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Aggregation of fuzzy opinions under group decision making

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Abstract

In this paper, a method is proposed for aggregating individual fuzzy opinions into a group fuzzy consensus opinion. This paper presents a procedure for aggregating the expert opinions. First, we define the index of consensus of each expert to the other experts using a similarity measure. Then, we aggregate the experts using the index of consensus and the importance of each expert. Finally, a numerical example is given to apply our model.

Keywords: Fuzzy individual opinions; Group consensus opinion; Agreement matrix; Fuzzy numbers

1. Introduction

In the multi-criteria decision making (MCDM) with group decision problems generally there arise situations of conflict and agreement among the experts as each expert has his own opinion or estimated rating under each criterion for each alternative. Hence, finding a group consensus function of aggregating these estimated ratings to represent a common opinion is an important issue. The purpose of this paper is to establish a procedure to combine the individual opinions to form a group consensus opinion. Since the subjectivity, imprecision and vagueness in the estimates of a given quantity enter into multi-criteria decision making problems, fuzzy set theory (FST) is helpful in dealing with the fuzziness of human judgement quantitatively.

Several aggregation methods based on fuzzy set theory have been proposed to combine the individual opinions on group decision making [1, 3, 5–7, 9, 11, 12, 14]. These authors [3, 6, 7, 9, 11, 12] propose assigning a fuzzy preference relation by each expert. Then, they derive a group fuzzy preference relation from individual fuzzy preference relations in order to determine the best alternative. Ishikawa et al. [5] and Xu and Zhai [14] proposed that each expert represents his subject judgement by an interval-value rating of each criterion for each alternative. Then, they constructed a cumulative frequency distribution to derive a group consensus judgement. Bardossy et al. [1] suggested that each expert's subjective estimate should be represented as a fuzzy number and combined in either an additive or a nonadditive manner.

In this paper, we propose a similarity aggregation method (SAM) to combine the individual subjective estimates which are represented by positive trapezoidal fuzzy numbers (PTFNs). First, we get

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the positive trapezoidal fuzzy numbers of each expert's estimate by the Delphi method and assumed that they have a common intersection at some α-level cut, $\alpha \in (0, 1]$. Then, we introduce a similarity measure function [2, 13, 15] to measure the degree of agreement between the opinions of the experts. According to the similarity measure function, we construct an agreement matrix which gives us insight into the agreement degree between expert opinions. We also consider the relative importance of the various experts. Based on the relative agreement degree and degree of importance, we develop a similarity aggregation method to combine the expert opinions.

This paper is divided into four sections. In Section 2, we introduce a similarity measure between the opinions of the experts and define an agreement matrix. And, we also propose a procedure to aggregate experts' fuzzy opinions into a fuzzy number. In Section 3, we discuss some properties of this aggregation method. In Section 4, we illustrate our procedure with an example.

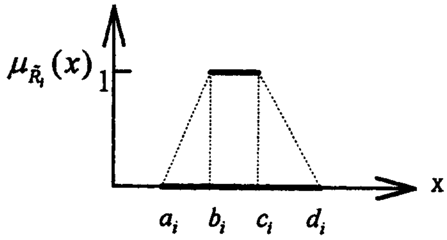

Fig. 1. Fuzzy estimates.

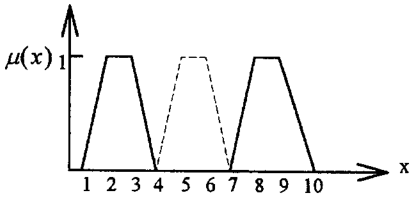

Fig. 2. No common intersection between expert A and B.

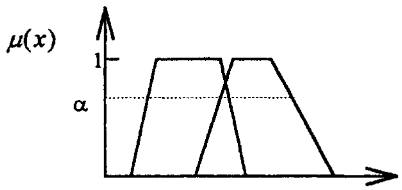

Fig. 3. Common intersection at a fixed α-level.

2. Aggregation procedure

According to the two intervals, the most likely interval $[b_i, c_i]$ and the largest interval $[a_i, d_i]$ where $a_i \leqslant b_i \leqslant c_i \leqslant d_i$ (see Fig. 1), each expert E_i $(i = 1, 2, \ldots, n)$ constructs a positive trapezoidal fuzzy number $\tilde{R}_i$ with membership functions $\mu_{\tilde{R}_i}(x)$ to represent the subjective estimate of the rating to a given criterion and alternative. How to construct an aggregation function to combine these estimated ratings $\tilde{R}_i$ $(i = 1, 2, \ldots, n)$ to represent the common opinion $\tilde{R}$, $\tilde{R} = f(\tilde{R}_1, \tilde{R}_2, \ldots, \tilde{R}_n)$, is an important issue.

In this paper, we assume that the estimates $\tilde{R}_i$ of each expert E_i $(i = 1, 2, \ldots, n)$ have a common intersection at some α-level cut, $\alpha \in (0, 1]$. The following example explains why we make this assumption. Suppose expert A and expert B construct their estimates as $\tilde{R}_A = (1, 2, 3, 4)$ and $\tilde{R}_B = (7, 8, 9, 10)$, respectively (see Fig. 2). In this case, their estimates have no common intersection. By Delphi method, the two experts insist that they do not adjust their estimates and if the aggregation result of two expert estimates falls under the interval [4, 7] then the aggregation result is not accepted by the two experts. In such a case, the aggregation result is unreasonable. When such a condition arises, the two experts must resume discussion or they must get new information and adjust their estimates. Therefore, we require that the expert estimates have a common intersection at some α-level cut. This is a necessary condition to obtain an aggregation result accepted by experts. In other words, the assumption is $\tilde{R}_i^{\alpha} \cap \tilde{R}_j^{\alpha} \neq \emptyset$, $\forall i, j \in \{1, 2, \ldots, n\}$. If the initial estimates of the kth expert and the lth expert have no intersection, then we use Delphi method [10] or get more information to adjust a_i, b_i, c_i, d_i by each expert in order to obtain a common intersection at the α-level cut (see Fig. 3).

In general, the relative importance of each decision maker or expert may not be equal. Sometimes there are important experts in decision group, such as the executive manager of a company, or some experts who are more experienced than others, the final decision is influenced by the different importance of each expert. Therefore, a good method of aggregating multi-expert opinions must consider the degree of importance of each expert in the aggregation procedure.

Referring to Fig. 4, $\tilde{R}_1$, $\tilde{R}_2$, $\tilde{R}_3$ are the estimates of experts E_1, E_2 and E_3, respectively. Area $a + b$ is the intersection of $\tilde{R}_1$ and $\tilde{R}_2$ which is greater than the intersection area b of $\tilde{R}_1$ and $\tilde{R}_3$. Then we can say, the agreement degree between expert E_1 and expert E_2 is higher than that between expert E_1 and expert E_3. Similarly, the agreement degree of expert E_2 has the highest agreement degree among others, we must put much emphasis on $\tilde{R}_2$. Based on the degree of importance and the agreement degree, we develop an aggregation method.

Suppose two experts have their estimates $\tilde{R}_i$ and $\tilde{R}_j$ (see Fig. 5), then the shaded area denotes the consistent area between expert i and expert j. The agreement degree $S(\tilde{R}_i, \tilde{R}_j)$ between expert E_i and expert E_j can be determined by the proportion of the consistent area (i.e. $\int_x \min\{\mu_{\tilde{R}_i}(x), \mu_{\tilde{R}_j}(x)\}\,dx$) to the total area (i.e. $\int_x \max\{\mu_{\tilde{R}_i}(x), \mu_{\tilde{R}_j}(x)\}\,dx$).

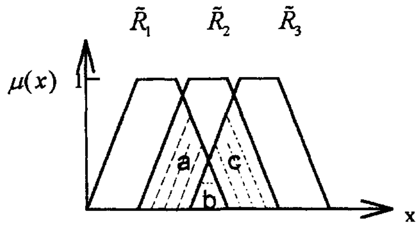

Fig. 4. The intersection of three experts' estimates.

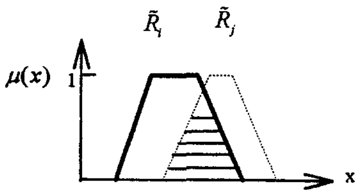

Fig. 5. The overlap of two expert opinions.

That is

$$S(\tilde{R}_i, \tilde{R}_j) = \frac{\int_x (\min\{\mu_{\tilde{R}_i}(x), \mu_{\tilde{R}_j}(x)\})\,dx}{\int_x (\max\{\mu_{\tilde{R}_i}(x), \mu_{\tilde{R}_j}(x)\})\,dx}, \tag{1}$$

where $S(\tilde{R}_i, \tilde{R}_j)$ is also called as similarity measure function by Zwick et al. [15].

If two experts have the same estimates, that is $\tilde{R}_i = \tilde{R}_j$, we get $S(\tilde{R}_i, \tilde{R}_j) = 1$. In other words, the two experts estimates are consistent, then the agreement degree between them is one. If two experts have completely different estimates, the agreement degree is zero. The higher the percentage of overlap, the higher the agreement degree.

After all the agreement degrees between the experts are measured, we can construct an agreement matrix (AM), which gives us insight into the agreement between the experts.

$$\mathrm{AM} = \begin{bmatrix} 1 & S_{12} & \cdots & S_{1j} & \cdots & S_{1n} \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ S_{i1} & S_{i2} & \cdots & S_{ij} & \cdots & S_{in} \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nj} & \cdots & 1 \end{bmatrix},$$

where $S_{ij} = S(\tilde{R}_i, \tilde{R}_j)$, if $i \neq j$ and $S_{ij} = 1$, if $i = j$.

By the definition of $S(\tilde{R}_i, \tilde{R}_j)$, the diagonal elements of AM are unity.

The average agreement degree of expert E_i $(i = 1, 2, \ldots, n)$ is given by

$$A(E_i) = \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} S_{ij}. \tag{2}$$

Then we compute the relative agreement degree of expert E_i $(i = 1, 2, \ldots, n)$ using Eq. (3),

$$\mathrm{RAD}_i = \frac{A(E_i)}{\sum_{i=1}^{n} A(E_i)}. \tag{3}$$

In some cases, the relative importance of experts is widely different. Some are more important than the others, such as the president of a nation, the executive manager of a company and some experts are more experienced than others. Therefore, we consider the relative importance weight of each expert. First, we select the most important person among experts and assign him weight one, i.e.

$r_i = 1$. Then we compare the jth expert with the most important person and get a relative weight for the jth expert $r_j, j = 1, 2, \ldots, n$. So we have $\max\{r_1, r_2, \ldots, r_n\} = 1$ and $\min\{r_1, r_2, \ldots, r_n\} > 0$. Finally, we define the degree of importance w_i as follows:

$$w_i = \frac{r_i}{\sum_{i=1}^{n} r_i}, \quad i = 1, 2, \ldots, n. \tag{4}$$

If the importance of each expert is equal then $w_1 = w_2 = \cdots = w_n = 1/n$.

As stated above, we get the relative agreement degree and the degree of importance of each expert. Now we can define the consensus degree coefficient of expert E_i $(i = 1, 2, \ldots, n)$ as

$$\mathrm{CDC}_i = \beta \cdot w_i + (1 - \beta) \cdot \mathrm{RAD}_i, \tag{5}$$

where $0 \leqslant \beta \leqslant 1$.

Let $\tilde{R}$ be an "overall" fuzzy number of combining experts' opinions. By the definition of the consensus degree coefficient of expert E_i $(i = 1, 2, \ldots, n)$, the aggregation result $\tilde{R}$ can be defined as

$$\tilde{R} = \sum_{i=1}^{n} (\mathrm{CDC}_i (\cdot) \tilde{R}_i), \tag{6}$$

where $(\cdot)$ is the fuzzy multiplication operator [8].

The consensus degree coefficient (CDC_i) of each expert is a good measure for evaluating the relative worthiness of each expert's estimates. Now, we have proposed an aggregation procedure, called as similarity aggregation method (SAM), to combine the fuzzy opinion of each expert into a fuzzy number to represent the common opinion of these experts. This procedure will be summarized by the following steps.

Step 1: Each expert E_i $(i = 1, 2, \ldots, n)$ constructs a positive trapezoidal fuzzy number $\tilde{R}_i$, according to the most likely interval $[b_i, c_i]$ and the largest interval $[a_i, d_i]$, where $a_i \leqslant b_i \leqslant c_i \leqslant d_i$ (see Fig. 1), to represent the subjective estimate of the rating to a given criterion and alternative. If the initial estimates of some experts have no intersection, then we use the Delphi method [10] to adjust the values a_i, b_i, c_i, d_i by each expert and to get the common intersection at a fixed α-level cut.

Step 2: Calculate the agreement degree $S(\tilde{R}_i, \tilde{R}_j)$ of the opinions between each pair of experts.

Step 3: Construct the agreement matrix (AM).

Step 4: Calculate the average agreement degree $A(E_i)$ of expert E_i $(i = 1, 2, \ldots, n)$.

Step 5: Calculate the relative agreement degree RAD_i of expert E_i $(i = 1, 2, \ldots, n)$.

Step 6: Define the degree of importance w_i of expert E_i $(i = 1, 2, \ldots, n)$.

Step 7: Calculate the consensus degree coefficient CDC_i of expert E_i $(i = 1, 2, \ldots, n)$.

Step 8: Aggregate the fuzzy opinions by the consensus degree coefficient CDC_i of expert E_i $(i = 1, 2, \ldots, n)$. The resultant is $\tilde{R} = \sum_{i=1}^{n} (\mathrm{CDC}_i (\cdot) \tilde{R}_i)$.

3. Properties of similarity aggregation method

The similarity aggregation method (SAM) preserves some important properties. These properties are as follows:

Property 1. Agreement preservation [1]. *If $\tilde{R}_i = \tilde{R}_j$ for all i, j, then $\tilde{R} = \tilde{R}_i$. In other words, if all estimates are identical the combined result should be the common estimate.*

Proof.

$$\begin{aligned} \because \tilde{R} &= \sum_{i=1}^{n} (\mathrm{CDC}_i (\cdot) \tilde{R}_i) = \tilde{R}_i (\cdot) \sum_{i=1}^{n} \mathrm{CDC}_i \\ &= \tilde{R}_i (\cdot) \sum_{i=1}^{n} [\beta \cdot w_i + (1 - \beta) \cdot \mathrm{RAD}_i] \\ &= \tilde{R}_i (\cdot) \left[\beta \cdot \sum_i w_i + (1 - \beta) \cdot \sum_i \mathrm{RAD}_i \right] \\ &= \tilde{R}_i (\cdot) [\beta + (1 - \beta)] = \tilde{R}_i. \end{aligned}$$

Agreement preservation is a consistency requirement.

Property 2. Order independence [1]. *Obviously, the result of the similarity aggregation method would not depend on order with which individual opinions or estimates are combined. That is, if $\{(1), (2), \ldots, (n)\}$ is a permutation of $\{1, 2, \ldots, n\}$ then $\tilde{R} = f(\tilde{R}_1, \tilde{R}_2, \ldots, \tilde{R}_n) = f(\tilde{R}_{(1)}, \tilde{R}_{(2)}, \ldots, \tilde{R}_{(n)})$. The result is also a consistency requirement.*

Property 3. *Let the uncertainty measure $H(\tilde{R}_i)$ of individual estimate $\tilde{R}_i$ be defined as the area under its membership function* [1].

$$H(\tilde{R}_i) = \int_{-\infty}^{\infty} \mu_{\tilde{R}_i}(x)\,\mathrm{d}x. \tag{7}$$

The uncertainty measure H defined in Eq. (7) fulfills the following equation:

$$H(\tilde{R}) = \sum_{i=1}^{n} \mathrm{CDC}_i \times H(\tilde{R}_i). \tag{8}$$

This means that the uncertainty after combination is a 'mean' of the uncertainties of each expert. Therefore, the uncertainty of the aggregation result by similarity aggregation method can be computed between the uncertainties of all experts, i.e. $\min_i H(\tilde{R}_i) \leqslant H(\tilde{R}) \leqslant \max_i H(\tilde{R}_i)$. This is a reasonable result for combining the opinions of all experts.

Property 4. *If an expert's estimate is far from the others, then his estimate is less important.*

Referring to Fig. 6, $\tilde{R}_1, \tilde{R}_2, \tilde{R}_3$ are the estimates of experts E_1, E_2 and E_3, respectively. Obviously, $\tilde{R}_3$ is the extreme estimate of the three, so the estimate of expert E_3 is less important than the estimates of experts E_1 and E_2. That is, the aggregation result is less influenced by expert E_3.

Property 5. *Because the experts have common intersection at a fixed α-level, we have $\mu_{\tilde{R}}(x) > 0$ for all x which implies that there exists at least one i for which $\mu_{\tilde{R}_i}(x) > 0$.*

This means that if a value was considered to be possible for the combination, then it should be possible for at least one estimate. This property will

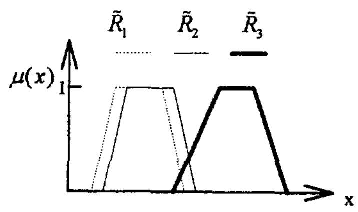

Fig. 6. The extreme estimate of the three.

increase the confidence degree of the aggregation result for each expert in the decision group. This is an important and reasonable property for the similarity aggregation method.

Property 6. *The common intersection area of all experts' estimates is included in the aggregation result. It means that $\bigcap_{i=1}^{n} \tilde{R}_i \subseteq \tilde{R}$.*

Proof. Suppose $\tilde{R}_i$, $i = 1, 2, \ldots, n$ have a common intersection at λ-level cut, that is $\bigcap_{i=1}^{n} \tilde{R}_i^{\lambda} \neq \emptyset$, where $\tilde{R}_i^{\lambda} = [a_i^{\lambda}, b_i^{\lambda}]$. Let $\bigcap_{i=1}^{n} \tilde{R}_i^{\alpha} = [a^{\alpha}, b^{\alpha}]$ for $\alpha \in (0, \lambda]$, then we have $a^{\alpha} = \max_i a_i^{\alpha}$ and $b^{a} = \min_i b_i^{\alpha}$.

By the definition of $\tilde{R}$,

$$\tilde{R} = \sum_{i=1}^{n} (\mathrm{CDC}_i \,(\cdot)\, \tilde{R}_i),$$

that leads to

$$\tilde{R}^{\alpha} = \sum_{i=1}^{n} \mathrm{CDC}_i \,(\cdot)\, \tilde{R}_i^{\alpha} = [a_*^{\alpha}, b_*^{\alpha}],$$

where $a_*^{\alpha} = \sum_{i=1}^{n} \mathrm{CDC}_i \cdot a_i^{\alpha}$ and $b_*^{\alpha} = \sum_{i=1}^{n} \mathrm{CDC}_i \cdot b_i^{\alpha}$.

Then we have $\min_i a_i^{\alpha} \leqslant a_*^{\alpha} \leqslant \max_i a_i^{\alpha}$ and $\min_i b_i^{\alpha} \leqslant b_*^{\alpha} \leqslant \max_i b_i^{\alpha}$, i.e., $[a^{\alpha}, b^{\alpha}] \subseteq [a_*^{\alpha}, b_*^{\alpha}]$, $\forall \alpha \in (0, \lambda)$. We prove that $\bigcap_{i=1}^{n} \tilde{R}_i \subseteq \tilde{R}$.

Property 7. *If the fuzzy opinions of all experts can be represented by a positive trapezoidal fuzzy number, then the membership function of the combination is also a positive trapezoidal fuzzy number. This property will reduce the complexity of mathematical analysis process in group decision making.*

4. Numerical example

Let us consider a problem with a given alternative and criterion using the opinions of three experts. The data for the opinions are given as positive trapezoidal fuzzy numbers as follows:

$\tilde{R}_1 = (1, 2, 3, 4)$,

$\tilde{R}_2 = (1.5, 2.5, 3.5, 5)$,

$\tilde{R}_3 = (2, 2.5, 4, 6)$.

Then we consider two cases:

1. Do not consider the degree of importance of each expert; i.e. $\beta = 0$.

2. Considering the degree of importance of each expert; i.e. $0 < \beta < 1$.

The result of $\tilde{R}$ is calculated in full detail in both cases and is also represented in graphical form.

Case 1: Do not consider the importance degree of each expert.

The agreement degrees between each expert are determined, using Eq. (1)

$$S(\tilde{R}_1, \tilde{R}_2) = S(\tilde{R}_2, \tilde{R}_1) = 0.55,$$

$$S(\tilde{R}_1, \tilde{R}_3) = S(\tilde{R}_3, \tilde{R}_1) = 0.36,$$

$$S(\tilde{R}_2, \tilde{R}_3) = S(\tilde{R}_3, \tilde{R}_2) = 0.67.$$

Then the agreement matrix is represented by

$$\mathrm{AM} = \begin{bmatrix} 1 & 0.55 & 0.36 \\ 0.55 & 1 & 0.67 \\ 0.36 & 0.67 & 1 \end{bmatrix}.$$

The average agreement degrees of the experts E_1, E_2 and E_3 are, respectively,

$$A(E_1) = 0.455, \quad A(E_2) = 0.61, \quad A(E_3) = 0.515.$$

Thus the relative agreement degrees of the experts E_1, E_2 and E_3 are given by

$$\mathrm{RAD}_1 = 0.455/(0.455 + 0.61 + 0.515) = 0.288,$$

$$\mathrm{RAD}_2 = 0.61/(0.455 + 0.61 + 0.515) = 0.386,$$

$$\mathrm{RAD}_3 = 0.515/(0.455 + 0.61 + 0.515) = 0.326.$$

Because we do not consider the degree of importance of each expert in this case ($\beta = 0$), the consensus degree coefficients of the experts E_1, E_2 and E_3 are

$$\mathrm{CDC}_1 = \mathrm{RAD}_1 = 0.288,$$

$$\mathrm{CDC}_2 = \mathrm{RAD}_2 = 0.386,$$

$$\mathrm{CDC}_3 = \mathrm{RAD}_3 = 0.326.$$

The "overall" fuzzy number of combining experts' opinions is (see Fig. 7)

$$\tilde{R} = 0.288(\cdot)\tilde{R}_1 + 0.386(\cdot)\tilde{R}_2 + 0.326(\cdot)\tilde{R}_3$$
$$= (1.519, 2.356, 3.519, 5.038).$$

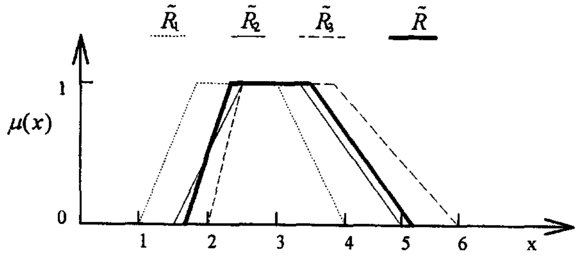

Fig. 7. Result of the case 1.

Case 2: Considering the degree of importance of experts

Suppose expert E_1 is the most important expert; i.e. $r_1 = 1$, and the relative weights of expert E_2 and E_3 to E_1 are $r_2 = 0.6$ and $r_3 = 0.8$, respectively. Then, we can define the degree of importance of three experts are $w_1 = 0.42$, $w_2 = 0.25$ and $w_3 = 0.33$, respectively. If the degree of importance is more important than the relative agreement degree, we can set $\beta = 0.4$. Therefore, the consensus degree coefficients of the experts E_1, E_2 and E_3 can be computed as

$$\mathrm{CDC}_1 = (0.4 \times 0.42 + 0.6 \times 0.288) = 0.34,$$

$$\mathrm{CDC}_2 = (0.4 \times 0.25 + 0.6 \times 0.386) = 0.33,$$

$$\mathrm{CDC}_3 = (0.4 \times 0.33 + 0.6 \times 0.326) = 0.33.$$

The "overall" fuzzy number of combining experts' opinions is (see Fig. 8)

$$\tilde{R} = 0.34(\cdot)\tilde{R}_1 + 0.33(\cdot)\tilde{R}_2 + 0.33(\cdot)\tilde{R}_3$$
$$= (1.495, 2.33, 3.495, 4.99).$$

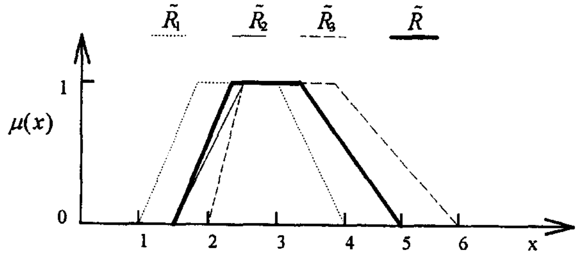

Fig. 8. Result of the case. 2.

Five aggregation techniques were proposed by Bardossy et al. [1] ; namely, crisp weighting, fuzzy weighting, minimal fuzzy extension, convex fuzzy extension, and mixed linear extension method. The fuzzy weighting combination is not agreement preserving because of the fuzzy number arithmetic operations involved, namely, multiplication and addition. Although the combination results of the minimal fuzzy extension and convex fuzzy extension methods can include the common intersection area of all experts' estimates, the width of the resultant fuzzy number $\tilde{R}$ are larger than the resultant of our method. For above example, with no preference assumption, the minimal fuzzy extension and convex fuzzy extension lead here to the same aggregation result $\tilde{R}' = (1, 2, 4, 6)$. The uncertainty of the aggregation result $(H(\tilde{R}') = 3.5)$ is larger than the uncertainty of the aggregation result for SAM in case 1 $(H(\tilde{R}) = 2.341)$ and case 2 $(H(\tilde{R}) = 2.33)$. The mixed linear extension is a combination technique of crisp weighting and the minimal fuzzy extension (or convex fuzzy extension). However, the preference of each estimate is difficult to determine and the computation is sophisticated. Besides, these five aggregation techniques did not provide a systematic process to determine the weight or the preference of each estimate. SAM provides a systematic and objective way to aggregate the individual fuzzy opinions in group decision making.

5. Conclusions

In this paper we have proposed a similarity aggregation method to aggregate fuzzy individual opinions into a fuzzy group consensus opinion, according to their consensus degree coefficient, in MCDM with group decision problems. We consider the difference of importance of each expert as a crisp value in our method. The degree of importance of each expert also can be represented by a linguistic variable; i.e. a fuzzy number. However, if the importance degree of each expert is a fuzzy number, then the aggregation method will not satisfy the consistency requirement [1].

Through the use of fuzzy set theory and positive trapezoidal fuzzy numbers, the similarity aggregation method provides a systematic and objective way to aggregate the individual fuzzy opinions in group decision making. In addition, by means of this aggregation procedure, we get the consensus information and construct the fuzzy judgement matrix for multi-criteria decision making with group decision problems reasonably [4, 14].

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