Optimal Capacity Expansion in the Presence of Capacity Options

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Abstract

This paper studies optimal long-term capacity strategies when there exist markets for capacity options. The essential ingredients of the problem derive from the strategic interaction of contract markets with spot markets. This interaction provides the foundation for the short-term pricing and contracting strategies for market participants. Sellers in this market can sign long-term (e.g., forward) contracts with Buyers, where such long-term contracts take the form of capacity options that may or may not be executed by Buyers at some pre-specified maturation date. Capacity not offered in the options market, or for which options by Buyers are not executed, can then be offered in the spot market. As in our earlier work, Wu et al. (2001a,b,c), we assume that there is some residual risk that such capacity may not find last-minute buyers. This risk of not being able to sell in the spot market, together with the benefits of long-term contracting, lead to an equilibrium in the pricing and demand for capacity options. The details of this equilibrium have been fully worked out in Wu et al. (2001c). However, this equilibrium is a short-term equilibrium and assumes that capacity of each Seller in the market is fixed. The purpose of this paper is to derive the optimal capacities for Sellers, given full knowledge of the short-term equilibria that would result from any set of capacity decisions they might take. We determine the best response strategies for each Seller in the game derived from the short-term outcome resulting from capacity decisions. We then characterize the long-run equilibrium, when it exists. This allows us also important insights into the nature of technologies that can survive in the long run. As we show, the factors determining such survival depend on the characteristics of both the costs of the technologies as well as the structure and volatility of the markets in which they operate. Numerical examples show that computing long-run equilibrium capacities (and associated short-run market equilibrium) are straightforward using algorithms developed in this paper.

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1 Introduction

The papers by Wu et al. (2001a,b,c), hereafter cited as WKZ have characterized the necessary and sufficient conditions of the short-term equilibrium, i.e. with a fixed capacity for every Seller, for a market in which Buyers can reserve capacity through options obtained from individual Sellers. Output on the day can be either obtained through executing such options or in a spot market. Examples of such contract-spot markets abound, and certainly include electric power, natural gas, various commodity chemicals, semiconductors and transportation services (see e.g., Kleindorfer et al. 2001; Kleindorfer and Wu 2001; Wu et al. 2001c). These markets can be expected to become more prominent under e-Business (Araman et al. 2001; Geoffrion and Krishnan 2001; Mendelson and Tunca 2001). See Kleindorfer and Wu (2001) for a survey on integrating contracting with business-to-business exchanges for capital-intensive industries. In this paper, we extend the WKZ short-term equilibrium results to determine optimal capacity strategies and study the equilibrium issues related to these strategies. To make this paper manageable in size, we will rely entirely on the framework and notation of WKZ (2001c).

Linking capacity expansion games with short-term pricing has been an important area of study in industrial organization, with a major contribution coming in the Kreps-Scheinkman (1983) paper, showing that precommitments of capacity, followed by Bertrand competition, gives rise to Cournot outcomes. This paper shows that something like these same results hold in the more realistic context in which long-term capacity planning must be integrated with short-term pricing and contracting strategies. This paper builds on previous research from economics, operations research and marketing science concerned with the integration of operating decisions (capacity and production decisions) with a two-tiered market structure supporting both contracts and spot purchases from suppliers.

While the focus of this paper is on integrating pricing and capacity decisions, an interesting by-product of this research is the characterization of efficient technology mixes in long-run equilibrium, where technologies are characterized by different unit variable and unit capacity costs. The efficient technology mix problem has been discussed by Allaz (1992), Allaz and Vila (1993), Crew and Kleindorfer (1976), and Gardner and Rogers (1999). The conditions characterizing the efficient mix are extended here to account for the integration of contract and exchange/spot markets. The usual cost conditions (indicating tradeoffs between unit capacity costs and unit variable costs across different technologies) need to be extended in the present context to account for the interaction of each technology with the characteristics (especially the volatility) of the spot market.

We proceed as follows. In Section 2, we first define some necessary notation and summarize assumptions and conditions needed for our model. In Section 3, building on the short-term results of WKZ (2001c), we structure the long-term capacity game among Sellers. This game is determined by the expected profits for each Seller in the short-term game of participating in the combined contract-spot markets. These profits, of course, depend on the capacity decisions made by Sellers prior to the play of this short-run game. We determine best response and equilibrium strategies for the long-run capacity game and show some properties of the price and capacities that result in equilibrium. We then consider the characteristics of efficient technology mixes in the long-run equilibrium. In section 4, we give some numerical examples to illustrate key insights derived in this paper. In section 5, we further characterize the long-run market segmentation for Sellers and their supporting technologies. Interestingly, this segmentation indicates a ranked ordering of technologies such that the first segment participates in both the contract and the spot market, the second segment participats only in the spot market, and the third segment is forced out of the market. Section 6 concludes the paper with some extensions and directions for future research.

2 Preliminaries

We assume a set of I Sellers, Ξ, and any number of Buyers. Following WKZ (2001c), we use the following notation.

P_s: spot market price. Its cumulative distribution function $F(P_s)$ is assumed to be common knowledge

β_i: Seller i's unit capacity cost per period

b_i: Seller i's short-run marginal cost of providing a unit

K_i: Seller i's total capacity. Let $K = (K_1, \ldots, K_I)$

s_i: Seller i's reservation cost per unit of capacity if the contract is signed

g_i: Seller i's execution cost per unit of output actually used from the contract. Recall from WKZ (2001c) that the (optimal) price of $g_i = b_i$

Q_i: Contract market demand for Seller i's output. Recall that in WKZ (2001c), we have shown when there are multiple Sellers, *Greedy Contracting* in order of the index $s_1 + G(g_1) \leq s_2 + G(g_2) \leq \ldots \leq s_I + G(g_I)$ is optimal for the Buyers

$U(z)$: Buyers' aggregate Willingness-To-Pay for output level z

Define $p = s_i + G(g_i)$ as the contract market price, symmetric for all Sellers at equilibrium, with the effective price function $G(x)$ defined as

$$G(x) = \int_0^x (1 - F(y))dy = \mathrm{E}\{\min(P_s, x)\}$$

and G^{-1} as the inverse function of G

$D_s(x)$: Buyer's normal demand function when there exists only the spot market, so $D_s(x) = U'^{-1}(x)$. Let $D(p) = D_s(G^{-1}(p))$

Define $c_i = \underline{s_i} + G(b_i)$, in which $\underline{s_i} = \mathrm{E}\{m(P_s - b_i)^+\}$ is Seller i's unit opportunity cost on the spot market if the buyer chooses to exercise his contract

m: the probability that the Seller can find a last-minute buyer on the spot market[2]

[2]To minimize notational complexity, we analyze only the case where m is uniform and fixed for all Sellers. It is straightforward to generalize these results to allow m to vary as a function of P_s and to vary across Sellers. See WKZ (2001c) for such an extension.

Define $X(M) = \sum_{i \in M} K_i$ as the total capacity of all Sellers in set M.

We make the following assumptions.

A1: The Buyer's WTP $U(z)$ is strictly concave and increasing so that $U'(z) > 0,\ U''(z) < 0,\ \text{for}\ z \geq 0$

A2: $zD_s''(z) + 2D_s'(z) \leq 0, \quad z \geq 0$

A3: $Q_i[D_s(g_i) - \sum_{l=1}^{i} Q_l] \geq 0, \quad i = 1, \ldots, I$, assuming $g_1 \leq g_2 \leq \ldots \leq g_I$

A4: When there is a bid-tie among Sellers, then the Buyers' demand for Seller i's output is proportionally allocated to the Sellers according to their bid capacity, thus $Q_i = D(p)\frac{K_i}{X(M)}$.

Concerning A1, these are standard assumptions on the Willingness-to-Pay function. From A1, we can easily know that $D(p)$ is monotonously descending. A2 is equivalent to $R' > 0$ and $R < 1$ where $R = -U''(Q)Q/U'(Q)$ is the Arrow-Pratt measure of relative risk aversion. This, too, is standard in the financial economics literature (e.g., Rothschild and Stiglitz 1970, 1971). A3, noted as the *No Excess Capacity Condition* in WKZ (2001c), implies that Buyers will not contract for more than what they are sure they will use if they buy under contract on the day, i.e., if $Q_i > 0$ then the sum of all contracted capacity with execution fees less than or equal to g_i must not exceed $D_s(g_i)$.

In WKZ (2001c, Theorem 2), we characterized the short-term equilibrium as the following. Let $K, \hat{p}, \hat{M}$ be any short-term equilibrium, where $\hat{M}(K) \subseteq \Xi$ is the equilibrium set of all Sellers having positive capacity contracts, i.e., $Q_i(\hat{p}) > 0, i \in \hat{M}$ and $Q_j(\hat{p}) = 0, j \in \Xi \setminus \hat{M}$. Assume $\hat{M}$ is non-singleton such that $|\hat{M}| > 1$ and $\text{Min}\{c_i \mid i \in \Xi\} < G(U'(0))$. Then the necessary and sufficient conditions for an equilibrium $\hat{p}$ to exist are[3]

SC1: $D(\hat{p}) = \sum_{i \in \hat{M}} K_i = X(\hat{M})$;

[3] In the singleton case when $|\hat{M}| = 1$, the only Seller providing positive contract output (which we denote as Seller 1) satisfies $c_1 = \min\{c_i | i \in \Xi\} < G(U'(0))$. The necessary and sufficient conditions for a single-supplier short-term equilibrium $\hat{p}$ to exist are $(i)\ \hat{p} = \max\{p^H, x^H\}$, where $p^H = \text{argmax}(p - c_1)D(p_1)$, and $x^H = D^{-1}(K_1)$, and $(ii)\ \hat{p} < \min\{c_i | i \in \Xi \setminus \{1\}\}$.

SC2: $\partial f_k(p_k)/\partial p_k < 0$ if $p_k > \hat{p}$, where $f_k(p_k)$ is defined as $f_k(p_k) = (p_k - c_k)(D(p_k) - \sum_{i \in \hat{M}_k^0} K_i)$, and $\hat{M}_k^0 = \hat{M} - \{k\}$; and

SC3: $\forall j \in \Xi \setminus M, \hat{p} < c_j$.

Condition SC1, noted as the "symmetry condition" in WKZ (2001c), says that in the short-term equilibrium, for any Seller "in the money", i.e., for any $k \in \hat{M}$, its entire capacity will be contracted in the contract market. Condition SC2 is a special case of the standard economic assumption (see, e.g., Friedman, 1988) for the behavior of the profit function. In Friedman (1988), it is assumed to be strictly concave, here we only require the function to be non-increasing to the right of the equilibrium price $\hat{p}$, where $\hat{p} \geq \text{argmax} f_k(p_k)$. Condition SC3 implies that any Seller "out of the money" does not have any incentive to join in the short-term contract market equilibrium, as doing so results a net loss in its profit.

Further discussion of these assumptions and conditions is in WKZ (2001a,b,c). We seek a sub-game perfect equilibrium (Fudenberg and Tirole, 1991) for the long-run capacity game, given the full knowledge of the short-run contracting and pricing game that will follow.

3 Optimal Capacity Expansion

The outcome of the short-term options-pricing game in the integrated contract and spot markets leads to the following profit function for any Seller k:

$$\text{E}\pi_k(\hat{p}, K) = (\hat{p} - c_k)Q_k + (c_k - \beta_k - G(b_k))K_k \tag{1}$$

Seller k's problem is to choose an optimal capacity K_k^* to maximize k's long-run expected profit, i.e.,

$$\text{Maximize}_{K_k} \text{E}\pi_k(\hat{p}, K).$$

Lemma 1: Let $\hat{p}(K)$ be the short-run equilibrium price and let $\hat{M}(K)$ be the set of Sellers in the contract-spot equilibrium. Assume $\hat{M}(K)$ is non-singleton so that $|\hat{M}(K)| > 1$[4]. Then the best response capacity strategy for each Seller $k \in \hat{M}(K)$ is

$$K_k^* = \max\{\frac{\hat{p} - \beta_k - G(b_k)}{\hat{p} - c_k} X(\hat{M}), 0\}. \tag{2}$$

Proof: Take any capacity vector K and let $\hat{M}(K)$ be the short-term equilibrium set (assuming it exists, and is non-singleton). Substituting $Q_k = D(\hat{p})\frac{K_k}{X}$ into the profit function, we obtain the following expression for the profit function for Seller $k \in \hat{M}(K)$:

$$\mathrm{E}\pi_k(\hat{p}, K) = (\hat{p} - c_k)D(\hat{p})\frac{K_k}{X} + (c_k - \beta_k - G(b_k))K_k$$

The FOC condition for maximizing $\mathrm{E}\pi_k(\hat{p}, K)$ gives

$$K_k^* = X - \frac{\beta_k + G(b_k) - c_k}{(\hat{p} - c_k)D(\hat{p})} X^2$$

From WKZ (2001c), we know that a necessary condition (SC1) in equilibrium is that $D(\hat{p}) = X$. This, coupled with the above FOC, results in the identity (2). It is straightforward to check that the Seller's profit function w.r.t. K_k is concave, as we see from the SOC

$$-2(\hat{p} - c_k)D(\hat{p})\frac{X - K_k}{X^3} < 0$$

Given this concavity, if the first term in (2) is negative, then the optimal capacity choice is $K_k^* = 0$. Hence the above solution is indeed optimal. □

We note that the above proof takes the equilibrium set $\hat{M}(K)$ as given, and determines the best response strategy for every Seller in $\hat{M}(K)$, assuming that the set $\hat{M}(K)$ does not change as K is adjusted. In the long-term equilibrium, where K is adjustable, what is required is that all best capacity responses K^*, given $\hat{M}(K^*)$, result in a short-run equilibrium p^* with $p^* = \hat{p}(K^*)$ and M^* with $M^* = \hat{M}(K^*)$. Thus, the long-run (subgame-perfect) equilibrium we seek to characterize is defined as follows.

[4] The case of singleton when $|\hat{M}| = 1$ is dealt with later.

Definition: A long-run non-singleton contract market equilibrium K^*, p^*, M^* is a vector such that $p^* = \hat{p}(K^*)$ and $M^* = \hat{M}(K^*)$ and such that $K^*_k > 0$ for all $k \in M^*$, where K^*_k satisfies the best-response condition (2), i.e.,

$$K^*_k = \frac{p^* - \beta_k - G(b_k)}{p^* - c_k} X^* \tag{3}$$

where $X^*(M^*) = \sum_{i \in M^*} K^*_i$.

Definition: Let $c_1 = \min\{c_i \mid i \in \Xi\}$ so that Seller 1 has the lowest c_i index among all Sellers. A long-run singleton contract market equilibrium K^*_1, p^*, M^* is a triple such that the following conditions are satisfied: (i) $p^* = \text{argmax}(p - \beta_1 - G(b_1))D(p)$; (ii) $K^*_1 = D(p^*)$; (iii) $c_1 \leq \beta_1 + G(b_1)$; (iv) $p^* < \min\{\max\{c_i, \beta_i + G(b_i)\} \mid i \in \Xi \setminus \{1\}\}$.

Definition: Define ζ_k - a modified Tobin's marginal q (Abel 1983; 1990; Abel et al. 1996) for Seller k as:

$$\zeta_k = \frac{\partial((p - c_k)D(p)\frac{K_k}{X})/\partial K_k}{\partial((\beta_k + G(b_k) - c_k)K_k)/\partial K_k} \tag{4}$$

Corollary 1: Let K^*, p^*, M^* be a long-run equilibrium solution. Then for any Seller $k \in M^*$, $\zeta^*_k = 1$ where ζ_k is Tobin's marginal q for Seller k, whether M^* is singleton or not.

Proof: (a) First we show the claim is true in the singleton case when $|M^*| = 1$. Since $p^* = \text{argmax}(p - \beta_1 - G(b_1))D(p)$, we have the FOC

$$(p^* - \beta_1 - G(b_1))\frac{\partial D}{\partial K_1} + D = 0,$$

since $p^* = D^{-1}(K_1)$, we have

$$\frac{D}{\partial D/\partial K_1} = \beta_1 + G(b_1) - p^* = \beta_1 + G(b_1) - D^{-1}. \tag{5}$$

thus Tobin's marginal q is

$$\zeta_1 = \frac{\partial((p^* - c_1)K_1)/\partial K_1}{\partial((\beta_1 + G(b_1) - c_1)K_1)/\partial K_1} = \frac{\partial((D^{-1}(K_1) - c_1)K_1)/\partial K_1}{\partial((\beta_1 + G(b_1) - c_1)K_1)/\partial K_1}$$

$$= \frac{D^{-1} - c_1 + \frac{\partial D^{-1}}{\partial K_1}K_1}{\beta_1 + G(b_1) - c_1} = \frac{D^{-1} - c_1 + \frac{\partial D^{-1}}{\partial K_1}D}{\beta_1 + G(b_1) - c_1}. \tag{6}$$

Since $\frac{\partial D^{-1}}{\partial K_1}\frac{\partial D}{\partial K_1} = 1$, so (6) can be rewritten as

$$\zeta_1 = \frac{D^{-1} - c_1 + \frac{D}{\partial D/\partial K_1}}{\beta_1 + G(b_1) - c_1}. \tag{7}$$

Substitute (5) into (7), and we get

$$\zeta_1^* = \frac{D^{-1} - c_1 + \beta_1 + G(b_1) - D^{-1}}{\beta_1 + G(b_1) - c_1} = 1.$$

(b) Second we show the claim hold when $|M^*| > 1$. We can write Tobin's marginal q for each supplier $k \in M^*$ as:

$$\zeta_k = \frac{\partial((p^* - c_k)D(p)\frac{K_k}{X})/\partial K_k}{\partial((\beta_k + G(b_k) - c_k)K_k)/\partial K_k} = \frac{(p^* - c_k)D(p^*)\frac{X - K_k}{X^2}}{\beta_k + G(b_k) - c_k}.$$

The first equality is the definition of Tobin's marginal q. The second equality is simply the result of taking derivatives w.r.t. K_k. Since at equilibrium, $D(p^*) = X^*$ and $\frac{K_k^*}{X^*} = \frac{p^* - \beta_k - G(b_k)}{p^* - c_k}$ (From Lemma 1), we can rewrite the above as

$$\zeta_k^* = \frac{(p^* - c_k)(1 - \frac{K_k^*}{X^*})}{\beta_k + G(b_k) - c_k} = \frac{(p^* - c_k)(1 - \frac{p^* - \beta_k - G(b_k)}{p^* - c_k})}{\beta_k + G(b_k) - c_k} = \frac{(p^* - c_k)\frac{\beta_k + G(b_k) - c_k}{p^* - c_k}}{\beta_k + G(b_k) - c_k} = 1.$$

□

Corollary 2: For any Seller $k \in M^*$, whether M^* is singleton or not, if $\exists K_k^* > 0$, then $p^* > \beta_k + G(b_k) > c_k$.

Proof: This is a direct consequence of Lemma 1 and the definition of singleton contract market equilibrium. □

It should be noted that the above lemma and corollaries characterize capacity conditions only for the long-term contract market. It may very well be the case that some Sellers build capacity and only participate in the spot market. Corollary 2 says that those who participate in the contract market in the long run, $\beta_k + G(b_k) > c_k$ or equivalently $\beta_k > \underline{s_k}$. This implies that in any long-term contract market equilibrium, every Seller (with positive capacity) satisfies $\beta_k + mG(b_k) > m\mu$, where μ is the mean of the spot market price. This is very intuitive. As the mean of the spot market price increases, or access conditions improve to the spot market (m increases), Sellers are less interested in participating in the contract market and more interested in participating in the spot market.

Corollary 3: Let K^*, p^*, M^* be a long-run equilibrium solution. Assume $\mid M^* \mid > 1$, then p^* must satisfy (in addition to being a short-run equilibrium price corresponding to K^*)

$$\sum_{i \in M^*} \frac{p^* - \beta_i - G(b_i)}{p^* - c_i} = 1 \tag{8}$$

or equivalently,

$$\sum_{i \in M^*} \frac{\beta_i + G(b_i) - c_i}{p^* - c_i} = \mid M^* \mid -1. \tag{9}$$

Proof: Summing over M^* on both sides of (3) results in (8). It is straightforward to get (9) from (8). □

Lemma 2: Let K^*, p^*, M^* be any long-run equilibrium solution. For the given K^*, p^*, the equilibrium set $M(p^*) \subseteq \Xi$, is unique.

Proof: It is trivial for the case when $\mid M^* \mid = 1$. Suppose $\mid M^* \mid > 1$. We prove this in two steps: (a) Given any set $M^* \subseteq \Xi$, p^* is unique; then we show (b) that $M^*(p^*) \subseteq \Xi$, is unique.

First, we prove (a) is true. Take any subset of $M^* \subseteq \Xi$. Assume (a) is not true, i.e., there are at least two pricing equilibria p_1^* and p_2^* corresponding to M^* satisfying (8) or (9). W.l.o.g. assume that $p_2^* > p_1^* > \max\{c_i \mid i \in M^*\}$. From (9) of Corollary 3, we know that

$$\sum_{i \in M^*} \frac{\beta_i + G(b_i) - c_i}{p_1^* - c_i} = \mid M^* \mid -1. \tag{10}$$

$$\sum_{i \in M^*} \frac{\beta_i + G(b_i) - c_i}{p_2^* - c_i} = \mid M^* \mid -1.$$

However, since $\forall i \in M^*$, $p_2^* > p_1^* > \max\{c_i \mid i \in M^*\}$ by assumption and $\beta_i + G(b_i) - c_i > 0$ by Corollary 2, we have

$$\sum_{i \in M^*} \frac{\beta_i + G(b_i) - c_i}{p_1^* - c_i} > \sum_{i \in M^*} \frac{\beta_i + G(b_i) - c_i}{p_2^* - c_i} = \mid M^* \mid -1.$$

This contradicts the assumption that p_1^* is an equilibrium since (10) is violated. Thus, we must have $p_1^* = p_2^*$, as asserted in claim (a).

Second, we show (b) is true. First we note that from WKZ (2001c), for any equilibrium set (short-run or long-run) M^*, if $j \in M^*$ and $c_i \leq c_j$, then $i \in M^*$, so that any equilibrium set for the long-term contract market consists of Sellers with contiguous indices c_i. Now assume there are two equilibrium sets $M_1^* = \{1, \ldots, l\}$ and $M_2^* = \{1, \ldots, l, l+1, \ldots, n\}$ with respective equilibrium prices p_1^*, p_2^*. Moreover, $p_1^* \leq c_{l+1}$ because otherwise $l+1$ would have an incentive to participate in the contract market and M_1^* would not be an equilibrium set. From (9) of Corollary 3,

$$\sum_{i \in M_1^*} \frac{\beta_i + G(b_i) - c_i}{p_1^* - c_i} = \mid M_1^* \mid -1; \tag{11}$$

$$\sum_{k \in M_2^*} \frac{\beta_k + G(b_k) - c_k}{p_2^* - c_k} = \mid M_2^* \mid -1. \tag{12}$$

Subtract (12) from (11), and rearrange terms, we get

$$\sum_{i \in M_1^*} (\beta_i + G(b_i) - c_i) \frac{p_1^* - p_2^*}{(p_1^* - c_i)(p_2^* - c_i)} = \mid M_2^* \mid - \mid M_1^* \mid - \sum_{k \in M_2^* \setminus M_1^*} \frac{\beta_k + G(b_k) - c_k}{p_2^* - c_k}. \tag{13}$$

Since from Corollary 1, we know that $\forall k \in M_2^* \setminus M_1^*$, $p_2^* > \beta_k + G(b_k) > c_k$, we have

$$\sum_{k \in M_2^* \setminus M_1^*} \frac{\beta_k + G(b_k) - c_k}{p_2^* - c_k} < \mid M_2^* \mid - \mid M_1^* \mid$$

or

$$| M_2^* | - | M_1^* | - \sum_{k \in M_2^* \setminus M_1^*} \frac{\beta_k + G(b_k) - c_k}{p_2^* - c_k} > 0.$$

Thus the LHS of (13) must be positive, i.e.,

$$\sum_{i \in M_1^*} (\beta_i + G(b_i) - c_i) \frac{p_1^* - p_2^*}{(p_1^* - c_i)(p_2^* - c_i)} = (p_1^* - p_2^*) \sum_{i \in M_1^*} \frac{\beta_i + G(b_i) - c_i}{(p_1^* - c_i)(p_2^* - c_i)} > 0$$

Since $\forall i \in M_1^*$, $\beta_i + G(b_i) - c_i > 0$ and $p_2^* - c_i > 0$, also $p_1^* > c_i$, the above inequality implies $p_2^* < p_1^*$. Thus $p_2^* < c_{l+1}$ since (as noted above) $p_1^* \leq c_{l+1}$. This contradicts the fact that Seller $l + 1$ is a member of M_2^*, so that claim (b) holds. Coupling (a) and (b), we have the uniqueness of $M^*(p^*)$. □

Theorem: The long-term equilibrium set $M^* \subseteq \Xi$, which may be empty, is characterized by the following algorithm. Index Sellers in the order of c_i, i.e., $c_1 \leq c_2 \leq \ldots \leq c_I$ and set $M^* = \phi$.

(i) $p^* = \text{argmax}(p - \beta_1 - G(b_1))D(p)$. If $c_1 > \beta_1 + G(b_1)$ then exit else if $p^* \leq c_2$, then $M^* = \{1\}$ exit. Else $M^* = \{1\}$ and $i = 2$.

(ii) Loop While $((p^* > \beta_i + G(b_i))$ and $(\beta_i + G(b_i) > c_i))$

begin

$M^* = M^* \bigcup \{i\}$.

compute $p^*(M^*)$ via (9).

if $i < I$ then $i = i + 1$ else exit.

end.

(iii) If $((p^* > c_i)$ and $(c_i \geq \beta_i + G(b_i)))$ then $M^* = \phi$.

(iv) If $\partial f_i(p_i)/\partial p_i \geq 0$ and $p_i > p^*$ then $M^* = \phi$.

Proof: This is direct consequence of Lemmas 1, 2 and Corollary 3. □

The above algorithm generates the equilibrium essentially by testing, in increasing order of c_i, the compatibility between the short-run pricing equilibrium and the long-run capacity equilibrium at the best-response strategies characterized in Lemma 1. An equilibrium can, of course, fail to exist. As embodied in the above algorithm, this occurs when adding a further Seller k to the contract market, at the long-term capacities implied by the best-response capacity strategies, the short-term equilibrium price drops below the required feasibility index c_k for Seller k. Thus, without Seller k in the contract market, Buyers' demand intensity signals that entry is desirable beyond the current participants in the contract market. But adding k drops the price below that which is sustainable in this market. Let us consider some examples to illustrate these points.

4 Numerical Examples

Numerical Example 1. Assume there are five Sellers with technology parameters $(G(b), \beta, K)$ as shown in Table 1 and the risk factor $m = 0.5$. We can compute $(\underline{s}, c, \beta + G(b))$ as shown in Table 1. Suppose the spot market price follows an exponential distribution, $f(y) = \frac{1}{30}e^{-y/30}$, so the mean of the spot market price is $\mu = 30$. Then the effective price function is $G(x) = -30(e^{-x/30} - 1)$, where $0 \leq x < \infty$, and thus we have $G^{-1}(p) = 30ln\frac{30}{30-p}$, where $0 \leq p < 30$. Suppose the WTP function is $U(z) = 30z(ln\frac{30}{z} + 1)$, where $0 < z \leq 30$, it is obvious that this function satisfies Assumption 1 as follows: $U'(z) = 30ln\frac{30}{z} \geq 0$ and $U''(z) = -\frac{30}{z} < 0$; thus we have $D_s(x) = U'^{-1}(x) = 30e^{-x/30}$, where $0 \leq x < \infty$. So the contract market demand function is $D(p) = D_s(G^{-1}(p)) = U'^{-1}(G^{-1}(p)) = 30 - p$, where $p \in [0, 30)$. It is straightforward to compute that in the short term, four Sellers, namely 1, 2, 3, and 4 achieve an equilibrium at a price $\hat{p} = 26$. Seller 5 is not in the short-term equilibrium even though 5 has strong incentives to participate since 5 can not make any money on the spot market due to very high short-run marginal cost $G(b_5) = 30$. Using the above Theorem, we can compute that there are only two survivors in the long run, namely

Table 1: Summary of Parameters and Results for Numerical Example 1. $m = 0.5$, $\mu = 30$, $D(p) = 30 - p$.

Seller	$G(b_i)$	β_i	K_i	$\underline{s_i}$	c_i	$\beta_i + G(b_i)$	$\hat{p}$	p^*	$\hat{\pi}_i$	π_i^*	K_i^*
1	6	14	1	12	18	20	26	23.2	6	13.4	4.2
2	10	12	1	10	20	22	26	23.2	4	3.1	2.6
3	18	4	1	6	24	22	26	–	4	–	–
4	20	2	1	5	25	22	26	–	4	–	–
5	30	1	1	0	30	31	–	–	–	–	–

1 and 2, with the equilibrium price, $p^* = 23.236$. The optimal capacity investments for these two Sellers are $K_1^* = 4.180$ and $K_2^* = 2.584$. Seller 3 and Seller 4 find themselves out of the contract market, and both participate only in the spot market. Seller 5 is "out of business" in the long run, and is better off by shutting down all its plants. Figure 1 depicts these results graphically. A more general result is summarized in Corollary 5.

The reader will note that equilibrium does not always exist. Here's another example.

Numerical Example 2. Assume there are three Sellers in the contract market. Seller 1's and 2's technology parameters and all the other market conditions are the same as in Example 1, except that Seller 3's $G(b_3) = 16$ and $\beta_3 = 6$ as in Table 2. The short-term equilibrium price is 27. However, there is no long-term equilibrium in this example, because the long-term contract price formed by Seller 1 and 2, 23.2, is higher than Seller 3's index $c_3 = 23$, at this contract market price $p = 23.2$, Seller 3 finds participation in the contract market is more profitable than staying in the spot market per unit capacity, since $p - (\beta_3 + G(b_3)) = 23.2 - 22 = 1.2 > \underline{s_3} - \beta_3 = 7 - 6 = 1$. However, if Seller 3 does participate in the contract market, the contract market price drops to 22.2, this makes participation undesirable, since net profit per unit capacity is less than staying in the spot market, i.e., $p - (\beta_3 + G(b_3)) = 22.2 - 22 = 0.2 < \underline{s_3} - \beta_3 = 7 - 6 = 1$. This is an example that shows there need be no long-term equilibrium in the contract market.

Table 2: Summary of Parameters and Results for Numerical Example 2. $m = 0.5$, $\mu = 30$, $D(p) = 30 - p$.

Seller	$G(b_i)$	β_i	K_i	$\underline{s_i}$	c_i	$\beta_i + G(b_i)$	$\hat{p}$	K_i^*	p,without S3	p,with S3
1	6	14	1	12	18	20	27	4.2	23.2	22.2
2	10	12	1	10	20	22	27	2.6	23.2	22.2
3	16	6	1	7	23	22	27	1	23.2	22.2

Numerical Example 3. It is interesting now to conduct a game-theoretical analysis of the investment game in Numerical Example 1. Assume Sellers 3, 4, 5 would not invest, and their capacity will be fixed throughout, i.e., $K_3 = K_4 = K_5 = 1$. Seller 1 and Seller 2 each has to decide whether to invest or not. Seller 3, 4, and 5 decide whether or not to participate in the contract market based on the resulting contract equilibrium price due to the capacity adjustment of Sellers 1 and 2.

In the above numerical example 1, we computed the profits for both parties in short-term equilibrium (K_1, K_2) and in long-term equilibrium (K_1^*, K_2^*). Now we compute the profits of both parties when only one Seller is using the best response strategy, (K_1^*, K_2) and (K_1, K_2^*). When Seller 1 expands its capacity to $K_1^* = 4$ but Seller 2 does not, $p = 24$, so Seller 4 is out, Seller 1's profit increases while Seller 2's profit decreases. Seller 3 is indifferent between participating in the contract market or in the spot market. On the other hand, when Seller 2 raises his capacity level to $K_2^* = 3$ while Seller 1 does not, $p = 25$, Seller 4 is indifferent between participating in the contract market and in the spot market. However, Seller 3 now finds the contract market more profitable than the spot market alone. Seller 2's profit increases while Seller 1's profit decreases due to Seller 2's capacity expansion. Table 2 contains the payoff matrix for Sellers 1 and 2. Clearly, the Nash equilibrium of this investment game is that both Seller 1 and 2 choose to invest, as characterized in our Theorem.

Table 3: A Two Seller Investment Game

	K_2(Don't Invest)	K_2^*(Invest)
K_1(Don't Invest)	6, 4	5, 9
K_1^*(Invest)	16, 2	13.4, 3.1

Our analysis leads to a somewhat different conclusion than that advanced by Allaz (1992), Allaz and Vila (1993) based on a simpler model in which capacity is assumed to be completely scalable within the timeframe of the contract market. They show that a prisoner's dilemma results in the forward-spot market with Oligopoly. They show, in the context of their model, that Sellers always prefer to stay in the spot-market, but are forced to enter into the contract market because of competition, even though doing so results in a net loss of profit. The above example clearly shows such a dilemma is not a general result in the richer model studied here in which capacity precommitments (at a cost) are present! The long-run equilibrium we obtained in this example is indeed Pareto efficient. Whether this Pareto optimality holds in general for the richer market studied here remains an open question.

5 Long-Run Market Segmentation and Conclusions

Corollary 5: Assume a long-term equilibrium K^*, p^*, M^* exists. The market segmentation of Sellers in the long-run is the following. (i) For any Seller k "in the money", then $\underline{s_k} < \beta_k$, i.e., $\forall k \in M^*$, k participates in both the contract and the spot market. (ii) For any Seller k "out of the money", i.e., $\forall k \in \Xi \setminus M^*$, the necessary and sufficient condition for k to participate in the spot market is $\underline{s_k} > \beta_k$. (iii) For any Seller $k \in \Xi \setminus M^*$ but $\underline{s_k} \leq \beta_k$; k will be "out of business" in the long-run.

Proof: From Corollary 2, we know that for any Seller $k \in M^*$, $\beta_k + G(b_k) > c_k$, since by definition, $c_k = \underline{s_k} + G(b_k)$, we have $\beta_k > \underline{s_k}$, thus claim (i) holds. Claim (ii) holds, since for any Seller $k \in \Xi \setminus M^*$, the necessary and sufficient condition for k to participate in the spot market is $p^* < c_k > \beta_k + G(b_k)$. Claim (iii) holds since for any Seller $k \in \Xi \setminus M^*$, if $p^* < c_k$ and $\underline{s_k} \leq \beta_k$, thus $p^* < c_k = \underline{s_k} + G(b_k) \leq \beta_k + G(b_k)$, then k can make money neither in the contract market nor in the spot market; k is better off by closing its business. □

A further interesting implication of the above Corollary is obtained if we assume the standard efficient technology conditions hold, i.e., $b_1 < b_2 < \ldots < b_I$ and $\beta_1 > \beta_2 > \ldots > \beta_I$ [5]. Under this assumption, if $k, k+1 \in \Xi \setminus M^*$ and if $\underline{s_{k+1}} > \beta_k$, then $\underline{s_{k+1}} > \beta_{k+1}$ implies $\underline{s_k} > \beta_k$. This means that if k is "out of the money" w.r.t. the contract market, but still competes in the spot market, then every other Seller not in the money with lower capacity costs than k will also only participate in the spot market.

Corollary 5 implies that the index line of $c_1 < c_2 < \ldots < c_I$ can be used to identify the unique group of Sellers who participate in the contract and spot market, a further disjoint and unique group of Sellers who only participate in the spot market, and, lastly, a remaining group of Sellers who will be "out of business" in the long-run.

Unsurprisingly, the nature of the spot market (volatility and price level) as well as both variable and capital costs and the access parameter (m) are factors affecting which technologies survive in the long run. The above results provide the key insights on how these cost and market factors interact strategically to determine which markets will exist in the long run and which Sellers will be able to survive in each respective market. Our results capture the interaction of competing technologies with alternative market structures, which accommodate both the extent of competition (in terms of the number of suppliers) as well as the relative cost and access advantages of alternative suppliers. Other results in the literature either ignore supplier heterogeneity (e.g., Allaz and Vila 1993) or competition

[5]If these conditions do not hold, then some technology has both higher capacity and higher operating costs than some other technology and would be dominated in the long-run; see, e.g., Crew and Kleindorfer (1976)

among suppliers (e.g. Gardner and Rogers 1999).

6 Some Extensions and Future Research

6.1 Efficiency of the Two-Part Options Contract

It is well known (e.g., Allaz and Vila 1993) that forward markets are generally inefficient under Cournot competition among suppliers (though these results typically ignore capacity constraints), unless there are many suppliers and many trading/contracting periods prior to the spot market. Our results provide a richer framework for analyzing the efficiency of forward contracts for the case of Bertrand-Nash competition with capacity constraints in the short run (arguably the natural form of competition for electronic markets). The reader should note that our two-part options $[s^*, g]$ are equivalent to forward contracts when $g = 0$; if $g = 0$, then clearly the Buyers will always exercise the contracts on the day (since $U'(z) > 0, \forall z$), and Sellers will therefore be forced to deliver the full amount of any option committed with $g = 0$. Such a contract is therefore a "must-produce, must-take" contract, i.e. a forward. However, as shown in Wu et al. (2001c), this contract is strictly dominated by an appropriately designed options contract from the Seller's perspective without changing the Buyer's utility. Thus, any such forward contract is Pareto dominated by some options contract when both contract and spot markets are active. Naturally, if custom features of a product make spot markets infeasible, then forward contracts can still be efficient, especially if they allow better, cheaper planning of production through advance reservation[6]. In a similar fashion, one should note the more intuitively obvious fact that contracts that precommit capacity without a reservation fee, of the form $[s, g] = [0, g]$ are also Pareto dominated.

A related interesting question would be whether generalizations to allow for state-

[6]This problem is analyzed in detail in Levi, Kleindorfer and Wu (2001), and includes an extension of the WKZ framework to allow different production costs for the same Seller producing for the contract market and the spot market.

dependent options contracts would perform better than the simpler contracts studied here. Such a contract would take the form $[s, g(\omega), Q(\omega)]$ where g and Q both may depends explicitly on the state of the world ω. However, such contracts are easily shown to be dominated by the two-part options contract studied here. But such contracts might be of interest if either Sellers' costs b depend on ω or if Buyers' demands depend on ω, for example, if the strength of Buyer demands depends on the "weather". However, the characterization of the optimal Buyer's choices are considerably more complicated when costs or demands are state-dependent, as worked out in detail in Spinler, Huchzermeier and Kleindorfer (2000) generalizing the single-Seller results of Wu, Kleindorfer and Zhang (2001b) to the state-dependent case.

6.2 Future research

The above characterizes long-run equilibrium in the usual "putty-putty" world of completely flexible capacity investments, or investments at least which could be evaluated and changed to any level before the fact. In many markets, capacity is a "putty-clay" investment, i.e., irreversible (see Dixit and Pindyck, 1994). In such markets, it would be interesting to characterize the long-run equilibria that would result if the only capacity choice options were complete withdrawal from the market or expansion of capacity. Similarly, following the logic of the single-supplier case in Wu et al. (2001a), it would be interesting to extend these results to a multi-supplier, continuous time options framework.

Another interesting matter to study is the dynamics of commitment and profitability in those cases in which contract market equilibria do not exist because of the cycling phenomena illustrated in the above numerical example 2. For some initial results on this matter, see Wu and Sun (2001).

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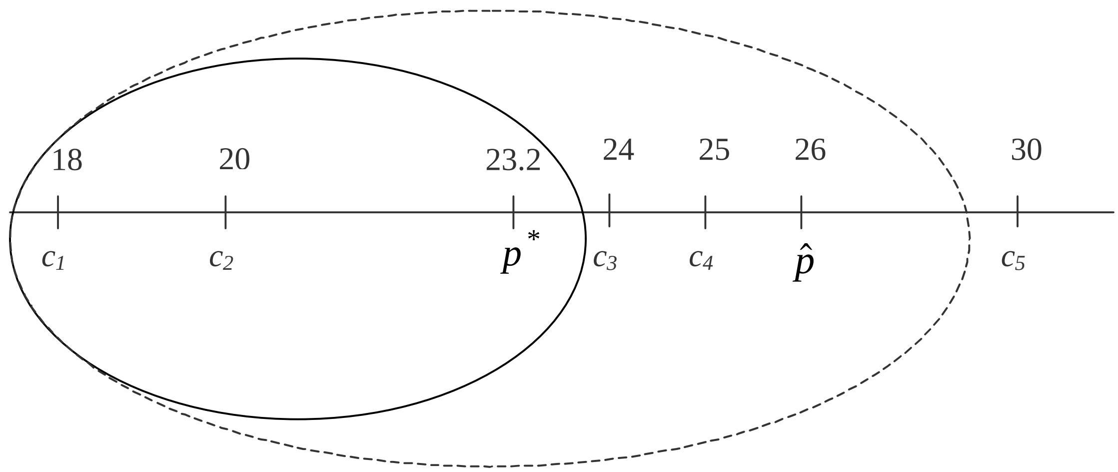

Figure 1: Illustration of Numerical Example 1. The dotted line indicates the short-term equilibrium set $\hat{M}$ = {1, 2, 3, 4} with the short-term equilibrium price $\hat{p}$ = 26, and the solid line indicates the long-term equilibrium set M^* = {1, 2} with the long-term equilibrium price of p^* = 23.2 .