Interference Channel With a Causal Relay Under Strong and Very Strong Interference

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***Abstract*— In this paper, we study a two-user interference channel with a causal relay, where the relay's transmit symbol depends not only on its past received symbols, but also on its present received symbol. This is an appropriate model for studying amplify-and-forward type relaying when the bandwidth delay-spread product is much smaller than one. For the discrete memoryless interference channel with a causal relay, we derive a genie-aided outer bound. For the Gaussian interference channel with a causal relay, we define strong and very strong interference conditions and propose an outer bound for each case. We also propose an achievable scheme based on instantaneous amplify-and-forward (AF) relaying for the Gaussian interference channel with a causal relay and so it achieves capacity under some conditions. Our result extends the previous result by El Gamal, Hassanpour, and Mammen on the optimality of instantaneous AF relaying for the Gaussian relay channel with a causal relay to that of the Gaussian interference channel with a causal relay under strong and very strong interference.**

***Index Terms*— Interference channel, interference relay channel, instantaneous relay, causal relay, and amplify-and-forward relaying.**

I. Introduction

The performance of wireless communication systems is significantly affected by interference since resources such as time, frequency, and space are often shared. The two-user interference channel (IC) is the simplest communication channel with multiple source-destination pairs interfering with each other. Single-letter capacity characterization for discrete memoryless and Gaussian interference channels is still unknown in general and known only for some limited cases including strong [1], [2] and very strong [3] interference conditions.

On the other hand, relays can be used to help communication for interference channels [4], [5]. Recently, various interference relay channel models, relaying strategies, and bounds have been studied in [6]–[16]. In Gaussian interference relay channel with an out-of-band reception/in-band transmission relay, sum capacity result is shown for very strong relay-interference regime [8]. For the interference relay channel where the relay's transmit signal can be heard from only one source, the capacity was characterized under a certain condition using an interference forwarding scheme [9]. The achievable rate regions for amplify-and-forward, decode-and-forward, and estimate-and-forward relaying schemes applied to interference relay channels were analyzed in [10]. A generalized hash-and-forward scheme that is a combination of compress-and-forward and hash-and-forward was studied in [11]. Rate splitting and decode-and-forward relaying approach was considered in [12] showing that transmitting the common message only at the relay achieves the maximum achievable sum rate for symmetric Gaussian interference relay channels. Improved compress-and-forward and compute-and-forward relaying schemes were proposed in [13] and the sum-rate capacity under weak and strong interference conditions was studied. For a Gaussian interference channel with a cognitive relay having access to the messages transmitted by both sources, [14] proposed a new achievable scheme by combining the Han-Kobayashi scheme and dirty paper coding. Gaussian Z-interference channels with a relay with a rate-limited digital link and out-of-band relay were studied in [15] and [16], respectively.

In many results studying interference relay channels, it is typically assumed that the relay's operation is strictly causal, i.e., the relay's transmit symbol depends only on its past received symbols. However, sometimes it makes more sense to assume that the relay's operation is causal, i.e., its transmit symbol depends also on its present received symbol. This is a better model for studying AF type relaying if the overall delay spread including the path through the relay is much smaller than the inverse of the bandwidth. Such a scenario can easily happen when cell sizes are small, e.g., femto cell environments. Such a channel has recently been studied in [17], [18]. For the Gaussian relay channel with a causal relay, instantaneous amplify-and-forward relaying achieves the capacity if the relay's transmit power is greater than a certain threshold [17], [18].

For the cognitive interference channel where one of the transmitters knows the message of the other user non-causally, capacity results have been shown for some channel classes [19]. Cognitive Z-interference channel where one transmitter and receiver pair is interference-free, an achievable scheme was proposed using superposition and Gel'fand-Pinsker encoding, which was shown to be optimal when the channel for the non-cognitive transmitter-receiver pair is noiseless [20]. For the broadcast channels with cognitive relays, a new achievable scheme was proposed in [21].

In this paper, we consider an interference channel with a causal relay where two source nodes want to transmit messages to their respective destination nodes and a causal

Manuscript received August 15, 2011; revised June 17, 2012; accepted December 16, 2012. Date of publication November 26, 2013; date of current version January 15, 2014. This work was supported by the KCC, Korea under Grant KCA-2012-08-911-04-001.

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Communicated by E. Erkip, Associate Editor for Shannon Theory.

Digital Object Identifier 10.1109/TIT.2013.2291401

relay helps their communications. We study both discrete memoryless and Gaussian memoryless channels. We present a genie-aided outer bound for the discrete memoryless interference channel with a causal relay. For the Gaussian interference channel with a causal relay, we define strong and very strong interference conditions motivated by strong and very interference conditions for the Gaussian interference channel. For the Gaussian interference channel with a causal relay under strong and very strong interference, we show new genie-aided outer bounds such that both receivers know the received sequence at the relay. We also propose an achievable scheme using instantaneous amplify-and-forward relaying. The complexity of the proposed relaying is very low compared to other more complicated relaying schemes including decode-and-forward and compress-and-forward since it only performs symbolwise operations. Despite its simplicity, we show that it can achieve the capacity exactly when the relay's transmit power is greater than a certain threshold for the very strong interference case. For the strong interference case, the same is true if some additional conditions are satisfied. This is surprising since it means that such a simple symbolwise relaying can be optimal in a complicated communication scenarios with two source-destination pairs interfering with each other.

This paper is organized as follows: In Section II, we describe the discrete memoryless interference channel with a causal relay and derive an outer bound. In Section III, we define the Gaussian interference channel with a causal relay under strong and very strong interference and propose an outer bound for each case. We propose an achievable scheme based on instantaneous amplify-and-forward relaying and show that it achieves the capacity under certain conditions.

II. Discrete Memoryless Interference Channel With a Causal Relay

The discrete memoryless interference channel with a causal relay has two senders transmitting $X_1 \in \mathcal{X}_1$ and $X_2 \in \mathcal{X}_2$, two receivers receiving $Y_1 \in \mathcal{Y}_1$ and $Y_2 \in \mathcal{Y}_2$, a relay receiving $Y_R \in \mathcal{Y}_R$ and transmitting $X_R \in \mathcal{X}_R$. The channel is specified by a set of conditional probability mass functions $p(y_R|x_1, x_2)$ and $p(y_1, y_2|x_1, x_2, x_R, y_R)$. The channel is denoted by $(\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_R, p(y_R|x_1, x_2)p(y_1, y_2|x_1, x_2, x_R, y_R), \mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_R)$.

A $(2^{nR_1}, 2^{nR_2}, n)$ code for the discrete memoryless interference channel with a causal relay consists of two message sets $[1 : 2^{nR_1}] \triangleq \{1, 2, \ldots, 2^{nR_1}\}$ and $[1 : 2^{nR_2}]$, two encoders that produce codewords $x_1^n(m_1)$ and $x_2^n(m_2)$ as functions of messages $m_1 \in [1 : 2^{nR_1}]$ and $m_2 \in [1 : 2^{nR_2}]$, respectively. The messages M_1 and M_2 are assumed to be independent of each other and uniformly distributed in $[1 : 2^{nR_1}]$ and in $[1 : 2^{nR_2}]$, respectively. The encoding function of the relay is defined as $x_{R,i} = f_i(y_R^i)$, for $1 \le i \le n$, where y_R^i denotes $\{y_{R,1}, \ldots, y_{R,i}\}$. For decoding, decoder i produces a message estimate $\hat{m}_i$ as a function of the received sequence $y_i^n \in \mathcal{Y}_i^n$, for $i \in \{1, 2\}$. We assume the channel is memoryless, i.e., $(X_1^{i-1}, X_2^{i-1}, Y_R^{i-1}) \to (X_{1,i}, X_{2,i}) \to Y_{R,i}$ and $(X_1^{i-1}, X_2^{i-1}, X_R^{i-1}, Y_R^{i-1}, Y_1^{i-1}, Y_2^{i-1}) \to (X_{1,i}, X_{2,i}, X_{R,i}, Y_{R,i}) \to (Y_{1,i}, Y_{2,i})$ form Markov chains.

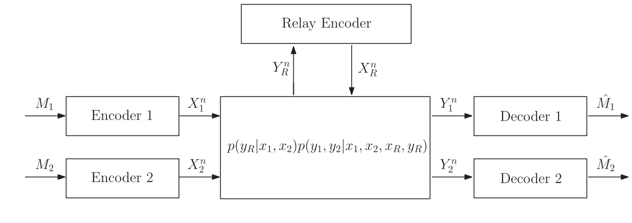

Fig. 1. Discrete Memoryless Interference Channel with a Causal Relay.

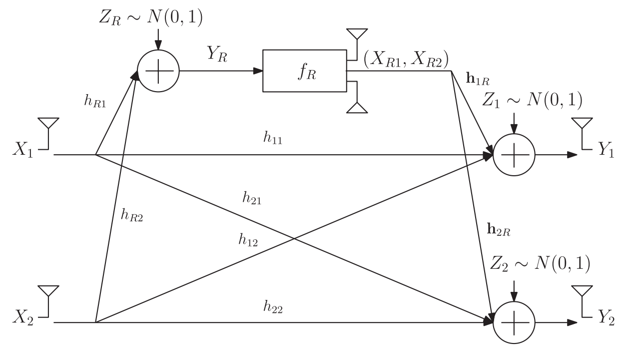

Fig. 2. Gaussian Interference Channel with a Causal Relay.

The discrete memoryless interference channel with a causal relay is depicted in Figure 1.

The average probability of error is defined to be $P_e^{(n)} = P\left((\hat{M}_1, \hat{M}_2) \neq (M_1, M_2)\right)$. We say a rate pair (R_1, R_2) is achievable if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes such that $\lim_{n\to\infty} P_e^{(n)} = 0$. The capacity region of the discrete memoryless interference channel with a causal relay is defined as the closure of the set of achievable rate pairs (R_1, R_2).

For the discrete memoryless interference channel with a causal relay, we get the following outer bound on the capacity region whose proof is given in Appendix A.

Theorem 1: For the discrete memoryless interference channel with a causal relay, the capacity region is contained in the set of rate pairs (R_1, R_2) such that

$$R_1 \le I(X_1; Y_R|X_2, Q) + I(X_1; Y_1|X_2, Y_R, X_R, Q), \quad \text{(1a)}$$

$$R_2 \le I(X_2; Y_R|X_1, Q) + I(X_2; Y_2|X_1, Y_R, X_R, Q), \quad \text{(1b)}$$

for some $p(q)p(x_1|q)p(x_2|q)p(x_R|x_1, x_2, y_R, q)$.

III. Gaussian Interference Channel With a Causal Relay

We consider the Gaussian interference channel with a causal relay depicted in Figure 2, which is a Gaussian version of the discrete memoryless interference channel with a causal relay considered in Section II. For simplicity, suppose the relay node is equipped with 3 antennas, one for receiving and two for transmitting signals. Our results can be easily extended to more general cases with more antennas. Receiving and transmitting antennas are assumed to be isolated so that they do not interfere with each other. Then the relay's received signal Y_R can be expressed as follows:

$$Y_R = h_{R1}X_1 + h_{R2}X_2 + Z_R \quad (2)$$

where h_{Rj} is the channel gain from source j to the relay, $Z_R \sim N(0,1)$ is a zero-mean unit-variance Gaussian noise at the relay. Suppose an average power constraint P on source nodes and P_R on the relay node. The received signal Y_i of destination i can be expressed as follows:

$$Y_1 = h_{11}X_1 + h_{12}X_2 + \mathbf{h}_{1R}\mathbf{X}_R + Z_1 \quad (3a)$$
$$Y_2 = h_{21}X_1 + h_{22}X_2 + \mathbf{h}_{2R}\mathbf{X}_R + Z_2 \quad (3b)$$

where $\mathbf{h}_{iR} = [h_{iR}^{[1]}, h_{iR}^{[2]}]$, $h_{iR}^{[k]}$ is the channel gain from the k-th antenna of the relay to destination i, h_{ik} is the channel gain from source k to destination i, for $i,k \in \{1,2\}$, $\mathbf{X}_R = [X_{R1}, X_{R2}]^T$ is the transmitting signal of the relay, where T means transpose, and $Z_i \sim N(0,1)$ is the Gaussian noise at destination i.

If the relay's power exceeds a certain threshold, then the capacity of the Gaussian interference channel with a causal relay can be achieved by a simple instantaneous amplify-and-forward relaying in some cases. We say the relay is potent if the relay's power exceeds the threshold.

Definition 1: We call the relay is *potent* if

$$\frac{(h_{2R}^{[1]}h_{22}h_{R1} - h_{1R}^{[1]}h_{11}h_{R2})^2 + (h_{2R}^{[2]}h_{22}h_{R1} - h_{1R}^{[2]}h_{11}h_{R2})^2}{(h_{1R}^{[2]}h_{2R}^{[1]} - h_{1R}^{[1]}h_{2R}^{[2]})^2 h_{11}^2 h_{22}^2} \le \frac{P_R}{(h_{R1}^2 + h_{R2}^2)P + 1}. \quad (4)$$

A. Gaussian Interference Channel With a Causal Relay Under Very Strong Interference

Definition 2: A Gaussian interference channel with a causal relay is said to have *very strong interference* if

$$\left(h_{11}^2 + h_{R1}^2\right) P \le \frac{(h_{21}h_{22} + h_{R1}h_{R2})^2 P}{\left(h_{22}^2 + h_{R2}^2\right)^2 P + h_{22}^2 + h_{R2}^2} \quad (5a)$$
$$\left(h_{22}^2 + h_{R2}^2\right) P \le \frac{(h_{11}h_{12} + h_{R1}h_{R2})^2 P}{\left(h_{11}^2 + h_{R1}^2\right)^2 P + h_{11}^2 + h_{R1}^2} \quad (5b)$$

Remark 1: It is easy to check that the above condition falls back to the very strong interference condition for the Gaussian interference channel if there is no relay, i.e., if $h_{R1} = h_{R2} = 0$.

The capacity region of the Gaussian interference channel with a causal relay under very strong interference can be characterized under some conditions as shown in the following theorem whose proof is in Appendix B.

Theorem 2: The capacity region of the Gaussian interference channel with a causal potent relay under very strong interference is given by the set of rate pairs (R_1, R_2) such that

$$R_1 \le \frac{1}{2}\log\left(1 + \left(h_{11}^2 + h_{R1}^2\right)P\right) \quad (6a)$$
$$R_2 \le \frac{1}{2}\log\left(1 + \left(h_{22}^2 + h_{R2}^2\right)P\right). \quad (6b)$$

Remark 2: The above result recovers the capacity result of the Gaussian interference channel without relay, i.e., when $h_{R1} = h_{R2} = 0$.

For achievability, we use instantaneous amplify-and-forward relaying. Unlike the conventional relay channel where the relay's encoding function depends only on the past received signals, in our channel the relay's transmit symbol depends also on the current received signal. This enables coherent combining at the destination for each symbol, which is why a simple relaying such as an instantaneous amplify-and-forward scheme can be optimal. Similarly, such a scheme can also be optimal in other cases including [17] and its generalization in [18].

B. Gaussian Interference Channel With a Causal Relay Under Strong Interference

Definition 3: A discrete memoryless interference channel with a causal relay is said to have *strong interference* if

$$I(X_1; Y_1|X_2, Y_R, X_R) \le I(X_1; Y_2|X_2, Y_R, X_R), \quad (7a)$$
$$I(X_2; Y_2|X_1, Y_R, X_R) \le I(X_2; Y_1|X_1, Y_R, X_R) \quad (7b)$$

for all $p(x_1)p(x_2)p(x_R|y_R)$.

Lemma 1: For the Gaussian interference channel with a causal relay, the strong interference condition is equivalent to $|h_{11}| \le |h_{21}|$ and $|h_{22}| \le |h_{12}|$.

The proof of the above lemma is in Appendix C. Lemma 1 means the strong interference condition for the Gaussian interference channel with a causal relay has the same form as that for the conventional Gaussian interference channel.

Lemma 2: If the Gaussian interference channel with a causal relay has strong interference, then

$$I(X_1^n; Y_1^n|X_2^n, Y_R^n) \le I(X_1^n; Y_2^n|X_2^n, Y_R^n), \quad (8a)$$
$$I(X_2^n; Y_2^n|X_1^n, Y_R^n) \le I(X_2^n; Y_1^n|X_1^n, Y_R^n) \quad (8b)$$

for all $p(x_1^n)p(x_2^n)$ and $x_{R,j} = f_j(y_R^j)$ for all $j \in \{1,2,...n\}$ and $n \ge 1$.

The proof of the above lemma is in Appendix D.

Using Lemma 2, we get the following outer bound for the capacity region of the Gaussian interference channel with a causal relay under strong interference.

Theorem 3: For the Gaussian interference channel with a causal relay under strong interference, the capacity region is contained in the set of rate pairs (R_1, R_2) such that

$$R_1 \le \frac{1}{2}\log\left(1 + (h_{11}^2 + h_{R1}^2)P\right) \quad (9a)$$
$$R_2 \le \frac{1}{2}\log\left(1 + (h_{22}^2 + h_{R2}^2)P\right) \quad (9b)$$
$$R_1 + R_2 \le \frac{1}{2}\log\Big(1 + \left(h_{11}^2 + h_{12}^2 + h_{R1}^2 + h_{R2}^2\right)P + (h_{11}h_{R2} - h_{12}h_{R1})^2 P^2\Big) \quad (9c)$$
$$R_1 + R_2 \le \frac{1}{2}\log\Big(1 + \left(h_{21}^2 + h_{22}^2 + h_{R1}^2 + h_{R2}^2\right)P + (h_{22}h_{R1} - h_{21}h_{R2})^2 P^2\Big) \quad (9d)$$

The proof is in Appendix E.

Remark 3: Interestingly, if $h_{11}h_{R2} = h_{12}h_{R1}$ and $h_{22}h_{R1} = h_{21}h_{R2}$, then the capacity region of the Gaussian interference channel with a causal potent relay under strong interference can be characterized and is given as the set of rate pairs

(R_1, R_2) such that

$$R_1 \le \frac{1}{2}\log\left(1+(h_{11}^2+h_{R1}^2)P\right) \quad (10a)$$

$$R_2 \le \frac{1}{2}\log\left(1+(h_{22}^2+h_{R2}^2)P\right) \quad (10b)$$

$$R_1+R_2 \le \frac{1}{2}\log\left(1+\left(h_{11}^2+h_{12}^2+h_{R1}^2+h_{R2}^2\right)P\right) \quad (10c)$$

$$R_1+R_2 \le \frac{1}{2}\log\left(1+\left(h_{22}^2+h_{21}^2+h_{R1}^2+h_{R2}^2\right)P\right) \quad (10d)$$

It can be easily proved by combining the outer bound in Theorem 3 and the achievable rate region of the instantaneous amplify-and-forward relaying used in proving Theorem 2.

IV. Conclusion

In this paper, we studied the interference channel with a causal relay and showed capacity results under some conditions. The main difference between our channel and the conventional interference relay channel is in that the relay's current transmit symbol depends on its current received symbol as well as on the past received symbols. For this channel, we defined very strong and strong interference conditions motivated by the strong and very strong interference conditions for the Gaussian interference channel. We presented new genie-aided outer bounds where both receivers know the received sequence of the relay. Using the characteristics of the strong interference, we suggested a tighter outer bound for the Gaussian interference channel with a causal relay under strong interference. We also proposed an achievable scheme based on instantaneous amplify-and-forward relaying. Surprisingly, the proposed instantaneous amplify-and-forward relaying scheme actually achieves the capacity exactly under certain conditions.

Appendix A
Proof of Theorem 1

An upper bound on nR_1 can be expressed as follows:

$$nR_1 = H(M_1) = H(M_1|M_2) \quad (11)$$

$$\overset{(a)}{\le} I(M_1; Y_1^n, Y_R^n|M_2) + n\epsilon_n \quad (12)$$

$$= \sum_{i=1}^{n} I(M_1; Y_{Ri}, Y_{1,i}|M_2, Y_R^{i-1}, Y_1^{i-1}) + n\epsilon_n \quad (13)$$

$$\overset{(b)}{=} \sum_{i=1}^{n} I(M_1, X_{1i}; Y_{Ri}, Y_{1,i}|M_2, X_{2i}, Y_R^{i-1}, Y_1^{i-1}) + n\epsilon_n \quad (14)$$

$$\le \sum_{i=1}^{n} I(M_1, M_2, X_{1i}, Y_1^{i-1}; Y_{Ri}, Y_{1,i}|X_{2i}, Y_R^{i-1}) + n\epsilon_n \quad (15)$$

$$= \sum_{i=1}^{n} I(M_1, M_2, X_{1i}, Y_1^{i-1}; Y_{Ri}|X_{2i}, Y_R^{i-1}) + \sum_{i=1}^{n} I(M_1, M_2, X_{1i}, Y_1^{i-1}; Y_{1,i}|X_{2i}, Y_R^{i}) + n\epsilon_n \quad (16)$$

$$\overset{(c)}{\le} \sum_{i=1}^{n} I(M_1, M_2, X_{1i}, Y_1^{i-1}, Y_R^{i-1}; Y_{Ri}|X_{2i}) + \sum_{i=1}^{n} I(M_1, M_2, X_{1i}, Y_1^{i-1}, Y_R^{i-1}; Y_{1,i}|X_{2i}, Y_{Ri}, X_{Ri}) + n\epsilon_n \quad (17)$$

$$\overset{(d)}{=} \sum_{i=1}^{n} I(X_{1i}; Y_{Ri}|X_{2i}) + \sum_{i=1}^{n} I(X_{1i}; Y_{1,i}|X_{2i}, Y_{Ri}, X_{Ri}) + n\epsilon_n \quad (18)$$

$$= nI(X_{1Q}; Y_{R,Q}|X_{2Q}, Q) + nI(X_{1Q}; Y_{1Q}|X_{2Q}, Y_{RQ}, X_{RQ}, Q) + n\epsilon_n \quad (19)$$

$$= nI(X_1; Y_R|X_2, Q) + nI(X_1; Y_1|X_2, Y_R, X_R, Q) + n\epsilon_n \quad (20)$$

where (a) follows from Fano's inequality, where ϵ_n tends to zero as $n \to \infty$, (b) and (c) follow since X_{ji} is a function of M_j for $j \in \{1, 2\}$, respectively and $X_{Ri} = f_i(Y_R^i)$, and (d) holds since $(M_1, M_2, Y_R^{i-1}, Y_1^{i-1}) \to (X_{1i}, X_{2i}) \to Y_{Ri}$ and $(M_1, M_2, Y_1^{i-1}, Y_R^{i-1}) \to (X_{1i}, X_{2i}, X_{Ri}, Y_{Ri}) \to Y_{1i}$. In the above, Q is a time sharing random variable such that $Q \sim \text{Unif}[1:n]$ which is independent of $(X_1^n, X_2^n, Y_1^n, Y_2^n, Y_R^n, X_R^n)$ and we define $X_1 = X_{1Q}$, $X_2 = X_{2Q}$, $Y_1 = Y_{1Q}$, $Y_2 = Y_{2Q}$, $Y_R = Y_{RQ}$, $X_R = X_{RQ}$.

Similarly, an upper bound on R_2 can be expressed as follows:

$$R_2 \le I(X_2; Y_R|X_1, Q) + I(X_2; Y_2|X_1, Y_R, X_R, Q) \quad (21)$$

Appendix B
Proof of Theorem 2

Achievability: Achievability follows by instantaneous amplify-and-forward scheme at the relay. In this case, the transmit signal of the relay can be expressed as follows:

$$\mathbf{X}_R = [\alpha_{R1} \quad \alpha_{R2}]^T\, Y_R \quad (22)$$

where α_{Ri} is the amplification factor of the i-th antenna of the relay. Let sources 1 and 2 transmit codewords using independent Gaussian codebooks with power P. Then, the received power of the relay becomes $(h_{R1}^2 + h_{R2}^2)P + 1$ and the transmit power constraint for the relay can be expressed as follows:

$$\alpha_{R1}^2 + \alpha_{R2}^2 \le \frac{P_R}{(h_{R1}^2+h_{R2}^2)P+1} \quad (23)$$

We set the amplification factors of the relay as follows:

$$\alpha_{R1} = \frac{h_{11}h_{R2}h_{1R}^{[2]} - h_{22}h_{R1}h_{2R}^{[2]}}{h_{11}h_{22}(h_{1R}^{[2]}h_{2R}^{[1]} - h_{1R}^{[1]}h_{2R}^{[2]})} \quad (24a)$$

$$\alpha_{R2} = \frac{h_{22}h_{R1}h_{2R}^{[1]} - h_{11}h_{R2}h_{1R}^{[1]}}{h_{11}h_{22}(h_{1R}^{[2]}h_{2R}^{[1]} - h_{1R}^{[1]}h_{2R}^{[2]})} \quad (24b)$$

It is easy to check α_{R1} and α_{R2} satisfy the following:

$$\alpha_{R1}h_{1R}^{[1]} + \alpha_{R2}h_{1R}^{[2]} = \frac{h_{R1}}{h_{11}} \quad (25a)$$

$$\alpha_{R1}h_{2R}^{[1]} + \alpha_{R2}h_{2R}^{[2]} = \frac{h_{R2}}{h_{22}}. \quad (25b)$$

Using the above $\alpha_{R,1}$ and $\alpha_{R,2}$, the received signal of destination i can be expressed as follows:

$$Y_1 = h_{11}X_1 + h_{12}X_2 + \mathbf{h}_{1R}\mathbf{X}_R + Z_1 \tag{26a}$$
$$= \left(\frac{h_{11}^2 + h_{R1}^2}{h_{11}}\right)X_1 + \left(\frac{h_{11}h_{12} + h_{R1}h_{R2}}{h_{11}}\right)X_2 + \left(\frac{h_{R1}}{h_{11}}\right)Z_R + Z_1$$
$$Y_2 = h_{21}X_1 + h_{22}X_2 + \mathbf{h}_{2R}\mathbf{X}_R + Z_2 \tag{26b}$$
$$= \left(\frac{h_{21}h_{22} + h_{R1}h_{R2}}{h_{22}}\right)X_1 + \left(\frac{h_{22}^2 + h_{R2}^2}{h_{22}}\right)X_2 + \left(\frac{h_{R2}}{h_{22}}\right)Z_R + Z_2$$

If the very strong interference condition (5) is satisfied, then the resulting Gaussian interference channel given in (26) has very strong interference and the capacity region is given by (6a).

Converse: Applying Theorem 1 to the Gaussian interference channel with a causal relay, the upper bound on R_1 in Theorem 1 can be expressed as follows:

$$R_1 \le I(X_1; Y_R|X_2, Q) + I(X_1; Y_1|X_2, Y_R, X_R, Q) \tag{27}$$
$$\overset{(a)}{\le} h(Y_R|X_2) - h(Y_R|X_1, X_2) + h(Y_1|X_2, Y_R, X_R) - h(Y_1|X_1, X_2, Y_R, X_R) \tag{28}$$
$$\overset{(b)}{=} h(h_{R1}X_1 + h_{R2}X_2 + Z_R|X_2) - h(Z_R) + h(h_{11}X_1+h_{12}X_2+h_{1R}X_R+Z_1|X_2, Y_R, X_R) - h(Z_1) \tag{29}$$
$$\overset{(c)}{\le} h(h_{R1}X_1+Z_R) - h(Z_R) + h(h_{11}X_1+Z_1|h_{R1}X_1+Z_R) - h(Z_1) \tag{30}$$
$$= h(h_{11}X_1 + Z_1, h_{R1}X_1 + Z_R) - h(Z_R) - h(Z_1) \tag{31}$$
$$\overset{(d)}{\le} \frac{1}{2}\log(1 + (h_{11}^2 + h_{R1}^2)P) \tag{32}$$

where (a) holds since conditioning reduces entropy and Markovity of the channel, (b) holds since Z_R is independent of X_1 and X_2, Z_1 is independent of X_1, X_2, Y_R, X_R, (c) holds since conditioning reduces entropy, and (d) holds since $h(h_{11}X_1 + Z_1, h_{R1}X_1 + Z_R) \le \frac{1}{2}\log(2\pi e)^2\left(1 + h_{11}^2P + h_{R1}^2P\right)$.

Similarly, an upper bound on R_2 can be expressed as follows:

$$R_2 \le \frac{1}{2}\log\left(1 + \left(h_{22}^2 + h_{R2}^2\right)P\right) \tag{33}$$

Appendix C
Proof of Lemma 1

$I(X_1; Y_1|X_2, Y_R, X_R)$ can be expressed as follows:

$$I(X_1; Y_1|X_2, Y_R, X_R) \tag{34}$$
$$= h(Y_1|X_2, Y_R, X_R) - h(Y_1|X_1, X_2, Y_R, X_R) \tag{35}$$
$$= h(h_{11}X_1+h_{12}X_2+h_{1R}X_R+Z_1|X_2, h_{R1}X_1+h_{R2}X_2+Z_R, X_R) - h(Z_1) \tag{36}$$
$$= h(h_{11}X_1+h_{12}X_2+Z_1|X_2, h_{R1}X_1+Z_R, X_R) - h(Z_1) \tag{37}$$
$$\overset{(a)}{=} h(h_{11}X_1 + h_{12}X_2 + Z_1|X_2, h_{R1}X_1 + Z_R) - h(Z_1) \tag{38}$$
$$\overset{(b)}{=} h(h_{11}X_1 + Z_1|h_{R1}X_1 + Z_R) - h(Z_1) \tag{39}$$

where (a) holds since $h_{11}X_1 + h_{12}X_2 + Z_1 \to (X_2, h_{R1}X_1 + Z_R) \to X_R$ and (b) holds since X_2 is independent of (X_1, Z_R, Z_1).

Similarly, $I(X_1; Y_2|X_2, Y_R, X_R)$, $I(X_2; Y_2|X_1, Y_R, X_R)$, and $I(X_2; Y_1|X_1, Y_R, X_R)$ can be expressed as follows:

$$I(X_1; Y_2|X_2, Y_R, X_R) = h(h_{21}X_1+Z_2|h_{R1}X_1+Z_R) - h(Z_2) \tag{40a}$$
$$I(X_2; Y_2|X_1, Y_R, X_R) = h(h_{22}X_2+Z_2|h_{R2}X_2+Z_R) - h(Z_2) \tag{40b}$$
$$I(X_2; Y_1|X_1, Y_R, X_R) = h(h_{12}X_2+Z_1|h_{R2}X_2+Z_R) - h(Z_1) \tag{40c}$$

If $|h_{11}| \le |h_{21}|$ and $|h_{22}| \le |h_{12}|$, then the Gaussian BC's from X_1 to $[(h_{21}X_1 + Z_2, h_{R1}X_1 + Z_R), (h_{11}X_1 + Z_1, h_{R1}X_1 + Z_R)]$ and from X_2 to $[(h_{12}X_2 + Z_1, h_{R2}X_2 + Z_R), (h_{22}X_2 + Z_2, h_{R2}X_2 + Z_R)]$ are both degraded, and therefore more capable. Thus, we get $I(X_1; h_{21}X_1 + Z_2, h_{R1}X_1 + Z_R) \ge I(X_1; h_{11}X_1 + Z_1, h_{R1}X_1 + Z_R)$ and $I(X_2; h_{12}X_2 + Z_1, h_{R2}X_2 + Z_R) \ge I(X_2; h_{22}X_2 + Z_2, h_{R2}X_2 + Z_R)$ for all $p(x_1)p(x_2)p(x_R|y_R)$. From this, it follows $I(X_1; Y_2|X_2, Y_R, X_R) \ge I(X_1; Y_1|X_2, Y_R, X_R)$ and $I(X_2; Y_1|X_1, Y_R, X_R) \ge I(X_2; Y_2|X_1, Y_R, X_R)$ for all $p(x_1)p(x_2)p(x_R|y_R)$. To show the other direction, assume $h(h_{11}X_1 + Z_1|h_{R1}X_1 + Z_R) \le h(h_{21}X_1 + Z_2|h_{R1}X_1 + Z_R)$, $h(h_{22}X_2 + Z_2|h_{R2}X_2 + Z_R) \le h(h_{12}X_2 + Z_1|h_{R2}X_2 + Z_R)$, $X_1 \sim N(0, P)$, and $X_2 \sim N(0, P)$, then we get $|h_{11}| \le |h_{21}|$ and $|h_{22}| \le |h_{12}|$.

Appendix D
Proof of Lemma 2

For the proof, we modify the induction steps done in [2]. For $n = 1$, Lemma 2 holds since

$$I(X_{1,1}; Y_{2,1}|X_{2,1}, Y_{R,1}) - I(X_{1,1}; Y_{1,1}|X_{2,1}, Y_{R,1}) \tag{41}$$
$$\overset{(a)}{=} I(X_{1,1}; Y_{2,1}|X_{2,1}, Y_{R,1}, X_{R,1}) - I(X_{1,1}; Y_{1,1}|X_{2,1}, Y_{R,1}, X_{R,1}) \tag{42}$$
$$\overset{(b)}{\ge} 0, \tag{43}$$

where (a) holds since $X_{R,1} = f_1(Y_{R,1})$ and (b) holds from the strong interference condition.

Now assume that Lemma 2 holds for $n = k$, i.e.,

$$I(X_1^k; Y_1^k|X_2^k, Y_R^k) \le I(X_1^k; Y_2^k|X_2^k, Y_R^k) \tag{44}$$

for all $p(x_1^k)p(x_2^k)$ and $x_{R,j} = f_j(y_R^j)$ for all $j \in \{1, 2, ...n\}$.

Let $\hat{Y}_{i,j} = h_{i1}X_{1,j} + h_{i2}X_{2,j} + Z_{i,j}$ and $\hat{Y}_i^k = (\hat{Y}_{i,1}, \hat{Y}_{i,2}, ..., \hat{Y}_{i,k})$ where $i \in \{1, 2\}$, $j \in \{1, 2, ..., n\}$. Then, the above condition is equivalent to the following:

$$I(X_1^k; \hat{Y}_1^k|X_2^k, Y_R^k) \le I(X_1^k; \hat{Y}_2^k|X_2^k, Y_R^k) \tag{45}$$

for all $p(x_1^k)p(x_2^k)$ since $X_R^k = f^k(Y_R^k)$. This implies that

$$I(X_1^k; \hat{Y}_1^k|X_2^k, Y_R^k, U) \le I(X_1^k; \hat{Y}_2^k|X_2^k, Y_R^k, U) \tag{46}$$

for all $p(u)p(x_1^k|u)p(x_2^k|u)$.

$$I(X_1^{k+1};Y_2^{k+1}|X_2^{k+1},Y_R^{k+1})-I(X_1^{k+1};Y_1^{k+1}|X_2^{k+1},Y_R^{k+1})$$

can be expressed as follows:

$$I(X_1^{k+1};Y_2^{k+1}|X_2^{k+1},Y_R^{k+1})-I(X_1^{k+1};Y_1^{k+1}|X_2^{k+1},Y_R^{k+1}) \quad (47)$$

$$\stackrel{(a)}{=} I(X_1^{k+1};\hat{Y}_2^{k+1}|X_2^{k+1}Y_R^{k+1})-I(X_1^{k+1};\hat{Y}_1^{k+1}|X_2^{k+1}Y_R^{k+1}) \quad (48)$$

$$= I(X_1^{k+1};\hat{Y}_2^{k}|X_2^{k+1}Y_R^{k+1})+I(X_1^{k+1};\hat{Y}_{2,k+1}|X_2^{k+1}Y_R^{k+1}\hat{Y}_2^k) \quad (49)$$

$$-I(X_1^{k+1};\hat{Y}_{1,k+1}|X_2^{k+1},Y_R^{k+1}) -I(X_1^{k+1};\hat{Y}_1^{k}|X_2^{k+1},Y_R^{k+1},\hat{Y}_{1,k+1}) \quad (50)$$

$$= I(X_1^{k+1}\hat{Y}_{1,k+1};\hat{Y}_2^{k}|X_2^{k+1},Y_R^{k+1}) +I(X_1^{k+1};\hat{Y}_{2,k+1}|X_2^{k+1},Y_R^{k+1},\hat{Y}_2^k) \quad (51)$$

$$-I(X_1^{k+1},\hat{Y}_2^k;\hat{Y}_{1,k+1}|X_2^{k+1},Y_R^{k+1}) -I(X_1^{k+1};\hat{Y}_1^{k}|X_2^{k+1},Y_R^{k+1},\hat{Y}_{1,k+1}) \quad (52)$$

$$= I(\hat{Y}_{1,k+1};\hat{Y}_2^{k}|X_2^{k+1}Y_R^{k+1})+I(X_1^{k};\hat{Y}_2^{k}|X_2^{k+1}Y_R^{k+1}\hat{Y}_{1,k+1}) \quad (53)$$

$$\begin{aligned}&+I(X_{1,k+1};\hat{Y}_2^k|X_1^k,X_2^{k+1},Y_R^{k+1},\hat{Y}_{1,k+1})\\&+I(X_{1,k+1};\hat{Y}_{2,k+1}|X_2^{k+1},Y_R^{k+1},\hat{Y}_2^k)\\&+I(X_1^k;\hat{Y}_{2,k+1}|X_{1,k+1},X_2^{k+1},Y_R^{k+1},\hat{Y}_2^k)\\&-I(\hat{Y}_2^k;\hat{Y}_{1,k+1}|X_2^{k+1},Y_R^{k+1})\\&-I(X_{1,k+1};\hat{Y}_{1,k+1}|X_2^{k+1},Y_R^{k+1},\hat{Y}_2^k)\\&-I(X_1^k;\hat{Y}_{1,k+1}|X_{1,k+1},X_2^{k+1},Y_R^{k+1},\hat{Y}_2^k)\\&-I(X_1^k;\hat{Y}_1^k|X_2^{k+1},Y_R^{k+1},\hat{Y}_{1,k+1})\\&-I(X_{1,k+1};\hat{Y}_1^k|X_1^k,X_2^{k+1},Y_R^{k+1},\hat{Y}_{1,k+1})\end{aligned} \quad (54)$$

$$\begin{aligned}&\stackrel{(b)}{=} I(X_1^k;\hat{Y}_2^k|X_2^{k+1},Y_R^{k+1},\hat{Y}_{1,k+1})\\&+I(X_{1,k+1};\hat{Y}_{2,k+1}|X_2^{k+1},Y_R^{k+1},\hat{Y}_2^k)\\&-I(X_{1,k+1};\hat{Y}_{1,k+1}|X_2^{k+1},Y_R^{k+1},\hat{Y}_2^k)\\&-I(X_1^k;\hat{Y}_1^k|X_2^{k+1},Y_R^{k+1},\hat{Y}_{1,k+1})\end{aligned} \quad (55)$$

$$\begin{aligned}&\stackrel{(c)}{\ge} I(X_{1,k+1};\hat{Y}_{2,k+1}|X_2^{k+1},Y_R^{k+1},\hat{Y}_2^k)\\&-I(X_{1,k+1};\hat{Y}_{1,k+1}|X_2^{k+1},Y_R^{k+1},\hat{Y}_2^k)\end{aligned} \quad (56)$$

$$\stackrel{(d)}{\ge} 0 \quad (57)$$

where (a) holds since $X_R^k = f^k(Y_R^k)$, (b) holds since $(X_{1,k+1},X_{2,k+1},Y_R^{k+1},\hat{Y}_{1,k+1}) \to (X_1^k,X_2^k) \to \hat{Y}_2^k$, $(X_1^k,X_2^k,Y_R^{k+1},\hat{Y}_{2,k}) \to (X_{1,k+1},X_{2,k+1}) \to \hat{Y}_{2,k+1}$, $(X_1^k,X_2^k,Y_R^{k+1},\hat{Y}_{2,k}) \to (X_{1,k+1},X_{2,k+1}) \to \hat{Y}_{1,k+1}$, and $(X_{1,k+1},X_{2,k+1},Y_R^{k+1},\hat{Y}_{1,k+1}) \to (X_1^k,X_2^k) \to \hat{Y}_1^k$, (c) follows from (46) since $(X_{2,k+1},Y_{R,k+1},\hat{Y}_{1,k+1}) \to (X_2^k,X_1^k) \to (\hat{Y}_1^k,\hat{Y}_2^k,Y_R^k)$, $X_1^k \to (X_{2,k+1},Y_{R,k+1},\hat{Y}_{1,k+1}) \to X_2^k$, and the memoryless channel property, and (d) follows from the strong interference condition, i.e., $I(X_{1,k+1};\hat{Y}_{2,k+1}|X_{2,k+1},Y_{R,k+1},X_{R,k+1},V) \ge I(X_{1,k+1};\hat{Y}_{1,k+1}|X_{2,k+1},Y_{R,k+1},X_{R,k+1},V)$ for all $p(v)p(x_{1,k+1}|v)p(x_{2,k+1}|v)p(x_{R,k+1}|y_{R,k+1},v)$, where $V=(X_2^k,Y_R^k,\hat{Y}_2^k)$, which satisfies $V \to (X_{1,k+1},X_{2,k+1}) \to (\hat{Y}_{1,k+1},\hat{Y}_{2,k+1},Y_{R,k+1})$ and $X_{1,k+1} \to V \to X_{2,k+1}$. Similarly, we can prove that $I(X_2^n;Y_2^n|X_1^n,Y_R^n) \le I(X_2^n;Y_1^n|X_1^n,Y_R^n)$.

Appendix E
Proof of Theorem 3

An upper bound on nR_1+nR_2 can be expressed as follows:

$$nR_1+nR_2 = H(M_1)+H(M_2) \quad (58)$$

$$\stackrel{(a)}{\le} I(X_1^n;Y_1^n,Y_R^n)+I(X_2^n;Y_2^n,Y_R^n)+2n\epsilon_n \quad (59)$$

$$\le I(X_1^n;Y_1^n,Y_R^n|X_2^n)+I(X_2^n;Y_2^n,Y_R^n)+2n\epsilon_n \quad (60)$$

$$= I(X_1^n;Y_R^n|X_2^n)+I(X_1^n;Y_1^n|Y_R^n,X_2^n)+I(X_2^n;Y_R^n) + I(X_2^n;Y_2^n|Y_R^n)+2n\epsilon_n \quad (61)$$

$$= I(X_1^n,X_2^n;Y_R^n)+I(X_1^n;Y_1^n|Y_R^n,X_2^n)+I(X_2^n;Y_2^n|Y_R^n) +2n\epsilon_n \quad (62)$$

$$\stackrel{(b)}{\le} I(X_1^n,X_2^n;Y_R^n)+I(X_1^n;Y_2^n|Y_R^n,X_2^n)+I(X_2^n;Y_2^n|Y_R^n) +2n\epsilon_n \quad (63)$$

$$= I(X_1^n,X_2^n;Y_R^n)+I(X_1^n,X_2^n;Y_2^n|Y_R^n)+2n\epsilon_n \quad (64)$$

$$= \sum_{i=1}^n I(X_1^n,X_2^n;Y_{R,i}|Y_R^{i-1})+\sum_{i=1}^n I(X_1^n,X_2^n;Y_{2,i}|Y_2^{i-1},Y_R^n) +2n\epsilon_n \quad (65)$$

$$\stackrel{(c)}{\le} \sum_{i=1}^n I(X_1^n,X_2^n,Y_R^{i-1};Y_{R,i}) + \sum_{i=1}^n I(X_1^n,X_2^n,Y_2^{i-1},Y_R^{i-1},Y_{R,i+1}^n;Y_{2i}|Y_{R,i},X_{R,i})+2n\epsilon_n \quad (66)$$

$$\stackrel{(d)}{=} \sum_{i=1}^n I(X_{1i},X_{2i};Y_{R,i})+\sum_{i=1}^n I(X_{1i},X_{2i};Y_{2i}|Y_{R,i},X_{R,i}) +2n\epsilon_n \quad (67)$$

$$= nI(X_{1Q}X_{2Q};Y_{R,Q}|Q)+nI(X_{1Q}X_{2Q};Y_{2Q}|Y_{R,Q},X_{R,Q},Q) +2n\epsilon_n \quad (68)$$

$$= nI(X_1,X_2;Y_R|Q)+nI(X_1,X_2;Y_2|Y_R,X_R,Q)+2n\epsilon_n \quad (69)$$

where (a) follows from Fano's inequality, (b) follows from Lemma 2, (c) holds since $X_{R,i}=f_i(Y_R^i)$, and (d) holds since $(X_1^{i-1},X_{1,i+1}^n,X_2^{i-1},X_{2,i+1}^n,Y_2^{i-1},Y_R^{i-1},Y_{R,i+1}^n) \to (X_{1i},X_{2i},Y_{R,i},X_{R,i}) \to Y_{2i}$ and $(X_1^{i-1},X_{1,i+1}^n,X_2^{i-1},X_{2,i+1}^n,Y_R^{i-1}) \to (X_{1i},X_{2i}) \to Y_{R,i}$. In the above, Q is a time sharing random variable such that $Q \sim \mathrm{Unif}[1:n]$ and independent of $(X_1^n,X_2^n,Y_1^n,Y_2^n,Y_R^n,X_R^n)$ and we define $X_1=X_{1Q}$, $X_2=X_{2Q}$, $Y_1=Y_{1Q}$, $Y_2=Y_{2Q}$, $Y_R=Y_{RQ}$, and $X_R=X_{RQ}$. Similarly, we get the following upper bound on R_1+R_2.

$$R_1+R_2 \le I(X_1,X_2;Y_R|Q)+I(X_1,X_2;Y_1|Y_R,X_R,Q) \quad (70)$$

Combining this result with Theorem 1, we get the following outer bound for the Gaussian interference channel with a causal relay under strong interference:

$$R_1 \le I(X_1;Y_R|X_2,Q)+I(X_1;Y_1|X_2,Y_R,X_R,Q), \quad (71a)$$

$$R_2 \le I(X_2;Y_R|X_1,Q)+I(X_2;Y_2|X_1,Y_R,X_R,Q), \quad (71b)$$

$$R_1+R_2 \le I(X_1,X_2;Y_R|Q) +\min\{I(X_1,X_2;Y_2|Y_R,X_R,Q),I(X_1,X_2;Y_1|Y_R,X_R,Q)\} \quad (71c)$$

for all $p(q)p(x_1|q)p(x_2|q)p(x_R|x_1, x_2, y_R, q)$. Following the same procedure as the converse proof for Theorem 2, we get the following upper bounds on R_1 and R_2

$$R_1 \le \frac{1}{2}\log\left(1+(h_{11}^2+h_{R1}^2)P\right) \quad (72a)$$

$$R_2 \le \frac{1}{2}\log\left(1+(h_{22}^2+h_{R2}^2)P\right). \quad (72b)$$

The upper bound on $R_1 + R_2$ for the Gaussian case can be expressed as follows:

$$\begin{aligned} R_1+R_2 &\le I(X_1, X_2; Y_R|Q)+I(X_1, X_2; Y_1|Y_R, X_R, Q) && (73)\\ &\le h(Y_R) - h(Y_R|X_1, X_2) + h(Y_1|Y_R, X_R) \\ &\quad -h(Y_1|X_1, X_2, Y_R, X_R) && (74)\\ &= h(h_{R1}X_1 + h_{R2}X_2 + Z_R) - h(Z_R) \\ &\quad +h(h_{11}X_1+h_{12}X_2+h_{1R}X_R+Z_1|Y_R, X_R) \\ &\quad -h(Z_1) && (75)\\ &\overset{(a)}{\le} h(h_{R1}X_1 + h_{R2}X_2 + Z_R) - h(Z_R) \\ &\quad +h(h_{11}X_1+h_{12}X_2+Z_1|h_{R1}X_1+h_{R2}X_2+Z_R) \\ &\quad -h(Z_1) && (76)\\ &= h(h_{R1}X_1 + h_{R2}X_2 + Z_R, h_{11}X_1 + h_{12}X_2 + Z_1) \\ &\quad -h(Z_R) - h(Z_1) && (77)\\ &\le \frac{1}{2}\log(1 + \left(h_{11}^2 + h_{12}^2 + h_{R1}^2 + h_{R2}^2\right) P \\ &\quad + (h_{11}h_{R2} - h_{12}h_{R1})^2 P^2) && (78) \end{aligned}$$

where (a) holds since conditioning reduces entropy. Similarly, we get,

$$R_1 + R_2 \le \frac{1}{2}\log\Big(1 + \left(h_{21}^2 + h_{22}^2 + h_{R1}^2 + h_{R2}^2\right) P + (h_{22}h_{R1} - h_{21}h_{R2})^2 P^2\Big),$$

which concludes the proof.

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