How much Spectrum Sharing is Optimal in Cognitive Radio Networks?

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Abstract

We explore the performance tradeoff between opportunistic and regulated access inherent in the design of multiuser cognitive radio networks. We consider a multichannel cognitive radio system with sensing limits at the secondary users and interference tolerance limits at the primary and secondary users. Our objective is to determine the optimal amount of spectrum sharing, i.e., the number of secondary users that maximizes the total deliverable throughput. We begin with perfect primary user detection and zero interference tolerance at the primary and secondary nodes. With identical primary and secondary traffic statistics, we find that the optimal fraction of licensed users lies between the two extremes of fully opportunistic and fully licensed operation and is equal to the traffic duty cycle. When the secondary users can vary their transmission probabilities based on the number of active primary users, we find that the optimal number of opportunistic users is equal to the average number of unoccupied channels. For the more involved case of imperfect sensing and non-zero interference tolerance constraints, we provide numerical simulation results to study the tradeoff between licensing and autonomy and the impact of sensing and interference tolerance on the throughput for different subchannel selection strategies at the secondary users.

Index Terms

Cognitive Radio, Multiple access, Regulation-Autonomy tradeoff

I. Introduction

The growing popularity of diverse wireless technologies has generated a huge demand for more bandwidth. The traditional 'divide and set aside' approach to spectrum regulation has ensured that the licensed (primary) users cause minimal interference to each other. However, it has created a very crowded spectrum with most frequency bands already assigned to different licensees [1]–[3]. The term 'cognitive radio' encompasses several techniques [4]–[12] that seek to overcome the spectral shortage problem by enabling secondary (unlicensed) wireless devices to communicate without interfering with the primary users. Our work will exclusively focus on the 'interweave' (interference avoidance) approach [6]–[12] to cognitive radio, wherein the secondary radio periodically monitors the radio spectrum, intelligently detects occupancy in the different frequency bands and then opportunistically communicates over the spectrum holes with minimal interference to the active primary users.

Opportunistic communication with interference avoidance faces a multitude of challenges in the detection of primary users and spectrum access, coexistence and sharing in multiuser environments. The literature for the study of spectral sensing for cognitive radio systems is extensive [7]–[9] (and references therein). Analysis in [7] shows that robust detection of primary users at low SNRs necessitates very sensitive detectors with infeasibly long observation times. In scenarios with noise uncertainty at the receiver, it is seen that detection may not even be possible, even with infinite sensing times. One solution to the sensing problem is to take a collaborative approach to the detection of primary users [7]–[9].

A major issue in a multiple secondary user environment is dynamic spectrum access and sharing, a topic that has generated a lot of research interest in the recent past [13]–[25]. This problem is similar to that of multiple access in multichannel wireless networks - in both these cases multiple independent transmitters need to access a set of shared channel resources. Many access protocols for cognitive networks have therefore been derived from conventional MAC protocols like ALOHA and CSMA. [15], [16] discuss a multichannel CSMA based decentralized cognitive MAC protocol, wherein the secondary users access subchannels based on a partial observation of the instantaneous spectrum availability. The performance of a channel statistics based access strategy capable of channel aggregation is studied in [20]. In the 'ADP' (Asynchronous Distributed Pricing) spectrum access scheme proposed in [19], the secondary users select the channel to transmit and the transmit power based on knowledge of the 'interference prices' of the other receivers in their respective subchannels. A few centralized and control channel based access strategies have also been explored in recent work. [14] considers a centralized server that coordinates and schedules transmissions for a group of links in such a way that the sum rate is maximized while each link is guaranteed a certain minimum rate. [18] presents a control channel dependent MAC protocol for secondary users operating over a primary cellular network.

Cognitive radio operation in practical multiuser environments is governed by interference tolerance and sensing limits at the primary and secondary users. The interference limits at the primary and secondary users indicate the amount of protection needed at each (primary or secondary) user from the multiuser interference to maintain a certain rate. In other words, the interference limit is a measure of how *tolerant* the users are to multiuser interference. On the other hand, the sensing limits (minimum SNR needed for detection) at the secondary users reflect the amount of protection that each secondary user is individually able to provide to the primary users. Put differently, the sensing limit is a measure of how *aggressively* the secondary users transmit their signals. In these scenarios, the key is to strike a balance between the two conflicting goals - minimizing the interference to the primary users, and maximizing the performance of the entire system - by limiting the number of secondary users. Therefore, the natural question that arises is: *What is the optimal number of secondary users (opportunistic access) relative to the number of primary users (licensed access) that maximizes the sum throughput in the* *system?* This is reminiscent of the familiar debate of licensing versus autonomy, a tradeoff that is fundamental to many areas of systems and control theory. The generality of this tradeoff is evident through an analogy with traffic control: Too much regulation, i.e., too few secondary users (traffic lights at every intersection) and the system is inefficient due to unoccupied spectral holes. On the other hand, too much autonomy/opportunistic behavior, i.e., too many secondary users, (no traffic lights) and the system becomes self-disruptive due to collisions between the secondary users.

The main goal of this work is to characterize the optimal amount of opportunistic use that maximizes the *sum* throughput in the system given the sensing and the tolerable interference limits. We consider a time slotted multichannel cognitive radio network where delay intolerant and bursty traffic is generated at the primary and secondary users at certain rates. While the primary users can utilize their channels whenever they have data to transmit, the secondary users will first have to monitor all the channels and then pick one of the unoccupied channels for secondary transmissions. The following is a summary of our results:

1) For the case of perfect sensing, zero interference tolerance:
   - With equal data generation probabilities at the primary and secondary users, we find that neither pure autonomy nor pure regulation is throughput optimal. Interestingly, the optimal fraction of licensed users is very close to the duty cycle of the users.
   - When the secondary users can vary their data transmission probabilities (while the primary users have fixed data generation probabilities), we find that the optimal number of secondary users is very close to the average number of unoccupied slots.
2) For the case of imperfect sensing and non-zero interference tolerance limits, numerical results exhibit an interplay between the tolerance of the primary users and the aggression of the secondary users. We find that the secondary users can exploit the tolerance of the primary users to transmit more aggressively, i.e., have a less sensitive primary user detector.

We begin with the mathematically tractable case of perfect sensing at the secondary systems and zero interference tolerance at the primary and secondary users in Section II. In Section III, we explore the general scenario of imperfect sensing and non-zero interference tolerance. For this more involved case, we develop throughput expressions and present some numerical results for different channel selection strategies in Section IV.

II. Perfect Sensing, Zero Interference Tolerance

Consider a certain channel resource that is equally shared among N_p primary users (primary transmitter-receiver pairs), i.e., each primary radio is licensed to transmit on a subchannel that spans $\left(\frac{1}{N_p}\right)^{th}$ of the available bandwidth. We divide each subchannel into time slots and assume that data traffic is generated at each primary user in an i.i.d fashion with a probability p during each time slot, i.e., on the average each primary user has data to transmit for a fraction p of the time. Consequently, each subchannel is unoccupied for a fraction $(1-p)$ of the time. To allow for higher spectral efficiencies, the channel is also open to be used *opportunistically* by N_s secondary users.

Any secondary user that has data to transmit monitors the N_p subchannels for primary users and randomly chooses one (if any) of the channels detected free for secondary communication. We consider *perfect primary radio detection* at the secondary users, i.e., the secondary users can detect whether or not the primary users are active in each of the N_p subchannels without any error. Further, we assume *zero interference tolerance* at each of the primary and secondary receivers, i.e., even the smallest amount of interference at any of the receivers will result in the loss of a packet. The case of imperfect detection and finite interference tolerance will be developed in the next section.

A. Fixed Secondary Traffic Generation Probability

We first consider the case where the primary and secondary users have identical traffic generation rates, i.e., delay intolerant packet data is generated at the secondary users with the same probability p per slot. Transmission at the primary and secondary users takes place at a rate of C bits/subchannel/time-slot. While the primary users can reliably transmit their data at this rate, the data of the secondary users is considered lost if either

- no free subchannel is available for secondary transmissions, or
- two or more secondary users select the same unoccupied licensed subchannel (i.e., when a *collision* occurs).

The performance metric of interest to us is the total amount of data (primary and secondary) that is *successfully delivered* per unit time, which we refer to as the *goodput*.

Notice that perfect sensing at the secondary users precludes any collisions between primary and secondary users. Therefore, the sum goodput of the primary users is:

$$C_p^{sum} = CN_p \text{Prob}\left[\text{PU is active}\right] = CN_p p \tag{1}$$

The sum goodput of the secondary users depend on the number of unoccupied subchannels. The sum secondary goodput can be written as

$$C_s^{sum} = \sum_{i=1}^{N_p} \binom{N_p}{i} p^{N_p - i} (1-p)^i C_s^{sum}(i) \tag{2}$$

where $C_s^{sum}(i)$ is the secondary goodput given that i of the N_p primary users do not have data to transmit. Conditioning on the number of secondary users having data to transmit, $C_s^{sum}(i)$ can be expressed as

$$C_s^{sum}(i) \;=\; \sum_{j=1}^{N_s} \binom{N_s}{j} p^j (1-p)^{N_s - j} C_s^{sum}(i,j), \tag{3}$$

where $C_s^{sum}(i,j)$ is the secondary goodput given that i subchannels are unoccupied and j secondary users are active. A secondary user's data is successfully transmitted only if there are no other secondary users in the slot it has chosen for transmission, the probability of which is $i\frac{1}{i}\left(1-\frac{1}{i}\right)^{j-1}$ and therefore we have

$$C_s^{sum}(i,j) = Cj\left(i\frac{1}{i}\left(1-\frac{1}{i}\right)^{j-1}\right) = Cj\left(1-\frac{1}{i}\right)^{j-1}.$$

Substituting $C_s^{sum}(i,j)$ and equation (3) in equation (2) and combining with equation (1), the sum goodput $C^{sum} = C_p^{sum} + C_s^{sum}$ simplifies to

$$C^{sum} = pCN_p\left(1+\frac{N_s}{N_p}\sum_{i=1}^{N_p}\binom{N_p}{i}p^{N_p-i}(1-p)^i\left(1-\frac{p}{i}\right)^{N_s-1}\right) \quad (4)$$

Figure 1(a) plots the normalized goodput ($\frac{C^{sum}}{pN_p}$) with increasing fraction of licensed users ($\lambda = \frac{N_p}{N_p+N_s}$ with decreasing N_s) for different values of the duty cycle p (with $N_p = 9$ primary users and $C = \frac{1}{N_p}$). The interesting observation from Figure 1(a) is that neither full autonomy ($\lambda = 0$, i.e., large N_s) nor fully regulated operation ($\lambda = 1$, i.e., $N_s = 0$) is goodput optimal. Instead the optimal fraction of primary users is an *intermediate value* $\lambda^*(p)$ that increases with the data generation rate p. Consistent with intuition, we note that licensing is good for high duty cycle (always ON, $p \to 1$) traffic while opportunistic operation is more suited for low duty cycle (rarely ON, $p \to 0$) cases.

It can also be seen from Figure 1(a) that the optimal fraction of licensed users ($\lambda^* = \frac{N_p}{N_p+N_s^*}$) is seen to be nearly equal to the duty cycle p, i.e., $\lambda^* \approx p$. Figure 1(b) demonstrates that this approximation is very tight, i.e., the optimal number of secondary users N_s^* (calculated from equation (4)) closely matches the fraction $\frac{N_p(1-p)}{p}$ for all p. This observation can be intuitively explained with a first order approximation as follows. The average number of primary users with data to transmit is $N_p p$. The average number of unoccupied subchannels is therefore $N_p(1-N_p)$. The average number of secondary users who have data to send is $N_s p$. If $N_s p > (N_p - N_p p)$, there is a high possibility of collisions. On the other hand, if $N_s p < (N_p - N_p p)$, there is a high chance that some of the subchannels remain unoccupied. Therefore one might expect the best value to be $N_s^* = \frac{(N_p - N_p p)}{p}$, as validated by Figure 1(b). The same trend is exhibited regardless of the value of N_p and p and C. This directly leads us to the following proposition:

Proposition 1: *With equal packet generation probabilities* p *at all users, if the number of primary users is* N_p*, the optimal number of secondary users* N_s^* *that maximizes the sum goodput with perfect primary user detection and non-zero interference tolerance at each of the users is* $N_s^* = \frac{N_p(1-p)}{p}$.

B. Adaptive Secondary Traffic Generation Probability

In the previous section, we have assumed that the traffic generation probability of the secondary users is fixed and equal to that of the primary users. In practice, the primary users are licensed owners of the spectrum and their traffic statistics are known, i.e., only the primary data generation rate is fixed. The secondary nodes in the system can, similar to p-persistent CSMA, employ a probabilistic access protocol to reduce the number of collisions and increase the sum throughput. In other words, the secondary users can attempt transmission with a probability that depends on the number of free primary user slots detected.

In this section, we assume that each secondary user, depending on the number of free slots i generates packets with a probability $q(i)$, i.e., the secondary user chooses to transmit with a probability $q(i)$. We wish to jointly determine:

1) The optimal access (or data generation) probability $q(i)$, and
2) The optimal number of secondary users N_s^*,

that maximize the sum throughput in this scenario. Since the sum goodput of the primary users is independent of N_s, maximizing the sum throughput is therefore equivalent to maximizing the sum throughput of only the secondary users. The optimization problem can therefore be written as

$$\max_{N_s} \sum_{i=1}^{N_p} \binom{N_p}{i} p^{N_p - i} (1-p)^i \left(\max_{q(i):0 \leqslant q(i) \leqslant 1} \left[\sum_{j=1}^{N_s} \binom{N_s}{j} q(i)^j (1-q(i))^{N_s - j} j \left(1 - \frac{1}{i}\right)^{j-1} C \right] \right)$$
$$= \max_{N_s} \sum_{i=1}^{N_p} \binom{N_p}{i} p^{N_p - i} (1-p)^i \left(\max_{q(i):0 \leqslant q(i) \leqslant 1} \left[N_s C q \left(1 - \frac{q(i)}{i}\right)^{N_s - 1} \right] \right) \quad (5)$$

The optimal transmission probability given the number of unoccupied primary subchannels, i.e., the solution to the inner maximization of equation (5), is provided by Theorem 1.

Theorem 1: *Given that* i *primary slots are unoccupied and that* N_s *secondary users exist in the system, the optimal traffic load that maximizes the sum goodput is given by* $q^*(i) = \min\left(1, \frac{i}{N_s}\right)$.

Proof: The maximization problem is

$$q^*(i) = \arg \max_{q:0 \leqslant q \leqslant 1} N_s q C \left(1 - \frac{q}{i}\right)^{N_s - 1} \quad (6)$$

Constructing the derivative of equation (6) w.r.t q and setting it to zero, we have

$$N_s C \left(1 - \frac{q}{i}\right)^{N_s - 2} \left(1 - \frac{q N_s}{i}\right) = 0, \quad (7)$$

whose root is $q = \frac{i}{N_s}$. Since $0 \leqslant q \leqslant 1$, the solution $q^*(i) = \min\left(1, \frac{i}{N_s}\right)$. Further, it can be seen that the second derivative,

$$\begin{aligned} \frac{d^2 C_s}{dq^2} &= N_s C \left[\left(1 - \frac{q}{i}\right)^{N_s - 2} \left(-\frac{N_s}{i}\right) + (N_s - 2)\left(1 - \frac{q}{i}\right)^{N_s - 3} \left(1 - \frac{q N_s}{i}\right) \right] \\ &= N_s C \frac{(N_s - 1)}{i^2} \left(1 - \frac{q}{i}\right)^{N_s - 3} \left(q - 2\frac{i}{N_s}\right), \end{aligned}$$

is negative (for all i) at $q = \min\left(1, \frac{i}{N_s}\right)$. ■

Theorem 1 shows that the optimal transmission probability at each secondary user is simply the ratio of the number of free slots to the total number of secondary users in the system. This result is intuitive - if $q(i) > \frac{i}{N_s}$, the system will be unstable, i.e., average number of transmitting secondary users will be higher than the number of unoccupied subchannels and the probability of collision is high. On the other hand, when $q(i) < \frac{i}{N_s}$ there is a high probability that some of the unoccupied subchannels remain unused by the active secondary users.

With the result of Theorem 1, the optimization problem of equation (5) reduces to

$$C = \max_{N_s} \left[CN_s \left(\sum_{i=1}^{N_p} \binom{N_p}{i} p^{(N_p-i)} (1-p)^i \min\left(1, \frac{i}{N_s}\right) \left(1 - \frac{\min\left(1, \frac{i}{N_s}\right)}{i}\right)^{N_s-1} \right) \right] \quad (8)$$

Theorem 2: The value of N_s that maximizes equation (8), N_s^*, is less than or equal to N_p.

Proof: The proof proceeds by contradiction. If $N_s^* > N_p$, $q^*(i) = \min\left(1, \frac{i}{N_s}\right) = \frac{i}{N_s}$ for all i. Substituting this into equation (8) yields

$$\begin{aligned} C &= \max_{N_s} \left[C \sum_{i=1}^{N_p} \binom{N_p}{i} p^{(N_p-i)} (1-p)^i i \left(1 - \frac{1}{N_s}\right)^{N_s-1} \right] \\ &= \left(C \sum_{i=1}^{N_p} \binom{N_p}{i} p^{(N_p-i)} (1-p)^i i \right) \max_{N_s} \left(1 - \frac{1}{N_s}\right)^{N_s-1}, \end{aligned}$$

which is maximum at $N_s = 1$. Since $N_p \geqslant 1$, this is a contradiction and therefore $N_s^* \leqslant N_p$. ■

An analytical solution to equation (8) is mathematically intractable. A numerical comparison of the solution of equation (8) with the average number of unoccupied channels $N_p(1-p)$ is shown in Figure 2. This directly leads us to the following proposition:

Proposition 2: *The optimal number of secondary users that maximize the sum capacity when the secondary users can adapt their transmission probabilities is given by* $N_s^* = N_p(1-p)$.

To determine the effects of sensing and interference tolerance on the goodput maximizing number of secondary users, we now consider a more general model in Section III.

III. Imperfect Sensing, Non-zero Interference Tolerance

We scale space and consider $(N_p + N_s)$ independent users (node-pairs) distributed uniformly in a circular area with unit radius, i.e, the probability that any node is located at a distance $r \in (0, 1)$ from the center of the disc is given by $p_R(r) = 2r$. We assume that each node-pair is a time-slotted half duplex system, i.e., communication can take place in both directions in a node-pair, albeit not simultaneously. Each time slot is considered to be long enough that arbitrary rates lower than the channel capacity can be achieved over a single slot. In any time slot, a delay intolerant data packet is generated at the transmitting node of a node-pair (the node that is designated to transmit in that particular time slot) with a probability p, i.e., the nodes within a node-pair have data to exchange for a fraction p of the time.

a) **Primary and secondary users:** The available channel bandwidth is divided into N_p equal subchannels and is licensed to N_p of the users (primary users). When a primary node has any data to be sent, it transmits it in its own subchannel. The rest of the users (N_s secondary users) only have opportunistic access to the spectrum and have to monitor all the N_p subchannels for primary activity before data transmission.

We capture the locations of the primary node-pairs in the following set of random variables $D_{p,i} = \left\{r_i^{(1)}, \theta_i^{(1)}, r_i^{(2)}, \theta_i^{(2)}, d_{p,i}\right\}, \quad 1 \leqslant i \leqslant N_p$, where $d_{p,i}$ is the link distance[1]. Similarly we define $D_{s,j} = \left\{r_j^{(1)}, \theta_j^{(1)}, r_j^{(2)}, \theta_j^{(2)}, d_{s,j}\right\}, \quad 1 \leqslant j \leqslant N_s$ for the secondary node-pairs. Further, we collect $\mathcal{D} = \left\{D_{p,1}, \ \cdots, \ D_{p,N_p}, \ D_{s,1}, \ \cdots, D_{s,N_s}\right\}$. The *minimum* distance between the i^{th} primary and j^{th} secondary node pairs (corresponding to the worst case interference) is denoted by the variable x_{ij}, and similarly, that between the i^{th} and j^{th} secondary users by y_{ij}.

b) **Interference Limit:** We consider a path-loss only signal propagation model of the form d^{-2} with distance d. The transmitting nodes (primary and secondary) use independent Gaussian codebooks with an average power constraint P. Transmission between any node-pair (primary and secondary) takes place at a data rate which is set so that the receiving node can tolerate a total interference of I, i.e., the data rate is given by $\log\left(1 + \frac{P/d^2}{1+I}\right)$, where d is the distance between the transmitter and receiver nodes in the node-pair.

c) **Sensing Limit:** We assume that each secondary node can detect primary nodes within a radius R_s around it, as shown in Figure 3. A subchannel is assumed to be free if *both* the secondary transmitter and receiver do not detect any primary users within their respective sensing regions. Since we consider nodes within a unit radius disc, a sensing radius $R_s = 2$ corresponds to perfect primary detection, while $R_s = 0$ corresponds to no detection. The observation time

[1]In terms of the polar coordinates of the two nodes, $d_{p,i} = \sqrt{r_i^{(1)2} + r_i^{(2)2} - 2r_i^{(1)} r_i^{(1)} \cos\left(\theta_i^{(1)} - \theta_i^{(2)}\right)}$

for sensing primary users is assumed to be very small compared to the length of the time slot.

d) **Spectrum access model:** : We consider two different selection strategies at each secondary node pair in choosing the subchannel for communication:

1) *Hard Decision Based Selection*: In this strategy, the primary user sensors at each secondary transmitter and receiver output a *binary* decision based on the received primary SNR at each end. In any subchannel, if no active primary user exists within the sensing radius R_s of the secondary transmitter and receiver, the subchannel is deemed free by the secondary node-pair. If more than one subchannel is detected to be free, the secondary node-pair *randomly choose one* of the subchannels for secondary communication.
2) *Soft Decision Based Selection*: In this strategy, the primary user sensors output soft values of the receive SNR on each of the subchannels. To minimize interference to the active primary users, the secondary node pair chooses the subchannel in which the minimum distance between the secondary and *active* primary nodes is the largest (inactive primary users are assumed to be located at $r_i^{(1)} = r_i^{(2)} = \infty$). Further, subchannels with primary users within the sensing radius are not considered, i.e., the subchannel chosen by the j^{th} primary user is given by $\arg\max_{\{i:|x_{ij}|>R_s\}}\{x_{ij}\}$. If there are two or more subchannels with the same maximum distance, *one* of them is randomly picked.

In the case of perfect sensing, we observe that both these selection strategies will have the same sum throughput.

Notice that in both schemes, since there is no cooperation between the secondary users, two or more secondary radios can choose the same subchannel. The set of all secondary users transmitting in subchannel i $(1 \leqslant i \leqslant N_p)$ is captured in the set $\mathcal{B}_i$. A data packet is considered lost if:

- The interference at the receiving node (from transmissions in the same subchannel) is larger[2] than I, or
- No free subchannels are detected.

The sum goodput of the primary and the secondary users can be expressed as in equations (9) and (10), where the indicator functions $\mathbb{I}\left[\cdot\right]$ determine whether or not there is an outage. The binary variable $Z_i = \mathbb{I}\left[i^{th} \text{ PU is active}\right]$ indicates whether or not the i^{th} primary user is using its subchannel. $\mathcal{B}_i$ is decided by the location of the primary and secondary nodes and also the subchannel selection strategy adopted by the secondary nodes.

$$C_p^{sum} = \mathsf{E}_{\mathcal{D}}\left[\mathsf{E}_{\{\mathcal{B}_1,\ \mathcal{B}_2,\ \cdots \mathcal{B}_{N_p}\}|\mathcal{D}}\left[\sum_{i=1}^{N_p} p\mathbb{I}\left[\sum_{j\in\mathcal{B}_i}\frac{P}{x_{ij}^2}\leqslant I\right]\log\left(1+\frac{P/d_{p,i}^2}{1+I}\right)\right]\right], \tag{9}$$

$$C_s^{sum} = \mathsf{E}_{\mathcal{D}}\left[\mathsf{E}_{\{\mathcal{B}_1,\ \mathcal{B}_2,\ \cdots \mathcal{B}_{N_p}\}|\mathcal{D}}\left[\sum_{i=1}^{N_p} p\left(\sum_{k\in\mathcal{B}_i}\mathbb{I}\left[\sum_{j\in\mathcal{B}_i/k}\frac{P}{y_{kj}^2}+Z_i\frac{P}{x_{ik}^2}\leqslant I\right]\log\left(1+\frac{P/d_{s,k}^2}{1+I}\right)\right)\right]\right]. \tag{10}$$

While equations (9) and (10) express the sum goodput in non-ideal, realistic scenarios with imperfect sensing and non-zero interference limits, the additional complexity makes analysis very difficult. Therefore, we numerically evaluate equations (9) and (10) to gain insights into the optimal number of secondary users in scenarios with imperfect sensing and non-zero interference tolerance limits.

IV. Simulation Results

We consider the system model of Section III discussed above with $N_p = 5$ primary users and plot the goodput for different values of the interference tolerance I and the sensing radius R_s. We assume traffic generation probabilities $p = 0.5$ at each of the primary and secondary users. We first consider the hard decision based subchannel selection scheme.

[2]We emphasize that the interference limit I is *not* a constraint imposed on the system, i.e., there is no guarantee that the interference at the receivers is less than I.

A. Hard Decision Based Selection

Figure 4(a) plots the sum goodput with increasing number of secondary users for different values of the sensing radius R_s. The interference tolerance I is equal to 0, i.e., even a small amount of interference to the primary or secondary users results in undecodable data and goodput loss. For a given number of secondary users N_s, the probability of a secondary user colliding with other primary users increases as the sensing radius R_s decreases. The goodput is therefore maximum for $R_s = 2$ (perfect sensing), as Figure 4(a) shows. Notice that the optimal number of secondary users $N_s^*(R_s)$ decreases as R_s decreases. This reflects the importance of sensing in a zero interference tolerance environment - for the specific case of $R_s = 0$ (no sensing), the presence of even a single secondary user introduces sufficient interference to the primary users to cause a *decrease* in the goodput, i.e., $N_s^*(R_s = 0, I = 0) = 0$. Since sensing takes place at both the secondary transmitter and receiver, most of the primary users are detected even with a moderate sensing radius ($R_s > 1$). The sum goodput difference between the $R_s = 1$ case and perfect sensing is therefore not very large.

Figure 4(b) considers a scenario where the primary and secondary users have a interference tolerance $I = 2$. Since each of the users is transmitting at a lower rate ($I = 2$, equations (9) and (10)), the sum goodput for the same N_s and sensing radius R_s is lower than that for the $I = 0$ (zero tolerance) case. Further, the higher interference tolerance at each of the users implies that, compared to the $I = 0$ case, more secondary users can be accommodated for the same sensing radius R_s, i.e., $N_s^*(R_s, I = 2) > N_s^*(R_s, I = 0)$. The interesting observation from Figure 4(b) is that secondary systems can *exploit the tolerance of the primary links to transmit more aggressively*, i.e. use a lower sensing radius. As Figure 4(b) shows, a smaller sensing radius ($R_s = 0.65$) provides the secondary users with more opportunities to transmit while maintaining the interference level below the limit. Notice that for $N_s \leqslant 11$, the optimal sensing radius is $R_s = 0.65$. Similar trends are shown even at higher interference tolerance values.

Figure 5(a) compares the sum throughput with increasing number of secondary users for different interference tolerance values. It can be seen that the throughput advantages of transmission at higher rates (lower I) decrease as the number of secondary users increase because of the higher number of collisions between the primary and secondary users. Specifically, at higher loads ($N_s > 15$), a larger sum goodput can be achieved by increasing the interference tolerance.

To study the effect of the sensing radius on the primary users, Figure 5(b) plots the primary goodput C_p with decreasing R_s for a tolerance level of $I = 1$. It can be seen that even with $N_s = 20$ secondary users, a sensing radius of $R_s = 1.1$ is sufficient to guarantee that the interference caused to primary users is minimal.

B. Soft Decision Based Selection

Figures 6(a) and 6(b) show the sum goodput with increasing number of secondary users for different sensing radii. Notice that there is a very high probability ($= 0.97$ at $p = 0.5$) that there will be at least one subchannel in which the corresponding primary user is inactive. In such situations, the secondary users in the soft decision based selection scheme will always choose the unoccupied subchannel. This explains the very small variation of the goodput with the sensing radius R_s, as Figures 6(a) and 6(b) show. For the case of perfect primary user detection ($R_s = 2$), the sum goodputs in both the hard decision and soft decision based selection schemes are the same, as expected. However, as R_s decreases, the secondary users will pick frequency bands to decrease the interference caused to active primary users and the sum throughput achieved is higher than that in the hard decision based selection scheme (Figures 4(a) and 4(b)).

Since each secondary user picks a subchannel so that the resulting interference to the associated active primary user is as small as possible, the throughput of the primary users is larger than in the hard decision based subchannel selection case, as shown in Figure 7(b).

V. Conclusion

Spectrum sharing and access are important issues facing opportunistic communication in multiuser cognitive radio systems. Because of the presence of user priority (primary and secondary), they pose unique design challenges that are not faced in conventional wireless systems. In an environment with multiple primary and secondary users, the tradeoff between sum throughput maximization and primary user interference minimization is a result of the well known interplay between regulation and autonomy. In scenarios with perfect primary user sensing and transmissions at the channel capacity, we characterize this tradeoff and identify the optimal amount of spectrum sharing that maximizes the total system throughput. We observe that the optimal fraction of primary users is very close to the duty cycle of the data traffic. When the secondary users adapt their transmission probability to decrease the number of collisions, we find that the optimal transmission probability is exactly equal to the fraction of unoccupied subchannels. The optimal number of secondary users in this scenario is found to be very close to the average number of unoccupied subchannels.

The more general case of imperfect sensing and finite interference tolerance at each of the users manifests similar trends and the optimal number of secondary users is again found to be between the two extremes of complete regulation and complete autonomy. Numerical results show that in a zero interference tolerance environment, the optimal number of secondary users increases as the sensing ability of the secondary nodes increases. Overall, the sensitivity of the sum goodput to primary user sensing is found to decrease as the interference tolerance at the primary and secondary users increases.

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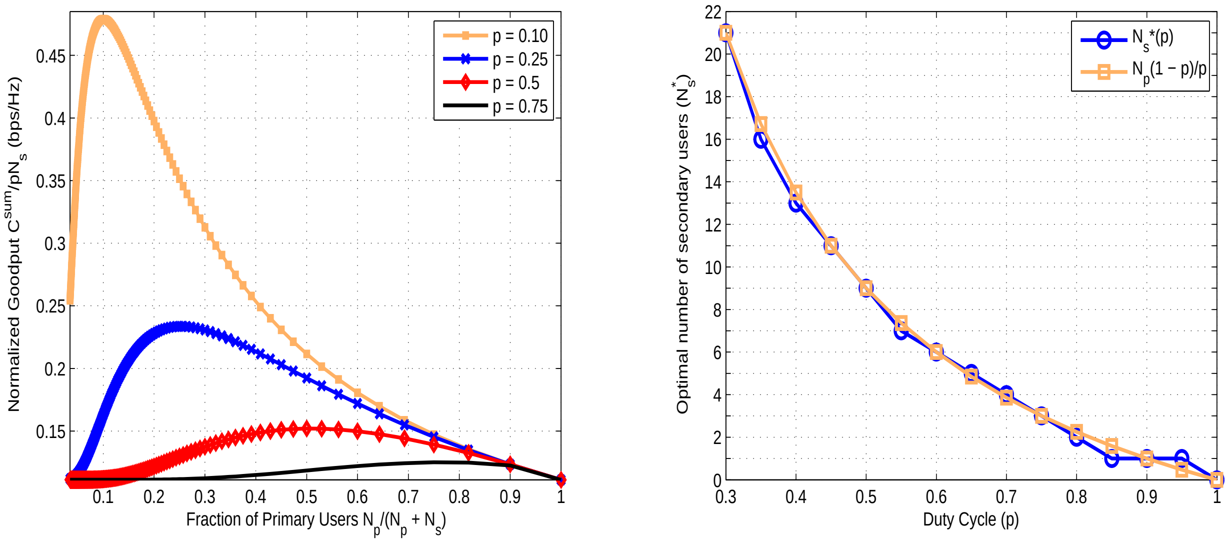


(a) Normalized goodput versus fraction of primary users (b) Optimal number of secondary users versus p

Fig. 1: Figure 1(a) plots the normalized goodput ($\frac{C^{sum}}{pN_p}$) versus the fraction of licensed users ($\frac{N_s}{N_p}$) with $N_p = 9$ users for different values of p. The optimal fraction of primary users can be seen to be equal to the duty cycle p. For different values of p, Figure 1(b) compares the optimal number of secondary users $N_s^*(p)$ with the value of $\frac{N_p(1-p)}{p}$.

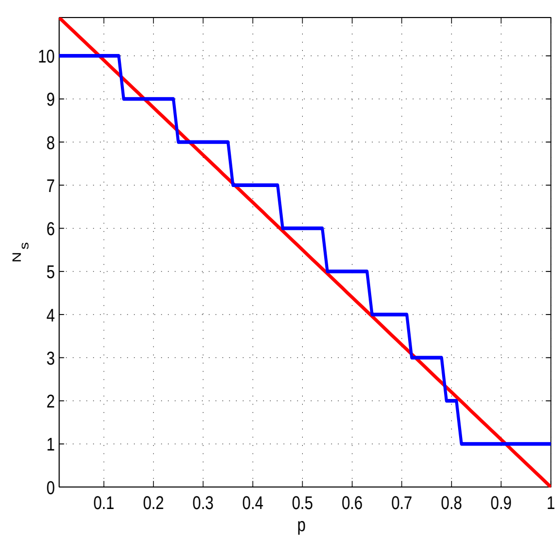


Fig. 2: Comparison of the optimal number of secondary users vs. $N_p(1-p)$.

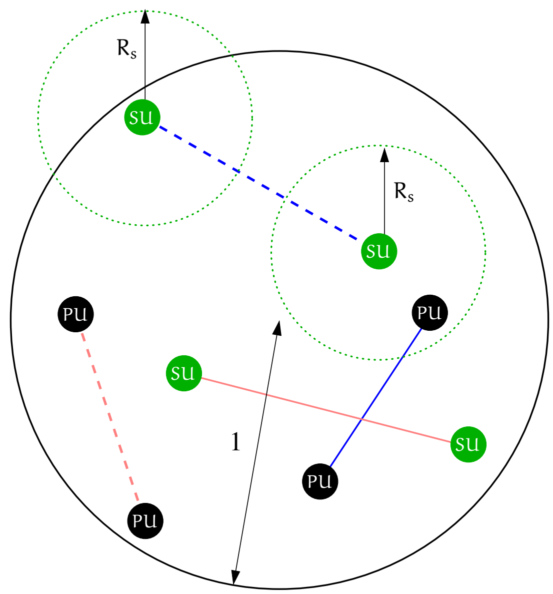
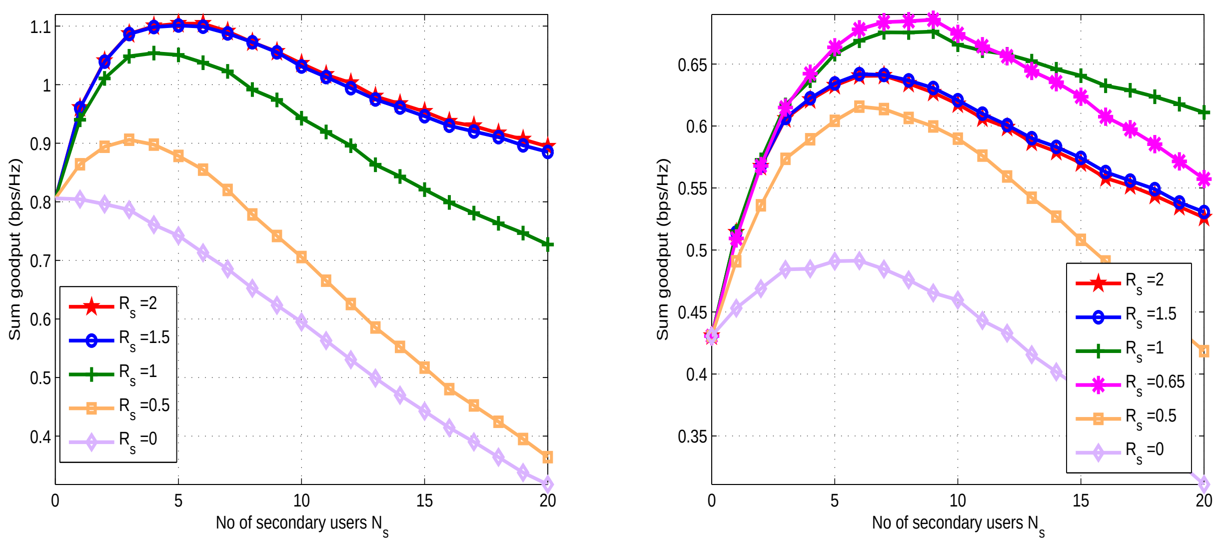


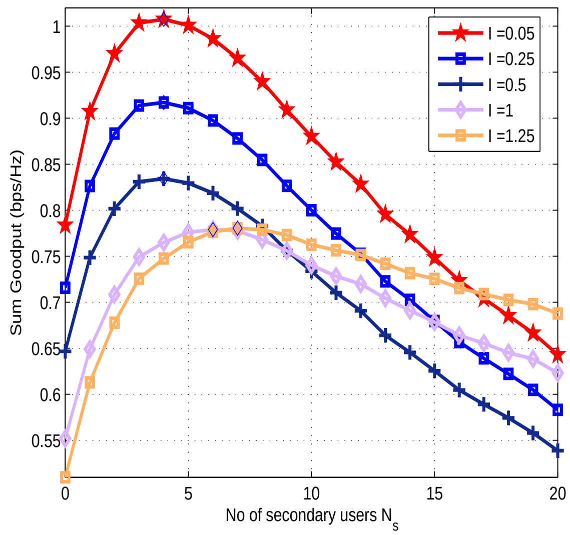
Fig. 3: PU and SU represent the primary and secondary users respectively. The circles around the secondary nodes are the sensing regions. The different subchannels are distinguished with colored links. Dotted lines indicate that the corresponding primary/secondary user does not have data to transmit.



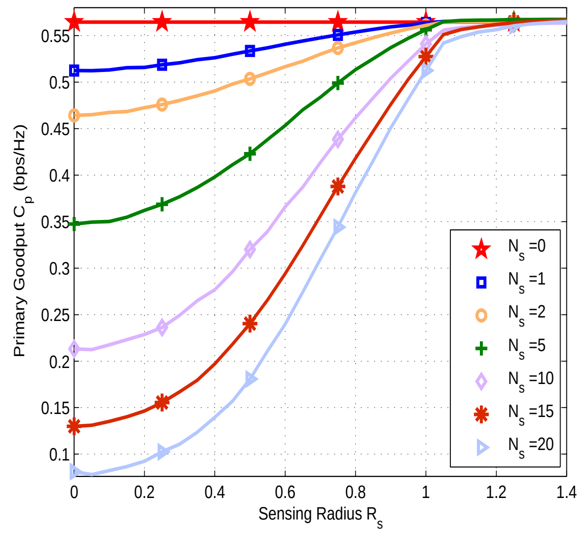
(a) Sum goodput vs. N_s for different R_s $(I = 0)$

(b) Sum goodput vs. N_s for different R_s $(I = 2)$

Fig. 4: Figures 4(a) and 4(b) plot the goodput versus increasing number of secondary users (N_s) for different sensing radii (R_s) for $I = 0$ and $I = 2$ respectively.

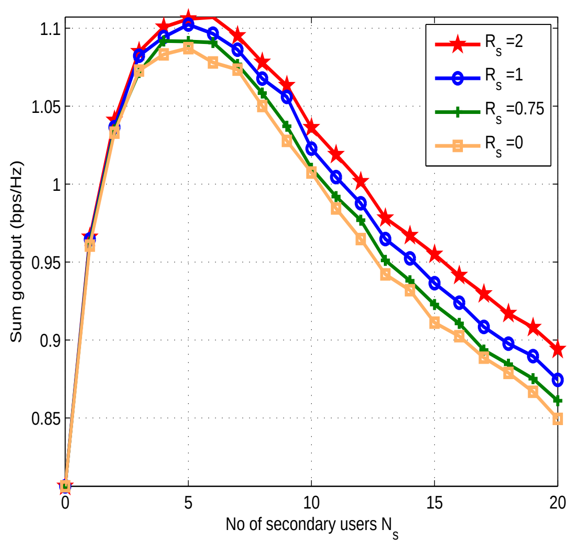


(a) Sum goodput vs. N_s for different I ($R_s = 0.9$)

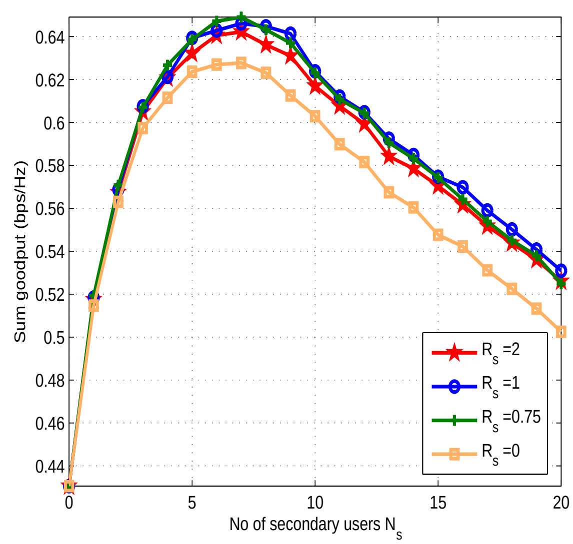


(b) Sum primary goodput vs. R_s for different N_s ($I = 1$)

Fig. 5: Figure 5(a) plots the sum goodput with increasing number of secondary users N_s for a sensing radius $R_s = 0.9$. Figure 5(b) shows the primary user goodput vs. sensing radius R_s for different number of secondary users N_s. The interference margin is fixed at $I = 1$.

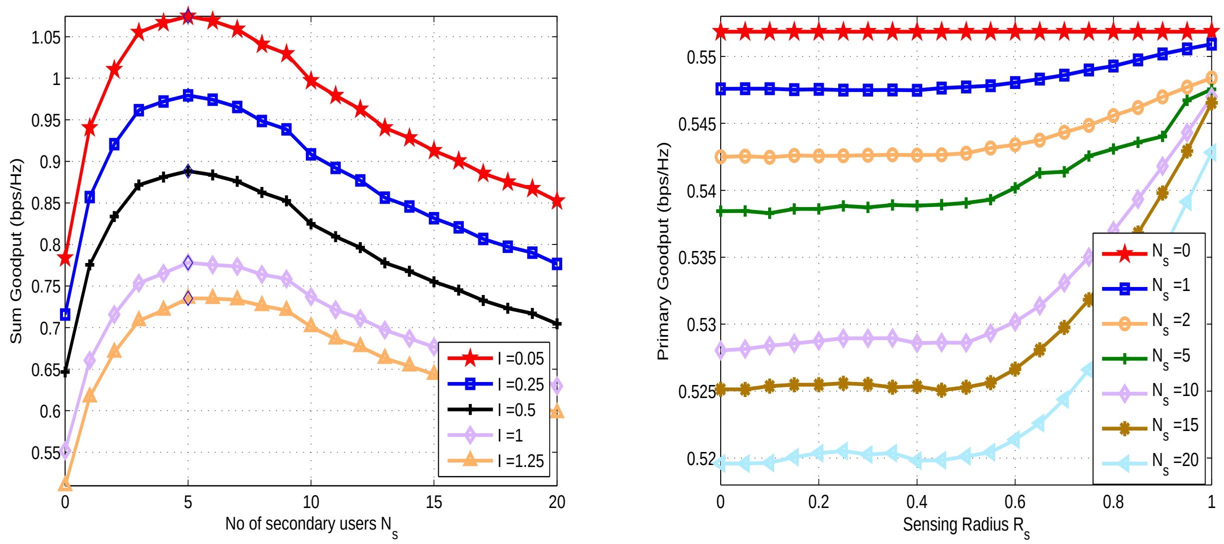


(a) Sum goodput vs. N_s for different R_s ($I = 0$)



(b) Sum goodput vs. N_s for different R_s ($I = 2$)

Fig. 6: Figures 6(a) and 6(b) plot the sum goodput with increasing number of secondary users N_s for $I = 0$ and $I = 2$ respectively.



(a) Sum goodput vs. N_s for different I ($R_s = 1$)

(b) Sum primary goodput vs. R_s for different N_s ($I = 1$)

Fig. 7: Figures 7(a) plots the sum goodput with increasing number of secondary users N_s for a sensing radius $R_s = 1$. Figure 7(b) shows the primary user sum throughput for $I = 1$.