Smart Antennas in Wireless Systems: Uplink Multiuser Blind Channel and Sequence Detection

Hui Liu, *Member, IEEE*, and Guanghan Xu, *Member, IEEE*

***Abstract*— Recently, space-division multiple-access (SDMA) schemes [1], [2] have been proposed to increase the capacity of wireless communication systems by simultaneously transmitting and receiving multiple co-channel signals through different *spatial channels*. In this paper, we address the uplink (remote users to the base station antenna array) blind channel and sequence identification problem for an SDMA system. We show that multiuser *blind* identification can be accomplished by exploiting the spatial and temporal diversities of an antenna array system. In particular, a recursive estimation algorithm is developed to recover multiple signals from intersymbol interference (ISI) and co-channel interference (CCI) by taking advantage of a special structure of the array output and the finite-alphabet property of digital communication signals. The implementation of the proposed approach in practical applications is discussed, and field experiments have been conducted to demonstrate the effectiveness of the proposed algorithm.**

***Index Terms*—Equalization, multiple-access.**

I. Introduction

SMART ANTENNA systems (SAS) [1]–[5], i.e., wireless systems that exploit the spatial dimension in signal processing by employing multiple antennas, have shown their prominence in overcoming some of the major difficulties in current wireless systems, e.g., capacity limitation, co-channel interference, multipath fading, etc. Among the many utilizations of a smart antenna system, the most compelling application is probably the space-division multiple-access (SDMA) system, in which multiple co-channel users communicate with the base station simultaneously without mutual interference. In principle, one can integrate SDMA with any existing multiple access standard and gain significantly in channel capacity with limited increase in system complexity. For example, by applying smart antennas to a time-division multiple-access (TDMA) system, two or more users are allowed to occupy the same time slots, leading to two or more times increase in total capacity. In this paper, we consider the uplink SDMA operation for linearly modulated digital wireless systems, e.g., IS-54, with frequency-selective channels. More specifically, we address the multiuser source recovery problem for array outputs suffering from both intersymbol interference (ISI) and co-channel interference (CCI).

Paper approved by J. H. Winters, the Editor for Equalization of the IEEE Communications Society. Manuscript received December 20, 1994; revised July 18, 1995.

H. Liu is with the Department of Electrical Engineering, University of Virginia, Charlottesville, VA 22903-2442 USA (e-mail: hl3r@virginia.edu).

G. Xu is with the Department of Electrical and Computer Engineering, The University of Texas at Austin, Austin, TX 78712-1084 USA (e-mail: xu@globe.utexas.edu).

Publisher Item Identifier S 0090-6778(97)01407-4.

Optimum detection and equalization, as an effective means for recovering multiple signals and overcoming the adverse effects of dispersion, has been extensively studied by various researchers [5]–[10]. To perform such operations, the spatial channel associated with each user needs to be reliably estimated. Since in many current systems the same training sequence is assigned to different users, without changing the existing system protocols, channel estimation approaches that rely on training sequences are inherently prohibited. Furthermore, the use of training sequences leads to a significant decrease in the bandwidth efficiency, especially in a fast changing mobile environment. Moreover, the occasional breakdown in communication links requires the system to have certain self-starting abilities. All these factors make it particularly desirable for a *blind* estimation algorithm which is capable of identifying multiple channels or separate co-channel signals based solely on the array outputs.

Since the antenna outputs are composed of upcoming signals from different directions, Andersson *et al.* [3] proposed to use directional beam forming to separate multiple co-channel signals. Their model approximates all coherent multipath signals with a *point source*, thus allows the direct use of many subspace-based direction of arrival (DOA) estimation algorithms, e.g., MUSIC [11] and ESPRIT [12]. The problem with this model, however, is its applicability to real communication scenarios. Talwar *et al.* [13] attacked this problem from a different direction and introduced a blind estimation approach which estimates the users' array response vectors (often referred to as their spatial signatures) by exploiting the finite-alphabet property of a digital communication signal. The algorithm is theoretically simple and efficient. Unfortunately, it cannot be applied to a long-delay multipath environment. Besides, the algorithm requires that signals from all users to be perfectly synchronized at the bit-level, which is impractical due to the different delays from remote users to the base-station.

In the presence of long-delay multipath components, the received signal suffers from ISI which can only be cured by channel equalization. Blind channel identification (BCI) [14] provides a possible solution to this problem. In wireless applications, the search for data-efficient algorithms has led

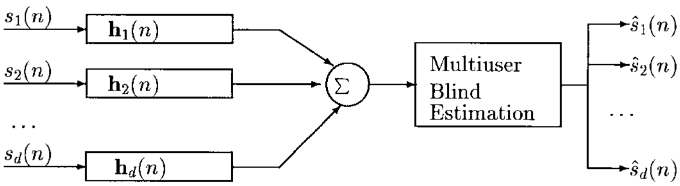

Fig. 1. Multiuser blind identification.

to the recent development of *oversampling* techniques that are potentially of great practical importance. Various newly developed approaches have shown great promise in studies and simulations performed to date [2], [15]–[22]. However, all these methods can only handle a single source with an exception of [23].

In this paper, we first establish a general framework for an antenna array system with co-channel signals, each of which suffers from ISI, so that a systematic approach for SDMA blind estimation can be developed. We assume that the channel characteristics remain constant over many information symbols—a condition which clearly depends on the fading rate of a specific application [24]. The prime objective of this paper is to introduce a new blind channel and sequence estimation algorithm for the multiuser system illustrated in Fig. 1. Our approach combines and subsequently extends the techniques of [25] and [13] to deconvolve the input sequences and then determine each individual symbol and channel. The method we propose is parametric, and therefore can accomplish blind estimation using a small number of data samples. In addition, we derive the identifiability condition of the new method and discuss its implementations. Field experiments have been conducted at the J. J. Pickle Research Campus using a Smart Antenna Testbed developed at the University of Texas at Austin. The results show that in an outdoor nearly stationary environment, the new approach is effective in both flat fading and frequency-selective fading (artificially created) scenarios.

II. Data Formulation

Let us first list some notational conventions to be used in this paper. $\mathbf{A}^T$, $\mathbf{A}^H$, $\mathbf{A}^\dagger$ denote the transpose, conjugate transpose, and the pseudo-inverse of $\mathbf{A}$, respectively. $\mathcal{R}\{\mathbf{A}\}$ represents the row span of $\mathbf{A}$ and $\mathbf{A}^\perp$ is its orthogonal complement (null space). $\otimes$ denotes the convolution operator.

Next, we introduce some basic assumptions on our problem. We consider the case where the composite channel of a wireless system can be perfectly modeled as a finite impulse response (FIR) filter. We also assume that the information-bearing symbols are drawn from a finite alphabet, e.g., binary phase shift keying (BPSK) or quaternary phase shift keying (QPSK). Both assumptions are plausible for most digital wireless communication scenarios.

Under the above assumptions, the output of a linearly modulated communication system can be expressed as a convolution of the transmitting symbol $s(n)$ and the channel response $h(t)$

$$y(t) = \sum_{m=-\infty}^{\infty} h(t - mT)s(m)$$

where T is the symbol period. Temporally oversample $y(t)$ by a factor of P, and denote by $\Delta = T/P$ the sampling period, a set of sequences with period T can be constructed according to $y^i(n) = y(t_0 + i\Delta + nT), 1 \le i \le P$. Assume that the channel response is limited to $L+1$ symbol periods, we have

$$y^i(n) = \sum_{l=0}^{L} h(t_0 + i\Delta + lT)s(n-l), \quad i = 1, \cdots, P.$$

Denoting $h^i(l) = h(t_0 + i\Delta + lT)$ yields

$$y^i(n) = \sum_{l=0}^{L} h^i(l)s(n-l), \quad i = 1, \cdots, P. \tag{1}$$

In an antenna array system, the system output is also *spatially* oversampled. An M-element array can produce M set of sequences given in (1)

$$y^{ij}(n) = \sum_{l=0}^{L} h^{ij}(l)s(n-l), \quad \begin{matrix} i = 1, \cdots, P \\ j = 1, \cdots, M \end{matrix}$$

where j denotes the antenna index.

Define

$$\begin{aligned} \mathbf{y}(n) &= \left[y^{11}(n), y^{12}(n), \cdots, y^{1M}(n), \cdots, \right. \\ &\quad \left. y^{P1}(n), y^{P2}(n), \cdots, y^{PM}(n)\right]^T; \\ \mathbf{h}(n) &= \left[h^{11}(n), h^{12}(n), \cdots, h^{1M}(n), \cdots, \right. \\ &\quad \left. h^{P1}(n), h^{P2}(n), \cdots, h^{PM}(n)\right]^T. \end{aligned} \tag{2}$$

A standard single-input and multiple-output (SIMO) system which accounts for both temporal and spatial oversampling results

$$\mathbf{y}(n) = \sum_{l=0}^{L} \mathbf{h}(l)s(n-l). \tag{3}$$

Clearly, the effective oversampling rate, e.g., the number of elements in the output or channel vector, is now PM.

In the presence of $d(d > 1)$ co-channel users, (3) becomes

$$\mathbf{y}(n) = \sum_{i=1}^{d} \mathbf{h}_i \otimes s_i(n) \tag{4}$$

where the subscript denotes the user's index. The problem under consideration is to estimate $\{\mathbf{h}_i\}$ and $\{s_i(n)\}$ from a finite number of system outputs $\mathbf{y}(1), \cdots, \mathbf{y}(N)$ without any statistical knowledge of the inputs.

For a single user system, there are several different ways of blind estimation. Optimal solutions in a bit error sense require joint estimation of the channels and inputs, and often involve Viterbi-type searching [26]–[28]. However, their advantages might be negated by the computational cost. Tong, Xu, and Kailath [15] showed that the output of an *oversampled* system as in (3) contains sufficient information for closed-form solutions of $\mathbf{h}(\cdot)$ and $s(\cdot)$. A class of data-efficient subspace-based blind estimation algorithms have been developed to

TABLE I
RECURSIVE BLIND IDENTIFICATION

Initialization:
Choose the smoothing factor K and the highest order of the channels L. Construct $\mathbf{Y}(K)$
Computer $\mathbf{V}_o(K)$ the null space of $\mathbf{Y}(K)$
For $l = L, \cdots, 1$, **if** $d_l \neq 0$, **do:**
Construct $\mathbf{V}(K+l)$ as in (7)
Calculate its null subspace $\mathbf{V}^{\perp}(K+l)$
Apply $\mathbf{V}^{\perp}(K+l)$ and $\mathbf{S}^{l+1}(2), \cdots, \mathbf{S}^{L}(L-l+1)$ to the Partial ILSP algorithm, identify $\mathbf{S}^{l}(1)$
Output:
The symbols from each user
The channels, which can be estimated by least-squares fitting: $\mathbf{H}(1) = \mathbf{Y}(1)\mathbf{S}(1)^H(\mathbf{S}(1)\mathbf{S}(1)^H)^{-1}$

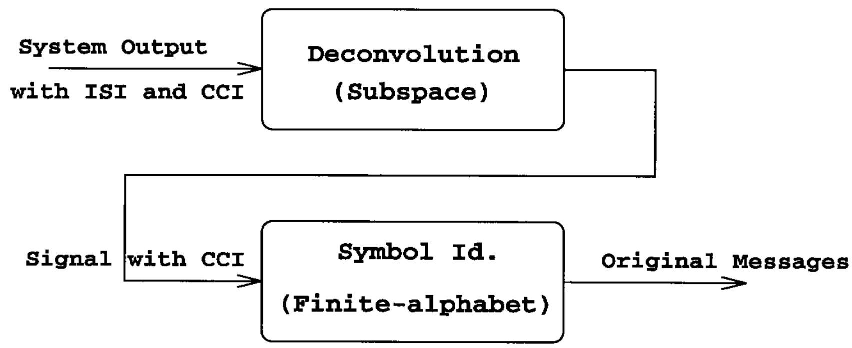

Fig. 2. Identification block diagram.

deconvolve the ISI effect and estimate the channels and/or input sequences directly from the data matrix. Unfortunately, for a multiuser system described in (4) where the outputs are clouded by both ISI and CCI, the subspace of the data matrix is no longer adequate to resolve the system [2], [25].

III. MULTIUSER BLIND CHANNEL IDENTIFICATION

We begin our derivation by considering the simplest case of a multiuser system where all channels have the same order. We present a two-stage blind estimation scheme by combining the techniques developed in [13] and [23]. Next, we extend our study to a general system and focus on the deconvolution of channels with nonidentical orders. Finally, we establish the complete identification procedure.

A. A Simple Case: Channels with Identical Orders

Given a finite number of data samples, the channel output vectors can be arranged in a matrix form in accordance with (4)

$$\mathbf{Y}(K) = \begin{bmatrix} \mathbf{y}(1) & \mathbf{y}(2) & \cdots & \mathbf{y}(N-K+1) \\ \mathbf{y}(2) & \mathbf{y}(3) & \cdots & \mathbf{y}(N-K+2) \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{y}(K) & \mathbf{y}(K+1) & \cdots & \mathbf{y}(N) \end{bmatrix} = \underbrace{[\mathbf{H}_1(K) \ \cdots \ \mathbf{H}_d(K)]}_{\mathbf{H}(K)} \underbrace{\begin{bmatrix} \mathbf{S}_1(K+L) \\ \vdots \\ \mathbf{S}_d(K+L) \end{bmatrix}}_{\mathbf{S}(K+L)} \quad (5)$$

where K is defined as the *smoothing factor*,

$$\mathbf{H}_i(K) = \underbrace{\begin{bmatrix} \mathbf{h}_i(L) & \cdots & \mathbf{h}_i(0) & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{h}_i(L) & \cdots & \mathbf{h}_i(0) \end{bmatrix}}_{K+L \text{ blocks}}$$

$$\mathbf{S}_i(K+L) = \begin{bmatrix} s_i(1-L) & \cdots & s_i(N-L-K+1) \\ s_i(2-L) & \cdots & s_i(N-L-K+2) \\ \vdots & \cdots & \vdots \\ s_i(K) & \cdots & s_i(N) \end{bmatrix}. \quad (6)$$

Note that $\mathbf{H}(K)$ is $PMK \times d(K+L)$ and $\mathbf{S}(K+L)$ is $d(K+L) \times (N-K+1)$. Clearly, if the effective oversampling rate, i.e., PM, is larger than the number of users d, there exists a finite K such that $\mathbf{H}(K)$ has more rows than columns. In the remainder of this paper, we shall refer to $\mathbf{H}(K)$ and $\mathbf{S}(K+L)$ as the multiuser *channel* matrix and *input* matrix, respectively. If either is identified, the other can easily be estimated by simple least-squares fitting.

Our approach to resolving a multiuser system is as follows: *Given the multiuser system outputs with both ISI and CCI, we first deconvolve the ISI using the subspace structure of the data matrix, and then determine the inputs from CCI by exploiting the finite-alphabet property. After all the inputs are available, the system is readily determined.* The cascade operations are depicted in Fig. 2.

In the following, we briefly describe the techniques which play a key role in our approach.

1) Multiuser Channel Deconvolution (MCD): With the assumption that the channel matrix $\mathbf{H}(K)$ is of full column rank, the row span of $\mathbf{S}(K+L)$ is equal to that of $\mathbf{Y}(K)$, i.e., $\mathcal{R}\{\mathbf{Y}(K)\} = \mathcal{R}\{\mathbf{S}(K+L)\}$. A least-squares approach that accomplishes channel deconvolution is provided in [25]. The method is summarized as follows.

1) Compute $\mathbf{V}_o(K)$—the null space of the row vectors of $\mathbf{Y}(K)$ and $\mathbf{S}(K+L)$.
2) Construct the following matrix:

$$\mathbf{V}(K+L) = \underbrace{\begin{bmatrix} \mathbf{V}_o(K) & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_o(K) & \cdots & \vdots \\ \vdots & \cdots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{V}_o(K) \end{bmatrix}}_{K+L \text{ blocks}} \quad (7)$$

where $\mathbf{0}$ is a column vector with number of zeros equalling the number of rows of $\mathbf{V}_o(K)$.
3) Calculate $\mathcal{R}\{\mathbf{S}(1)\}$ as the null space of $\mathbf{V}(K+L)$. In other words, $\mathbf{V}^{\perp}(K+L) = \mathbf{W}\mathbf{S}(1)$, where $\mathbf{W}$ is a $d \times d$ full rank matrix.
Note that

$$\mathbf{S}(1) = \begin{bmatrix} s_1(1-L) & s_1(2-L) & \cdots & s_1(N) \\ \vdots & \vdots & \vdots & \vdots \\ s_d(1-L) & s_d(2-L) & \cdots & s_d(N) \end{bmatrix}.$$

The inputs are readily determined by the last step if there is only one user. When $d > 1$, each individual symbol in $\mathbf{S}(1)$ is yet to be identified. However, the convolution effect of the channels that causes ISI has been removed. $\mathbf{V}^{\perp}(K+L)$ is now a linear transformation of a *synchronous* multiuser system $\mathbf{S}(1)$! The blind estimation problem is reduced to the identification of $\mathbf{S}(1)$ and $\mathbf{W}$ from $\mathbf{V}^{\perp}(K+L)$.

2) Symbol Identification Using the Finite-Alphabet Property: To identify $\mathbf{S}(1)$ from $\mathbf{V}^{\perp}(K+L)$, notice that for most digital communication signals, the information bearing symbols are from a finite alphabet. It is proved in [13] and [29] that given sufficient data samples, blind symbol estimation can be achieved almost surely. Let $\mathbf{U} = \mathbf{V}^{\perp}(K+L)$, we adopt the iterative least squares with projection (ILSP) algorithm proposed in [13] for symbol identification:

1) given $[\mathbf{S}_k(1)]_{ij}$ with each element a member of the finite alphabet;
2) $k := k+1$;
   a) $\mathbf{S}_{k+1}(1) = \mathbf{S}_k(1)\mathbf{S}_k^H(1)(\mathbf{U}\mathbf{S}_k^H(1))^{-1}\mathbf{U}$;
   b) project $[\mathbf{S}_{k+1}(1)]_{ij}$ to the closest symbol in the alphabet;
3) continue until $(\mathbf{S}_{k+1}(1) - \mathbf{S}_k(1)) = \mathbf{0}$.

3) Estimation Procedure: With MCD and ILSP, the blind estimation of a system with identical channel orders becomes more or less a solved problem. We can: 1) estimate $\mathbf{V}_o(K)$ from the data matrix and construct $\mathbf{V}(K+L)$ as in (7); then 2) calculate its null space and apply it to ILSP and identify the inputs; 3) if the channels need to be identified, they can be easily determined using least-squares fitting.

The above estimation scheme, although being simple and effective, may only have limited applications since natural channels can be more complex. 1) The channels may be of different orders and 2) the orders of the channels are not known *a priori*. In the following, we extend our analysis to general channels and present a recursive scheme to address the above practical problems.

B. General Case—Channel with Nonidentical Orders

The strategy for identifying a general system is similar to that of a system with identical channel orders, i.e., Step 1) deconvolve the outputs to remove the ISI, and Step 2) identify each symbol from CCI. Since the channels causing ISI are now much more complex, our focus in the following will be on Step 2).

Again, let d be the total number of users and $L+1$ be the maximum channel length. Rearrange the inputs and channels into L groups such that each group has d_l users with the same channel length, $l+1$. Obviously, d_l can be zero and $\sum_{l=1}^{L} d_l = d$.

Since each group is now a subsystem with identical channel orders, we thus define $\mathbf{H}^l(K)$ and $\mathbf{S}^l(K+l)$ as the channel and input matrix for the lth subsystem in accordance with (5), where the superscript l denotes the channel order. Using such notation, (5) can be modified accordingly for a general system

$$\begin{aligned} \mathbf{Y}(K) &= \sum_{l=1, d_l \neq 0}^{L} \mathbf{H}^l(K)\mathbf{S}^l(K+l) \\ &= \underbrace{[\mathbf{H}^1(K) \quad \cdots \quad \mathbf{H}^L(K)]}_{\mathbf{H}(K)} \underbrace{\begin{bmatrix} \mathbf{S}^1(K+1) \\ \vdots \\ \mathbf{S}^L(K+L) \end{bmatrix}}_{\mathbf{S}(K)}. \end{aligned} \quad (8)$$

The total number of rows of $\mathbf{S}(K)$ is $\sum_{l=1}^{L}(K+l)d_l$. It is not difficult to see that if the effective oversampling rate is larger than d, the overall channel matrix $\mathbf{H}(K)$ can be smoothed to have more rows than columns. We assume that $\mathbf{H}(K)$ is of full column rank and at the same time, there are sufficient data samples such that $\mathbf{S}(K)$ has many more columns than rows.

A *recursive* identification procedure is established in Table I. The details of each recursion are explained in the following sections.

1) Channel Deconvolution: First, we need to examine the feasibility of deconvolving the channel effects using the subspace structure of the data matrix. More specifically, we need to examine the identifiability of the row span of the input vectors using the null space $\mathbf{V}_o(K)$. This requires some additional discussion since unlike in the system with identical channel orders, $\mathbf{V}_o(K)$ is now only a subset of the null space of $\mathbf{S}^l(K+l), l = 1, \cdots, L$. The $\mathbf{V}(\cdot)$ matrices constructed from $\mathbf{V}_o(K)$ may not be sufficient to determine $\mathcal{R}\{\mathbf{S}^l(1)\}, l = 1, \cdots, L$.

To build intuition on the use of $\mathbf{V}(\cdot)$ for estimation, let $K = 1$ and consider a simple two-user system with channels of length two and three ($L=2, d_1=1, d_2=1$). By (8)

$$\mathbf{V}(1)=\mathbf{V}_o(1)\perp\begin{bmatrix} s_1(0) & s_1(1) & \cdots & s_1(N-1)\\ s_1(1) & s_1(2) & \cdots & s_1(N)\\ s_2(-1) & s_2(0) & \cdots & s_2(N-2)\\ s_2(0) & s_2(1) & \cdots & s_2(N-1)\\ s_2(1) & s_2(2) & \cdots & s_2(N)\end{bmatrix} = \begin{bmatrix}\mathbf{S}^1(2)\\ \mathbf{S}^2(3)\end{bmatrix}. \tag{9}$$

Recognizing the Hankel structure of $\mathbf{S}^1(2)$ and $\mathbf{S}^2(3)$, it can be easily shown that by some rearrangement similar to (7)

$$\begin{aligned}&\begin{bmatrix}\mathbf{V}_o(1) & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_o(1)\end{bmatrix}\perp\begin{bmatrix} s_1(0) & s_1(1) & \cdots & s_1(N)\\ s_2(-1) & s_2(0) & \cdots & s_2(N-1)\\ s_2(0) & s_2(1) & \cdots & s_2(N)\end{bmatrix}\\ &\quad\Rightarrow \mathbf{V}(2)\perp\begin{bmatrix}\mathbf{S}^1(1)\\ \mathbf{S}^2(2)\end{bmatrix}\\ &\begin{bmatrix}\mathbf{V}_o(1) & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_o(1) & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{V}_o(1)\end{bmatrix}\perp[s_2(-1)\quad s_2(0)\quad\cdots\quad s_2(N)]\\ &\quad\Rightarrow \mathbf{V}(3)\perp\mathbf{S}^2(1).\end{aligned} \tag{10}$$

Apparently, one can use $\mathbf{V}(3)$ to determine $\mathbf{S}^2(1)$, provided that they are complementary (for this particular example, ILSP is not necessary since $d_2=1$). Such is not the case for $\mathbf{S}^1(1)$ since $\mathbf{V}(2)$ is not its complete null space. However, $\mathbf{S}^2(\cdot)$ can be constructed from the already identified $\mathbf{S}^2(1)$; thus, it is possible to determine $\mathbf{S}^1(1)$ using the knowledge of $\mathbf{V}(2)$ and $\mathbf{S}^2(2)$.

Following this idea, let us consider a general system with more users. By the Hankel structure of $\mathbf{S}^l(K+l)$ and the fact that $\mathbf{V}(1)=\mathbf{V}_o(K)\perp\{\mathbf{S}^L(K+L),\cdots,\mathbf{S}^1(K+1)\}$, we may properly rearrange the corresponding blocks and obtain the following:

$$\begin{array}{cccccc}\mathbf{V}(K+1) & \perp & \mathbf{S}^L(L+1), & \mathbf{S}^{L-1}(L), & \cdots & \mathbf{S}^1(1);\\ \vdots & \vdots & \vdots & \vdots & & \\ \mathbf{V}(K+L-1) & \perp & \mathbf{S}^L(2), & \mathbf{S}^{L-1}(1); & & \\ \mathbf{V}(K+L) & \perp & \mathbf{S}^L(1). & & & \end{array} \tag{11}$$

It is seen that $\{\mathbf{V}(K+l), l=1,\cdots,L\}$ have *inherent* null vectors:

$$\mathbf{N}(l)\stackrel{\text{def}}{=}[\mathbf{S}^l(1)^T\quad \mathbf{S}^{l+1}(2)^T\quad\cdots\quad \mathbf{S}^L(L-l+1)^T]^T. \tag{12}$$

Given knowledge of $\mathcal{R}\{\mathbf{N}(l)\}, l=1\cdots,L$, one can expect to *recursively* identify the subsystems from $\mathbf{S}^L(1)$ to $\mathbf{S}^1(1)$. The question is whether or not $\mathcal{R}\{\mathbf{N}(l)\}$ can be uniquely determined from $\mathbf{V}_o(K)$. In other words, is $\mathcal{R}\{\mathbf{N}(l)\}$ the complete null space of $\mathbf{V}(K+l)$? The following theorem provides the answer to this question.

2) Theorem 1: $\mathcal{R}\{\mathbf{N}(l)\}=\mathcal{R}\{\mathbf{V}^{\perp}(K+l)\}$ provided that $\mathbf{S}(K+1)$ has full row rank.

Proof: See the Appendix.

Since most communication sequences are random in nature, the full rank condition of $\mathbf{S}(K+1)$ can easily be satisfied. Although more studies are required to find out how uncorrelatedness of the symbols ensures full-rank, our extensive computer simulations show that $\mathbf{S}(K+1)$ is *almost always* of full rank. The above theorem asserts that we can determine the row span of the inherent null vectors from the data matrix, which means we can achieve channel deconvolution with the subspace information. The significance of Theorem 1 also lies in the fact that it leads to a practical method of order detection for a system with nonidentical channel orders. More specifically, it is seen from (5) and (6) that $\mathbf{S}^l(i-l+1), l\le i\le L$ has $(i-l+1)d_i$ rows, consequently, the number of inherent null vectors of $\mathbf{V}(K+l)$ is $\Sigma_{i=l}^L(i-l+1)d_i$. Therefore, using this formula and the fact that $\text{Rank}\{\mathbf{V}^{\perp}(K+L)\}=d_L$ and $\text{Rank}\{\mathbf{V}^{\perp}(K+L+i)\}=0,\ i\ge 1$, we can recursively determined the number of users d_l with channels order l by observing the change of the dimension of the null space of $\mathbf{V}(K+l), l=1,\cdots,L$.

3) Partial ILSP Algorithm: Once the row spaces of $\{\mathbf{N}(l), l=1,\cdots,L\}$ are available, the determination of $\mathbf{S}_L(1)$ is no different than the identification of a system with identical channel orders. However, the ILSP algorithm in the succeeding recursions ($l<L$) needs to be modified, since set of the symbols in $\mathbf{N}(l)$ that corresponds to the subsystems with higher order channels has already been identified. Therefore, this part of symbols should be used as *a priori* knowledge to help identifying the unknown symbols.

Let $\mathbf{P}$ be the *unknown* symbol matrix and let $\mathbf{Q}$ be the *known* symbol matrix. Denote $\mathbf{O}=\mathbf{W}[\mathbf{P}^T\quad\mathbf{Q}^T]^T$, where $\mathbf{W}$ is a full rank square matrix, a partial ILSP method which identifies $\mathbf{P}$ given $\mathbf{O}$ and $\mathbf{Q}$ can be easily extended from the original ILSP algorithm. Thus

1) given $[\mathbf{P}_k]_{ij}$ with each element a member of the finite alphabet;
2) $k := k+1$;
a) $\mathbf{P}_{k+1}=\mathbf{P}_k[\mathbf{P}_k^H\quad\mathbf{Q}^H](\mathbf{O}[\mathbf{P}_k^H\quad\mathbf{Q}^H])^{-1}\mathbf{O}$;
b) project $[\mathbf{P}_{k+1}]_{ij}$ to the closest symbol in the alphabet;
3. continue until $(\mathbf{P}_{k+1}-\mathbf{P}_k)=\mathbf{0}$.

In each iteration, the fixed symbol matrix $\mathbf{Q}$ guides $\mathbf{P}_k$ to the right direction. This ILSP may converge faster than its original version.

4) Recursive Blind Identification: We have explained all the key operations for the recursive blind identification (RBI) scheme for general systems. The estimation procedures in Table I can now be well understood.

1) The aim of the initialization is: 1) to choose a proper smoothing factor such that the channel matrix has full column rank. This allows the row span of the input matrix $\mathbf{S}(K)$ to be shared by the data matrix $\mathbf{Y}(K)$. 2) Preselect L, which is the highest order of the channels, based on previous knowledge. 3) Calculate the null space of $\mathbf{Y}(K)$, $\mathbf{V}_o(K)$, which contains information of the span of the *inherent* null vectors.
2) The initial step provides all necessary information to deconvolve the CCI and remove the ISI. The next part of RBI is a loop indexed by l which gradually reduces the channel orders and determine all the subsystems. In each loop, the rank of $\mathbf{V}(K+l)$ is first examined to determine whether or not $d_l=0$, $\mathcal{R}\{\mathbf{N}(l)\}$ can then be calculated as the null space of $\mathbf{V}(K+l)$. Since $\mathbf{S}^{l+1}(1),\cdots,\mathbf{S}^L(1)$

have already been identified from the previous loops, we can apply $\mathbf{V}^{\perp}(K+l)$ and the identified inputs to partial ILSP algorithm and obtain $\mathbf{S}^{l}(1)$.

3) When the loop is completed, the channels can be identified by least-squares fitting.

IV. IMPLEMENTATIONS

So far, most of our attention has been focused on the development of the estimation algorithm. While the RBI scheme is quite self-contained for blind estimation in most scenarios, certain types of channelc may be degenerate and cause a failure in estimation. In this section, we study in more detail the implementation aspects of the proposed algorithm.

In order for the proposed BCI scheme to be valid, the channel matrix $\mathbf{H}(K)$ must be of full column rank. Since $\mathbf{H}(K)$ can be rearranged into a generalized Sylvester resultant matrix by padding zeros to shorter channels, its rank can be calculated from [30, eq. (30)], where the rank condition of a generalized Sylvester resultant matrix is studied. However, such a formula is not so helpful without a physical interpretation. In the following, we simplify the discussion by analyzing the fundamental structure of the channel matrix for a *single* user. It is reasonable to believe that if each single channel matrix is of full column rank, then the overall multiuser channel matrix should be of full rank, since channels from different users are not correlated in general.[1]

For later reference, we recall an important lemma regarding the rank condition of the channel matrix [15]:

Lemma 1: Let $\mathbf{H}(K)$ be the channel matrix constructed from $\mathbf{h}(0),\cdots,\mathbf{h}(L)$, and $h_i(z) = h_i(0) + h_i(1)z + \cdots + h_i(L)z^{L-1}$, $i = 1,\cdots,M$, where $h_i(n)$ is the ith element of $\mathbf{h}(n)$. $\mathbf{H}(K)$ can be smoothed to be of full column rank *iff* polynomials $\{h_i(z)\}$ do not share any common roots.

A. Short-Delay Multipath Channels

In some wireless systems, e.g., an indoor wireless system, the propagation channels are frequency-nonselective [31]. In other words, all the multipath components corresponding to one user can be regarded as coherent. In such cases, the continuous antenna output can be represented as

$$\mathbf{y}(t) = \mathbf{a}_i w_i(t)$$

where $\mathbf{a}_i$ is defined as the *spatial signature* of the ith user and $w_i(t)$ is the signal waveform from the ith user. For linear modulation with pulse function $p(t)$, $w_i(t) = \sum_{n=-\infty}^{\infty} p(t - nT)s(n)$.

One of the most important applications of the proposed approach is to estimate the spatial signatures without knowing the inputs. It is worth pointing out that in practical situations, $\{w_i(t)\}$ from different users cannot be synchronous at a bit-level in general. Therefore, in the case of multiple users, $\mathbf{y}(n)$, the discrete counterpart of $\mathbf{y}(t)$, contains both CCI and ISI regardless of the sample timing. The spatial signature estimation problem is a special case of BCI.

[1] If a single user channel matrix is of full column rank, it is always of full column rank with a larger smoothing factor [15].

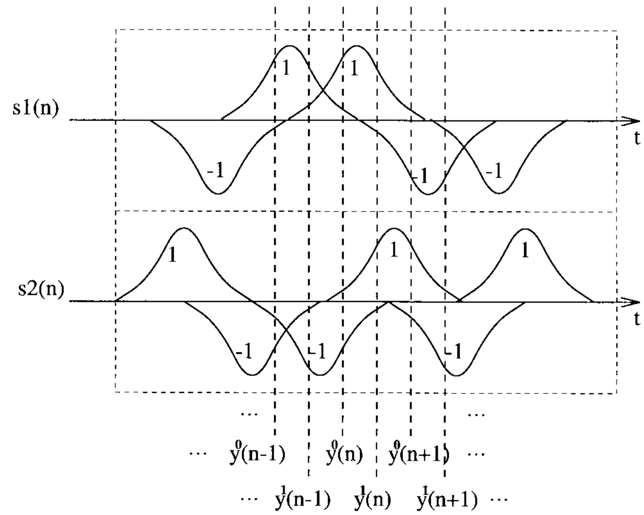

Fig. 3. Asynchronous BPSK signals with $P = 2$.

1) Spatial Oversampling Only: Fig. 3 depicts the waveforms of two asynchronous BPSK signals. Let us examine the effective channels for each user. If the output from one antenna $y(t)$ is sampled at the symbol rate, each point in the data sequence ($\{y^0(n)\}$ or $\{y^1(n)\}$) will be affected by only two adjacent symbols, provided that the energy of the pulse function is limited to two symbol periods. Therefore, the effective channel order is almost always two. The discrete array output can thus be writted as

$$\begin{aligned}\mathbf{y}(n) &= \mathbf{h}(0)s(n) + \mathbf{h}(1)s(n-1)\\ &= \mathbf{a}(\alpha_1 s(n) + \alpha_2 s(n-1))\end{aligned} \tag{13}$$

where α_1, α_2 are complex scalars. Equation (13) suggests that the channel vectors $\mathbf{h}(0) = \alpha_1\mathbf{a}$ and $\mathbf{h}(1) = \alpha_2\mathbf{a}$ are aligned. Clearly, by Lemma 1, the channel matrix constructed from $\mathbf{h}(0)$ and $\mathbf{h}(1)$ is always rank deficient regardless of the smoothing factor.

2) Spatial and Temporal Oversampling: Now let us consider the case with both temporal and spatial oversampling. When the antenna output is temporal oversampled by a factor of two, as illustrated in Fig. 3, then by stacking two neighboring array output vectors as in (2), we obtain

$$\mathbf{y}(n) = \begin{bmatrix}\mathbf{y}^0(n)\\ \mathbf{y}^1(n)\end{bmatrix} = \begin{bmatrix}\mathbf{a}(\alpha_{11}s(n) + \alpha_{12}s(n-1))\\ \mathbf{a}(\alpha_{21}s(n) + \alpha_{22}s(n-1))\end{bmatrix} \tag{14}$$

or

$$\mathbf{y}(n) = \begin{bmatrix}\mathbf{y}^1(n)\\ \mathbf{y}^0(n+1)\end{bmatrix} = \begin{bmatrix}\mathbf{a}(\alpha_{21}s(n) + \alpha_{22}s(n-1))\\ \mathbf{a}(\alpha_{11}s(n+1) + \alpha_{12}s(n))\end{bmatrix} \tag{15}$$

depending on the choice of the starting point. Consequently, the channels can be expressed as

$$[\mathbf{h}(0) \quad \mathbf{h}(1)] = \begin{bmatrix}\alpha_{11}\mathbf{a} & \alpha_{21}\mathbf{a}\\ \alpha_{12}\mathbf{a} & \alpha_{22}\mathbf{a}\end{bmatrix} \tag{16}$$

or

$$[\mathbf{h}(0) \quad \mathbf{h}(1) \quad \mathbf{h}(2)] = \begin{bmatrix}\mathbf{0} & \alpha_{21}\mathbf{a} & \alpha_{22}\mathbf{a}\\ \alpha_{11}\mathbf{a} & \alpha_{12}\mathbf{a} & \mathbf{0}\end{bmatrix}. \tag{17}$$

TABLE II
EXPERIMENTAL RESULTS FOR SHORT-DELAY MULTIPATH CHANNELS

# Data Vectors	M	P	K	# Sources	Baud Rate	Modulation
50	8	2	1	2	50 KHz	BPSK
Exp. #	delay 1 [T]	delay 2 [T]	$\|\|\hat{\mathbf{a}}_1 - \mathbf{a}_1\|\|$	$\|\|\hat{\mathbf{a}}_2 - \mathbf{a}_2\|\|$	RMSE (SIR): $\hat{s}_1$	RMSE (SIR): $\hat{s}_2$
1	0.4	0.7	0.31%	1.32 %	0.0326 (29.7[dB])	0.0339 (29.4[dB])
2	0.2	0.9	0.28%	0.67 %	0.0219 (33.2[dB])	0.0537 (25.4[dB])

Either way, the channel vectors are not aligned, which means that the channel matrix can be smoothed to be of full column rank.

3) Property 1: A system with only coherent multipath CANNOT be identified using the spatial oversampled data. Temporal oversampling has to be used to restore the rank of the channel matrix.

By applying the proposed algorithm, the channels for each user can be determined. One can rearrange its corresponding parts according to (16) or (17) into

$$[\alpha_{11}\mathbf{a} \quad \alpha_{12}\mathbf{a} \quad \alpha_{21}\mathbf{a} \quad \alpha_{22}\mathbf{a}].$$

The spatial signature $\mathbf{a}$ can be estimated as the left principal singular vector of the constructed matrix.

4) Long-Delay Multipath Channels: For wireless systems with long-delay multipath (frequency-selective fading), denote $\tau_1 \leq \tau_2 \leq \cdots \leq \tau_r, \tau_1 < T$ the ordered delays. For the single-user case, it is straightforward to show that the array output without temporal oversampling is given by

$$\mathbf{y}(n) = \sum_{i=1}^{r} \mathbf{a}(i)\Big(\alpha_{i1}s\Big(n - \Big\lceil\frac{\tau_i}{T}\Big\rceil\Big) + \alpha_{i2}s\Big(n - \Big\lceil\frac{\tau_i}{T}\Big\rceil - 1\Big)\Big) \tag{18}$$

where $\mathbf{a}(i)$ is the array response vector corresponding to the ith delay and $\lceil x \rceil$ stands for the smallest integer that is greater than or equal to x. The effective channel length is $\lceil\frac{\tau_r}{T}\rceil + 2$. In contrast to (13), uncorrelated $\{\mathbf{a}(i)\}$ are involved, so the composite channels $\mathbf{h}(0)\cdots\mathbf{h}(\lceil\frac{\tau_r}{T}\rceil)$ are thus coprime in general.

5) Property 2: A system with long-delay multipath components can be identified from the spatially oversampled data. Temporal oversampling is not required.

V. EXPERIMENTAL RESULTS

To emphasize the practicality of the RBI scheme, we applied it to some real data collected from RF field experiments. Our facilities include an eight-element uniform linear antenna array at the base station, several remote transceivers and a central control unit equipped with data acquisition boards. The RF system operates at 900 MHz. The baseband signals are 50 K symbols/s BPSK sequences with raised-cosine pulse shaping. Multiple transmitters, positioned at roughly 30 m away from the antenna array, were used to generate a co-channel multiple access environment.

In each experiment, 50 data vectors were applied to the proposed RBI algorithm. In order to evaluate the new approach, we used the identified channels to equalize the system outputs. The scatter plot of the equalized signals provides a graphic illustration of the estimation results.[2] Root mean square error (RMSE) of the symbol estimates $(\sqrt{\sum_{i=1}^{50}\|\hat{s}(n) - s(n)\|^2/50})$ is utilized to quantify the estimation performance. Equivalently, one can calculate the signal-to-interference ratio (SIR) as $20\log_{10}(\mathrm{RMSE})$.

A. Short-Delay Multipath Channels

We transmitted two asynchronous BPSK sequences with equal power from two remote antennas placed 20° apart. The SIR for each signals before spatial separation was about 0 dB. Reflections from surrounding generated a typical flat fading scenario. The superposition of these two co-channel signals and their multipath components was received by the antenna array and sampled at twice the symbol rate. Therefore, the effective oversampling rate is 16. The experimental setup and processing results, e.g., percentage errors of the spatial signature estimates, RMSE's of the signal estimates and SIR's after separation, are summarized in Table II.

The top plot of Fig. 4 depicts the singular value distribution of the acquired data matrix $\mathbf{Y}(1)$. The first four values are distinctively larger than the others which suggests that the total channel order is four. It is adequate to conclude that both channels have a channel order of two. The singular values distribution of $\mathbf{V}(K+1)$ provides ever finer discretization (Fig. 4, bottom plot). The last two singular values, which are much smaller than the others, imply that both channels are of order two.

The estimation procedures in Sections III-A and IV-A were applied to estimate the spatial signatures, $\hat{\mathbf{a}}_1$ and $\hat{\mathbf{a}}_2$. We measured the spatial signatures $\mathbf{a}_1$ and $\mathbf{a}_2$ by letting one user transmit at a time and calculating its spatial signature based on the array outputs. Using $(1 - \frac{\|\hat{\mathbf{a}}_i^H \mathbf{a}_i\|^2}{\|\hat{\mathbf{a}}_i\|^2\|\mathbf{a}_i\|^2})$ as the performance measure, the estimation errors are only 0.31% and 1.32%. Fig. 5 compares the scatter plots from the antenna outputs with the scatter plots of the separated sources using the spatial filter designed from the spatial signatures [32]. Clearly, both sources have been successfully recovered (above 25 dB gain in SIR; see Table II).

We repeat the above experiment with different parameters and the results are listed in Table II.

B. Long-Delay Multipath Channels

In the same experimental environment, we added two more transmitters to generate *artificial* long-delay multipath signals. Specifically, we let transmitters 1 and 2 transmit $s_1(t)$ with

[2] There exists a perfect finite length equalizer which can invert a multichannel FIR filter. This specificity was already pointed out in several places [20], [21].

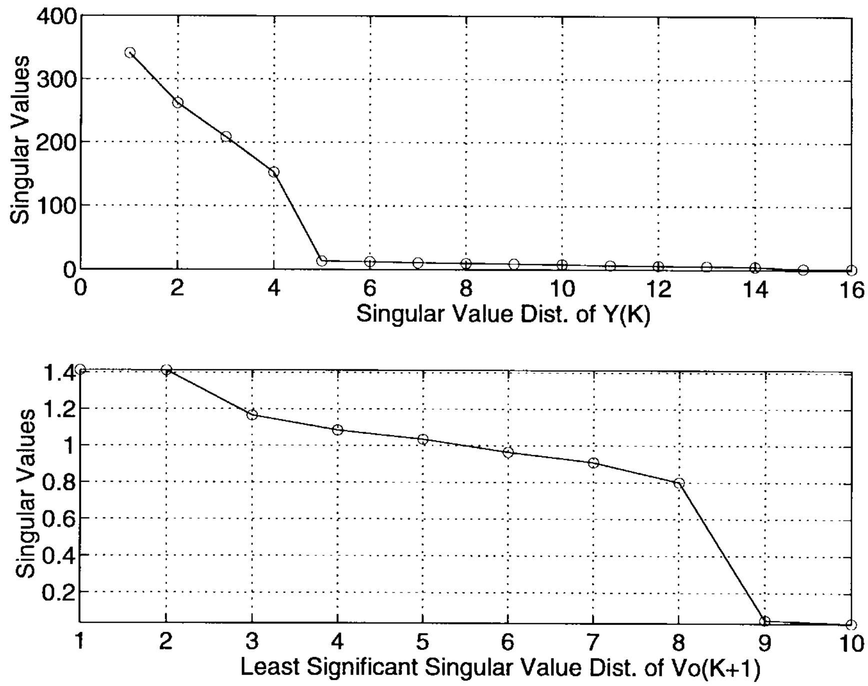

Fig. 4. Singular value distribution: short-delay multipath channels.

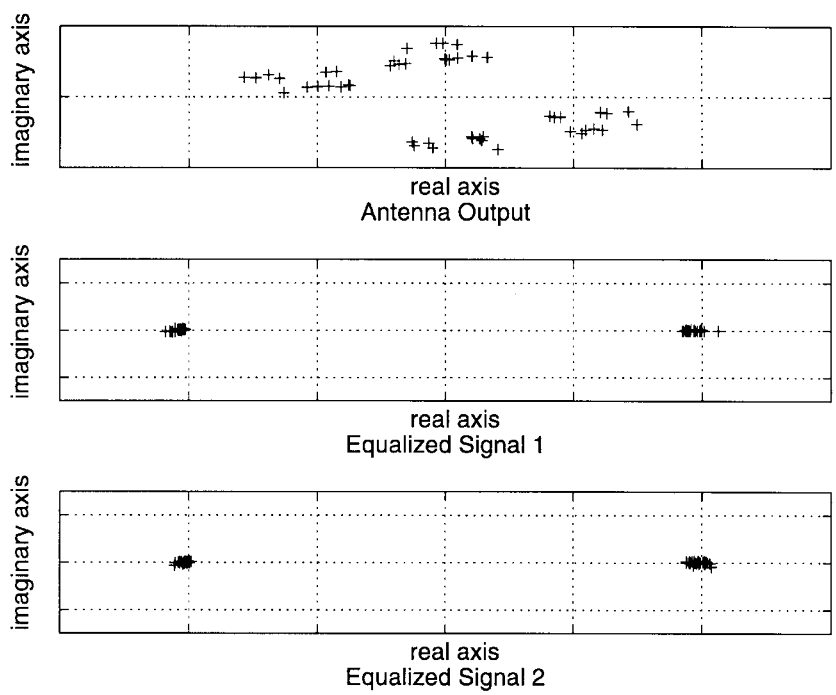

Fig. 5. Signal constellation: short-delay multipath channels.

different time delays. At the same time, transmitter 3 and 4 were used to transmit $s_2(t)$ with another set of delays. The delay parameters were selected according to [33], in which the authors showed that almost all delays in a wireless environment are within 10 μs (10 μs equals 0.5 symbol period at our transmitting rate, and around 0.26 and 2.0 symbol periods for IS-54 and GSM, respectively). The transmitting power from each transmitter was adjusted according to the delays and the distance between the transmitters and the base-station. The experimental setup and results are summarized in Table III. It is important to point out that our scenarios do not necessarily represent the wireless environments in real applications. Our sole purpose here is to evaluate the proposed algorithm on real data.

In the first two experiments, only spatial oversampling was used. In each experiment, we studied the rank conditions of

TABLE III
EXPERIMENTAL RESULTS FOR LONG-DELAY MULTIPATH CHANNELS

# Data Vectors	M	P	K	# Sources	Baud Rate	Modulation
50	8/2	1/5	1	2	50KHz	BPSK

	Source # 1		Source #2			
Exp. #	delay 1 [T]	delay 2 [T]	delay 1 [T]	delay 2 [T]	RMSE (SIR): $\hat{s}_1$	RMSE (SIR): $\hat{s}_2$
1	0.1	0.7	0.2	0.8	0.0365 (29.8[dB])	0.0747 (22.5[dB])
2	0.3	1.2	0.1	0.2	0.0897 (20.1[dB])	0.0383 (28.3[dB])
3	0.1	0.7	0.2	0.8	0.0566 (25.0 [dB])	0.0745 (22.6[dB])

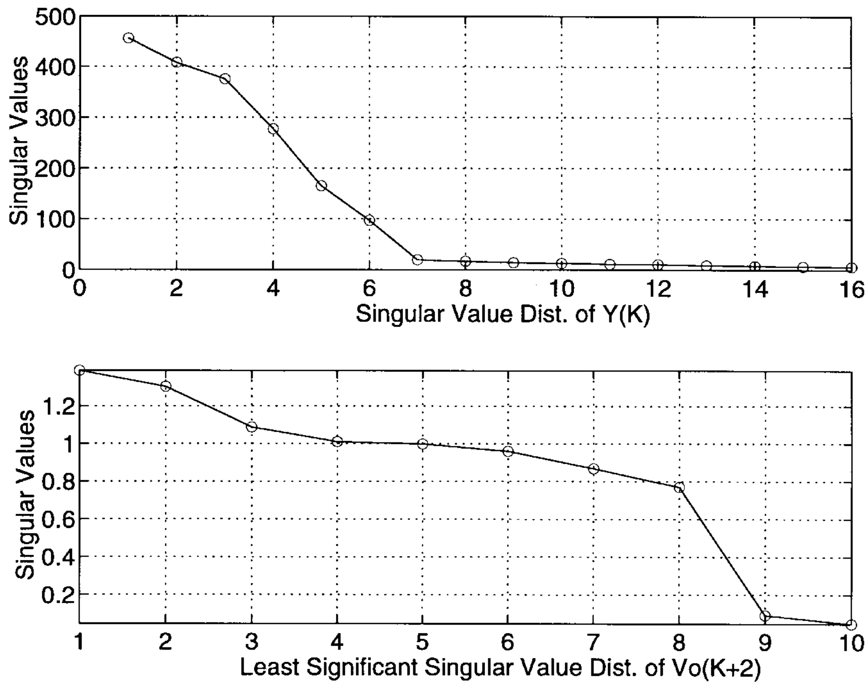

Fig. 6. Singular value distribution: long-delay multipath channels.

the data matrix $\mathbf{Y}(K)$ and $\{\mathbf{V}(\cdot)\}$ to determine the channel orders.

The singular value distribution of $\mathbf{Y}(K)$ and $\mathbf{V}(K+2)$ for the first experiment are shown in the top and bottom plots of Fig. 6. The six distinctively larger singular values of $\mathbf{Y}(K)$ suggests that both channels are of order three. This is verified by the two least significant singular values of $\mathbf{V}(K+2)$. Therefore, no recursion is necessary. ILSP was applied to the null space of $\mathbf{V}(K+2)$ and Fig. 7 gives the comparison of scatter plots before and after source separation and equalization were performed.

In the second experiment, we first examined the singular value distribution of the data matrix depicted on the top part of Fig. 8. The decay after the fifth value implied that the total channel order is five. We thus knew that there must be a channel with order three and the other with order two. This was verified by the singular value distribution of $\mathbf{V}(K+2)$ and $\mathbf{V}(K+1)$ in the middle and bottom of Fig. 8. We applied the proposed RBI method to identify the inputs and channels. The channels were then used to separate and equalize the original data from the antenna outputs. The comparison of the results are illustrated in Fig. 9.

Finally, we repeated the first experiment with a different oversampling rate. This time, the outputs from only two antennas from the array were used, and the temporal oversampling rate was raised to five. Since we already knew that the total order of the channels was six, which was much smaller than the effective oversampling rate ten, we could directly use the original data without smoothing. Fig. 10 gives the comparison results. It is seen that the performance is comparable to that of the first experiment, which indicates that there is sufficient temporal diversity among the channels.

VI. CONCLUSION

In this paper, we have studied the uplink multiuser channel and sequence estimation problem for an SDMA wireless system. We developed a general framework for the array output with both ISI and CCI, based on which we showed that source separation can be accomplished by exploiting the spatial and temporal diversities among the users. A data-

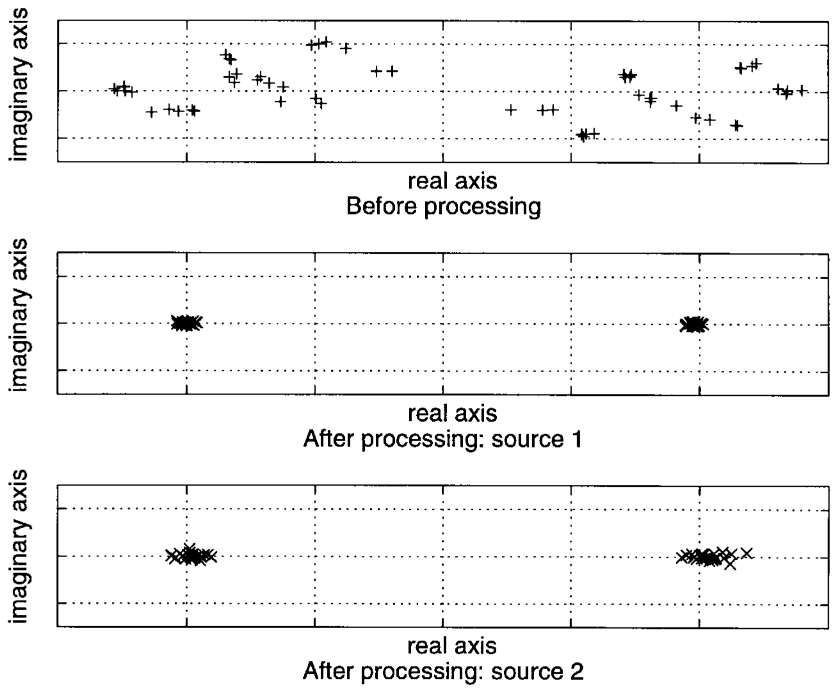

Fig. 7. Signal constellation: long-delay multipath channels.

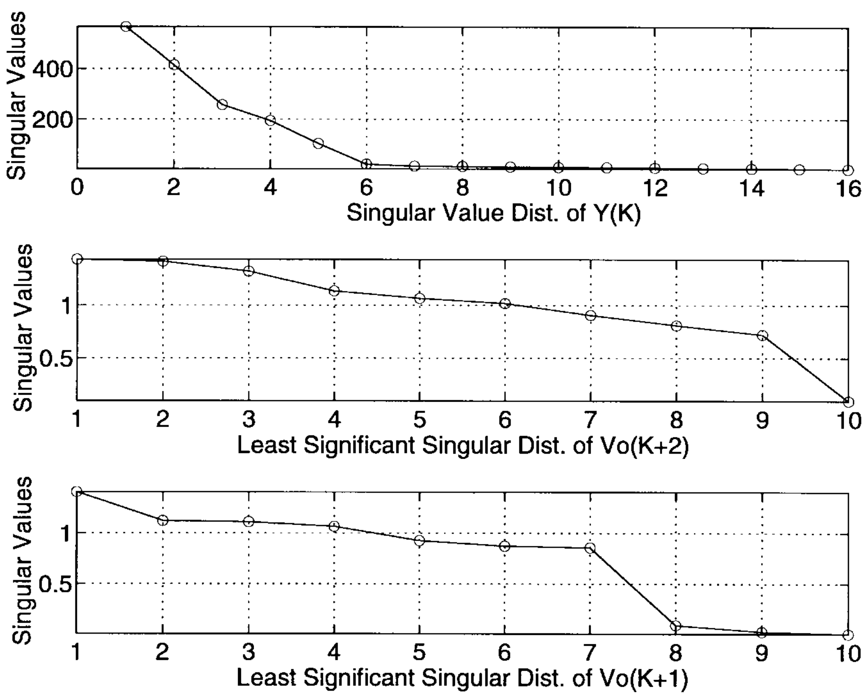

Fig. 8. Singular value distribution: long-delay multipath channels.

efficient algorithm was derived to recursively identify each spatial channel and the uplink symbols. Experimental results show the proposed method can effectively resolve systems with both short-delay multipath (flat fading) and long-delay multipath (frequency-selective fading).

Appendix
Proof of Theorem 1

To prove this theorem, we first introduce an important lemma concerning multiple Hankel matrices.

Lemma 2: Define $\mathbf{D}(l, \mathbf{a})$ as a *finite* Hankel matrix with N columns formed by the elements of a vector $\mathbf{a}$

$$\mathbf{D}(l, \mathbf{a}) = \begin{bmatrix} a(1) & a(2) & \cdots & a(N) \\ a(2) & a(3) & \cdots & a(N+1) \\ \vdots & \vdots & \cdots & \vdots \\ a(l) & a(l+1) & \cdots & a(N+l-1) \end{bmatrix}.$$

Assume that $[\mathbf{D}^T(l_1+1, \mathbf{a}_1), \cdots, \mathbf{D}^T(l_P+1, \mathbf{a}_P)]^T$ is of full row rank. If there exists another vector $\mathbf{b}$ such that

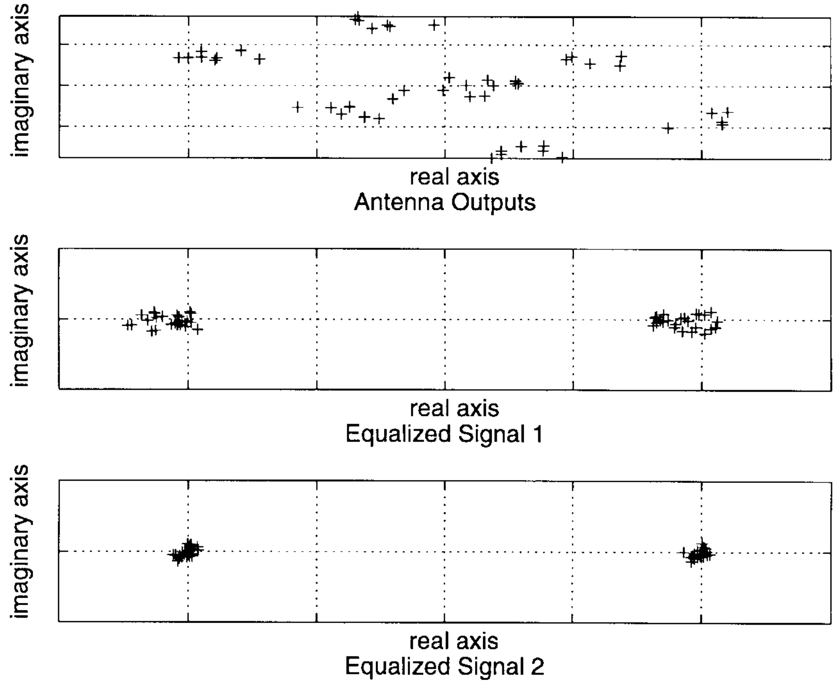

Fig. 9. Signal constellation: long-delay multipath channels.

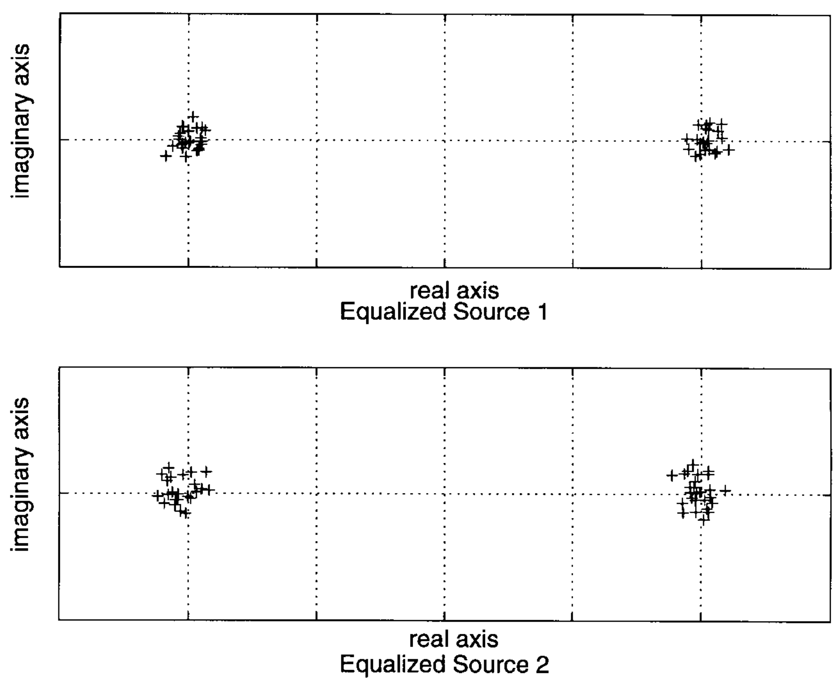

Fig. 10. Signal constellation: long-delay multipath channels.

$\mathbf{D}(L, \mathbf{b}) = \sum_{j=1}^{P} \mathbf{W}_j \mathbf{D}(l_j, \mathbf{a}_j),\ l \geq l_j$, then

$$\begin{cases} \mathbf{W}_j &= \alpha_j \mathbf{I}, \quad \text{if } l_j = L \\ \mathbf{W}_j &= \mathbf{0}, \quad \text{if } l_j < L. \end{cases}$$

Proof: We use $\bar{\mathbf{c}}$ ($\underline{\mathbf{c}}$) to denote the vector $\mathbf{c}$ without the first (last) element. Define $\mathbf{d}(\mathbf{a}_j)_i$ the ith row of $\mathbf{D}(l_j, \mathbf{a}_j)$. Since $\mathbf{D}(L, \mathbf{b}) = \sum_{i=1}^{P} \mathbf{W}_i \mathbf{D}(l_i, \mathbf{a}_i)$.

For any consecutive rows in the above equation ($i = 2, \cdots, L$), we have

$$\mathbf{d}(\mathbf{b})_{i-1} = \sum_{j=1}^{P} \left(\sum_{k=1}^{l_j} \mathbf{W}_j(i-1, k) \mathbf{d}(\mathbf{a}_j)_k \right)$$
$$\mathbf{d}(\mathbf{b})_{i} = \sum_{j=1}^{P} \left(\sum_{k=1}^{l_j} \mathbf{W}_j(i, k) \mathbf{d}(\mathbf{a}_j)_k \right) \tag{A.1}$$

where $\mathbf{W}_j(i,k)$ is the element of $\mathbf{W}_j$ at the ith row and kth column. Consequently,

$$\begin{aligned}\bar{\mathbf{d}}(\mathbf{b})_{i-1} &= \sum_{j=1}^{P}\left(\sum_{k=1}^{l_j}\mathbf{W}_j(i-1,k)\bar{\mathbf{d}}(\mathbf{a}_j)_k\right)\\ \underline{\mathbf{d}}(\mathbf{b})_i &= \sum_{j=1}^{P}\left(\sum_{k=1}^{l_j}\mathbf{W}(i,k)\underline{\mathbf{d}}(\mathbf{a}_j)_k\right).\end{aligned}\tag{A.2}$$

Using the fact that $\bar{\mathbf{d}}(\cdot)_{i-1} = \underline{\mathbf{d}}(\cdot)_i$ for a Hankel matrix, we obtain

$$\begin{aligned}\sum_{j=1}^{P}\Bigg(\mathbf{W}_j(i,1)\underline{\mathbf{d}}(\mathbf{a}_j)_1 + \sum_{k=2}^{l_j}(\mathbf{W}(i,k) - \mathbf{W}(i-1,k-1))\\ \times\, \underline{\mathbf{d}}(\mathbf{a}_j)_k + \mathbf{W}(i-1,l_j)\bar{\mathbf{d}}(\mathbf{a}_j)_{l_j}\Bigg) = \mathbf{0}.\end{aligned}\tag{A.3}$$

Under the assumption that the rows of $[\mathbf{D}^T(l_1+1,\mathbf{a}_1),\cdots,\mathbf{D}^T(l_P+1,\mathbf{a}_P)]^T$ are linearly independent, and noting that $\underline{\mathbf{d}}(\mathbf{a}_j)_1, \underline{\mathbf{d}}(\mathbf{a}_j)_2,\cdots,\underline{\mathbf{d}}(\mathbf{a}_j)_{l_j}, \bar{\mathbf{d}}(\mathbf{a}_j)_{l_j}$ are the l_j+1 rows of $\mathbf{D}(l_j+1,\mathbf{a}_j)$, all the coefficients in (A.3) must be zero

$$\begin{cases}\mathbf{W}_j(i,1) = 0;\\ \mathbf{W}_j(i,k) = \mathbf{W}_j(i-1,k-1), \quad k=2,\cdots,l_j;\ i=2,\cdots,L\\ \mathbf{W}_j(i-1,l_j) = 0.\end{cases}\tag{A.4}$$

Considering all the possible i values, we can easily verify that $\mathbf{W}_j = \alpha\mathbf{I}$ if $l_j = L$ and $\mathbf{W}_j = \mathbf{0}$ if $l_j < L$. □

Theorem 1 is equivalent to that $\mathbf{V}(K+l)\mathbf{a}^H = \mathbf{0}$ leads to $\mathbf{a} = \sum_i \alpha_i \mathbf{n}_i$ for any $\mathbf{a}$, where $\mathbf{n}_i$ is one of the inherent vectors.

From $\mathbf{V}(K+l)\mathbf{a}^H = \mathbf{0}$ and the Hankel block structure of $\mathbf{V}(K+l)$, one can easily obtain that

$$\underbrace{\begin{bmatrix} a(1) & a(2) & \cdots & a(N)\\ a(2) & a(3) & \cdots & a(N+1)\\ \vdots & \vdots & \cdots & \vdots\\ a(K+l) & a(K+l+1) & \cdots & a(N+K+l-1)\end{bmatrix}}_{\mathbf{D}(K+l,\mathbf{a})} \times \mathbf{V}_o(K)^H = \mathbf{0}.$$

Therefore, $\mathcal{R}\{\mathbf{D}(l+K,\mathbf{a})\} \subset \mathcal{R}\{\mathbf{S}(K)\}$.

Note that the row span of $\mathbf{S}(K)$ is the union of the $(K+l)$-row Hankel matrices formed from the inherent null vectors $\{\mathbf{n}_i\}$, and the $(K+l_i)$-row Hankel matrices formed from input vectors $\{s_i\}$ whose corresponding channel order $l_i < l$. Lemma 2 asserts that $\mathbf{D}(K+l,\mathbf{a}) = \sum_i \alpha_i \mathbf{D}(\mathbf{n}_i, K+l)$. Comparing each element of both size, we obtain $\mathbf{a} = \sum_i \alpha_i \mathbf{n}_i$, which completes the proof.

Acknowledgment

The authors would like to thank S. Wilson at the University of Virginia and the anonymous reviewers for insightful comments and helpful critiques of the manuscript.

References

[1] J. H. Winters, J. Salz, and R. D. Gitlin, "The capacity of wireless communication systems can be substantially increased by the use of antenna diversity," in *Proc. Conf. Inform. Sci. Syst.*, Princeton, NJ, 1992.

[2] G. Xu, H. Liu, L. Tong, and T. Kailath, "A least-squares approach to blind channel identification," *IEEE Trans. Signal Processing*, vol. 43, pp. 2982–2993, Dec. 1995.

[3] S. Andersson, M. Millnert, M. Viberg, and B. Wahlberg, "An adaptive array for mobile communication systems," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 230–236, Feb. 1991.

[4] S. C. Swales, M. A. Beach, D. J. Edwards, and J. P. McGreehan, "The performance enhancement of multibeam adaptive base-station antennas for cellular land mobile radio systems," *IEEE Trans. Veh. Technol.*, vol. 39, pp. 56–67, Feb. 1990.

[5] J. H. Winters, J. Salz, and R. D. Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Trans. Commun.*, vol. 42, pp. 1740–1751, Feb./Mar./Apr. 1994.

[6] P. Balaban and J. Salz, "Optimum diversity combining and equalization in digital data transmission with applications to cellular mobile radio—Part I: Theoretical considerations," *IEEE Trans. Commun.*, vol. 40, pp. 885–894, May 1992.

[7] M. V. Clark, L. J. Greenstein, W. K. Kennedy, and M. Shafi, "NMSE diversity combining for wide-band digital cellular radio," *IEEE Trans. Commun.*, vol. 40, pp. 1128–1135, June 1992.

[8] P. S. Henry and B. S. Glance, "A new approach to high capacity digital mobile radio," *Bell Syst. Tech. J.*, vol. 60, Oct. 1981.

[9] J. Yang and S. Roy, "Joint transmitter-receiver optimization for multi-input multi-output systems with decision feedback," *IEEE Trans. Inform. Theory*, vol. IT-40, pp. 1334–1347, Sept. 1994.

[10] M. L. Honig, P. Crespo, and K. Steiglitz, "Optimization of pre- and post-filters in the presence of near- and far-end crosstalk," *IEEE J. Select Areas Commun.*, vol. 10, pp. 614–629, Apr. 1992.

[11] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," in *Proc. RADC Spectrum Estimat. Workshop*, Griffiss AFB, NY, 1979, pp. 243–258.

[12] R. Roy and T. Kailath, "ESPRIT—Estimation of signal parameters via rotational invariance techniques," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, pp. 984–995, July 1989.

[13] S. Talwar, M. Viberg, and A. Paulraj, "Blind estimation of multiple co-channel digital signals using an antenna array," *IEEE Signal Processing Lett.*, vol. 1, pp. 29–31, Feb. 1994.

[14] Y. Sato, "A method of self-recovering equalization for multilevel amplitude-modulation," *IEEE Trans. Commun.*, vol. COM-23, pp. 679–682, June 1975.

[15] L. Tong, G. Xu, and T. Kailath, "Blind identification and equalization based on second-order statistics: A time domain approach," *IEEE Trans. Inform. Theory*, vol. 40, Mar. 1994.

[16] H. Liu and G. Xu, and L. Tong, "A deterministic approach to blind equalization," in *Proc. 27th Asilimar Conf. Signals, Syst. Comput.*, Pacific Grove, CA, Nov. 1993, pp. 751–755.

[17] J. K. Tugnait, "On blind identifiability of multipath channels using fractional sampling and second-order cyclostationary statistics," in *Proc. Global Telecom. Conf.*, 1993, pp. 2000–2005.

[18] L. Tong, G. Xu, B. Hassibi, and T. Kailath, "Blind identification and equalization based on second-order statistics: A frequency domain approach," *IEEE Trans. Inform. Theory*, vol. 41, no. 1, pp. 329–333, Jan. 1995.

[19] S. V. Schell, D. L. Smith, and S. Roy, "Blind channel identification using subchannel response matching," in *Proc. 1994 Conf. Inform. Sci. Syst.*, Princeton, NJ, Mar. 1994.

[20] E. Moulines, P. Duhamel, J. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," in *Proc. IEEE ICASSP'94*, Apr. 1994, pp. IV573–IV576.

[21] D. T. M. Slock, "Blind fractionally-spaced equalization, perfect-reconstruction filter banks and multichannel linear prediction," in *Proc. IEEE ICASSP'94*, Apr. 1994, pp. IV585–IV588.

[22] L. A. Baccala and S. Roy, "A new time-domain blind channel identification method based on cyclostationarity," *IEEE Signal Processing Lett.*, vol. 1, pp. 89–91, June 1994.

[23] H. Liu and G. Xu, "A deterministic approach to blind symbol estimation," *IEEE Signal Processing Lett.*, vol. 1, Dec. 1994.

[24] J. H. Winters, "Signal acquisition and tracking with adaptive arrays in the digital mobile radio system IS-54 with flat fading," *IEEE Trans. Veh. Technol.*, vol. 42, pp. 377–384, Nov. 1993.

[25] H. Liu and G. Xu, "Closed-form blind symbol estimation in digital communications," *IEEE Trans. Signal Processing*, vol. 43, pp. 2714–2723, Nov. 1995.

[26] L. Tong, "Blind sequence estimation," *IEEE Trans. Commun.*, submitted for publication.
[27] N. Seshadri, "Joint data and channel estimation using fast blind trellis seach techniques," in *Proc. GLOBECOM'90*, 1991, pp. 1659–1663.
[28] E. Zervas, J. Proakis, and V. Eyuboğlu, "A Quantized channel approach to blind equalization," in *Proc. ICC'92*, Chicago, IL, June 1992, pp. 1539–1643.
[29] D. Yellin and B. Porat, "Blind identification of FIR systems excited by discrete-alphabet inputs," *IEEE Trans. Signal Processing*, vol. 41, pp. 1331–1339, Mar. 1993.
[30] S. Y. Kung, T. Kailath, and M. Morf, "A generalized resultant matrix for polynomial matrices," in *Proc. IEEE Conf. Decision Contr.*, Florida, 1976, pp. 892–895.
[31] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1989, 2nd ed.
[32] B. Ottersten, R. Roy, and T. Kailath, "Signal waveform estimation in sensor array processing," in *Proc. 23rd Asilomar Conf. Signals, Syst., Comput.*, Pacific Grove, CA, Nov. 1989, vol. 2, pp. 787–791.
[33] T. S. Rappaport, S. Y. Seidel, and R. Singh, "900-MHz multipath propagation measurements for U.S. digital cellular radiotelephone," *IEEE Trans. Veh. Technol.*, vol. 39, pp. 132–139, May 1990.

Hui Liu (S'92–M'96) received the B.S. degree from Fudan University, Shanghai, China, in 1988, the M.S. degree from Portland State University, Portland, OR, in 1992, and the Ph.D. degree from The University of Texas at Austin, in 1995, all in electrical engineering.

From September 1992 to December 1992, he was a Software Engineer at the Quantitative Technology Corporation, Beaverton, OR. During the summer of 1995, he was a Consultant for Bell Northern Research. In the fall of 1996, he held the position of Director of Engineering at CWiLL Telecommunications, Inc., where he directed the development of the CWiLL smart antenna prototype. He joined the Faculty of the Department of Electrical Engineering, University of Virginia, Charlottesville, VA, in September 1995. His current research interests include wireless communications, array signal processing, system identification, and DSP applications.

Guanghan Xu (S'86–M'92) was born in Shanghai, China, on November 10, 1962. He received the B.S. degree with honors in biomedical engineering from Shanghai Jiao Tong University, Shanghai, China, in 1985, the M.S. degree in electrical engineering from Arizona State University, Tempe, AZ, in 1988, and the Ph.D. degree in electrical engineering from Stanford University, Stanford, CA, in 1991.

During the summer of 1989, he was a Research Fellow at the Institute of Robotics, Swiss Institute of Technology, Zurich, Switzerland. From 1990 to 1991, he was a General Electric Fellow of the Fellow–Mentor–Advisor Program at the Center of Integrated Systems, Stanford University, Stanford, CA. From 1991 to 1992, he was a Research Associate with the Department of Electrical Engineering, Stanford University, Stanford, CA, and a short term as a Visiting Scientist at the Laboratory of Information and Decision Systems, Massachusetts Institute of Technology, Cambridge, MA. In 1992, he joined the faculty of the Department of Electrical and Computer Engineering, The University of Texas at Austin. He has worked in several areas including signal processing, communications, numerical linear algebra, multivariate statistics, and semiconductor manufacturing. His current research interest is focused on smart antenna systems for wireless communications.

Dr. Xu received the 1995 NSF career Award and is a member of Phi Kappa Phi.