entropy generation rate with horizontal inlet flow, results in less value than that with vertical inlet flow.

2) The constraint design of a vertical inlet flow model creates an improved thermal performance with lower base temperature than that of unconstrained design model.

3) In the design of minimizing the entropy generation rate with the prescribed heat dissipation, the optimal aspect ratio of fins goes to the highest allowable limit.

4) The design of minimizing the entropy generation rate with a prescribed surface temperature results in an optimal aspect ratio instead of the highest allowable limit.

5) Under the prescribed heat dissipation, the design of the presented method yields to an economic structural size and enhanced thermal performance with the lower surface temperature. The heat sink volume increases as the surface temperature decreases.

6) Under the prescribed surface temperature, the design of the presented method can increase heat dissipation with less structural volume. The increasing structural volume of the heat sink increases the heat dissipation.

7) In general, the optimization method, using the minimization of entropy generation rate, can improve the thermal performance of heat sink, and the mass (structural volume) required is accordingly reduced.

8) The methodology and design process presented in this paper can be further applied to other finned type heat sink designs.

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