Control of linear systems subject to input constraints: a polynomial approach. Part I – SISO plants*

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Abstract

A polynomial approach is pursued for locally stabilizing discrete-time linear systems subject to input constraints. Using the Youla-Kučera parametrization and the extended Farkas lemma, the problem of guaranteeing closed-loop stability for any initial condition chosen in a given polyhedron of the state-space is formulated as a linear programming feasibility problem. This paper, as the first of a series of two, collects introductory material and may be regarded as tutorial. For the sake of clarity, only SISO plants and stabilization in given polyhedral regions are considered.

Keywords

Linear Systems, Input Constraints, Polynomial Approach, Youla-Kučera Parametrization, Extended Farkas Lemma.

*This work was supported by the Barrande Project No. 97/005-97/026, by the Grant Agency of the Czech Republic under contract No. 102/99/1368 and by the Ministry of Education of the Czech Republic under contract No. VS97/034.

1 Introduction

The problem of control constraints appears in most practical control systems. Due to technological and safety reasons, the actuators cannot drive an unlimited energy to the controlled plant. This fact can be translated into bounds on control and state variables.

Control systems are often linearly designed. The modern theory of linear control provides efficient methods for computing control laws that guarantee stability and some performance requirements with respect to the linear closed-loop system. In general, this kind of design does not directly consider amplitude limitations on the control inputs. Then, the presence of input bounds can be source of parasitic equilibrium points and limit cycles, or can even lead the closed-loop system to an unstable behavior. In past years, this fact has motivated study of both analysis and design techniques taking control bounds into account, see [1, 7, 24] and references therein.

Control limitation may be handled implicitly, or a posteriori, through the so-called anti-windup strategies [14]. Alternatively, input constraints may be handled explicitly, or a priori, pursuing one of the two following approaches:

- Saturation avoidance methods – They consist in preventing input saturation. The closed-loop system therefore stays in a region of linear behavior. On the one hand, an interesting approach developed in the literature consists in determining a feedback control law that ensures positive invariance of a set included in a region of closed-loop linear behavior and including all admissible initial states. This positive invariant set is then considered as a linear local region of stability, see [11, 2] for comprehensive overviews. These methods may rely on the extended Farkas lemma [10], linear programming [25, 6], eigenstructure assignment [5, 6] or set-induced norms [22]. On the other hand, a more general convex programming approach can also be pursued [3]. It relies upon the Youla-Kučera parametrization of all stabilizing controllers [15, 26] and has been extended to handle $\mathcal{H}_2$ or $\mathcal{H}_\infty$ performance criteria using state-space arguments [23].
- Saturation allowance methods – They consist in letting the saturation occur. The closed-loop system is therefore non-linear. In this sense, significant results have lately emerged in the scope of global stabilization [21, 20] and semi-global stabilization [18]. They inherently require stability of the open-loop system. Relaxing this stability assumption, results addressing the local stabilization problem have also been obtained [24, 13, 12, 8, 9].

In this paper, we focus on a saturation avoidance method mixing some of the above mentioned techniques. On the one hand, we use the Youla-Kučera parametrization of all stabilizing controllers in the context of the polynomial approach to control systems [16, 17]. On the other hand, we use the extended Farkas lemma, traditionally invoked when studying positive invariance, to come up with a convex programming formulation of the constrained stabilization problem, as in [3]. To the authors knowledge, our development is the first application of the polynomial approach to the control of linear systems subject to input constraints. Moreover, as a natural outcome, we can guarantee that we describe the whole set of stabilizing controllers under input constraints.

This paper, as the first of a series of two, is intended to be of tutorial nature. We deliberately restricted the study to SISO plants and stabilization within a given polyhedron of the state-space. As a consequence, the arguments used throughout the paper are kept relatively simple and the resulting convex programming problem turns out to reduce to a mere linear programming problem. In the second paper of the series, more involved topics will be touched on, such as simultaneous computation of the control law and the stabilization domain, or maximization of the size of the stabilization domain for MIMO plants.

The outline of the paper is as follows. In Section 2, the problem to be solved is stated. Some preliminary material is described in Section 3 that will be instrumental to the derivation in Section 4 of a linear programming formulation of the problem. Two illustrative examples are eventually proposed in Section 5.

2 Problem Statement

Consider a single-input single-output observable discrete-time linear system

$$\begin{array}{rcl} \xi_{k+1} & = & F\xi_k + Gu_k \\ z_k & = & H\xi_k \end{array} \tag{1}$$

where $\xi_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}$, $z_k \in \mathbb{R}$ stand for the state vector, input and output signals, respectively. System input u_k is subject to hard constraints

$$-u^- \leq u_k \leq u^+ \tag{2}$$

where u^-, u^+ are given positive scalars. Moreover, initial system state ξ_0 is supposed to belong to a given polyhedron. It holds

$$\xi_0 \in \mathcal{P}_N = \{\xi : N\xi \leq \nu\} \tag{3}$$

where N is a given matrix and ν is a given vector.

Performing z-transform, linear system (1) can equivalently be written in a transfer function setting as

$$z(\mathtt{d}) = \frac{b(\mathtt{d})}{a(\mathtt{d})}u(\mathtt{d}) + \frac{c(\mathtt{d})}{a(\mathtt{d})}\xi_0 \tag{4}$$

where $\mathtt{d}$ stands for the usual backward shift operator [16]. Polynomials $a(\mathtt{d})$, $b(\mathtt{d})$ and polynomial vector $c(\mathtt{d})$ verify

$$\frac{b(\mathtt{d})}{a(\mathtt{d})} = H(\mathbf{I} - F\mathtt{d})^{-1}G\mathtt{d} \qquad \frac{c(\mathtt{d})}{a(\mathtt{d})} = H(\mathbf{I} - F\mathtt{d})^{-1}.$$

System (4) should have no unstable hidden modes, so couples $a(\mathtt{d})$, $b(\mathtt{d})$ and $a(\mathtt{d})$, $c(\mathtt{d})$ are assumed to be coprime for $|\mathtt{d}| \leq 1$. System (4) is to be driven by a dynamic controller

$$u(\mathtt{d}) = -\frac{y(\mathtt{d})}{x(\mathtt{d})}z(\mathtt{d}) \tag{5}$$

where $x(\mathtt{d})$, $y(\mathtt{d})$ are polynomials.

With these notations, the problem to be solved in this paper is as follows.

Problem 1 *Find dynamic controller (5) such that linear system (4) subject to input constraints (2) is stabilized when initialized within polyhedron (3).*

3 Preliminaries

First, we need to recall two well-known and widely used results of linear system control theory, namely the Youla-Kučera parametrization of all stabilizing controllers [15, 26] and the extended Farkas lemma [10].

Lemma 1 (Youla-Kučera parametrization) *Let $\hat{x}(\mathtt{d})$, $\hat{y}(\mathtt{d})$ denote a particular solution to the polynomial Bézout equation*

$$a(\mathtt{d})x(\mathtt{d}) + b(\mathtt{d})y(\mathtt{d}) = 1 \tag{6}$$

where $a(\mathtt{d})$, $b(\mathtt{d})$ are given coprime polynomials. Then all linear controllers

$$u(\mathtt{d}) = -\frac{y(\mathtt{d})}{x(\mathtt{d})}z(\mathtt{d})$$

that stabilize the linear system

$$z(\mathtt{d}) = \frac{b(\mathtt{d})}{a(\mathtt{d})}u(\mathtt{d})$$

are parametrized by

$$\begin{array}{rcl} x(\mathtt{d}) & = & \hat{x}(\mathtt{d}) + b(\mathtt{d})q(\mathtt{d}) \\ y(\mathtt{d}) & = & \hat{y}(\mathtt{d}) - a(\mathtt{d})q(\mathtt{d}) \end{array} \tag{7}$$

where $q(\mathtt{d})$ is an arbitrary stable rational function, or converging infinite sequence.

Lemma 2 (Extended Farkas lemma) *Polyhedron $\mathcal{P}_N = \{\xi : N\xi \leq \nu\}$ is included within polyhedron $\mathcal{P}_M = \{\xi : M\xi \leq \mu\}$ if and only if there exists a matrix P with non-negative entries such that*

$$\begin{array}{rcl} PN & = & M \\ P\nu & \leq & \mu. \end{array}$$

4 Linear Programming Formulation

We now proceed toward a linear programming solution to the problem stated in Section 2, using the results of Section 3.

From equations (4), (5) and Lemma 1, the system input reads

$$u(\mathtt{d}) = -\frac{y(\mathtt{d})c(\mathtt{d})}{a(\mathtt{d})x(\mathtt{d}) + b(\mathtt{d})y(\mathtt{d})}\xi_0 = [a(\mathtt{d})q(\mathtt{d})c(\mathtt{d}) - \hat{y}(\mathtt{d})c(\mathtt{d})]\xi_0 \tag{8}$$

where $\hat{y}(\mathtt{d})$ is a particular solution to Bézout equation (6) and $q(\mathtt{d})$ is an arbitrary stable rational function. We denote sequences $u(\mathtt{d})$, $q(\mathtt{d})$ as

$$\begin{aligned} u(\mathtt{d}) &= u_0 + u_1\mathtt{d} + u_2\mathtt{d}^2 + \cdots \\ q(\mathtt{d}) &= q_0 + q_1\mathtt{d} + q_2\mathtt{d}^2 + \cdots \end{aligned}$$

and define

$$\begin{aligned} v(\mathtt{d}) &= a(\mathtt{d})c(\mathtt{d}) &&= v_0 + v_1\mathtt{d} + v_2\mathtt{d}^2 + \cdots \\ w(\mathtt{d}) &= -\hat{y}(\mathtt{d})c(\mathtt{d}) &&= w_0 + w_1\mathtt{d} + w_2\mathtt{d}^2 + \cdots \end{aligned} \tag{9}$$

From the above relations, upon equating coefficients at like powers of the indeterminate $\mathtt{d}$ in relation (8) it follows that

$$u_i = \bar{M}_i(q)\xi_0 \tag{10}$$

where

$$\bar{M}_i(q) = w_i + \sum_{k=0,1,2,\ldots} q_k v_{i-k} \tag{11}$$

is a row-vector parametrized by scalars q_k for $k = 0, 1, 2 \ldots$

Recall that input sequence $u(\mathtt{d})$ is subject to constraints (2). In view of relation (10), initial state vector ξ_0 is also constrained. This is captured by the following lemma.

Lemma 3 *Input $u(\mathtt{d})$ satisfies constraints (2) if and only if*

$$\xi_0 \in \mathcal{P}_M = \{\xi : M(q)\xi \leq \mu\} \tag{12}$$

where

$$M(q) = \begin{bmatrix} \bar{M}_0(q) \\ -\bar{M}_0(q) \\ \bar{M}_1(q) \\ -\bar{M}_1(q) \\ \vdots \end{bmatrix} \qquad \mu = \begin{bmatrix} u^+ \\ u^- \\ u^+ \\ u^- \\ \vdots \end{bmatrix}. \tag{13}$$

Recalling the statement of Problem 1, vector ξ_0 is assumed to belong to polyhedron $\mathcal{P}_N$. Hence polyhedron $\mathcal{P}_N$ must be included within polyhedron $\mathcal{P}_M$. Using Lemmas 2 and 3, we can now write necessary and sufficient conditions for Problem 1 to be solved.

Theorem 1 *Problem 1 is solved if and only if there exists P with non-negative entries and q such that*

$$\begin{aligned} PN &= M(q) \\ P\nu &\leq \mu. \end{aligned} \tag{14}$$

A stabilizing controller is then retrieved from parameter q.

Several remarks are in order.

Remark 1 (Polynomial Youla-Kučera parameter) In general, sequence $q(\mathtt{d})$ can be expressed as an infinite stable Laurent series. However, for practical purpose, we only consider polynomial, or finite-impulse-response $q(\mathtt{d})$ in the sequel.

Remark 2 (Linear programming problem) When $q(\mathtt{d})$ is a polynomial, relations (14) become a finite-dimensional linear programming problem that must be solved for a non-negative matrix P and scalar coefficients q_0, q_1, ... If $\deg p$ stands for the degree of a polynomial $p(\mathtt{d})$, it follows from relation (8) that

$$\deg u \leq \max\{\deg a + \deg q + \deg c, \deg \hat{y} + \deg c\}.$$

In linear programming problem (14), matrix $M(q)$ features $2(\deg u + 1)$ rows and n columns.

Remark 3 (Controller order) As in Remark 2, we deduce from relation (7) that

$$\begin{array}{rcl} \deg x & \leq & \max\{\deg \hat{x}, \deg b + \deg q\} \\ \deg y & \leq & \max\{\deg \hat{y}, \deg a + \deg q\}. \end{array}$$

Consequently, the order of stabilizing controller (5) directly depends on $\deg q$, the degree of Youla-Kučera parameter $q(\mathtt{d})$. Contrary to most state-space-based constrained stabilization methods found in the literature, our approach is thus not restricted to static controllers or dynamic output-feedback controllers of the same order than the controlled plant.

Remark 4 (Positive invariance) As pointed out in the introduction, most of the existing methods to deal with local stabilization of constrained input systems hinge upon the concept of positive invariance: system trajectories, once initialized in some set, are guaranteed to stay within this set while converging to the origin. Our approach does not rely on positive invariance. That is to say, the above polyhedra $\mathcal{P}_N$ or $\mathcal{P}_M$ are not necessarily positively invariant.

5 Numerical Examples

In the following examples, numerical computations on polynomials were performed with the Polynomial Toolbox Version 2.0 for MATLAB [19]. Linear programming problems were solved with the function `lp` of the Optimization Toolbox for MATLAB [4].

5.1 First Example

As a first illustrative example, we consider the discrete-time linear system studied in [25]

$$\xi_{k+1} = \begin{bmatrix} 0.8 & 0.5 \\ -0.4 & 1.2 \end{bmatrix} \xi_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k$$

when only the first component of the state vector is available for feedback, i.e.,

$$z_k = [1 \quad 0]\xi_k.$$

System input is constrained to

$$-7 \leq u_k \leq 7$$

and initial state vector ξ_0 is assumed to belong to convex polyhedron $\mathcal{P}_N = \{\xi : N\xi \leq \nu\}$ where

$$N = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ -1.5 & 2 \\ 1.5 & -2 \end{bmatrix} \quad \nu = \begin{bmatrix} 5 \\ 5 \\ 10 \\ 10 \end{bmatrix}.$$

We aim at finding a stabilizing feedback controller for the above system.

First, it is easily found that

$$\begin{aligned} a(\mathtt{d}) &= 0.8621 - 1.7241\mathtt{d} + \mathtt{d}^2 \\ b(\mathtt{d}) &= 0.4310\mathtt{d}^2 \\ c(\mathtt{d}) &= [0.8621 - 1.0345\mathtt{d} \quad 0.4310\mathtt{d}^2] \end{aligned}$$

in transfer function setting (4). A particular solution to Bézout equation (6) is then

$$\begin{aligned} \hat{x}(\mathtt{d}) &= 1.1600 + 2.3200\mathtt{d} \\ \hat{y}(\mathtt{d}) &= 6.5888 - 5.3824\mathtt{d}. \end{aligned}$$

Recalling Remark 1, we assume that $q(\mathtt{d})$ is a polynomial of degree δ. Using Theorem 1, we find that the lowest degree for which linear programming problem (14) turns out to be feasible is $\delta = 3$. Matrices of the linear programming problem read

$$P = \begin{bmatrix} 0 & 0.4667 & 0.4667 & 0 \\ 0.4667 & 0 & 0 & 0.4667 \\ 0.3076 & 0.1060 & 0 & 0.4932 \\ 0 & 0.2015 & 0.5462 & 0.0530 \\ 0.0192 & 0 & 0 & 0.1338 \\ 0 & 0.0192 & 0.1338 & 0 \\ 0.4116 & 0 & 0 & 0.4942 \\ 0 & 0.4116 & 0.4942 & 0 \\ 0.1891 & 0 & 0 & 0 \\ 0 & 0.1891 & 0 & 0 \\ 0 & 0 & 0.2742 & 0 \\ 0 & 0 & 0 & 0.2742 \\ 0 & 0.2986 & 0.5247 & 0 \\ 0.2986 & 0 & 0 & 0.5247 \end{bmatrix} \qquad M(q) = \begin{bmatrix} -1.1667 & 0 \\ 1.1667 & 0 \\ 0.9414 & -0.5833 \\ -0.9414 & 0.5833 \\ 0.2199 & -0.2293 \\ -0.2199 & 0.2293 \\ 1.1529 & -0.1653 \\ -1.1529 & 0.1653 \\ 0.1891 & 0.3781 \\ -0.1891 & -0.3781 \\ -0.4112 & 0.5483 \\ 0.4112 & -0.5483 \\ -1.0857 & 0.4524 \\ 1.0857 & -0.4524 \end{bmatrix}$$

for polynomial

$$q(\mathtt{d}) = 6.0731 + 5.2855\mathtt{d} + 3.0815\mathtt{d}^2 + 1.0495\mathtt{d}^3$$

and a vector μ whose entries are all equal to 7. Reporting $q(\mathtt{d})$ into relations (7), a stabilizing output controller in transfer function setting (5) is given by polynomials

$$\begin{aligned} x(\mathtt{d}) &= 1.1600 + 2.3200\mathtt{d} + 2.6177\mathtt{d}^2 + 2.2783\mathtt{d}^3 + 1.3282\mathtt{d}^4 + 0.4524\mathtt{d}^5 \\ y(\mathtt{d}) &= 1.3533 + 0.5320\mathtt{d} + 0.3834\mathtt{d}^2 - 0.8773\mathtt{d}^3 - 1.2721\mathtt{d}^4 - 1.0495\mathtt{d}^5. \end{aligned}$$

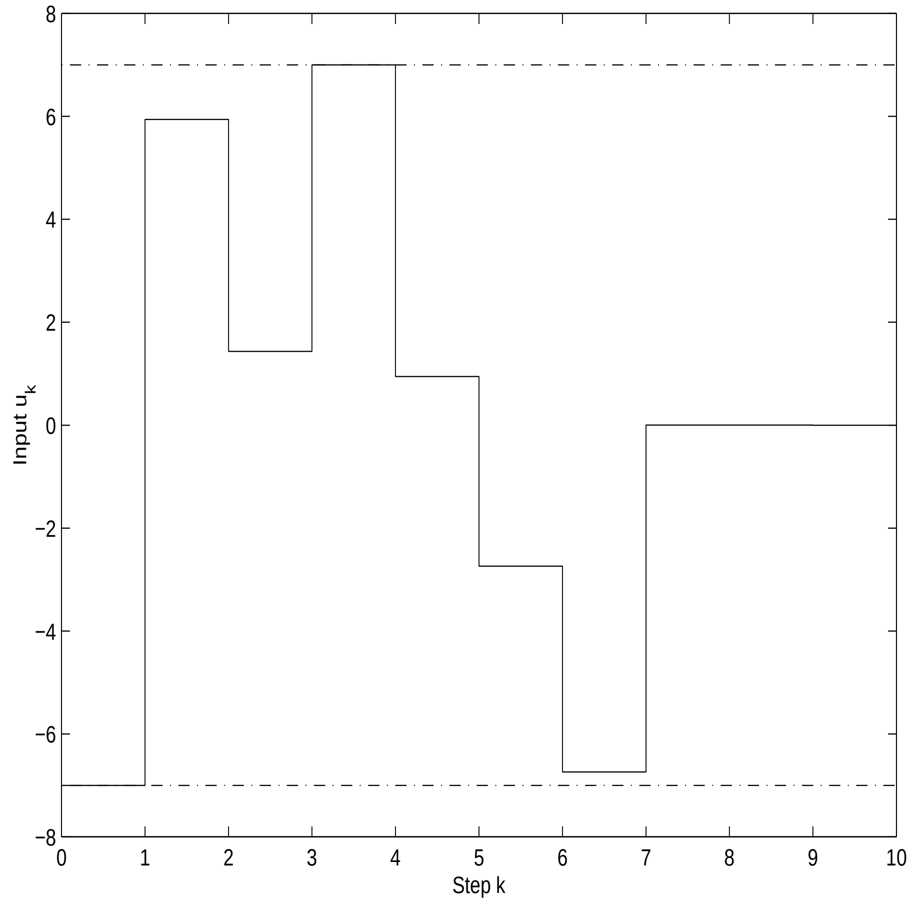

Figure 1 – System input.

The resulting closed-loop system is guaranteed to be stable for any initial condition chosen within polyhedron $\mathcal{P}_N$. The system input is represented in Figure 1 when the system state is initialized to

$$\xi_0 = \begin{bmatrix} 6 \\ -0.5 \end{bmatrix} \in \mathcal{P}_N.$$

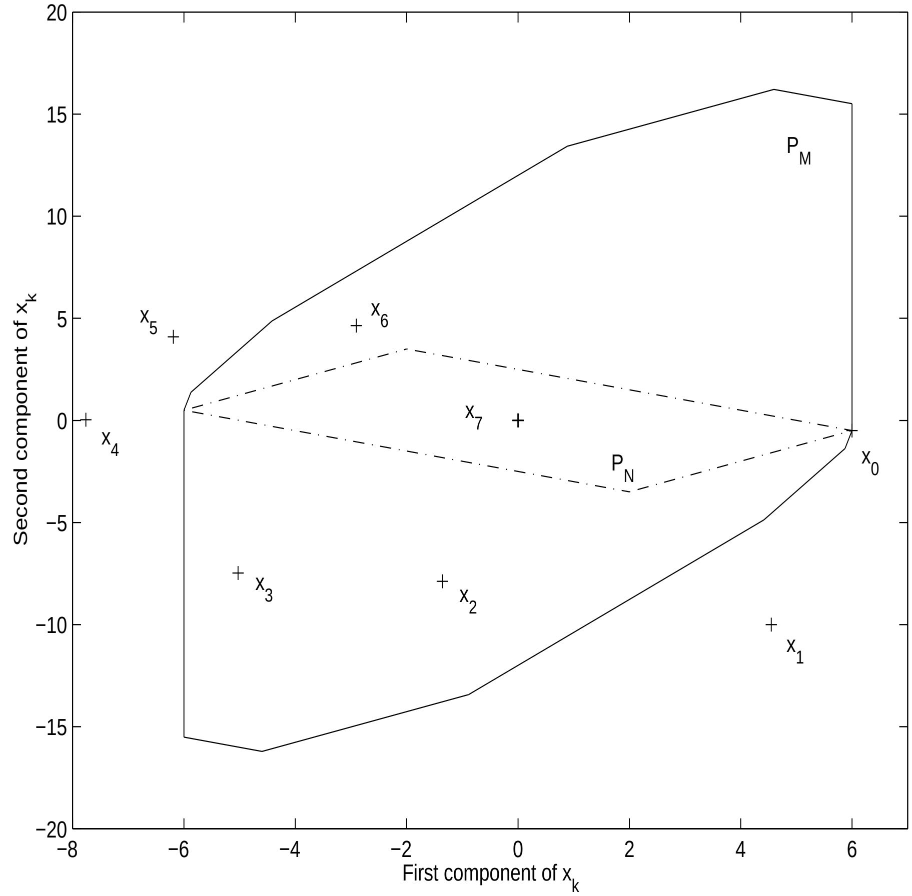

Figure 2 – Polyhedra $\mathcal{P}_M$ and $\mathcal{P}_N$ with system state ξ_k.

For comparison, Figure 2 shows the requested polyhedron of stabilizable initial conditions $\mathcal{P}_N$ together with the actual polyhedron of stabilizable initial conditions $\mathcal{P}_M$ corresponding to the controller found above. In contrast to the results achieved in [25], our approach does not require that polyhedron $\mathcal{P}_N$ be positively invariant. Indeed, system stability is ensured by the mere inclusion of polyhedron $\mathcal{P}_N$ into polyhedron $\mathcal{P}_M$. Another significant advantage of our approach with respect to that proposed in [25] is that we can design dynamic output feedback controllers. On the other hand, we may come up with controllers of relatively high order.

5.2 Second Example

We now illustrate on a very simple numerical example that the choice of polynomial $q(\mathtt{d})$ may influence the size of the polyhedron of stabilizable initial conditions.

Consider the unstable discrete-time system

$$\begin{aligned} \xi_{k+1} &= 2\xi_k + u_k \\ z_k &= \xi_k \end{aligned}$$

where system input is constrained to

$$-1 \leq u_k \leq 1.$$

Assume that the polyhedron of admissible initial conditions is given by

$$\mathcal{P}_N = \{\xi : -1/\alpha \leq \xi \leq 1/\alpha\}$$

where α is a positive scalar.

The system polynomials are readily computed as $a(\mathtt{d}) = 1 - 2\mathtt{d}$, $b(\mathtt{d}) = 1$ and $c(\mathtt{d}) = 1$. A particular solution to Bézout equation (6) is $\hat{x}(\mathtt{d}) = 0$ and $\hat{y}(\mathtt{d}) = 1$.

Then, we successively build linear programming problem (14) for increasing values of δ, the degree of polynomial $q(\mathtt{d})$. We can easily find parameters q_i such that the threshold α is minimized and hence the size of polyhedron $\mathcal{P}_N$ is maximized. In Table 3, we reported values of the q_is and the minimum achievable α for increasing values of δ. When degree δ tends to infinity, $q(\mathtt{d})$ tends to a converging infinite sequence and threshold α tends to a finite value, namely 1. This is not surprising since it is well-known that in general stabilization of an open-loop unstable system with a bounded control can be achieved only locally.

δ	α	q_0	q_1	q_2	q_3	q_4	q_5
–	2	0	0	0	0	0	0
0	4/3	2/3	0	0	0	0	0
1	8/7	6/7	4/7	0	0	0	0
2	16/15	14/15	4/15	8/15	0	0	0
3	32/31	30/31	28/31	24/31	16/31	0	0
4	64/63	62/63	60/63	56/63	48/63	32/63	0
5	128/127	126/127	124/127	120/127	112/127	96/127	64/127

Table 3 – Threshold α and parameters q_i for increasing values of degree δ.

The example clearly illustrates the fact that the size of the domain of stabilizable initial conditions can be enlarged using the degrees of freedom inherent to the approach. This topic is further developed in the second paper of the series.

6 Conclusion

We have provided a simple solution to the difficult problem of locally stabilizing a SISO discrete-time linear system subject to hard input constraints. Under the assumption that the domain of stabilizable initial conditions is a given polyhedron of the state-space, our technique results in an output feedback controller of given order straightforwardly derived upon solving a linear programming problem.

The ideas proposed in this paper are conceptually simple, for they rely on well-known results of control theory such as the Youla-Kučera parametrization and the extended Farkas lemma. Most importantly, they can be extended to the more general setting of

MIMO systems with an unknown, not necessarily polyhedral domain of stabilizable initial conditions. Such extensions will be covered in considerable detail in the second paper of the series.

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