Three-Dimensional Air Combat: Numerical Solution Using Randomised Trajectory

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Abstract. This paper investigates a complex pursuit-evasion game in three dimensions with complete information applied to two aircrafts in an air combat. Both aircrafts are simulated as point masses with limitations of the flight performance. To find an optimal trajectory for the evader, populations of trajectories are randomly generated for a given time length. The optimal evader's trajectory is a trajectory that gives the best payoff. The best payoff is a trajectory that guides the evader from being intercepted, and gives the maximum separation distance at the end of the given time length. The pursuer uses a proportional navigation guidance system to guide itself to the evader. As an illustrative example, the study considers the evasion of an aircraft, which is very agile but slower, from a pursuing missile, which is faster but less agile. The aircraft maneouvres are restricted by various control and state variable inequality constraints. Several factors are studied in this paper to see their relationship to interceptability. These factors are intercept radius, turning radius and speed. For the purpose of simplifying the analysis, it is assumes both players to fly at a constant speed. This technique is able to find an optimal trajectory for the evader in order to avoid interception. The optimal trajectories exhibit several well known tactical manoeuvres such as the horizontal-S and the vertical-S, but the manoeuvres need to be performed in a timely manner for a successful evasion.

1. Introduction

In this paper, a class of 3D optimal path-planning problems is considered for a fighter aircraft with kinematical and tactical constraints during air combat. Most of the problems that involve two opposing units can be described in the form of games. The theory of games is used extensively in various fields, such as economics, social studies and in military activities, to assist in understanding the complex nature of the problem and to find optimal strategies.

Military pursuit-evasion game simulations using evolutionary algorithms have been studied by many researchers. These cover naval warfare simulations [2], terrain-avoiding trajectory and missile avoidance [3, 4]. The basic key issues that need to be addressed in these simulations, as outlined by [5], are:

a. Minimizing the risk of aircraft detection by radar;
b. Minimizing the risk of submarine detection by sensors;
c. Minimizing cumulative radiation damage in passing through a contaminated area;
d. Finding optimal trajectories for multiple aircraft to avoid collision;
e. Maximizing the probability of target detecting by a searcher;
f. Minimizing the fuel consumption;

Optimal flight path trajectory problems for civil air traffic control was studied by [6] using non-linear programming with collocations on finite elements. The study of optimal trajectories for aircraft in a threat environment using calculus of variation was carried out by [5]. Other techniques, apart from evolutionary algorithms, are gradient-based algorithms, dynamic programming and network flow optimisation.

The key concern is whether the models can be implemented onboard an aircraft as real time solvers, able to produce relatively accurate results in the presence of errors, and are stable throughout the operating envelopes. As indicated by [5], the efficiency of discreet optimisation depends on the type of objective function, technological constraints, and the type of trajectory approximation schemes used.

According to [5], many previous studies on trajectory generation for military aircraft are concentrated on feasible direction algorithms and dynamic programming. These methods tend to be computationally intense and therefore are not well suited for onboard applications. In order to reduce the computation time, [7] uses a simple analytical risk function to further develop lateral and vertical algorithms to optimise the flight trajectory with respect to time, fuel, aircraft final position and exposed risks.

In this research, the development of a technique to study the optimal flight path for aircraft is presented. The high non-linearity of the problem makes it almost impossible to solve in the classical way. ModSAF, the US battlefield simulation system, uses evolutionary algorithms in its simulation [8] to demonstrate the potential of artificial intelligence techniques for human behaviour simulation.

The availability of high performance computer architectures, such as parallel processors, has opened the opportunity to use evolutionary algorithms to their fullest extend to solve real time flight path planning problems.

2. Methodology

The study uses techniques proposed by Istas [13]. In this technique, several populations of possible aircraft trajectories (or strategies) are randomly generated and tested.

In each population, there are 100 strategies. A strategy is actually an instruction for the aircraft to change its heading and flight path angle at every second. For example, at t=0s, the aircraft changes its heading angle by 20 degrees to the left and climb up by 15 degrees and at t=1s, again the aircraft has to change its heading angle and flight path angle to a new direction. The duration of the process is 100 seconds. The 100 seconds duration is chosen arbitrarily.

To represent a strategy in a computer program, the change of heading (ψ) and flight path (γ) angles, Figure 2, has to be coded. This is made possible by determining the maximum permissible range for the heading and flight path angles. In this research, the range of the angles is restricted between -30^{0} to 30^{0} for both heading and flight path angles.

Discrete angle interval of 2.5^{0} was used for heading and flight path angles. With this respect, we can now generate $(24+1)^2 = 625$ possible combinations of heading and flight path angles. Table 1 shows the coding of heading and flight path angles.

Table 1: Coding the heading and flight path angle.

ID	Heading Angle, ψ, deg.	Flight Path Angle, γ, deg.
1	-30	-30
2	-30	-27.5
3	-30	-25
:	:	:
625	30	30

Instead of directly using the angles, the strategy uses the values of IDs as shown in Table 1. A series of numbers valued between 001 and 625 are randomly constructed such as shown in Figure 1 with 100 three-digits integer were ordered in series. The first value is 477 means turn 17.5^{0} to the left and dive 27.5^{0}. Next manoeuvre is 474 which means ‘and then turn 15^{0} to the left and climb up 27.5^{0}’. This is repeated for the next sequence up to the last sequence, i.e. 425, see Figure 3 as an example. The whole process is called the trajectory of the aircraft or a strategy. A population consists of 100 strategies. There are 150 populations and each one of them is randomly generated.

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477474448142322034607318540047151
406470003015092600004405291367099
174485275509539601480460169513036
219135010442313183123283357081206
530185157185181400097499007495454
340235131068356373281336339418451
291489218178391346485616171039545
009487307336584594047294619385599
551399298027037447113477124099604
425
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Figure 1: An example of a coded strategy.

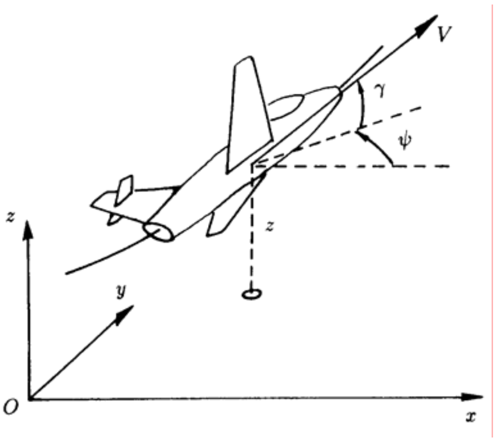

Figure 2: Definition of heading angle (ψ) and flight path angle (γ) [15].

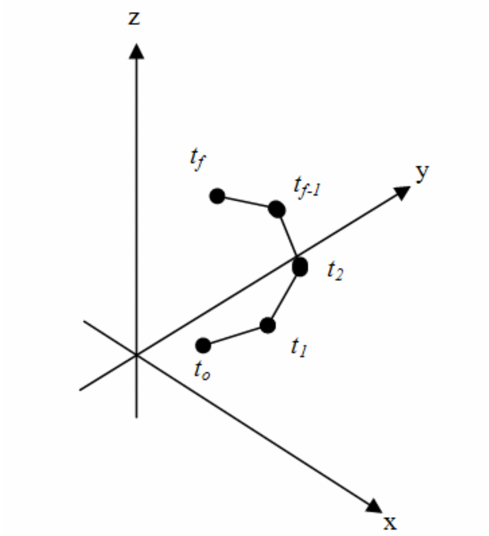

Figure 3: Example of a trajectory.

Each strategy is evaluated by running a game simulation with the pursuer chasing the evader. The evader has to use the given strategy to avoid interception.

The evader’s aerodynamic and performance capabilities where included by putting limits of the turning rate (TR), the maximum flight path angle, the maximum lift, the maximum load factor, maximum engine thrust, maximum speed and fuel available.

The adversary employs proportional navigation guidance (PNG) and the objective is to intercept the evader. It is assumed that the evader knows the pursuer's states, navigation guidance system and performance at any time.

Interception occurs when the distance between both players is less than a defined intercept radius. The cycle continues for 150 populations.

Basically, the steps of the optimisation routine is given by the following pseudo-code

1. At any given time, get the states of the evader and pursuer and start the optimisation routine by
    a. Generate strategies for the evader
    b. For each strategy, run the simulation
    c. Sort the results according to it's fitness value, the best strategy has the highest fitness value.
    d. Repeat (a) for the next population.
    e. After a predetermined number of populations are reached, the optimisation is stopped. The best result is evaluated, selected and ready to be used by the evader for 'actual' evasive maneuver.
2. The evader uses the best strategy against the pursuer.
3. The next cycle is repeated if the pursuer is still active.
4. The program stops if one of the termination criterions is reached, such as one of the players is running out of fuel.

Each trajectory's simulation time is 100 seconds. A game that expands up to 350 seconds would require the evader to search for four optimised trajectories.

2.1 Mathematical Model

Point mass models are used to represent both aircrafts. Both aircrafts are assumed to be fully controllable by their respective Augmented Stability Control System. Aircraft dynamics are presented by the following equations [14,15]:

$$v = \frac{1}{m}\left(T\cos\alpha - D\right) - g\sin\gamma$$

$$\dot{\gamma} = \frac{1}{mv}\left(L + T\sin\alpha\right)\cos\phi - \frac{g}{v}\cos\gamma$$

$$\dot{\psi} = \frac{\left(L + T\sin\alpha\right)}{mv\cos\gamma}\sin\phi$$

$$\dot{x} = v\cos\gamma\cos\psi$$

$$\dot{y} = v\cos\gamma\sin\psi$$

$$h = v\sin\gamma$$

where,

$$L = \tfrac{1}{s}\rho V^2 sC_L$$

$$C_L = C_{L_\alpha}(\alpha - \alpha_0)$$

$$D = \frac{1}{2}\rho v^2 sC_D$$

$$C_D = C_{D0} + kC_L^2$$

The payoff of the game is the final distance, *l*, between the two players at the final time, t_f, and the evader is not intercepted. A strategy is considered the best if *l* maximum and successfully steer the evader from interception.

The payoff is calculated by integrating the 3 degree-of-freedom equations of motions with respect to time. At every time step, the evader changes its heading and flight path angle which was given by the strategy. The pursuer 'sees' the evader and tries to chase and intercept it. The pursuer only knows the current state of the evader and does not know the evader's next move.

3. Simulation Conditions

The following aircraft parameters, as given in Table 1, were used for the simulation.

Table 1 Nominal Parameters for pursuer and evader aircraft

Pursuer	
Mass, *m*	6875 kg
Wing area,*S*	27.9 m^2
X-position, x_0	0.0 m
Y-position, y_0	0.0 m
max turnrate	10 deg/s
Max ceiling	15000 m
$C_{L\alpha}$	1.1
C_{D0}	0.412
k	0.9
T	160000 N
max + load	9
max – load	-4
max fuel weight	4000 kg
Evader	
Mass, *m*	8500 kg
Wing area, *S*	38.0 m^2
X-position, x_0	1000 m
Y-position, y_0	1000 m
max turnrate	20 deg/s
Max ceiling	17000 m
$C_{L\alpha}$	1.2
C_{D0}	0.45
k	0.9
T	18000 m

max + load	9
max – load	-4
max fuel weight	4000 kg
General	
Number of Generation	150
Population Size	100
Game's duration	200s

Simulations were conducted for a combination of interception radii, speeds and maximum turning rates.

To simplify the analysis, the velocity of both aircraft's are kept constant.

The fuel consumption is also considered by subtracting the fuel consumed at every time step. This is possible if the engine's thrust specific fuel consumption is known. For the time being, this value is fixed because of limited information of the engine.

4. Results

General

24 simulations have been carried out. Each simulation takes about 20 seconds to simulate 15000 trajectories on an Intel 1.5GHz machine.

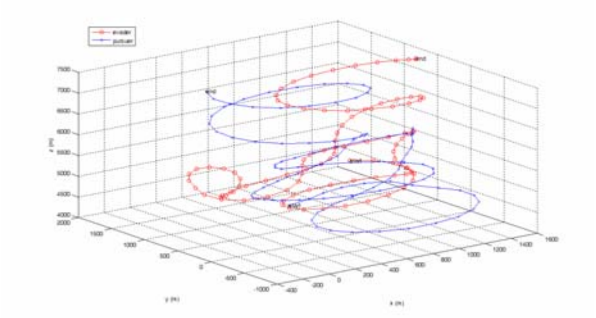

Figure 4: Example of 3D encounter

Figure 4 shows a 3D encounter between an evader and a pursuer. Instead of flying in a straight line, both players fly in almost seemingly circular manner but in 3D. It can also be visualised in 2D such as given in Figure 5.

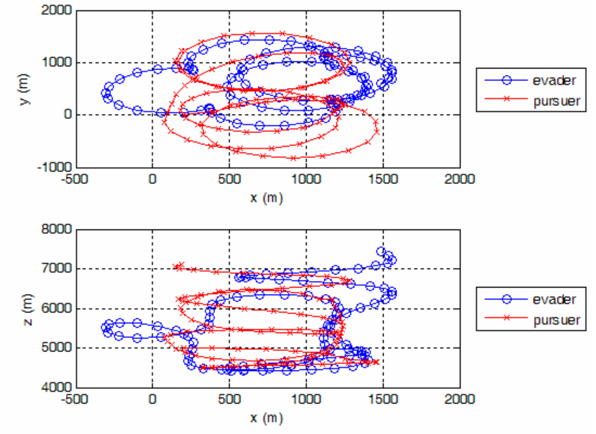

Figure 5: 2D representation of the encounter.

Effect Interception Radius

In this case, the pursuer speed and turning rate are kept constant at 200 m/s and 20deg/s, respectively. The evader's speed and maximum turning rate are 150 m/s and 30deg/s, respectively.

The results are plotted in Figure 6. A small percentage of trajectories with interception were found if the interception radius is small. As the interception radius increases, the number of trajectories with interception increases as well.

Small interception radius gives the evader more time to turn away from the pursuer and takes more effort by the pursuer to get closer to the evader.

But higher interception radius eases the pursuer effort to intercept the evader, whereas, the evader faces harder task to find the best time to turn away from the pursuer. A typical short range air-to-air missile's lethal radius is less than 9 m [16].

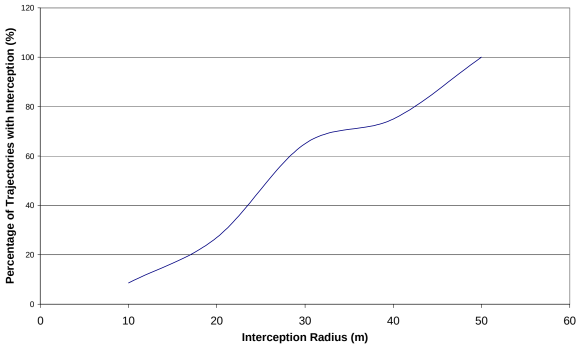

Figure 6: The percentage of trajectories with interception agains the pursuer's interception radius. (Number of trajectories = 15000, V_p=200m/s,V_E=150m/s, $TR_{max,P}$=20deg/s, $TR_{max,E}$=30deg/s

Effect of Evader's Speed

The speed of the evader could determine its ability to avoid interception. 12 optimisation simulations were carried out to see this effect. The results are also plotted as in Figure 7. The constants in this analysis are the pursuer's speed, V_p = 200 m/s, the maximum turning rates for the evader and the pursuer at 30 deg/s and 20 deg/s, respectively, and the interception radius of 20m.

The optimisation could not find trajectories that could save the evader from interception when the speed is 100 m/s. This speed is relatively too slow to evade interception.

As the speed increases, more trajectories without interception are found. The trend is maintained between 20% and 60% when the range of the speed is between 110 m/s and 180 m/s.

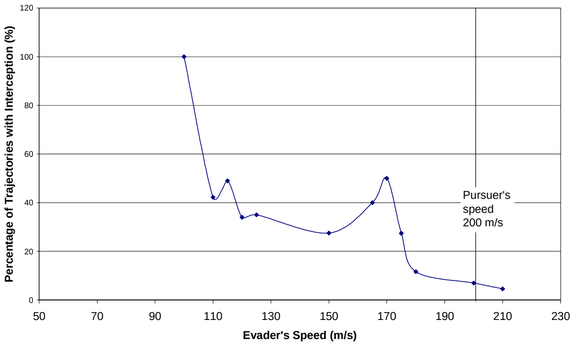

Figure 7: The percentage of trajectories with interception against evader's speed. (Number of trajectories = 15000, V_p=200m/s, $TR_{max,P}$=20deg/s, $TR_{max,E}$=30deg/s, Interception Radius=20m)

Effect of Maximum Turning Rates

In this case, no trajectory without interception was found for the first three combinations of maximum turning rates as shown in Figure 8. When the difference is small, it is very hard for the evader to out manoeuvre the pursuer. This can happen because every time the evader turns, the pursuer could easily turns with the almost the same rate and still also able to closing in.

However, when the difference is 10deg/s, the optimisation is able to find trajectories that give no interception to the evader, although their number is small.

As the difference goes larger, the percentage of trajectories with interception decreases. Many "good" trajectories were found for the evader. A good trajectory is a trajectory that guides the evader from being intercepted.

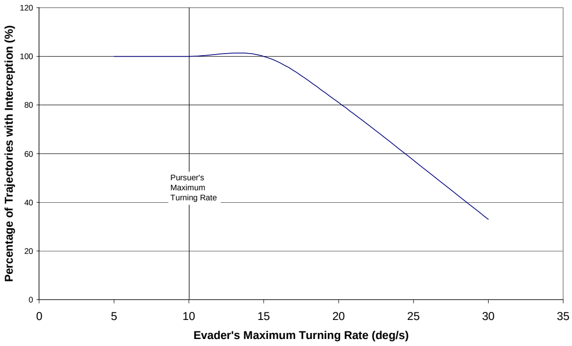

Figure 8: The percentage of trajectories with interception against evader's maximum turning rate. (Number of trajectories = 15000, V_p=200m/s,V_E=150m/s, $TR_{max,P}$=10deg/s, Interception Radius=20m)

5. Conclusion

Optimal trajectory in the evader's context is the trajectory that able for it to evade interception. The search for optimal trajectory using Games Theory, Differential Games and other analytical optimisation technique can be very difficult and time consuming. The nonlinearity of the aircraft's parameters add to the complexity of the problem.

This technique proved that through stochastic approach, it is possible to search for optimal solution in a short period of time.

The overall optimisation time is around 20 seconds. The shorter computing time means the aircraft has enough time to 'think optimally' against incoming pursuer such a missile. The computing time can be reduced further by parallelising the computation.

The optimisation could easily find "good" trajectories for the evader when the pursuer's interception radius is small, but as the interception radius increases, the search for the trajectories grow harder. If the pursuer's interception radius is less than 50 m, then there are good chances that the optimisation could find trajectories without interception.

The relative difference of speed between the players must be within 0.55 – 0.9. For example, when the pursuer speed is 200 m/s, the slowest speed for the evader should be 200(0.55) = 110 m/s.

And also, the maximum turning rate has an effect on the survivability of the evader. The evader has to have at least twice the maximum turning rate of the pursuer to evade interception. The higher the turning rate means the better for the evader's survivability.

In the simulations, the evader used typical air combat manoeuvres to evade interception. These manoeuvres are horizontal-S, vertical-S and high-g barrel roll.

These manoeuvres had to be performed in a timely manner by the evader in order to successfully evade interception.

In the future, actual aircraft aerodynamic and performance characteristics will be used to make the simulation more realistic. The randomised trajectory will be replaced with Evolutionary Algorithm to reduce the computing time by reducing the number of generated populations.

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