

MR2665782 (2011f:16011) 16D50 (16E50)

Herzog, Ivo (1-OHSL); **Rothmaler, Philipp** (1-CUNY)

When cotorsion modules are pure injective. (English summary)

J. Math. Log. **9** (2009), *no. 1*, 63–102.

Summary: “We characterize rings over which every cotorsion module is pure injective (Xu rings) in terms of certain descending chain conditions and the Ziegler spectrum, which renders the classes of von Neumann regular rings and of pure semisimple rings as two possible extremes. As preparation, descriptions of pure projective and Mittag-Leffler preenvelopes with respect to so-called definable subcategories and of pure generation for such are derived, which may be of interest on their own. Infinitary axiomatizations lead to coherence results previously known for the special case of flat modules. Along with pseudoflat modules we introduce quasiflat modules, which arise naturally in the model-theoretic and the category-theoretic contexts.”

References

1. M. Auslander and I. Reiten, Applications of contravariantly finite subcategories, *Adv. Math.* **86**(1) (1991) 111–152. [MR1097029 \(92e:16009\)](#)
2. G. Azumaya, Locally pure-projective modules, *Contemporary Math.* **124** (1992) 17–22. [MR1144024 \(93e:16027\)](#)
3. Ş. Basarab, The models of the elementary theory of finite abelian groups, *Stud. Cerc. Mat.* **27**(4) (1975) 381–386 (in Romanian). [MR0403961 \(53 #7770\)](#)
4. Ş. Basarab, On the elementary theories of abelian profinite groups and abelian torsion groups, *Rev. Roumaine Math. Pures Appl.* **22**(3) (1977) 299–309. [MR0439622 \(55 #12508\)](#)
5. W. W. Crawley-Boevey, Locally finitely presented additive categories, *Commun. Algebra* **22**(5) (1994) 1641–1674. [MR1264733 \(95h:18009\)](#)
6. N. Dubrovin and G. Puninski, Classifying projective modules over some semilocal rings, *J. Algebra Appl.* **6**(5) (2007) 839–866. [MR2355623 \(2008h:16004\)](#)
7. E. E. Enochs and O. M. G. Jenda, *Relative Homological Algebra*, de Gruyter Expositions in Mathematics, Vol. 30 (Walter de Gruyter, 2000). [MR1753146 \(2001h:16013\)](#)
8. A. Facchini, *Module Theory: Endomorphism Rings and Direct Sum Decompositions in Some Classes of Modules*, Progress in Mathematics, Vol. 167 (Birkhäuser, 1998). [MR1634015 \(99h:16004\)](#)
9. P. A. Guil Asensio, M. C. Izurdiaga, Ph. Rothmaler and B. Torrecillas, Strict Mittag–Leffler modules, in preparation.
10. I. Herzog, Elementary duality of modules, *Trans. Amer. Math. Soc.* **340**(1) (1993) 37–69. [MR1091706 \(94a:03060\)](#)
11. I. Herzog and Ph. Rothmaler, Pure projective approximations, *Math. Proc. Cambridge Philos. Soc.* **146** (2009) 83–94. [MR2461869 \(2010c:16002\)](#)
12. W. Hodges, *Model Theory*, Encyclopedia of Mathematics and Its Applications, Vol. 42 (Cam-

- bridge University Press, 1993). [MR1221741 \(94e:03002\)](#)
13. H. Lenzing, *Homological Transfer from Finitely Presented to Infinite Modules*, Lecture Notes Math., Vol. 1006 (Springer, 1983), pp. 734–761. [MR0722664 \(85f:16034\)](#)
 14. S. Mac Lane and I. Moerdijk, *Sheaves in Geometry and Logic* (Springer, 1992). [MR1300636 \(96c:03119\)](#)
 15. M. Makkai, Full continuous embeddings of toposes, *Trans. Amer. Math. Soc.* **269** (1982) 167–196. [MR0637034 \(83c:03058\)](#)
 16. M. Prest, *Model Theory and Modules*, London Mathematics Society Lecture Note Series, Vol. 130 (Cambridge University Press, 1988). [MR0933092 \(89h:03061\)](#)
 17. M. Prest, Remarks on elementary duality, *Ann. Pure Appl. Log.* **62** (1993) 183–205. [MR1226307 \(94d:03072\)](#)
 18. M. Prest, *Purity, Spectra and Localisation*, Encyclopedia of Mathematics and its Applications, Vol. 121 (Cambridge University Press, 2009). [MR2530988 \(2010k:16002\)](#)
 19. M. Prest and G. Puninski, Some model theory over hereditary noetherian domains, *J. Algebra* **211** (1999) 268–297. [MR1656581 \(2000b:03132\)](#)
 20. M. Prest, Ph. Rothmaler and M. Ziegler, Absolutely pure and flat modules and ‘indiscrete’ rings, *J. Algebra* **174** (1995) 349–372. [MR1334216 \(96d:16002\)](#)
 21. M. Prest, Ph. Rothmaler and M. Ziegler, Extensions of elementary duality, *J. Pure Appl. Algebra* **93** (1994) 33–56. [MR1268781 \(95c:03087\)](#)
 22. G. Puninski and Ph. Rothmaler, Pure-projective modules, *J. London Math. Soc. (2)* **71** (2005) 304–320. [MR2122430 \(2006d:16009\)](#)
 23. J. Rada and M. Saorin, Rings characterized by (pre)envelopes and (pre)covers of their modules, *Commun. Algebra* **26**(3) (1998) 899–912. [MR1606190 \(99f:16002\)](#)
 24. M. Raynaud and L. Gruson, Critères de platitude et de projectivité, Seconde partie, *Invent. Math.* **13** (1971) 52–89. [MR0308104 \(46 #7219\)](#)
 25. Ph. Rothmaler, Purity in model theory, in *Proc. Conf. Model Theory and Algebra, Essen/Dresden, 1994/95*, eds. M. Droste and R. Göbel, Algebra, Logic and Application Series, Vol. 9 (Gordon & Breach, 1997), pp. 445–469. [MR1687736 \(2000f:03113\)](#)
 26. Ph. Rothmaler, Mittag–Leffler modules and positive atomicity, Habilitationsschrift, Kiel (1994).
 27. Ph. Rothmaler, Mittag–Leffler modules, in *Proc. Conf. Model Theory and Algebra, Florence 1995*, *Ann. Pure Appl. Log.* **88** (1997) 227–239. [MR1600915 \(99b:16006\)](#)
 28. R. Wisbauer, *Foundations of Module and Ring Theory* (Gordon and Breach, 1991). [MR1144522 \(92i:16001\)](#)
 29. J. Xu, *Flat Covers of Modules*, Lecture Notes Mathematics, Vol. 1634 (Springer, 1996). [MR1438789 \(98b:16003\)](#)
 30. M. Ziegler, Model theory of modules, *Ann. Pure Appl. Log.* **26** (1984) 149–213. [MR0739577 \(86c:03034\)](#)
 31. W. Zimmermann, Rein injektive direkte Summen von Moduln, *Commun. Algebra* **5**(10) (1977) 1083–1117. [MR0450327 \(56 #8623\)](#)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

© *Copyright American Mathematical Society 2011, 2013*