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Provably Safe Coordinated Strategy for Distributed Conflict Resolution

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This paper proposes a pair-wise distributed conflict resolution strategy that resolves aircraft encounters in a provably safe coordinated way. In this approach, each aircraft independently computes a list of resolution advisories based only on its state information and the state information of the traffic aircraft. Effective coordination between the two aircraft is achieved without communicating their intentions or negotiating a solution. The strategy is safe in the sense that the aircraft will not maneuver into each other while following their respective resolution advisories. Furthermore, the strategy will keep the aircraft separated even if only one aircraft maneuvers. This feature provides an additional layer of safety that mitigates potential conflicts when one aircraft does not maneuver due to equipment failure or other factors. The algorithm that implements the resolution strategy is not computationally intensive and is ideally suitable for distributed airborne deployment. The mathematical development has been formalized and mechanically verified in the Program Verification System (PVS). Low fidelity simulations have shown that the proposed approach is effective even in cases of multiple aircraft conflicts in a heavily crowded airspace.

I. Introduction

Emerging air traffic management concepts such as *Free flight*,[1,2] *Distributed Air/Ground Traffic Management*,[3] and *Advanced Airspace Concept*[4] target the increasing complexity of the future airspace system. Common to these concepts is the use of more reliable surveillance and communication technologies that enable the redistribution of roles for traffic separation. To support these new modes of operation, several decision support systems are being developed.

Conflict detection and resolution (CD&R) systems are designed to increase traffic awareness and provide corrective maneuvers for impending conflicts.[5] On-board CD&R systems are particularly attractive as they support decentralized decision making of new air traffic management concepts. However, if several aircraft are equipped with CD&R systems, aircraft coordination is a major safety issue. The words *cooperative* and *non-cooperative* are often used in CD&R literature to convey concepts related to coordination. For example, *cooperation* may express exchange of intent information,[6–9] degree of coordination,[2,5] sharing of effort to achieve separation,[10] and temporary delegation of responsibility for separation.[11] To avoid confusion, we

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only use in this paper the term *coordination* and we said that two resolution maneuvers are *coordinated* if they ensure that aircraft will not fly into each after maneuvering to solve the conflict.

In a distributed CD&R system, coordination can be achieved in several ways. If intent information is exchanged, the approach is called *strategic*. Otherwise, it is called *tactical*. Distributed strategic planning algorithms that generate conflict free trajectories are proposed in Refs. 6, 8, 9. Tactical schemes that achieve separation when all aircraft independently maneuver are presented in Refs. 2, 7, 10, 12. Coordination can also be achieved by assigning priorities to the conflicting aircraft. In this case, the aircraft with lower priority maneuvers to separate from the aircraft with higher priority. The priority can be negotiated during the conflict as in the strategic algorithm proposed in Ref. 13 or it can be pre-determined as in the tactical approaches proposed in Refs. 10, 14.

Strategic CD&R systems allow for large look-ahead times and they are, in general, significantly more complex than tactical ones. On the other hand, tactical systems are appropriate for short look-ahead times. In this paper, we consider tactical *geometric* algorithms. These algorithms predict the future position of an aircraft based only on the linear projection of its current state.[2,10,12,14] Geometric algorithms use analytical methods and, therefore, are very efficient. This is an advantage for on-board deployment as aircraft have limited access to computational resources.

We propose a distributed tactical conflict resolution strategy that achieves coordination without exchanging intent information. This approach is implemented on top of a previous developed pair-wise CD&R algorithm called KB3D.[14] The input to KB3D is the state information of the aircraft, i.e., 3-D position and velocity vector. We distinguish the host aircraft as the *ownship* and the traffic aircraft as the *intruder*. The output is a list of resolution maneuvers, in the form of 3-D velocity vectors, for the *ownship*. The novelty of this work is a distributed strategy for KB3D that guarantees coordinated maneuvers for both aircraft. As an added layer of safety, the resolution strategy keeps the aircraft separated even if one aircraft does not maneuver, for example due to equipment failure. All the mathematical development presented in this paper has been formalized and mechanically checked in the verification system PVS.[15]

This paper is organized as follows. Section II introduces the notation and formulates the problem. Section III gives a short overview to the tactical CD&R algorithm KB3D. Section IV presents a coordinated strategy for KB3D and gives rigorous proofs of its properties. Section V illustrates the use of the coordinated strategy with a concrete example. Performance results of a low fidelity simulation are shown in Section VI. The last section discusses our approach and summarizes the work.

II. The Problem

The state of an aircraft A is given by its position vector $\mathbf{s}_A = (s_{Ax}, s_{Ay}, s_{Az})$ and its velocity vector $\mathbf{v}_A = (v_{Ax}, v_{Ay}, v_{Az})$. A cylinder of diameter D and height H is centered around each aircraft. Typical values for D and H are 5nm and 1000ft. Two aircraft A and B are *in violation* if their protected zones overlap, i.e., if their horizontal separation $\sqrt{(s_{Bx} - s_{Ax})^2 + (s_{By} - s_{Ay})^2}$ is less than D and their vertical separation $|s_{Bz} - s_{Az}|$ is less than H.

The aircraft are *in conflict* if for some time $0 \le t \le T$, their predicted positions $\mathbf{s}_A + t\mathbf{v}_A$ and $\mathbf{s}_B + t\mathbf{v}_B$ are in violation. The time T is the look-ahead time, a typical value for a tactical algorithm is 5 minutes.

Given the states of two conflicting aircraft A and B, that are not in violation, a velocity vector $\mathbf{v'}_A$ is an *independent* maneuver for A (with respect to B) if for all $0 \le t \le T$, $\mathbf{v'}_A$ achieves separation (assuming that aircraft B does not maneuver), i.e.,

$$\begin{aligned}\sqrt{(s_{Bx} - s_{Ax} + t(v_{Bx} - v'_{Ax}))^2 + (s_{By} - s_{Ay} + t(v_{By} - v'_{Ay}))^2} &\ge D, \quad \text{or}\\ |s_{Bz} - s_{Az} + t(v_{Bz} - v'_{Az})| &\ge H.\end{aligned}$$

Furthermore, velocity vectors $\mathbf{v'}_A$ and $\mathbf{v'}_B$ are *coordinated* maneuvers for A and B, respectively, if for all

$0 \le t \le T$, $\mathbf{v'}_A$ and $\mathbf{v'}_B$ achieve separation, i.e.,

$$\begin{aligned}\sqrt{(s_{Bx} - s_{Ax} + t(v'_{Bx} - v'_{Ax}))^2 + (s_{By} - s_{Ay} + t(v'_{By} - v'_{Ay}))^2} &\ \ge\ D, \quad \text{or} \\ |s_{Bz} - s_{Az} + t(v'_{Bz} - v'_{Az})| &\ \ge\ H.\end{aligned}$$

This paper proposes a distributed resolution strategy that provides coordinated maneuvers $\mathbf{v'}_A$ and $\mathbf{v'}_B$ for aircraft A and B, where both $\mathbf{v'}_A$ and $\mathbf{v'}_B$ are independent. The strategy assumes that the state information of both aircraft is known, but no other communication between the aircraft is required.

III. KB3D in a Nutshell

KB3D[14] is a state-based geometric conflict detection and resolution algorithm. It is a 3-D extension of an algorithm independently developed by K. Bilimoria.[10] KB3D computes independent maneuvers for the ownship, each of which solves the conflict assuming that the ownship maneuvers. The maneuvers computed by KB3D involve the modification of a single parameter of the ownship velocity vector: vertical speed, heading, or ground speed.

Let O and I be the ownship and intruder aircraft, respectively. The predicted positions of the aircraft at time t are given by $\mathbf{s}_O + t\mathbf{v}_O$ and $\mathbf{s}_I + t\mathbf{v}_I$. Their *predicted separation vector* $\mathbf{s}(t)$ is $\mathbf{s} + t\mathbf{v}$, where $\mathbf{s} = \mathbf{s}_O - \mathbf{s}_I$ and $\mathbf{v} = \mathbf{v}_O - \mathbf{v}_I$ are the relative position and velocity vectors, respectively, of the ownship with respect to the intruder. Instead of two individual protected zones centered around each aircraft, we consider one *relative protected zone* centered at the intruder aircraft. This relative protected zone is a cylinder of diameter $2D$ and height $2H$. It is easy to see that the aircraft are *in violation* if the ownship's position $\mathbf{s}$ is in the relative protected zone.[a] Furthermore, they are *in conflict* if for some time t, $0 \le t \le T$, the predicted separation vector $\mathbf{s}(t)$ is in the relative protected zone. The time to loss of separation defines a vector on the envelope of the relative protected zone, i.e., a vector where either $x^2 + y^2 = D^2$ and $z^2 < H^2$, or $x^2 + y^2 \le D^2$ and $z^2 = H^2$. This vector is called the *impact* vector. Figure 1 illustrates a conflict situation, and the relative position $\mathbf{s}$ and impact vector $\mathbf{p}$.

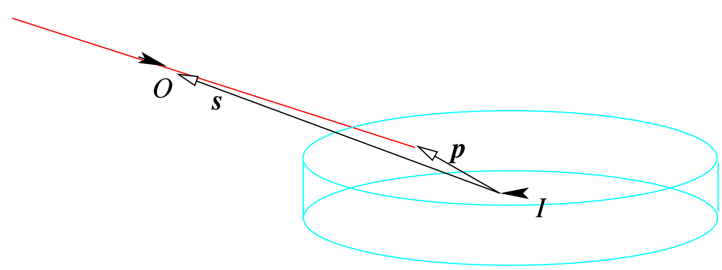


Figure 1. Conflict situation

Resolution maneuvers are new velocity vectors $\mathbf{v}'_O$ such that the half-line $\mathbf{s}_O - \mathbf{s}_I + t(\mathbf{v}'_O - \mathbf{v}_I)$ does not intersect the relative protected zone. We are interested on solutions that minimize the change on the velocity vector of the ownship $||\mathbf{v}'_O - \mathbf{v}_O||$. These solutions are tangent to the relative protected zone, i.e., they intersect the envelope of the cylinder, but not its interior. Each one of the intersection points determines a *contact vector*.

There are two type of contact vectors, those that intersect the top or bottom circles of the cylinder, and those that intersect a vertical line on the side of the cylinder. It can be proved that the contact vectors

[a] The protected zone of the ownship and intruder aircraft overlap if and only if the ownship is in the relative protected zone centered around the intruder.

on the circles satisfy $x^2 + y^2 = D^2$ and $z = \varepsilon H$, with $\varepsilon = \pm 1$, and the contact vectors on the lines satisfy $x = \alpha s_x + \varepsilon \beta s_y$, $y = \alpha s_y - \varepsilon \beta s_x$, and $z^2 < H^2$, with $\varepsilon = \pm 1$ and

$$\alpha = \frac{D^2}{s_x^2 + s_y^2}, \tag{1}$$

$$\beta = \frac{D\sqrt{s_x^2 + s_y^2 - D^2}}{s_x^2 + s_y^2}. \tag{2}$$

Solutions can be classified as *circle solutions* and *line solutions* according to their contact vector. The circle solutions corresponding to $\varepsilon = 1$ are called *top circle solutions*, those corresponding to $\varepsilon = -1$ are called *bottom circle solutions*. The line solutions corresponding to $\varepsilon = 1$ are called *left line solutions*, those corresponding to $\varepsilon = -1$ are called *right line solutions*. Not all type of solutions exist for all conflicts. For example, it can be proved that if $s_x^2 + s_y^2 < D^2$ then there are no line solutions. The thick lines in Figure 2 are contact vectors that define circle and line solutions for different relative positions of the ownship.

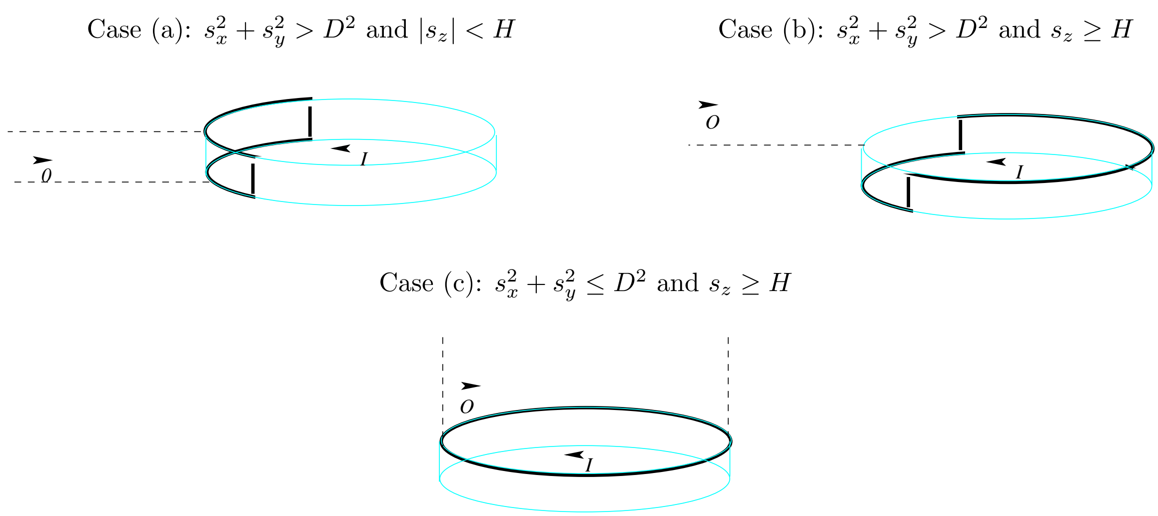


Figure 2. Circle and line solutions

KB3D computes resolution maneuvers where only one parameter of the ownship velocity vector is modified, i.e., vertical speed, heading, and ground speed. Vertical speed solutions are always circle solutions. Heading and ground speed solutions can be circle or line solutions. However, KB3D only computes heading and ground solutions that are line solutions.

IV. Coordinated Strategy for KB3D

A *strategy* is *coordinated* if it provides resolution maneuvers that achieve separation when both aircraft maneuver. It is *distributed* if each aircraft independently computes the resolution maneuver. A strategy can be distributed but may not be coordinated. For example, the distributed strategy "each aircraft maneuvers to its right" is coordinated when the conflicting aircraft are flying in opposite directions, but it may not achieve separation when the aircraft are flying in the same direction. The distributed strategy "the higher aircraft climbs, the lower aircraft descends" is coordinated in every situation except when both aircraft are at the same altitude.

The resolutions provided by KB3D are independent, i.e., each one of them solves the conflict when the ownship maneuvers. However, the resolutions are not always coordinated, i.e., if the intruder also implements a resolution maneuver provided by KB3D, then the two aircraft may end up in another conflict situation. For instance, if both aircraft climb to solve the conflict, they may just create another conflict higher in the airspace.

A simple strategy to coordinate the resolutions provided by KB3D is to define a priority rule such that one aircraft maneuvers while the other continues on its original path. For example, it could be pre-determined that the aircraft with the smaller identifier always maneuvers. These kinds of rules are not fair, and worst, they are not robust. If for some reason, the aircraft with the lower identifier does not maneuver, the other aircraft will have to improvise an avoidance maneuver.

In the general case, KB3D computes 6 independent solutions for the ownship: 2 vertical solutions (top circle and bottom circle) 2 heading solutions (left line and right line), and 2 ground speed solutions (left line and right line). We propose a strategy that selects KB3D solutions that are guaranteed to be coordinated.

A. Strategy

Let $\mathbf{p}$ be the impact vector on the relative protected zone.

- *Vertical speed*: if $p_z > 0$ or ($p_z = 0$ and $s_z > 0$) or ($p_z s_z = 0$ and $s_x > 0$) or ($p_z s_z s_x = 0$ and $s_y > 0$), then select the top circle solution ($\varepsilon = 1$). Otherwise, select the bottom circle solution ($\varepsilon = -1$).
- *Heading and ground speed*: if $s_y p_x - s_x p_y > 0$, then select the left line solution ($\varepsilon = 1$). Otherwise, select the right line solution ($\varepsilon = -1$).

This strategy selects at most 3 solutions, i.e., one KB3D solution of each type: vertical speed, heading, and ground speed. Furthermore, it has been formally proved that this strategy selects at least one coordinated vertical solution.

Henceforth, we will call KB3D$_2$ the algorithm that implements this strategy on top of KB3D. As the set of solutions selected by KB3D$_2$ is a subset of KB3D solutions, all of them are independent, i.e., they achieve separation when only the ownship maneuvers. The rest of this section presents a rigorous proof that KB3D$_2$ solutions are also coordinated, i.e., they also achieve separation when both aircraft maneuver. Formal proofs of these properties in PVS, and prototypes of the algorithm in C++ and Java are available from `http://research.nianet.org/fm-at-nia/KB3D`.

B. Rationale

Let A and B be two conflicting aircraft. The aircraft A considers itself the ownship and computes the impact vector $\mathbf{p}_A$, i.e., the vector at the time of loss of separation t when the half-line $\mathbf{r}_A(t) = \mathbf{s}_A - \mathbf{s}_B + t(\mathbf{v}_A - \mathbf{v}_B)$ intersects the envelope of the relative protected zone. The aircraft B also considers itself the ownship and computes the impact vector $\mathbf{p}_B$, i.e., the vector at the time of loss of separation t when the half-line $\mathbf{r}_B(t) = \mathbf{s}_B - \mathbf{s}_A + t(\mathbf{v}_B - \mathbf{v}_A)$ intersects the envelope of the relative protected zone. Note that $\mathbf{r}_B(t) = -\mathbf{r}_A(t)$. As the time of loss of separation is the same for both aircraft, $\mathbf{p}_B = -\mathbf{p}_A$.

The relative positions computed by each aircraft are opposite. Hence, it is not difficult to see that KB3D$_2$ selects opposite ε for circle solutions for aircraft A and B, i.e., when KB3D$_2$ selects a top circle solution for A, it selects a bottom circle solution for B. Conversely, if KB3D$_2$ selects a bottom circle solution for A, it selects a top circle solution for B. For heading and ground speed, KB3D$_2$ always selects the same ε for line solutions for aircraft A and B, i.e., left line solutions or right line solution for both aircraft.

C. Proof

Each aircraft may chose a vertical solution (vertical speed) or a horizontal one (heading or ground speed). Thus, there are three cases to consider:

1. one aircraft chooses a vertical solution and the other a horizontal solution,
2. both aircraft choose a vertical solution, or
3. both aircraft choose a horizontal solution.

In each of these cases, we prove that, if both aircraft maneuver, the conflict is solved.

1. One aircraft chooses a vertical solution and the other a horizontal solution

Call A the aircraft that chooses the horizontal solution and B the aircraft that chooses the vertical solution. By hypothesis, B only modifies its vertical speed, then the horizontal separation between the aircraft is the same as if B does not maneuver. Since the horizontal solution for A is independent, horizontal separation is achieved for all times.

2. Both aircraft choose a vertical solution

Call A the aircraft that chooses the top circle solution ($\varepsilon = 1$) and B that that chooses the bottom circle solution ($\varepsilon = -1$). The relative position of A is $\mathbf{s} = \mathbf{s}_A - \mathbf{s}_B$ and that of B is $-\mathbf{s}$.

The time t corresponding to the contact vector $\mathbf{p}_A$ satisfies the equations

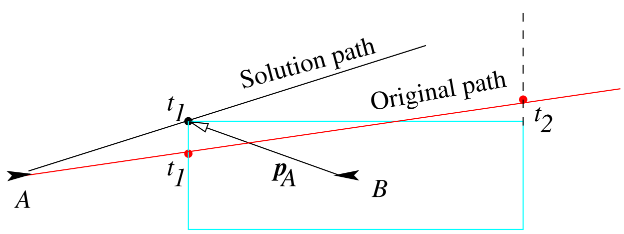
$$(s_x + t(v_{Ax} - v_{Bx}))^2 + (s_y + t(v_{Ay} - v_{By}))^2 = D^2, \quad \text{and} \tag{3}$$

$$s_z + t(v'_{Az} - v_{Bz}) = H. \tag{4}$$

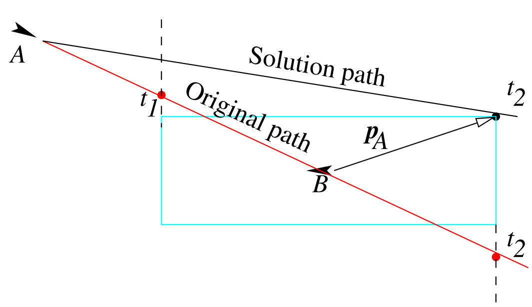
Moreover, according to the KB3D$_2$ strategy, $p_{Az} \geq 0$.

As the initial situation is a conflict situation, Formula (3) has two solutions at times t_1 and t_2, where $t_1 < t_2$. The time of the contact vector $\mathbf{p}_A$ depends on the value of s_z:

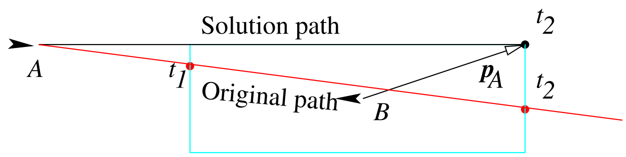
- If $s_z < H$ then the contact vector $\mathbf{p}_A$ is at time t_1:



- If $s_z > H$ then the contact vector $\mathbf{p}_A$ is at time t_2:



- If $s_z = H$ then we define the contact vector $\mathbf{p}_A$ at time t_2:



Let τ be the time corresponding to the contact vector $\mathbf{p}_A$.

The time t corresponding to the contact vector $\mathbf{p}_B$ satisfies the equations

$$(-s_x + t(v_{Bx} - v_{Ax}))^2 + (-s_y + t(v_{By} - v_{Ay}))^2 = D^2, \quad \text{and} \tag{5}$$

$$-s_z + t(v'_{Bz} - v_{Az}) = -H. \tag{6}$$

Formula (5) is the same as Formula (3), and thus the solutions t_1 and t_2 are the same. The time corresponding to the contact vector $\mathbf{p}_B$ is t_1 if $-s_z > -H$, i.e., $s_z < H$, and t_2 otherwise. Thus, the time of contact vector $\mathbf{p}_B$ is also τ.

The solutions of A and B satisfy

$$s_z + \tau(v'_{Az} - v_{Bz}) = H, \quad \text{and} \tag{7}$$

$$-s_z + \tau(v'_{Bz} - v_{Az}) = -H. \tag{8}$$

Subtracting formula (7) and (8), we get

$$s_z + \tau(v'_{Az} - v'_{Bz}) + s_z + \tau(v_{Az} - v_{Bz}) = 2H. \tag{9}$$

To finish the proof, we consider two cases.

- Case $s_z < H$. In this case, $\tau = t_1$. As the initial situation is a conflict situation and we have horizontal separation before τ, we must lose separation after τ. Therefore,

$$s_z + \tau(v_{Az} - v_{Bz}) < H. \tag{10}$$

  Formulas (9) and (10) yield

$$s_z + \tau(v'_{Az} - v'_{Bz}) \geq H. \tag{11}$$

  Therefore, if both aircraft maneuver, vertical separation is achieved after τ. As we have horizontal separation before τ, separation is achieved for all times.

- Case $s_z \geq H$. In this case, $\tau = t_2$. As the initial situation is a conflict situation and we have horizontal separation after τ, we must lose separation before τ. Therefore,

$$s_z + \tau(v_{Az} - v_{Bz}) < H. \tag{12}$$

  As in the previous case, we get

$$s_z + \tau(v'_{Az} - v'_{Bz}) \geq H. \tag{13}$$

  Therefore, if both aircraft maneuver, vertical separation is achieved before τ. As we have horizontal separation after τ, separation is achieved for all times.

3. *Both aircraft choose a horizontal solution*

The vertical speeds play no role and the problem is a purely two dimensional problem. We call $\mathbf{s}_A$, $\mathbf{s}_B$, $\mathbf{v}_A$, and $\mathbf{v}_B$ the horizontal part of position and velocity vectors of A and B. The relative position of A is $\mathbf{s} = \mathbf{s}_A - \mathbf{s}_B$ and that of B is $-\mathbf{s}$.

We write $\mathbf{u} \cdot \mathbf{v}$ for the scalar product of two vectors $\mathbf{u}$ and $\mathbf{v}$, and $\Delta(\mathbf{u}, \mathbf{v})$ for their determinant.

A PROPERTY OF SOLUTIONS: Let $\mathbf{v}_A$ and $\mathbf{v}_B$ be two velocity vectors and $\mathbf{v} = \mathbf{v}_A - \mathbf{v}_B$. If the half-line $\mathbf{s} + t\mathbf{v}$ intersects the envelope of the relative protected zone but not the protected zone itself, the equation

$$(\mathbf{s} + t\mathbf{v})^2 = D^2, \quad \text{i.e.,} \tag{14}$$
$$t^2\mathbf{v}^2 + 2t(\mathbf{s} \cdot \mathbf{v}) + \mathbf{s}^2 - D^2 = 0, \tag{15}$$

has a single solution. In this case, the discriminant of this equation is null, i.e.,

$$\mathbf{v}^2(\mathbf{s}^2 - D^2) = (\mathbf{s} \cdot \mathbf{v})^2.$$

Therefore,

$$\mathbf{s}^2\mathbf{v}^2(\mathbf{s}^2 - D^2) = \mathbf{s}^2(\mathbf{s} \cdot \mathbf{v})^2.$$

Using

$$\mathbf{s}^2\mathbf{v}^2 = (\mathbf{s} \cdot \mathbf{v})^2 + \Delta(\mathbf{s}, \mathbf{v})^2,$$

we get

$$((\mathbf{s} \cdot \mathbf{v})^2 + \Delta(\mathbf{s}, \mathbf{v})^2)(\mathbf{s}^2 - D^2) = \mathbf{s}^2(\mathbf{s} \cdot \mathbf{v})^2, \quad \text{i.e.,} \tag{16}$$
$$\Delta(\mathbf{s}, \mathbf{v}) = \varepsilon r(\mathbf{s} \cdot \mathbf{v}), \tag{17}$$

where $\varepsilon = \pm 1$ and $r = \frac{D}{\sqrt{\mathbf{s}^2 - D^2}}$. This can be geometrically interpreted by the fact that the tangent of the angle ϕ in Figure 3 is either $-r$ or r. The contact vectors given in Section III for line solutions can be

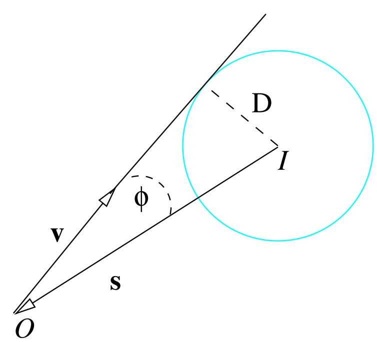


Figure 3. Top view of a solution

computed from Formula (17).

A PROPERTY OF CONFLICTS: If for some time t the half-line intersects the relative protected zone, the Formula (14) has two solutions and one is positive. In this case, the discriminant of the equation is positive, i.e.,

$$\mathbf{v}^2(\mathbf{s}^2 - D^2) < (\mathbf{s} \cdot \mathbf{v})^2.$$

Therefore,

$$((\mathbf{s} \cdot \mathbf{v})^2 + \Delta(\mathbf{s}, \mathbf{v})^2)(\mathbf{s}^2 - D^2) < \mathbf{s}^2(\mathbf{s} \cdot \mathbf{v})^2, \quad \text{i.e.,}$$
$$\left|\frac{\Delta(\mathbf{s}, \mathbf{v})}{\mathbf{s} \cdot \mathbf{v}}\right| < r$$

Note that both roots have the same sign as $\mathbf{v}^2$ and $\mathbf{s}^2 - D^2$ are both positive. Since one root is positive, both are. Therefore, $\mathbf{s} \cdot \mathbf{v} < 0$ and

$$|\Delta(\mathbf{s}, \mathbf{v})| < -r(\mathbf{s} \cdot \mathbf{v}). \tag{18}$$

PROOF OF CASE 3: Let $\mathbf{v}'_A$ and $\mathbf{v}'_B$ be the line solutions proposed by KB3D$_2$ to aircraft A and B, respectively. We show that if both aircraft maneuver then separation is achieved.

We define the following relative velocity vectors:

$$\begin{aligned}\mathbf{v} &= \mathbf{v}_A - \mathbf{v}_B, \\ \mathbf{v}'_{AB} &= \mathbf{v}'_A - \mathbf{v}_B, \\ \mathbf{v}'_{BA} &= \mathbf{v}'_B - \mathbf{v}_A, \quad \text{and} \\ \mathbf{v}' &= \mathbf{v}'_A - \mathbf{v}'_B.\end{aligned}$$

The initial situation is a conflict. Therefore, by Formula (18),

$$|\Delta(\mathbf{s}, \mathbf{v})| < -r(\mathbf{s} \cdot \mathbf{v}). \tag{19}$$

Velocity vectors $\mathbf{v}'_A$ and $\mathbf{v}'_B$ are independent solutions. Hence, by Formula (17),

$$\Delta(\mathbf{s}, \mathbf{v}'_{AB}) = \varepsilon_A r(\mathbf{s} \cdot \mathbf{v}'_{AB}), \quad \text{and} \tag{20}$$

$$\Delta(-\mathbf{s}, \mathbf{v}'_{BA}) = \varepsilon_B r(-\mathbf{s} \cdot \mathbf{v}'_{BA}), \tag{21}$$

where $\varepsilon_A = \pm 1$ and $\varepsilon_B = \pm 1$. According to the KB3D$_2$ strategy, $\varepsilon_A = \varepsilon_B$.

Adding Formulas (20) and (21) yield

$$\Delta(\mathbf{s}, \mathbf{v}') + \Delta(\mathbf{s}, \mathbf{v}) = \varepsilon_A r(\mathbf{s} \cdot \mathbf{v}') + \varepsilon_A r(\mathbf{s} \cdot \mathbf{v}). \tag{22}$$

We only consider the case $\varepsilon_A = 1$ as the other case is similar. By Formula (22), we get

$$\Delta(\mathbf{s}, \mathbf{v}') + \Delta(\mathbf{s}, \mathbf{v}) = r(\mathbf{s} \cdot \mathbf{v}') + r(\mathbf{s} \cdot \mathbf{v}). \tag{23}$$

Furthermore, from Formula (19),

$$r(\mathbf{s} \cdot \mathbf{v}) < \Delta(\mathbf{s}, \mathbf{v}). \tag{24}$$

Finally, from Formulas (23) and (24),

$$\Delta(\mathbf{s}, \mathbf{v}') \leq r(\mathbf{s} \cdot \mathbf{v}'). \tag{25}$$

Since Formula (25) does not satisfy Formula (18), the vector $\mathbf{v}' = \mathbf{v}'_A - \mathbf{v}'_B$ maintains separation.

V. Example

Assume that aircraft N321QL and N654FT are approaching on perpendicular paths, one at level flight and one climbing. Aircraft N321QL has a ground speed of 237 knots, at flight level FL420, at level flight, and due east (bearing 91.22 degrees). Aircraft N654FT has a ground speed of 252 knots, at FL409, climbing at 531 feet per minute, and due north (bearing 359.40 degrees). The aircraft are 10.1 nautical miles apart and the relative heading from aircraft N321QL to N654F is 136.81 degrees. For a protected zone of $D = 0.5$ nautical miles and $H = 500$ feet, a conflict is detected 1.64 minutes, for a duration of 0.17 minutes, from the current state.

For each aircraft, KB3D$_2$ computes three resolution maneuvers: vertical speed, ground speed, and heading. Figure 4 shows the top and side views of the encounter, and the vertical and heading resolution maneuvers.

- The vertical speed maneuver is for aircraft N321QL to climb at 200 feet per minute and for aircraft N654FT to reduce its climb rate to less than 333 feet per minute.

- The ground speed maneuver requires aircraft N321QL to reduce its speed from 237 knots to 216 knots or less and for N654FT to increase its speed from 252 to 277 knots or more.
- The heading resolution is for aircraft N321QL to change its course to a bearing of 96.6 degrees and for aircraft N654FT to change its course to a bearing of 4.2 degrees.

The conflict will be avoided for any combination of N321QL and N654FT resolutions if one of the aircraft maneuvers or if both aircraft maneuver.

VI. Low Fidelity Simulation

The performance and characteristics of $KB3D_2$ have been evaluated in a low fidelity simulation. The objectives of the simulation are

- to determine if the solutions generated by $KB3D_2$ are physically and/or operationally implementable by a generic commercial aircraft, and
- to evaluate the performance of a pair-wise CD&R algorithm in a multiple aircraft environment.

The simulation comprises a rectangular section of en-route air space consisting of 200 by 200 nautical miles horizontally and 32 thousand feet vertically (18 to 50 thousand feet). A simple 3 dimensional kinematic point model is used to represent the aircraft. The number of aircraft in the volume is selected at the start of the simulation and it is maintained constant throughout the simulation run. When aircraft depart the airspace volume, new aircraft enter the simulation volume through random points of entry. Aircraft can be initialized with user selected ground speeds, vertical speeds and other state parameters or can be left unspecified. When left unspecified, the parameters will be initialized at random, using value ranges. Each aircraft in the simulation has an implementation of the $KB3D_2$ algorithm. Resolutions generated by the algorithm go through a selection constraint logic. The physical and operational constraints are shown in Table 1.

Max. altitude for resolution	40 Kft
Max. vertical speed	2000 ft/m
Maneuver time	12% of time to conflict
Max. acceleration	1.42 knots/s
Max. deceleration	1.25 knots/s
Max. ground speed	550 knots
Min. ground speed	250 knots
Max. speed increase	50 knots
Max. speed decrease	100 knots
Max. heading change	17 deg
Max. bank angle	35 deg

Table 1. Operational and Physical Constraints

The maximum turn rate of the aircraft is also constrained but it is not a constant bound as in the table above but rather a function of its ground speed. The constraint is given by:

$$\mathit{max\ turn\ rate} \quad = \quad \frac{180 \cdot g \cdot \tan(\phi)}{v \cdot \pi},$$

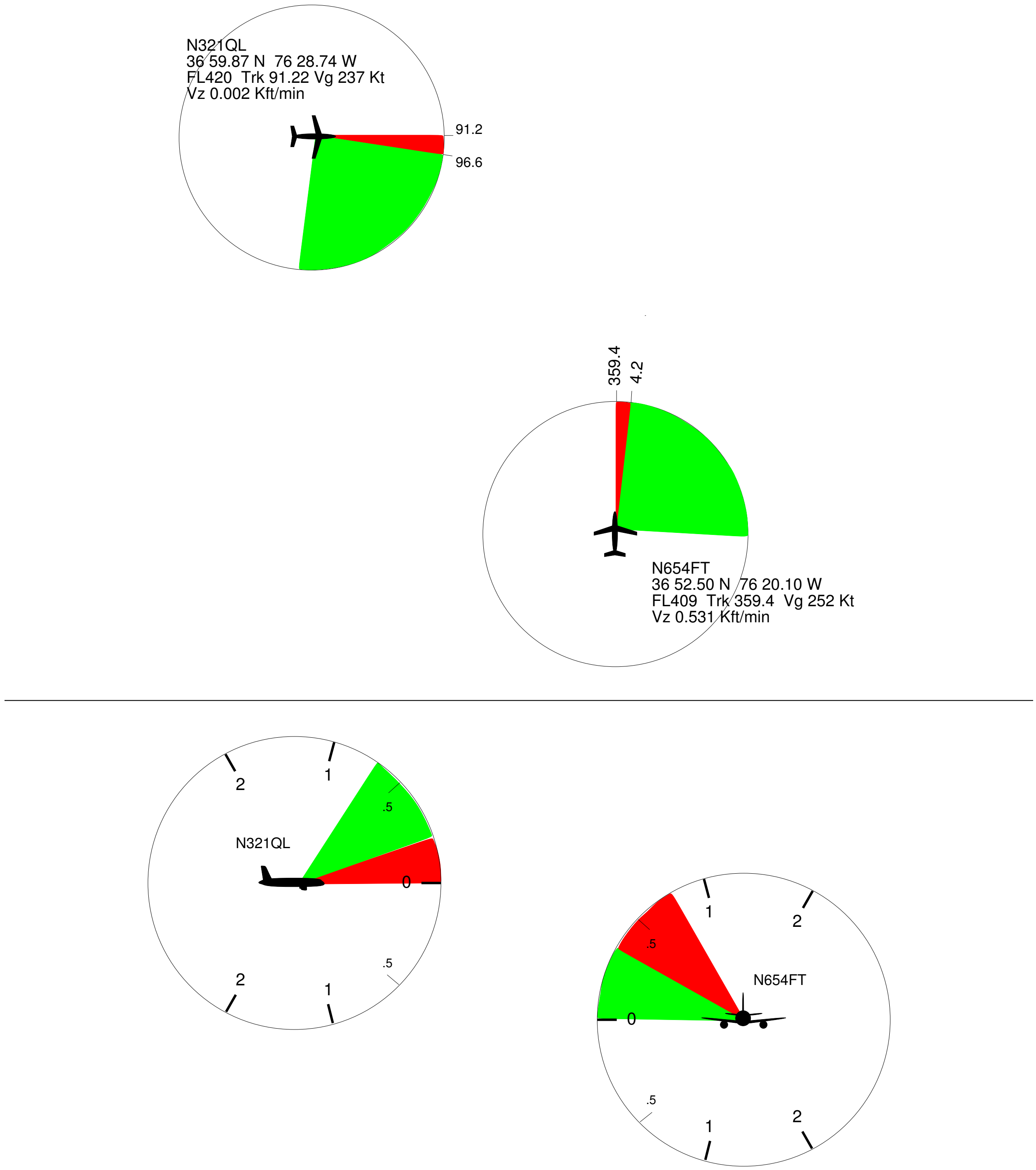


Figure 4. Top and side views of crossing paths, one aircraft climbing encounter

where g is the gravitational force, v is the ground speed, and ϕ is the bank angle of the aircraft. If all three solutions generated by $KB3D_2$ exceed the operational or physical constraints, then the change of heading solution is arbitrarily selected and implemented to achieve as close as possible the desired maneuver.

In one simulation step, each aircraft runs the CD&R algorithm against all other $N-1$ aircraft in the airspace. Therefore, the algorithm is executed $N(N-1)$ times per simulation step. Multiple conflicts are handled in a naive way. For example, if an aircraft detects and starts executing a resolution maneuver and subsequent conflicts are detected, the resolution of the subsequent conflicts can override the first resolution. This is one of the objectives of the experiment: to evaluate the performance of a pair-wise CD&R algorithm in a multiple conflict scenario. The number of aircraft in the airspace is varied from 18 to 250. The lookahead time is set to 5 minutes. The protected zone (separation minima) is set to 5 nautical miles horizontally and 1000 feet vertically. The duration of each simulation is 100 hours.

Figure 5 shows a graph of the number of conflicts as a function of number of aircraft. Eighteen aircraft

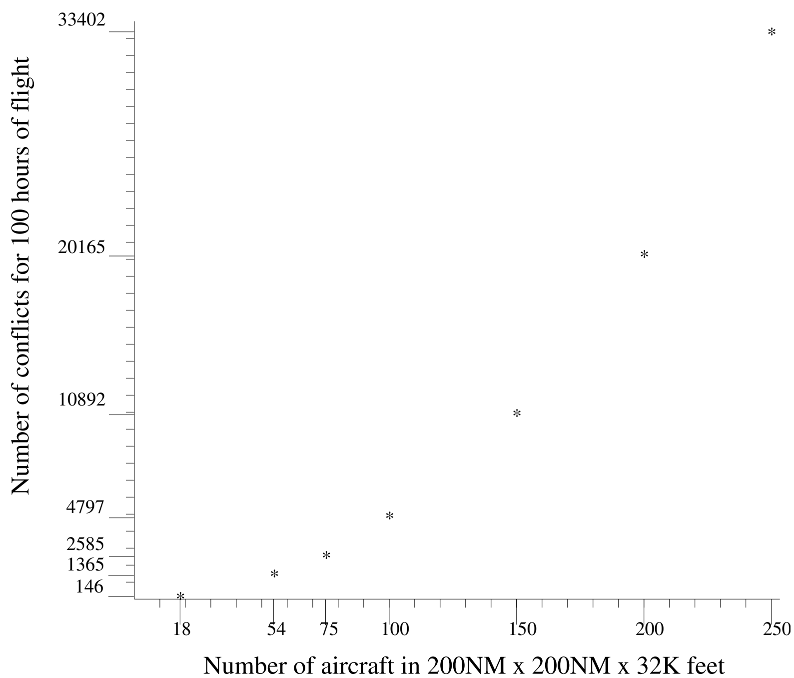


Figure 5. Conflicts as function of number of aircraft

in a 200 by 200 nautical miles airspace corresponds approximately to the traffic density over the continental United States in the year 1997. Fifty four aircraft in the same airspace is a figure used in previous experiments[16] as 3 times 1997 densities.

Figure 6 shows number of space violations (loss of separation minima) as a function of number of aircraft per simulation. A space violation occurs when an aircraft is less than 5 nautical miles horizontally *and* less than 1000 vertically from another aircraft. None of the violations recorded during simulation presents a collision threat. Table 2 summarizes the characteristics of the loss of separation for the 250 aircraft simulation.

The effectiveness of the $KB3D_2$ algorithm is also evaluated by running the simulation with the CD&R enabled and disabled. The results are given in Table 3.

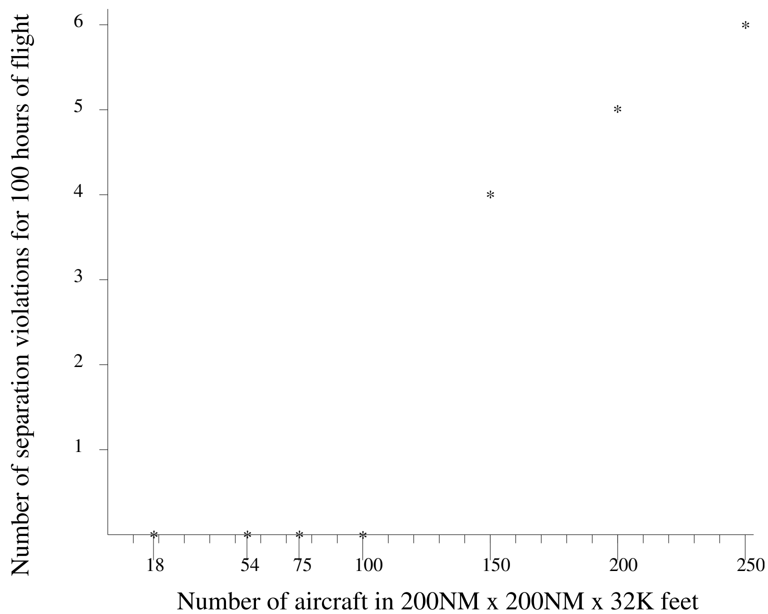


Figure 6. Separation minima violations as function of number of aircraft

Number of aircraft	250
Closest separation	4.650 nm, horiz. 252 ft, vert.
Average horizontal	4.935 nm
Average vertical	474 ft
Average duration	14.2 s

Table 2. Loss of separation summary: 250 aircraft

VII. Conclusion

In this paper, we have proposed a distributed strategy for geometric conflict resolution that provides provably safe coordinated maneuvers for two aircraft. The strategy guarantees that if both aircraft maneuver, separation is achieved without the need of communication for resolution coordination. Geometric algorithms for distributed coordination have been already proposed in Refs. 2, 10, 12. However, in all these cases, separation is achieved if both aircraft maneuver. The novelty of our approach is that each resolution maneuver selected by the proposed strategy is independent, i.e., the strategy guarantees separation if one aircraft maneuvers or if both aircraft maneuver. This features provides an added layer of safety and makes for a more robust CD&R system.

The proposed strategy is implemented on top of KB3D.[14] However, it is sufficiently general to be used with other geometric algorithms. For example, one configuration of the algorithm presented in Ref. 10 computes the same horizontal solutions as KB3D. Although the two algorithms are based on very different mathematical techniques, the coordinated strategy is applicable to both algorithms.

Our strategy is just an instance of a family of possible coordinated strategies. Indeed, the coordination

Number of aircraft	18	18
CD&R	Disabled	Enabled
Number of violations	81	0
Closest separation	0.212 nm, horiz. 165 ft, vert.	N/A
Average horizontal	2.88 nm	N/A
Average vertical	455.8 ft	N/A
Average duration	122.1 s	N/A

Table 3. Comparison of simulation with CD&R enabled and disabled

property holds for *any strategy* that selects circle solutions with opposite ε and line solutions with the same ε, for each conflicting aircraft. In future work, we will study other schemes for choosing the values of ε that, for instance, return optimal coordinated maneuvers with respect to a given cost function.

Although KB3D$_2$ is a pair-wise algorithm, a low fidelity simulation study shows that it is effective even in cases of multiple aircraft conflicts in a heavily crowded airspace. Despite these positive results, on-going work is looking for extensions of the approach presented in this paper to distributed multiple aircraft coordination.[17]

Finally, we stress that the complete mathematical development presented in this paper has been formally verified. Given the critical nature of CD&R systems, we believe that this formal verification is a major step toward the safety analysis of distributed air traffic management concepts.

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