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# On the evolution of an optical pulse with initial chirp in a nonlinear fiber at the zero dispersion point

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**Abstract**

It is shown that the process of pulse shaping in a nonlinear fiber with zero dispersion can be controlled by means of appropriate choice of initial chirp. A simplified model was used which admits the general solution of an initial value problem with arbitrary initial profiles of intensity and chirp in terms of elementary functions. A particular example of an initially chirped pulse is discussed in order to demonstrate the qualitative features of the process. © 1999 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

The propagation of pulses in one-mode optical fibers with positive dispersion is usually described by some form of the nonlinear Schrödinger (NLS) equation

$$i\varepsilon q_x + \tfrac{1}{2}\varepsilon^2 q_{tt} - g\left(|q|^2\right)q = 0, \tag{1}$$

where $x$ and $t$ are dimensionless coordinates of space and retarded time, respectively, $q$ is the slowly varying electric field envelope, and the parameter $\varepsilon$ measures the relative strength of the dispersion term (see, e.g., Refs. [1–3]). The usual case of the Kerr nonlinearity corresponds to the nonlinearity function $g(|q|^2) = |q|^2$. It is well known that Eq. (1) can be transformed into the hydrodynamic-type system by means of the so-called Madelung transformation [4–7]

$$q(x,t) = \sqrt{\rho(x,t)}\,\exp\left(\frac{i}{\varepsilon}\int^t u(x,t')\,\mathrm{d}t'\right). \tag{2}$$

Indeed, simple calculation yields

$$\rho_x + (\rho u)_t = 0, \tag{3}$$

$$u_x + \frac{\partial}{\partial t}\left[\frac{u^2}{2} + g(\rho) - \varepsilon^2\left(\frac{\rho_{tt}}{4\rho} - \frac{\rho_t^2}{8\rho^2}\right)\right]. \tag{4}$$

Eq. (3) can be treated as a continuity equation with exchanged space and time coordinates for a fluid with density $\rho$ and velocity $u$. Eq. (4) corresponds to the Euler hydrodynamical equation for inviscid fluid with the following dependence of the pressure on density

$$p(\rho) = \int^\rho \rho' \frac{\mathrm{d}g(\rho')}{\mathrm{d}\rho'}\,\mathrm{d}\rho', \tag{5}$$

and additional 'quantum mechanical pressure' proportional to $\varepsilon^2$. So in the limit $\varepsilon \to 0$, which means that nonlinearity dominates over dispersion, we come to the usual Euler hydrodynamics with 'material' equation of state (5). These equations have been investigated intensively in classical dynamics of compressible fluids (see, e.g., Refs. [8,9]), and

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can be applied to describe the evolution of pulses as long as their $t$-dependence is smooth enough. However, strong nonlinearity leads inevitably to the so-called wave-breaking point, when such a dependence becomes very sharp with infinite derivative $\mathrm{d}\rho/\mathrm{d}t$ at this point. Hence, some other physical mechanism must be taken into account at the wave-breaking point. In classical fluid dynamics it is viscosity, which leads to the formation of the well-known shock waves. If viscosity is small enough, as it occurs in optical systems, then the dispersion becomes crucial for the description of the fluid behavior in the vicinity of the wave-breaking point and leads to the so-called 'dissipationless shock waves', i.e., an expanding region of fast oscillations which connects two regions with different values of density $\rho$. Such dispersive hydrodynamics was investigated in the cases of the Korteweg-de Vries equation [10–13], NLS equation [14–16], and derivative NLS equation [17,18] with the use of the Whitham approach [19]. A similar approach has been applied to modulationally unstable systems (see Refs. [20–23]). Recently these ideas were applied to non-return-to-zero (NRZ) optical communication systems in Refs. [24–26].

In Ref. [24] Kodama and Wabnitz used the simple self-similar solution of the hydrodynamical equations corresponding to the zero dispersion limit of the NLS equation for the description of the evolution of an initially rectangular pulse in an optical fiber. It has been shown that with increasing space coordinate the pulse becomes longer and acquires chirp. This corresponds to the 'dam' problem in the fluid dynamics terms and means that the gaseous layer of finite width begins to expand after removal of the container's walls and acquires some distribution of velocity in the direction perpendicular to the layer. This intuitively attractive picture shows immediately that initial chirp (velocity distribution) of opposite sign can prevent to some extent the expansion of the pulse. This idea was realized in Refs. [25,26] in the case of simple initial conditions (at $x=0$) with initially rectangular intensity distribution (dependence on $t$) and constant chirp of opposite signs in both halves of the pulse. In this case the edges of the pulse do not expand as fast as without chirp, but such simple choice leads to immediate creation of the dispersive shock wave in the center of the pulse due to the 'collision' of its two halves. It is clear that one can avoid this problem using a smoother chirp function vanishing at the center of the pulse. However, such an approach demands the development of a more complete theory than the simple self-similar solutions used in Refs. [24–26]. In principle, one can obtain the general solution of the Cauchy problem for the hydrodynamical equations corresponding to the NLS equation (see Ref. [16]), but this solution is very complicated and has not been received in a sufficiently convenient form. Therefore we want to present here a complete solution of the problem of evolution of the pulse with arbitrary initial intensity profiles and chirp for such a choice of Eq. (1) which admits a solution in elementary functions and can be easily applied to a wide class of initial conditions. Though such modeling of real situation is not exact, it preserves the main physical features of the problem and gives a simple description of the behavior of the pulse. The general solution is illustrated by one particular example which reveals the qualitative features of the process.

## 2. The general solution of the model hydrodynamical equations

The propagation of a pulse in different physical systems is often modeled by the generalized NLS equation with power nonlinearity function

$$g\left(|q|^2\right)=|q|^h, \tag{6}$$

which in the dispersionless limit leads to the following hydrodynamical equations (see (3), (4)),

$$\rho_x+\rho u_t+u\rho_t=0, \tag{7}$$

$$u_x+uu_t+(h/2)\rho^{h/2-1}\rho_t=0. \tag{8}$$

In hydrodynamical terms Eq. (8) corresponds to the equation of state (see Eq. (5))

$$p(\rho)=\frac{h}{h+2}\rho^{h/2+1}. \tag{9}$$

It is well known (see Refs. [8,9]) that the general solution of Eqs. (7), (8) can be expressed in terms of elementary functions, if the exponent $\kappa=h/2+1$ is such that $(3-\kappa)/(\kappa-1)$ is an even integer number $2n$, i.e. $h=4/(2n+1)$, $n=0,1,2,\ldots$. Unfortunately, the NLS equation with $h=2$ corresponds to $\kappa=2$ and does not satisfy this condition. But the choice $n=1$, i.e. $h=4/3$, gives the value $\kappa=5/3$ rather close to $\kappa=2$, and one can hope that this choice leads to a qualitatively satisfactory approximation to the real situation[2]. Therefore we choose $h=4/3$ with Eq. (8) replaced by

$$\rho u_x+\rho uu_t+c^2\rho_t=0, \tag{10}$$

where

$$c^2=\frac{\mathrm{d}p}{\mathrm{d}\rho}=\tfrac{2}{3}\rho^{2/3},$$

$c$ being the 'sound velocity' variable.

[2] This expectation is confirmed by a comparison of simple self-similar solutions for both cases $h=2$ and $h=4/3$. Such a comparison is presented in the Appendix.

To solve the system (7), (10), we shall use the classical hodograph method described in Ref. [9]. To this end, it is convenient to introduce instead of $\rho$ the variable

$$v=\int_0^{\rho}\frac{c(\rho')}{\rho'}\,\mathrm{d}\rho'=\sqrt{6}\,\rho^{1/3}=3c, \tag{11}$$

so that Eqs. (7) and (10) take the form

$$\tfrac{1}{3}v u_t+u v_t+v_x=0,\quad u u_t+\tfrac{1}{3}v v_t+u_x=0. \tag{12}$$

Now we execute the hodograph transformation, considering $t$ and $x$ as functions of independent variables $v$ and $u$:

$$t=t(v,u),\quad x=x(v,u). \tag{13}$$

Simple calculation leads to the system

$$-(v/3)\partial x/\partial v+u\partial x/\partial u-\partial t/\partial u=0,$$

$$-u\partial x/\partial v+(v/3)\partial x/\partial u+\partial t/\partial v=0, \tag{14}$$

linear with respect to the unknown functions (13). Taking into account Eq. (11), we can write this system in the form

$$\partial(t-ux)/\partial u+\left(1/2v^2\right)\partial(v^3x)/\partial v=0,$$

$$\partial(t-ux)/\partial v+\left(1/2v^2\right)\partial(v^3x)/\partial u=0. \tag{15}$$

The second equation in (15) allows one to introduce the potential $V(v,u)$, so that

$$t-ux=\frac{\partial V}{\partial u},\quad \frac{v}{2}x=-\frac{\partial V}{\partial v}. \tag{16}$$

Then substitution of these expressions into the first equation in (15) yields the equation for the potential:

$$\frac{\partial^2 V}{\partial v^2}-\frac{\partial^2 V}{\partial u^2}+\frac{2}{v}\frac{\partial V}{\partial v}=0. \tag{17}$$

It is easy to see that $vV$ satisfies the wave equation and, hence, the general solution of Eq. (17) has the form

$$V(v,u)=\left[f(v+u)+g(v-u)\right]/v, \tag{18}$$

where $f(\xi)$ and $g(\eta)$ ($\xi=v+u$, $\eta=v-u$) are arbitrary functions. Thus, the general solution of the system (12) can be written in an implicit form

$$t-ux=\left[f'(\xi)-g'(\eta)\right]/v,\quad \xi=v+u,\eta=v-u,$$

$$vx/2=\left[f(\xi)+g(\eta)\right]/v^2-\left[f'(\xi)+g'(\eta)\right]/v, \tag{19}$$

and contains two arbitrary functions which have to be determined from initial conditions.

## 3. The general solution of the initial value problem

Let the time dependence of intensity and chirp at $x=0$ be described by the functions

$$v(0,t)=v_0(t),\quad u(0,t)=u_0(t). \tag{20}$$

Our task is to determine the functions $f(\xi)$ and $g(\eta)$ in terms of these two initial value functions $v_0(t)$ and $u_0(t)$ or, equivalently, in terms of

$$\xi(t)=v_0(t)+u_0(t),\quad \eta(t)=v_0(t)-u_0(t). \tag{21}$$

To this purpose we have to fulfil Eqs. (19) specified to $x=0$,

$$f'(\xi)-g'(\eta)=\tfrac{1}{2}(\xi+\eta)t,$$

$$f'(\xi)+g'(\eta)=2\frac{f+g}{\xi+\eta}\equiv 2\mathscr{B}(t). \tag{22}$$

Both these equations are identities in $t$, and in the second we introduced a new function $\mathscr{B}(t)$. Then we may write

$$f'(\xi)=\mathscr{B}+\frac{t}{4}(\xi+\eta),\quad g'(\eta)=\mathscr{B}-\frac{t}{4}(\xi+\eta). \tag{23}$$

Due to the identity $f+g=(\xi+\eta)\mathscr{B}$, c.f. the second of Eqs. (22), it is natural to write

$$f(\xi)=\mathscr{A}(t)+\xi\mathscr{B}(t),\quad g(\eta)=-\mathscr{A}(t)+\eta\mathscr{B}(t) \tag{24}$$

with another new function $\mathscr{A}(t)$. By differentiation of the first of Eqs. (24) with respect to $\xi$ where $t=t(\xi)$ is understood as the reverse function to $\xi(t)$ and, similarly, differentiation of the second equation with respect to $\eta$ we find

$$f'=\mathscr{B}+(\mathscr{A}'+\xi\mathscr{B})\frac{\mathrm{d}t}{\mathrm{d}\xi},\quad g'=\mathscr{B}+(-\mathscr{A}'+\eta\mathscr{B})\frac{\mathrm{d}t}{\mathrm{d}\eta}. \tag{25}$$

For simplicity we assume that both $\xi(t)$ and $\eta(t)$ are monotonic functions. Otherwise we had to cut out pieces of the $t$ interval where these conditions are fulfilled. The combination of Eqs. (25) with Eqs. (23) leads to

$$\mathscr{A}'=\tfrac{1}{4}(\xi'\eta+\xi\eta')t=\tfrac{1}{2}(v_0'v_0-u_0'u_0)t,$$

$$\mathscr{B}'=\tfrac{1}{4}(\xi'-\eta')t=\tfrac{1}{2}u_0't, \tag{26}$$

and $\mathscr{A}$, $\mathscr{B}$ are determined by quadrature,

$$\mathscr{A}(t)=\int^t\tfrac{1}{2}\left[v_0'(\bar t)v_0(\bar t)-u_0'(\bar t)u_0(\bar t)\right]\bar t\,\mathrm{d}\bar t,$$

$$\mathscr{B}(t)=\int^t\tfrac{1}{2}u_0'(\bar t)\bar t\,\mathrm{d}\bar t. \tag{27}$$

Finally $f(\xi)$, $g(\eta)$ are found from Eq. (24) with $t$ expressed respectively in terms of $\xi$ or $\eta$. One should notice that this solution is valid within the region in the $(u,v)$-plane, enclosed by the initial value curve (20) and two characteristics $v+u=\xi$ and $v-u=\eta$ going from the end points of this curve (see Ref. [9]).

## 4. Example

We are interested in the investigation of the influence of an initial chirp on the evolution of the pulse. Therefore let us choose the initial profiles of intensity and chirp in the following simple form

$$v_0(t) = \begin{cases} a\left(1-(t/T)^2\right), & |t| \le T \\ 0, & |t| \ge T, \end{cases} \tag{28}$$

and

$$u_0(t) = \begin{cases} b(t/T)^2, & t<0, \\ -b(t/T)^2, & t>0. \end{cases} \tag{29}$$

Here $2T$ is the pulse duration, $a$ measures the intensity in the center of the pulse ( $\rho(0) = a^3/6^{3/2}$), and $b$ describes the chirp. To diminish distortion of the pulse caused by nonlinearity, we take $b>0$, i.e. chirp is negative at $t>0$, and positive at $t<0$. The pulse is symmetric (chirp is antisymmetric), so it is enough to consider the evolution of the part of the pulse with $t>0$ only.

According to definitions (21) we find

$$t(\xi) = T\sqrt{\frac{a-\xi}{a+b}}, \quad t(\eta) = T\sqrt{\frac{a-\eta}{a-b}}. \tag{30}$$

From (27) we obtain

$$\mathscr{A}(t) = -\frac{a^2}{3T^2}t^3 + \frac{a^2-b^2}{5T^4}t^5, \quad \mathscr{B}(t) = -\frac{b}{3T^2}t^3.$$

Their substitution into (24) leads to the expressions for the functions $f(\xi)$ and $g(\eta)$ and, subsequently, Eqs. (19) yield the solution of Eqs. (12):

$$t - ux = \frac{T}{6v}\left[\left(\frac{a-\xi}{a+b}\right)^{1/2}\frac{ab+(3a+2b)\xi}{a+b} - \left(\frac{a-\eta}{a-b}\right)^{1/2}\frac{ab-(3a-2b)\eta}{a-b}\right], \tag{31}$$

$$\begin{aligned} x = &-\frac{2T}{15v^3}\left\{\left(\frac{a-\xi}{a+b}\right)^{3/2}\left[(2a+3b)a+(3a+2b)\xi\right] - \left(\frac{a-\eta}{a-b}\right)^{3/2}\left[(2a-3b)a+(3a-2b)\eta\right]\right\} \\ &-\frac{T}{3v^2}\left[\left(\frac{a-\xi}{a+b}\right)^{1/2}\frac{ab+(3a+2b)\xi}{a+b} + \left(\frac{a-\eta}{a-b}\right)^{1/2}\frac{ab-(3a-2b)\eta}{a-b}\right], \end{aligned} \tag{32}$$

in an implicit form.

Expressions (31) and (32) give us the space $x$ and time $t$ coordinates as functions of the variables $v$ and $u$. If we fix the value of space coordinate ( $x=\text{const}$), then these expressions define some curve in the $(u,v)$-plane in a parametric form with $t$ as a parameter. For example, at $x=0$ we obtain the initial value curve defined by Eqs. (28) and (29), which in our case is the segment of the straight line (recall that we consider only the region with $t>0$)

$$v = a(1+u/b). \tag{33}$$

At the end points $(-b,0)$ and $(0,a)$ of this segment the values of $\xi$ and $\eta$ are equal to $\xi=-b$ or $\xi=a$ and $\eta=b$ or $\eta=a$, respectively. The solution found above is valid inside the rectangle in the $(u,v)$-plane with the sides defined by the equations for characteristics

$$v+u=-b, \quad v+u=a, \quad v-u=b, \quad v-u=a. \tag{34}$$

(If $a=\pm b$, then one of two sets of characteristic lines becomes parallel to the initial value line (33), the rectangle reduces to this line, and the solution (32) loses its region of applicability.) The initial value curve (33) is a diagonal of this rectangle and divides it into two triangles: one triangle corresponds to the negative values of space variable ( $x<0$) and the other to the positive values ( $x>0$). The triangle in the $(u,v)$-plane for the case $T=1$, $a=1$, $b=0.2$ is shown in Fig. 1 (its angles are distorted because of different scales of $u$ and $v$ axes). The solid lines represent the solution curves (31), (32) for fixed values of $x$ as already discussed. The straight line $x=0$ is the initial curve (33), and the two dashed lines $v+u=1$ and $v-u=0.2$ repre-

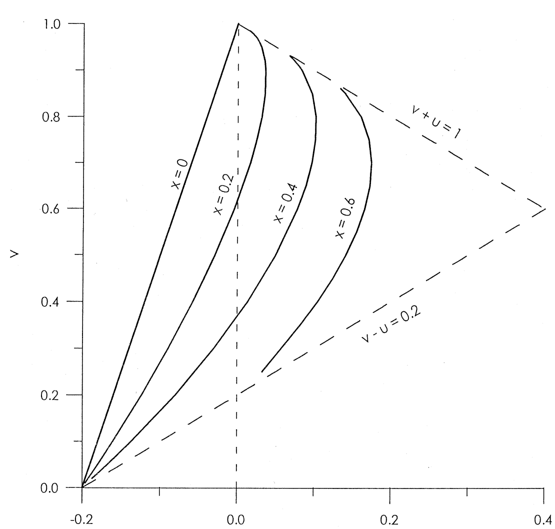


Fig. 1. The hodograph $(u,v)$-plane for the evolution of the pulse with initial chirp ($b=0.2$). Solid lines represent the solution (31), (32) at different values of $x$. The solution is valid inside the triangle with sides defined by the initial value curve $x=0$ and the characteristics $v-u=0.2$ and $v+u=1$. The vertical dashed line at $u=0$ distinguishes regions with positive and negative chirp.

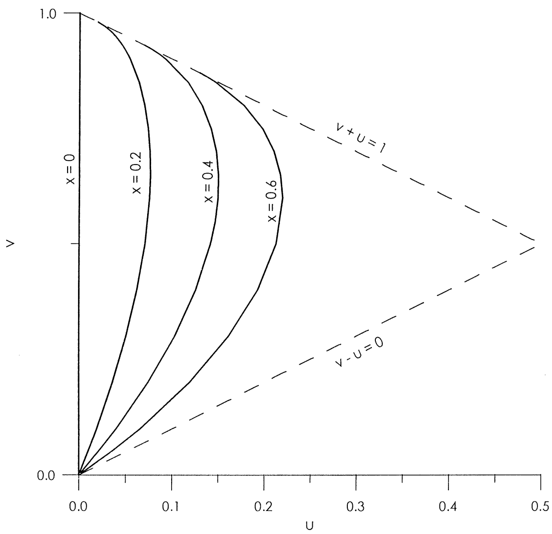


Fig. 2. The same as in Fig. 1, but for the pulse without initial chirp $(b=0)$.

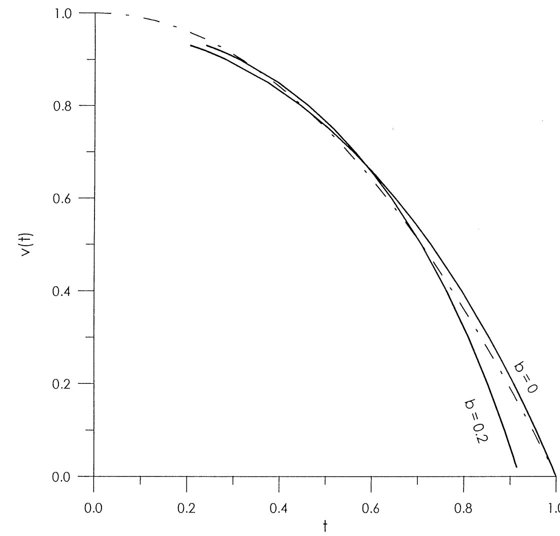


Fig. 3. Solid lines show the dependence of the 'intensity' variable $v$ on time $t$ at distance $x=0.4$ for pulses with initial chirp $(b=0.2)$ and without it $(b=0)$. The dashed line represents the initial profile of $v(t)$ at $x=0$.

sent the two characteristics (34). The vertical dashed line at $u=0$ distinguishes two regions of negative $(u<0)$ and positive $(u>0)$ chirp. As we can see, at sufficiently small $x$ a considerable part of the pulse has negative chirp what means its contraction. But the part of the pulse with a higher intensity (large $v$), where initial negative chirp was small, acquires some positive chirp already at small $x$ due to nonlinearity effects, and this region of positive chirp increases with increasing $x$. At sufficiently large $x$ almost all pulse has positive chirp what means its steepening and expansion in the direction of positive $x$. This figure should be compared with a similar plot corresponding to the pulse without initial chirp (Fig. 2). Now the initial value curve corresponds to the segment of $v$-axis $(0 \le v \le 1)$ and from the very beginning the whole pulse acquires positive chirp and steepens faster than in the case with initial chirp shown in Fig. 1.

The dependence of the 'intensity' variable $v$ on time $t$ for one half of the pulse with and without chirp is shown in Fig. 3 at $x=0.4$. (Only the parts corresponding to the solution (31), (32) are depicted.) One can see that the lower intensity part of the pulse with chirp $(b=0.2)$ moves in the negative direction of $x$, whereas the pulse without chirp steepens compared to the initial profile (dashed line).

The dependence of the chirp variable $u$ on $t$ at $x=0.4$ is shown in Fig. 4. In accordance with the above results, the non-zero initial chirp (dashed line) evolves into the curve with negative values at large $t$ (weak influence of nonlinearity and strong influence of initial chirp) and positive values at small and intermediate $t$ (strong influence of nonlinearity and weak influence of initial chirp), whereas the zero initial chirp evolves into the curve with positive chirp along the whole pulse due to the influence of nonlinearity.

This example shows how one can control the pulse evolution by means of change of initial chirp. The initial functions (28), (29) were chosen due to their analytical simplicity and other (maybe, more efficient or realistic) initial functions can be considered in a similar way.

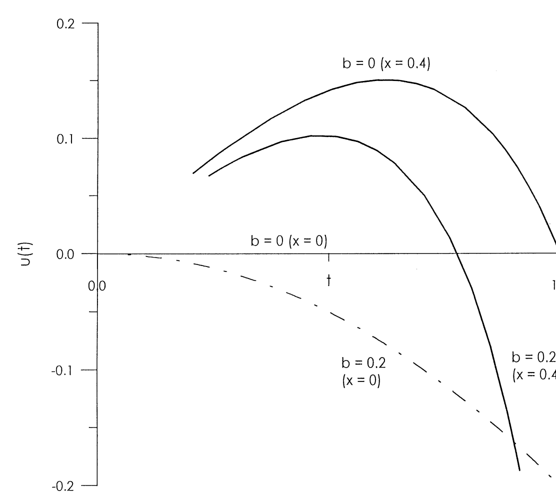


Fig. 4. Solid lines show the dependence of the chirp variable $u$ on time $t$ at distance $x=0.4$ for pulses with initial chirp $(b=0.2$, initial chirp is presented by the dashed line) and without it $(b=0$, initial chirp is presented by the $t$-axis).

## 5. Conclusion

The particular example discussed above leads to some general conclusion. One can see from Fig. 2 that the maximal chirp arising due to nonlinear effects in the pulse without initial chirp is almost proportional to $x$ even at rather large $x$,

$$u \simeq \alpha x.$$

The proportionality constant $\alpha$ depends on the nonlinearity function in (1) and in our case is equal to $\alpha \simeq 0.36$–$0.38$ in the region of $x$ under consideration. The initial chirp $u_0$ influences the pulse evolution as long as it has the same order of magnitude as the chirp due to nonlinearity effects, i.e. $u_0 \simeq \alpha x$, what gives us an estimation of distance

$$x \simeq u_0/\alpha,$$

within which one can control the evolution of the pulse. Stronger nonlinearity (e.g., Kerr-like one) will increase the value of the constant $\alpha$, but one may expect that the general qualitative picture will remain the same. For investigation of details of the pulse evolution for other types of nonlinearity function, a complete theory analogous to the one worked out here should be developed.

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## Appendix A

To justify the qualitative validity of our approximation with replacement of the exponent $h=2$ (corresponding to the Kerr nonlinearity) to the exponent $h=4/3$ (admitting a complete analytical solution in an elementary form), let us compare the solutions of the same simple problem of rarefaction wave for both these cases. Let the initial condition at $x=0$ be

$$\rho = \begin{cases} \rho_0 & \text{at } t<0, \\ 0 & \text{at } t>0, \end{cases} \tag{A.1}$$

i.e. the step-like pulse without initial chirp ''enters'' into the medium at $x=0$ (note that we use a coordinate system moving with the group velocity of the pulse).

(a) In case of $h=2$ Eqs. (6) and (8) read

$$\rho_x + \rho u_t + u\rho_t = 0, \quad u_x + uu_t + \rho_t = 0. \tag{A.2}$$

Since the initial condition (A.1) does not contain any parameters with time dimension (i.e. we consider a pulse with infinitely sharp front), the variables $\rho$ and $u$ can only depend on the self-similar variable $\xi = t/x$. Whence Eqs. (A.2) reduce to

$$(u-\xi)\rho' + \rho u' = 0, \quad \rho' + (u-\xi)u' = 0, \tag{A.3}$$

where the prime denotes differentiation with respect to $\xi$. This system has nontrivial solution only if

$$u - \xi = \pm\sqrt{\rho}, \tag{A.4}$$

and our initial condition corresponds to the ''plus'' sign. Then any equation of (A.4) gives $\sqrt{\rho}\,u' + \rho' = 0$ and integration yields

$$u = 2\left(\sqrt{\rho_0} - \sqrt{\rho}\right), \tag{A.5}$$

where $\rho_0$ is an integration constant. Thus Eqs. (A.4) and (A.5) give us the desired solution

$$\rho = \left(\tfrac{2}{3}\right)^2 \left(1 - \frac{t}{2\sqrt{\rho_0}\,x}\right)^2 \rho_0, \quad u = \tfrac{2}{3}\left(\sqrt{\rho_0} + \frac{t}{x}\right), \tag{A.6}$$

for the rarefaction wave problem in the case $h=2$. Just this solution was used by Kodama and Wabnitz [24–26] in their theory of evolution of a rectangular pulse in nonlinear optical fibers. We see that the point with vanishing intensity ($\rho=0$) moves with the ''velocity'' $(t/x)_0 = 2\sqrt{\rho_0}$ which describes the pulse widening due to ''nonlinear pressure'', and the opposite point of the non-uniform region which matches with the region of constant intensity $\rho=\rho_0$ propagates into the latter region with the ''sound velocity'' $(t/x)_{\rho_0} = -\sqrt{\rho_0}$. The whole non-uniform region acquires the chirp $u$ linearly dependent on $\xi = t/x$.

(b) In case of $h=4/3$, instead of (A.2) we obtain the system

$$\rho_x + \rho u_t + u\rho_t = 0, \quad u_x + uu_t + (2/3)\rho^{-1/3}\rho_t = 0. \tag{A.7}$$

Again $\rho$ and $u$ depend only on $\xi = t/x$, and simple calculation analogous to that of the case $h=2$ leads to the solution

$$\rho = \left(\tfrac{3}{4}\right)^3 \left(1 - \frac{t}{\sqrt{6}\,\rho_0^{1/3}x}\right)^3 \rho_0, \quad u = \tfrac{3}{4}\left(\sqrt{\tfrac{2}{3}}\,\rho_0^{1/3} + \frac{t}{x}\right). \tag{A.8}$$

Qualitatively, this solution has the same structure as (A.6) and describes the same ''rarefaction'' wave but for different ''material equation'' $p = (2/5)\rho^{5/3}$ ($h=3/4$) instead of $p=(1/2)\rho^2$ ($h=2$). Correspondingly, the characteristic velocities are changed: the pulse widens with the velocity $\sqrt{6}\,\rho_0^{1/3}$, and the ''sound velocity'' is equal now to $\sqrt{2/3}\,\rho_0^{1/3}$. Nevertheless, the qualitative picture remains the same: the pulse widens due to nonlinearity and acquires chirp $u$ linearly dependent on $\xi = t/x$ inside the non-uniform region. Therefore this chirp due to the nonlinearity can be compensated to some extend by the initial chirp as it was considered in the present paper.

The general validity of the hydrodynamical approach was confirmed in Refs. [24,25] by means of a numerical solution of the NLS equation. It was found that small dispersion can be neglected as long as the hydrodynamical solution does not lead to the wave-breaking and formation of the shock wave.

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