



SPECIAL SECTION: THE DEVELOPMENT OF MATHEMATICAL COGNITION

Development of number combination skill in the early school years: when do fingers help?

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Abstract

Children's change over time in frequency of finger use on number combinations was examined in relation to their change in accuracy. Performance was tracked longitudinally over 11 time points, from the beginning of kindergarten (mean age = 5.7 years) to the end of second grade (n = 217). Accuracy in number combinations increased steadily during the time period while frequency of finger use declined. Correlations between finger use and accuracy decreased gradually, ranging from 0.60 in kindergarten to -0.15 at the end of second grade. Low-income children showed linear growth in frequency of finger use while middle-income children slowed down by second grade and even started to decline. Although girls and boys showed similar growth patterns in frequency and accuracy, boys used their fingers less often than girls and were more accurate. The findings indicate that finger use is most adaptive when children are first learning number combinations, but this benefit lessens over time.

Introduction

Facility with addition and subtraction number combinations is a hallmark of elementary school math (Jordan, Hanich & Uberti, 2003). Number combination skill at the beginning of kindergarten is the single strongest predictor of math achievement at the end of first grade (Jordan, Kaplan, Locuniak & Ramineni, 2007). Moreover, deficient fact mastery is a signature feature of math difficulties and disabilities (Gersten, Jordan & Flojo, 2005).

Developmental studies show that children use a variety of strategies to solve number combinations until they achieve fact mastery (Ginsburg & Russell, 1981; Kerkman & Siegler, 1993; Siegler & Shrager, 1984; Siegler & Shipley, 1995; Siegler & Jenkins, 1989). Early on children might simply guess, or they might approximate answers based on their intuitive knowledge of adding and taking away. They also might form a mental model of the number representations and transformations (Huttenlocher, Jordan & Levine, 1994). As children become knowledgeable about calculation principles (e.g. commutativity), they learn to count on from the first or larger addend in addition and form rules for solving difficult combinations (e.g. $7 + 10 = 17$ so $7 + 9 = 16$, or $8 + 5 = 13$ so $13 - 5 = 8$) (Baroody, 1999; Dowker, 1998). Children master some combinations more quickly than others and make adaptive strategy choices when an answer cannot be retrieved automatically.

Along the path to mastery, fingers provide a natural scaffold for calculation. In fact, archeological studies reveal that use of fingers for arithmetic dates back to ancient times (Williams & Williams, 1995). It has been suggested that calculation skills were derived from finger sequencing, and that finger knowledge and calculation have common neurological underpinnings (Ardila, 1993). Finger counting strategies, which help children represent and manipulate quantities, facilitate the transition between early nonverbal representations and conventional symbolic representations that are more dependent on culture. Finger counting can lead to accurate associations between a combination and its solution (Siegler & Shipley, 1995).

There are questions, however, about the timing of finger use. When do children benefit from using their fingers and when do fingers potentially get in the way of development? Jordan and colleagues (Jordan, Huttenlocher & Levine, 1992; Jordan, Levine & Huttenlocher, 1994) reported that finger use was associated with accuracy on number combinations in kindergarten and first grade. Kindergartners with the worst performance on number combinations, who also tended to be from low-income backgrounds, almost never used their fingers spontaneously. Many of these children, however, successfully solved the same calculations when the problems were presented in a nonverbal format, suggesting they had fundamental knowledge of number transformations. In

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first grade, low-income children started using their fingers and made substantial gains in calculation accuracy. Middle-income children used their fingers less often in first grade than they did in kindergarten, indicating that early use of fingers helped them shift more quickly to accurate mental calculation or retrieval.

Suggestively, studies show that children with MD (mathematical difficulties) use their fingers more for number combinations and retrieve facts less than children without MD (Hanich, Jordan, Kaplan & Dick, 2001; Jordan, Kaplan & Hanich, 2002; Jordan, Hanich & Kaplan, 2003a; Ostad, 1997, 1999).

In the present longitudinal investigation, we examined change over time in frequency of finger use on number combinations in relation to change over time in accuracy. Children's performance was assessed on 11 occasions, from the beginning of kindergarten through the end of second grade. Earlier work suggests that deficits in fact mastery can be identified reliably by second grade and that these deficits persist throughout elementary school (Jordan, Hanich & Kaplan, 2003b). Children in the present study were from middle and low SES families, but all were taught with the same curricular approach, one that neither encouraged nor discouraged fingers.

Our hypothesis was that fingers are most helpful when children are first learning to compute with small number sets (totals of 10 or less) but become less so over time. By second grade, children must compute with larger numbers, where other strategies are more advantageous (Siegler & Shipley, 1995). We expected low-income children to start using their fingers later than their middle-income counterparts and to continue to use them longer. Because it has been suggested that girls rely more on concrete objects than boys when solving number combinations (Carr & Jessup, 1997; Fennema, Carpenter, Jacobs, Franke & Levi, 1998; Jordan *et al.*, 2003a) and that a male advantage in math reasoning is mediated by calculation fluency (Geary, Saults, Liiu & Hoard, 2000), gender was also considered in our analyses.

Method

Participants

Drawn from the same school district in Northern Delaware, US, 414 participants were recruited at the beginning of kindergarten for our longitudinal study of children's math (Jordan, Kaplan, Olah & Locuniak, 2006). The district served both low-income urban and middle-income suburban communities. From this group, 217 children remained in the study at the end of second grade. Background characteristics of the participating children at the end of second grade are presented in Table 1. The demographics for children in second grade mirror those for the original kindergarten sample (Jordan *et al.*, 2006). All children were taught math with the same curricular approach in kindergarten and first

Table 1 Demographic information by income status for participants at time 11 ($n = 217$)

	Low income 58 (27%)	Middle income 159 (73%)
Gender		
Male	30 (52%)	88 (55%)
Female	28 (48%)	71 (45%)
Race		
Minority ^a	51 (88%)	49 (31%)
Non-minority	7 (12%)	110 (69%)
English language learners	11 (19%)	2 (1%)
Mean kindergarten start age (<i>SD</i>)	67 months (4 months)	67 months (4 months)

^a Low income was defined as participating in the free or reduced lunch program in school. Minority refers to African-American ($n = 33$), Asian ($n = 2$), or Hispanic ($n = 16$) for low-income group and African-American ($n = 28$), Asian ($n = 11$), or Hispanic ($n = 10$) for middle-income group.

grade (Teaching Integrated Mathematics and Science Curriculum, 2004).

Materials and procedure

Children were assessed longitudinally on number combinations over 11 time points: four times in kindergarten (September, November, February, and April); four times in first grade (September, November, February, and April); and three times in second grade (November, February, and April). Children were tested individually in school by a trained undergraduate or graduate student researcher. The data were collected as a part of a larger study on the development of children's math proficiency. Although the measures were given in English, children participating in the English Language Learners program were assessed by a researcher fluent in both English and Spanish and were allowed to have instructions clarified in Spanish and/or to respond in Spanish.

Number combinations were read to children as: 'How much is m and n ?' and 'How much is n take away m ?' Children answered orally. Children were told they could do anything they wanted to help them get an answer and were asked to keep their hands in full view of the examiner. An item was repeated once at the child's request. On each trial, the examiner recorded the child's exact answer and whether or not fingers were used to assist calculation. Inter-rater reliability for a fingers/no-fingers strategy classification on number combinations is 98% (Hanich *et al.*, 2001).

In kindergarten, children were given number combinations involving sums and minuends of 7 or less ($n = 8$). To prevent ceiling effects, four combinations were added in the middle of first grade, which involved sums and minuends of 10 and 13 ($n = 12$). In second grade, all of the sums and minuends were between 7 and 17 ($n = 10$). Answers were coded as correct/incorrect, and the raw scores for each child were converted to percentage correct to give a common metric for the analyses. Likewise, fingers scores represented the percentage of trials on which fingers were used at a given time point.

Table 2 Correlations between percentage of trials on which fingers were used and percentage correct on number combinations at each time point

	T1NC	T2NC	T3NC	T4NC	T5NC	T6NC	T7NC	T8NC	T9NC	T10NC	T11NC
T1fing	.58**										
T2fing		.60**									
T3fing			.57**								
T4fing				.59**							
T5fing					.47**						
T6fing						.32**					
T7fing							.41**				
T8fing								.29**			
T9fing									.17**		
T10fing										−0.02	
T11fing											−0.15*

** $p < .01$.* $p < .05$.

Results

The correlations between the percentage of trials on which fingers were used on number combinations (*frequency of finger use*) and percentage of trials on which number combinations were solved correctly (*accuracy*) for each time point are presented in Table 2. The data show a progression from relatively strong and significant positive correlations in kindergarten (.57 to .60), to decreasing positive correlations in first and second grades (.47 to .17), and finally to a small but significant negative correlation by the end of second grade (−.15). The pattern was similar for addition and subtraction problems.

A parallel process growth curve model was used to estimate the correlations among the growth parameters of frequency of finger use and accuracy on number combinations. A parallel process growth curve model joins two conventional growth curve models but allows the growth parameters of each process to correlate. Thus, we were able to examine whether the growth parameters associated with frequency of finger use correlated with growth parameters associated with number combinations accuracy.

A path diagram of the parallel growth curve model is given in Figure 1. The path diagram shows how the parallel process model is specified. The squares indicate the repeated measures of frequency of finger use and accuracy while the circles represent the intercept, slope, and quadratic term for frequency of finger use, and the intercept and slope for accuracy. The diagram also shows how the growth factors are regressed on the intercept and slope for gender and income. Remaining paths are specified to provide estimates of the growth parameters for the two processes. The correlations among the growth factors are not presented here.

Table 3 shows the parameter estimates for the parallel growth curve model. Based on inspection of the empirical trajectories of finger use and number combinations, we specified a linear growth curve model for accuracy and a quadratic growth curve for frequency of finger use.

The status parameter (intercept) was estimated for Time 11, corresponding to the end of second grade. We present the results for frequency of finger use and accuracy without covariates (Model 0) and then add the covariate background variables (Model 1) of income (1 = low-income children, 0 = middle-income children), and gender (1 = male, 0 = female).

Frequency of finger use

The observed and fitted growth trajectories for mean percentage of trials fingers were used on number combinations are plotted in Figure 2a. Holding constant change over time in accuracy, the results showed that frequency of finger use at the end of second grade was significantly different from zero (frequency intercept = 41.72, $z = 17.10$, $p < .05$). The linear slope of finger use was non-significant (frequency slope = −0.48, $z = -1.74$, $p > .05$). The quadratic term was found to be negative and statistically significant (frequency quadratic = −0.05, $z = -6.63$, $p < .05$), indicating that finger use slows down over time.

Accuracy

The observed and fitted growth trajectories for mean percentage correct on number combinations are plotted in Figure 2b. Holding constant change over time in frequency of finger use, the results showed that number combinations accuracy at the end of second grade was statistically different from zero (accuracy intercept = 80.54, $z = 57.18$, $p < .05$). The linear slope for number combinations was positive and statistically significant (accuracy slope = 1.56, $z = 26.58$, $p < .05$), indicating that accuracy in number combinations increases linearly over time.

Income and gender

The *estimated* means for frequency of finger use and accuracy at each time point are plotted for income in Figure 3 (a and b) and for gender in Figure 4 (a and b).

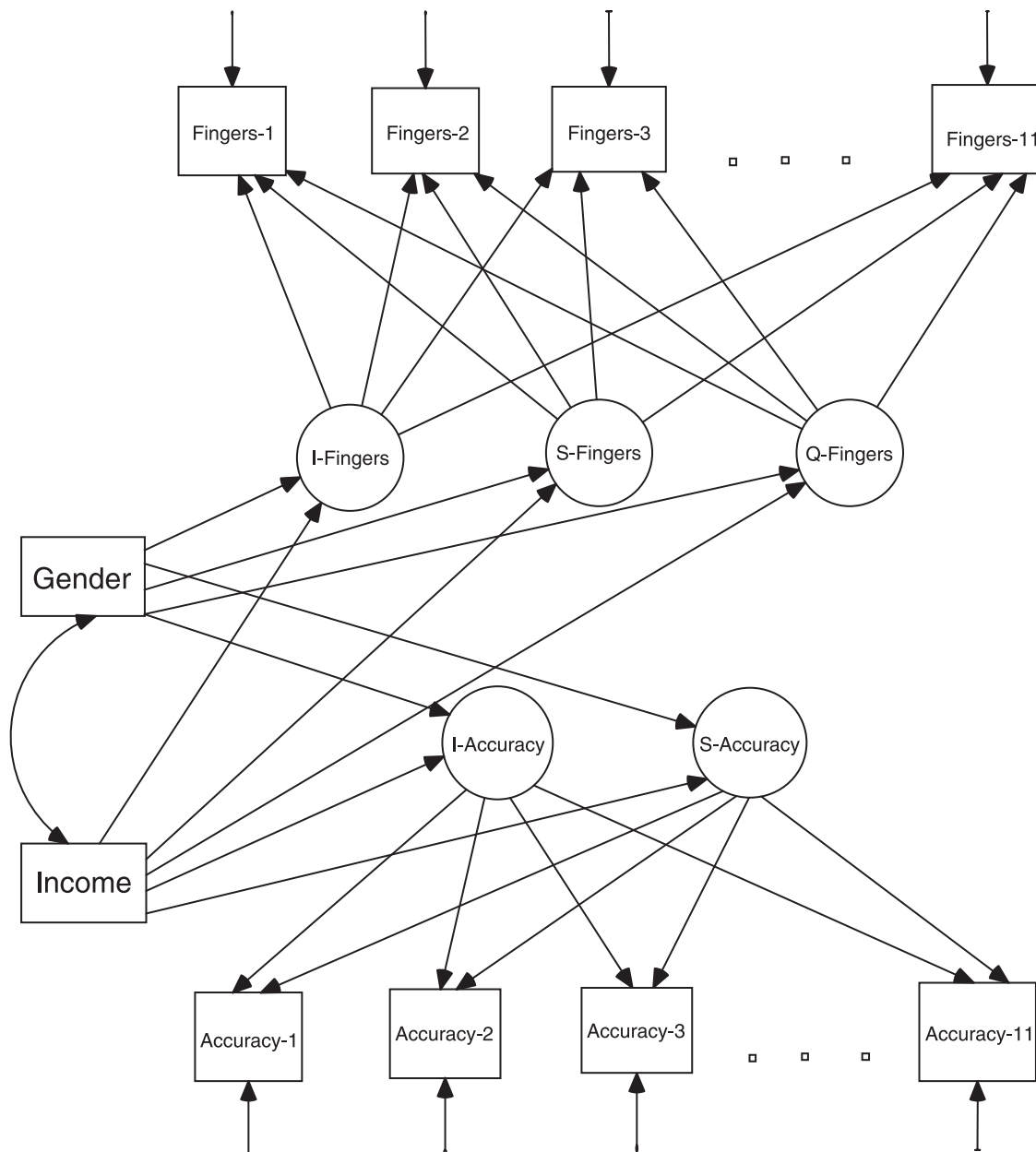


Figure 1 Path diagram of parallel growth curve model for finger use and number combinations. 'Fingers' stands for the percentage of trials on which fingers were used and 'Accuracy' stands for percentage correct on number combinations. Note that for ease of reading, correlations among growth parameters are not presented but are estimated in the analysis. (I: intercept, S: linear slope, Q: quadratic slope).

When adding predictors to the model, the mean growth parameters are associated with the group coded zero on income and gender. For middle-income girls, there is a statistically significant linear decrease and deceleration in frequency of finger use and a statistically significant linear increase in accuracy. In general, though, at the end of second grade, low-income children used their fingers more often than middle-income children (frequency intercept = 12.66, $z = 2.66$, $p < .05$) and showed a greater linear increase in the frequency of finger use over time (frequency slope = 1.81, $z = 3.41$, $p < .05$), with their frequency of finger use accelerating

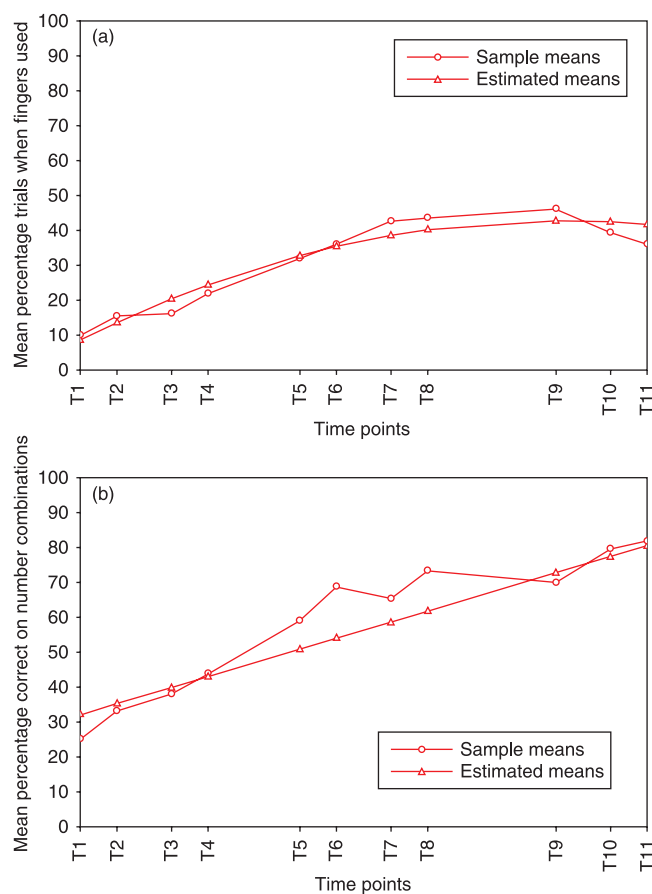
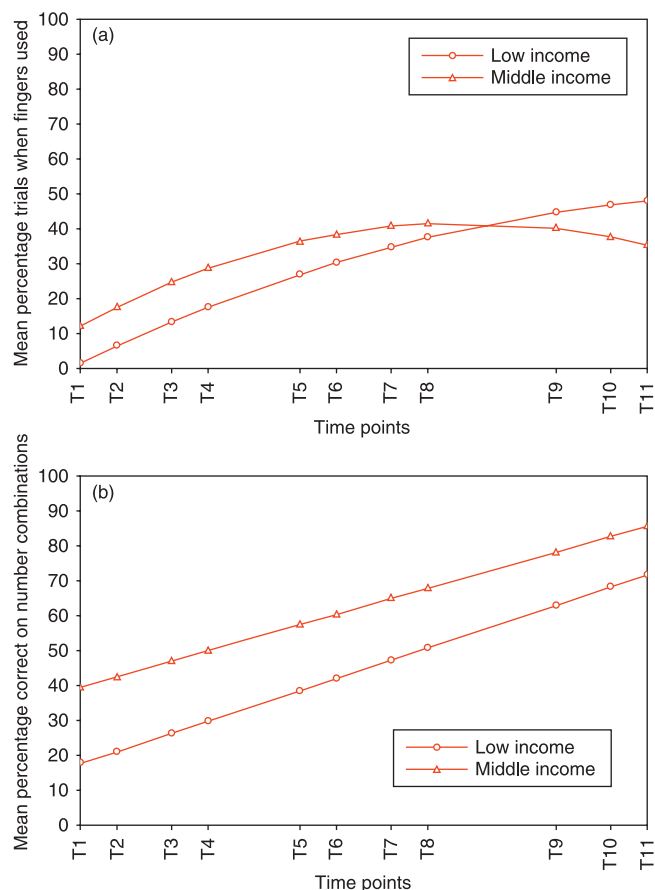
more quickly than for middle-income children (frequency quadratic = 0.34, $z = 2.18$, $p < .05$) (see Figure 3a). The mean percentage of trials on which low-income children used their fingers at time 11 was 44% ($SD = 36$) versus 33% ($SD = 36$) for middle-income children.

With regard to accuracy, low-income children showed a greater linear increase over time than middle-income children (accuracy slope = 0.26, $z = 2.32$, $p < .05$), but ultimately exhibited poorer performance by the end of second grade (accuracy intercept = -13.97, $z = -5.2$, $p < .05$). This can be seen in Figure 3b. The mean percent correct at the end of second grade was 73% ($SD = 24$)

Table 3 Parameter estimates for parallel growth curve model: the percentage of trials on which fingers were used and percentage correct on number combinations

	Percentage of trials on which fingers were used		Percentage correct on number combinations	
	Model 0	Model 1	Model 0	Model 1
Growth parameters				
Intercept	41.72*	45.45*	80.54*	82.42*
Slope	-0.48	-0.88*	1.56*	1.41*
Quadratic	-0.05*	-0.06*		
Var(intercept)	924.83*	816.42*	448.42*	351.02*
Var(slope)	9.54*	8.11*	0.56*	0.51*
Var(quadratic)	0.01*	0.01*		
Regression coefficients				
Intercept on male		-19.03*		5.97*
Slope on male		-0.84		0.15
Quadratic on male		-0.01		
Intercept on low income		12.66*		-13.97*
Slope on low income		1.81*		0.26*
Quadratic on low income		0.03*		
BIC	61591.13	62946.51	61591.13	62946.51

Note: Var () stands for the conditional variance of the parameters in parentheses.
* $p < .05$.

**Figure 2** (a) Observed and fitted growth trajectories for mean percentage of trials on which fingers were used on number combinations. (b) Observed and fitted growth trajectories for mean percentage correct on number combinations.**Figure 3** (a) Fitted growth trajectories for mean percentage of trials on which fingers were used on number combinations, by income status. (b) Fitted growth trajectories for mean percentage correct on number combinations, by income status.

for low-income children and 85% ($SD = 16$) for middle-income children.

In terms of gender, girls used their fingers more often than boys at the end of second grade (frequency intercept = -19.03 , $z = -4.46$, $p < .05$). The mean percentage of trials on which girls used their fingers was 48% ($SD = 38$) versus 26% ($SD = 31$) for boys. There was no significant gender difference in the linear trend, and no significant gender difference in the acceleration for finger use. With regard to accuracy, boys performed significantly better than girls at the end of second grade (accuracy intercept = 5.97 , $z = 2.43$, $p < .05$). The mean percent correct at the end of second grade was 78% ($SD = 20$) for girls and 85% for boys ($SD = 18$). However, there were no differences in their linear rates of growth.

Correlations between finger use and number combinations growth parameters

For the model without predictors (Model 0) the correlation between the linear trend in finger use and the linear trend in accuracy was not significant ($r = -0.03$, $p > .05$), indicating that increasing reliance on fingers does not

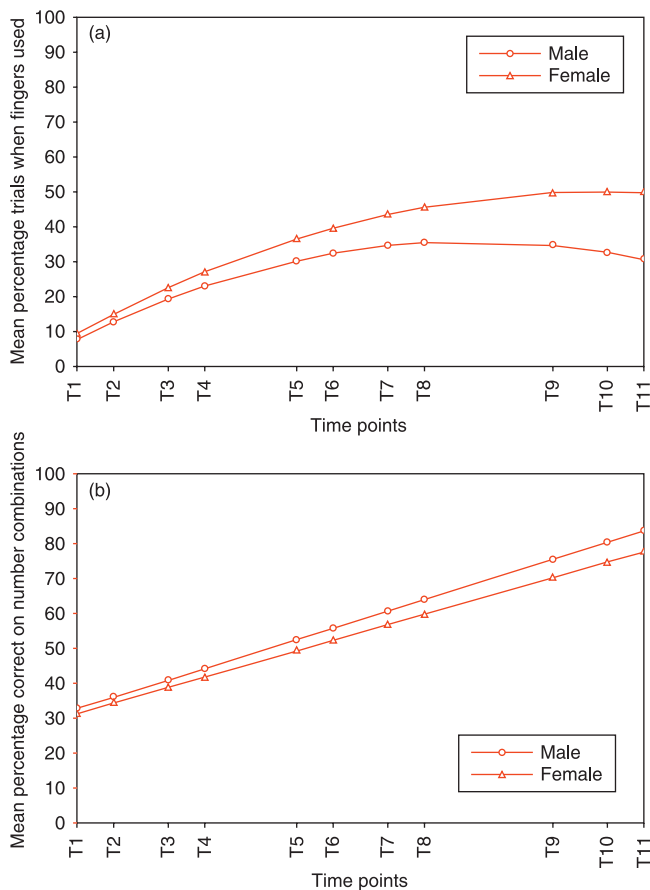


Figure 4 (a) Fitted growth trajectories for mean percentage of trials on which fingers were used on number combinations, by gender. (b) Fitted growth trajectories for mean percentage correct on number combinations, by gender.

confer an advantage to linear development in number combinations accuracy. There was a negative and significant correlation between the quadratic and linear trends in Model 0 ($r = -0.36$, $p < .05$), indicating that as finger use decelerates, accuracy in number combinations increases.

Discussion

Finger use for math calculation is natural and intuitive. Fingers are readily available and well suited to our base-10 number system (Ardila, 1993). However, many teachers and parents question the value and appropriateness of finger use in elementary school, even though most contemporary math curricula encourage concrete representation and manipulation of quantities. It is important to be mindful that our number combinations task allowed children to choose their own strategies, but we did not provide external supports or concrete manipulatives.

Overall, accuracy in number combinations increased steadily from the beginning of kindergarten through the end of second grade, while frequency of finger use

declined over the same time period. In kindergarten, frequency of finger use was a strong and reliable predictor of number combination accuracy. By the end of second grade, however, there was a small but significant negative correlation between finger use and accuracy. Although we did not observe specifically how children used their fingers, finger counting errors are more common on combinations with larger totals (e.g. > 10) (Geary, 1994).

At the end of second grade, low-income children used their fingers more but were less accurate than their middle-income counterparts. Between kindergarten and second grade, low-income children showed a steady increase in finger use whereas middle-income children started to decrease around second grade. Although low-income children's greater finger use was associated with more accurate performance, they never caught up with the middle-income group in accuracy. Over a decade ago, when children were taught with a less contemporary math curriculum, Jordan *et al.* (1992, 1994) reported a strikingly similar crossover finding, namely, that low-income children use their fingers less than middle-income children in kindergarten but more in first grade.

Girls and boys showed similar developmental paths in frequency of finger use and calculation accuracy, holding income level constant. At the end of second grade, however, boys used their fingers less often than girls and retained their small but reliable edge in accuracy. These results are in keeping with previous reports (Carr & Jessup, 1997; Fennema *et al.*, 1998; Jordan *et al.*, 2003a) showing that girls use their fingers or other concrete representations more than boys in arithmetic. The question is, why are boys less reliant on fingers than girls? Data collected in kindergarten (Jordan *et al.*, 2006) show gender differences favoring boys in number sense overall. It has been suggested that basic number sense (e.g. knowledge of numerical magnitudes) underpins fact mastery and fluency (Jordan *et al.*, 2003b; Gersten *et al.*, 2005), which may lead to less reliance on fingers. There also might be gender differences related to strategy choices and style. Carr and Jessup (1997) found that boys prefer retrieval to other calculation strategies, including fingers.

Our findings suggest that in kindergarten children should be encouraged, or even explicitly taught, to use their fingers. Although some children will always have difficulties with automatic fact retrieval, they might eventually be better served by calculating in their heads rather than on their hands.

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References

- Ardila, C. (1993). On the origins of calculation abilities. *Behavioural Neurology*, **6**, 89–97.
- Baroody, A.J. (1999). Children's relational knowledge of addition and subtraction. *Cognition and Instruction*, **17** (2), 137–175.
- Carr, M.C., & Jessup, D.L. (1997). Gender differences in first-grade mathematics strategy use: social and metacognitive influences. *Journal of Educational Psychology*, **89** (2), 318–328.
- Dowker, A.D. (1998). Individual differences in arithmetical development. In C. Donlan (Ed.), *The development of mathematical skills* (pp. 275–302). London: Taylor and Francis.
- Fennema, E., Carpenter, T.P., Jacobs, V.R., Franke, M.L., & Levi, L.W. (1998). A longitudinal study of gender differences in young children's mathematical thinking. *Educational Researcher*, **27** (5), 6–11.
- Geary, D.C. (1994). *Children's mathematical development*. Washington, DC: American Psychological Association.
- Geary, D.C., Saults, S.J., Liu, F., & Hoard, M.K. (2000). Sex differences in spatial cognition, computational fluency, and arithmetical reasoning. *Journal of Experimental Child Psychology*, **77**, 337–353.
- Gersten, R., Jordan, N.C., & Flojo, J.R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities*, **38**, 293–304.
- Ginsburg, H.P., & Russell, R.L. (1981). Social class and racial influences on early mathematical thinking. *Monographs of the Society for Research in Child Development*, **46** (6 Serial No. 193).
- Hanich, L., Jordan, N.C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology*, **93** (3), 615–626.
- Huttenlocher, J., Jordan, N.C., & Levine, S.C. (1994). A mental model for early arithmetic. *Journal of Experimental Psychology: General*, **123**, 284–296.
- Jordan, N.C., Hanich, L.B., & Kaplan, D. (2003a). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with co-morbid mathematics and reading difficulties. *Child Development*, **74** (3), 834–850.
- Jordan, N.C., Hanich, L.B., & Kaplan, D. (2003b). Arithmetic fact mastery in young children: a longitudinal investigation. *Journal of Experimental Child Psychology*, **85**, 103–119.
- Jordan, N.C., Hanich, L.M., & Uberti, H.Z. (2003). Mathematical thinking and learning disabilities. In A. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Recent research and theory* (pp. 359–383). Mahwah, NJ: Erlbaum.
- Jordan, N.C., Huttenlocher, J., & Levine, S.C. (1992). Differential calculation abilities in young children from middle- and low-income families. *Developmental Psychology*, **28**, 644–653.
- Jordan, N.C., Kaplan, D., & Hanich, L.B. (2002). Achievement growth in children with learning difficulties in mathematics: findings of a two-year longitudinal study. *Journal of Educational Psychology*, **94** (3), 586–597.
- Jordan, N.C., Kaplan, D., Locuniak, M.N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research and Practice*, **22** (1), 36–46.
- Jordan, N.C., Kaplan, D., Olah, L., & Locuniak, M.N. (2006). Number sense growth in kindergarten: a longitudinal investigation of children at risk for mathematics difficulties. *Child Development*, **77**, 153–175.
- Jordan, N.C., Levine, S.C., & Huttenlocher, J. (1994). Development of calculation abilities in middle- and low-income children after formal instruction in school. *Journal of Applied Developmental Psychology*, **15**, 223–240.
- Kerkman, D.D., & Siegler, R.S. (1993). Individual differences and adaptive flexibility in lower-income children's strategy choices. *Learning and Individual Differences*, **5**, 113–136.
- Ostad, S.A. (1997). Developmental differences in addition strategies: a comparison of mathematically disabled and mathematically normal children. *British Journal of Educational Psychology*, **67**, 345–357.
- Ostad, S.A. (1999). Developmental progression of subtraction strategies: a comparison of mathematically normal and mathematically disabled children. *European Journal of Special Needs Education*, **14** (1), 21–36.
- Siegler, R.S., & Jenkins, E. (1989). *How children discover new strategies*. Hillsdale, NJ: Erlbaum.
- Siegler, R.S., & Shipley, C. (1995). Variation, selection, and cognitive change. In T. Simon & G. Halford (Eds.), *Developing cognitive competence: New approaches to process modeling* (pp. 31–76). Hillsdale, NJ: Erlbaum.
- Siegler, R.S., & Shrager, J. (1984). Strategy choice in addition and subtraction: how do children know what to do? In C. Sophian (Ed.), *Origins of cognitive skills* (pp. 229–293). Hillsdale, NJ: Erlbaum.
- Teaching Integrated Math and Science Curriculum (2004). *Math Trailblazers* (2nd edn.). Chicago, IL: Kendall/Hunt Publishing Company.
- Williams, B.P., & Williams, R.S. (1995). Finger numbers in the Greco-Roman world and the early middle ages. *Isis*, **86** (4), 587–608.