Decentralized Cognitive Radio Control based on Inference from Primary Link Control Information

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Abstract

This work on cognitive radio access ventures beyond the more traditional "listen-before-talk" paradigm that underlies many cognitive radio access proposals. We exploit the bi-directional interaction of most primary communication links. By intelligently controlling their access parameters based on the inference from observed link control signals of primary user (PU) communications, cognitive secondary users (SUs) can achieve higher spectrum efficiency while limiting their interference to the PU network. In one specific implementation, we let the SUs listen to the PU's feedback channel to assess their own interference on the primary receiver, and adjust radio power accordingly to satisfy the PU's interference constraint. We propose a discounted distributed power control algorithm to achieve non-intrusive secondary spectrum access without either a centralized controller or active PU cooperation. We present an analytical study of its convergence property. We show that the link control feedback information inherent in many two-way primary systems can be used as important reference signal among multiple SU pairs to distributively achieve a joint performance assurance for primary receiver's quality of service.

Index Terms

Wireless communications, inference for opportunistic spectrum access, dynamic spectrum access control, distributed algorithm, cognitive radio networks.

I. Introduction

Because of its potential to alleviate spectrum scarcity, the overlay of cognitive radio networks over the spectrum of high priority primary user (PU) networks has recently attracted a high level of research interest. Most existing works on cognitive overlay centers on the listen-before-talk (LBT) concept which relies on (cooperative) spectrum sensing of primary user activities (e.g., [1], [2], [3]). LBT requires secondary users (SUs) to detect the absence of primary user signals before channel access. The finding by the FCC that most LBT-based devices do not degrade TV reception quality [4] provided a major boost to the cognitive radio concept. Though conceptually simple, LBT focuses on the sensing of primary transmission signals instead of on determining the potential effect of secondary user access on primary receiver's performance.

Because of the focus on primary transmitters rather than their receivers, LBT-based cognitive protocols need to be more conservative in limiting SU transmission for the protection of primary users (PUs) from SU interference. First, it needs to assume the least favorable fading environment (i.e., strong interference channel against weak primary channel). For instance, the threshold for the LBT devices was set at 30dB below the DTV reception threshold in the FCC TV white space testing [4]. Second, it has to anticipate the worst sum interference at the primary receiver (PU-Rx) from multiple potential SU devices. Third, it does not allow SU systems to exploit any extra capacity when a PU system, not fully loaded, can tolerate substantial interference (e.g., through forward error correction, beamforming, or spectrum spreading). On the other hand, LBT may also be too aggressive within the well known hidden node environment.

To overcome the shortcomings of LBT, we propose and advocate a different framework that incorporates the inherent feedback information in typical **two-way** PU communication links. Such link control information is available in many practical systems in the form of, e.g., power control feedback in CDMA cellular [5], channel quality indicator feedback in HSDPA [5], ACK/NAK feedback in cellular or WiFi networks [5], [6]. Such feedback information from the PU-Rx can provide a good indicator of the actual (often aggregated) impact of the SU interferences on the reception quality of the PU communication link.

Figure 1 provides a simple illustration of SUs being able to overhear the feedback from the PU-Rx to the PU transmitter (PU-Tx). This feedback information enables an SU to monitor the performance of PU-Rx (affected by one or more SUs), and adjust accordingly its own access parameters based on inference. The benefits of inference based on such link level feedback information are multi-folds: (i) It enables explicit protection of the PU-Rx through feedback monitoring, especially in the presence of multiple SU pairs; (ii) It facilitates *distributed access control* of multiple SUs based on the PU-Rx response to the sum SU interference; (iii) It permits different levels of interaction between PUs and SUs; (iv) It leads to more efficient spectrum usage through learning; (v) It is robust and adaptive to changes due to network load fluctuation and radio environment dynamics.

The proposed new framework requires that secondary radios be "cognitive" enough to receive and decode link control information from primary networks requiring strict interference constraint. This new framework is particularly suitable for cases where both primary and secondary networks belong to the same operator or interest group. In particular, given the ability to decode PU-Rx feedback information, secondary networks of lower priority opportunistically access spectrum nominally but not fully occupied by (legacy) PUs of higher QoS/access priorities. The DARPA XG project is one such example where secondary cognitive radios access underutilized spectrum of legacy users. Another example involves cognitive femto cells to improve indoor cellular coverage. Such applications offer clear incentives for PUs to conditionally allow cognitive SU access and to permit more flexible and effective spectrum sharing, often without fundamental regulatory changes.

We focus on decentralized control of SUs in order to accommodate broader applications for which centralized control may be costly or infeasible. In other cases, the cost to retrofit existing infrastructure may be expensive, time consuming, or disruptive. These cases call for distributed intelligent SU access protocols. As a concrete step toward this goal, we study a primary system with an outage probability QoS requirement. The PU pairs exchange 1-bit outage feedback information that can be overheard and exploited by (multiple) SUs. The objective of the SUs is to maximize its utility while satisfying the outage probability constraint set by the PU operator. The key challenge is to achieve this goal in a distributed manner at the multiple SUs without PU cooperation and with minimum SU coordination.

Our contributions are as follows: 1) we present a novel framework for cognitive spectrum access under PU link quality constraint based on PU-Rx feedbacks; 2) we formulate the cognitive spectrum utilization problem as a convex optimization problem through practical approximation; 3) we propose a discounted distributed power control (DDPC) algorithm and analyze its performance for individual SUs without explicit central control; 4) we show the convergence property of the proposed DDPC algorithm for networks of synchronous and asynchronous SUs.

II. Related Work

Distributed power control for cellular systems has been studied in the literature. In [7], the authors studied the convergence of a simple distributed power control algorithm to a feasible solution that satisfies the target signal-to-interference ratio (SIR) requirement for each user. The authors of [8] proposed a framework for the joint optimization on cell selection and power control of cellular uplinks. For wireless multihop networks, the authors of [9] proposed setting both power price and external interference price/compensation to adjust transmit power in a distributed way. The work of [10] involves a joint optimization problem of adjusting the flow rate and the transmit power. In these works, either the Lagrange multiplier on the resource constraints or the external interference price must be exchanged among different participating nodes [9], [10]. While the aforementioned works assume static wireless channels, power control with outage probability requirement has been considered for Rayleigh fading channels in [11].

There are also power control algorithms for cognitive radios to achieve efficient and fair usage on the shared spectrum resources without explicit protection constraint on the PU QoS (e.g., [12]). In our problem setting, the PUs have strict QoS requirement and do not participate in the power control algorithm.

There are also studies on overlaying cognitive radio networks that propose the SUs participate the transmission of the PU traffic in forms of dirty paper coding (e.g., [13], [14], [15]), distributed space-time coding (e.g, [16]), cognitive relay (e.g., [17]) among others. Such schemes require that the SU decode the signal transmitted from the PU-Tx and perform precoding to guarantee the transmission rate of the PU in the exchange of spectrum opportunities. They also require the awareness of the PU-Rx on the existence of SU transmission. Here, we assume that the SUs transmit simultaneously with the PU and do not help PU transmission. The PU treats the received signal from all SUs as interference. This simplifies the transceiver design at the SU-Tx since it does not require the knowledge on the codebook used by the PU as well as complicated precoder implementation. Here we focus on the design of distributed power control algorithm for multiple SUs under the protection requirement from the PU.

Closely related to the framework in this manuscript is the use of power control for mitigating the interference on PUs while maximizing the spectrum usage of SUs. For example, the authors of [18] quantified the relationship between SU transmit power and the probability of spectrum opportunity based on a Poisson model of the primary network traffic. They also studied the subtle interaction between detecting the PU transmit signal and locating the spectrum opportunity. In [19], the authors proposed adjusting SU transmit power based on the spectrum sensing results. In essence, these works belong to the LBT category. In [20], the authors proposed an auction-based power allocation framework for spread spectrum users to share spectrum with an interference temperature sensed at a measurement point, whereas a central manager needs to collect bids from distributed users. The authors of [21], formulated a power allocation game considering both the interference temperature constraint at the PUs and the QoS requirement at the SUs. Monitor stations are required to report the value of the dual variable at every iteration for the proposed algorithms.

The idea of applying PU feedback channel information has also been previously considered. For example, in our preliminary work [22], we presented results on utilizing the PU ACK/NAK information to maximize the utility of a single SU without considering the interaction of multiple SUs. In [23], power control message of the primary systems is used to improve the SU's spectrum sensing accuracy. The idea is to pro-actively send sounding signals and adjust its maximum transmit power based on the reaction from the PU-Rx. In [24], the authors proposed the use of the ACK/NAK information on the PU feedback channel by the SU to adjust its input rate in order to improve the spectrum efficiency while meeting the PU's target rate. However, both [23] and [24] consider only a single SU.

Our work differs from previous works on cognitive radio power control in at least one of the following aspects: 1) our system does not require a centralized controller or an interference monitor; instead, it depends explicitly on the inherent PU link feedback; 2) our SUs are not required to exchange coordination information among themselves, thereby significantly lowering the overhead cost and protocol complexity; 3) our access objective is to maximize the total SU spectrum utility instead of individual SU utility while jointly but distributively satisfying the outage requirement of the PU.

In our earlier work [25], we presented a framework for multiple SUs to perform distributed power control based on observation from PU feedbacks. In this paper, we expand the framework and provide convergence analysis on the discounted distributed power control (DDPC) algorithm to tackle the lack of parameter update synchronization typically among users in cognitive radio networks [1]. Other related works on utility optimization without message passing among multiple participating nodes can be found in [26], [27].

III. System Model and Basic Assumptions

A. Notations

We use bold fonts to denote vectors, the curly inequality symbol $\preceq$ to denote the component-wise inequality, and $\mathbf{1}$ to represent a vector with all of its elements 1 with appropriate length. Symbol $E\{\cdot\}$ denotes expectation operation, and $\Pr[\cdot]$ represents the probability of a random event. In this paper, $\log(\cdot)$ is a natural logarithm function.

B. Power Control of Cognitive Overlay

We consider the scenario of a cognitive radio network overlaying on top of a legacy PU network in which multiple SUs are allowed to share the spectrum designated for the high priority PU network in a non-intrusive manner. The non-intrusiveness requirement of the cognitive users has dual meanings. First, PU, given a higher spectrum access priority, is able to set the permissible level of interference or disruption from SU transmissions. This level of tolerance is controllable and can be used by the PU networks to set a price for SU access. Second, the legacy PUs do not actively cooperate with or help the SUs. In other word, such a cognitive network overlay requires no modification to PU's normal operation. Deployment of such SU networks is easier for legacy networks to accommodate as they are usually easier to set up, less disruptive, and less costly.

Let us consider a PU link comprising two network nodes that communicate via a forward link and a reverse link. The forward link carries primary traffic data from the PU-Tx to the PU-Rx, whereas the reverse link returns feedback control information from the PU-Rx to the PU-Tx. Denote the forward

[1] All results in Section V are new.

transmission power of the PU-Tx as P_0. For the cognitive network, we consider M secondary transmitters (SU-Tx's) geographically distributed around the PU nodes. This cognitive network overlays on the primary forward link spectrum and desires to access the forward link channel with minimum disruption to the primary forward link. Let $\mathbf{P} = [P_1, \cdots, P_M]$ denote the transmit powers of the M SUs, respectively. The SUs control their own access of the shared spectral band on the forward link through power control. For convenience here, we further assume that the primary channel utilization is time-slotted. The cognitive SUs have synchronized their spectrum access to the time-slot clock by, e.g., listening to the timing pilot on a broadcast control channel of the PU network (see, e.g., [28]).

C. Primary Outage Constraint

To determine the interference level at the PU-Rx due to SU transmissions, let G_i be the transceiver processing gain between the SU-Tx i and the PU-Rx. Let F_i further denote its corresponding small-scale fading channel gain due to multipath and mobility. Note that we reserved the special index $i = 0$ for the PU. We consider cases in which G_i remains almost unchanged whereas F_i may vary from slot to slot. We assume a non-line-of-sight (NLOS) radio transmission environment among all transmitters and the PU-Rx. For example, the PU-Rx may be a mobile device covered by a wireless hotspot in urban areas. In this case, we can adopt a Rayleigh fading channel model, in which F_i follows independent exponential distribution with unit mean. In other words, the average power gains of the fading channels all equal to 1. Thus, the received power at the PU-Rx from SU i and its average are, respectively,

$$P_i G_i F_i, \qquad \text{and} \qquad \mathbf{E}[P_i G_i F_i] = P_i G_i. \tag{1}$$

Let N_0 denote the white Gaussian noise power at the PU-Rx. The (random) signal-to-interference-noise-ratio (SINR) at the PU-Rx as a result of the fading channel environment is:

$$\gamma = \frac{P_0 G_0 F_0}{N_0 + \sum_{i=1}^{M} P_i G_i F_i}. \tag{2}$$

Even without the SUs, random channel fading renders zero outage impossible, in which case the exponential distribution of F_0 leads to the baseline PU-Rx outage probability of

$$\eta_0 = 1 - \exp(-\frac{N_0 \gamma_{th}}{P_0 G_0}). \tag{3}$$

where γ_{th} is the desired SINR value. Obviously, SUs' transmission when the PU is busy will result in an increase on the outage probability perceived by the PU-Rx. To maintain its quality of service (QoS), the PU would require its outage probability in the presence of SUs to stay below a certain threshold η ($\eta \geq \eta_0$) to control secondary access. This constraint can be expressed as:

$$\Pr[\gamma < \gamma_{th}] \leq \eta. \tag{4}$$

We assume that the PU protection requirement η was announced *a priori* to the SUs. However, our proposed DDPC algorithm does not need the SUs to know η_0, and it is able to adapt the change in η_0.

D. Tradeoff between Primary QoS and Secondary Utility

There clearly exists a trade-off between the signal quality at the PU-Rx characterized by its outage probability and the spectrum utilization of the SUs. A desired trade-off can be achieved by choosing an appropriate value for η to allow satisfactory QoS at the PU-Rx while still receiving maximum possible compensation from SUs that may be permitted to opportunistically access the shared bandwidth for communications.

For the SUs to protect PU QoS, we exploit the feedback information from the PU-Rx to the PU-Tx for its link quality control. Often, the PU-Rx sends 1-bit feedback to the PU-Tx, indicating whether or not an outage (i.e., $\gamma < \gamma_{th}$) has occurred during the last packet transmission. We assume that this information is strong enough to be overheard (with potential error) by all the participating SUs. Based on inference from such feedback information, SUs can then make learned decisions on their transmit powers in a distributed manner to satisfy the PU outage probability.

We assume that SUs are deployed without coordination. They may not be aware of one another and may have limited information on the overall overlay network. There is no central controller that has all the channel information to perform joint power control for SUs. Each SU only knows its own channel statistics. Nevertheless, interferences from multiple SU-Tx's to the PU-Rx would accumulate. This imposes a great challenge to the design of cognitive spectrum access since the joint PU protection guarantee has to be achieved without all the interference channel gain information. The challenge is more severe when both the number of SUs and the PU channel statistics are time-varying.

Note that for SUs that are within the interference range of each other, the optimal design of the multiple access scheme under the PU protection constraint is an open question. Indeed, even for Gaussian interference channels, the exact capacity region remains unknown [29]. It would be interesting to study the impact of contentions and/or cooperation among SUs on the primary system and vice versa in our future work. On the other hand, existing schemes such as TDMA, CSMA/CA or distributed power control with message passing among SUs (e.g., [10]) proposed in literature can be used in combination of our proposed DDPC algorithm (though suboptimal) to control the medium access among these interfering SUs. Auction based approaches (e.g., [20]) can also be used.

In the rest of the paper, we focus on tackling the challenge of maximizing the total SU utility subject to the constraint on the accumulated interruption to the PU-Rx via distributed power control. For simplicity, we ignore the mutual interference among SUs. The results obtained can be directly used in judicious deployments of SU networks in which the SUs sharing the same spectrum opportunities are located far away from each other. To quantify the spectral utility of SUs, we assume that the satisfaction of the i-th SU user pair can be characterized by $\log(1 + h_i P_i)$, where h_i is the effective channel gain reflecting the impacts such as modulation, interference level, and transmission distance of the SU transceiver pair. It is clear that this utility definition is related to the information rate that can be reliably conveyed on the i-th SU link. We also impose a physical limit to the transmission power, $P_i \leq P_{\max}, i = 1, \cdots, M$.

IV. Distributed Power Control Algorithms

The challenges to implementing distributed power control algorithm for cognitive radio networks are multi-folds: 1) PU is oblivious to SU activities and only reports its own outage; 2) SUs do not exchange channel information among themselves; 3) Dynamic PU/SU traffic activities require SUs to adapt their access algorithms.

A. Constrained Optimization Framework

For the SUs, the objective of performing power control is to maximize the total utility of all SU pairs while satisfying the PU outage requirement. Define $f(\mathbf{P}) = \Pr[\gamma < \gamma_{th}]$, the formal description of the optimization problem is

$$\begin{aligned} \underset{\mathbf{P}}{\text{maximize}} \quad & \sum_{i=1}^{M} \log(1 + h_i P_i) \\ \text{subject to} \quad & f(\mathbf{P}) \leq \eta, \\ & \mathbf{P} \preceq P_{max} \mathbf{1}. \end{aligned} \tag{5}$$

Define b_i as the unit interference effect from SU-Tx i to the PU-Rx, i.e.,

$$b_i = \frac{G_i \gamma_{th}}{P_0 G_0}. \tag{6}$$

For Rayleigh fading channel, the outage probability at the PU-Rx for a given SU transmit power vector $\mathbf{P}$ is [11]:

$$f(\mathbf{P}) = 1 - \exp(-\frac{N_0 \gamma_{th}}{P_0 G_0}) \prod_{i=1}^{M} (1 + b_i P_i)^{-1}. \tag{7}$$

To simplify notations, we define: $\mu = (1 - \eta_0)/(1 - \eta)$, which can be interpreted as the relative outage margin to accommodate SU transmissions. Clearly, we expect $\mu \geq 1$. Thus, we can modify the outage constraint on $\mathbf{P} = [P_1, \cdots, P_M]$ into

$$\prod_{i=1}^{M} (1 + b_i P_i) \leq \mu, \tag{8}$$

which is an upper bound on a posynomial function of $\mathbf{P}$.

B. Algorithm Development

Note that the feasible set defined by (8) is non-convex. As a result, the optimization problem in (5) remains nonlinear and non-convex. Adopting an approach similar to [10], we approximate the utility function of the i-th SU by $\log(h_i P_i)$. The approximation on the utility function is justifiable when the processing gain (e.g., using spreading spectrum, multiuser detection, or beamforming techniques) at each secondary transceiver pair is large and when there are not too many nearby secondary stations. In addition, such an approximation enables us to transform the original problem into a convex optimization problem (via variable transformation) for which we can effectively find the global optimum and derive corresponding distributed algorithms. Another way to transform problem (5) into a convex optimization problem is to keep the objective function as $\log(1 + h_i P_i)$, by modifying the PU outage probability constraint and imposing a constraint on the average interference power perceived at the PU-Rx using the so-called certainty-equivalent margin (CEM) relaxation [11]. A lower bound and upper bound on the average interference power $\sum_i^M P_i G_i$ is obtained in [25]. As a result, an upper bound and a lower bound on the optimal total SU utility function can be obtained and used as comparison in Section VI to show the effectiveness of the adopted approximation. The disadvantage of using CEM relaxation is that it requires the PU-Rx to feedback the measured interference power on the control channel. In comparison, we only need a 1-bit outage event feedback. We also note that such an approximation has several limitations since $h_i P_i \gg 1$ may not always hold, especially when there are many SUs in the neighborhood. In this case, the solution obtained with the approximation can serve as an initial searching point for the original non-convex utility maximization problem (5).

With the approximation, the objective function reduces to $\sum_i \log(h_i) + \log(P_i)$. Henceforth, without loss of optimality, we can ignore the constants $\{h_i\}$. Adopting the technique of geometric programming [30], we can perform the following variable transformation,

$$x_i = \log(P_i), i = 1, \cdots, M,$$

and transform the constraint (8) into $\log$-scale[2]. Denoting $\mathbf{x} = [x_1, \cdots, x_M]^T$, and $\bar{x} = \log P_{\max}$, the resulting optimization becomes

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & -\sum_{i=1}^{M} x_i \\ \text{subject to} \quad & \sum_i \log\left(1 + b_i e^{x_i}\right) \le \log \mu, \\ & \mathbf{x} \preceq \bar{x}\mathbf{1}. \end{aligned} \tag{9}$$

[2]Such a transformation leads to an equivalent solution due to the monotonic increasing property of $\log(\cdot)$ function.

The objective function in (9) is affine in $\mathbf{x}$, and the constraint is convex in $\mathbf{x}$ (the Hessian matrix of the first constraint is a diagonal matrix of positive elements and the second constraint is affine in $\mathbf{x}$). As a result, we now have (9) as a convex optimization problem that can be solved numerically with efficiency in a centralized manner and may be amenable to a distributed implementation.

We define the Lagrange function associated with the problem (9) as

$$L(\lambda, \mathbf{x}) = -\sum_{i=1}^{M} x_i + \lambda(\sum_{i=1}^{M} \log(1 + b_i e^{x_i}) - \log \mu), \tag{10}$$

where $\mathbf{x} \preceq \bar{x}\mathbf{1}$. The dual function can be obtained as

$$\begin{aligned} q(\lambda) &= \inf_{\mathbf{x} \preceq \bar{x}\mathbf{1}} L(\lambda, \mathbf{x}) \\ &= -\lambda \log \mu + \inf_{\mathbf{x} \preceq \bar{x}\mathbf{1}} \{\sum_{i=1}^{M} \lambda \log(1 + b_i e^{x_i}) - \sum_{i=1}^{M} x_i\} \\ &= -\lambda \log \mu + \sum_{i=1}^{M} \inf_{x_i \leq \bar{x}} \{\lambda \log(1 + b_i e^{x_i}) - x_i\}. \end{aligned} \tag{11}$$

Note that here we exploit the advantageous structure of the separable problems: the minimization involved in the calculation of the dual function is decomposed into M simpler minimizations. Each minimization requires only local channel information, i.e., b_i. We can then solve the minimization problem in (11) with regard to any given $\lambda \geq 0$ as:

$$x_i^*(\lambda) = \text{argmin}_{x_i \leq \bar{x}} \{\lambda \log(1 + b_i e^{x_i}) - x_i\} = \begin{cases} \min\{-\log((\lambda - 1)b_i), \bar{x}\}, & \text{if } \lambda > 1, \\ \bar{x}, & \text{if } 0 \leq \lambda \leq 1. \end{cases} \tag{12}$$

With $x_i^*(\lambda)$, we can solve the dual optimization problem which is expressed as:

$$\begin{aligned} &\text{maximize} \quad && q(\lambda) \\ &\text{subject to} \quad && \lambda \geq 0. \end{aligned} \tag{13}$$

Note that we can set the transmit power of each SU to (or close to) zero such that the constraints in the approximated optimization problem (9) can be satisfied strictly (assuming $\mu > 1$). By the Slater constraint qualification, the optimal duality gap is zero and there exists at least one (see Proposition 5.3.1 in [31]) geometrical multiplier λ^o. According to Proposition 5.1.5 in [31], the dual-primal pair $(\lambda^o, \mathbf{x}^o)$ is optimal if the following conditions are satisfied.

$$\mathbf{x}^o \preceq \bar{x}\mathbf{1}, \qquad \sum_{i=1}^{M} \log(1 + b_i e^{x_i^o}) - \log \mu \leq 0, (\text{Primal feasibility}), \tag{14}$$

$$\lambda^o \geq 0, (\text{Dual feasibility}), \tag{15}$$

$$\mathbf{x}^o \in \arg\min_{\mathbf{x} \preceq \bar{x}\mathbf{1}} L(\mathbf{x}, \lambda^o), (\text{Lagrangian optimality}), \tag{16}$$

$$\lambda^o[\sum_{i=1}^{M}\log(1+b_i e^{x_i^o})-\log\mu]=0, (\text{Complementary slackness}). \tag{17}$$

On one hand, if the effective channel gains b_is are such that $\sum_{i=1}^{M}\log(1+b_i e^{\bar{x}})<\log\mu$, the solution of the dual problem (13) is $\lambda^o=0$, and the corresponding transmit power vector is $\bar{x}\mathbf{1}$. It can be verified that $(0,\bar{x}\mathbf{1})$ satisfies the above optimality condition. In this case, the PU outage probability constraint is always loose. On the other hand, when $\sum_{i=1}^{M}\log(1+b_i e^{\bar{x}})\geq\log\mu$, we have by (12) that the outage probability constraint is tight. In other words, when $\sum_{i=1}^{M}\log(1+b_i e^{\bar{x}})\geq\log\mu$, we have the optimal pair of dual-primal variables $(\lambda^o,\mathbf{x}^o)$ satisfying the following relationship:

$$\begin{aligned}&\sum_{i=1}^{M}\log(1+b_i e^{x_i^o})=\log\mu\\&x_i^o=\begin{cases}\min\{-\log((\lambda^o-1)b_i),\bar{x}\}, & \text{if } \lambda^o>1,\\ \bar{x}, & \text{if } \lambda^o=0.\end{cases}\end{aligned} \tag{18}$$

Note that at least one SU transmits with power equals to $-\log((\lambda^o-1)b_i)$, and thus $\lambda^o\notin(0,1]$. Therefore, any method that finds the solution to the above equation set will give us the optimal power control for the SUs. As a special case, when the maximum power constraint at each SU is loose (i.e., $\bar{x}=\infty$), we can obtain a closed-form expression for the optimal dual and primal variable $(\lambda^o,\mathbf{x}^o)$ as follows (see also [25]):

$$\begin{aligned}&\lambda^o=(\mu^{1/M}-1)^{-1}+1,\\&x_i^o=\log[\frac{\mu^{1/M}-1}{b_i}], i=1,\cdots,M.\end{aligned} \tag{19}$$

This solution implies that the contributing interference from each SU to the PU-Rx, expressed as $b_iP_i^o=b_ie^{x_i^o}$, should be normalized. However, the solution expressed in (19) requires that each SU know the total number of SUs sharing the spectrum opportunities as well as the value of η_0 and thus requires a centralized controller.

To facilitate the distributed implementation of power control, we resort to iterative approaches to find the optimal solution. Recall that the PU-Rx transmits a 1-bit indicator to the PU-Tx to signify whether the SINR at the PU-Rx falls below the required SINR threshold in each time slot. Such information reflects the reception quality at the PU-Rx, and can be used to infer the aggregated interference from all SUs to the PU-Rx on the forward link. The key idea is for the SUs to update the dual variable λ iteratively based on the PU outage probability resulting from the SUs and adjust their transmit powers according to (12). There are many ways to control the update procedure for the dual variable λ while taking into account the noise in the observation of PU outage probability, such as stochastic subgradient method [32] and stochastic approximation method [33], [34]. Here we elect to use the stochastic approximation method for its flexibility and better convergence speed. We develop a distributed power control algorithm for multiple SUs next (also in [25]).

C. Distributed SU Access Control

Define $\tilde{g}(\lambda)$ as the excess of the PU outage probability constraint in log-scale when the transmit power of the SUs is given by $x_i^*(\lambda)$ as in (12). Specifically, let

$$\tilde{g}(\lambda) = \log f(\mathbf{x}^*(\lambda)) - \log(\eta), \tag{20}$$

where without causing confusion, we slightly abuse the notation of $f(\mathbf{x})$ by using it to represent $f(\mathbf{P})$. Suppose that $\sum_{i=1}^{M} \log(1 + b_i e^{\bar{x}}) > \log \mu$. In other words, if all SUs transmit with the maximum power, the outage probability constraint will then be violated. Under this assumption, it is easy to prove that, if $\eta_0 \le \eta < 1$, $\tilde{g}(\lambda) = 0$ is a necessary and sufficient condition for $\sum_{i=1}^{M} \log(1 + b_i e^{x_i(\lambda)}) = \log \mu$ to hold [3], and the optimality conditions are satisfied.

Ideally, if each SU is aware of the value of λ in each iteration and has the perfect knowledge of $f(\mathbf{x})$, then λ can be updated iteratively via

$$\lambda(k+1) = \lambda(k) + a(k)\tilde{g}(\lambda(k)), \tag{21}$$

where k is the iteration number, and $a(k)$ is the step-size for each iteration. The update will converge to the optimal solution given appropriate regulation on the step-size (by contraction mapping). However, such an ideal update is difficult to implement in a totally distributed way without information exchange among SUs. Let λ_i denote the local version of the dual variable λ at the i-th SU, and let $\Lambda = [\lambda_1, \cdots, \lambda_M]$. Suppose that each SU adjusts its transmit power by substituting λ of (12) by the local version of the Lagrangian multiplier λ_i. The PU outage probability now depends on the value of Λ since each SU adapts its transmit power based on λ_i according to:

$$x_i(\Lambda) = x_i^*(\lambda_i) = \begin{cases} \min\{-\log((\lambda_i - 1)b_i), \bar{x}\}, & \text{if } \lambda_i > 1, \\ \bar{x}, & 0 \le \lambda_i \le 1, \end{cases} \quad \text{for} \quad i = 1, \cdots, M. \tag{22}$$

The same applies to the log-scale residual excess of the PU outage probability constraint. Denote such dependence as $f(\Lambda)$ and $\tilde{g}(\Lambda)$, respectively. We then have

$$f(\Lambda) = 1 - (1 - \eta_0) \prod_{i=1}^{M} (1 + b_i e^{x_i(\Lambda)})^{-1}, \tag{23}$$

where $x_i(\Lambda)$ is given in (22); also

$$\tilde{g}(\Lambda) = \log(f(\Lambda)) - \log \eta. \tag{24}$$

[3]Here the proof is omitted for brevity.

Obviously $\Lambda = \lambda^o \mathbf{1}$ is a feasible solution to equation $\tilde{g}(\Lambda) = 0$; but there exist other values of Λ such that $\tilde{g}(\Lambda) = 0$ and λ_i may get trapped on the boundary of the feasible set, leading to suboptimal transmit powers. In particular, when $\{\lambda_i\}$ differ from user to user, $\{x_i\}$ (and thus transmit powers of the SU's) determined by (24) would no longer minimize the Lagrangian function of the primal optimization problem in (9). Indeed, we do not even have an appropriate definition of the Lagrangian function. Due to the possible discrepancy among $\{\lambda_i\}$, the conventional dual decomposition approach does not apply.

In order to mitigate this problem and achieve consistency among SUs, we resort to a discounted distributed power control (DDPC) algorithm as shown in Algorithm 1 in which we apply a discount factor β_i on the update of λ_i for each SU as in (27). *The key idea is to gradually diminish the impact of asynchronousness on the discrepancy among the local copies* $\{\lambda_i\}$*, and rely on the common observation (i.e., the outage event reported by the PU-Rx on the feedback channel) to determine the update in the correct direction.* Consequently, we introduce bias in the update direction and the convergent point (if exists) may not satisfy $\tilde{g}(\Lambda) = 0$ anymore. The resulting SU transmit power may not satisfy the outage probability constraint. However, the SUs can manage to satisfy the required outage probability constraint by using a suitably chosen and tighter constraint η_u (instead of η) in the DDPC algorithm, by applying the adaptive control scheme shown in Figure 2. Additionally, we cannot observe the PU outage probability directly from the 1-bit information on the PU feedback channel. Therefore we define an observation window with duration of T slots and count the outage events during such an observation window. For simplicity, we denote the PU outage probability during the k-th observation windows as $f(k) = f(\mathbf{x}_k)$. An unbiased estimate of $f(k)$ will be N_k/T, where N_k is the number of outage events within the k-th window. Here, to avoid pathological value in the estimate of the PU outage probability, we use a slightly biased estimator $\hat{f}(\mathbf{x}_k)$, also denoted as $\hat{f}(k)$ in (26). We also use $\hat{f}(\Lambda)$ to represent the implicit dependence of the estimate on Λ. Since the estimate is noisy, the direction and amount of update based on the estimate is random and is denoted by $g(\Lambda)$. Specifically, $g(\Lambda)$ is given by:

$$g(\Lambda) = \log \hat{f}(\mathbf{x}(\Lambda)) - \log \eta, \tag{25}$$

where $x_i(\Lambda)$ is given in (22). The key challenges are to understand the impact of the discount factor β_i and biased estimation as well as whether the algorithm converges, especially when different users have different values of $\lambda_i(k)$. In next section, we present the convergence result of this algorithm and study the design trade-off in face of observation errors.

Algorithm 1 Discounted Distributed Power Control (DDPC)

1: Initialize: $k = 0$, $\lambda_i(0) > 1$, $P_i(0) = \frac{1}{(\lambda_i(0)-1)b_i}$

2: Observe: for the k-th updating period with T time slots, record N_k, the number of outage events during time slot $[(k-1)T+1, kT]$.

3: Estimate:

$$\hat{f}(k) = \begin{cases} 1/T, & \text{if } N_k = 0, \\ N_k/T, & \text{otherwise.} \end{cases} \quad (26)$$

4: Update the local copies of the Lagrangian multiplier:

$$\lambda_i(k+1) = \max\{\beta_i(k)\lambda_i(k) + a_i(k)g(\lambda_1(k), \cdots, \lambda_M(k)), 0\}, \quad (27)$$

where $\beta_i(k)$, $0 < \beta_i(k) < 1$, is a forgetting factor within $(0, 1)$, $a_i(k) > 0$ is the step-size of the updating procedure, and $g(\lambda_1(k), \cdots, \lambda_M(k))$ is the commonly observed violation of the PU outage probability constraint in the last slot (in the log-scale) when the i-th SU-Tx power is determined by $\lambda_i(k)$ as:

$$g(\lambda_1(k), \cdots, \lambda_M(k)) = \log(\hat{f}(k)) - \log(\eta). \quad (28)$$

5: Update the transmit power:

$$P_i(k+1) = \begin{cases} \min\{\frac{1}{(\lambda_i(k+1)-1)b_i}, P_{\max}\}, & \text{if } \lambda_i(k+1) > 1; \\ P_{\max}, & \text{if } 0 \leq \lambda_i(k+1) \leq 1. \end{cases} \quad (29)$$

6: Return to Step 2.

V. DDPC Convergence Analysis

Having presented the DDPC algorithm for SU power control based on common PU-Rx feedback, we now consider several special operating environments and the corresponding variants of the Lagrangian update algorithm (27). We consider two cases. In the first case, we assume that SUs are time-synchronized and have the same global information on time index and/or the λ value. The goal is to understand the impact of the discount factor. In the second case, we do not assume synchronization among users and do not assume any communications among SUs. The goal is to understand the impact from the lack of synchronization. All proofs are relegated to the Appendix.

We introduce a new design parameter $c > 0$. A critical step in proving the convergence of DDPC is to have

$$\beta_i(k) = 1 - ca_i(k). \quad (30)$$

We assume the relationship of (30) holds throughout the paper. Then the update of Λ in (27) can be rewritten as:

$$\lambda_i(k+1) = \max\{\lambda_i(k) + a_i(k)[g(\Lambda(k)) - c\lambda_i(k)], 0\}, \quad i = 1, \cdots, M. \quad (31)$$

Define $\bar{g}(\Lambda) = E\{g(\Lambda)\}$, where the expectation is over the random value of the estimated PU outage probability $\hat{f}(\Lambda)$. We first have the continuity of the function $\bar{g}(\Lambda)$ as follows.

Lemma 1: $\bar{g}(\Lambda)$ is continuous on $\Lambda \in [0, \infty)^M$.

From the truncation operation on the estimation of the PU outage in (26), we have $\hat{f}(\Lambda) \in [1/T, 1]$, and thus we can construct bounds for $g(\Lambda)$ as:

$$\log(\frac{1}{\eta}) - \log(T) \leq g(\Lambda(k)) \leq \log(\frac{1}{\eta}). \quad (32)$$

Consequently, we have the following bounds on the sequence of $\lambda_i(k)$ for each SU generated by the update in (31).

Lemma 2: For $i = 1, \cdots, M$, if the initial values are chosen such that $\lambda_i(0) < \bar{\lambda}$, where $\bar{\lambda} = \frac{1}{c}\log(\frac{1}{\eta})$, then the sequence of $\lambda_i(k)$ generated by the update expressed in (31) is upper bounded by $\lambda_{\max} = (1/c + \bar{a})\log(\frac{1}{\eta})$, where $\bar{a} = \max_{i,k} a_i(k)$.

Denote the update direction function that involves the forgetting factor for the i-th SU as $y_i(\cdot)$, i.e.,

$$y_i(\Lambda) = g(\Lambda) - c\lambda_i, \quad (33)$$

which is random due to the uncertain $g(\Lambda)$. With the bounds on $g(\Lambda)$ expressed in (32) and Lemma 2, we can bound the update direction function for each SU as follows.

Lemma 3: For $i = 1, \cdots, M$, if the initial values are chosen such that $\lambda_i(0) < \bar{\lambda}$, where $\bar{\lambda} = \frac{1}{c}\log(\frac{1}{\eta})$, then the sequence of update direction $y_i(\Lambda)$ is bounded as: $|y_i(\Lambda)| < b$, where $b = \log(\frac{1}{\eta}) + \log(T) + c\lambda_{\max}$.

In the rest of the paper, when $\lambda_1 = \cdots = \lambda_M = \lambda$, we use notations $g(\lambda)$ and $\bar{g}(\lambda)$ to replace $g(\lambda\mathbf{1})$ and $\bar{g}(\lambda\mathbf{1})$, respectively. We also remove the subscript of $\lambda_i(k)$ to write $\lambda(k)$. Similar shorthands apply to $a_i(k)$, $\beta_i(k)$, and $y_i(k)$. We have:

Lemma 4: $\bar{g}(\lambda)$ is a decreasing function of λ.

A. Synchronized SU Power Control

In this scenario, we assume that a network clock, i.e., the value of k, is broadcast to the SUs. Each SU applies the same updating algorithm and has the same value of $\lambda(k)$ for time slot k. When a new SU joins the network, it will acquire the network clock and the current value of λ. When an SU leaves the network, no action needs to be taken. We further assume a sufficient condition:

$$E\{\log \hat{f}(\bar{x})\} > \log \eta, \quad (34)$$

which can be obtained by setting the SU's maximum transmit power large enough.

In practical systems, there inevitably will be errors when the SUs estimate the outage probability perceived by the PU-Rx. In [25], we studied the relationship between such observation errors and the length of observation T. Here, we focus on the asymptotic behavior of the noisy update algorithm. To this end, we consider the following updating modification with time-varying forgetting factor and step-size. Starting from $k = 0$, all SUs update the Lagrange multiplier via

$$\begin{aligned}\lambda(k+1) &= \beta(k)\lambda(k) + a(k)g(\lambda(k)) \\ &= \lambda(k) + a(k)y(\lambda(k)).\end{aligned} \tag{35}$$

Because the SU observation of the PU outage probability ($\hat{f}(k)$) is noisy, the update in (35) is random. It turns out that this update algorithm is akin to the classic stochastic approximation method [34]. Define λ_* is such that

$$\bar{g}(\lambda_* \mathbf{1}) = c\lambda_*, \tag{36}$$

we arrive at the following convergence result:

Proposition 1: For the adaptive synchronized update algorithm of (35), the Lagrangian multiplier converges to λ_* with probability 1 when the condition (34) holds and when the non-negative step-size $a(k)$ is chosen such that

$$\lim_{n\to\infty} \sum_{k=0}^{n} a(k) = \infty \quad \textit{and} \quad \lim_{n\to\infty} \sum_{k=0}^{n} a^2(k) < \infty.$$

Actually, even if different SUs update their λ_is with different initial values, from the continuity property of $\bar{g}(\Lambda)$ and the conditions stated in Proposition 1, we can show that with synchronized step-size $a(k)$ among all SUs, the update will converge to λ_* with probability 1 by Theorem 2.3.1 in [33] (page 39). In addition, due to the independence of the estimate error $\xi(k) = y(\Lambda(k)) - \bar{y}(\Lambda(k))$ (by independent channel fading assumption) and the uniform boundedness of $E\{|\xi(\Lambda)|^{2m}\}$ (by the boundedness of $y(\Lambda(k))$ in Lemma 3), the step-size selection can be loosen to $\sum_{j=0}^{\infty} a^{m+1}(k) < \infty$, where m is some integer (see example 6 in Chapter 2 of [33], page 37).

A few remarks are in order. Note that for the optimization problem (9), the optimal Lagrangian multiplier, denoted by λ^o, satisfies the condition $\tilde{g}(\lambda^o) = 0$. However, the convergent point of the DDPC with synchronized update, λ_*, may not satisfy this condition. Due to the concavity of $\log(\cdot)$, we have

$$E\{\log(\hat{f}(\mathbf{x}))\} \leq \log(E\{\hat{f}(\mathbf{x})\}). \tag{37}$$

Together with the truncation operation of estimating the outage probability, even if $E\{\log(\hat{f}(\mathbf{x})) \leq \log(\eta)\}$, we cannot guarantee that $f(\mathbf{x}) \leq \eta$ after convergence. In other words, the outage probability perceived by the PU-Rx may be greater or smaller than the required η after convergence. To mitigate this problem, we can introduce an adjustable parameter, η_u to be used as the PU outage probability constraint for our DDPC algorithm. The value of η_u can be estimated from numerical simulations with regard to all possible values of M. An alternative is to update the value of η_u online based on the observed outage probability after the algorithm converges, as depicted in Figure 2, where Δ_u is the update step-size.

We also observe the trade-off when choosing parameter c. From the monotonicity of $\bar{g}(\lambda)$, $f(\lambda)$, and $\bar{g}(\lambda_*) = c\lambda_*$, the smaller the value of c, the smaller the amount of excess outage perceived by the PU-Rx. On the other hand, for a given $a(k)$, a smaller c indicates a larger $\beta(k)$ (closer to 1), which leads to slower convergence.

We can also use other forms of update direction function and obtain variances of the proposed DDPC algorithm. For instance, we can use a convex function of $\hat{f}$ such as $\log(1-\eta) - \log(1-\hat{f}(\mathbf{x}))$; thus, we can potentially obtain transmit power that is more conservative in terms of protecting PU when the convergent point satisfies:

$$E\{\log(1-\hat{f}(\mathbf{x}))\} = \log(1-\eta)$$

by Jensen's inequality. Another option is to use $N_k/T - \eta$, which is unbiased in terms of achieving the desired outage probability constraint. For this update direction, we can set $T = 1$ and reduce the time required to obtain the estimate of PU outage probability. However, the number of iterates required is large. In this paper, we elect to use update direction as $\log(\hat{f}(\mathbf{x})) - \log(\eta)$ so as to achieve faster convergence. Intuitively, since $0 < \eta \ll 1$, the adopted update direction is able to drag a small PU outage probability $f(\Lambda)$ within the neighborhood of η quickly.

B. Unsynchronized Secondary User Access

Now we study the more general case in which no information exchange among SUs is required. As described in Algorithm 1, each SU maintains its own transmit power control without knowledge on the existence of other SUs. The only connection among SUs is through the common observation on the outage event reported by the PU-Rx on its feedback control channel. Specifically, they have the same information on how much aggregated interruption caused to the PU-Rx at time k.

Our results are built around the simpler scenario with two SU pairs in the system. The two SU transmitters activate their transmission at different time instants (slots). Denote this activation time difference as integer $k_d > 0$. Without loss of generality, we assume that $k_1 = 0$, $k_2 = k_d$, i.e., the second user starts k_d slots later. We assume that each SU uses the same step-size generation rule of the following type

$$a_i(k) = a_0(k + k_i)^{-\upsilon}, i = 1, 2, \tag{38}$$

where both a_0 and υ are predefined positive constants, and k_i is the time instant that the i-th SU-Tx activates its algorithm. Both $a_1(k)$ and $a_2(k)$ are positive and decreasing functions of k. Since we require no information exchange among SUs, the second SU does not know the current value of the first SU's step-size $a_1(k)$ or its Lagrangian $\lambda_1(k)$. In fact, the second SU updates its local version of the "Lagrangian multiplier" [4], λ_2, using its own step-size $a_2(k)$ and initial point $\lambda_2(0)$ asynchronously from $\lambda_1(k)$.

We now investigate the convergence property of the updating algorithm for the Lagrangian multiplier in (27). We first look at the asymptotic property of the discrepancy between $\lambda_1(k)$ and $\lambda_2(k)$ to derive sufficient conditions for its convergence to zero. For simplicity, we also write $g(\lambda_1(k), \lambda_2(k))$ as $g(k)$ so long as there is no confusion. When the observations on the PU feedback channel are error-free, the two SUs use the same update direction and amount, and thus we have

$$\lambda_1(k+1) - \lambda_2(k+1) = (1 - ca_2(k))(\lambda_1(k) - \lambda_2(k)) + (a_1(k) - a_2(k))(g(k) - c\lambda_1(k)), \tag{39}$$

Define $\delta(k) = \lambda_1(k) - \lambda_2(k)$, and $u(k) = a_2(k) - a_1(k)$. Recall from (30) that $\beta_i(k) = 1 - c \cdot a_i(k)$.

Lemma 5: If the step-size sequences $a_1(k)$ and $a_2(k)$ are chosen such that

$$\begin{aligned} &\lim_{n\to\infty} \prod_{j=1}^{n} \beta(j) = 0, \qquad 0 < \beta(j) < 1, \quad i = 1, 2, \\ &\lim_{n\to\infty} \sum_{j=1}^{n} |u(j)| < \infty. \end{aligned} \tag{40}$$

then

$$\lim_{k\to\infty} \delta(k) = 0.$$

This result establishes the diminishing discrepancy between the Lagrange multipliers for the proposed DDPC algorithm with consideration of biased and noisy estimate on the update algorithm. For $\upsilon \in (0.5, 1]$, $a_i(k)$ defined in Eq. (38) satisfies these two sufficient conditions. Since we assume all SUs adopt the same step-size generation scheme of (38) we can guarantee the conditions in Lemma 5 without information exchange among them. But different SUs may have a different local index of $k + k_i$ and thus actually use different step-size at any particular time instance. This is different from the distributed utility-optimal CSMA schemes for random access stations in [27], where the step-size at different users must be the same at every instance k.

Next we present the convergence result of $\lambda_i(k)$ for the two SU case. Since we focus here on the convergence property of the Lagrangian multiplier λ_1 and λ_2, we make further assumptions as follows:

I. The maximum transmit power constraint is always loose. This leads to a closed-form expression of $\tilde{g}(\lambda_1, \lambda_2)$ and the establishment of its differentiability.

[4]Since the two SU-Tx's have different views on the dual problem, $\lambda_i, i = 1, 2$ are not the true Lagrangian in the rigorous sense. The use of "Lagrangian multiplier" is a minor abuse of the term.

II. The noisy estimate of $\tilde{g}(\lambda_1, \lambda_2)$ is unbiased. Specifically, we assume that

$$g_s(\lambda_1(k), \lambda_2(k)) = \tilde{g}(\lambda_1(k), \lambda_2(k)) + w(k), \tag{41}$$

where $w(k)$ is a bounded zero mean noise. Here for distinction, we use $g_s(\cdot)$ instead of $g(\cdot)$.

III. For $i = 1, 2$, $a_i(k)$ is a diminishing sequence with $\sum_{k=1}^{\infty} a_i^2(k) < \infty$, and $\sum_{k=1}^{\infty} a_i(k) = \infty$.

IV. $0 < c < \log(\frac{1}{\eta})$.

Following the result of Lemma 5, we then have the uniform convergence result of $\lambda_i(k)$ given below.

Proposition 2: If Assumptions I-IV hold, and the step-size sequence $a(k)$ satisfies the conditions specified in Lemma 5, then, with probability 1,

$$\lim_{k \to \infty} \lambda_i(k) = \lambda_s,$$

where λ_s is such that $\tilde{g}(\lambda_s) = c\lambda_s$.

The convergent point satisfies $\tilde{g}(\lambda_s) = c\lambda_s > 0$. By the monotone decreasing property of $\tilde{g}(\cdot)$, the resulting outage probability will exceed the constraint η. In order to satisfy the outage probability constraint, the SUs should use a tighter outage probability constraint $\eta_u < \eta$ in the update. We can derive an upper bound on the distance between the PU outage probability achieved by using λ_s and that by using λ_o. Denote η_s as the PU outage probability when Lagrangian multiplier λ_s is used by the SUs. An upper bound on the discrepancy is as follow:

$$\frac{\eta_s}{\eta} < \exp\left(cM[(\mu - 1) + \frac{1}{2}(\frac{1}{M} - 1)(\mu - 1)^2]^{-1} \right). \tag{42}$$

This upper bound can be proved via Taylor expansion and the detail is omitted due to length constraint.

Remarks:

- As discussed earlier, we can achieve different trade-offs between convergence speed and the gap from λ^o by choosing different values of c. The larger the value of c, the smaller the value of β [Eq. (30)], the faster the proposed DDPC algorithm "forgets" its asynchronous discrepancy, and consequently the better the capability to accommodate dynamic SU system changes. The price to pay is the larger difference between the convergent point λ_s and the optimum λ^o.
- Another trade-off lies in the choice of υ. For a faster convergence, a smaller υ is preferred. However, with noise in the estimation, a smaller υ will introduce a larger fluctuation in the updating iterations. Also related to this issue is the choice of observation period, T, as mentioned in [25]. In particular, a longer observation window provides more accurate estimate on the PU outage probability and thus less fluctuation in the update iterations at the expense of larger convergence time.

- The amount of time to reach convergence depends on both the initial discrepancy among $\{\lambda_i\}$'s and the convergence speed of the stochastic approximation method. For a given c, a large difference among $\{\lambda_i\}$'s (e.g., when a new SU joins the network) may lead to slow convergence since it takes long time for the DDPC algorithm to diminish such a difference. When difference SUs reach consistency on their updates, the update direction function $\log(\hat{f}(k)) - \log(\eta)$ provides good convergence speed as mentioned in Section V-A and results in short response time to the PU dynamics as shown in Section VI.

Although our convergence proof has not been generalized to an arbitrary number of SU, we expect that the convergence of DDPC for multiple asynchronous SU's can be established. Similar methodology as the two-SU case may be adopted, i.e., by first showing the maximum discrepancy among λ_i, diminishes, and then studying the distance of these λ_i components from the desired convergence point. Thus far, our numerical results (given in the next section) have been positive.

VI. Simulations

In this section, we present simulation studies on the performance of the proposed DDPC algorithm in the case of multiple asynchronous SU's. First, we provide the convergence result for the special case in which the observation noise additive to $\tilde{g}(\Lambda)$ is a zero-mean uniformly distributed random variable within [-0.5, 0.5]. In the simulation, we set $M = 3$, $\eta = 0.1$, $\eta_0 = 0.01$, $c = 0.0001$, $\upsilon = 0.4$, $a_0 = 50$, and $P_{\max} = 30$dBm (1000mW) (the corresponding $\bar{x} = \log(1000) = 6.9078$). The activation instants of the three SUs are 1, 100, and 200, respectively. The initial value of $\lambda_i(0)$ is set to 100 for each SU. The effective interference channel gain from each SU to the PU-Rx b_is are set to $[0.3568, 0.0197, 0.4432] \times 10^{-3}$. In other words, SU-2 has the best channel opportunity. We display the updates of λ_i and x_i over time, and the convergent point λ_s in Figure 4, from which we can confirm the convergence of more than two SUs without synchronization. After convergence, the transmit power of SUs is $[112, 1000, 90]$ mW. Specially, SU-2 transmits with the maximum power most of the time. We can observe that SUs with larger average interference channel gains transmit with smaller power. We also test the algorithm using 5 different random seeds, and the resulting average PU outage probabilities along each convergence process are smaller than 0.1004, i.e., only slightly larger than η.

We also show the difference between the convergent point λ_s and λ^o as the value of c varies in Figure 5 for cases when $M = 2,\ 4,\ 6$, $\eta = 0.1$, and $\eta_0 = 0.01$. We can observe that, as c increases, the difference increases. In addition, more SUs in the system lead to larger difference. The PU outage probability normalized by η after convergence and its upper bound derived in (42) are also shown for comparison in

Figure 6. We can see that by setting c small enough, the resulting PU outage probability is very close to its requirement. The result also indicates that, for a larger value of c, it may be helpful to have an outer loop to adjust the value of η_u as in Figure 2 to satisfy the original target protection constraint defined by η. Another way to guarantee that the PU outage probability is below the predefined threshold η along the convergence process is to use large enough initial points $\lambda_i(0)$ (but smaller than $\bar{\lambda} = \frac{1}{c}\log(\frac{1}{\eta})$ such that the update of $\lambda_i(k)$ is bounded).

Next, we evaluate the performance of our proposed DDPC in a more practical setting. We set up a system with multiple SU pairs and one PU pair with their locations shown in Figure 3. For SUs, only the transmitters are shown. The simulation parameters are set as: $\eta = 10\%$, N_0 = $-$100dBm, $P_0 = 33$dBm, $P_{\max} = 33$dBm, $M = 3$, $\gamma_{th} = 6$, and $G_i = d_i^{-4}, i = 0, 1, \cdots, 3$, where d_i denotes the distance from the i-th SU-Tx to the PU-Rx. The 3 SUs are activated at $k_1 = 0$, $k_2 = 100$, and $k_3 = 200$. The duration for one outage probability update is set to $T = 200$. Note here that the noise caused by the estimation of (26) is biased. To test the proposed algorithm under a more dynamic system, we also allow the distance of the PU-Rx from the PU-Tx (d_0) to jump from 500 meters to 600 meters at the middle of the simulation outage. As a result, the outage probability perceived by the PU-Rx without SU transmission changes from $\eta_0 = 0.0186$ to 0.0381 and the margin for SU transmission is reduced.

In Figure 7, we plot the the update process of the "Lagrangian multiplier" λ_i for $\eta_u = 0.10$. We can observe the convergence behavior of the proposed algorithm. Although we encounter noisy observations/estimations during outage sensing, the algorithm converges smoothly and fairly as each SU eventually acquires similar value of λ_i. Also note that there exists a small gap between the convergent point and λ_*. This difference is caused by the bias in the estimation of the outage probability (in log-scale). This gap can be reduced by adopting a longer observation period, i.e., a greater T. However, this may render the update less agile and less sensitive to the system dynamics.

In Figure 8 , we plot the outage probability perceived by the PU as a function of time by setting $\eta_u = 0.10$ and $\eta_u = 0.09$ for our DDPC algorithm. Note that the time index in Figure 8 aligns with that in Figure 7. In other words, the outage probability shown is along the convergence process. We can observe that with $\eta_u = 0.10$, the outage probability perceived by the PU over the whole simulation time is only slightly higher than required. As discussed earlier, this offset can be overcome by applying an outer-loop control mechanism to adjust the target outage probability requirement η_u in place of η used in our algorithm. This is confirmed by observing that the PU outage probability is under the constraint almost all the time with $\eta_u = 0.09$, .

In Figure 9, we show the total SU utility $\sum_i \log(1+h_iP_i)$ achieved by the DDPC algorithm as a function of time. For comparison, we plot the maximum SU utility of the transformed convex optimization problem (9) obtained by utility function approximation. We also plot the lower and upper bounds on the true optimal total SU utility of the original optimization problem (5) achieved by transforming the outage probability constraint using the certainty-equivalent margin (CEM) model as in ([11], [25]). The idea is to retain the $\log(1 + h_i P_i)$ utility function for each SU but use a lower/upper bound on the outage probability expression. The solutions to all the transformed convex optimization problems are obtained using the Matlab-based convex optimization modeling system CVX [35]. We can see that the gap in the total utility achieved by the three approximation methods (utility approximation, lower bound and upper bound with CEM model) are negligible. We can also observe that the total utility achieved by the SUs with $\eta_u = 0.10$ ($\eta_u = 0.09$) may be slightly above (or below) the optimal utility for the original problem in (5). This is caused by the slightly higher (or lower) outage probability produced by the DDPC algorithm. The advantage of the DDPC lies in its distributed implementation.

Note that when the PU's interference-free outage probability changes suddenly, the relative interference margin left for SUs to exploit is reduced at time instant 50000, when we observe a spike on the outage probability perceived by the PU. Nevertheless, our algorithm can quickly infer this change and adjust the SUs' transmit power promptly to reduce the deteriorate interruption to the PU-Rx's reception quality. This is due to the advantage of the chosen update direction function given that the discrepancy among the updates at different SUs is small as discussed in Section V-A. We also tested the convergence of the proposed DDPC with a smaller value of T, for which we observed a more bursty convergence procedure due to more noise in the estimate of PU outage probability but we can achieve convergence in a shorter time. The simulation results are omitted due to space limits.

VII. Conclusions

In this work, we proposed a discounted distributed power control (DDPC) algorithm for multiple SUs in a cognitive radio network. The proposed algorithm exploits the outage information from the PU-Rx on the PU feedback channel as an external inference signal for coordination among distributed SU transmitters. We proved the convergence property of the proposed DDPC algorithm for two secondary user case, and provided the promising convergence results for scenarios with more than two SUs. This distributed SU power control can tackle asynchronousness issue in a typical cognitive radio network and approximate the optimal solution without PU cooperation, central controller/monitor, or inter-SU message passing. In future works, we plan to generalize our framework to include the more dynamic scenarios involving adaptive PUs and SUs. We are also keen to assess the tradeoff between the security concerns and the revenue from cognitive users by allowing some unencrypted link control feedback among the PU pairs.

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APPENDIX

A. Proof of Lemma 1

Proof: Given $0 \preceq \Lambda$, we expand the expectation operation on $g(\Lambda)$ which is given by (25) and obtain

$$\begin{aligned}\bar{g}(\Lambda) &= \log(\frac{1}{T}) \Pr[N_k = 0|\lambda] + \sum_{i=1}^{T} \log(\frac{i}{T}) \Pr[N_k = i|\lambda] \\ &= \log(\frac{1}{T})(1 - f(\mathbf{x}(\Lambda)))^T + \sum_{i=1}^{T} \log(\frac{i}{T}) \binom{T}{i} (1 - f(\mathbf{x}(\Lambda)))^i f(\mathbf{x}(\Lambda))^{T-i},\end{aligned} \tag{43}$$

where N_k is a Binomial distributed random variable with successful trial probability $f(\mathbf{x}(\Lambda))$, which is determined by the transmit power vector $\mathbf{x}$, which is subsequently determined by Λ as in (22). Since $-\log((\lambda_i - 1)b_i)$ is a decreasing and continuous function for $\lambda_i > 1$, and it intersects $\bar{x}$ when $\lambda_i =$

$1+\frac{1}{b_i}e^{-\bar{x}} > 1$, we have that $x_i(\cdot)$ is a continuous mapping of Λ over $[0,\infty)^M \subset \mathcal{R}^M$ to $(-\infty,\bar{x}]^M \subset \mathcal{R}^M$. For Rayleigh channel model used in the paper,

$$f(\mathbf{x}) = 1-(1-\eta_0)\prod_{i=1}^{M}(1+b_ie^{x_i})^{-1}, \tag{44}$$

which is a continuous function of $\mathbf{x} \in (-\infty,\bar{x}]^M$. Observe that $\bar{g}(\cdot)$ is a polynomial function of f, we have $\bar{g}(\cdot)$ is continuous on $f \in [\eta_0, 1]$. By the composition rule of continuous functions (Theorem 4.7 in [36], we have $\bar{g}(\Lambda)$ a continuous function of Λ. □

B. *Proof of Lemma 2*

Proof: Let $\bar{\lambda} = -\log\eta/c$, and $\bar{a} = \max_{i,k}\{a_i(k)\}$. Both $\bar{\lambda}$ and $\bar{a}$ are positive. Suppose $\lambda_i(k) \le \bar{\lambda}$. Since $g(\Lambda(k)) \le \log(\frac{1}{\eta})$ (32) and $\lambda_i(k) \ge 0$, we have

$$\begin{aligned}\lambda_i(k+1) &= \lambda_i(k) + a_i(k)[g(\Lambda(k)) - c\lambda_i(k)] \\ &\le \lambda_i(k) + a_i(k)[\log(\frac{1}{\eta}) - c\lambda_i(k)] \\ &\le \lambda_i(k) + a_i(k)\log(\frac{1}{\eta}) \\ &\le \bar{\lambda} + \bar{a}\log(\frac{1}{\eta}).\end{aligned} \tag{45}$$

On the other hand, when $\lambda_i(k) > \bar{\lambda}$, we have

$$\lambda_i(k+1) \le \lambda_i(k) + a_i(k)[\log(\frac{1}{\eta}) - c\bar{\lambda}] = \lambda_i(k). \tag{46}$$

In other words, whenever $\lambda_i(k)$ becomes larger than $\bar{\lambda}$, the negative update direction will result in a smaller number of $\lambda_i(k)$ next.

Let $\lambda_{\max} = \bar{\lambda} + \bar{a}\log(\frac{1}{\eta})$. It then follows by induction from the two preceding equations that, if we choose $\lambda_i(0) < \bar{\lambda}$, then we have $\lambda_i(k) \le \lambda_{\max}$. The above argument holds for all $i = 1, \cdots, M$ and the proof completes.

□

C. *Proof of Lemma 3*

Proof: Since $y_i(\Lambda(k)) = g(\Lambda(k)) - c\lambda_i$, we have

$$|y_i(\Lambda(k)| \le |g(\Lambda(k))| + c|\lambda_i(k)|. \tag{47}$$

By the bounds on $g(\Lambda(k))$ as in (32) and $\lambda_i(k)$ as in Lemma 2, the proof is complete. □

D. Proof of Lemma 4

Proof: Due to the independence assumption on channel fading, the number of outage event within an observation window follows Bernoulli distribution with the PU outage probability determined by the SU transmit power vector $\mathbf{x}(\lambda\mathbf{1})$ for given λ. Observe that the transmit power of each SU, $x_i(\lambda\mathbf{1}) = x_i^*(\lambda)$ as in (22) is a decreasing (or non-increasing) function of λ and also note that here each SU uses the same λ to obtain its transmit power. Therefore, for any two values of λ with $0 \leq z_1 \leq z_2$, we have $\mathbf{x}^(z_2) \preceq \mathbf{x}^(z_1)$. Then we have the PU outage probabilities associated with z_1 and z_2 satisfying $f(\mathbf{x}(z_1)) \geq f(\mathbf{x}(z_2))$. Let random variables X_1 and X_2 denote the number of successful trials among T independent Bernoulli trials with successful probability as $f(\mathbf{x}(z_1))$ and $f(\mathbf{x}(z_2))$, respectively. Then we have that X_1 is stochastically larger than X_2 [37]. By the property of the stochastic ordering (Proposition 1.2 in Appendix of [37]), since $\log \hat{f}(z_i) = \log\{\max\{X_i/T, 1/T\}\}$ is an increasing function of X_i for $i = 1, 2$, we have:

$$E\{\log \hat{f}(\mathbf{x}(z_1))\} \geq E\{\log \hat{f}(\mathbf{x}(z_2))\}, \tag{48}$$

where the expectation operation is with regard to the distributions of X_1 and X_2, respectively. Therefore, we have $\bar{g}(z_1) \geq \bar{g}(z_2)$ and complete the proof. □

E. Proof of Proposition 1

Proof: As a special case of Lemma 1, we know that $\bar{g}(\lambda)$ is a continuous function of $\lambda \in [0, \infty)$. By Lemma 4, we have $\bar{g}(\lambda)$ is a decreasing function of λ, and thus $\bar{y}(\lambda) = \bar{g}(\lambda) - c\lambda$ is strictly decreasing for $c > 0$. When $\lambda = 0$, $\mathbf{x}^*(0) = \bar{x}\mathbf{1}$, by invoking condition (34), we have $\bar{g}(0) > 0$. On the other hand, we have by (32),

$$\log(\frac{1}{\eta}) - \log(T) \leq \bar{g}(\lambda) \leq \log(\frac{1}{\eta}), \tag{49}$$

and thus when $\lambda > \log(\frac{1}{\eta})/c$, $\bar{g}(\lambda) - c\lambda < 0$. Therefore, by the Intermediate Value Theorem for continuous function, we know there exists a number in $[0, \log(\frac{1}{\eta})/c]$ such that

$$\bar{y}(\lambda) = 0. \tag{50}$$

Denote this number as λ_*. By the monotonic decreasing property of $\bar{y}(\lambda)$, we also have the uniqueness of λ_*, and $\lambda_* > 0$. Then the condition A.2.3.1 in [33] holds since for each $\epsilon > 0$, there is a $\delta > 0$ such that

$$\bar{y}(\lambda) \leq -\epsilon \quad \text{for} \quad \lambda \in [\lambda_* + \delta, \infty) \tag{51}$$

and

$$\bar{y}(\lambda) \geq \epsilon \quad \text{for} \quad \lambda \in (0, \lambda_* - \delta]. \tag{52}$$

Define $\xi(k) = y(\lambda(k)) - \bar{y}(\lambda(k))$. Obviously, we have $E\{\xi(k)|\lambda(k)\} = 0$ and $\xi(k)$ is bounded since $y(\lambda(k))$ is bounded by Lemma 3. Then we have $E\{|\xi(k)|^2\} < \infty$. We can write the update of λ as follows:

$$\begin{aligned} \lambda(k+1) &= \lambda(k) + a(k)y(\lambda(k)) \\ &= \lambda(k) + a(k)[\bar{y}(\lambda(k)) + \xi(k)]. \end{aligned} \tag{53}$$

In addition, since $\xi(k)$ is independent due to the independent fading assumption, $\{\sum_{k=0}^{n} a(k)\xi(k)\}$ is a Martingale sequence. Therefore, by Martingale bound, for $a(k)$ such that $\sum_{k=1}^{\infty} a(k) < \infty$, we have for each $\epsilon > 0$,

$$\lim_{n\to\infty} \Pr[\sup_{m>n} |\sum_{i=n}^{m} a(i)\xi(i)| \geq \epsilon] \leq \lim_{n\to\infty} E\{|\xi(k)|^2\} \sum_{i=n}^{\infty} a^2(i)/\epsilon^2 = 0, \tag{54}$$

and thus the condition A.2.2.4' in [33] holds.

By the Theorem 2.3.2 in [33], we have for $a(k)$ such that $a(k) \to 0$, and $\sum_k a(k) = \infty$, the update in (53) converges to λ_* with probability 1 (Note that the actual update algorithm has a projection operation over $\lambda(k)$ into $[0, \infty)$ with which the convergence result carries since $\lambda_* > 0$ according to the remarks in Chapter 2 of [33]).

□

F. Proof of Lemma 5

Proof: By Lemma 3, we have $y_1(k)$ and $y_2(k)$ are both bounded by a constant b, i.e., $|y_i(k)| < b$. For any $\varepsilon > 0$, let $\varepsilon_1 = \varepsilon/2\lambda_{\max}$, $\varepsilon_2 = \varepsilon/2b$. If $a_1(k)$ and $a_2(k)$ satisfy the conditions specified in Lemma 5, there exists an integer K such that

$$\sum_{j=K}^{\infty} |u(j)| < \varepsilon_2. \tag{55}$$

In addition, we can also find a large enough $N \geq K$ such that, $\forall k > N$, we have

$$\prod_{j=K}^{k} \beta_2(j) < \varepsilon_1. \tag{56}$$

Since $|u(j)| > 0$, from (55) we can see that, for $k > N \geq K$,

$$\sum_{j=K}^{k} |u(j)| < \varepsilon_2. \tag{57}$$

Taking advantage of (39), we have, for $k > K$,

$$\begin{aligned} |\delta(k+1)| &\leq \beta_2(k)|\delta(k)| + |u(k)|b \\ &\leq \alpha(k)\left[\beta_2(k-1)|\delta(k-1)| + |u(k-1)|b\right] + |u(k)|b \\ &\leq \alpha(k)\alpha(k-1)|\delta(k-1)| + (|u(k-1)| + |u(k)|)b \\ &\ \ \vdots \\ &\leq [\prod_{j=K}^{k}\alpha(j)]|\delta(K)| + \sum_{j=K}^{k}|u(j)|b \end{aligned} \tag{58}$$

Note that $|\delta(K)| \leq \lambda_{\max}$. Relying on (55) and (56), for $k > N$, $|\delta(k+1)| < \varepsilon$ for any positive ϵ. Hence, $\lim_{k\to\infty}|\delta(k)| = 0$. □

G. Proof of Proposition 2

Proof: Since the maximum power constraint is always loose, the outage probability constraint is tight by the optimality conditions specified in Section IV-B. By using the expression of $x_i^*(\lambda_i)$ as in (12), we have $\lambda_i > 1$. Furthermore, we can write $\tilde{g}(\lambda_1, \lambda_2)$ as:

$$\tilde{g}(\lambda_1,\lambda_2) = \log\left(f(k)\right) - \log\eta = \log(1-(1-\eta_0)\frac{\lambda_1-1}{\lambda_1}\frac{\lambda_2-1}{\lambda_2}) - \log\eta, \tag{59}$$

with its first-order partial derivative with respect to λ_1 as

$$\frac{\partial\tilde{g}(\lambda_1,\lambda_2)}{\partial\lambda_1} = \frac{-(1-\eta_0)}{1-(1-\eta_0)\frac{\lambda_1-1}{\lambda_1}\frac{\lambda_2-1}{\lambda_2}} \cdot \frac{\lambda_2-1}{\lambda_2}\frac{1}{\lambda_1^2} \leq 0. \tag{60}$$

From the symmetry of $\tilde{g}(\lambda_1, \lambda_2)$, we can obtain similar expression for $\frac{\partial\tilde{g}(\lambda_1,\lambda_2)}{\partial\lambda_2}$, and we have for any given $x > 1$ and $y > 1$,

$$\frac{\partial\tilde{g}(x,y)}{\partial x} = \frac{\partial\tilde{g}(y,x)}{\partial x}. \tag{61}$$

Note that all poles of the partial derivative falls outside $(1, \infty)$. In addition, if there exists a solution to the following equation systems

$$\tilde{g}(\lambda_1,\lambda_2) - c\lambda_i = 0, \quad i = 1, 2$$

for a given positive number c, the solution should satisfy

$$\lambda_1 = \lambda_2 = \lambda_s,$$

where λ_s is such that $\tilde{g}(\lambda_s) = c\lambda_s$. Note that $\tilde{g}(\lambda) - c\lambda$ is continuous and strictly decreasing in λ. When $0 < c < \log(\frac{1}{\eta})$, the existence of such a solution is guaranteed by the Intermediate Value Theorem since $\lim_{\lambda\to 1}\tilde{g}(\lambda) = \log(\frac{1}{\eta})$.

By Lemma 5, we can find a large enough $K < \infty$ such that, for $\varepsilon > 0$, $\forall k > K$, we have

$$|\delta(k)| = |\lambda_1(k) - \lambda_2(k)| \leq \varepsilon. \tag{62}$$

Define $\epsilon_i(k) = \lambda_i(k) - \lambda_s$, and $d_i(k) = E\{\epsilon^2(k)\}$. Next we show that

$$\lim_{k\to\infty} d_i(k) = 0, i = 1, 2, \tag{63}$$

which implies the convergence in probability of $\lambda_i(k)$ to λ_s.

From (41) we have the update as

$$\lambda_i(k+1) - \lambda_i(k) = a_i(k)y_i(k) = a_i(k)[\tilde{g}(\lambda_1(k), \lambda_2(k)) + w(k)], \tag{64}$$

where $w(k)$ is a zero-mean and bounded random variable. Because $\log(\frac{\eta_0}{\eta}) \leq \tilde{g} \leq \log(\frac{1}{\eta})$, we have $y_i(k)$ is bounded and thus there exists a positive constant $\bar{C}$ such that for all $(\lambda_1(k), \lambda_2(k))$,

$$\Pr[|y_i(k)| \leq \bar{C}] = E\{\int_{-\bar{C}}^{\bar{C}} dH_i(y|\lambda_1(k), \lambda_2(k))\} = 1,$$

where $H_i(y|\lambda_1(k), \lambda_2(k))$ denotes the distribution function in $y_i(k)$ given $(\lambda_1(k), \lambda_2(k))$.

From (64), we have

$$\begin{aligned} d_i(k+1) &= E\{(\lambda_i(k+1) - \lambda_s)^2\} = E\{E\{(\lambda_i(k+1) - \lambda_s)^2|\lambda_1(k), \lambda_2(k)\}\} \\ &= E\{\int [(\lambda_i(k) - \lambda_s) + a_i(k)y]^2 dH_i(y|\lambda_1(k), \lambda_2(k))\} \\ &= d_i(k) + a_i^2(k)E\{\int y^2 dH_i(y|\lambda_1(k), \lambda_2(k))\} + 2a_i(k)E\{(\lambda_i(k) - \lambda_s)\bar{y}_i(k)\}. \end{aligned} \tag{65}$$

Setting

$$v_i(k) = E\{(\lambda_i(k) - \lambda_s)\bar{y}_i(k)\}, \tag{66}$$

$$e_i(k) = E\{\int y^2 dH_i(y|\lambda_1(k), \lambda_2(k))\}, \tag{67}$$

we can write

$$d_i(k+1) - d_i(k) = a_i^2(k)e_i(k) + 2a_i(k)v_i(k). \tag{68}$$

By the bound on $y_i(k)$ and convergent property of $\sum_{k=1}^n a_i^2(k)$, we have the positive-term series $\sum_{k=1}^n a_i^2(k)e_i(k)$ converges.

Next we prove that for $k > K$, we have $a_1(k)v_1(k) + a_2(k)v_2(k) \leq 0$. Since $|\lambda_1(k) - \lambda_2(k)| \leq \varepsilon$ for $k > K$, we have the domain of the expectation operation for $v_i(k)$ as a stripe with width ε. We further partition this stripe into four regions:

$$\begin{aligned} S_1 &= \{(\lambda_1, \lambda_2)|\lambda_1 \leq \lambda_s, \lambda_2 \leq \lambda_s, |\lambda_1(k) - \lambda_2(k)| \leq \varepsilon\}; \\ S_2 &= \{(\lambda_1, \lambda_2)|\lambda_1 < \lambda_s < \lambda_2, |\lambda_1(k) - \lambda_2(k)| \leq \varepsilon\}; \\ S_3 &= \{(\lambda_1, \lambda_2)|\lambda_2 < \lambda_s < \lambda_1, |\lambda_1(k) - \lambda_2(k)| \leq \varepsilon\}; \\ S_4 &= \{(\lambda_1, \lambda_2)|\lambda_1 > \lambda_s, \lambda_2 > \lambda_s, |\lambda_1(k) - \lambda_2(k)| \leq \varepsilon\}. \end{aligned} \tag{69}$$

For regions S_1 and S_4, relying on the decreasing property of $\tilde{g}(\lambda_1, \lambda_2)$ in both variables, we have:

$$(\lambda_i - \lambda_s)\bar{y}_i(k) \leq 0, \tag{70}$$

and thus

$$\sum_{i=1}^{2} a_i(k)(\lambda_i - \lambda_s)\bar{y}_i(k) \leq 0. \tag{71}$$

Note that both regions S_2 and S_3 falls into the vicinity of (λ_s, λ_s). We apply Taylor expansion on function $\tilde{g}(\lambda_1, \lambda_2)$ in the vicinity of (λ_s, λ_s):

$$\begin{aligned}
\tilde{g}(\lambda_1, \lambda_2) &= \tilde{g}(\lambda_s, \lambda_2) + \frac{\partial \tilde{g}(\lambda_s, \lambda_2)}{\partial \lambda_1}(\lambda_1 - \lambda_s) + O_1(\varepsilon^2) \\
&= \tilde{g}(\lambda_s, \lambda_s) + \frac{\partial \tilde{g}(\lambda_s, \lambda_s)}{\partial \lambda_2}(\lambda_2 - \lambda_s) + O_1(\varepsilon^2) \\
&\quad + (\frac{\partial \tilde{g}(\lambda_s, \lambda_s)}{\partial \lambda_1} + \frac{\partial^2 \tilde{g}(\lambda_s, \lambda_s)}{\partial \lambda_1 \partial \lambda_2}(\lambda_2 - \lambda_s) + O_2(\varepsilon^2))(\lambda_1 - \lambda_s) \\
&= \tilde{g}(\lambda_s, \lambda_s) + \frac{\partial \tilde{g}(\lambda_1 = \lambda_s, \lambda_s)}{\partial \lambda_1}(\lambda_1 - \lambda_s) + \frac{\partial \tilde{g}(\lambda_s, \lambda_2 = \lambda_s)}{\partial \lambda_2}(\lambda_2 - \lambda_s) + O(\varepsilon^2),
\end{aligned} \tag{72}$$

where $O(\varepsilon^2)$ denotes all the terms with order of ε higher than 1. This result requires $\tilde{g}(\lambda_1, \lambda_2)$ to have a bounded second-order derivative within the neighborhood of (λ_s, λ_s), which can be verified by investigating the poles of the second-order derivative. By the symmetry of $\tilde{g}(\lambda_1, \lambda_2)$, we have

$$\frac{\partial \tilde{g}(\lambda_1 = \lambda_s, \lambda_s)}{\partial \lambda_1} = \frac{\partial \tilde{g}(\lambda_s, \lambda_2 = \lambda_s)}{\partial \lambda_2} \doteq c'. \tag{73}$$

Clearly, $c' \leq 0$. As a result, we have within the neighborhood of (λ_s, λ_s),

$$\tilde{g}(\lambda_1, \lambda_2) = \tilde{g}(\lambda_s) + c'(\lambda_1 + \lambda_2 - 2\lambda_s) + O(\varepsilon^2). \tag{74}$$

For region S_3, we can express $\epsilon_1(k)$ and $\epsilon_2(k)$ for $k > K$ as:

$$\epsilon_1(k) = \frac{\varepsilon'}{2} + \delta', \quad \epsilon_2(k) = -\frac{\varepsilon'}{2} + \delta' \tag{75}$$

for some $0 \leq \varepsilon' \leq \varepsilon$ and $-\varepsilon'/2 \leq \delta' \leq \varepsilon'/2$. Recall that $u(k) = a_2(k) - a_1(k)$. Together with $\tilde{g}(\Lambda)$ expressed in (74), we have

$$\begin{aligned}
&\sum_{i=1}^{2} a_i(k)(\lambda_i - \lambda_s)\bar{y}_i(k) = \sum_{i=1}^{2} a_i(k)(\lambda_i - \lambda_s)(\tilde{g}(\Lambda(k)) - c\lambda_i(k)) \\
=& a_1(k)(\frac{\varepsilon'}{2} + \delta')[c(-\frac{\varepsilon'}{2} - \delta') + 2c'\delta'] + [a_1(k) + u(k)](-\frac{\varepsilon'}{2} + \delta')[c(\frac{\varepsilon'}{2} - \delta') + 2c'\delta'] \\
=& 2[2a_1(k) + u(k)]c'\delta'^2 \\
&- \frac{a_1(k)}{2}c[(\frac{\varepsilon'}{2} + \delta')^2 + (\frac{\varepsilon'}{2} - \delta')^2] - u(k)c(\frac{\varepsilon'}{2} - \delta')^2 \\
&- \frac{a_1(k)}{2}c[(\frac{\varepsilon'}{2} + \delta')^2 + (\frac{\varepsilon'}{2} - \delta')^2] - u(k)\varepsilon' c'\delta'.
\end{aligned} \tag{76}$$

When $|u(k)| < 2a_1(k)$, since $c' < 0$, the first line of the last equality is negative. The second line of the last equality is also negative when $|u(k)| < 2a_1(k)$. When $|u(k)| \le \frac{a_1(k)}{2}\frac{c}{|c'|}$, the last line of the above equation is negative. For $1/k$-like sequences, these conditions on $|u(k)|$ hold for $k > N_1$, where $N_1 > K$ is a large enough constant. Consequently, $(\lambda_i - \lambda_s)\bar{y}_i(k) \le 0$ is satisfied in region S_3. Similar arguments apply to region S_2. Therefore, we obtain $a_1(k)v_1(k) + a_2(k)v_2(k) \le 0$.

The rest of the proof follows similar arguments as in [34]. Summing over (68), we obtain

$$\sum_{i=1}^{2} d_i(k+1) = \sum_{i=1}^{2} d_i(N_1) + \sum_{i=1}^{2}\sum_{j=N_1}^{k} a_i^2(j)e_i(j) + \sum_{i=1}^{2}\sum_{j=N_1}^{k} a_i(k)v_i(k) \tag{77}$$

Since $d_i(k+1) \ge 0$, it follows that the positive-term series $-\sum_{i=1}^{2}\sum_{j=N_1}^{k} a_i(k)v_i(k)$ converges. It then follows $\lim_{k\to\infty}\sum_{i=1}^{2} d_i(k+1)$ exists, which is denoted by d.

Next we show that there exists two sequences $\{k_i(k)\}$ of non-negative constants such that for $k > N_1$,

$$-\sum_{i=1}^{2} a_i(k)v_i(k) \ge \sum_{i=1}^{2} a_i(k)k_i(k)d_i(k), \qquad \sum_{k=1}^{\infty} a_i(k)k_i(k) = \infty, i = 1, 2. \tag{78}$$

Following the same arguments of Lemma 1 in [34], this implies that $d = 0$. For region S_1 and S_4, it is easy to see that

$$\begin{aligned} &-\sum_{i=1}^{2} a_i(k)(\lambda_1(k) - \lambda_s)[\tilde{g}(\lambda_1(k), \lambda_2(k)) - c\lambda_i(k)] \\ =&-\sum_{i=1}^{2} a_i(k)(\lambda_1(k) - \lambda_s)^2\frac{\tilde{g}(\lambda_1(k), \lambda_2(k)) - c\lambda_i(k)}{\lambda_i(k) - \lambda_s} \\ \ge&\, c\sum_{i=1}^{2} a_i(k)(\lambda_1(k) - \lambda_s)^2 \end{aligned} \tag{79}$$

For region S_3, using the Taylor expansion in (74) and (75), we have:

$$\begin{aligned} &-\sum_{i=1}^{2} a_i(k)(\lambda_1(k) - \lambda_s)[\tilde{g}(\lambda_1(k), \lambda_2(k)) - c\lambda_i(k)] - \sum_{i=1}^{2} a_i(k)k_i(k)(\lambda_1(k) - \lambda_s)^2 \\ =&[-a_1(k)k_1(k) - a_2(k)k_2(k)]\delta'^2 \\ &+[a_1(k)(-2c' - k_1(k)) + a_2(k)(-2c' - k_2(k))](\frac{\varepsilon'}{2})^2 \\ &+[a_1(k)(-c' - k_1(k)) - a_2(k)(-c' - k_2(k))]\varepsilon'\delta' \end{aligned} \tag{80}$$

The minimum value of (80) as a function of δ' is obtained when $\delta' = \pm\frac{\varepsilon'}{2}$, and it is easy to verify that for $k_i(k) < -c'$, the minimum value is larger than 0. Similar arguments apply to region S_2. Therefore, if we set $k_i(k)$ as a constant sequence such that $k_i(k) < \min\{c, -c'\}$ and take expectation over regions S_1-S_4, we have:

$$-\sum_{i=1}^{2} a_i(k)v_i(k) \ge \sum_{i=1}^{2} a_i(k)k_i(k)d_i(k), \tag{81}$$

and thus both conditions in (78) hold. Therefore, $d = 0$. This completes the proof. $\square$

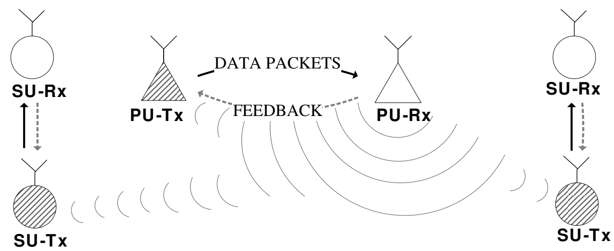

Fig. 1. Using PU feedback in multi-SU systems.

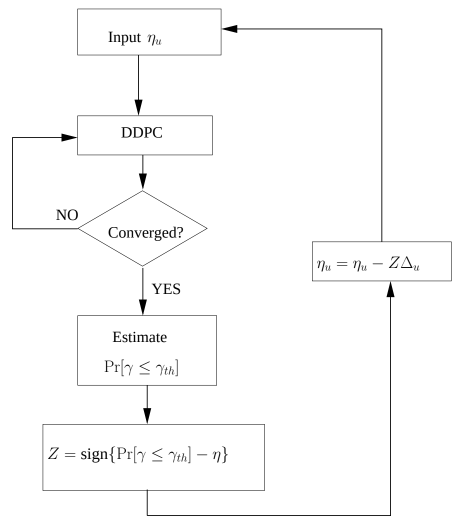

Fig. 2. Adaptive outage probability control.

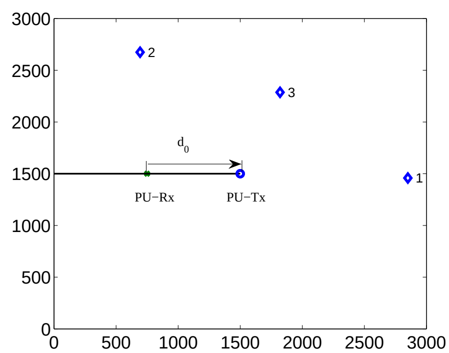

Fig. 3. User locations.

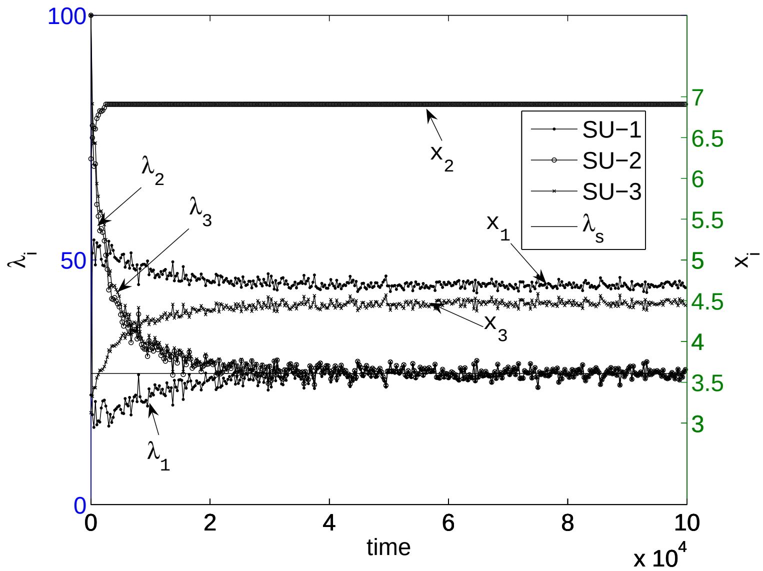

Fig. 4. Convergence behavior of the proposed DDPC algorithm with noisy observation.

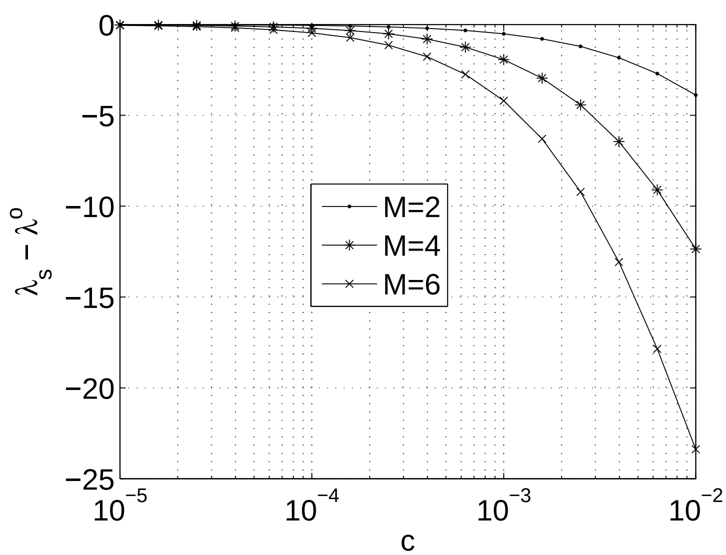

Fig. 5. Difference between the convergent point λ_s and the optimal Lagrangian multiplier λ^o.

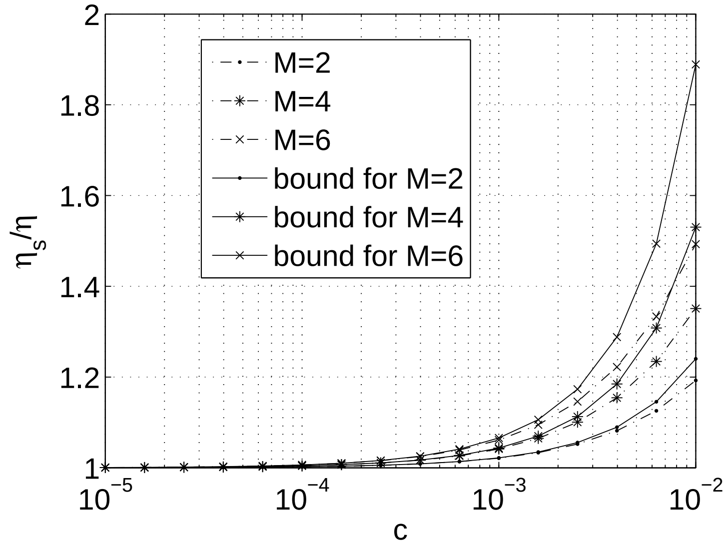

Fig. 6. PU Outage probability with different values of c after convergence.

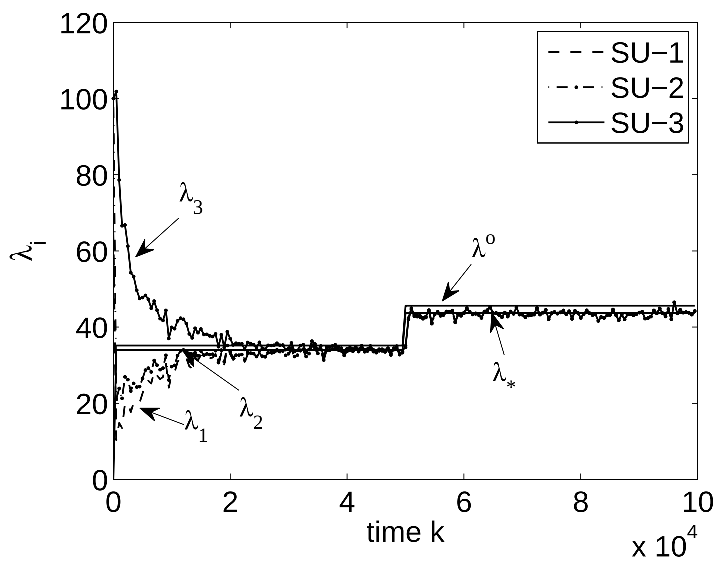

Fig. 7. Lagrangian multipliers

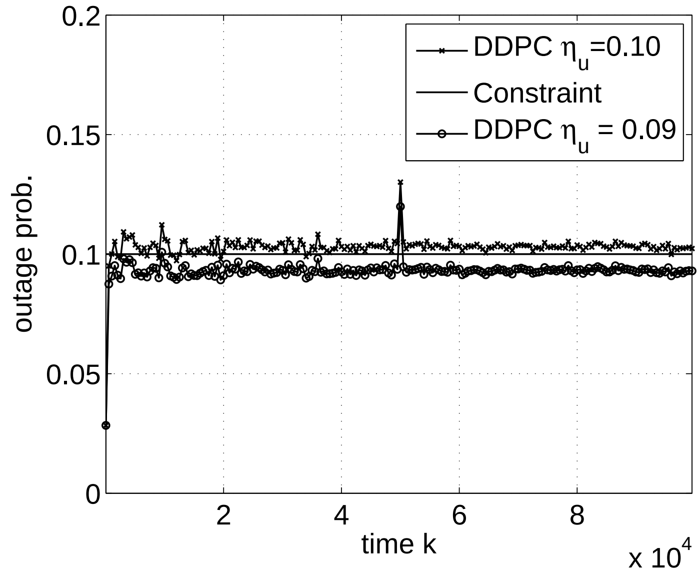

Fig. 8. Outage probability perceived by the PU

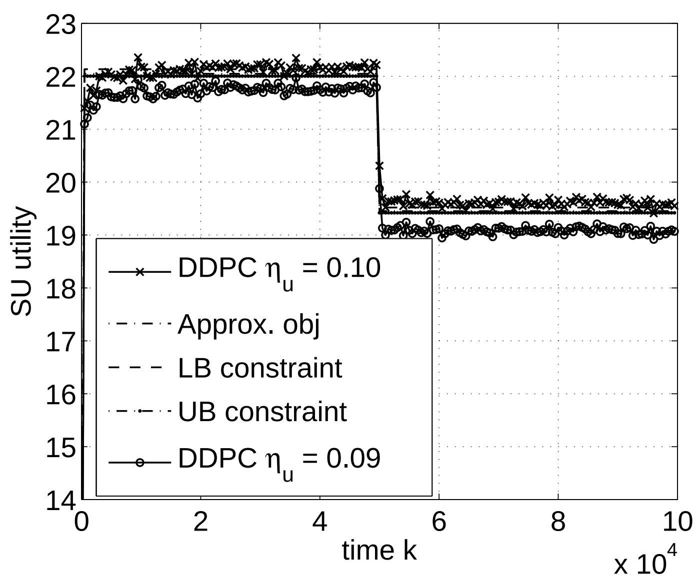

Fig. 9. Utility function of the SUs